

相関関数からハドロン間相互作用へ

基研 大西 明

京大原子核理論グループコロキウム、*May 12, 2021*

- 核力研究の広がり：NN 力からハドロン間相互作用へ
- 相関関数とハドロン間相互作用
 - 相関関数とは？
 - Survey of measured correlation function data: $\Lambda\Lambda$, $p\Omega^-$, pK^- , $p\Xi^-$.
- 相関関数とハドロンの束縛状態
 - 相関関数の源サイズ依存性
- Summary

自己紹介

- 履歴書： 1964 年神戸生まれ, 1992 年京都大学博士 (理学)
1992 年学振 (RCNP), 1993-2008 北大, 2008 年 ~ 基研

- 研究課題：核物質の相図と状態方程式の解明を目指す

- 重イオン衝突

非平衡グルーオン動力学 (松田、国広、高橋)

5次元時空でのレプリカ発展による量子場時間発展 (同上)

重イオン衝突での物質の回転にともなう奇妙な現象 (ExHIC-P)

フローによる状態方程式の軟化の探索 (奈良, Stöcker, Steinheimer, ...)

- QCD 相図と中性子星物質状態方程式

経路最適化法を用いた符号問題の研究 (森、柏、滑川)

重イオン衝突を用いた対称エネルギーの制限 (池野、小野、奈良)

対称エネルギーから中性子星物質状態方程式へ (Kolomeitsev, Lattimer, Tews)

- ストレンジネス & ハドロン物理

ハドロン相関によるハドロン間相互作用の探求

(神谷、兵藤、森田、初田・佐々木 (HAL QCD)、Weise、福井、緒方、...)

静止 Ξ -吸収からのダブルハイパー核生成 (石塚、椿原、平田)

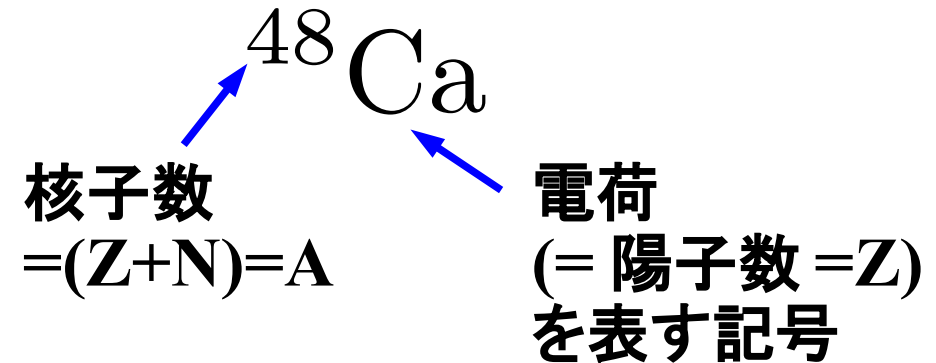
今回

核力研究の広がり NN 力からハドロン間相互作用へ

原子核とは

■ 定義 1: 原子の中心にある核子 (陽子・中性子) からなる系

- 陽子数 Z 、中性子数 N
- 安定な原子核 287 種
- 見つかっている原子核 ~ 3000 種
- 存在が予言 ~ 9000 種



■ 定義 2: 強い相互作用によって現れるハドロンの束縛・共鳴状態

- 構成要素 = ハドロン
核子 (p, n), ハイペロン (Λ, Σ, Ξ), 中間子 (K, η, \dots)
- 「原子の核」ではない励起状態も含む。
- ハドロン = 強い相互作用で結合したクォーク・ハドロンの複合粒子

核力の概観

■ 核力の3領域

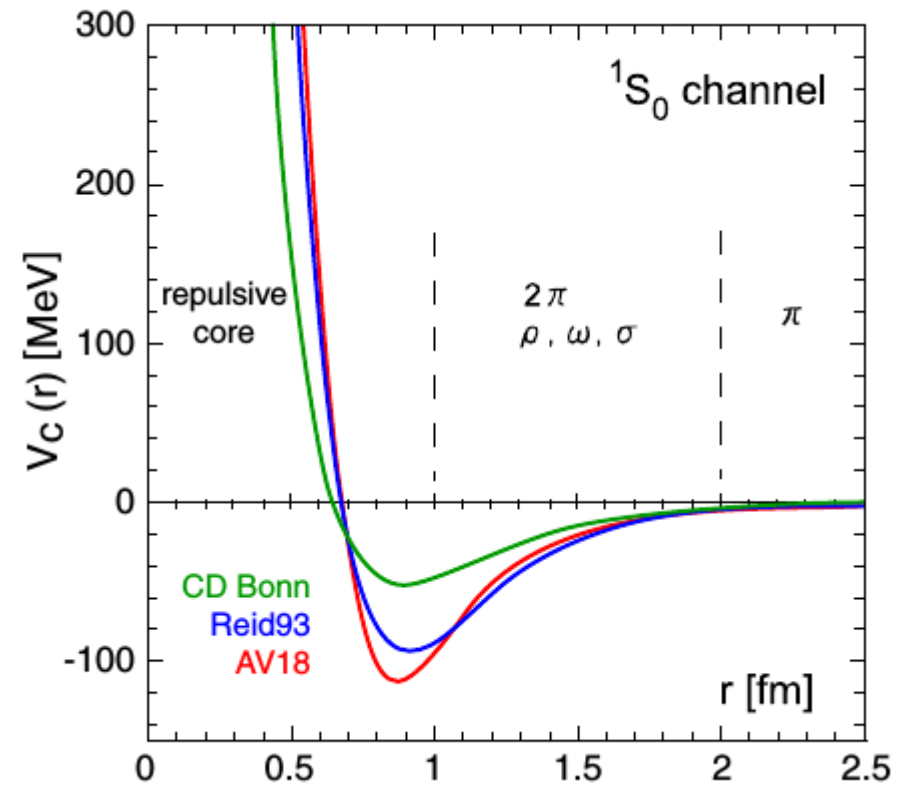
- 遠方 ($r > 2$ fm) \rightarrow 1π 交換力
Yukawa ('35)

- 中距離 \rightarrow ボソン交換 ($2\pi, \rho, \omega, \sigma, \dots$)
E.g. Machida, Toyoda ('56), Nambu-Jona-Lasinio ('61)

- 短距離 \rightarrow 斥力芯
Jastrow ('51)

■ 強いスピン・アイソスピン依存性

- 中心力、スピン軌道力、
テンソル力、...
- アイソスピン対称性
T=1 状態 ($pp, nn, (pn+np)/\sqrt{2}$)
T=0 状態 ($(pn-np)/\sqrt{2}$)



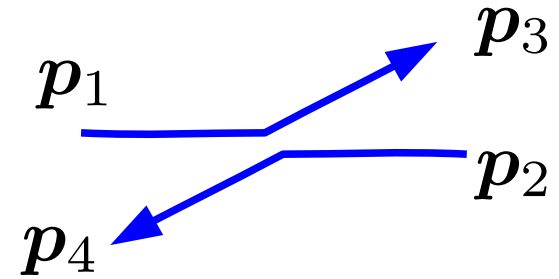
Aoki, Hatsuda, Ishii ('07)

核力と位相差

■ Radial wave function ($\psi_\ell(r) = u_\ell(r)/kr$)

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + \frac{\hbar^2 \ell(\ell+1)}{2mr^2} + V_\ell(r) \right] u_\ell(r) = E u_\ell(r)$$

$$u_\ell(r) \rightarrow \sin(kr - \ell\pi/2 + \delta_\ell) = \frac{1}{2i} \left(e^{i(kr - \ell\pi/2 + \delta_\ell)} - e^{-i(kr - \ell\pi/2 + \delta_\ell)} \right)$$



■ Boundary condition

$$\Psi(\mathbf{r}) = \sum_{\ell} (2\ell + 1) i^{\ell} \frac{A_{\ell} u_{\ell}(r)}{kr} P_{\ell}$$

$$\rightarrow \frac{1}{2ikr} \sum_{\ell} (2\ell + 1) i^{\ell} \left(S_{\ell}(k) e^{i(kr - \ell\pi/2)} - e^{-i(kr - \ell\pi/2)} \right) P_{\ell}$$

$$= e^{i\mathbf{k} \cdot \mathbf{r}} + \frac{e^{ikr}}{kr} \sum_{\ell} (2\ell + 1) \frac{S_{\ell}(k) - 1}{2i} P_{\ell} = e^{i\delta_{\ell}} \sin \delta_{\ell}$$

内向き = 平面波のまま $\rightarrow A_l = \exp(i\delta_l)$

$$A_{\ell} = e^{i\delta_{\ell}}, \quad S_{\ell}(k) = e^{2i\delta_{\ell}(k)}, \quad f(\theta) = \frac{1}{k} \sum_{\ell} (2\ell + 1) e^{i\delta_{\ell}} \sin \delta_{\ell} P_{\ell}(\cos \theta)$$

Phase shifts determine the scattering amplitude

$\delta_{\ell} = \text{phase shift (位相差)}$

ハドロン間相互作用を調べるには？

■ 理論からのアプローチ

- 核力模型：中間子交換、クォーク模型
- 第一原理：カイラル有効場理論 (χ EFT)、格子 QCD
 - ◆ χ EFT ではデータから決めるべき未定の定数あり。
 - ◆ LQCD は大きな計算資源が必要。

■ 実験からのアプローチ

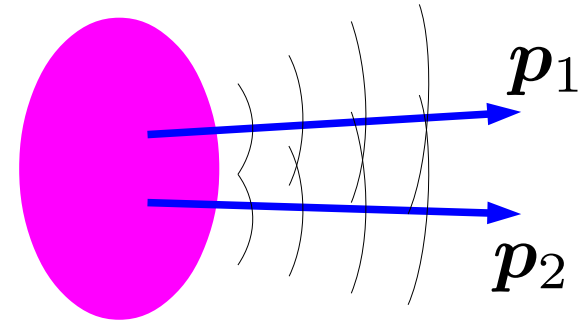
- 核力 (NN 力): 豊富な散乱実験データ → 位相差 → ポテンシャル
 - ハイペロン - 核子力 (YN 力)、中間子 (π , K)- 核子力:
限られた散乱データ、ハイパー核、ハドロン原子 (Σ^- , π^- , K^- など)
 - 他のハドロン間相互作用は？
 - ◆ 散乱実験ができず、原子核や核子と束縛状態を作らないハドロンと核子
 - ◆ 核子以外のハドロン間の相互作用
- 相関関数からハドロン間力へ

相関関数とハドロン間相互作用

2 粒子運動量相関関数

■ 粒子の放出点分布関数

$$N_i(\mathbf{p}) = \int d^4x S_i(x, \mathbf{p})$$



■ 2 粒子運動量分布

- 独立に作られた 2 粒子の波動関数に

対称性・終状態相互作用で相関が作られるとする。 *Koonin('77), Pratt+('86)*

$$N_{12}(\mathbf{p}_1, \mathbf{p}_2) \simeq \int d^4x d^4y S_1(x, \mathbf{p}_1) S_2(y, \mathbf{p}_2) \underline{|\Psi_{\mathbf{p}_1, \mathbf{p}_2}(x, y)|^2} \quad \text{2 粒子 w.f.}$$

$$\simeq \int d^4x d^4y S_1(x, \mathbf{p}_1) S_2(y, \mathbf{p}_2) \underline{|\varphi_q(\mathbf{r})|^2} \quad \text{相対波動関数}$$

■ 同種ボソン、ガウス源での相関関数

Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)

$$\Psi_{\mathbf{p}_1, \mathbf{p}_2}(\mathbf{x}, \mathbf{y}) = \frac{1}{\sqrt{2}} \left[e^{i(\mathbf{p}_1 \cdot \mathbf{x} + \mathbf{p}_2 \cdot \mathbf{y})} + e^{i(\mathbf{p}_1 \cdot \mathbf{y} + \mathbf{p}_2 \cdot \mathbf{x})} \right] = e^{i\mathbf{P} \cdot \mathbf{R}} \times \sqrt{2} \cos \mathbf{q} \cdot \mathbf{r}$$

$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{N_{12}(\mathbf{p}_1, \mathbf{p}_2)}{N_1(\mathbf{p}_1)N_2(\mathbf{p}_2)} \simeq \int d\mathbf{r} S_{12}(\mathbf{r}) |\varphi_q(\mathbf{r})|^2$$

$$\simeq 1 + \exp(-4q^2 R^2)$$

相関関数から
粒子放出源の
サイズが分かる！

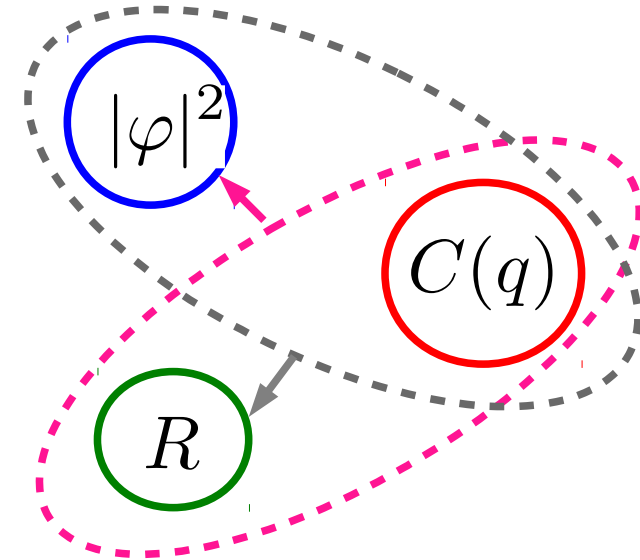
Correlation Function (CF): Non-standard usage

- HBT, GGLP: CF + w.f. \rightarrow Source Size
Another way: CF + Source Size \rightarrow w.f. \rightarrow hh interaction
- Effect of hadron-hadron interaction on the wave function
 - Assumption: Only s-wave ($L=0$) is modified.
 - Non-identical particle pair, Gauss source.

$$\varphi_{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{q}\cdot\mathbf{r}} - j_0(qr) + \chi_q(r)$$

$$\rightarrow C(\mathbf{q}) = \int d\mathbf{r} S(r) |\varphi_{\mathbf{q}}(\mathbf{r})|^2$$

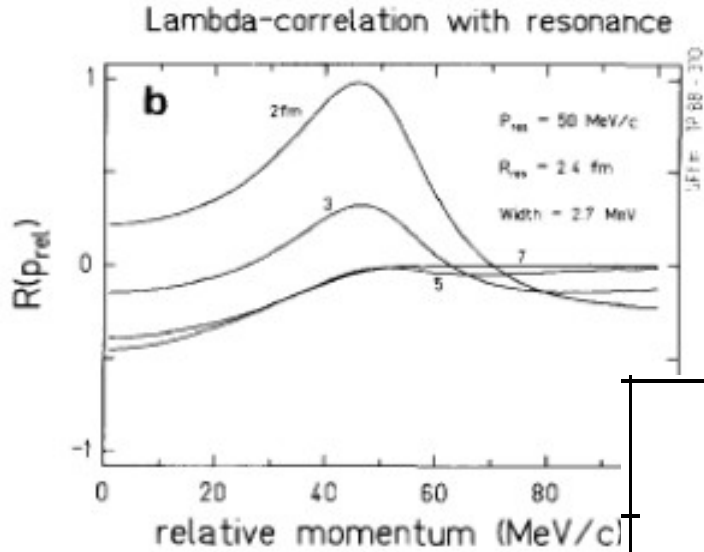
$$= 1 + \int d\mathbf{r} S(r) \{ |\chi_q(r)|^2 - |j_0(qr)|^2 \}$$



*R. Lednicky, V. L. Lyuboshits ('82);
K. Morita, T. Furumoto, AO, PRC91('15)024916*

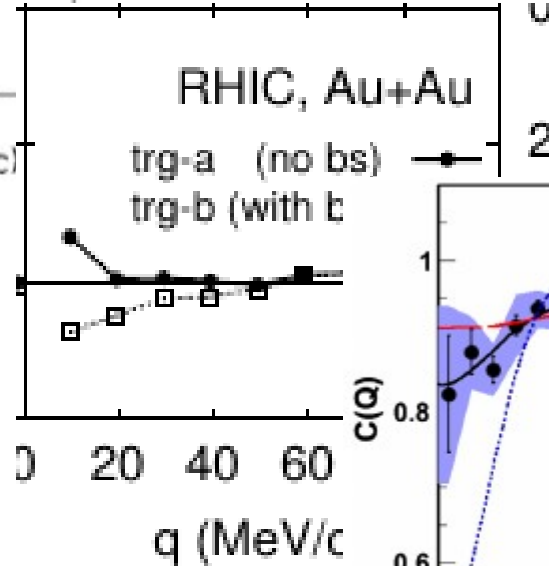
*Corr. Fn. shows how much squared w. f. is enhanced
 \rightarrow Large CF is expected with attraction*

