

Flavored hadron correlations and interactions from heavy ion collisions

Akira Ohnishi (YITP, Kyoto U.)

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- Introduction
- Basics in femtoscopic study of hadron-hadron interactions
- Recent & Future Femtoscopic Studies of HHI
- Summary

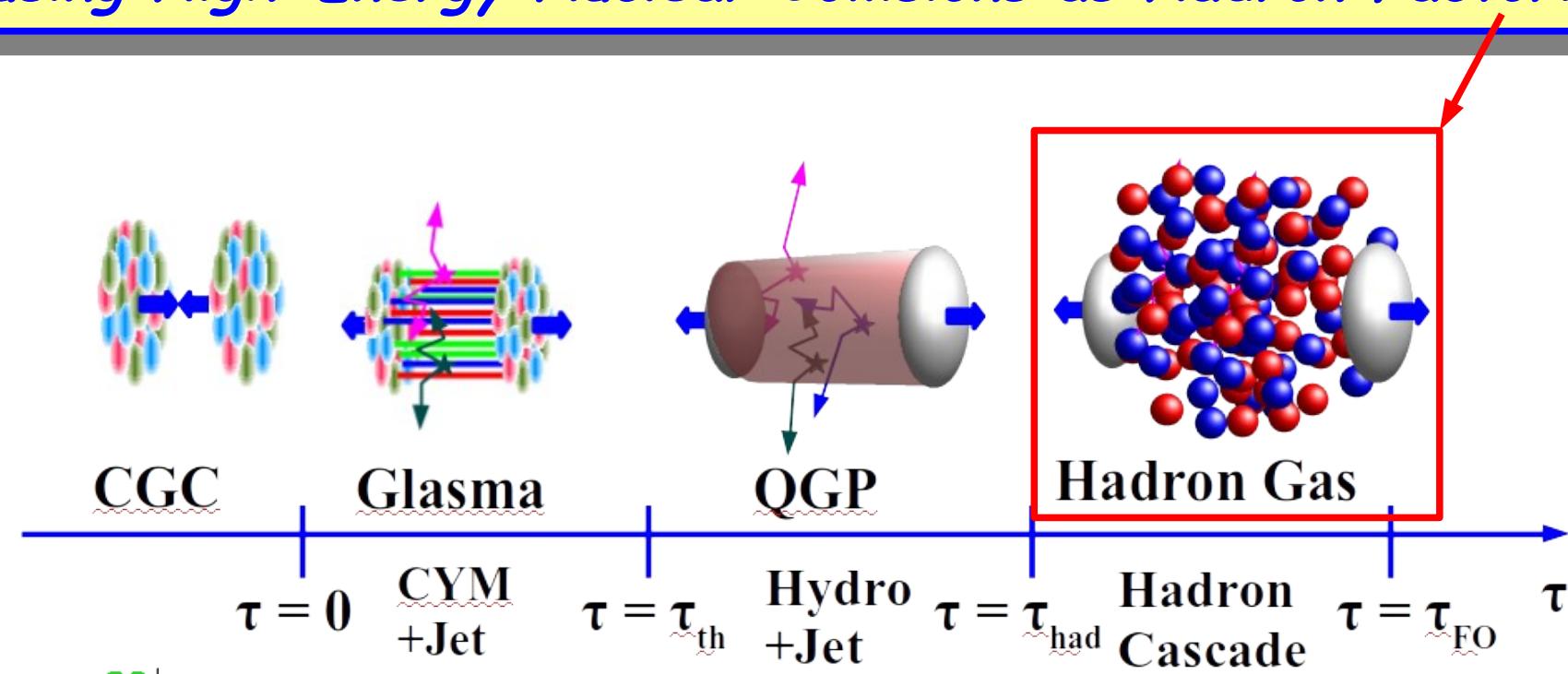
Hadron Physics using High-Energy HIC

■ High-Energy Heavy-Ion Collisions

- Too complex → Statistical → Simple and Clean !
- High T & Large volume → Abundant hadrons
- Nearly 4π detector / Vertex detector

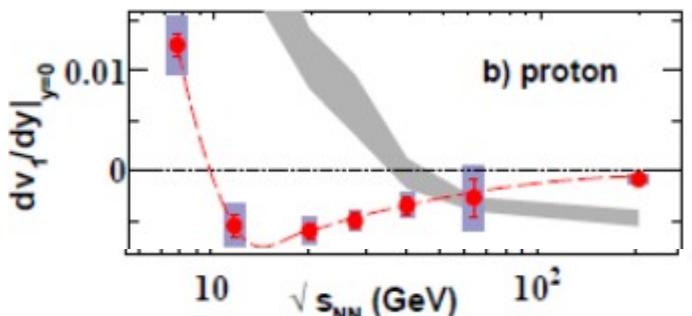
Let's study Hadron Physics

*(e.g. hadron-hadron interactions and hadronic matter EOS)
by using High-Energy Nuclear Collisions as Hadron Factories.*

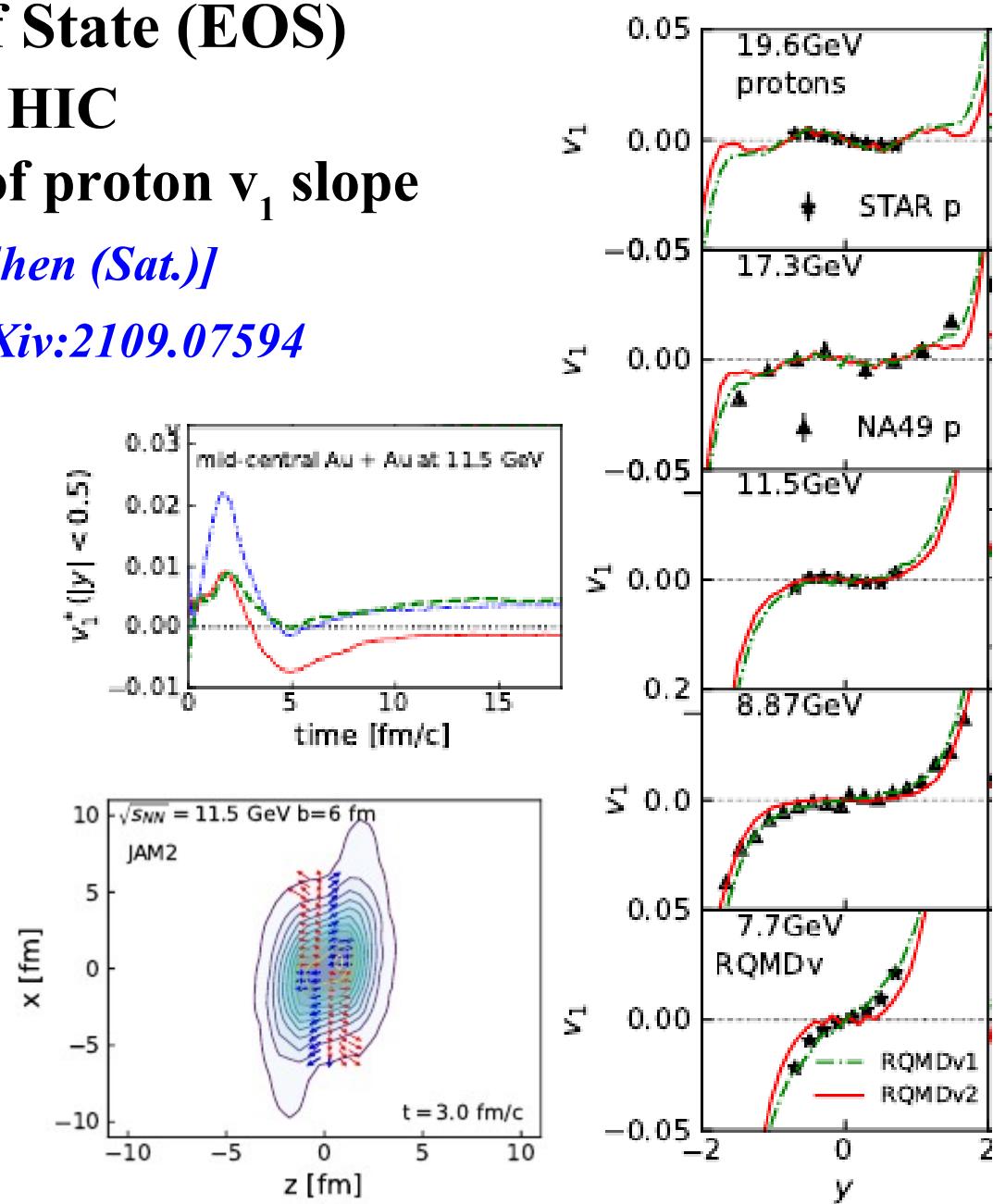


Revisiting dense hadronic matter EOS

- Collective flow → Equation of State (EOS)
 - Old but important subject in HIC
 - Non-monotonic energy dep. of proton v_1 slope
= Unsolved puzzle [c.f. Chun Shen (Sat.)]
- Recent discovery Y. Nara, AO, arXiv:2109.07594
 - JAM2+RQMDv (EOS effect)
 - Positive flow in compression,
Negative flow in expansion
due to tilted ellipsoid.
 - E-dep. of dv_1/dy is explained
 - ATHIC abst. was not accepted.



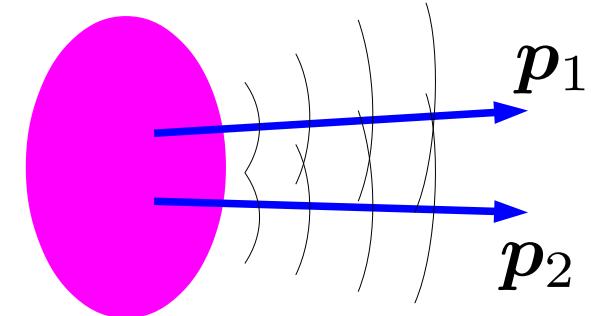
STAR, PRL112('14)162301



Femtoscopic study of hadron-hadron interaction

■ Correlation function (CF)

- One can access various hadron-hadron interactions using CFs (femtoscopy).
- CF=convolution of source fn. and |w.f.|² (Koonin-Pratt formula) *Koonin('77), Pratt('86), Lednicky+('82)*



$$C(p_1, p_2) = \frac{N_{12}(p_1, p_2)}{N_1(p_1)N_2(p_2)} \simeq \int dr S(r) |\varphi_q(r)|^2 \quad (\text{q=rel. mom.})$$

source fn. relative w.f.

■ Source size from quantum stat. + CF (HBT-GGLP effect)

Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)

