Femtoscopic study of NΞ interaction and search for the H dibaryon state around the NΞ threshold

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in collaboration with

Y. Kamiya, K. Sasaki, T. Fukui, T. Hatsuda, T. Hyodo, K. Morita, K. Ogata

APFB2020, Mar. 1-5, 2021, Kanazawa / Online (Hybrid)

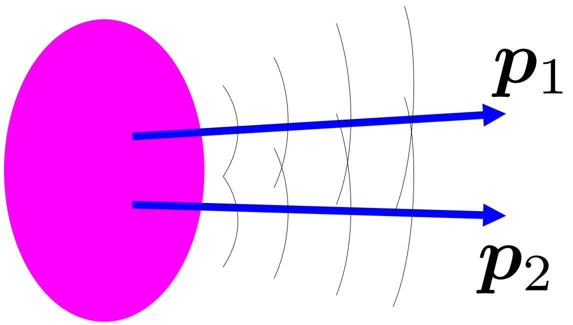
- **Introduction**
  - **Femtoscopic diagnosis of bound state existence**
- **NΞ-ΛΛ interaction and pΞ^- and ΛΛ correlation functions**
  - **HAL QCD potential / Comparison with data and previous results**
- **Summary**

Y. Kamiya, K. Sasaki, T. Fukui, T. Hatsuda, T. Hyodo, K. Morita, K. Ogata, AO, in prep.

Introduction

Femtoscopic study of hadron-hadron interaction

- **Koonin-Pratt formula**
  Koonin('77), Pratt+('86), Lednicky+('82)



$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r} \underline{S_{12}(\boldsymbol{r})} \underline{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}$$

source fn. **relative w.f.**

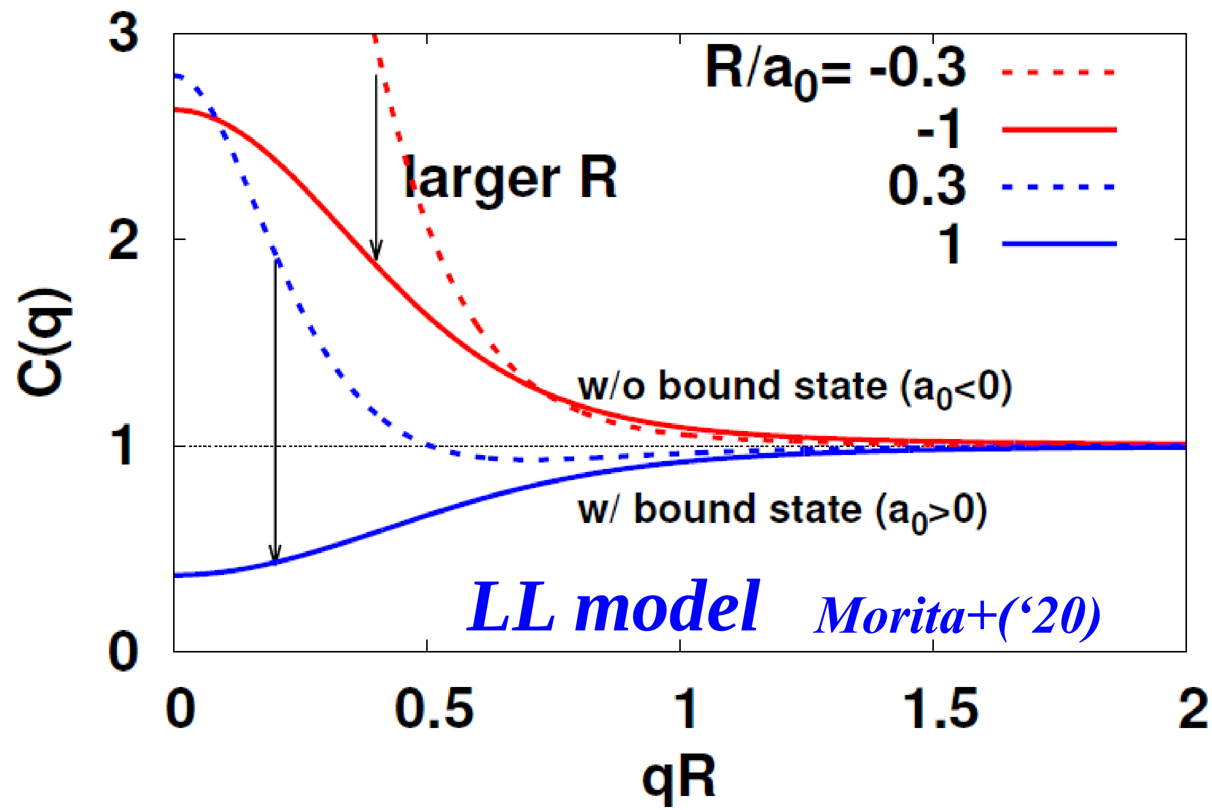
- **Standard usage = Source size using quantum statistics**
  Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)

- ***Non-Std. usage = Hadron-hadron interaction***
  CF shows how much $|\varphi|^2$ is enhanced → V_{hh} effects !
  R. Lednicky, V. L. Lyuboshits ('82); K. Morita, T. Furumoto, AO (1408.6682)

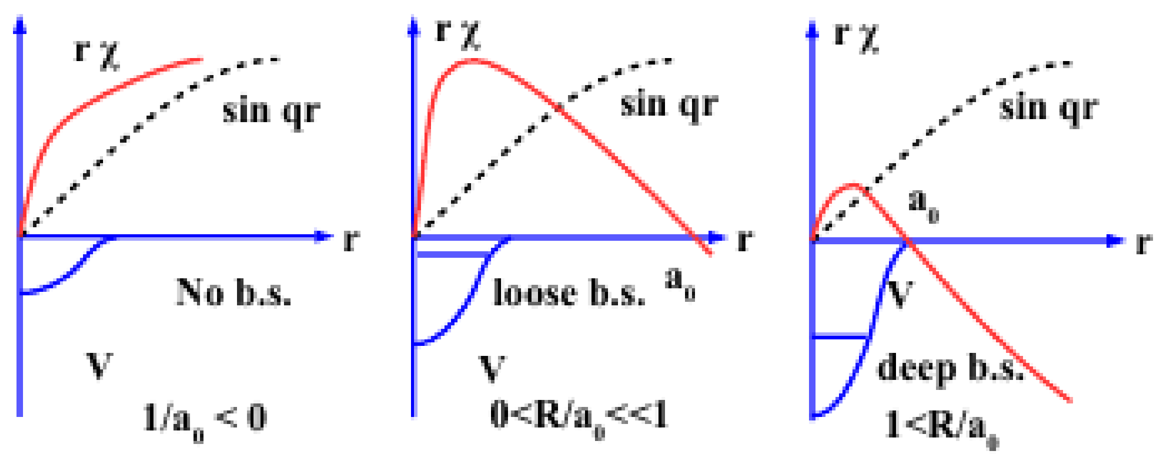
$$C(\boldsymbol{q}) = 1 + \int d\boldsymbol{r} S(r) \left\{ |\varphi_0(r)|^2 - |j_0(qr)|^2 \right\} \quad (\varphi_0 = \text{s-wave w.f.})$$

Bound state diagnosis by femtoscopy

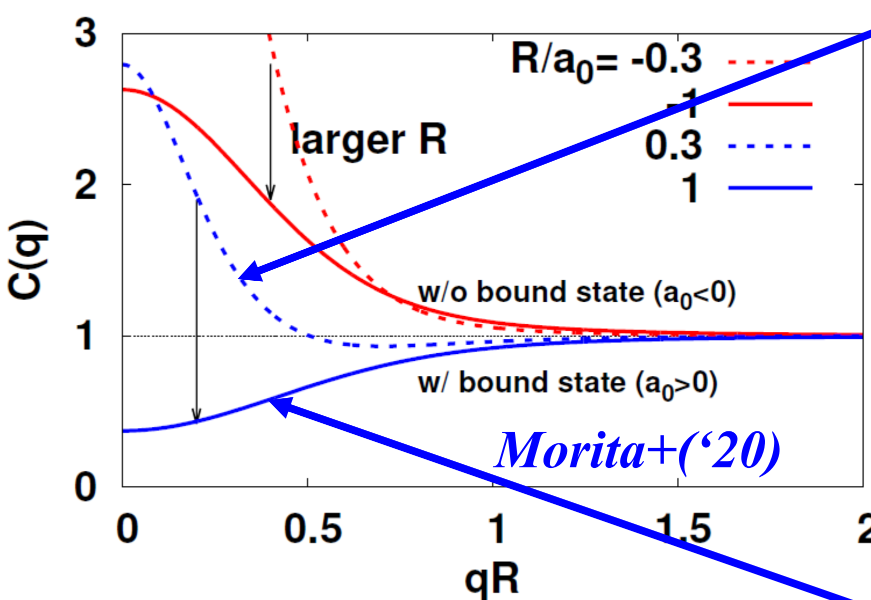
- **Source size dep. of CF tells the sign of the scattering length (a_0).**
  - **With attraction, CF is enhanced at small R.**
  - **With a_0 >0, CF is suppressed at R ~ a_0**



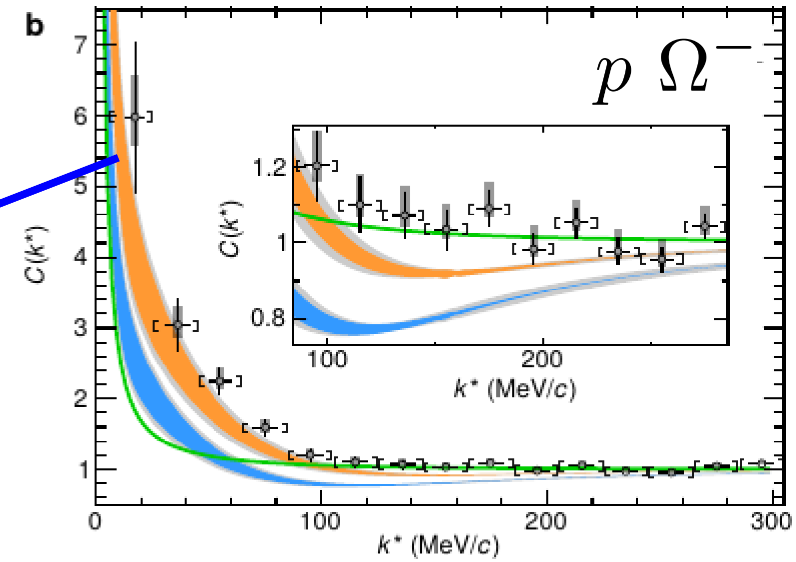
Source size dep. of CF
→ To be bound,
or not to be bound.