Example: Λ correlation and Λ interaction

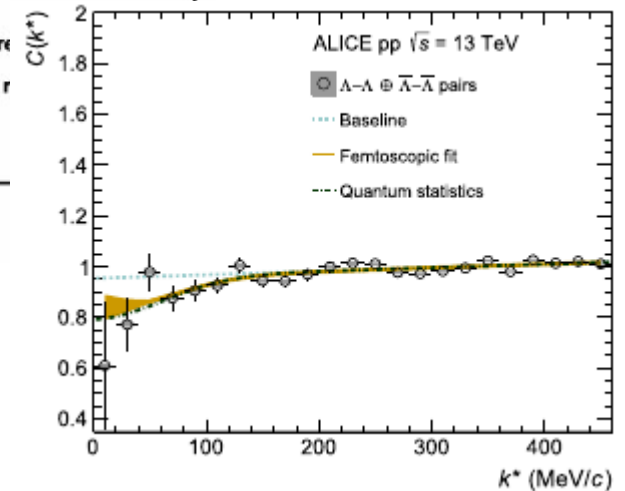
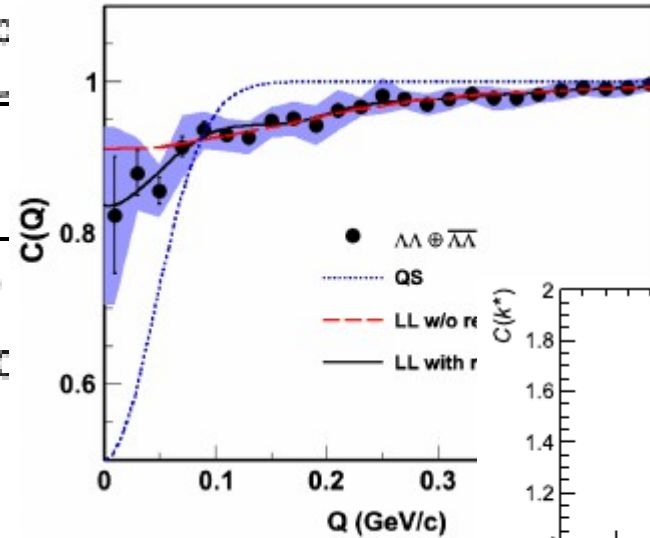


*AO, Hirata, Nara,
Shinmura, Akaishi,
NPA670('00)297c*

*C. Greiner, B. Muller,
PLB219('89)199.*



*L. Adamczyk+[STAR],
PRL114('15)022301*

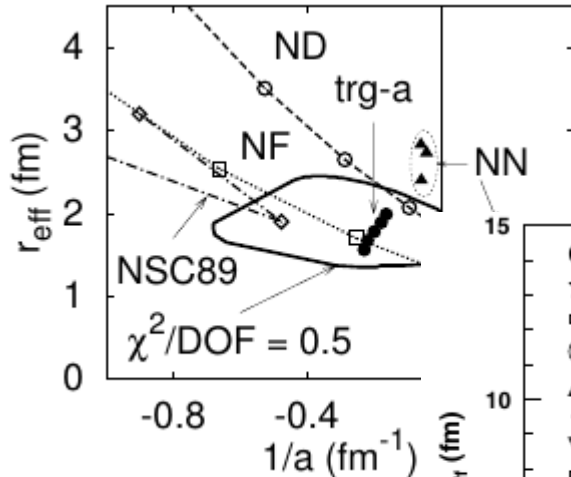


*Weak enh. over quantum
statistical (HBT) CF.*

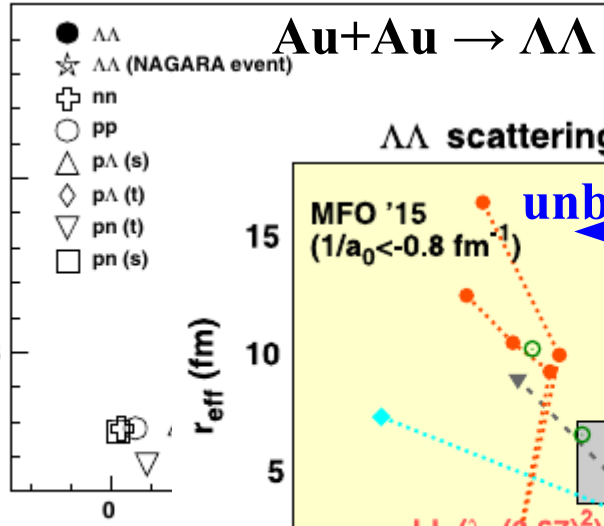
*S. Acharya+[ALICE],
PLB797('19)134822*

Example: $\Lambda\Lambda$ correlation and $\Lambda\Lambda$ interaction

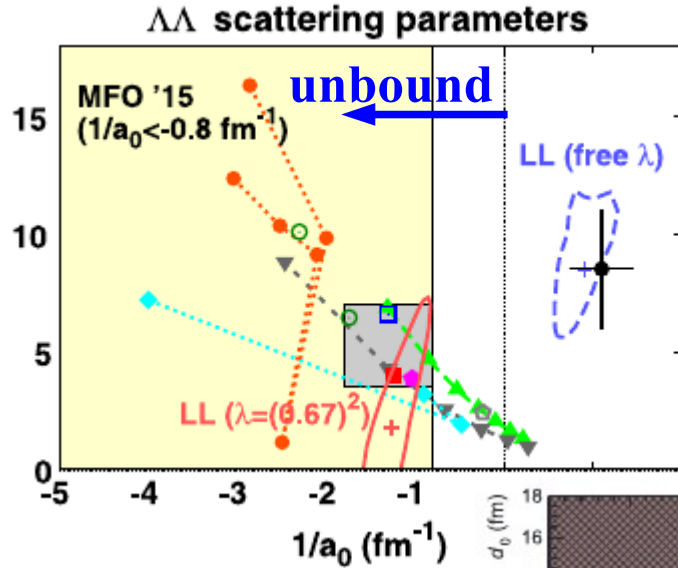
It is unlikely that $\Lambda\Lambda$ bound state exists.



L. Adamczyk+[STAR], PRL114('15)022301

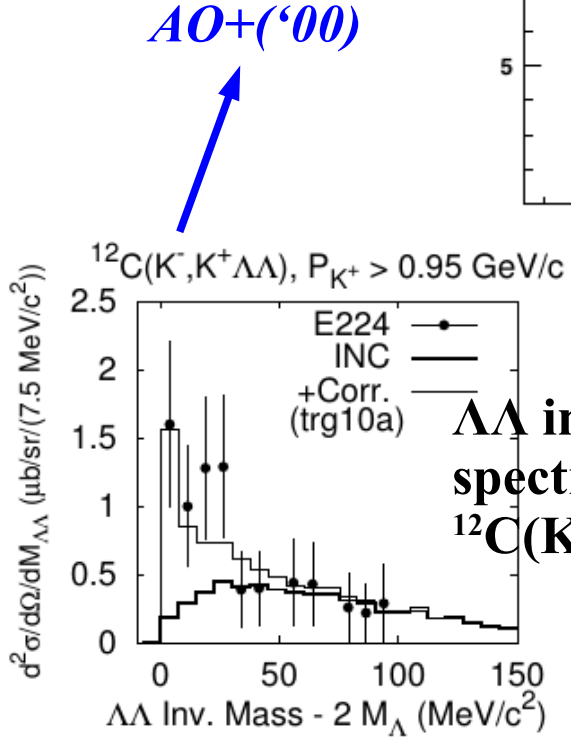


AO, K. Morita, K. Miyahara, T. Hyodo, NPA954('16)294



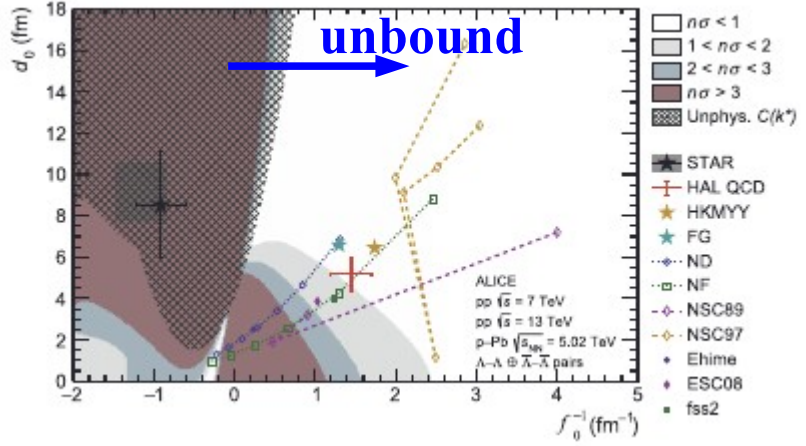
- ND ▲
- NF ▲
- NSC89 ◆
- NSC97 ◆
- ESC08c ◆
- Ehime ◊
- fss2 ■
- FG □
- HKMY ○
- STAR ●

$$\delta \sim - a_0 q$$



S. Acharya+[ALICE], PLB797('19)134822

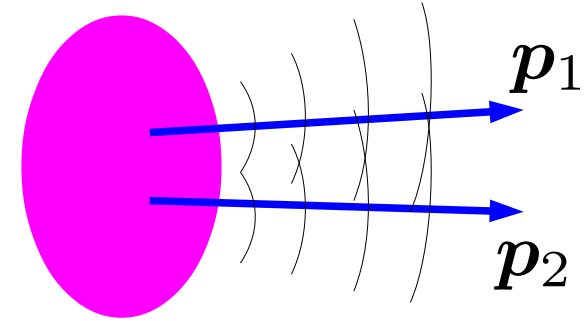
$$\delta \sim + a_0 q$$



Fermtoscopic Study of Hadron-Hadron Interactions

■ どのような粒子が観測できるか？

- 電荷をもち、寿命の長い粒子 (p, π^\pm, K^\pm)
- しばらく飛んで荷電粒子に崩壊する粒子 ($\Lambda, \Xi^-, D^\pm, \dots$)



散乱実験・原子核

Λ ハイパー核・YN 散乱

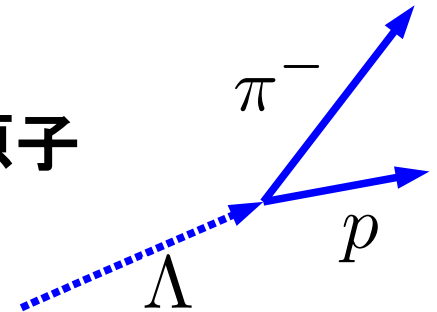
Ξ ハイパー核

散乱・mesic 原子

	n	p	Λ	Σ	Ξ^-	Ω^-	K^-	K^+	π^-	π^+
n										
p		O	O	O	O	O	O	O	Δ	Δ
Λ		O	O							
Σ		O								
Ξ^-		O								
Ω^-		O								
K^-		O								
K^+		O								
π^-		Δ								
π^+		Δ								

Σ 核
(${}^4_\Sigma\text{He}$)
スペクトル

$\Lambda\Lambda$ 核



Current
Femtoscscopy
(O: observed)

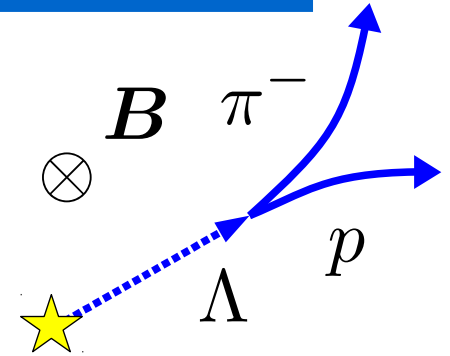
ハドロンはどれだけ飛ぶか？

■ 崩壊するまでに飛ぶ距離

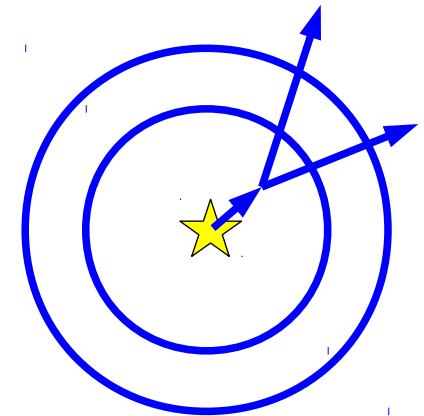
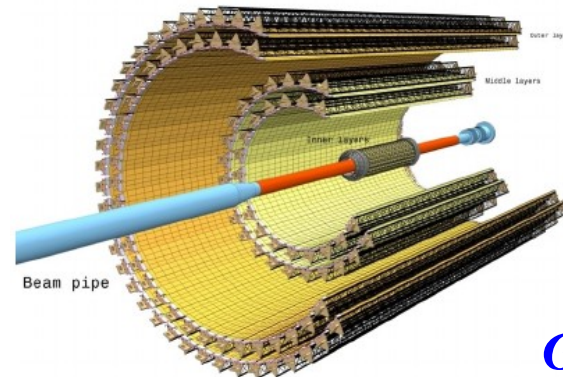
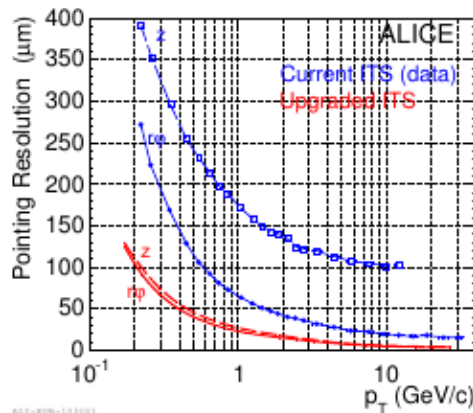
$$l = \gamma v \tau = \gamma \beta c \tau \simeq \gamma (c \tau) \quad (E \gg m)$$

■ ハドロンの $c\tau$

- Strange baryons \rightarrow A few cm \rightarrow Time Projection Chamber (TPC)
 - ◆ $c\tau(\Lambda)=7.9$ cm, $c\tau(\Sigma^+)=2.4$ cm, $c\tau(\Xi^-)=4.9$ cm.
- Charmed hadrons \rightarrow A few hundred μm \rightarrow Silicon Vertex Detector
 - ◆ $c\tau(D^\pm)=312$ μm , $c\tau(D^0)=123$ μm , $c\tau(\Lambda^+)=61$ μm ,



Charmed hadron も
(ほぼ) イベント毎に観測可能に！

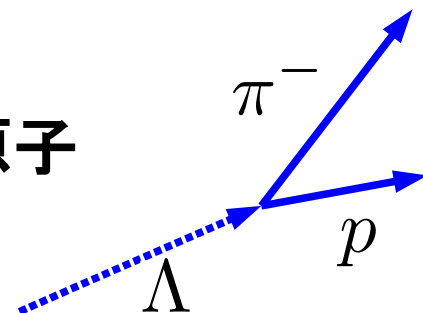
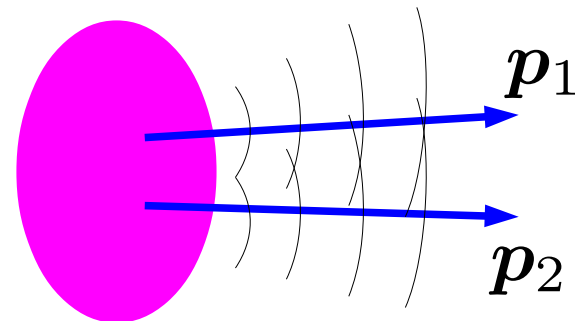


G. Contin+[ALICE]
PoS(Vertex2019)003

Fermtoscopic Study of Hadron-Hadron Interactions

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- しばらく飛んで荷電粒子に崩壊する粒子 ($\Lambda, \Xi^-, D^\pm, \dots$)



散乱実験・原子核

Λ ハイパー核・YN 散乱

Ξ ハイパー核

散乱・mesic 原子

	n	p	Λ	Σ	Ξ^-	Ω^-	K^-	K^+	π^-	π^+	D^-	D^+
n												
p												
Λ												
Σ												
Ξ^-												
Ω^-												
K^-												
K^+												
π^-												
π^+												
D^-												
D^+												

Σ 核
($^4_\Sigma\text{He}$)
スペクトル

$\Lambda\Lambda$ 核

Future
Femtoscscopy
(High-luminosity
+ Silicon Vertex
Detector)

相関関数とハドロンの束縛状態

To be, or not to be, that is the question.