■ Hadron-hadron interaction from source size + CF

- CF of non-identical pair from static spherical source

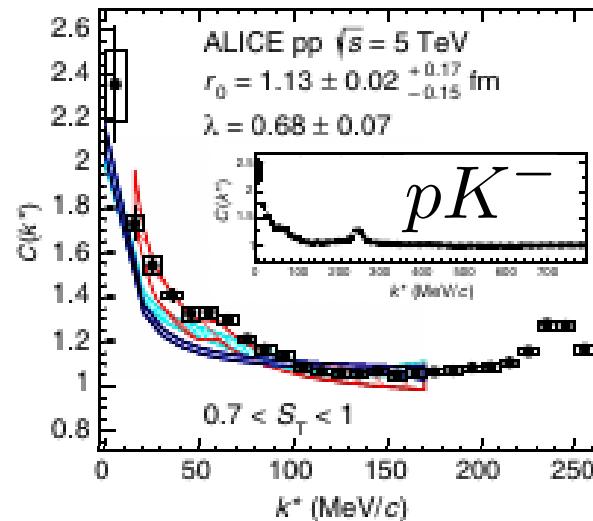
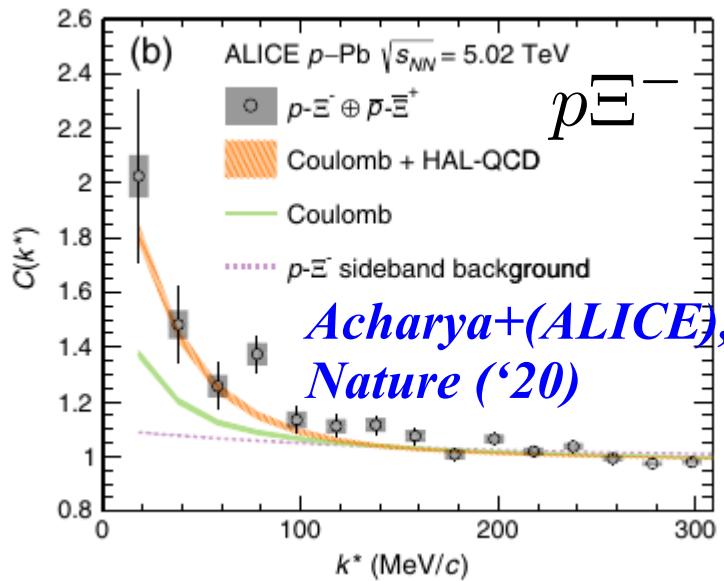
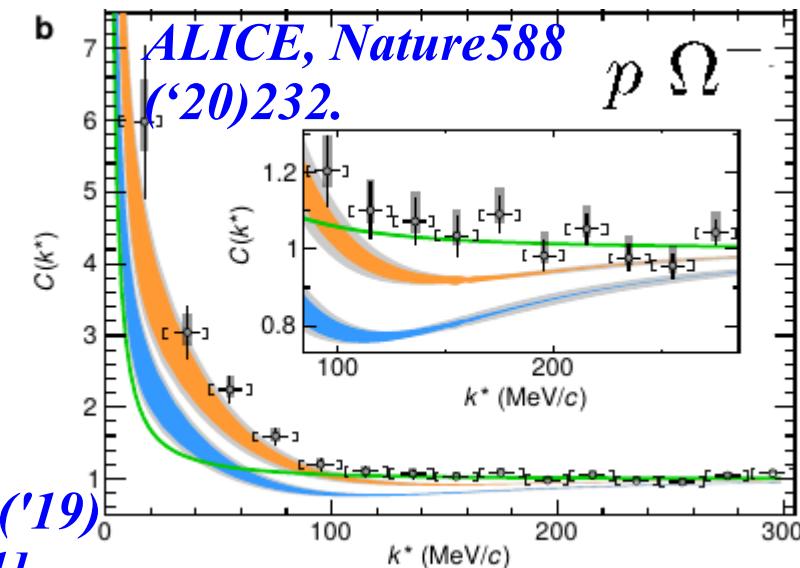
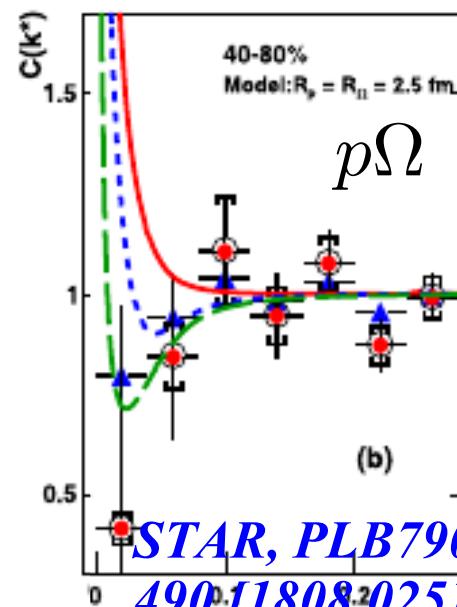
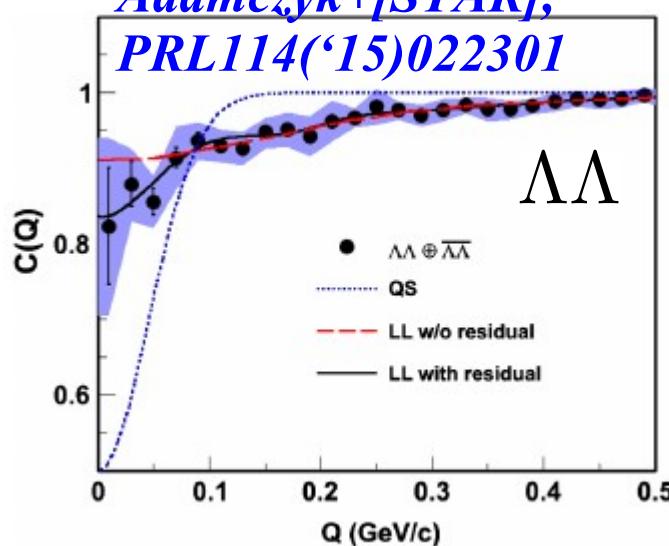
Lednicky, Lyuboshits ('82); Morita, Furumoto, AO ('15)

$$C(q) = 1 + \int dr S(r) \left\{ |\varphi_0(r)|^2 - |j_0(qr)|^2 \right\} \quad (\varphi_0 = \text{s-wave w.f.})$$

CF shows how much $|\varphi|^2$ is enhanced $\rightarrow V_{hh}$ effects !

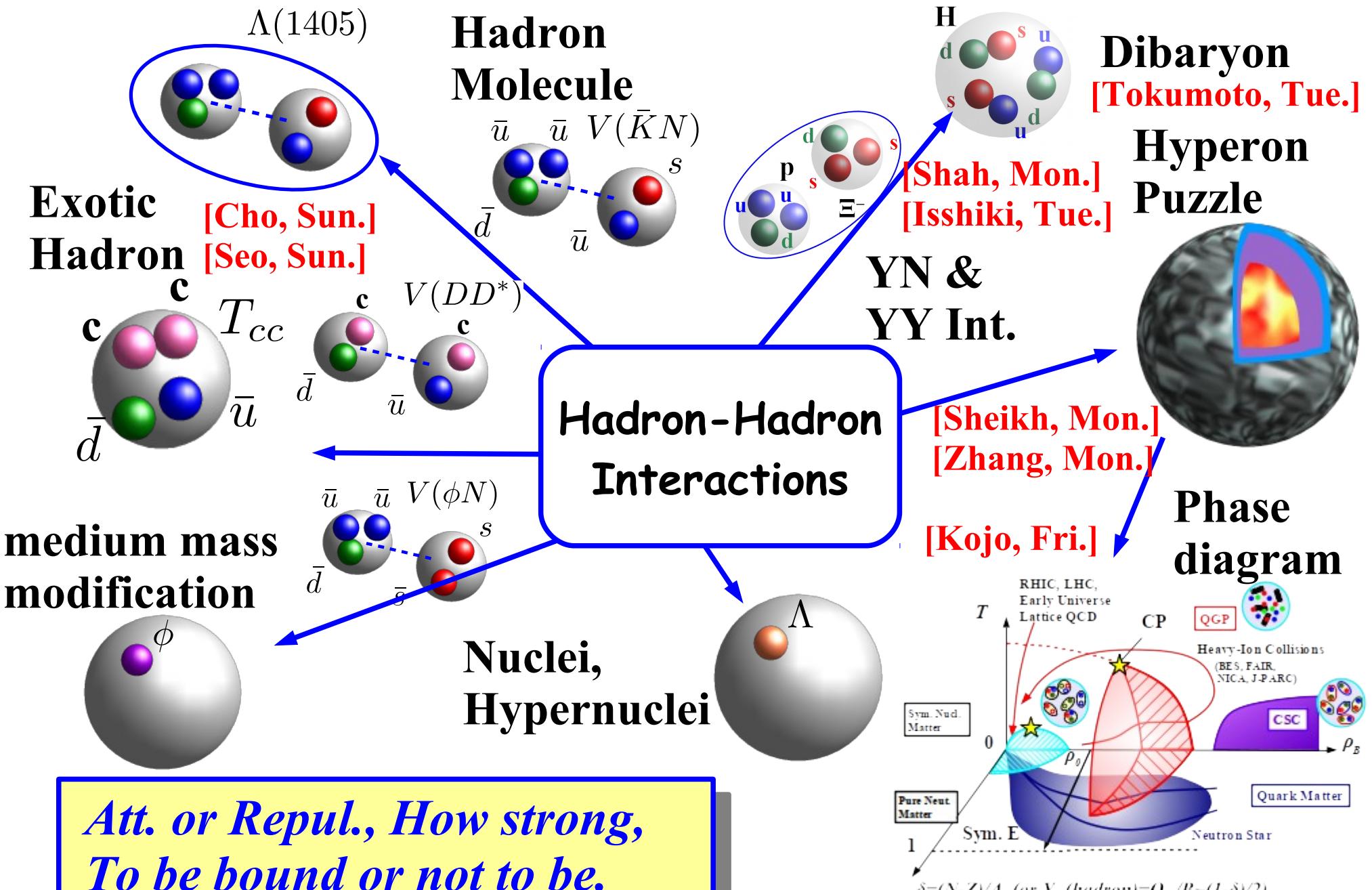
Measured Flavored Hadron CFs (examples)

*Adamczyk+[STAR],
PRL114('15)022301*



*S. Acharya+[ALICE],
PRL124('20)092301*

Hadron-Hadron Interactions: Relevance



Basic formulae in femtoscopic study of hadron-hadron interactions

Lednicky-Lyuboshits (LL) model

- Lednicky-Lyuboshits analytic model
(Asymp. w.f.+eff. range corr.+Gaussian source)
Lednicky, Lyuboshits ('82)

- CF = a known function of $f(q)$, R , r_{eff} , and q .

$$\psi_0(r) \rightarrow \psi_{\text{asy}}(r) = \frac{e^{-i\delta}}{qr} \sin(qr + \delta) = \mathcal{S}^{-1} \left[\frac{\sin qr}{qr} + f(q) \frac{e^{iqr}}{r} \right]$$

$$\begin{aligned} C_{\text{LL}}(q) &= 1 + \int dr S_{12}(r) (|\psi_{\text{asy}}(r)|^2 - |j_0(qr)|^2) \\ &= 1 + \frac{|f(q)|^2}{2R^2} F_3 \left(\frac{r_{\text{eff}}}{R} \right) + \frac{2\text{Re}f(q)}{\sqrt{\pi}R} F_1(2x) - \frac{\text{Im}f(q)}{R} F_2(2x) \end{aligned}$$

($x = qR$, R = Gaussian size, F_1, F_2, F_3 : Known functions)

- Scattering amplitude at low energies

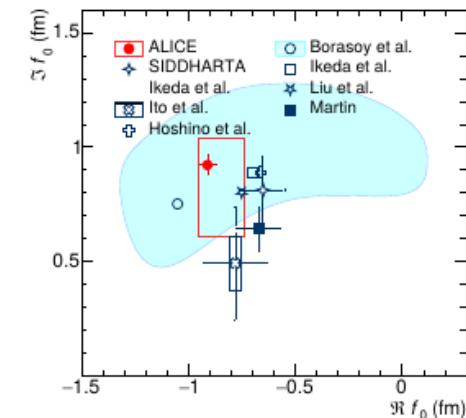
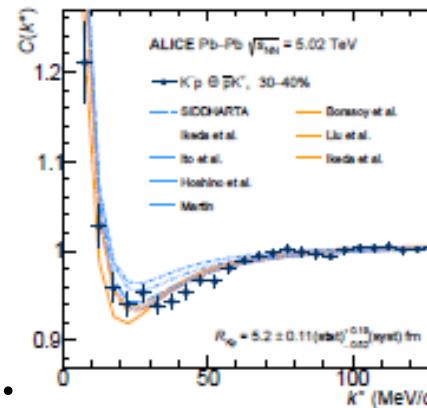
$$q \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 + \mathcal{O}(q^4) \rightarrow f(q) = (q \cot \delta - iq)^{-1}$$

From scattering length (a_0) and effective range (r_{eff}), one can calculate the correlation function !

Scattering length from CFs

K⁻p

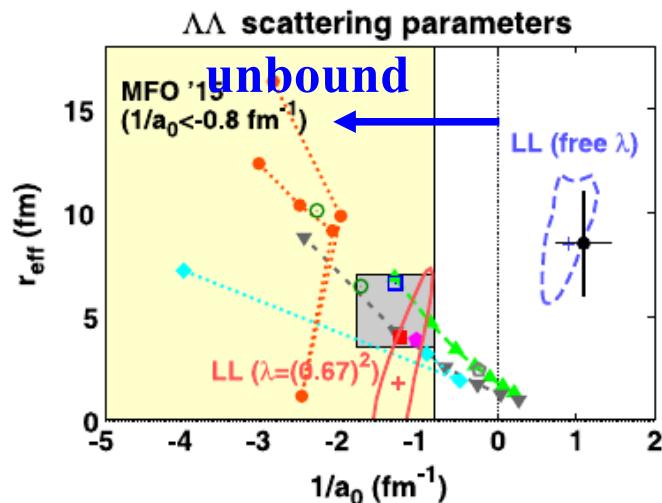
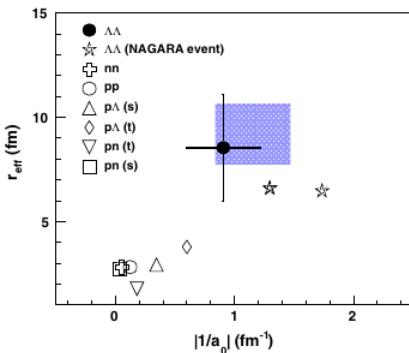
- LL (w/ Coulomb) gives a_0
- Comparable with kaonic atom data (SIDDHARTA).
- See also Siejka+[STAR], NPA982('19)359.