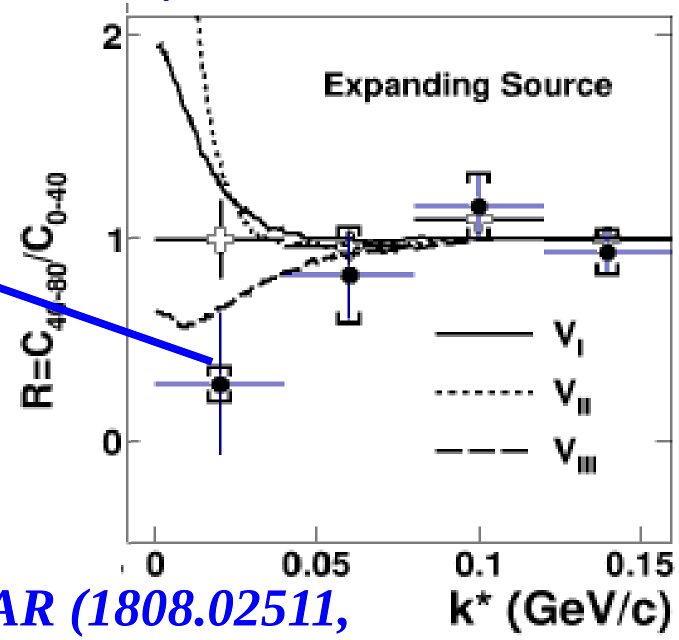
ALICE+STAR = NΩ Dibaryon



Morita+('20)



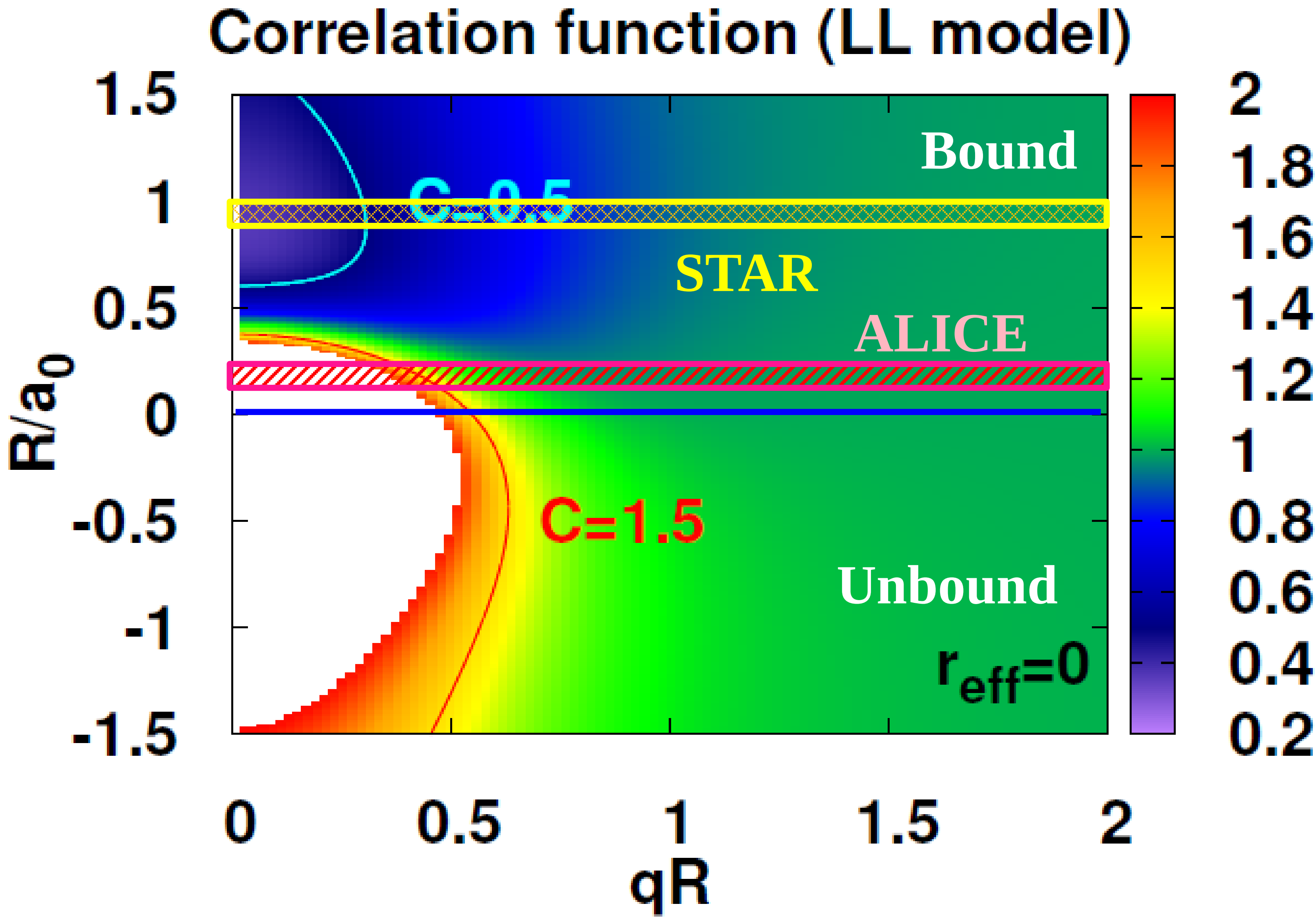
Acharya+(ALICE), Nature ('20)



STAR (1808.02511, PLB790 ('19) 490)

c.f. Fabbietti's talk on Monday

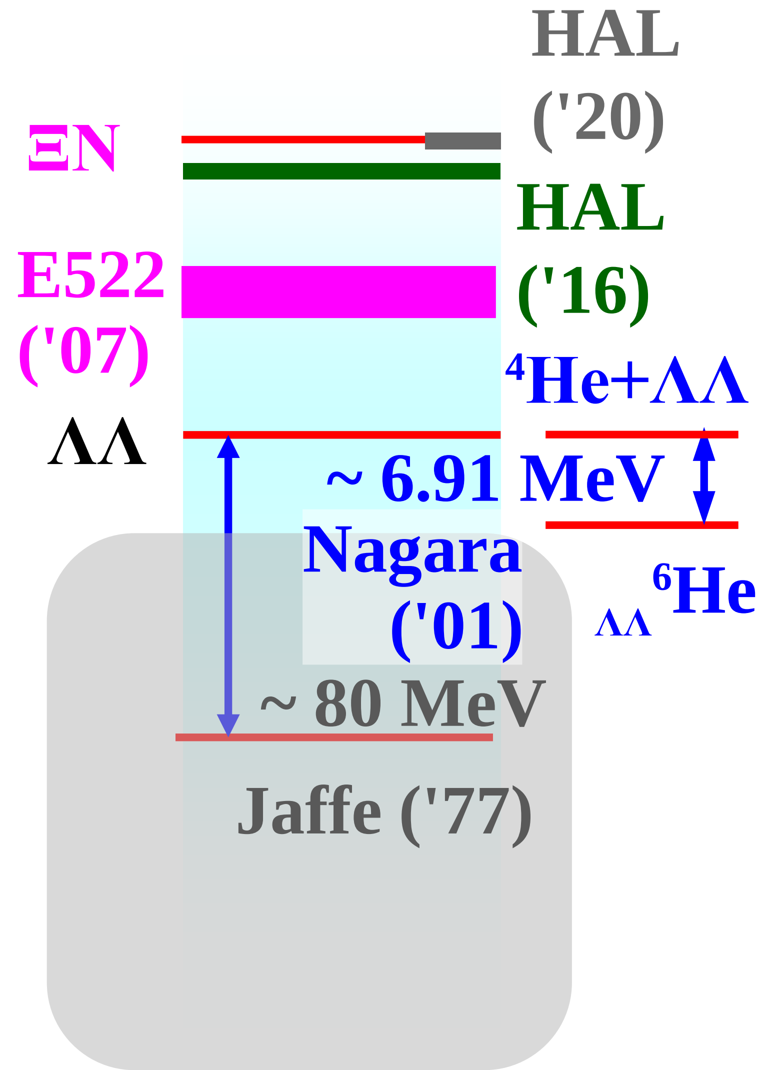
Source Size Dependence of Correlation Function



LL model: R. Lednicky, V. L. Lyuboshits ('82)
AO, K.Morita,K.Miyahara,T.Hyodo (1605.06765)

To be bound or not to be bound, the question for H

- **H-dibaryon: 6-quark state (uuddss)**
  - **Prediction:** ***R.L.Jaffe, PRL38(1977)195***
  - **Ruled-out by double Λ hypernucleus**
    Takahashi et al.,PRL87('01) 212502
  - **Resonance or Bound "H" ?**
    Yoon et al.(KEK-E522)+AO ('07)
- **Lattice QCD results**
  - **Bound:**
    HALQCD('11), NPLQCD('11,'13), Mainz('19)
    (heavier quark mass or SU(3) limit)
  - **Resonance (Bound state of NΞ):**
    HAL QCD ('16,18) **(heavier m_q)**
  - **Virtual Pole (around NΞ threshold)**
    HAL QCD ('20) **(almost physical m_q)**