■ “面白い”ハドロン間相互作用は？

- NN 力, YN 力 → 原子核や核物質、中性子星物質
- $\Lambda\Lambda$ - ΞN , ΩN , $\Delta\Delta$ → ダイバリオン状態を作る可能性あり
(quark 間のパウリ排他率が働かず、one-gluon 交換力が引力)
- $\bar{K}N$ ($\bar{s}q$ - qqq), DN ($\bar{c}q$ - qqq), DN ($\bar{c}q$ - qqq), ...
→ 5q 状態 (penta quark state) を作る可能性あり

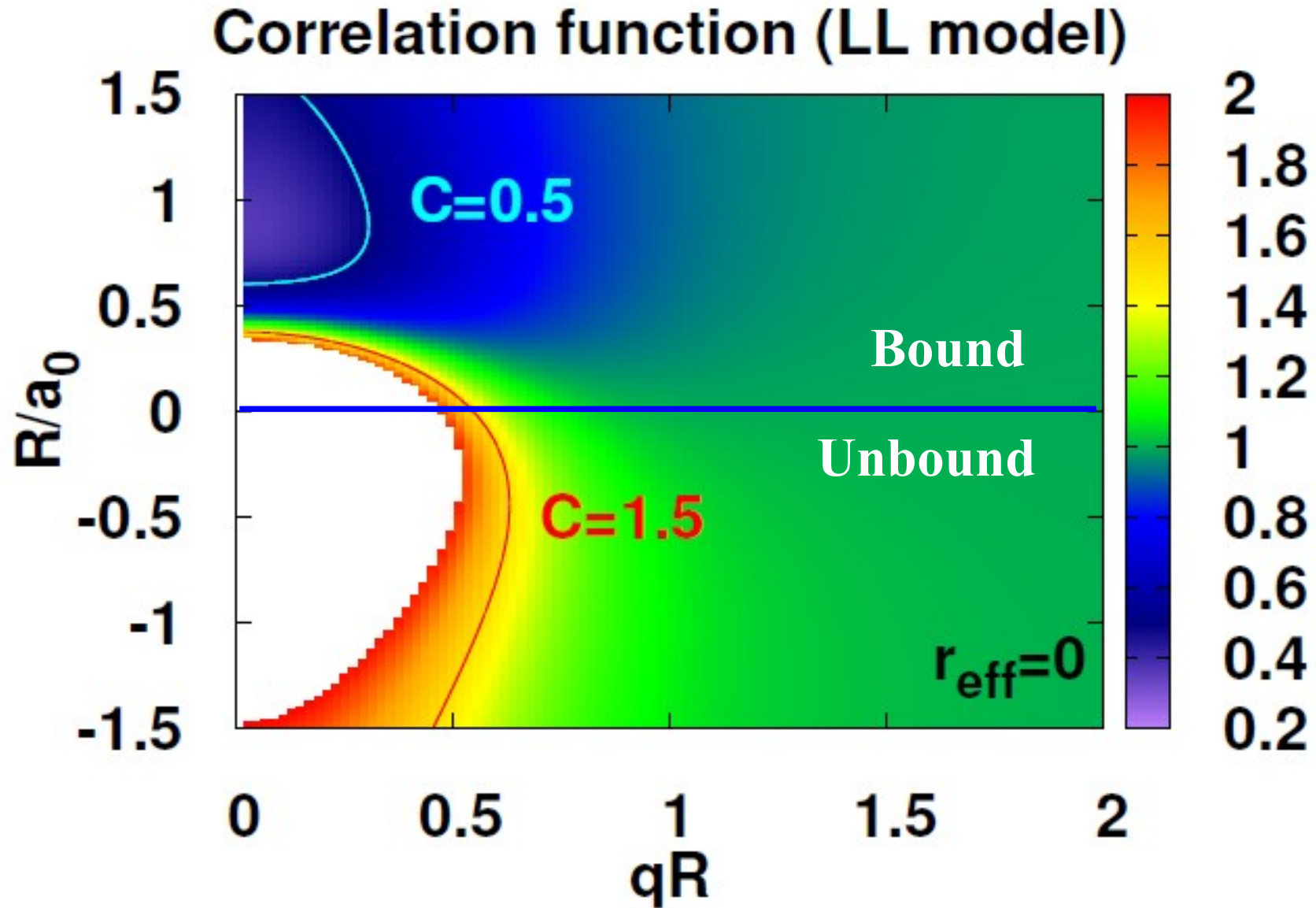
Table 1. Leading $6q$ $L = 0$ dibaryon candidates [12], their BB' structure and the CM interaction gain with respect of the lowest BB' threshold calculated by means of Eq. (2). Asterisks are used for the 10_f baryons $\Sigma^* \equiv \Sigma(1385)$ and $\Xi^* \equiv \Xi(1530)$. The symbol $[i,j,k]$ stands for the Young tableaux of the $SU(3)_f$ representation, with i arrays in the first row, j arrays in the second row and k arrays in the third row, from which \mathcal{P}_f is evaluated. The $\bar{10}$ $SU(3)_f$ representation is denoted here 10^* .

$-S$	$SU(3)_f$	I	J^π	BB' structure	$\frac{\Delta(V_{CM})}{M_0}$
0	$[3,3,0]$ 10^*	0	3^+	$\Delta\Delta$	0
1	$[3,2,1]$ 8	1/2	2^+	$\frac{1}{\sqrt{5}}(N\Sigma^* + 2\Delta\Sigma)$	-1
2	$[2,2,2]$ 1	0	0^+	$\frac{1}{\sqrt{8}}(\Lambda\Lambda + 2N\Xi - \sqrt{3}\Sigma\Sigma)$	-2
3	$[3,2,1]$ 8	1/2	2^+	$\frac{1}{\sqrt{5}}(\sqrt{2}N\Omega - \Lambda\Xi^* + \Sigma^*\Xi - \Sigma\Xi^*)$	-1

A. Gal ('16); M. Oka ('88)

束縛状態の有無によって
相関関数は変わるのか？
→ Yes!

Source Size Dependence of Correlation Function



E.g. AO, Morita, Miyahara, Hyodo ('16)
LL model: R. Lednicky, V. L. Lyuboshits ('82)

Wave function around threshold (S-wave, attraction)

Low energy w.f. and phase shift

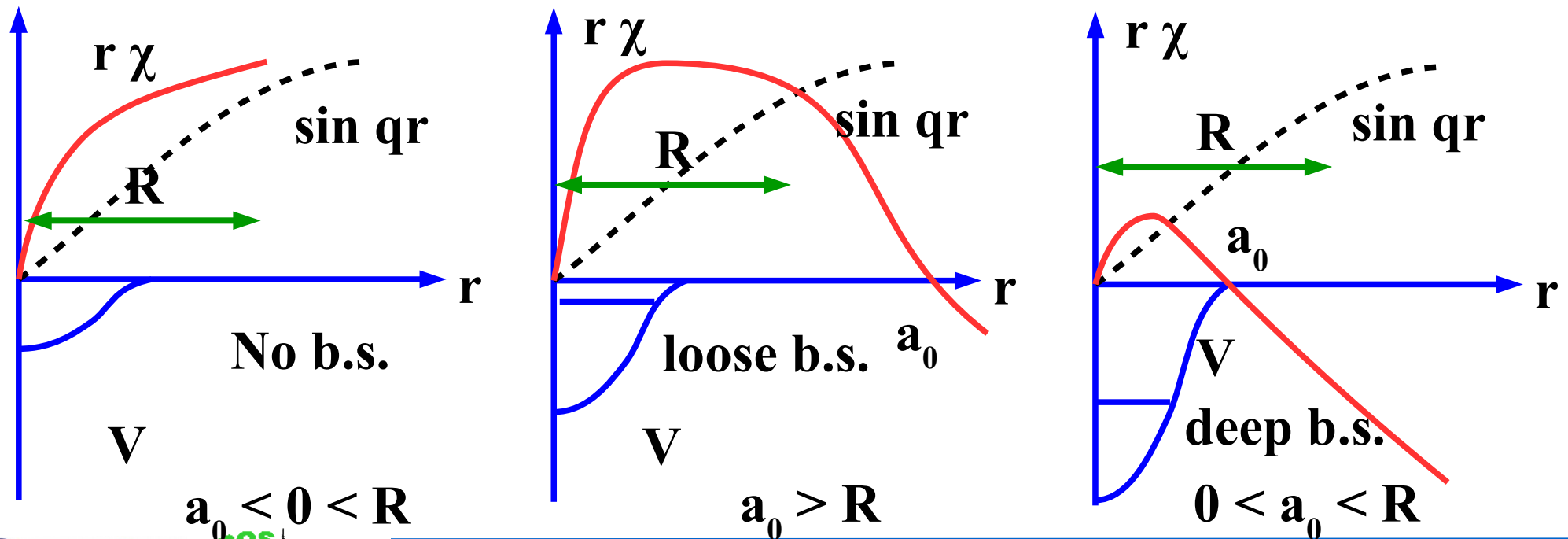
$$u(r) = qr\chi_q(r) \rightarrow \sin(qr + \delta(q)) \sim \sin(q(r - a_0))$$

$$q \cot \delta = -\frac{1}{a_0} + \frac{1}{2}r_{\text{eff}}q^2 + \mathcal{O}(q^4) \quad (\delta \sim -a_0q)$$

a_0 = scatt. length

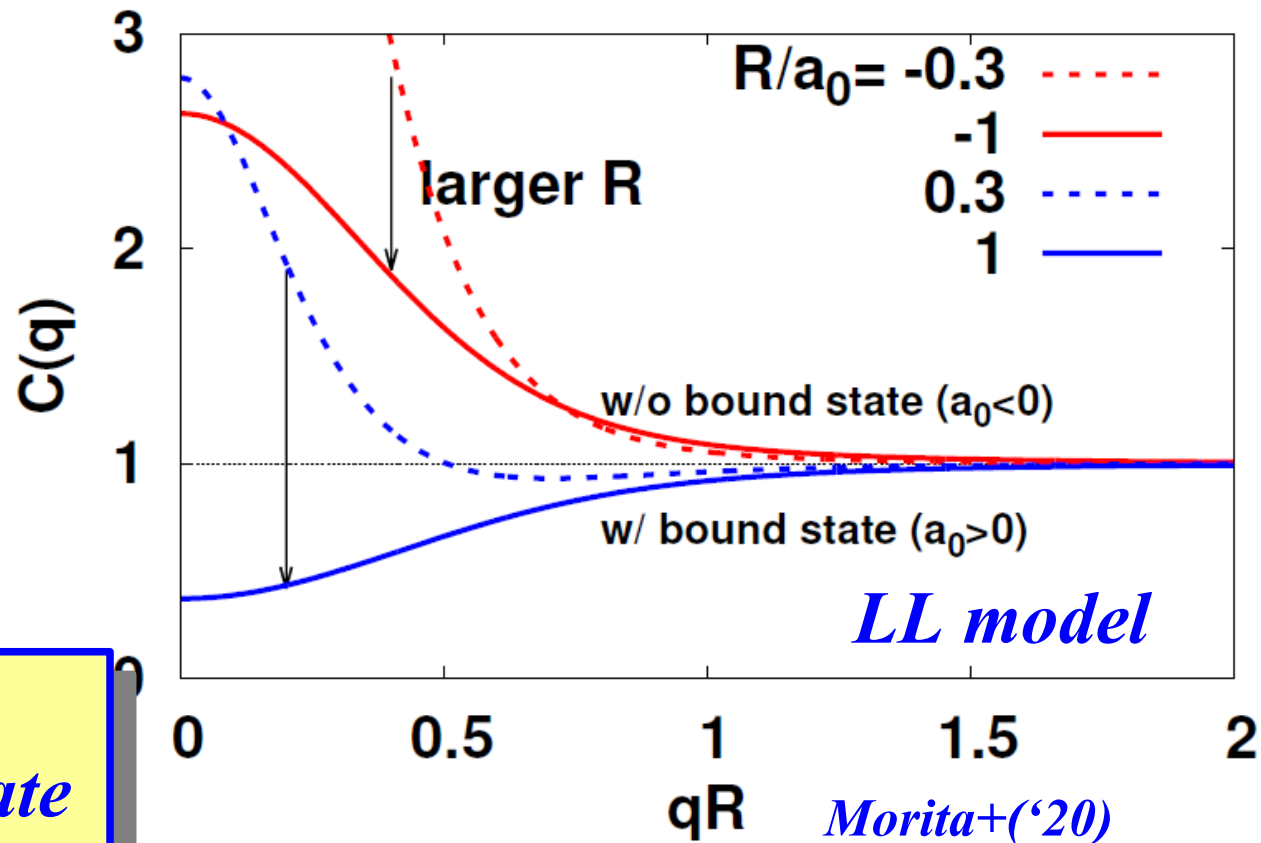
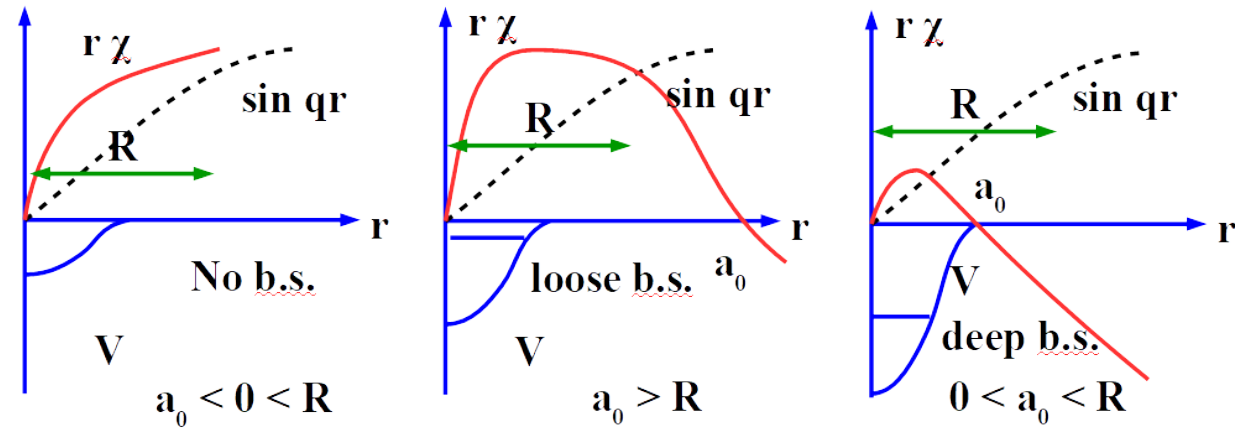
r_{eff} = eff. range

- Wave function grows rapidly at small r with attraction.
- With a bound state ($a_0 > 0$), a node appears around $r = a_0$



From correlation function to hadron-hadron interaction

- Large $|a_0|$ ($|a_0| > R$)
→ Large $C(q)$
(unitary regime)
- w/o bound state
($a_0 < 0$, $|a_0| \sim R$)
→ $C(q) > 1$
- With bound state
($a_0 > 0$, $|a_0| \sim R$)
→ Region with $C(q) < 1$ appears



Source size dep. of CF
→ Existence of bound state

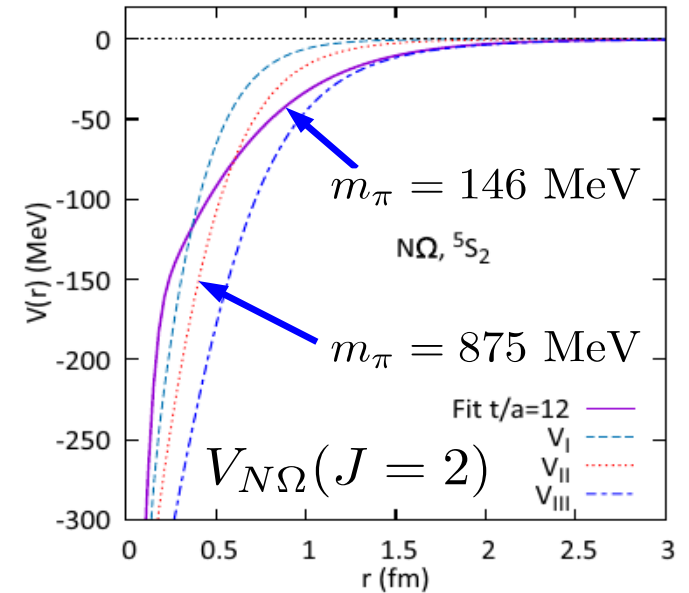
Modern Hadron-Hadron Interactions

■ Lattice QCD hh potential

- V_{hh} is obtained from the Schrödinger eq. for the Nambu-Bethe-Salpeter (NBS) amplitude.