Acharya+[ALICE], PLB822('21), 136708 [2105.05683] ($\delta \sim a_0 q$)

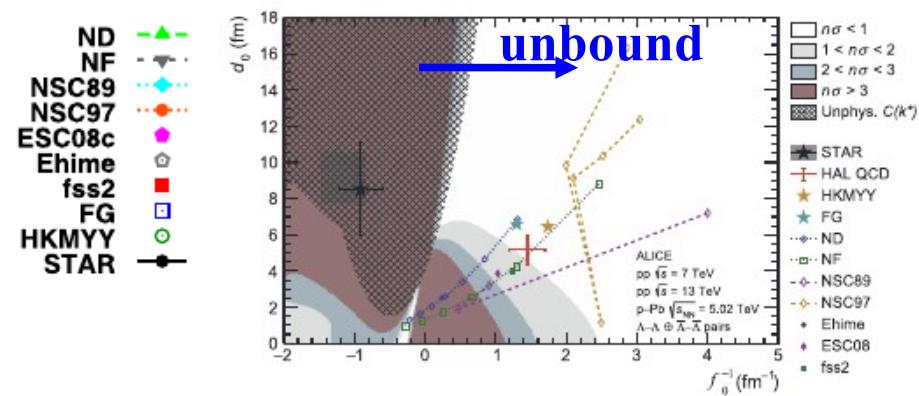
ΛΛ

- Quantum statistics + LL
 $\rightarrow (a_0, r_{\text{eff}})$ region



L. Adamczyk+
[STAR], PRL114 ('15)022301

K.Morita, T.Furumoto, AO, PRC91 ('15) 024916; AO, Morita, Miyahara, Hyodo, NPA954 ('16)294 ($\delta \sim -a_0 q$)



S. Acharya [ALICE], PLB797 ('19) 134822 ($\delta \sim a_0 q$).

Femtoscopic diagnosis of bound state existence

- Lednicky-Lyuboshits model with zero effective range

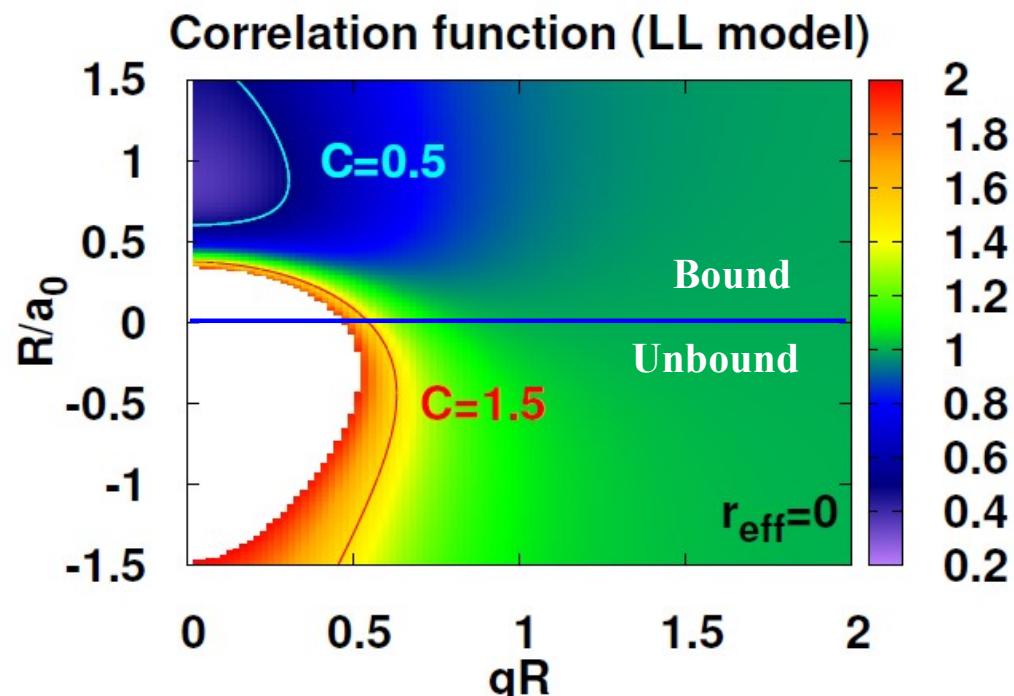
$$\rightarrow \text{CF} = C(qR, R/a_0)$$

$$r_{\text{eff}} = 0 \rightarrow q \cot \delta = -1/a_0 \rightarrow f(q) = (q \cot \delta - iq)^{-1} = -\frac{R}{R/a_0 + iqR}$$

$$C(x, y) = 1 + \frac{1}{x^2 + y^2} \left[\frac{1}{2} - \frac{2y}{\sqrt{\pi}} F_1(2x) - xF_2(2x) \right] \quad (x = qR, y = R/a_0)$$

$$= \frac{1}{2} \left(\frac{1}{y} - \frac{2}{\sqrt{\pi}} \right)^2 + 1 - \frac{2}{\pi} \quad (F_1 \rightarrow 1, F_2 \rightarrow 0 \text{ at } x \rightarrow 0)$$

- With a bound state, CF is suppressed at low q.
- Scattering w.f. needs to have a node to be orthogonal to b.s.

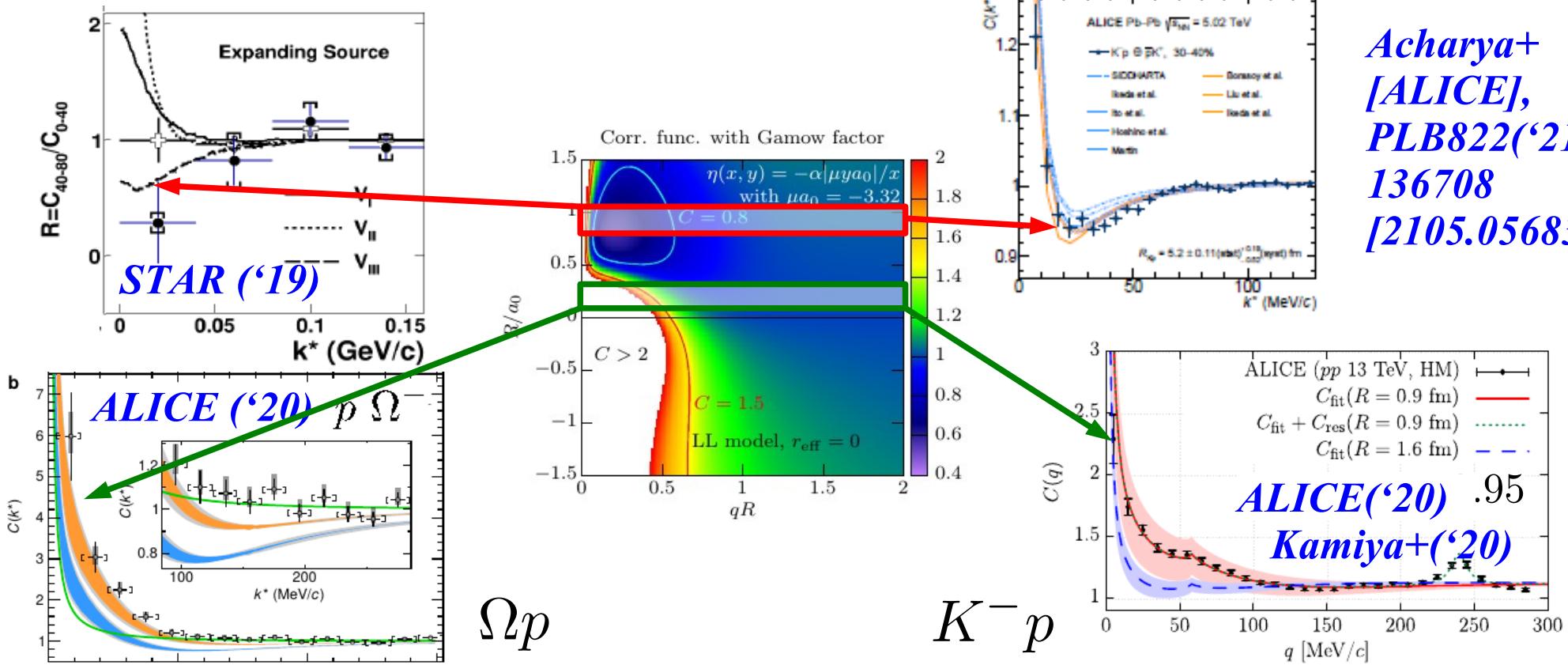


E.g. AO, Morita, Miyahara, Hyodo ('16)

Hadronic molecules suggested by CFs

■ Ωp and $K^- p$

- Bound states are expected.
Dalitz, Tuan ('59); Akaishi, Yamazamki ('02); Goldman+('87); Oka ('88); Etminan+[HAL QCD] ('14); Iritani+[HAL QCD] ('19).
- Dip is expected at $R \sim |a_0|$ *Morita+('16, '20); Kamiya+('20); Haidenbauer('18)*
- Data support the existence of a BS.



Coupled-Channel Effects

■ Correlation Function with Coupled-Channel Effects

- Koonin-Pratt-Lednicky-Lyuboshits-Lyuboshits (KPLLL) formula
Coupled-channel contributions with $\psi(-)$ boundary cond.
Lednicky, Lyuboshits, Lyuboshits, Phys. Atom. Nucl. 61 (1998), 2950;
J. Haudenbauer, NPA981('19) 1 [1808.05049];
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL('20).

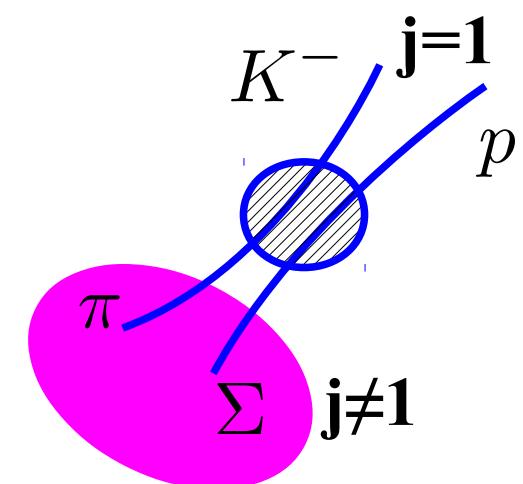
$$C(\mathbf{q}) = 1 - \int d\mathbf{r} S_1(r) |j_0(qr)|^2 + \int d\mathbf{r} \sum_j \omega_j S_j(r) |\psi_j^{(-)}(q; r)|^2$$

$$\psi_{j=1}(r) \rightarrow [e^{iqr} + A_1(q)e^{-iqr}]/2iqr \quad (\omega_1 = 1)$$

$$\psi_{j \neq 1}(r) \rightarrow A_j(q)e^{-iqr}/2iqr \quad [\Psi^{(-)} \text{ boundary condition}]$$

(No Coulomb case)

- Wave functions of other channels also contribute to correlation functions.
- Source size R and weight ω_j ($j \neq 1$) are taken as the parameter.

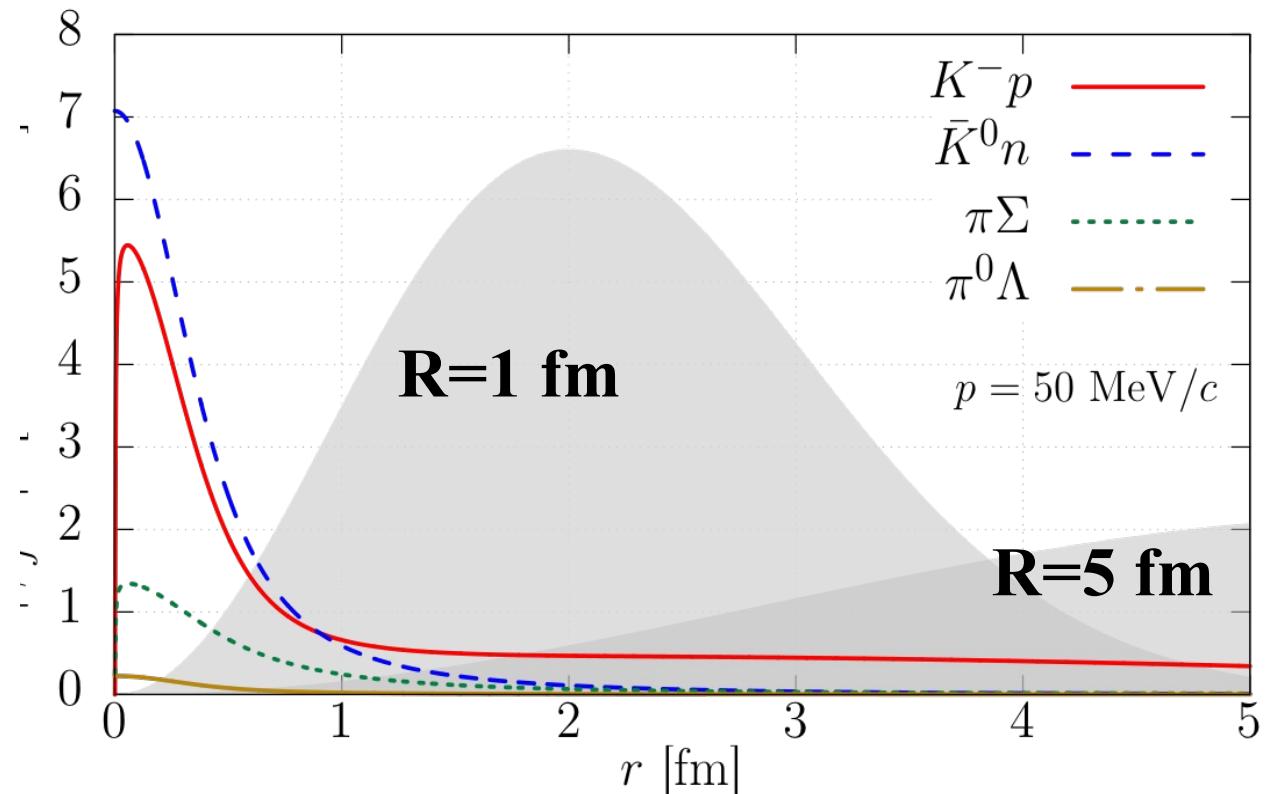


$\omega_j S_j(r) |\psi_j^{(-)}(q; r)|^2$
— — —
Source weight Normalized Source fn.

Discriminating Coupled-Channel Effects

■ Source size dependence again !