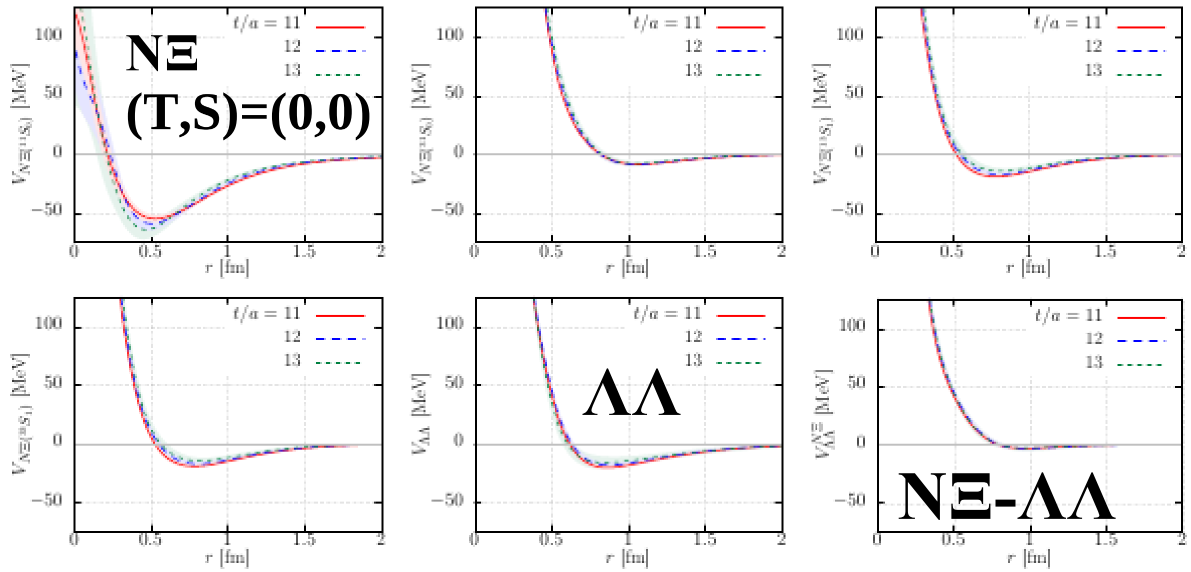


We examine LQCD NΞ-ΛΛ potential and discuss H using CF !

NΞ interaction and pΞ⁻ correlation function

NΞ-ΛΛ potential from Lattice QCD

- **NΞ-ΛΛ potential at almost physical quark mass (m_π=146 MeV)**
  K. Sasaki et al. (HAL QCD), NPA 998 ('20) 121737 (1912.08630)
  - **Strong attraction in (T,S)=(0,0) NΞ channel**
  - **Weak attraction in ΛΛ → Coupling with NΞ causes ΛΛ attraction.**



  - **There is no bound state in NΞ-ΛΛ system (except for Ξ^- atom), but there is a virtual pole around the NΞ threshold (3.93 MeV below $n\Xi^0$ threshold) on the irrelevant Riemann sheet, (+, –, +) [relevant=(–,+,+)]**

$$E_{\rm pole} = 2250.5 \pm i0.3\ {\rm MeV}$$

Sign of Im(eigen momentum)

Correlation Function with Coupled-Channel Effects

- **Correlation function formula (Koonin-Pratt formula) with CC**
  → **sum of channel contributions**
  to the relevant channel with outgoing momentum *q*

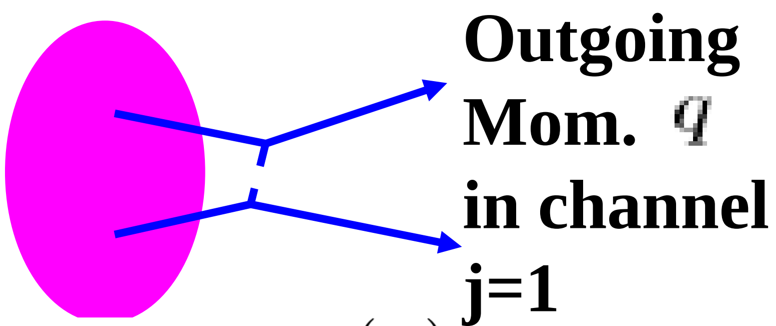
Lednicky, Lyuboshits ('81); Haudenbauer ('19)

$$C(\boldsymbol{q}) = \sum_j \omega_j \int d\boldsymbol{r} S_j(\boldsymbol{r}) |\Psi_j^{(-)}(\boldsymbol{r})|^2$$

$$\Psi_j^{(-)}(\boldsymbol{r}) = [e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr)]\delta_{1j} + \psi_j^{(-)}(r)$$

$$\psi_j^{(-)}(q) \propto e^{-iqr}/r \text{ or } e^{-\kappa r} \quad (r \to \infty)$$

(No Coulomb case)



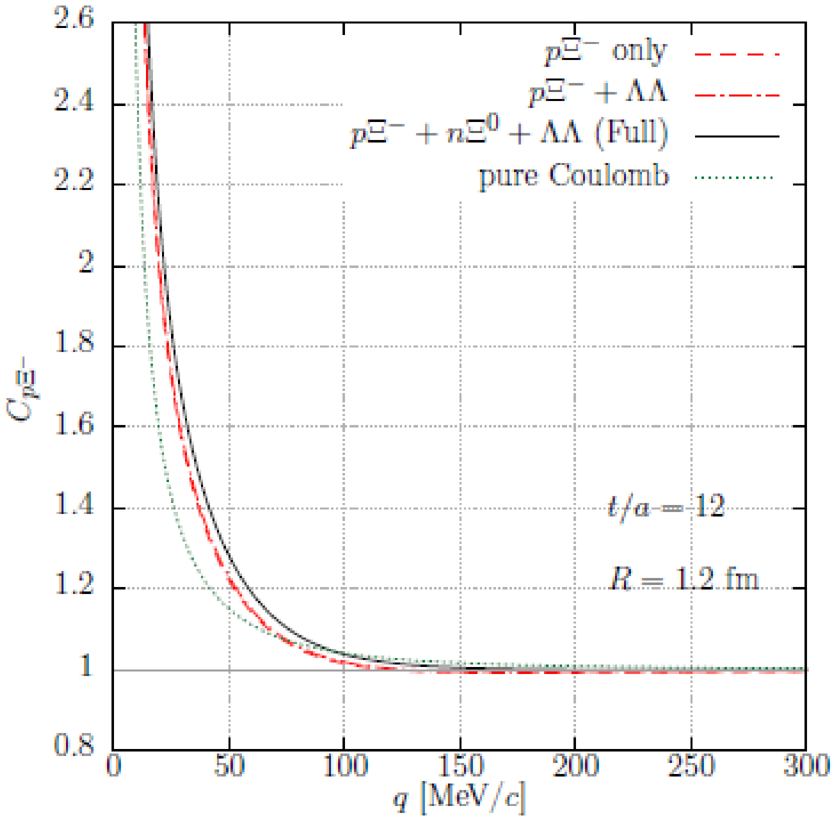
$$\omega_j S_j(\boldsymbol{r}) |\Psi_j^{(-)}(\boldsymbol{r})|^2$$

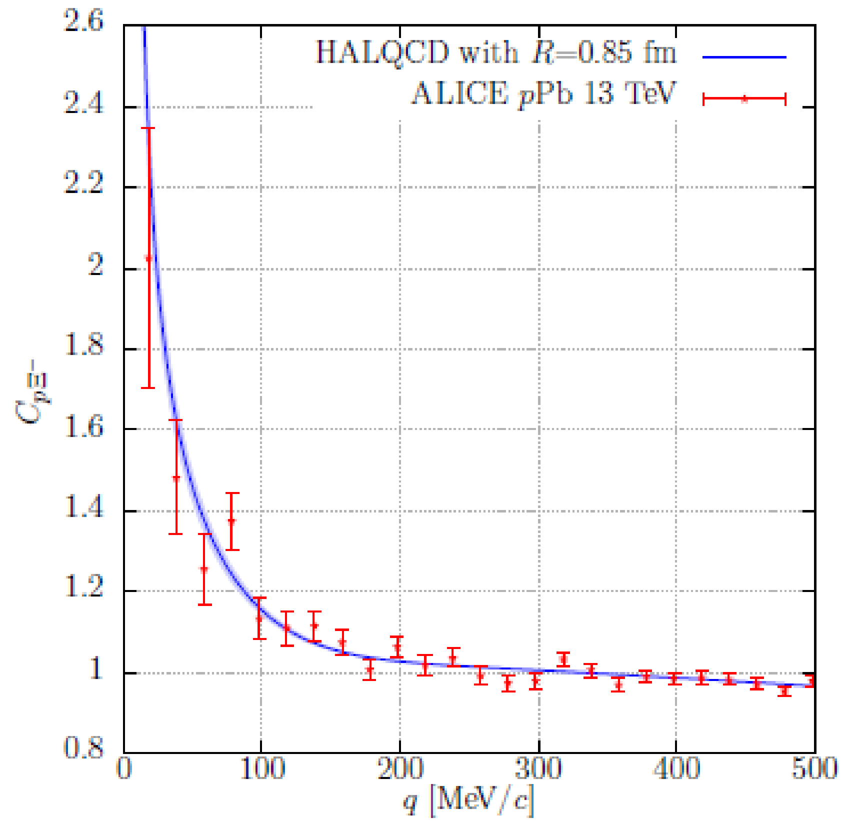
Source weight (ω_j) **Normalized Source fn.** ($S_j(\boldsymbol{r})$)

- **Effects of coupled-channel, strong & Coulomb pot., and threshold difference are taken into account in the charge base, pΞ^-, nΞ^0, $\Lambda\Lambda$.**
  Y. Kamiya+, PRL('20, K^- p)
- **Source weights (ω_j) are chosen to be unity.**
- **Source size R is taken as the parameter.**