N. Ishii, S. Aoki, T. Hatsuda, PRL99('07)022001.

→ $\Omega\Omega$, $N\Omega$, $\Lambda\Lambda$ - $N\Xi$ potentials
at phys. quark mass are published



■ Chiral EFT / Chiral SU(3) dynamics

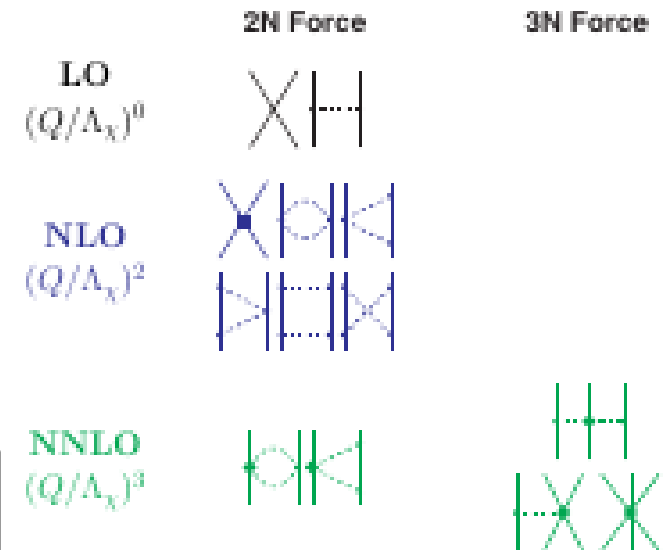
- V_{hh} at low E. can be expanded systematically in powers of Q/Λ .

S. Weinberg ('79); R. Machleidt, F. Sammarruca ('16);

Y. Ikeda, T. Hyodo, W. Weise ('12).

→ NN , NY , YY , $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$, ...

- Quark cluster models,
Meson exchange models,
More phenomenological models, ...



Let us examine modern hh interactions !

ΩN potential from lattice QCD

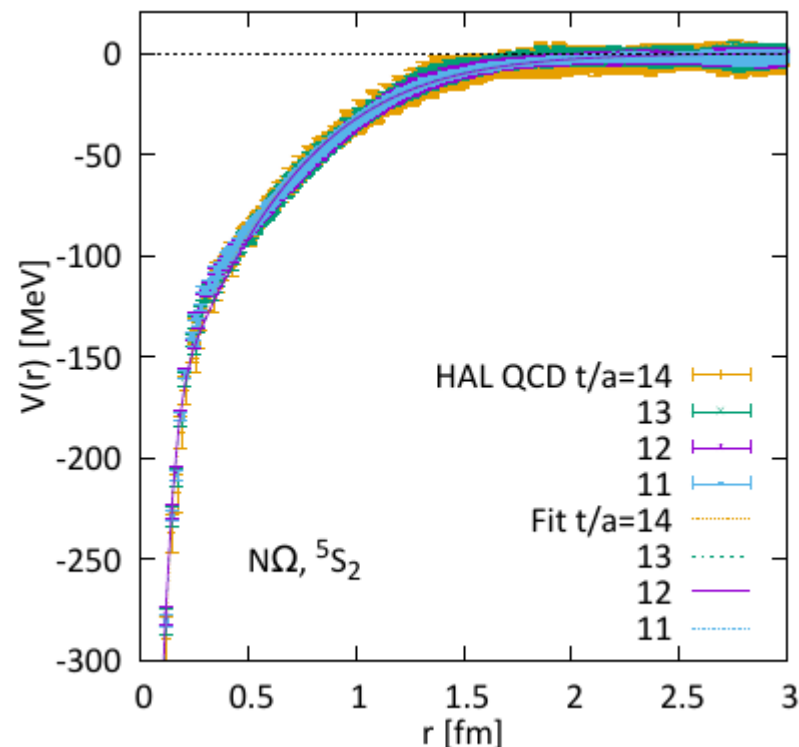
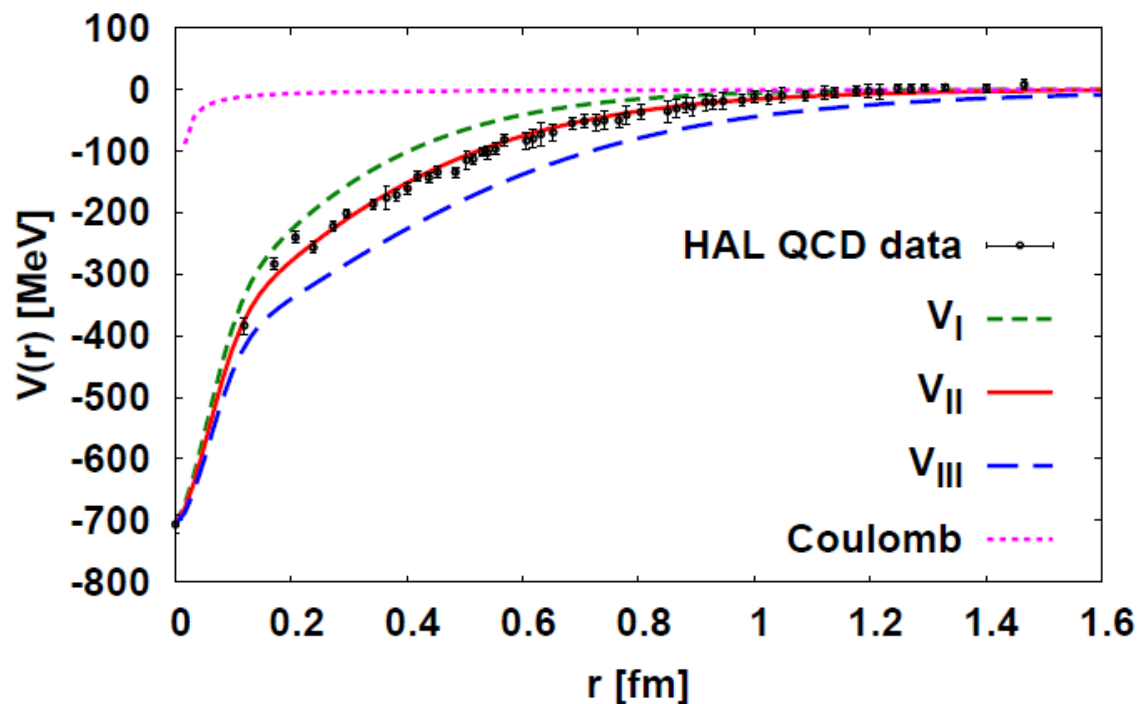
■ ΩN potential by HAL QCD Collab. ($J=2$)

- $m_\pi=875$ MeV, **B.E.~0.63 MeV**

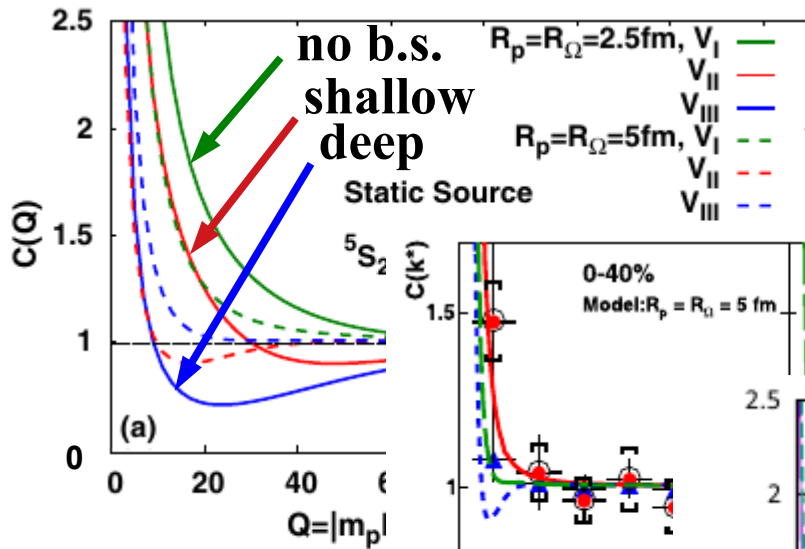
F.Etminan et al. (HAL QCD Collab.), NPA928('14)89.

- $m_\pi=146$ MeV, **B.E.~2.2 MeV**

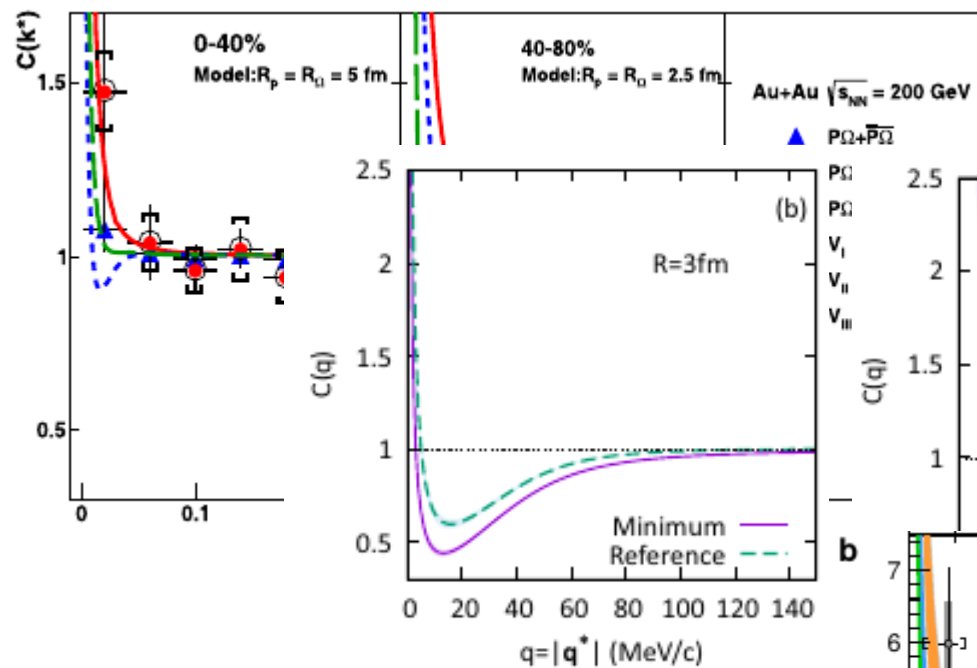
T. Iritani et al. (HAL QCD Collab.), PLB 792('19)284.



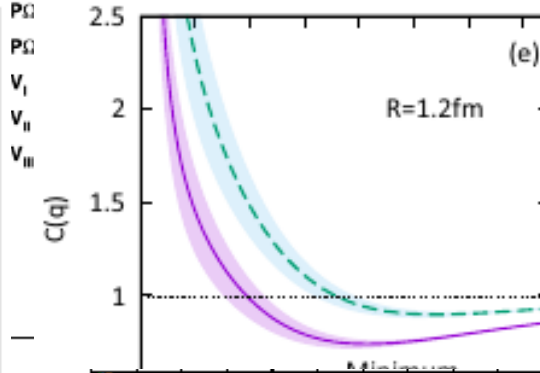
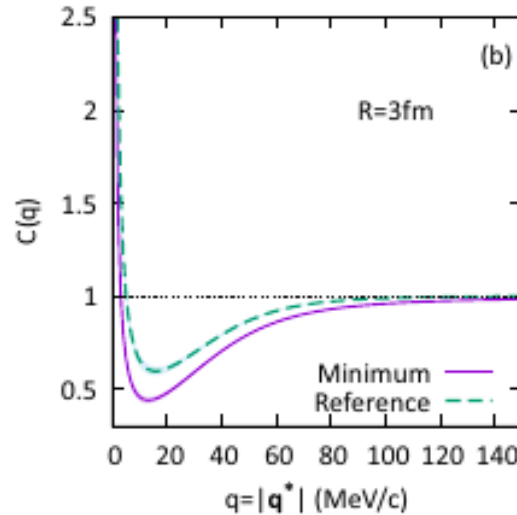
$p\Omega^-$ correlation



*K. Morita, AO, F. Etminan,
T. Hatsuda, PRC94('16)031901(R)
(w/ Lattice potential with heavier quark mass)*

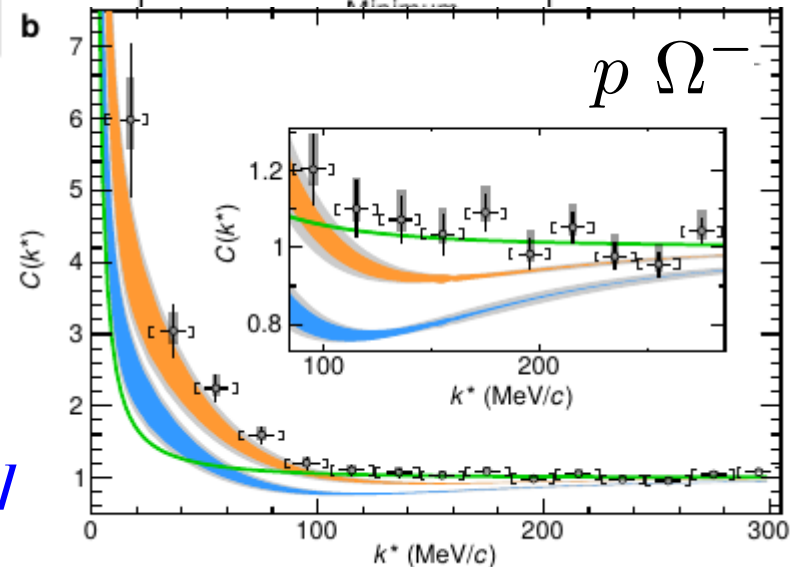


*J. Adam+[STAR],
PLB790('19)490.*

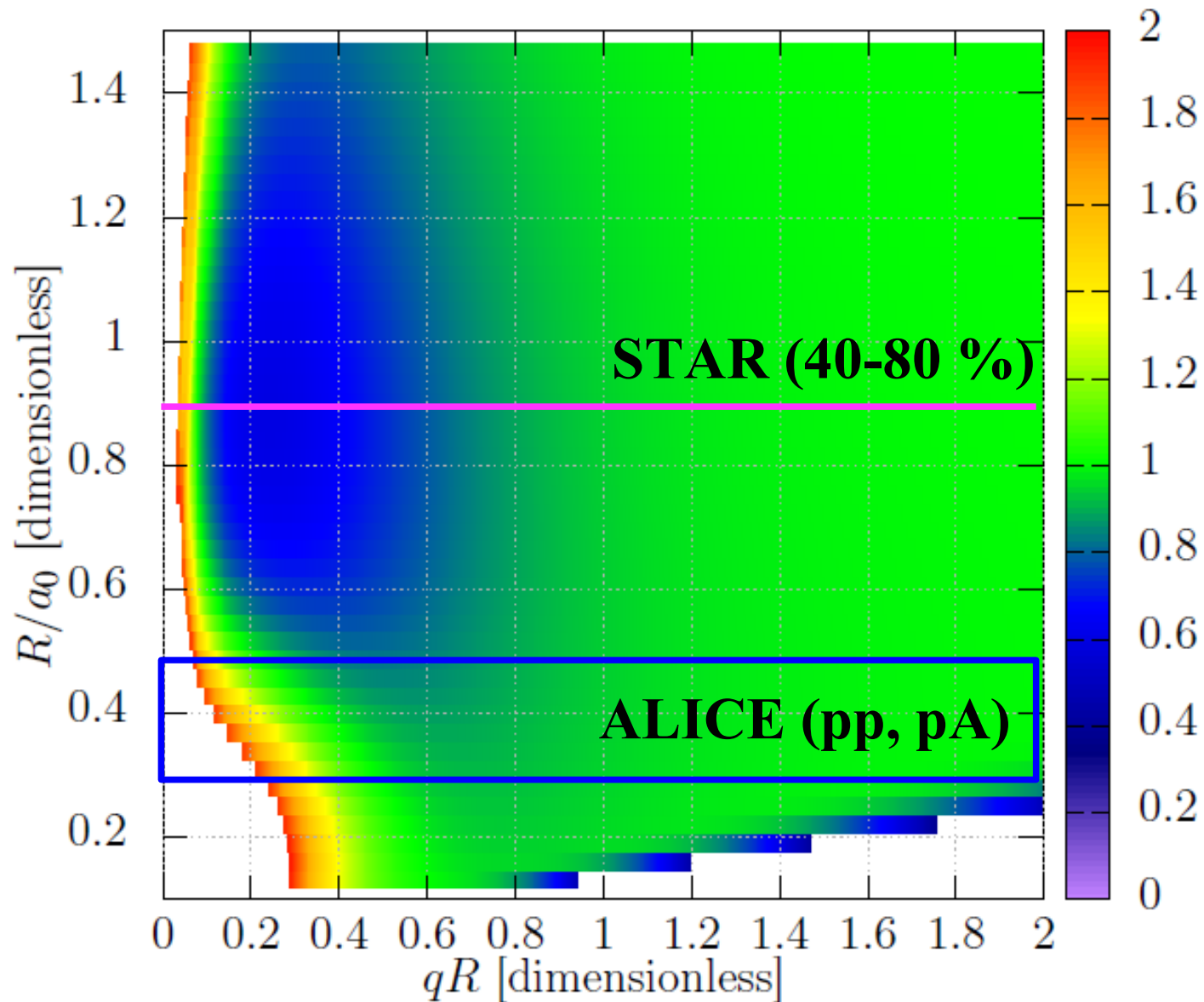


*K. Morita, S. Gongyo, T. Hatsuda,
T. Hyodo, Y. Kamiya, AO,
PRC 101('20)015201. (w/ Lattice
potential at physical quark mass)*