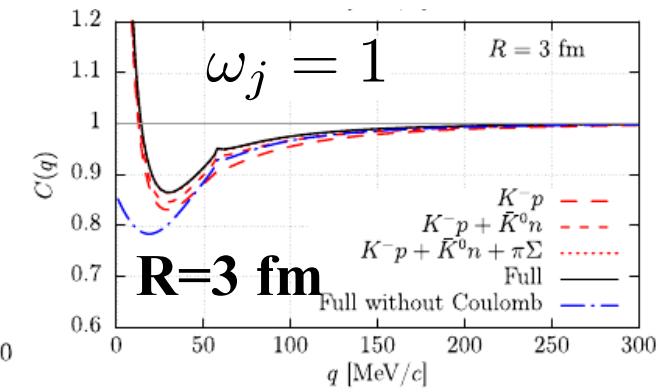
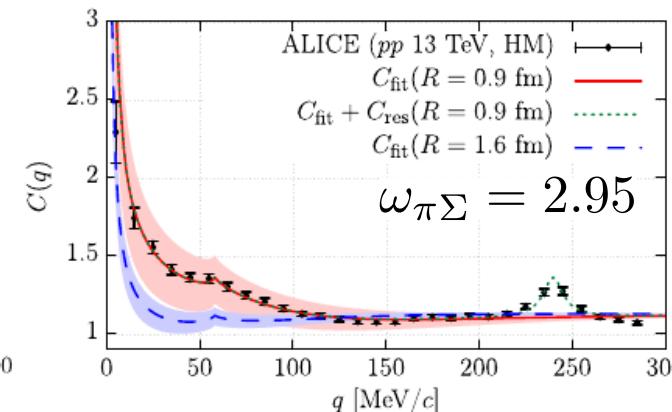
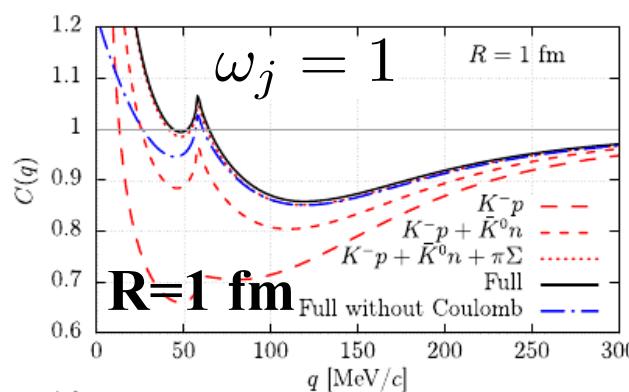
- Unmeasured coupled-channel wave functions disappear soon.
→ CFs with large source is dominated
by the measured channel wave function !
- Scattering parameters from CFs with large source
Coupled-channel effects from CFs with small source.



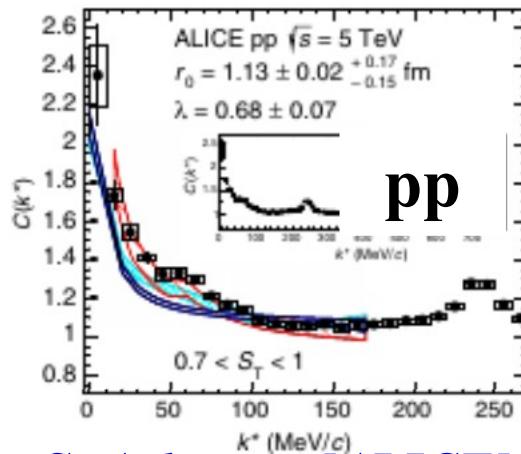
w.f. Kamiya+, arXiv:1911.01041v1

Source Size Dependence of $C(K^- p)$

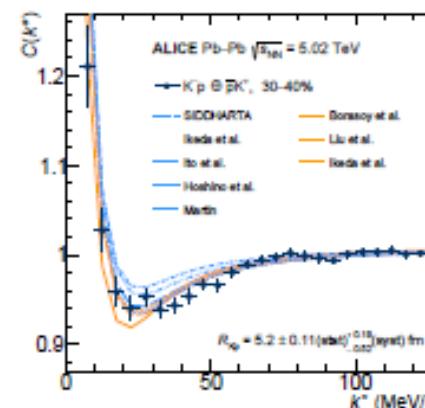
- Coupled-channel effects are suppressed when R is large, and “pure” $K^- p$ wave function may be observed in HIC.



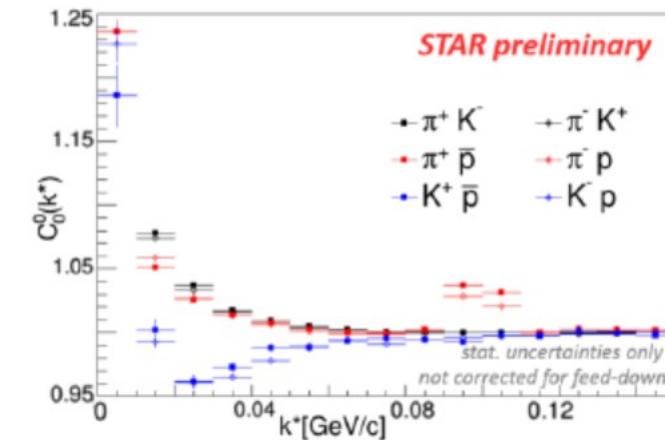
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.



*S. Acharya+[ALICE],
PRL124('20)092301*



*S. Acharya+[ALICE],
PLB822('21)136708*



*Siejka+[STAR, preliminary],
NPA982 ('19)359.*

Recent & Future Femtoscopic Studies of HHI

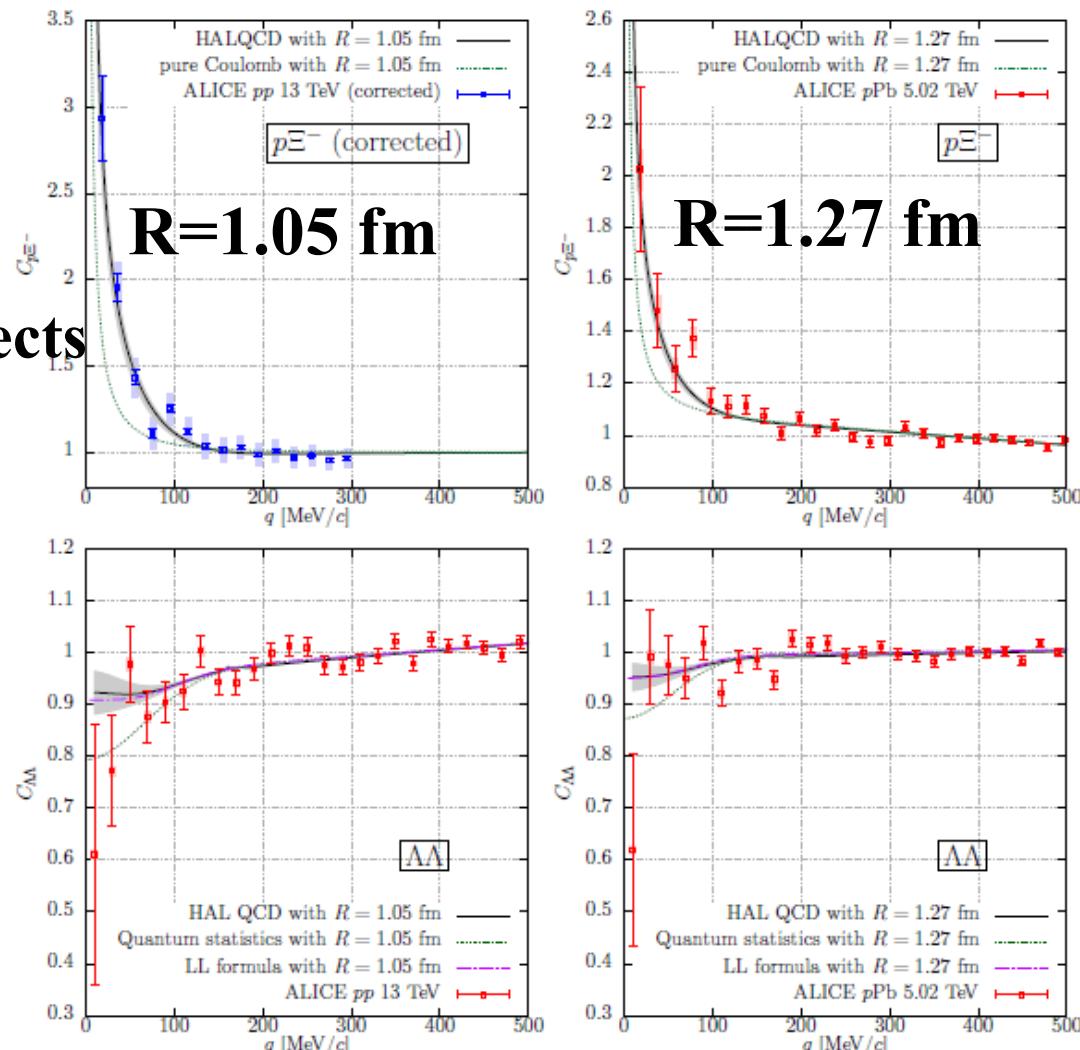
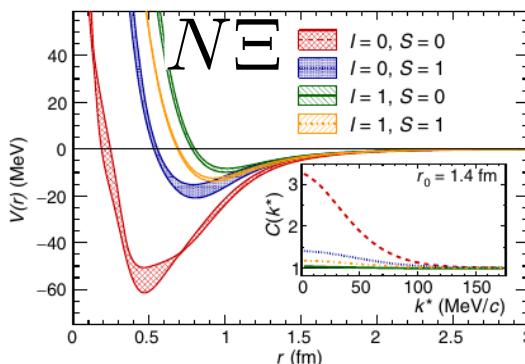
Ξ^-p & $\Lambda\Lambda$ correlation functions (pp and pA)

■ Correlation function data from pp and pA collisions

S. Acharya et al. [ALICE], PLB 797('19)134822 ($\Lambda\Lambda$); PRL123('19)112002 (Ξ^-p from pA); Nature 588('20)232 (Ξ^-p from pp).

- CF(Ξ^-p) is enhanced at low q . → Att. pot.
- CF($\Lambda\Lambda$) is slightly enhanced from quantum stat. result. → Weakly attractive pot.
- CFs with coupled-channel effects using lattice QCD potential explains the data well.

*K. Sasaki [HAL QCD]('20);
Y.Kamiya+, arXiv:2108.09644.*



Ξ^- -p & $\Lambda\Lambda$ correlation functions (AA)

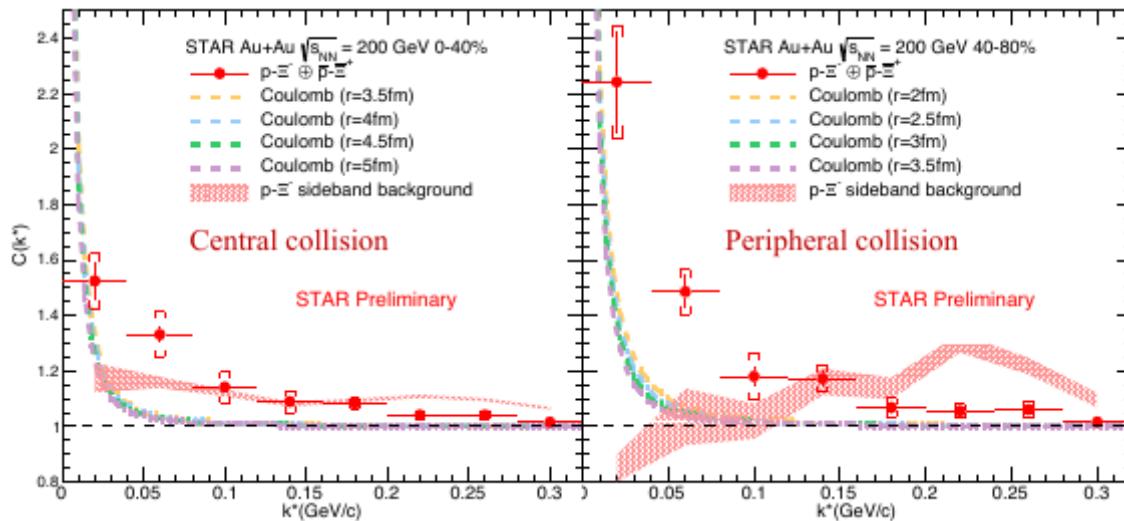
■ Correlation function data from AA collisions

[c.f. Shah, Mon., Isshiki, Tue.]

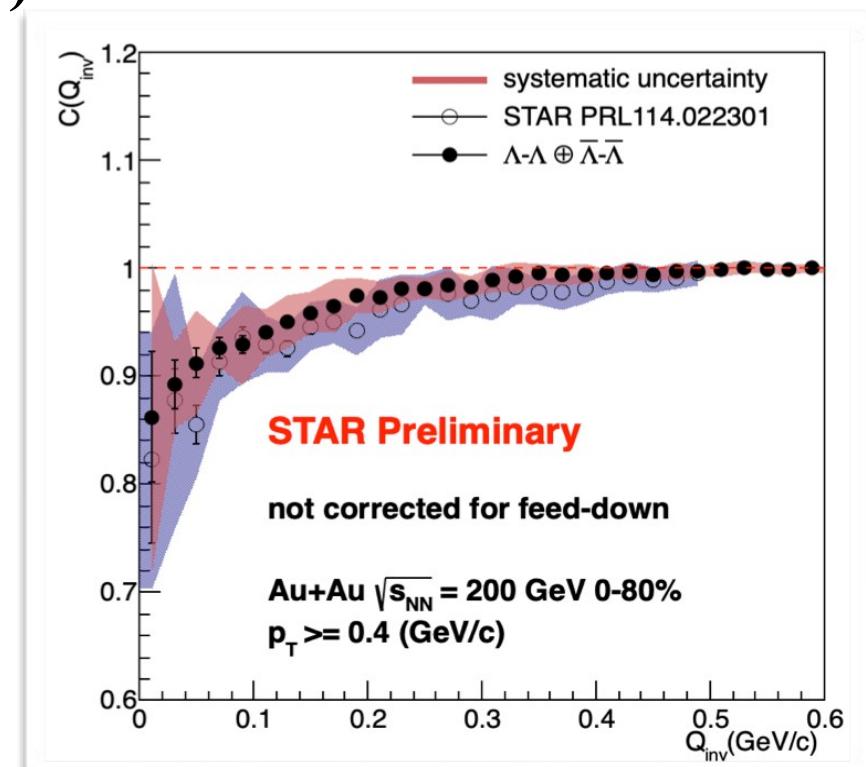
K. Mi+(STAR, preliminary), Au+Au 200 AGeV, APS2021.