$p\Xi^-$ correlation function

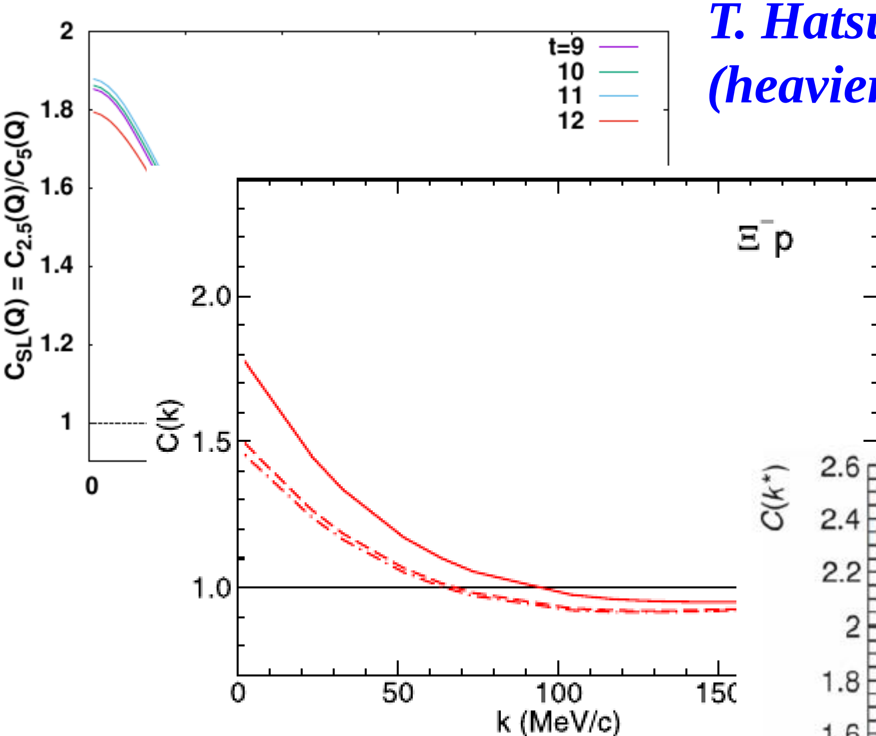
- **$p\Xi^-$ correlation function data implies attractive $N\Xi$ interaction.**
  - **Calculated CF agrees with ALICE data.**
  - **Strong enhancement from pure Coulomb CF**
  - **$\Lambda\Lambda$ source effect is negligible. $n\Xi^0$ source effect is visible.**





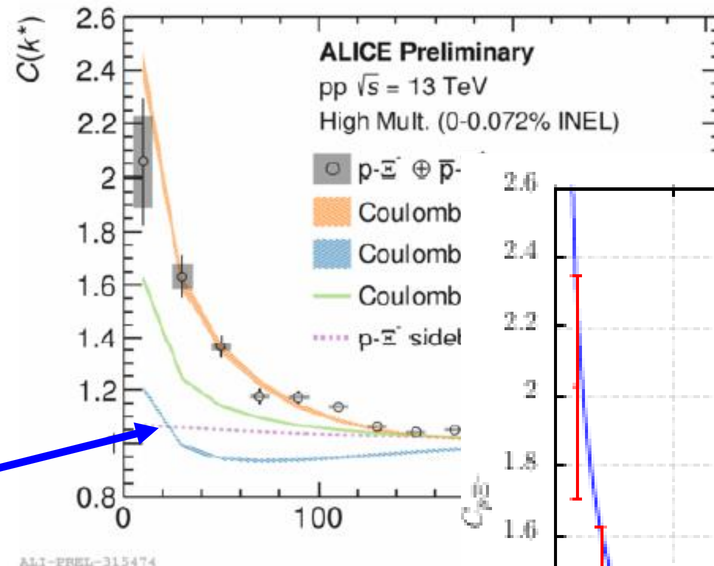
Kamiya+, in prep.; Acharya+(ALICE), Nature ('20)

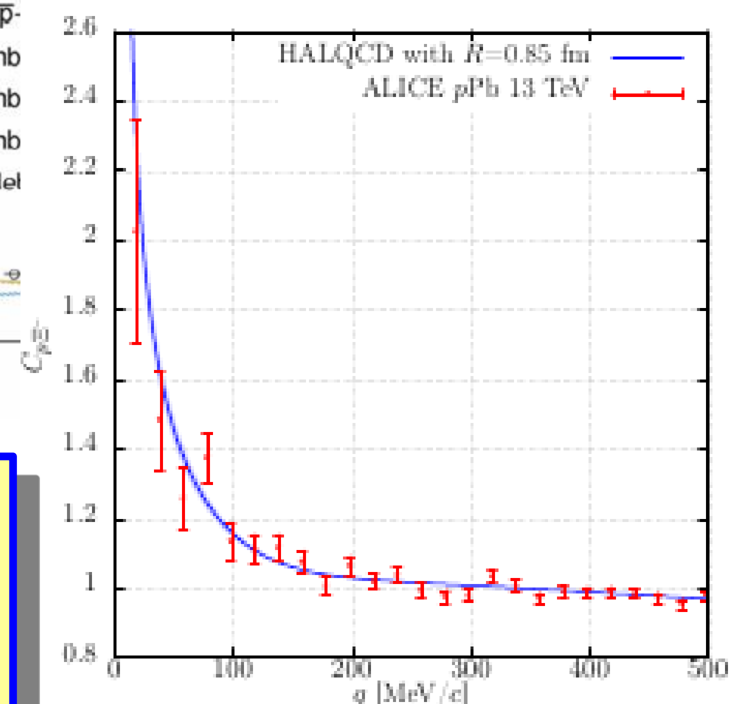
Comparison with other results



T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17)856.
(heavier quark mass)

J. Haidenbauer, NPA981('19)1.
(NLO(600), w/ CC effects, w/o Coulomb)
(w/ Coulomb, it will be comparable with data.)



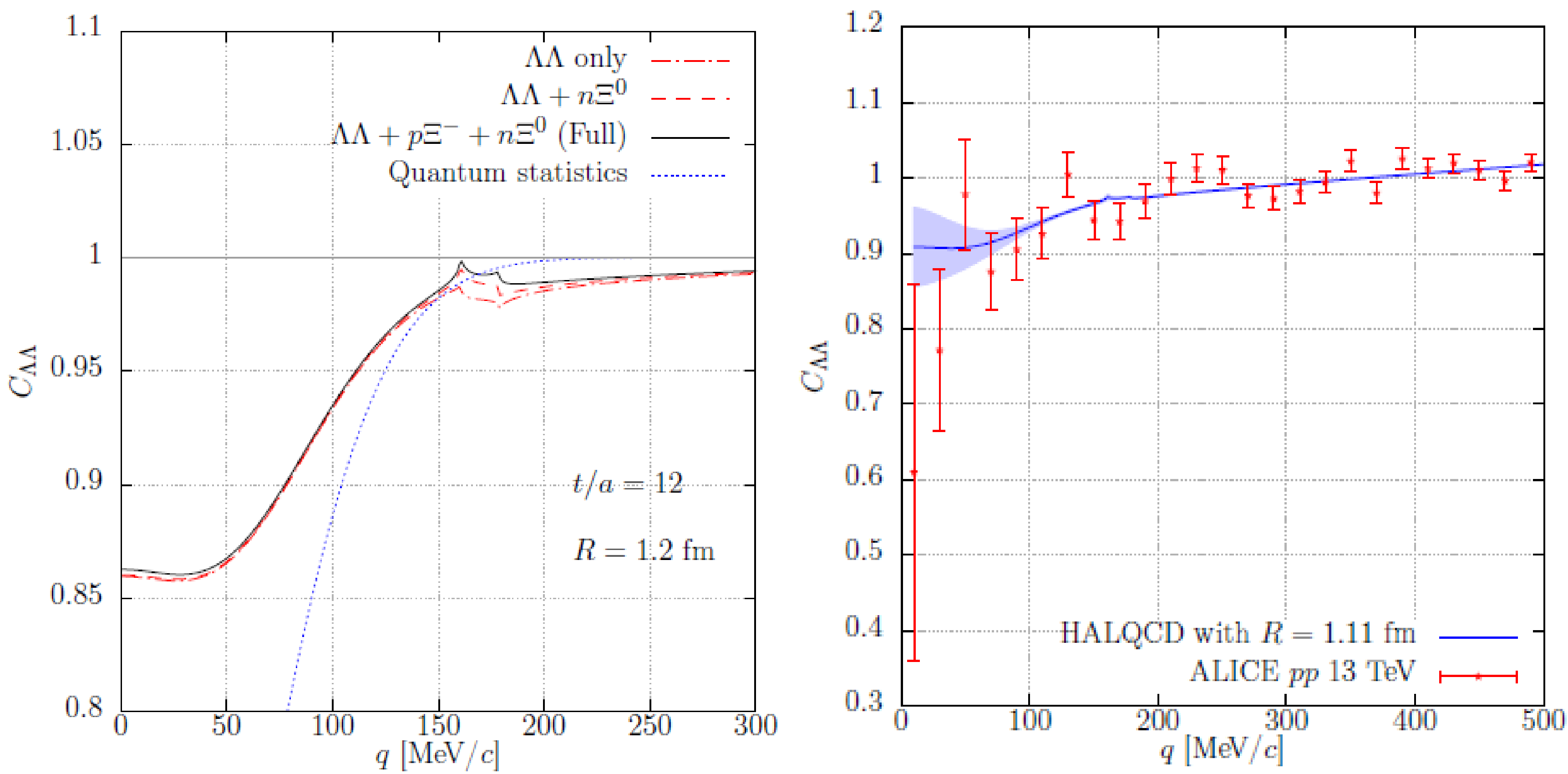


D. L. Mihairov+[ALICE],
NPA1005('21)121760 (QM2019).
(Nijmegen potential does not explain the data.)

Kamiya+(in prep.).
(w/ Lattice BB pot. at phys. m_q and CC effects with $\Lambda\Lambda$)

ΛΛ correlation function

- **ΛΛ correlation function**
  - **Calculated CF agrees with ALICE data.**
  - **NΞ source effect is visible only around threshold.**



Kamiya+, in prep.; Acharya+(ALICE), Nature ('20)