*S. Acharya+[ALICE],
Nature ('20), [2005.11495]
(pp 13 TeV)*

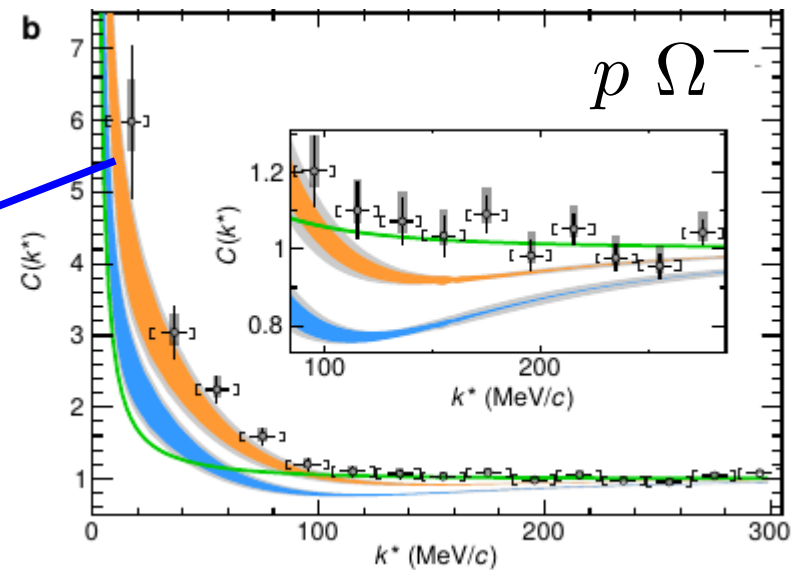
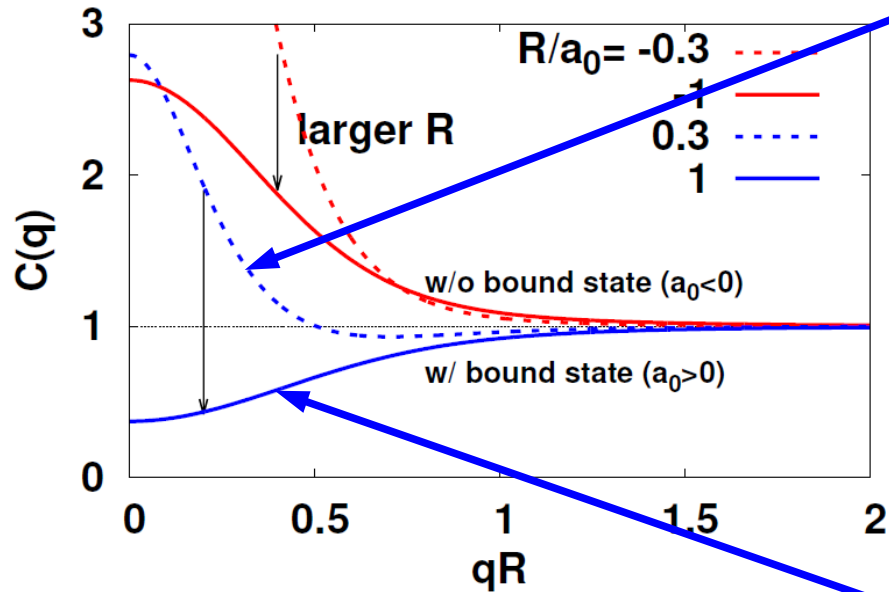


Correlation Function with Gaussian source

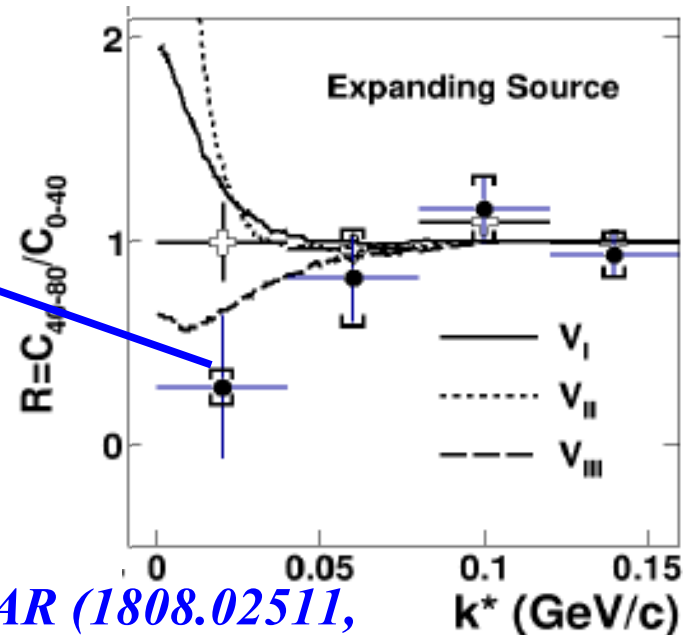


$N\Omega$ potential (J=2, HAL QCD, $a_0=3.4$ fm) + Coulomb

STAR + ALICE = $N\Omega$ Dibaryon



ALICE, 2005.11495



STAR (1808.02511,
PLB790 ('19) 490)

Do you know any mechanism to suppress $C(q)$ other than the existence of bound state, when the interaction is attractive ?
(Strong flow, ...)

Do I have 10 minutes ?

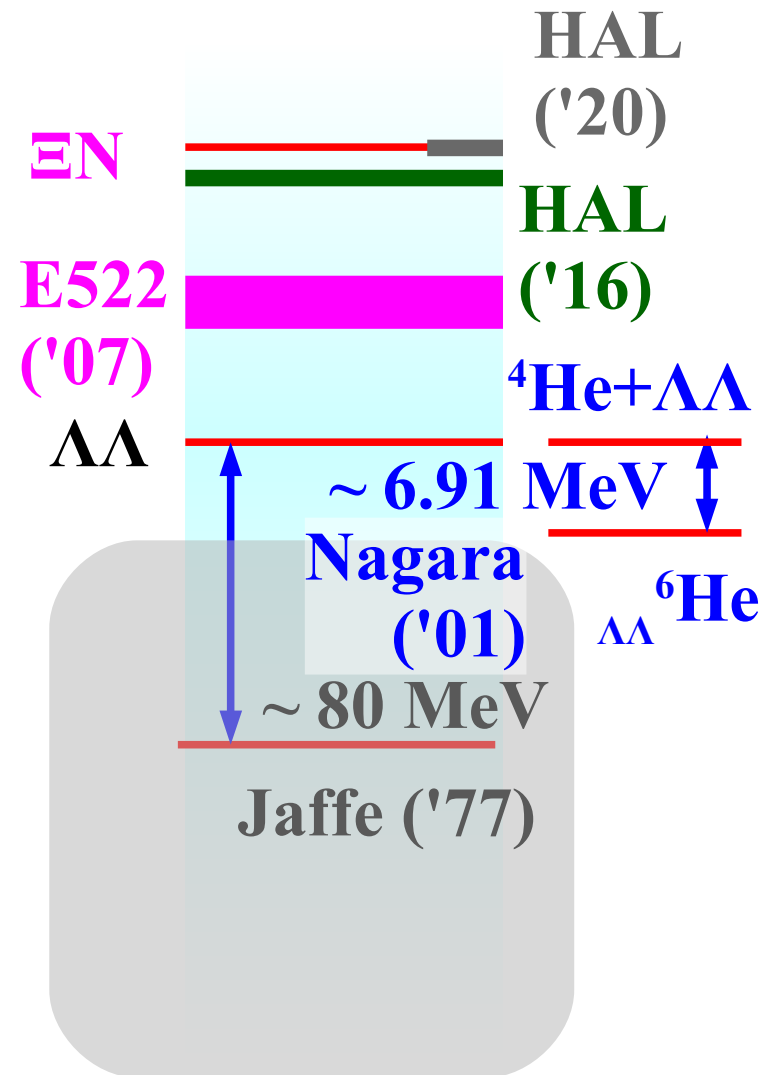
H dibaryon state, to be bound or not to be bound ?

■ H-dibaryon: 6-quark state (uuddss)

- Prediction: *R.L.Jaffe, PRL38(1977)195*
- Ruled-out by double Λ hypernucleus
Takahashi et al., PRL87('01) 212502
- Resonance or Bound “H” ?
Yoon et al.(KEK-E522)+AO ('07)

■ Lattice QCD results

- Bound:
HALQCD('11), NPLQCD('11,'13), Mainz('19)
(heavier quark mass or SU(3) limit)
- Resonance (Bound state of $N\Xi$):
HAL QCD ('16,18) (heavier m_q)
- Virtual Pole (around $N\Xi$ threshold)
HAL QCD ('20) (almost physical m_q)



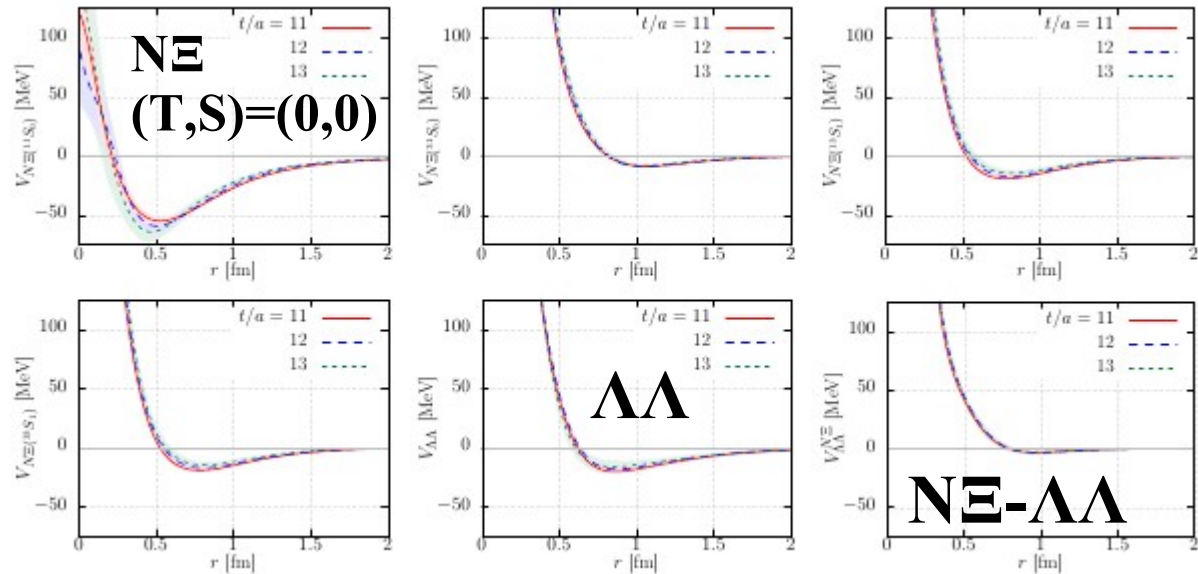
We examine LQCD $N\Xi$ - $\Lambda\Lambda$ potential and discuss H using CF !

$N\Xi-\Lambda$ potential from Lattice QCD

- $N\Xi-\Lambda\Lambda$ potential at almost physical quark mass ($m_\pi=146$ MeV) by HAL QCD Collaboration

K. Sasaki et al., NPA 998 ('20) 121737 (1912.08630)

- Strong attraction in $(T,S)=(0,0)$ of $N\Xi$
- Weak attraction in $\Lambda\Lambda$ (Coupling with $N\Xi$ causes $\Lambda\Lambda$ attraction)
- There is no bound state in $N\Xi-\Lambda\Lambda$ system (except for Ξ^- atom), but there is **a virtual pole around the $N\Xi$ threshold** (3.93 MeV below $n\Xi^0$ threshold) on the irrelevant Riemann sheet, $(+, -, +)$ [relevant= $(-, +, +)$]



$$E_{\text{pole}} = 2250.5 \pm i0.3 \text{ MeV}$$

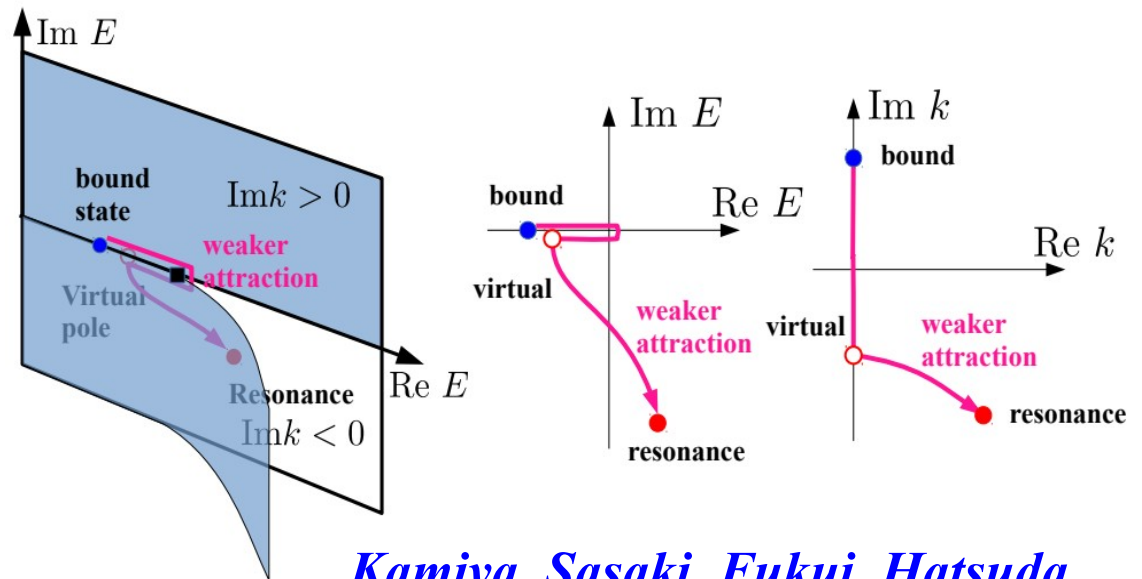
↑ sign of $\text{Im}(\text{eigen momentum})$

Fate of H dibaryon state \sim Virtual Pole ?

- Recent HAL QCD results at almost physical quark mass
 - There is no bound state in $N\Xi-\Lambda\Lambda$ system (except for Ξ^- atom), but there is **a virtual pole around the $N\Xi$ threshold** (3.93 MeV below $n\Xi^0$ threshold) on the irrelevant Riemann sheet, (+, -, +) [channels = 1($\Lambda\Lambda$), 2($n\Xi^0$), 3($p\Xi^-$)]
 - Wave function in $n\Xi^0$ channel diverges while the $\text{Re}(\text{energy})$ is lower than the threshold \rightarrow Virtual pole

$$u_i(r) \propto \exp(ik_i r) = \exp(i\text{Re}(k_i)r) \exp(-\text{Im}(k_i)r)$$

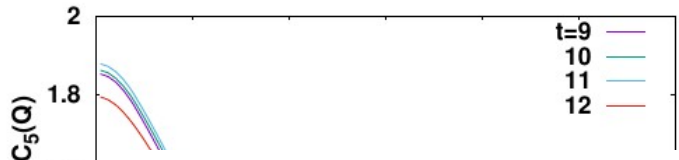
- If it appears in the (-, +, +) Riemann sheet, it is a $\Lambda\Lambda$ resonance (a $N\Xi$ bound state).



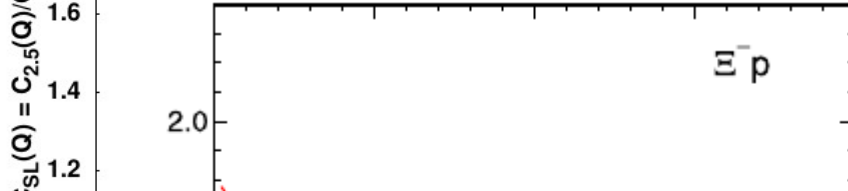
Kamiya, Sasaki, Fukui, Hatsuda, Hyodo, Morita, Ogata, AO, in prep.