Moe Isshiki+ (STAR, preliminary), Strangeness physics workshop, 2021.

- We do not see a dip in $C(\Xi^- p)$ from Au+Au.
→ There will be no bound state of $\Xi^- p$.
- Much higher statistics data of $C(\Lambda\Lambda)$ from Au+Au are obtained.
→ LL formula fit will be possible.



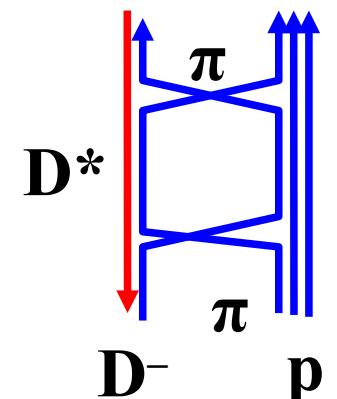
K. Mi+(STAR, preliminary),
Au+Au 200 AGeV, APS2021.
(No Dip at larger R)



Moe Isshiki+ (STAR, preliminary).

Charmed Hadron Interactions

- Charmed hadron interactions are extremely important in exotic hadron physics.
- Example: D⁻ p correlation
 - Proposed potentials generally predict weak or repulsive interaction.
Hofmann, Lutz ('05) (repulsive); Haidenbauer+('07) (repulsive); Yamaguchi+('11) (att., w/ bs); Fontoura+('13) (repulsive)
 - Attraction from pion exchange
S. Yasui, K. Sudoh, PRD80('09)034008.



Model	$a_{DN}^{l=0}$	$a_{DN}^{l=1}$	$a_{\bar{D}}$	
SU(4) contact [185]	-0.16	-0.26	-0.24	1. <i>Hoffmann, Lutz ('05)</i>
Meson exchange [194]	0.07	-0.45	-0.32	2. <i>Haidenbauer+ ('07)</i>
Pion exchange [192]	-4.38	-0.07	-1.15	3. <i>Yamaguchi+('11)</i>
Chiral quark model [219]	0.03–0.16	-(0.20–0.25)	-(0.14–0.15)	4. <i>Fontoura+('13)</i>

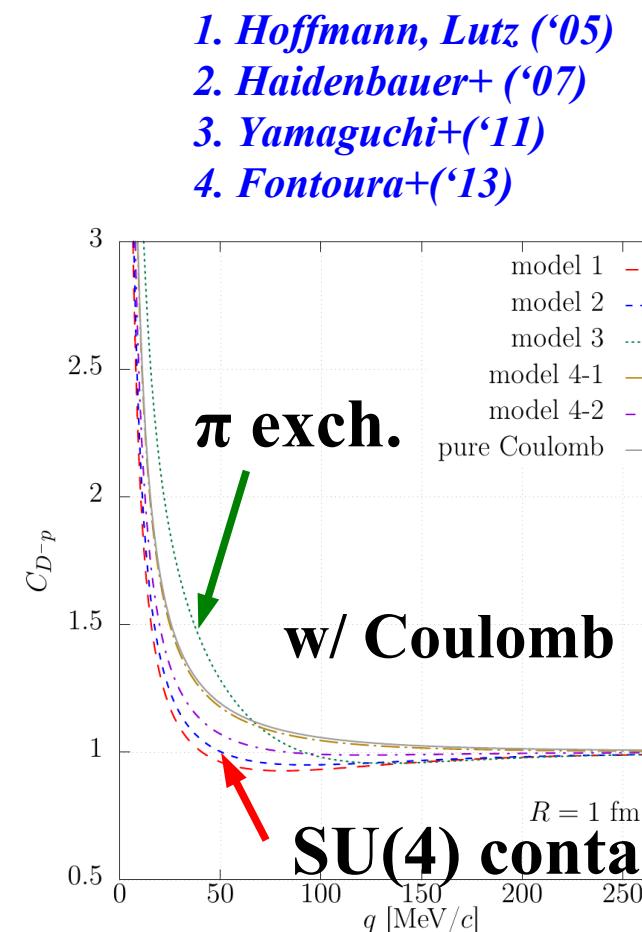
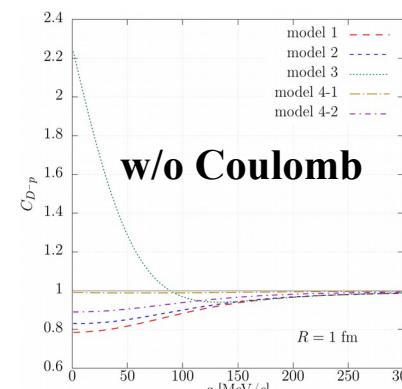
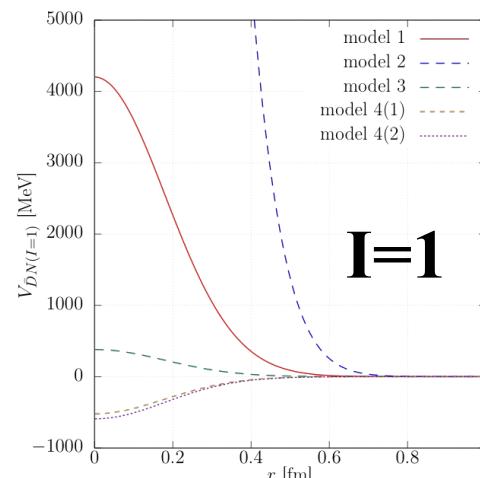
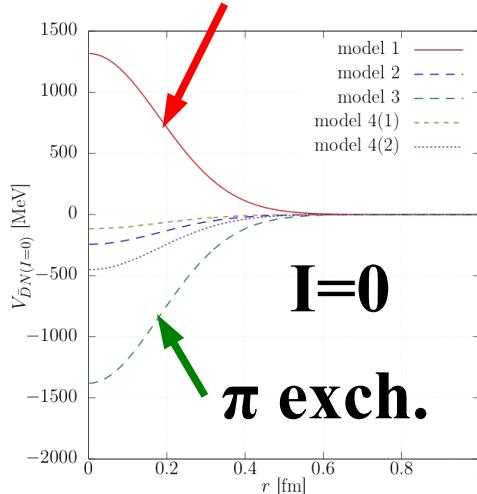
Hosaka, Hyodo, Sudoh, Yamaguchi, Yasui, PPNP96('17)88

Which picture is correct ?

Charmed Hadron Interactions

- D⁻ p CFs from proposed scattering length *Kamiya, Hyodo, AO (in prog.)*
 - One-range Gaussian potential strength is fitted to proposed a₀ with the range of ρ meson exchange.
 - Measurable difference is found.

SU(4) Contact

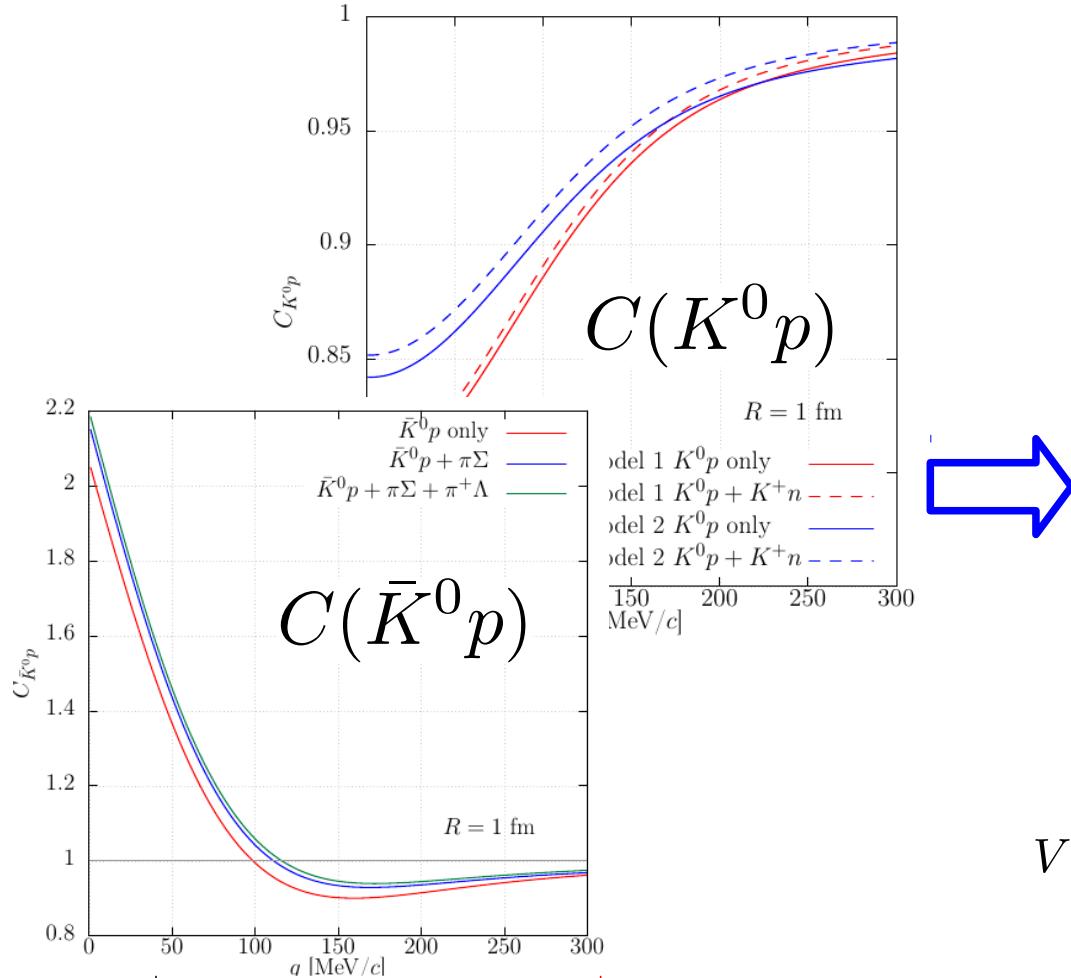


Data will discriminate
these potentials !

Ks p correlation function

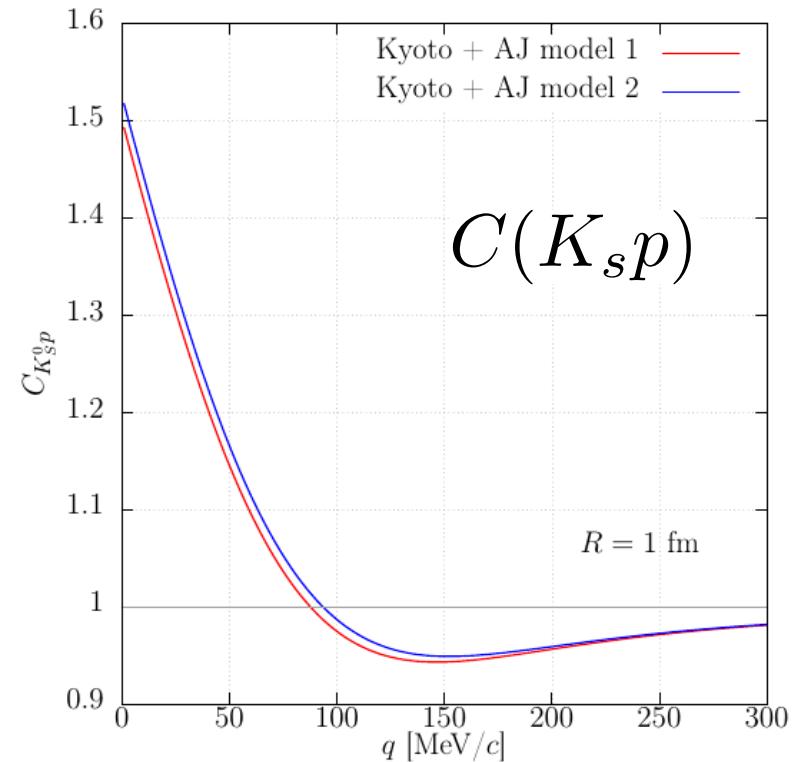
■ Ks wave function *Kamiya+ (in prep.)*

$$|K_s\rangle = \frac{1}{\sqrt{2}} [|K^0\rangle + |\bar{K}^0\rangle] \rightarrow C(K_s p) = \frac{1}{2} [C(K^0 p) + C(\bar{K}^0 p)]$$