Comparison with other results

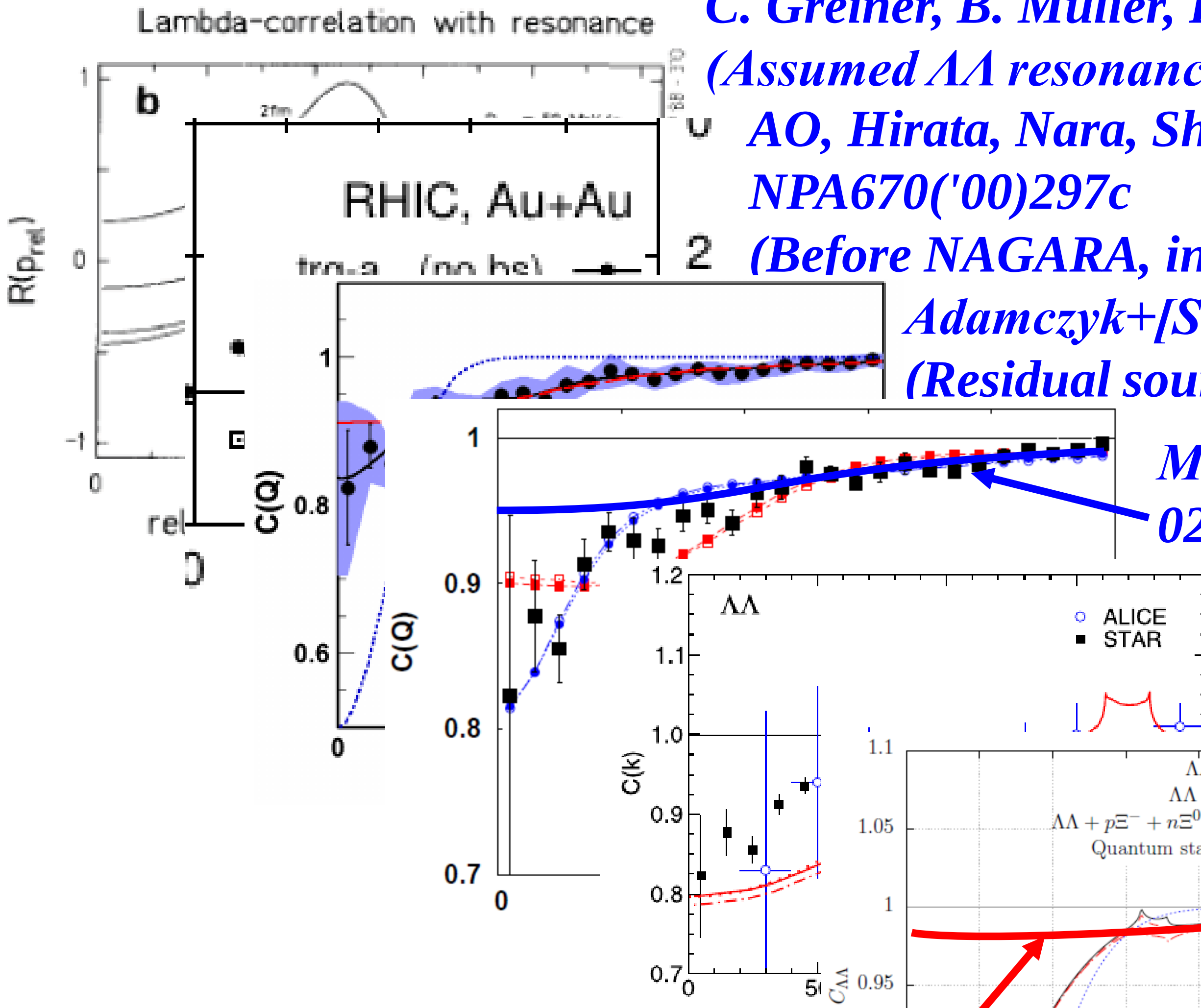
C. Greiner, B. Muller, PLB219('89)199.
(Assumed ΛΛ resonance)

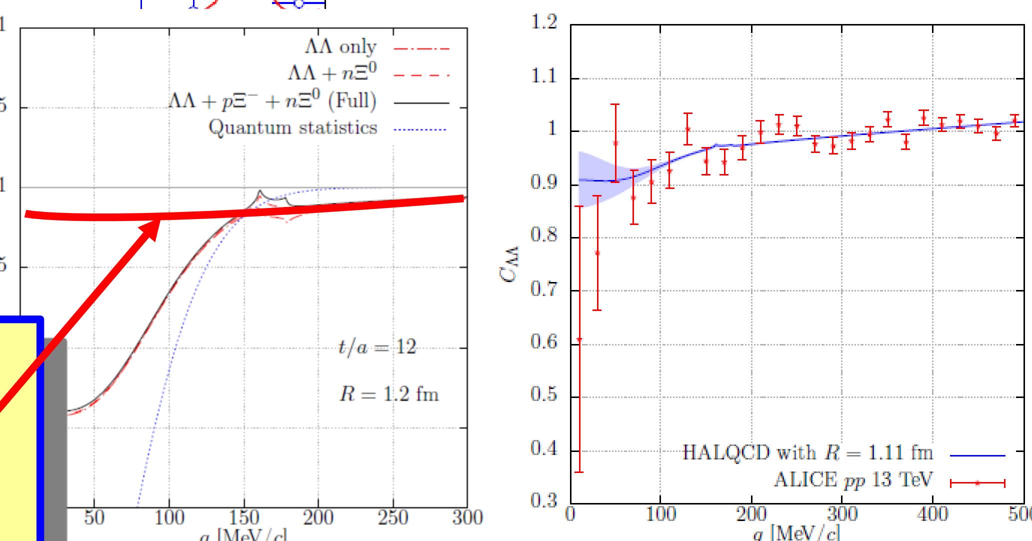
AO, Hirata, Nara, Shinmura, Akaishi, NPA670('00)297c
(Before NAGARA, interaction was too strong.)

Adamczyk+[STAR], PRL114('15)022301
(Residual source R ~ 0.5 fm was assumed.)

Morita, Furumoto, AO,PRC91('15) 024916. (Res.Source + flow)

J. Haidenbauer, NPA981('19)1.
(Larger cusp ?)





Kamiya+(in prep.).
(Smaller cusp than χEFT.
CC simulates res. source !)

Summary

- **Correlation functions (CFs) can be utilized**
  to constrain / examine the hadron-hadron interactions,
  as well as to deduce the pair is to be bound or not to be bound.
- **We have calculated pΞ^- and ΛΛ correlation functions by using lattice NΞ-ΛΛ coupled-channel (CC) potential with effects of CC, Coulomb potential, threshold difference.**
- **Recent ALICE data are consistent with the HAL QCD potential.**
  - **CC effect of NΞ-ΛΛ is not very big.**
  - **ΛΛ CF shows cusps at NΞ thresholds, and CC effect simulates small residual source assumed to interpret heavy-ion data by STAR.**
  - **The data may suggest the existence of the H dibaryon state as the virtual pole around the NΞ threshold.**
- **Pole position is sensitive to the detail of interaction, so the source size dependence would be desirable to measure.**

Homeworks

- Pole position is sensitive to the detail of interaction, so the source size dependence would be desirable to measure.
  - Larger size (STAR seems to be analyzing pΞ- correlation function data, private communication with Neha Shar)
  - Smaller size (J-PARC-E42 is measuring pπΛ, ΛΛ, $p\Xi^-$ invariant mass distribution, and effective source size is smaller than pp)
- Analytic Lednicky-Lyuboshits type formula should be developed / examined to confirm the pole position of the H dibaryon.
  - Explaining the data with a given potential is not enough ! Method to extract scattering parameters directly from data is desired.
- Further studies including charm, three-body, deuteron will enrich hadron interactions from femtoscopy.

Thank you for attention !

Coauthors of Y. Kamiya et al. ($p\Xi^-$), in prep.

Y. Kamiya

K. Sasaki

T. Fukui

T. Hatsuda

T. Hyodo

K. Morita

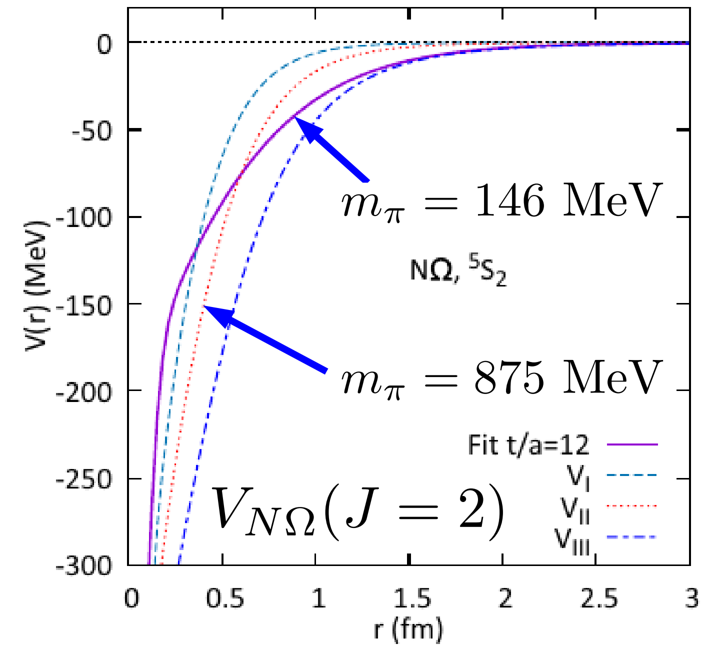
K. Ogata

AO

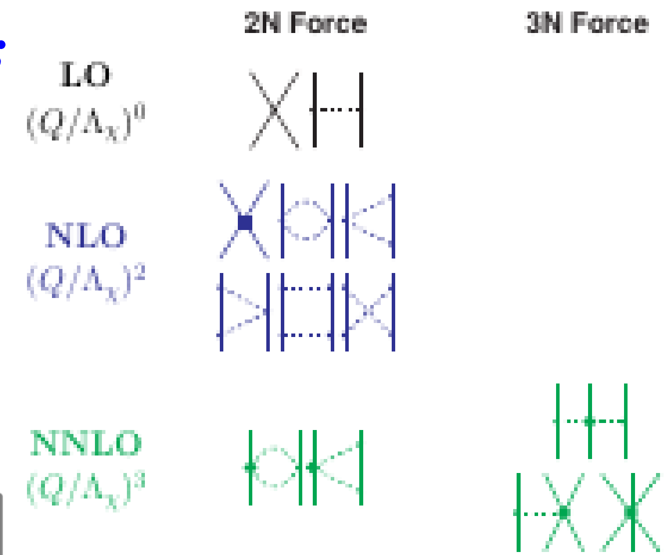
Y. Kamiya, K. Sasaki, T. Fukui, T. Hatsuda, T. Hyodo, K. Morita, K. Ogata, AO, in prep.