$p\bar{\Xi}$ correlation function

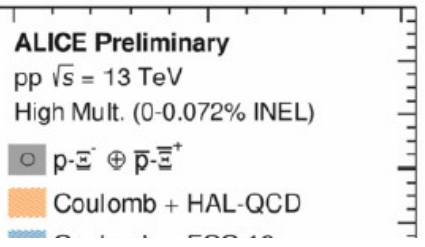
*T. Hatsuda, K. Morita, AO,
K. Sasaki, NPA967('17)856.
(heavier quark mass)*



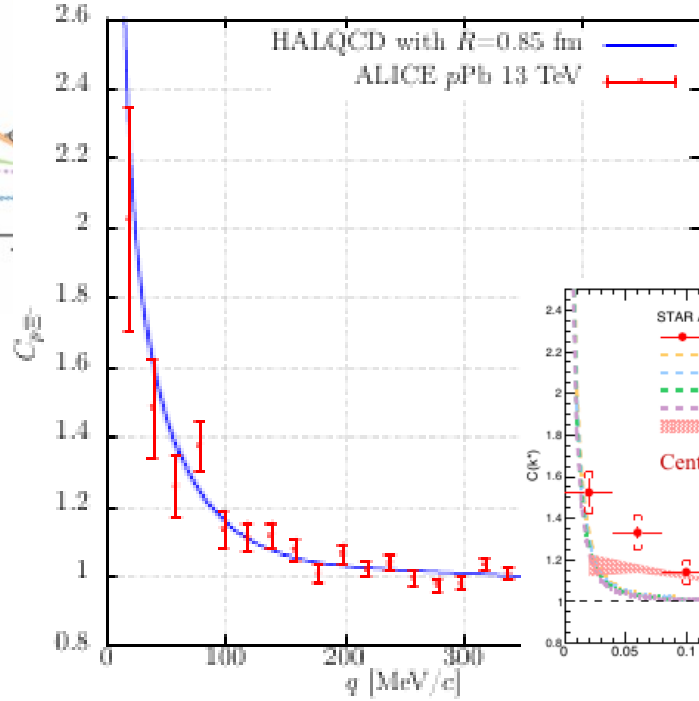
*J. Haidenbauer, NPA981('19)1.
(NLO(600), w/ CC effects, w/o Coulomb)
(w/ Coulomb, it will be comparable with data.)*



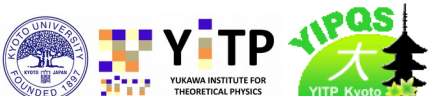
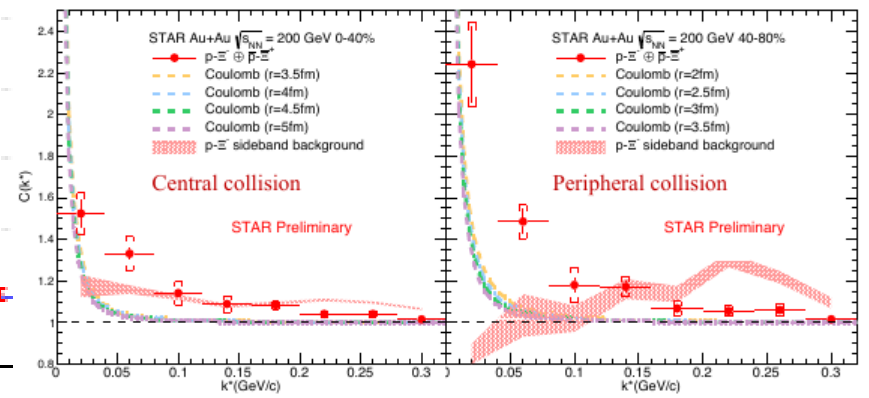
*D. L. Mihairov+[ALICE], NPA 1005 ('21)121760 (QM2019). (Nijmegen pot. does not explain the data. w/o CC)
Acharva+(ALICE), Nature ('20)*



*K. Mi+(STAR, preliminary),
Au+Au, 200 AGeV, APS2021.
(No Dip at larger R)*



*Kamiya+(in prep.),
w/ Lattice BB pot.
at phys. m_q
CC effects with $\Lambda\Lambda$.*



Λ correlation function

C. Greiner, B. Muller, PLB219('89)199.

(Assumed $\Lambda\Lambda$ resonance)

AO, Hirata, Nara, Shinmura, Akaishi,

NPA670('00)297c

(Before NAGARA, interaction was too strong.)

Adamczyk+[STAR], PRL114('15)022301

(Residual source $R \sim 0.5$ fm was assumed.)

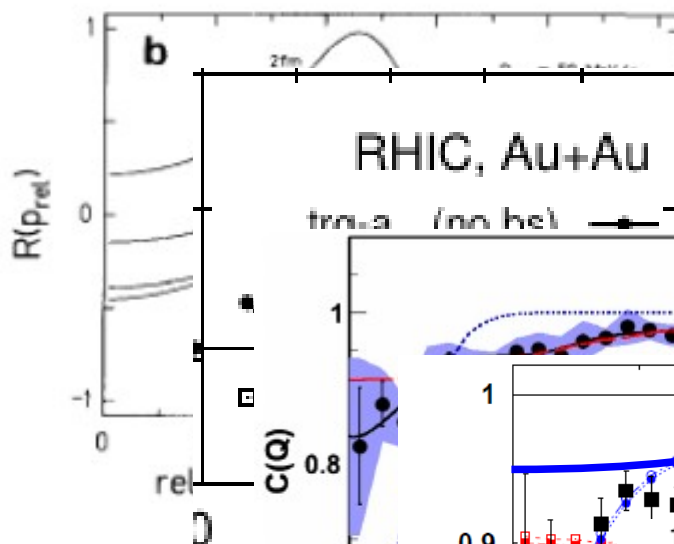
Morita, Furumoto, AO, PRC91('15)

024916. (Res. Source + flow)

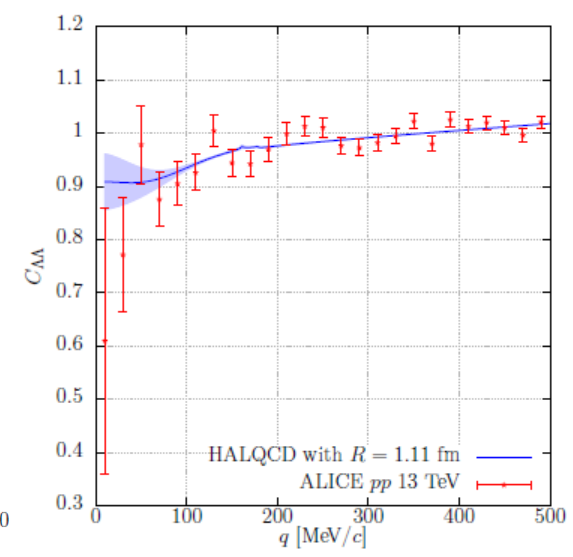
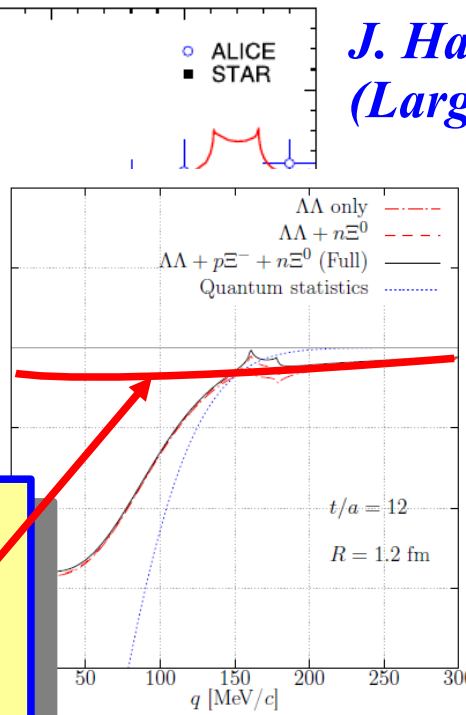
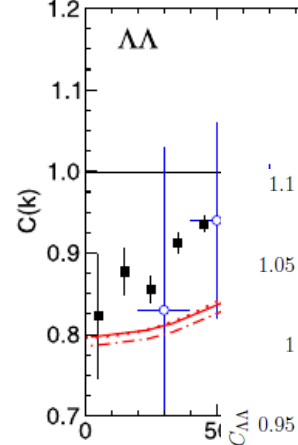
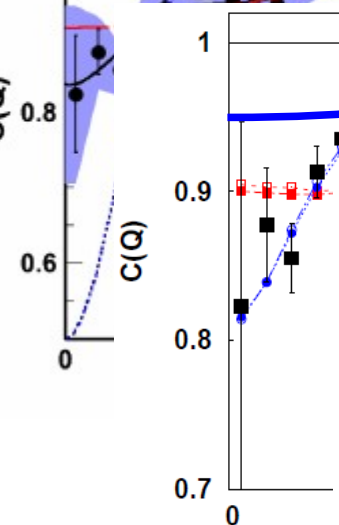
J. Haidenbauer, NPA981('19)1.

(Larger cusp ?)

Lambda-correlation with resonance



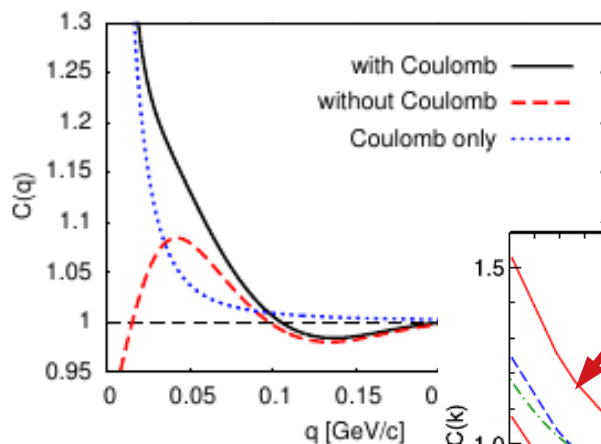
u
2



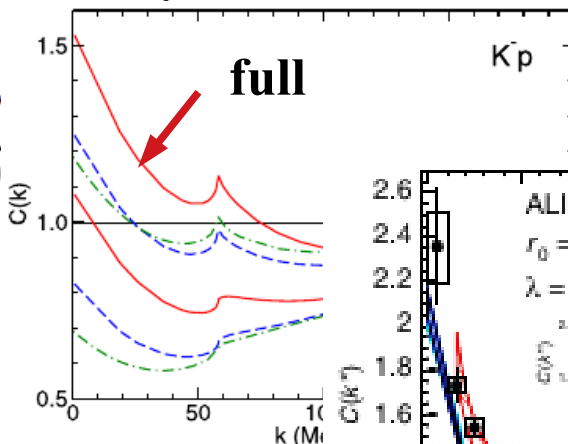
Kamiya+(in prep.).
(Smaller cusp than χ EFT.
CC simulates res. source !)

Do I have 3 minutes ?

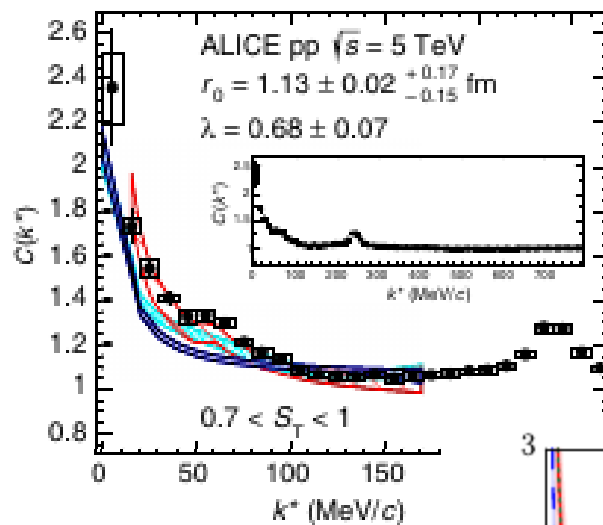
pK - correlation



*S. Cho+ [ExHIC], PPNP95('17)279.
(Insufficient coupled-channel effects)*



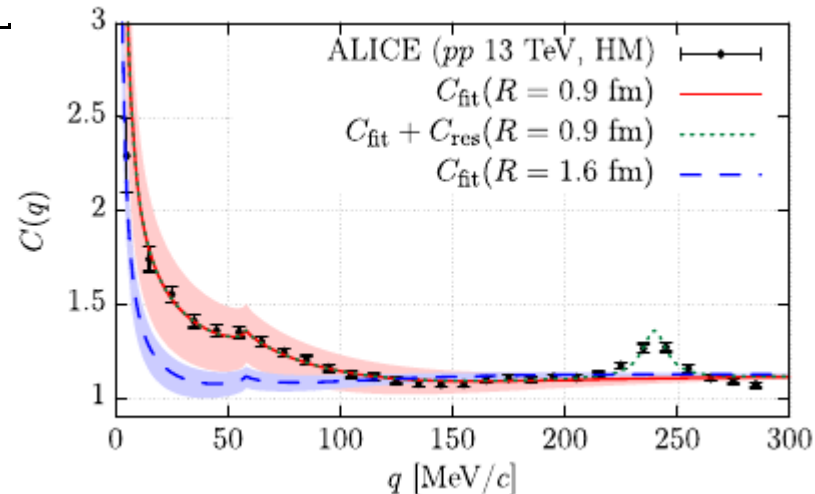
*J. Haidenbauer, NPA981('19)1.
(w/ CC effects, w/o Coulomb)*



*S. Acharya+[ALICE],
PRL124('20)092301*

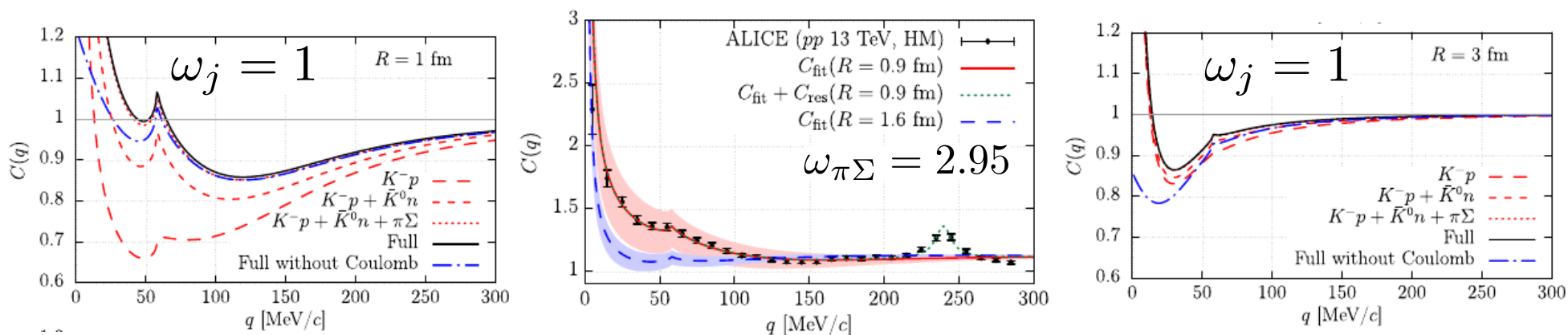
Source size dep. may clarify bound state nature of $\Lambda(1405)$.

*Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.
(Chiral SU(3) dynamics)*

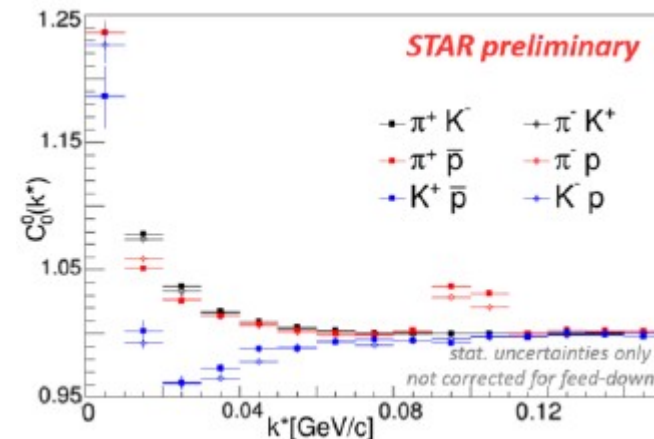
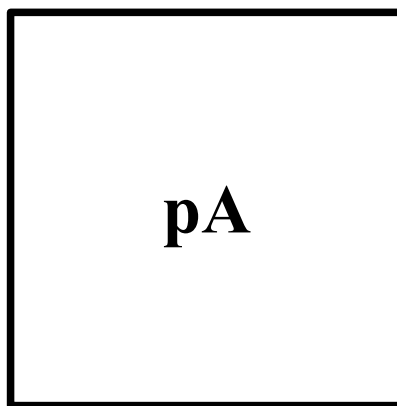
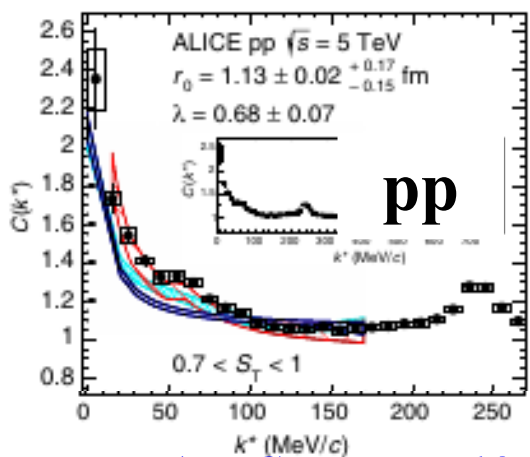


Source Size Dependence of $C(pK^-)$

- Coupled-channel effects are suppressed when R is large, and “pure” pK^- wave function may be observed in HIC.
- Can we deduce $(\text{Re } a_0, \text{Im } a_0)$ at precision comparable to that in SIDDHARTA kaonic hydrogen data ?



Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.



S. Acharya+[ALICE], PRL124('20)092301 Siejka+[STAR, preliminary], NPA982 ('19)359.

Summary

- 高エネルギー原子核衝突 (pp, pA, AA) からの2粒子運動量相関関数は未知のハドロン間相互作用の制限に有効である。
 - “初期条件”はきれいでないが、 $|\psi|^2$ の情報は得られる
 - 2015年～現在までに多くの相関関数が測定され、実験データ情報を持つハドロン間相互作用は年々増えている。(RHIC, LHC はハドロン工場！負けるな J-PARC！)
 - Exotic hadron (ハドロン束縛・共鳴状態) の探索にも役立つ。
- さらに理論的に調べるべき多くの「対」がある
 - 様々なハドロン対 (LQCD, χ EFT で予言がある対が better, Charmed hadron もあり)、ハドロン - 重陽子 (E.g., Ogata+(2103.00100))、**3体相関関数 (e.g. Λpp)**、...
- データ自体から散乱長などの散乱パラメータが引き出せないか？

Thank you for your attention !

Coauthors of *arXiv:1908.05414* ($p\Omega$, $\Omega\Omega$) and *arXiv:1911.01041* (pK^-)

K. Morita



S. Gongyo



T. Hatsuda



T. Hyodo



Y. Kamiya



K. Sasaki



ALICE

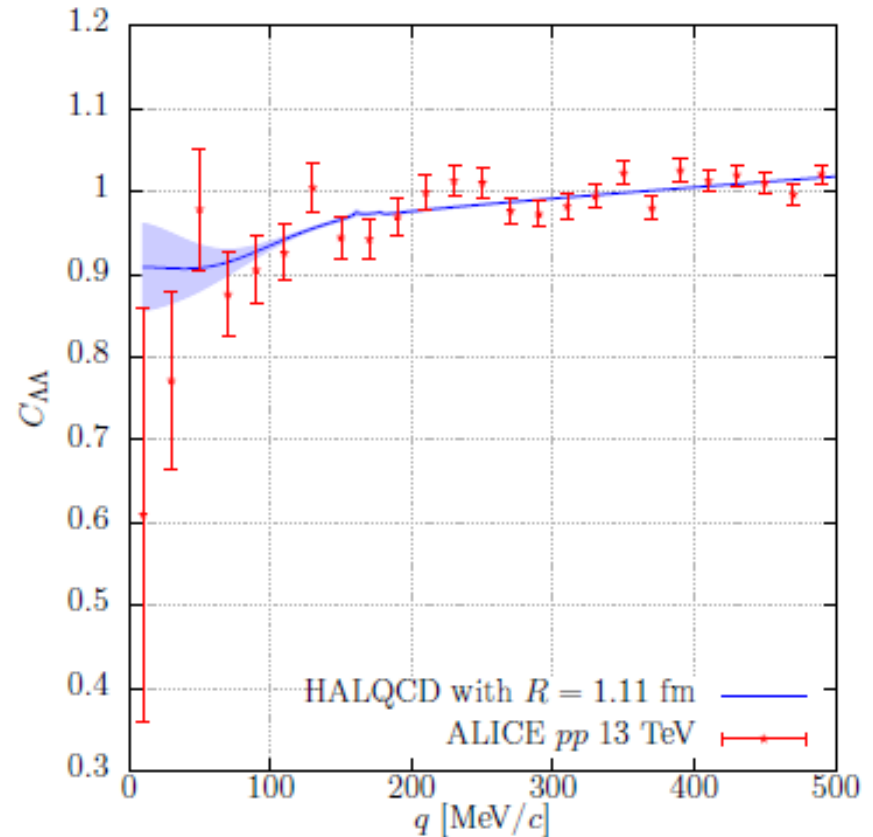
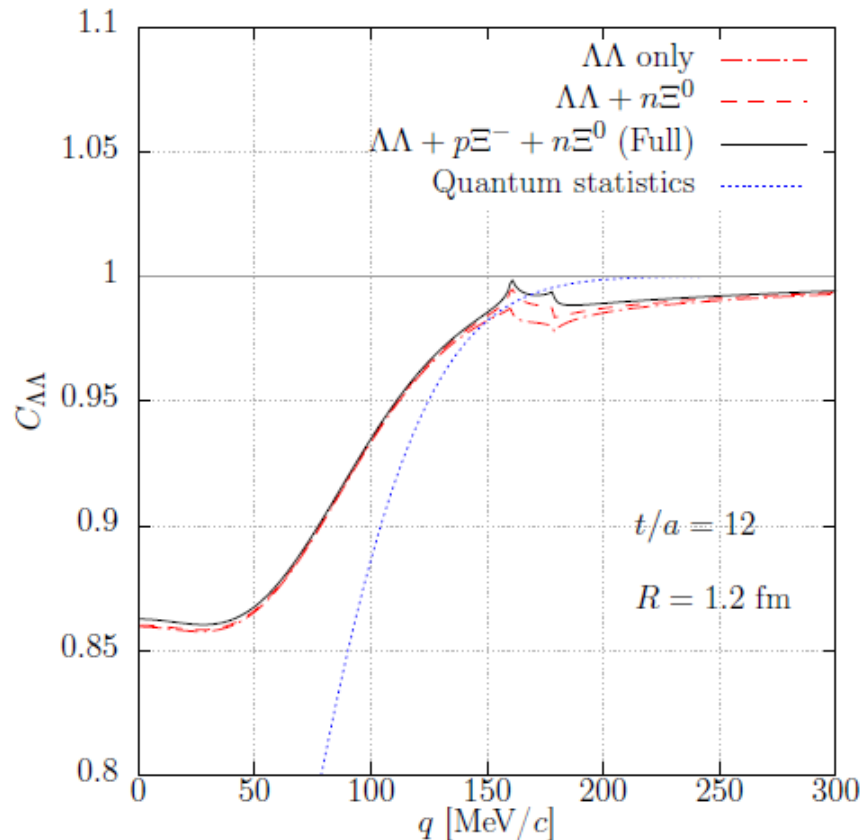
W. Weise



Λ correlation function

■ $\Lambda\Lambda$ correlation function

- Strong enhancement from pure Coulomb CF
- $N\Xi$ source effect is visible only around threshold.
- Calculated CF agrees with ALICE data.



Kamiya+, in prep.; Acharya+(ALICE), Nature ('20)

Lednicky-Lyuboshitz formula

Lednický-Lyuboshits (LL) model

■ Lednický-Lyuboshits analytic model

- **Asymp. w.f. + Eff. range corr. + $\psi^{(-)} = [\psi^{(+)}]^*$**

$$\psi_0(r) \rightarrow \psi_{\text{asy}}(r) = \frac{e^{-i\delta}}{qr} \sin(qr + \delta) = \mathcal{S}^{-1} \left[\frac{\sin qr}{qr} + f(q) \frac{e^{iqr}}{r} \right]$$

$$\begin{aligned} \Delta C_{\text{LL}}(q) &= \int dr S_{12}(r) (|\psi_{\text{asy}}(r)|^2 - |j_0(qr)|^2) \\ &= \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\text{eff}}}{R}\right) + \frac{2\text{Re}f(q)}{\sqrt{\pi}R} F_1(x) - \frac{\text{Im}f(q)}{R} F_2(x) \end{aligned}$$

($x = 2qR$, $R = \text{Gaussian size}$, F_1, F_2, F_3 : Known functions)

■ Phase shifts

$$q \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 + \mathcal{O}(q^4) \rightarrow \delta \simeq -a_0 q + \mathcal{O}(q^3)$$

$$\sin(qr + \delta) \simeq \sin(q(r - a_0) + \dots) \quad \text{Node at } \mathbf{r} \sim \mathbf{a}_0$$

for small q

$C(q)$ in the low momentum limit

- Correlation function at small q (and $r_{\text{eff}}=0$) $\rightarrow F_1=1, F_2=0, F_3=1$

$$\Delta C_{\text{LL}}(q) \rightarrow \frac{|f(0)|^2}{2R^2} + \frac{2\text{Re}f(0)}{\sqrt{\pi}R} \quad (q \rightarrow 0)$$

$$f(q) = (q \cot \delta - iq)^{-1} \simeq \left(-\frac{1}{a_0} + \frac{1}{2}r_{\text{eff}}q^2 - iq \right)^{-1} \rightarrow -a_0$$

$$C_{\text{LL}}(q \rightarrow 0) = 1 + \frac{a_0^2}{2R^2} - \frac{2a_0}{\sqrt{\pi}R} = 1 - \frac{2}{\pi} + \frac{1}{2} \left(\frac{a_0}{R} - \frac{2}{\sqrt{\pi}} \right)^2$$

$$1 - 2/\pi \simeq 0.36, \quad \sqrt{\pi}/2 \simeq 0.89$$

$C(q \rightarrow 0)$ takes a minimum of **0.36** at $R/a_0 = 0.89$ in the LL model.

$p\Omega^-$ correlation

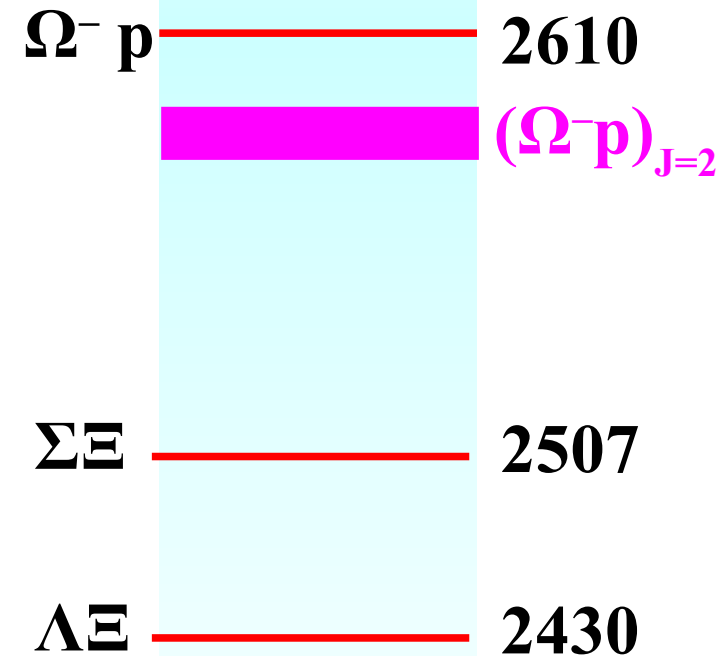
Where is dibaryon ?

- Deuteron = First dibaryon (pn bound state)
 - H-particle: 6-quark state (uuddss)
 - Predicted (*Jaffe ('77)*), Ruled-out ($\Lambda\Lambda$ nucl., *Takahashi+('01)*), Suggested as a resonance in exp. (*Yoon+ ('07)*) or as a bound state of ΞN (*HAL QCD ('16)*)
 - Dibaryon would appear in channels, where *Oka ('88)*, *Gal ('16)*
 - The Pauli blocking of quarks does not operate,
 - and the Color-magnetic interaction is attractive
- Examples: H(= $\Lambda\Lambda$ - $N\Xi$ - $\Sigma\Sigma$), $N\Omega$, $N\Sigma^*$, $d^*(=\Lambda\Lambda)$.

Let us examine the existence of dibaryon states by using the correlation function !

Ω N dibaryon

- Ω : sss, $J\pi=3/2^+$, $M=1672$ MeV
- Is there an ΩN bound state ($S=-3$ dibaryon) ?
 - Predicted as a dibaryon candidate
Goldman+ ('87), *Oka* ('88), *Gal* ('16)
 - Lattice QCD predicts a bound state with narrow width for $J=2$ (5S_2)
(Coupling to octet-octet with $L=2$)
Etminan+ (HAL QCD)('14),
Iritani+ (HAL QCD) ('19)
 - Meson exchange potential is also proposed
T. Sekihara, Y. Kamiya, T. Hyodo,
PRC98 ('18) 015205
 - Correlation function is measurable !
Adam+ (STAR)('19), ALICE, in prep.

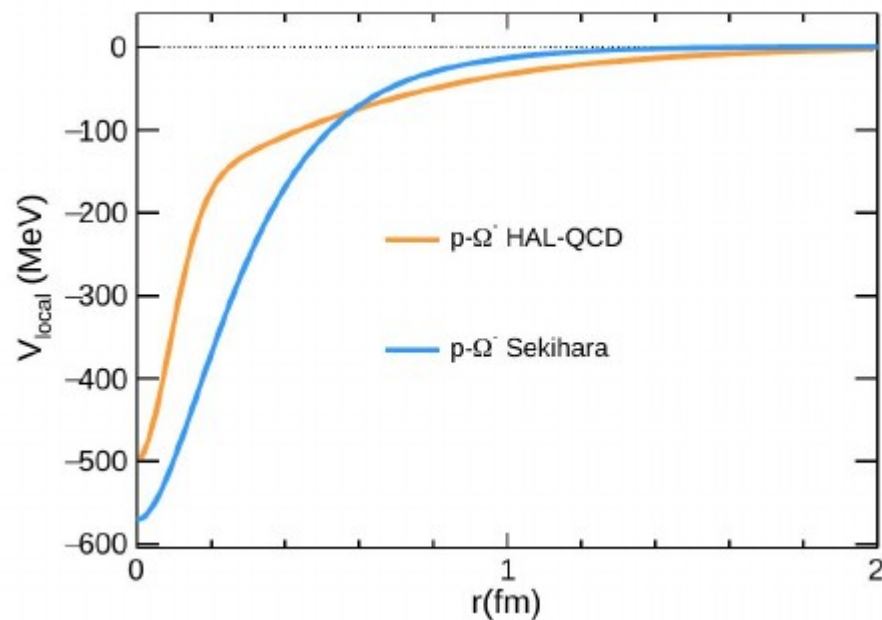
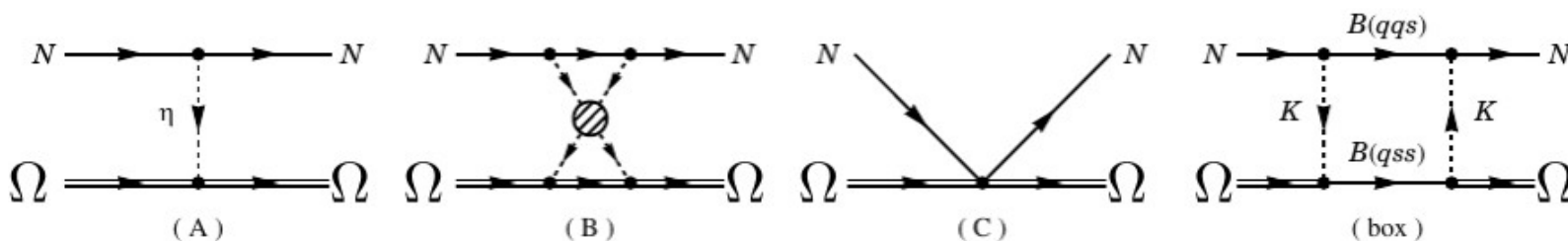


Meson Exchange Potential

■ Meson exchange $N\Omega$ potential

T. Sekihara, Y. Kamiya, T. Hyodo, PRC98 ('18) 015205

- η meson exchange, σ exchange, contact term, box diagram.
- Contact term is fitted to **the scatt. length** of HAL QCD potential.