$K^- p, K^+ p, K_s p$, and $K^- n$ or $K^+ \bar{n}$ (missing)

to obtain KN and $\bar{K}N$ interactions with $I = 0, 1$

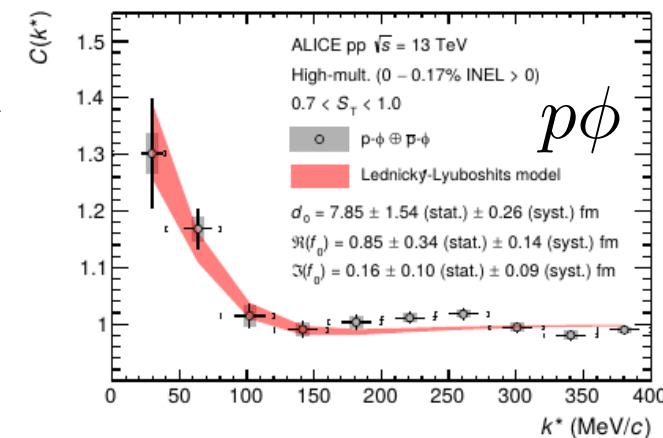
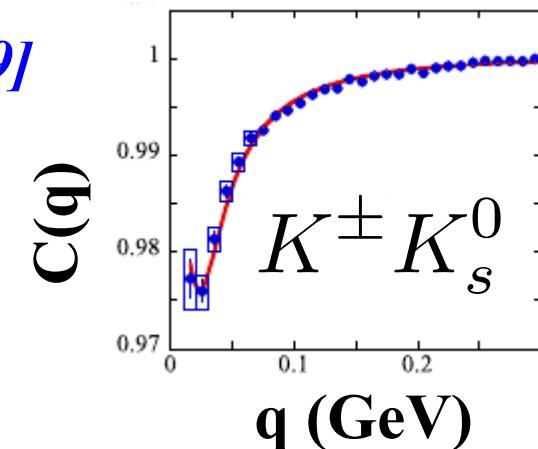


$V(\bar{K}N)$: Chiral SU(3) (Kyoto-Münich) model
(Miyahara, Hyodo, Weise ('18))

$V(KN)$: Chiral SU(3) based model
(Aoki, Jido ('19))

Other Correlation Functions

- $\bar{p}\bar{p}, p\bar{\Lambda}$ *E.g. A. Kisiel [ALICE], Acta Phys.Polon.Supp. 6 ('13)519*
- $K^\pm K_s^0$ *S.Acharya+ [ALICE], PLB774 ('17)64 [1705.04929]*
 - Slightly suppressed at low q
 - Tetraquark component of a_0 meson
- $p\phi$ *Acharya+[ALICE], PRL127('21)172301[2105.05578]*
 - Attractive potential which may be responsible to medium mass reduction
- $p\bar{\Lambda}, \Lambda\bar{\Lambda}$ [2105.05190],
 $p\Sigma^0$ ['20 [1910.14407]] [ALICE]
- deuteron-hadron CF
S. Mrówczyński and P. Słoń,
Acta Phys.Polon.B51('20)1739 [1904.08320];
F. Etminan, M. M. Firoozabadi, [1908.11484];
J. Haidenbauer, PRC102('20)034001 [2005.05012];
K.Ogata, T.Fukui, Y.Kamiya, AO [2103.00100].



*Acharya+[ALICE],
PRL127 ('21) 17,
172301[2105.05578]*

Summary

- Correlation function from heavy-ion collisions (including high-energy pp and pA collisions) are useful to access hadron-hadron interactions.
 - One can obtain the basic properties of HHIs such as Attractive or Repulsive, Strength, To be bound or not to be.
 - Source size dependence is useful in deducing the existence of a bound state and in discriminating the coupled-channel effects.
- We are sorry for using a static Gaussian source functions.
 - Flavored hadron CFs are measured in 1D (as a function of $|q|$), m_T dependence is not yet discussed, and errors are still large.
 - Effects of flow, elongated shape, and others are expected to be renormalized in changing the source size **at present**.
- Many hadron physicists are waiting for your data.

Thank you for your attention !

HBT puzzle

■ Calculating HBT radius in dynamical models is not easy.

*M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357
[nucl-ex/0505014]; S. Pratt, PRL102('09)232301 [0811.3363].*

choices then tends to exceed the number of experimental constraints. In fact, all **Lisa+('05)** the model results that we review in the current subsection remain unsatisfactory with this respect: **They either deviate significantly from femtoscopic data, or they reproduce these data at the price of missing other important experimental information.** In particular, there is so far no dynamically consistent model that reproduces quantitatively both the systematic trends discussed in Section 4 and the corresponding single inclusive spectra. In this situation, the scope of this subsection is

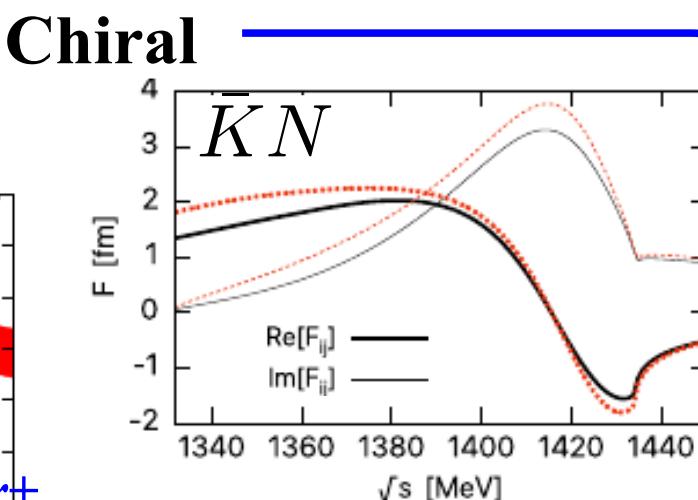
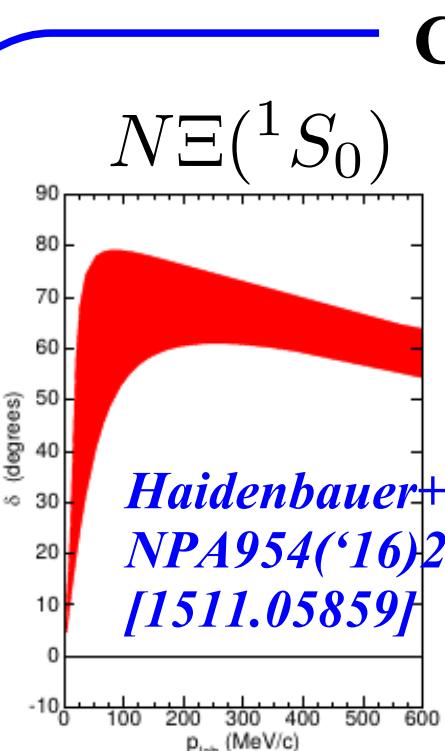
Pratt ('09)

Two particle correlation data from the BNL Relativistic Heavy Ion Collider have provided detailed femtoscopic information describing pion emission. In contrast with the success of hydrodynamics in reproducing other classes of observables, these data had avoided description with hydrodynamic-based approaches. This failure has inspired the term “HBT puzzle,” where HBT refers to femtoscopic studies which were originally based on Hanbury Brown–Twiss interferometry. **Here, the puzzle is shown to originate not from a single shortcoming of hydrodynamic models, but the combination of several effects:** mainly prethermalized acceleration, using a stiffer equation of state, and adding viscosity.

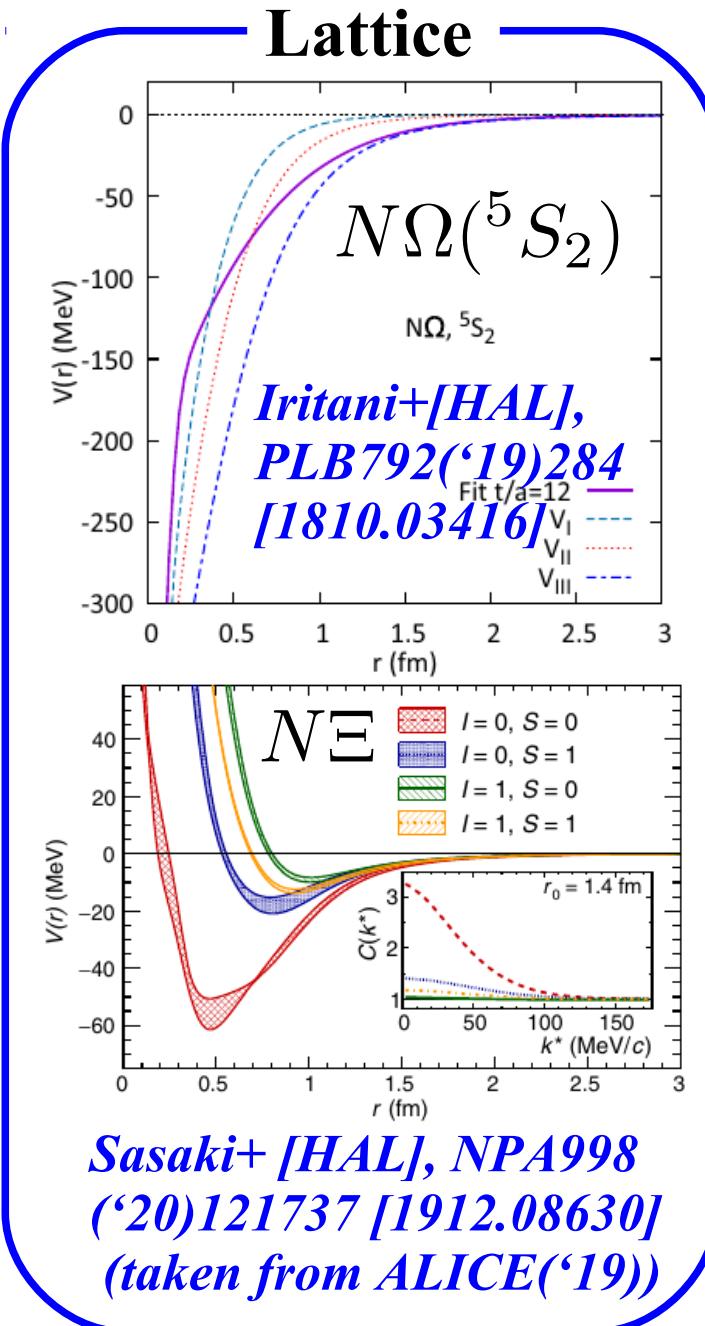
How about afterburner effects ?

Theoretical Approaches to HHI

- Nuclear force models: meson exch., quark model, ... (need data)
- *Ab initio*: chiral EFT, lattice QCD (need data or CPU resources)



Miyahara, Hyodo,
Weise, PRC98('18),
025201 [1804.08269]
(Ikeda-Hyodo-Weise
amplitude)



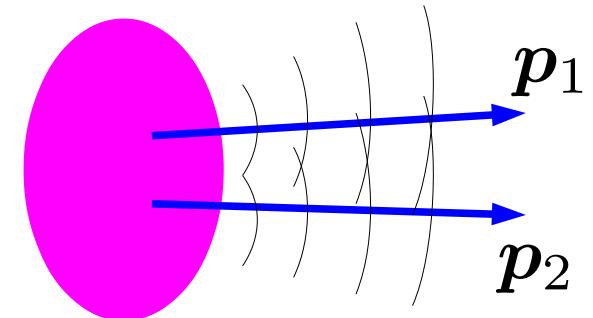
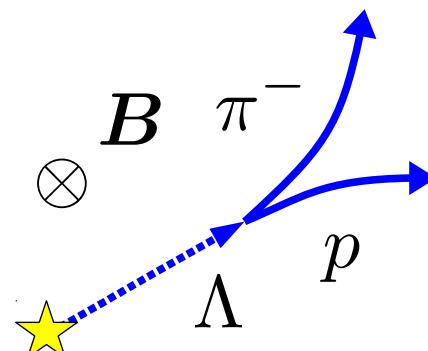
Experimental Approaches to HHI

■ Experimental approaches

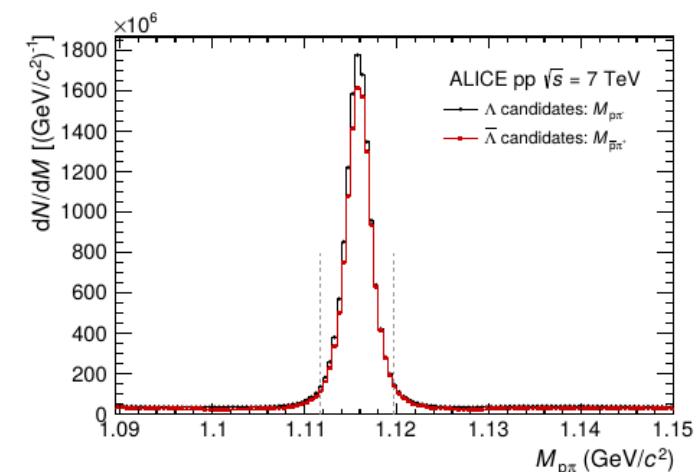
- hh scattering (NN, YN, π N, KN)
- Hadronic nuclei (normal nuclei, hypernuclei, kaonic nuclei)
- Hadronic atom (π^- , K $^-$, Σ^- , Ξ^- , ...)
- Femtoscopy

■ Femtoscopic study of hh interactions

- Applicable to various hh pairs (NN, YN, KN, DN, YY, Yd, YNN ...)
- Valid when the source is chaotic
- Weakly decaying particles
→ Good pair purity
- Future measurements:
Charmed hadron, hNN, ...



$$C(p_1, p_2) = \frac{N_{12}(p_1, p_2)}{N_1(p_1)N_2(p_2)}$$



ALICE [1805.12455]