Modern Hadron-Hadron Interactions

- **Lattice QCD *hh* potential**
  - **V_{hh} is obtained from the Schrödinger eq. for the Nambu-Bethe-Salpeter (NBS) amplitude.**
    N. Ishii, S. Aoki, T. Hatsuda, PRL99('07)022001.
    → ΩΩ, NΩ, ΛΛ-NΞ potentials at phys. quark mass are published



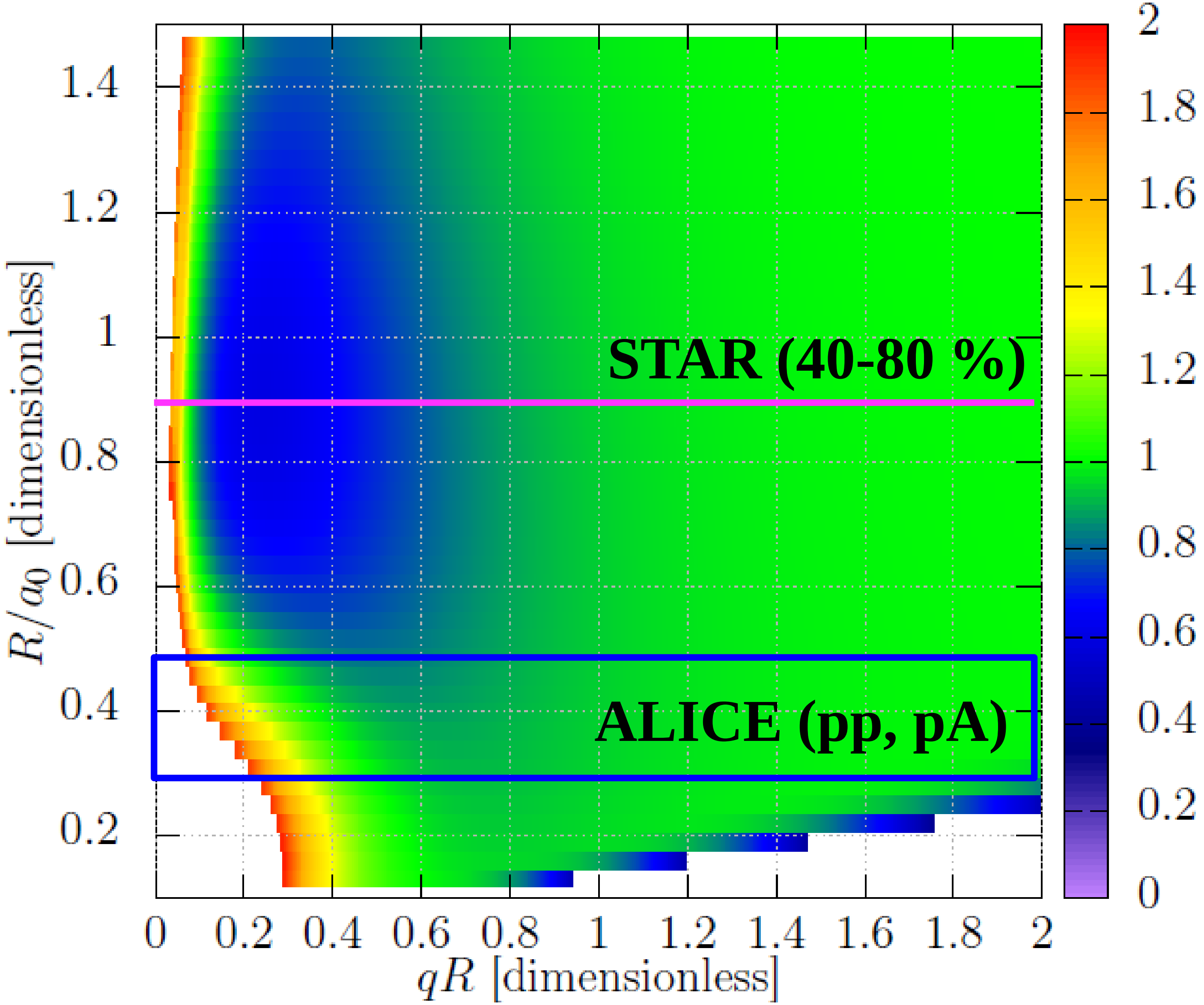
- **Chiral EFT / Chiral SU(3) dynamics**
  - **V_{hh} at low E. can be expanded systematically in powers of Q/Λ.**
    S. Weinberg ('79);R. Machleidt, F. Sammarruca ('16); Y. Ikeda, T. Hyodo, W. Weise ('12).
    → NN, NY, YY, KN-πΣ-πΛ, ...



- **Quark cluster models, Meson exchange models, More phenomenological models, ...**

Let us examine modern hh interactions !

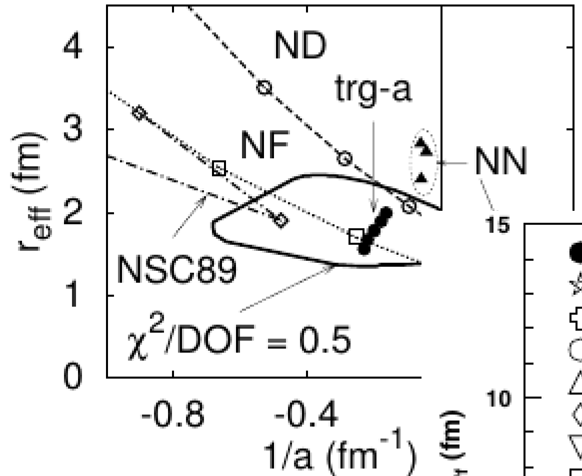
Correlation Function with Gaussian source



NΩ potential (J=2, HAL QCD, a =3.4 fm) + Coulomb

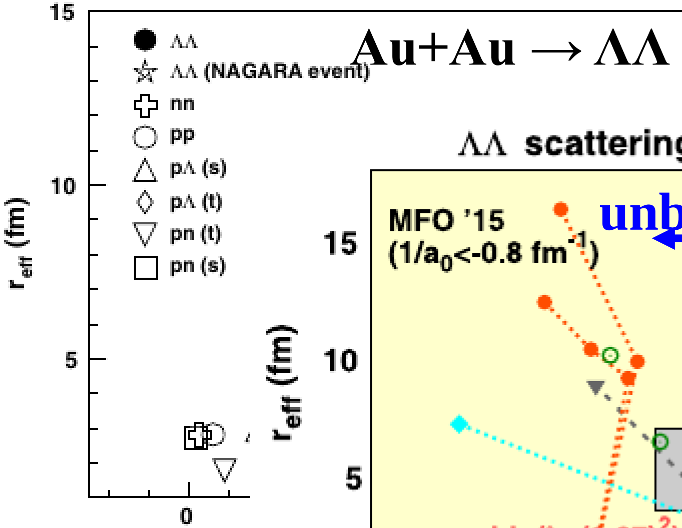
ΛΛ correlation and ΛΛ interaction

It is unlikely that ΛΛ bound state exists.

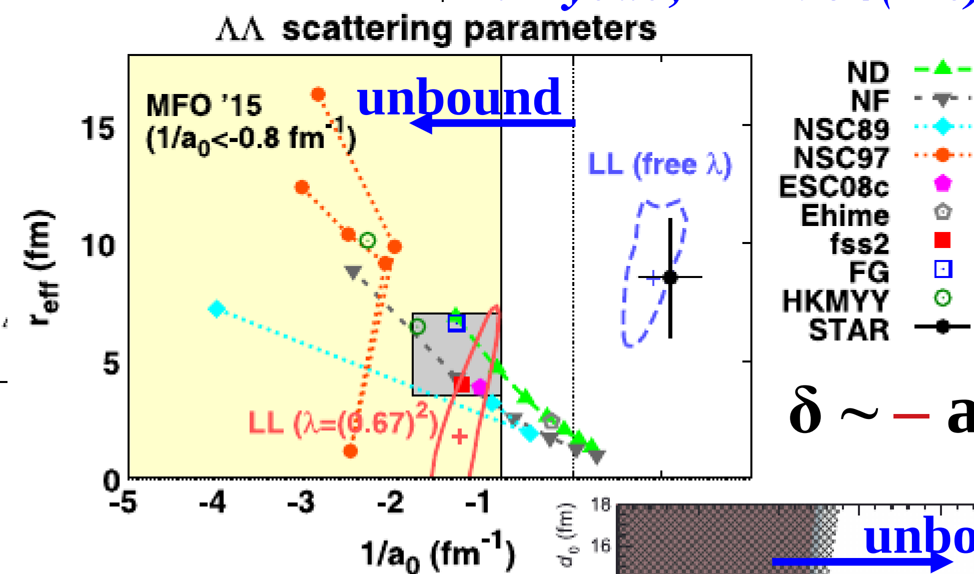


AO+('00)

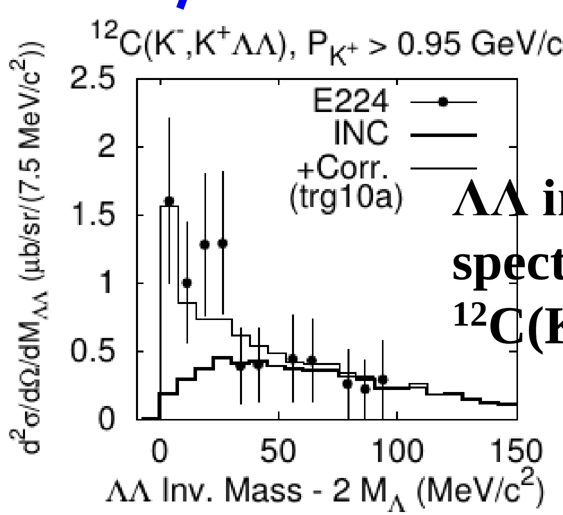
L. Adamczyk+[STAR], PRL114('15)022301



AO, K. Morita, K. Miyahara, T. Hyodo, NPA954('16)294

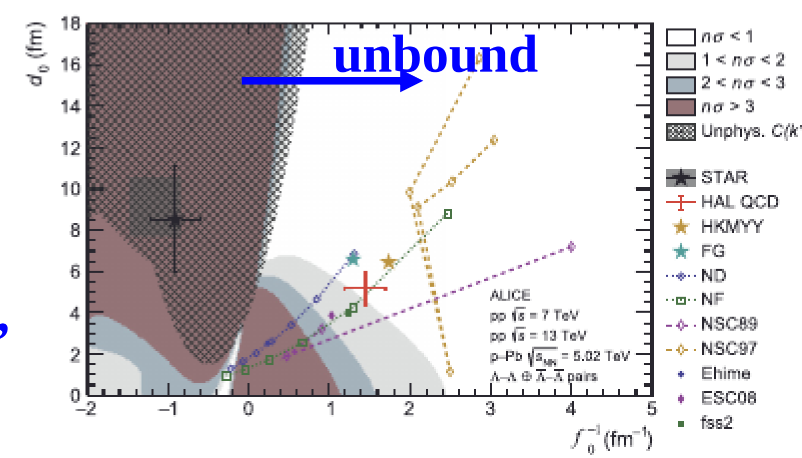


$\delta \sim - a_0 q$



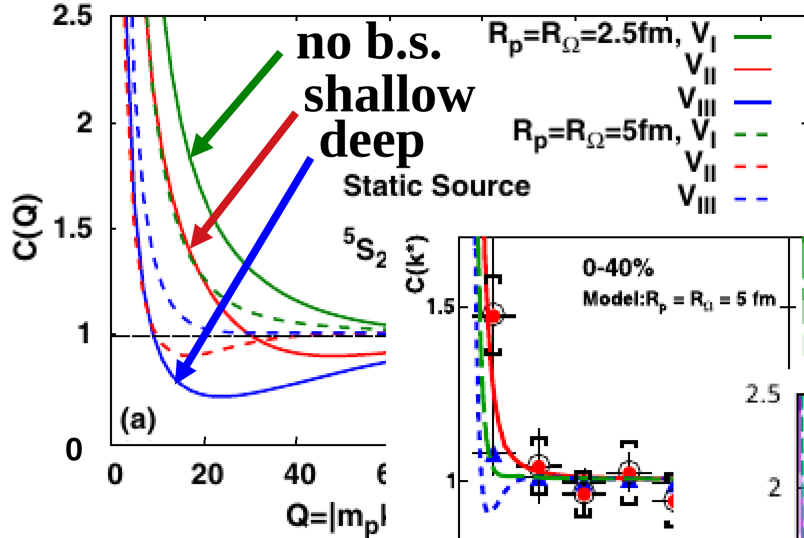
ΛΛ inv. mass spectrum from $^{12}C(K^-, K^+\Lambda\Lambda)$

S. Acharya+[ALICE], PLB797('19)134822

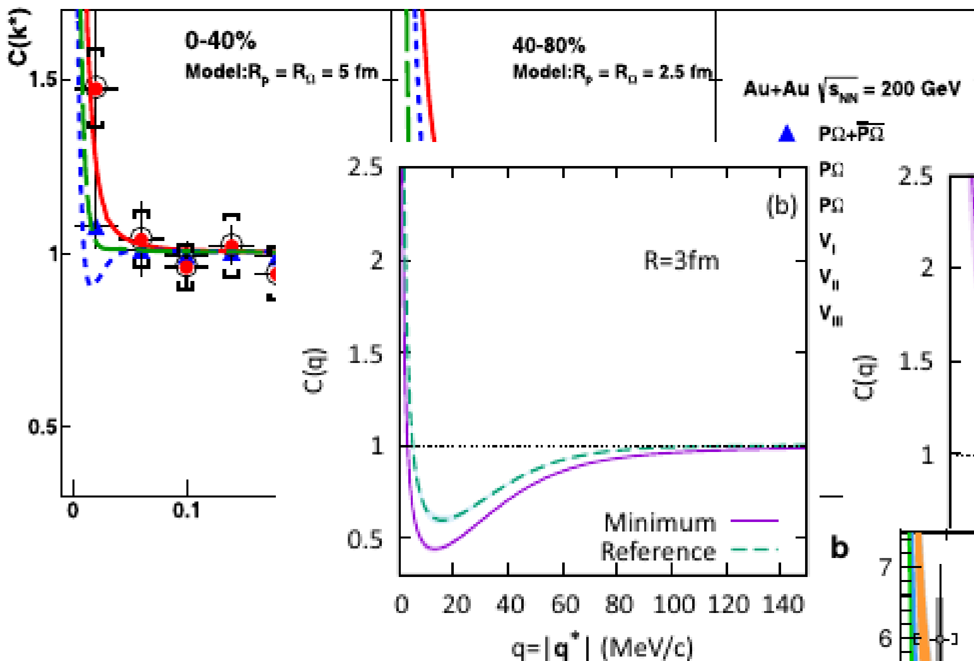


$\delta \sim + a_0 q$

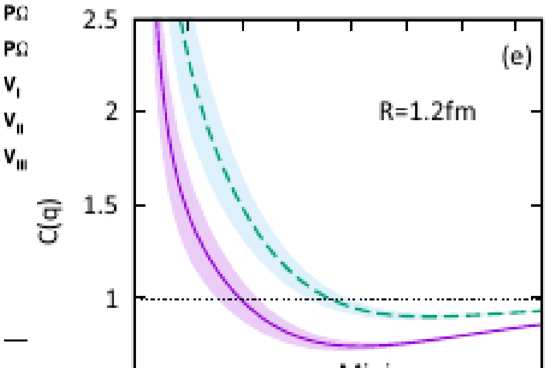
pΩ⁻ correlation



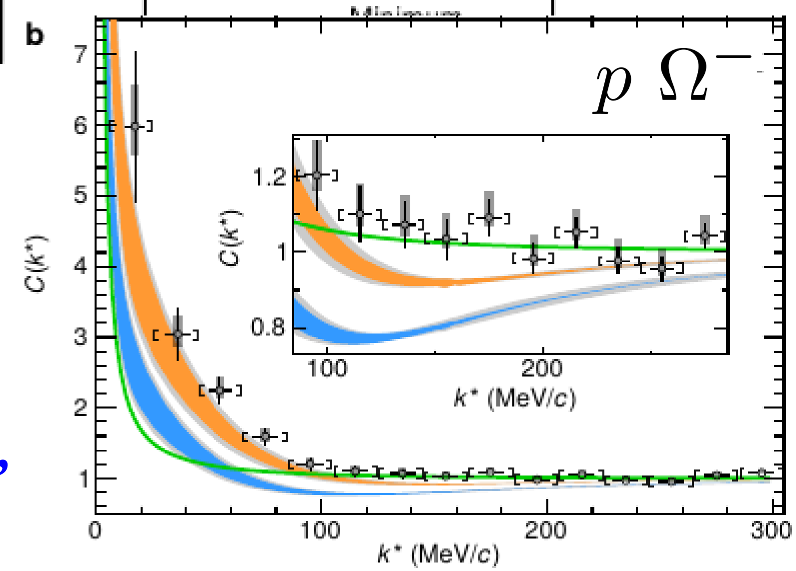
K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R) (w/ Lattice potential with heavier quark mass)



J. Adam+[STAR], PLB790('19)490.



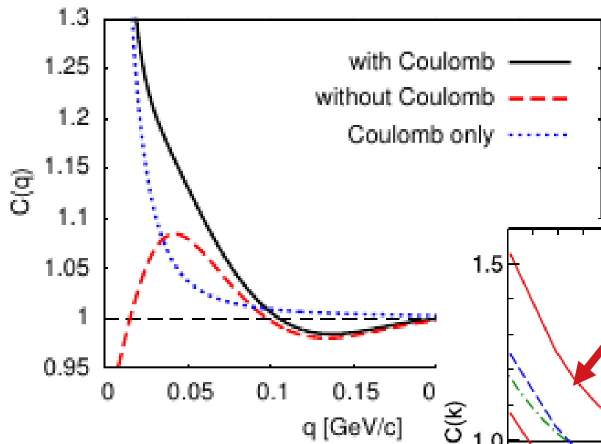
K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC 101('20)015201. (w/ Lattice potential at physical quark mass)



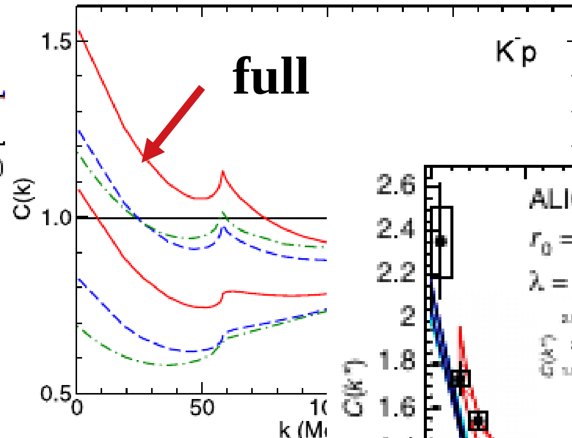
S. Acharya+[ALICE], 2005.11495 [nucl-ex] (pp 13 TeV)

Bound state ?