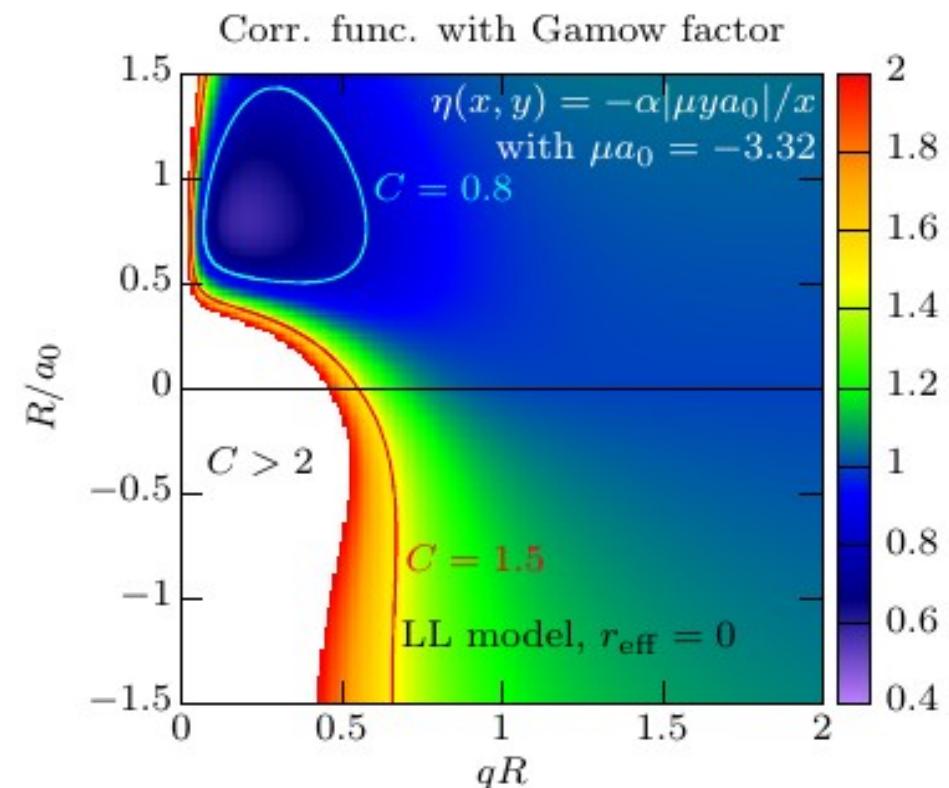
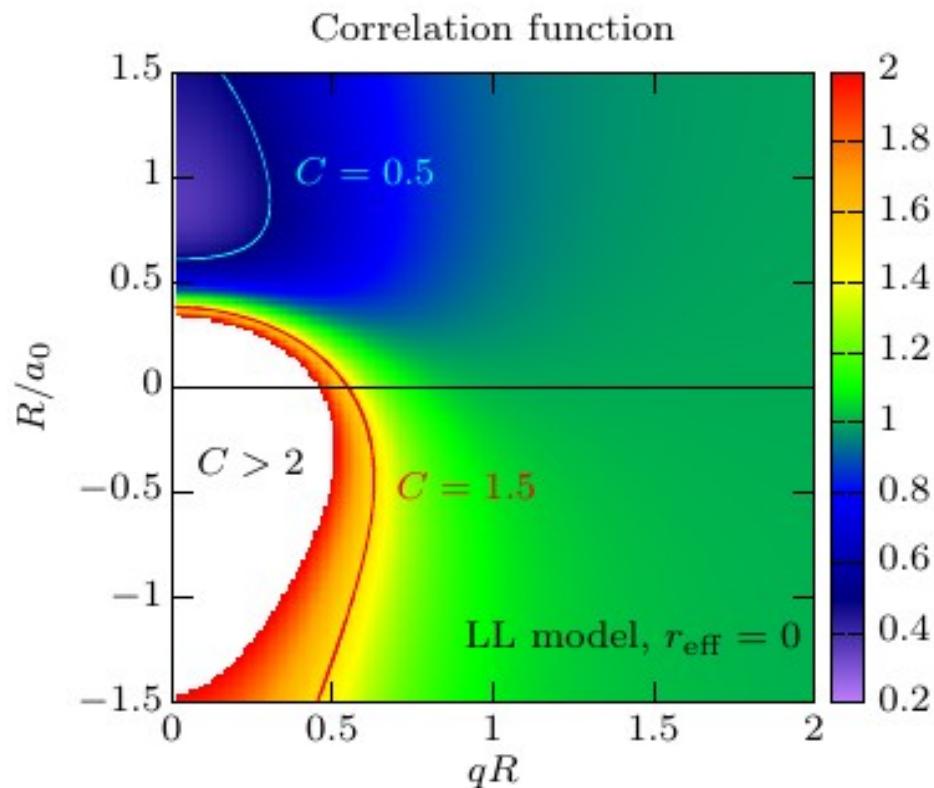
Femtoscopy Diagnosis of Bound State

- With Coulomb attraction, a bound state causes a dip.

- E.g. LL formula and the Gamow factor

$$C(q) \rightarrow C_{\text{strong}}(q) \times A_{\text{Gamow}}(\eta)$$

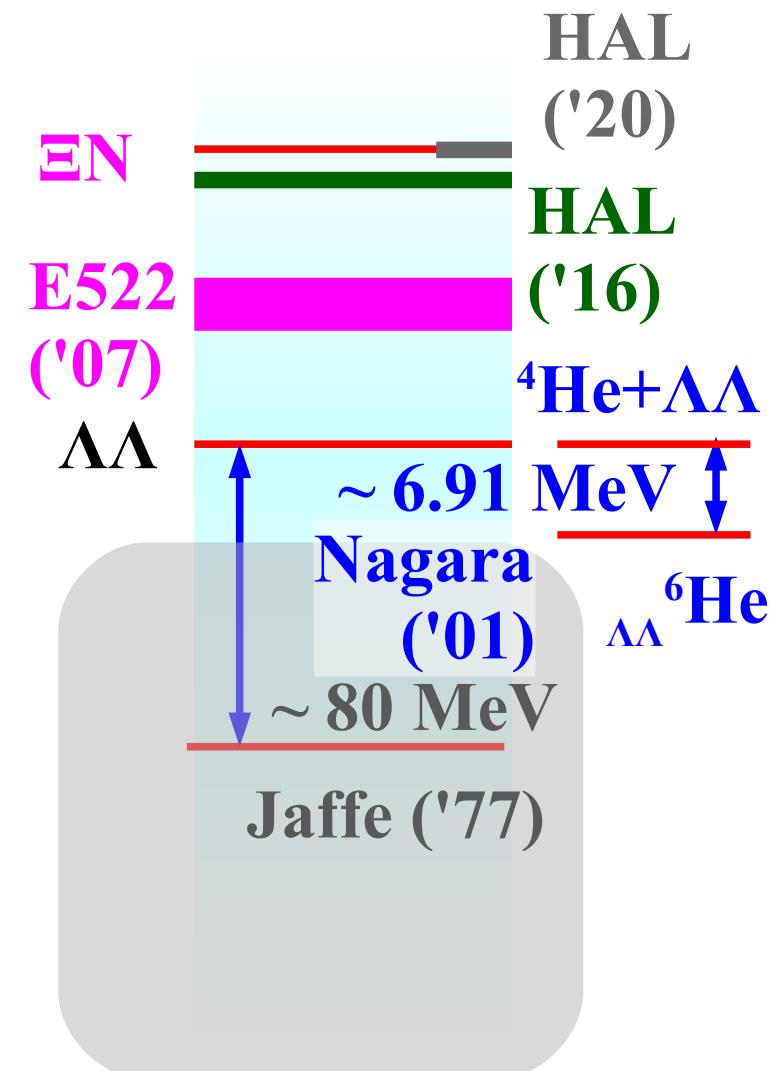
$$A_{\text{Gamow}}(\eta) = 2\eta / (\exp(2\eta) - 1) \quad (\eta = Z_1 Z_2 \mu |ya_0|/x)$$



Kamiya+('21), 2108.09644

H dibaryon state, to be bound or not to be bound ?

- H-dibaryon: 6-quark state (uuddss)
 - Prediction: *R.L.Jaffe, PRL38(1977)195*
 - Ruled-out by double Λ hypernucleus
Takahashi et al.,PRL87('01) 212502
 - Resonance or Bound “H” ?
Yoon et al.(KEK-E522)+AO ('07)
- Lattice QCD results
 - Bound (below $\Lambda\Lambda$ threshold):
HALQCD('11), NPLQCD('11,'13), Mainz('19)
(heavier quark mass or SU(3) limit)
 - Resonance (Bound state of $N\Xi$):
HAL QCD ('16,18) (HAL preliminary)
 - Virtual Pole (around $N\Xi$ threshold)
HAL QCD ('20) (almost physical m_q)



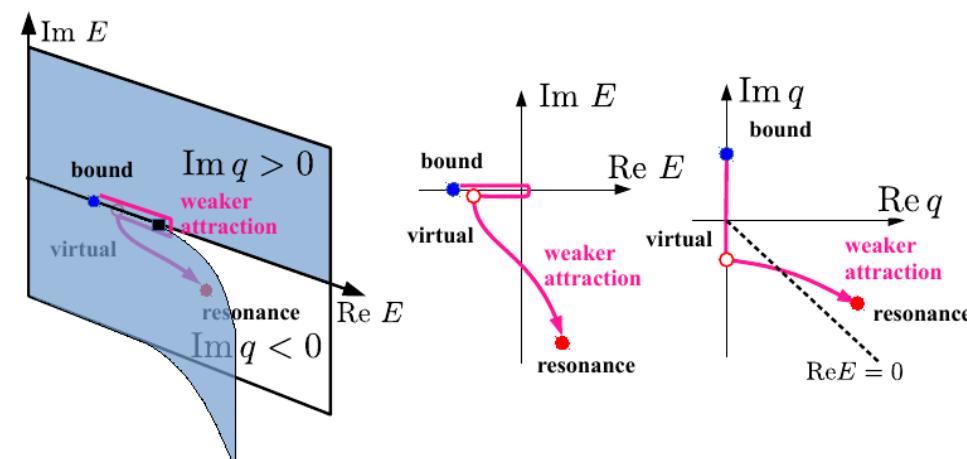
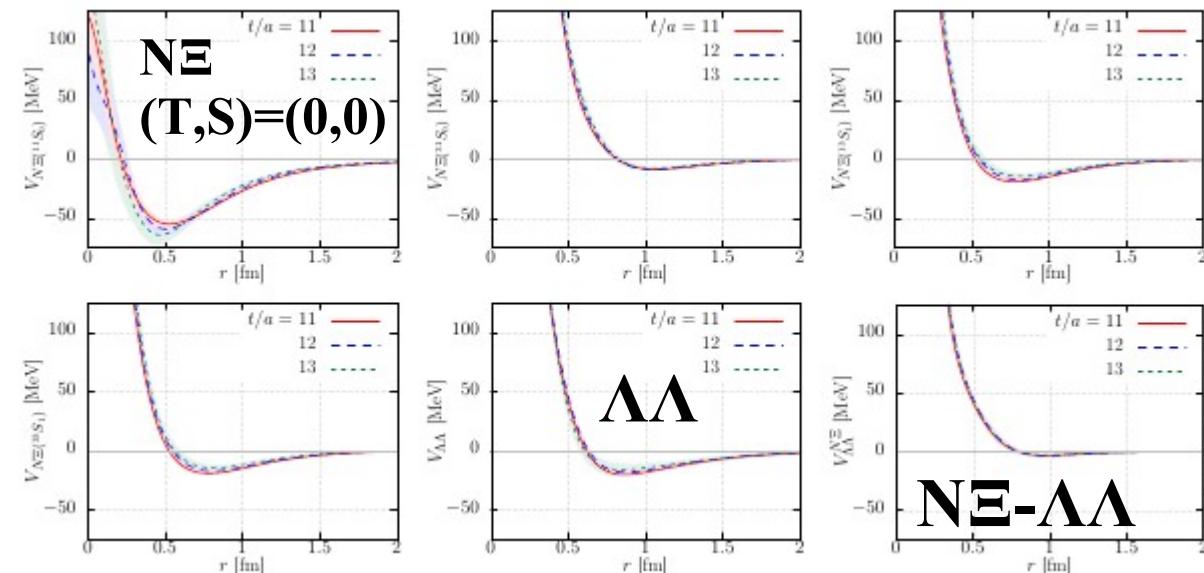
We examine LQCD $N\Xi$ - $\Lambda\Lambda$ potential and discuss H using CF !