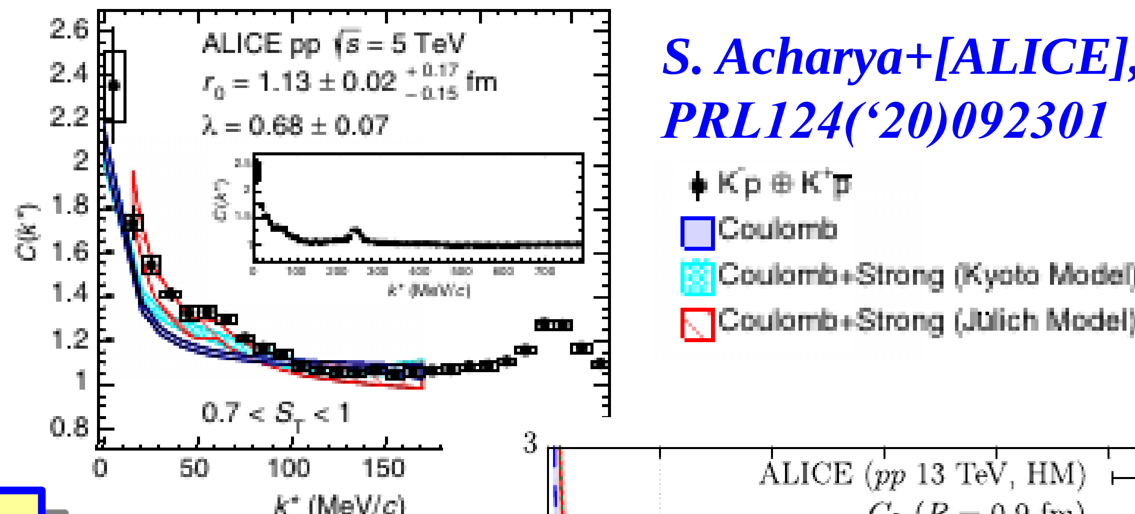
pK⁻ correlation



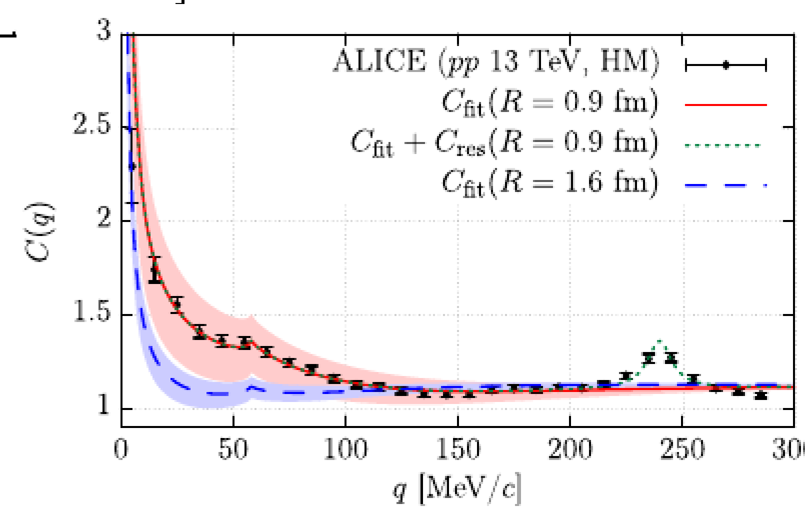
S. Cho+ [ExHIC], PPNP95('17)279.
(Insufficient coupled-channel effects)



J. Haidenbauer, NPA981('19)1.
(w/ CC effects, w/o Coulomb)



S. Acharya+[ALICE], PRL124('20)092301

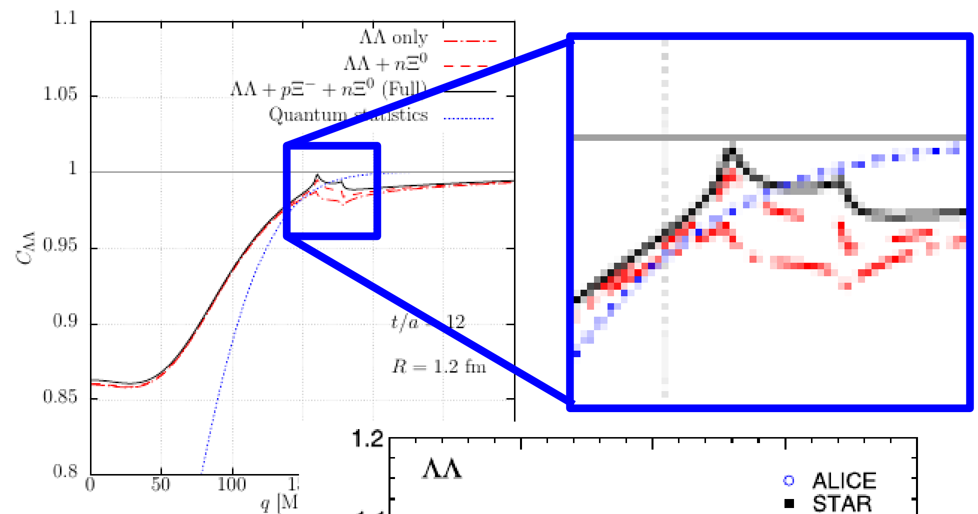


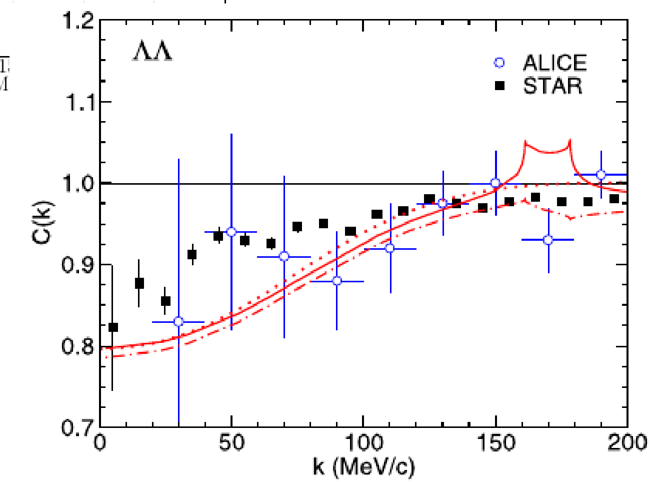
Source size dep. shows interesting feature.

Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.
(Chiral SU(3) dynamics)

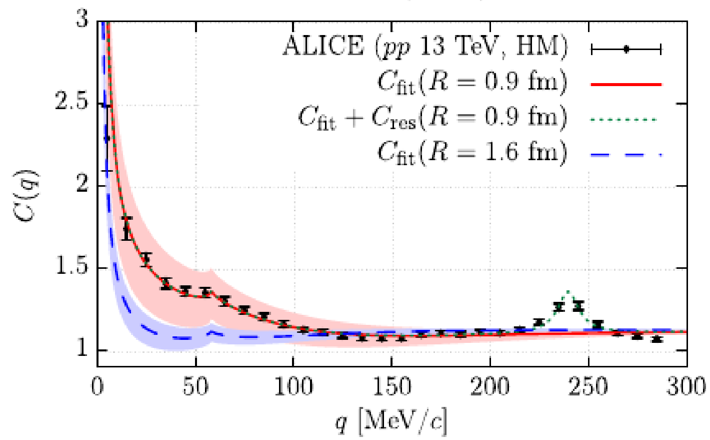
Other bound states ?

- **ΛΛ-NΞ**
  - **C (q) in AA(RHIC) and pp(LHC) are similar (No b.s. below ΛΛ).**
  - **LQCD predicts a virtual pole near NΞ threshold, which can be detected as the cusp in C (q). NLO(600) potential predicts the same. (The fate of H particle)**
    K. Sasaki+[HAL QCD], NPA998('20)121737; Y. Kamiya+, in prep.; Haidenbauer('19).





- **KN**
  - **Λ(1405) is believed to be the bound state of KN, and "dip" is expected at *R* ~ *a* .**
  - **However, Coulomb and coupled-channel effects modify the dip-like behavior.**
    Kamiya+ ('20).



Lednicky-Lyuboshits (LL) model

- **Lednicky-Lyuboshits analytic model**
  - **Asymp. w.f. + Eff. range corr. + $\psi^{(-)} = [\psi^{(+)}]^*$**

$$\psi_0(r) \to \psi_{\rm asy}(r) = \frac{e^{-i\delta}}{qr}\sin(qr+\delta) = \mathcal{S}^{-1}\left[\frac{\sin qr}{qr} + f(q)\frac{e^{iqr}}{r}\right]$$

$$\Delta C_{\rm LL}(q) = \int d\boldsymbol{r} S_{12}(r)\left(|\psi_{\rm asy}(r)|^2 - |j_0(qr)|^2\right)$$
$$= \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\rm eff}}{R}\right) + \frac{2{\rm Re}f(q)}{\sqrt{\pi}R}F_1(x) - \frac{{\rm Im}f(q)}{R}F_2(x)$$

$$(x = 2qR, R = \text{Gaussian size}, F_1, F_2, F_3 : \text{Known functions})$$

- **Phase shifts**

$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}q^2 + \mathcal{O}(q^4) \to \delta \simeq -a_0 q + {\rm O}(q^3)$$

$$\sin(qr+\delta) \simeq \sin(q(r-a_0)+\cdots)$$

Node at r ~ a_0 for small q

C(q) in the low momentum limit

- **Correlation function at small q (and r =0) → F =1, F =0, F =1**

$$\Delta C_{\mathrm{LL}}(q) \to \frac{|f(0)|^2}{2R^2} + \frac{2\mathrm{Re}f(0)}{\sqrt{\pi}R}\ (q \to 0)$$

$$f(q) = (q\cot\delta - iq)^{-1} \simeq \left(-\frac{1}{a_0} + \frac{1}{2}r_{\mathrm{eff}}q^2 - iq\right)^{-1} \to -a_0$$

$$C_{\mathrm{LL}}(q \to 0) = 1 + \frac{a_0^2}{2R^2} - \frac{2a_0}{\sqrt{\pi}R} = 1 - \frac{2}{\pi} + \frac{1}{2}\left(\frac{a_0}{R} - \frac{2}{\sqrt{\pi}}\right)^2$$

$$1 - 2/\pi \simeq 0.36,\ \ \sqrt{\pi}/2 \simeq 0.89$$

C(q → 0) takes a minimum of 0.36 at R/a = 0.89 in the LL model.

Correlation Function with Coupled-Channels Effects

J. Haidenbauer, NPA 981('19)1; R. Lednicky, V. V. Lyuboshits, V. L. Lyuboshits, Phys. At. Nucl. 61('98)2950.

- **Single channel, w/o Coulomb (non-identical pair)**

$$C(\boldsymbol{q}) = 1 + \int d\boldsymbol{r} S(\boldsymbol{r}) \left[|\chi^{(-)}(r,q)|^2 - |j_0(qr)|^2 \right]$$

- **Single channel, w/ Coulomb**

$$C(\boldsymbol{q}) = \int d\boldsymbol{r} S(\boldsymbol{r}) \left[|\varphi^{C,\mathrm{full}}(\boldsymbol{q},\boldsymbol{r})|^2 + |\chi^{C,(-)}(r,q)|^2 - |j_0^C(qr)|^2 \right]$$

Full free Coulomb w.f. **s-wave w.f. with Coul.** **s-wave Coul. w.f.**

- **Coupled channel, w/ Coulomb**

$$C_i(\boldsymbol{q}) = \int d\boldsymbol{r} S_i(\boldsymbol{r}) \left[|\varphi^{C,\mathrm{full}}(\boldsymbol{q},\boldsymbol{r})|^2 + |\chi_i^{C,(-)}(r,q)|^2 - |j_0^C(qr)|^2 \right]$$

$$+ \sum_{j \neq i} \omega_j \int d\boldsymbol{r} S_j(\boldsymbol{r}) |\chi_j^{C,(-)}(r,q)|^2$$

s-wave w.f. in j-th channel

Outgoing B.C. in the i-th channel, ω = Source weight (ω =1)