NΞ-ΛΛ potential from Lattice QCD

- NΞ-ΛΛ potential at almost physical quark mass ($m_\pi = 146$ MeV) by HAL QCD Collaboration

K. Sasaki et al. [HAL QCD Collab.], NPA 998 ('20) 121737 (1912.08630)

- Strong attraction in $(T,S)=(0,0)$ of NΞ
- Weak attraction in ΛΛ (Coupling with NΞ causes ΛΛ attraction)
- There is no bound state in NΞ-ΛΛ system (except for Ξ^- atom), but there is a virtual pole around the NΞ threshold (3.93 MeV below $n\Xi^0$ threshold) on the irrelevant Riemann sheet, $(+, -, +)$ [relevant= $(-, +, +)$] (sign of Im (eigenmomentum))

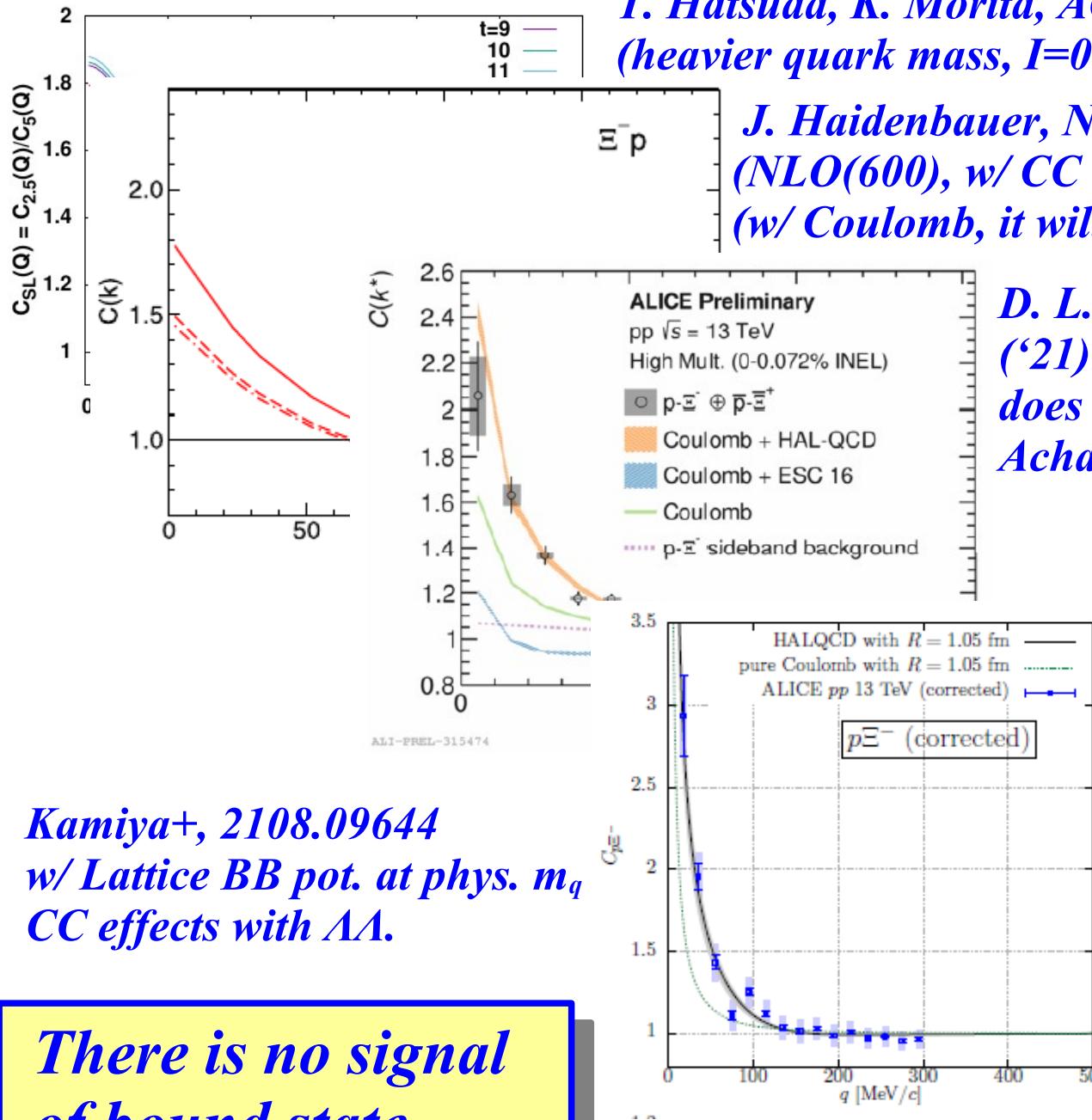


$\Xi^- p$ correlation function

T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17)856.
(heavier quark mass, $I=0$ only, w/o CC effects)

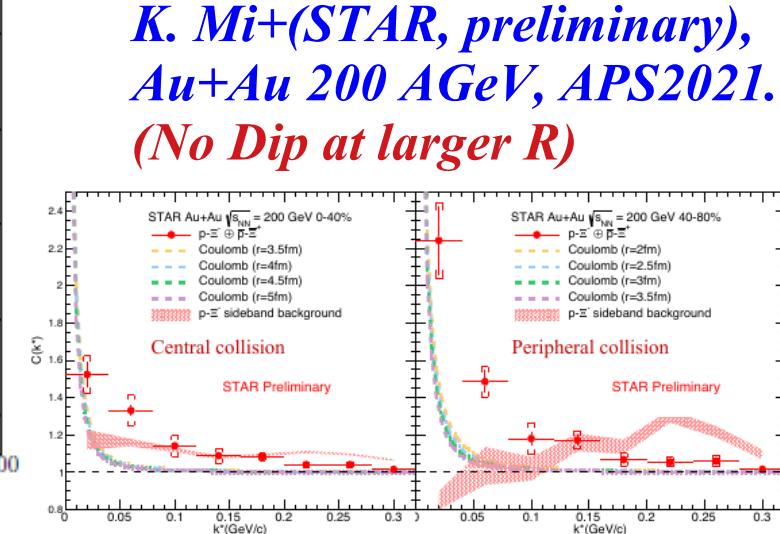
J. Haidenbauer, NPA981('19)1.
(NLO(600), w/ CC effects, w/o Coulomb)
(w/ Coulomb, it will be comparable with data.)

D. L. Mihairov+[ALICE], NPA 1005
('21)121760 (QM2019). (Nijmegen pot.
does not explain the data. w/o CC)
Acharya+(ALICE), Nature ('20)

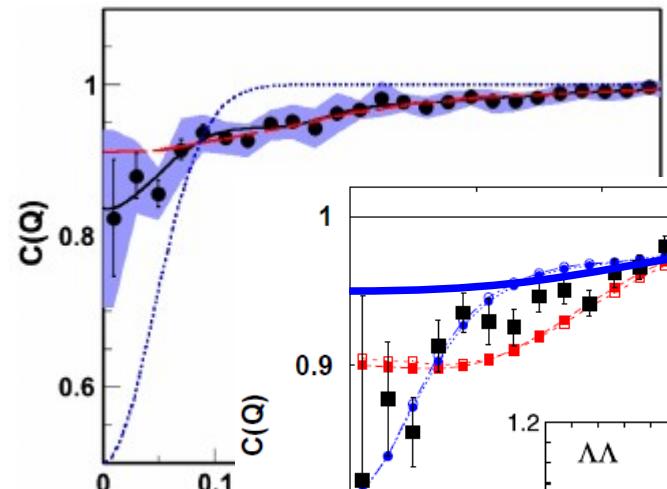


Kamiya+, 2108.09644
w/ Lattice BB pot. at phys. m_q
CC effects with AA.

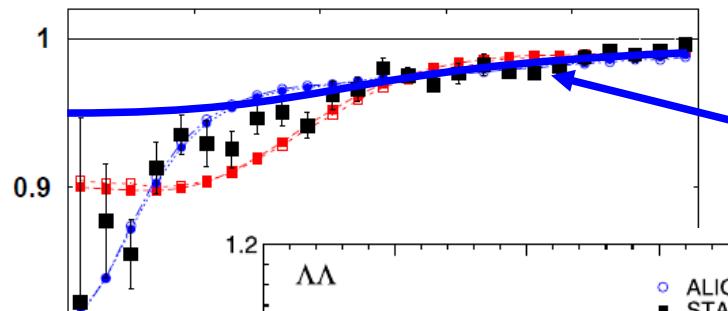
**There is no signal
of bound state.**



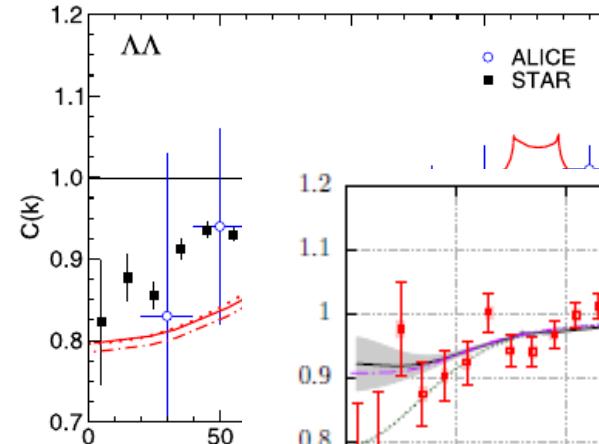
$\Lambda\bar{\Lambda}$ correlation function



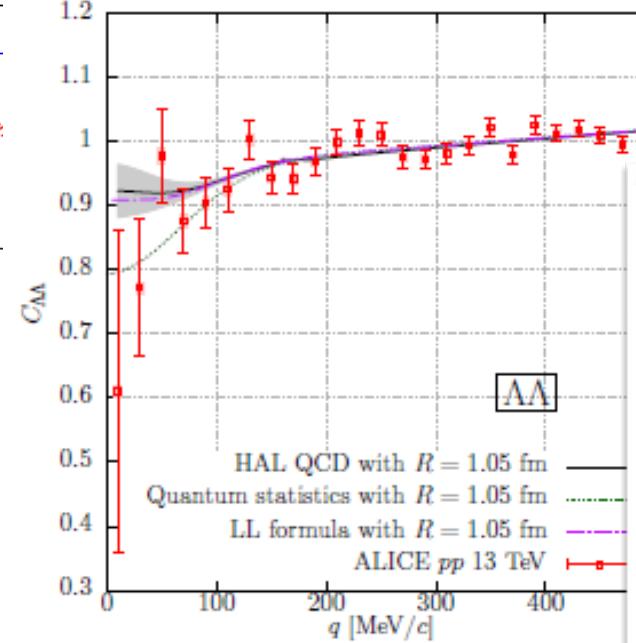
*Adamczyk+[STAR], PRL114('15)022301
(Residual source $R \sim 0.5$ fm was assumed.)*



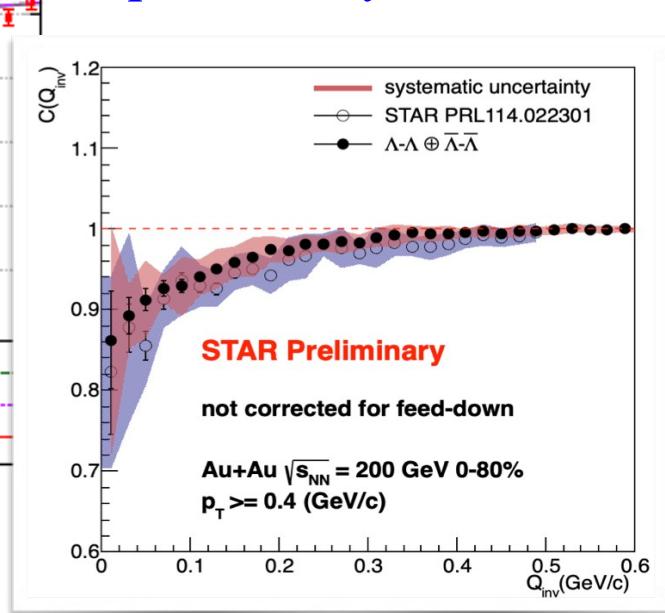
*Morita, Furumoto, AO, PRC91('15)
024916. (Res.Source + flow)*



*J. Haidenbauer, NPA981('19)1.
(NLO600)*



*Kamiya+, 2108.09644
w/ Lattice BB pot. at phys. m_q
CC effects with $\Lambda\Lambda$.*



Scope of Femtoscopic study of HHI

pK^-
*Chiral CC pot.
(examined)
Bound state
(favored)*

$p\Xi^-$
*Lattice QCD CC
pot. (examined)
Bound state
(disfavored)*

$p\Omega$
*Lattice QCD pot.
J=2 (examined)
Bound state
(favored)*

$\Lambda\Lambda$
*Scattering pars. (a_0 , r_{eff})
(constrained)
Bound state (disfavored)*

	n	p	K^-	K^+	π^-	π^+	Λ	Σ	Ξ^-	Ω^-	D^-	D^+	K_s	$+a$
n														
p		O	O	O	Δ	Δ	O	O	O	O	O	O	O	
K^-		O	O	O	O	O							O	
K^+		O	O	O	O	O							O	
π^-		Δ	O	O	O	O								
π^+		Δ	O	O	O	O								
Λ		O								O				
Σ		O												
Ξ^-			O											
Ω^-				O										
D^-					O									
D^+						O								
K_s							O	O						
$+a$														



pD^\pm
Chamed hadron-nucleon interaction (work in prog.)

$K^\pm K_s^0$
Tetraquark component in a_0 meson

Other bound states ?

■ $\Lambda\Lambda$ - $N\Xi$

- $C_{\Lambda\Lambda}(q)$ in AA(RHIC) and pp(LHC) are similar (No b.s. below $\Lambda\Lambda$).
- LQCD predicts a virtual pole near $N\Xi$ threshold, which can be detected as the cusp in $C_{\Lambda\Lambda}(q)$.
NLO(600) potential predicts the same.
(The fate of H particle)
K. Sasaki+[HAL QCD], NPA998('20)121737;
Y. Kamiya+, 2108.09644; Haidenbauer('19).

■ $K^{\bar{b}}N$

- $\Lambda(1405)$ is believed to be the bound state of $K^{\bar{b}}N$ and “dip” is expected at $R \sim a_0$.
- However, Coulomb and coupled-channel effects modify the dip-like behavior.
Kamiya+ ('20).