Calculation Details

K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC101('20)015201

■ $N\Omega$ potential from HAL QCD Collab.

Etminan+(HAL QCD) ('14), Iritani+ (HAL QCD)('19)

- $J=1$ potential is uncertain \rightarrow Three models

Strong abs. at $r < r_0$ ($r_0 \sim 2$ fm) (*Morita+('16)*) (Standard)

Complete absorption $\chi(J=1) = 0$ (Minimum)

Same w.f. as that with $J=2$, $\chi(J=1) = \chi(J=2)$ (Reference)

- Statistical Error can be evaluated by using Jackknife potentials.

■ Coulomb potential enhances CF even without strong int.

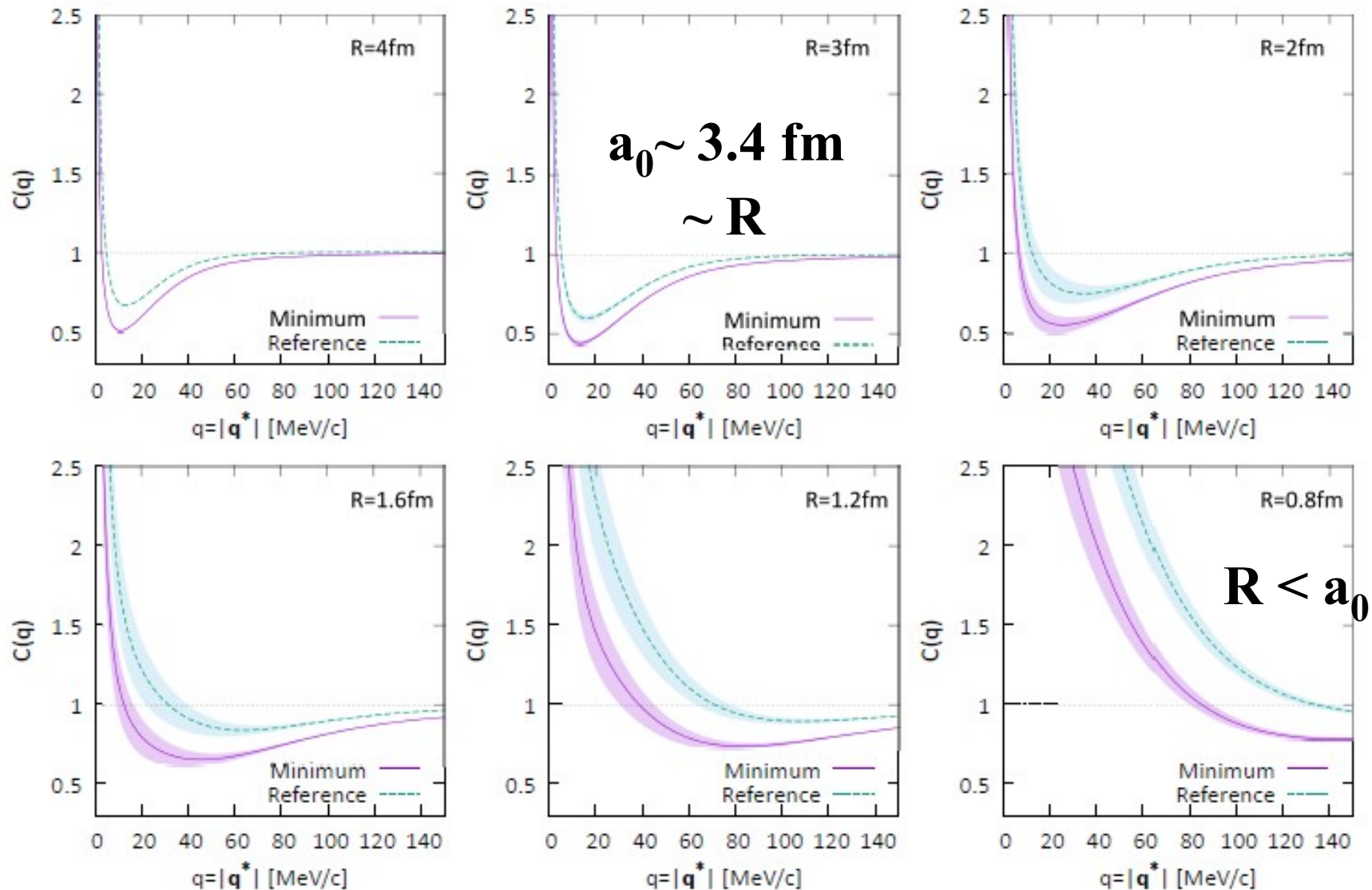
\rightarrow Small-Large ratio of CF (*Morita+('16)*)

- Large source \rightarrow Coulomb force dominate

Small source \rightarrow Visible strong interaction effects

■ Source function: Blast wave, Gaussian source

Source Size Dependence of Correlation Function



Gaussian Source

*K. Morita, S. Gongyo, T. Hatsuda,
T. Hyodo, Y. Kamiya, AO ('20)*

K^-p correlation

$K^- p$ interaction

■ $\Lambda(1405)$ $\bar{K}N$ quasi-bound state

*Dalitz, Tuan ('60); Koch ('94); Kaiser, Siegel, Weise ('95);
AO, Nara, Koch ('97); Jido+('03)*

● Positive scattering length in K^- atoms

*M.Iwasaki et al. PRL78('97)3067;
M.Bazzi et al. [SIDDHARTA Collab.],
PLB704('11)113.*

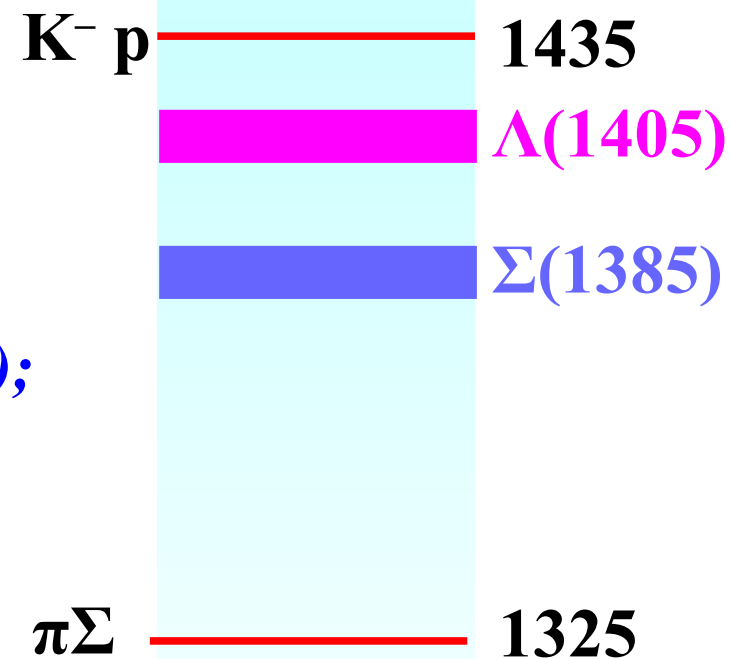
■ Kaonic nuclei ?

*Nogami ('63); Akaishi, Yamazaki ('02);
Shevchenko, Gal, Mares ('07); Ikeda, Sato ('07);
Dote, Hyodo, Weise ('09)*

→ Needs precise info. on $\bar{K}N$ int.

■ Scattering amplitude and Potential fitting scattering and SIDDARTA data in chiral approach

*Ikeda, Hyodo, Weise ('11,'12),
Miyahara, Hyodo ('16)*



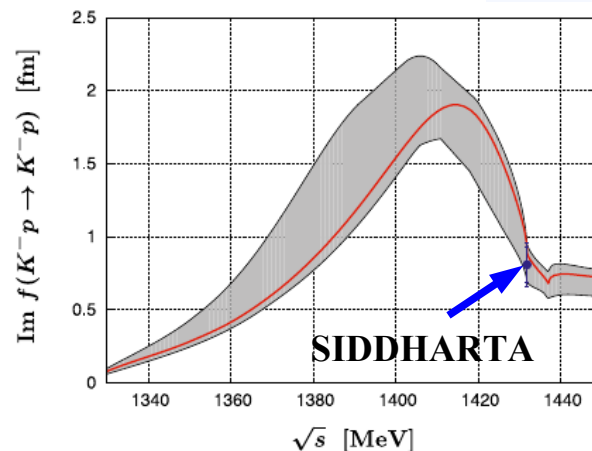
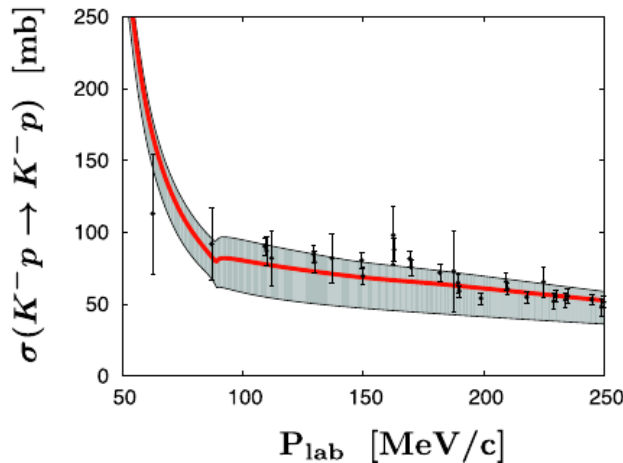
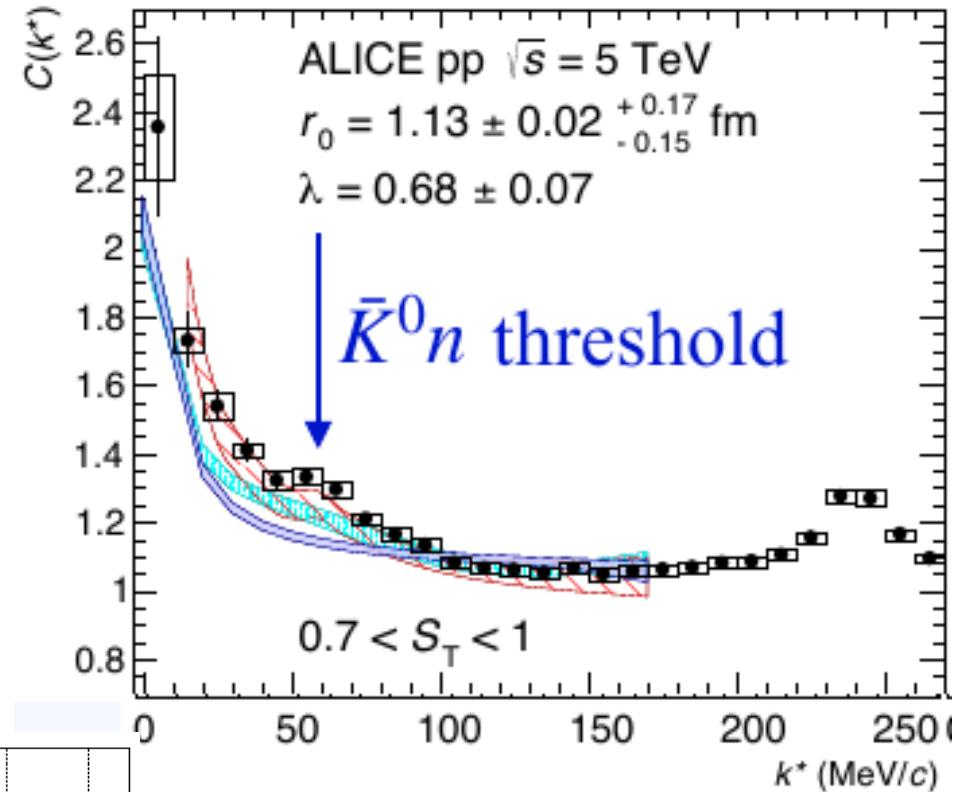
How about $K^- p$ correlation ?

$K^- p$ correlation function data

■ $K^- p$ correlation function from high-multiplicity events of pp collisions

S. Acharya et al. (ALICE), arXiv:1905.13470

- High precision data from low to high momentum !
c.f. Previous scatt. data & Kaonic atom data.
- Enhanced at low k , cusp, $\Lambda(1520)$, ...



Red: Kyoto model
 Blue: Julich model
 grey: Coulomb

Y. Ikeda, T. Hyodo, W. Weise, NPA881 ('12) 98

$\bar{K}N-\pi\Sigma-\pi\Lambda$ Scattering Amplitude and Potential

■ Amplitude in chiral SU(3) coupled-channels dynamics

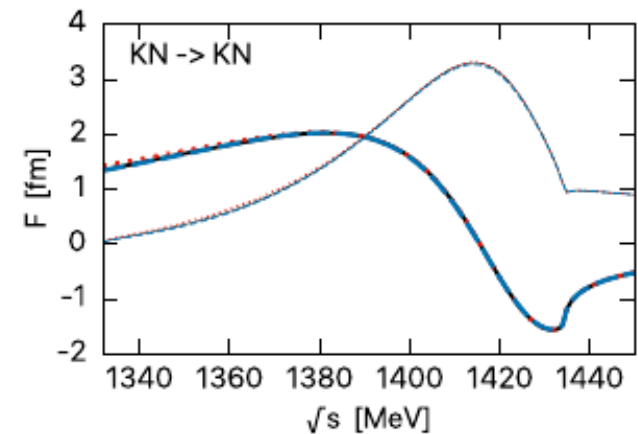
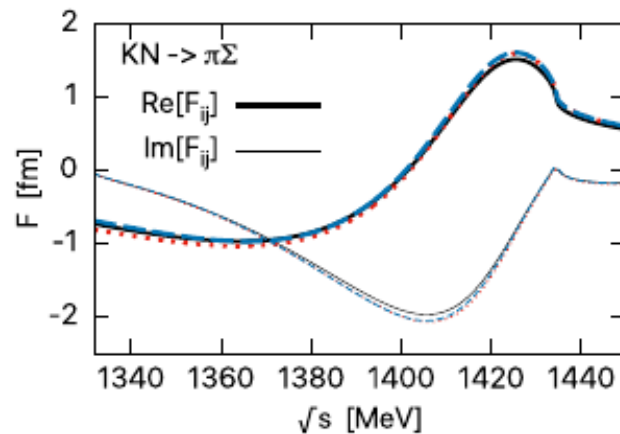
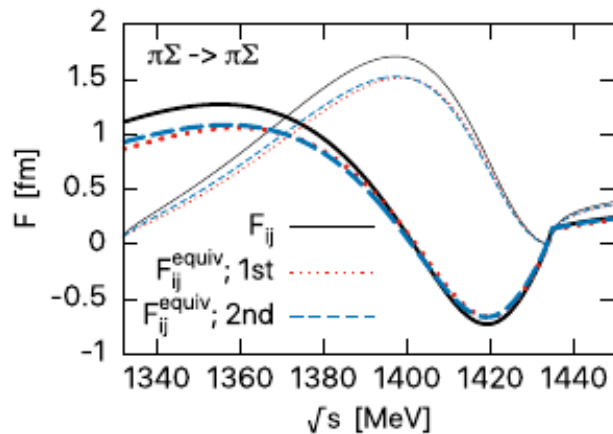
Y. Ikeda, T. Hyodo, W. Weise, NPA881 ('12) 98

- NLO meson-baryon effective Lagrangian ($\bar{K}N-\pi\Sigma-\pi\Lambda$)
+ fit of Kaonic Hydrogen, Cross Section, Threshold branching ratio

■ Coupled-channels potential

K. Miyahara, T. Hyodo, W. Weise, PRC98('18)025201

- Potential fitted to IHW amplitude



Y. Ikeda, T. Hyodo, W. Weise, NPA881 ('12) 98

K. Miyahara, T. Hyodo, W. Weise, PRC98('18)025201

Correlation Function with Coupled-Channels Effects

J. Haidenbauer, NPA 981('19)1; R. Lednicky, V. V. Lyuboshits, V. L. Lyuboshits, Phys. At. Nucl. 61('98)2950.

- **Single channel, w/o Coulomb (non-identical pair)**

$$C(\mathbf{q}) = \underline{1} + \int d\mathbf{r} S(\mathbf{r}) \left[\underline{|\chi^{(-)}(r, q)|^2} - \underline{|j_0(qr)|^2} \right]$$

- **Single channel, w/ Coulomb**

$$C(\mathbf{q}) = \int d\mathbf{r} S(\mathbf{r}) \left[\underline{|\varphi^{C,\text{full}}(\mathbf{q}, \mathbf{r})|^2} + \underline{|\chi^{C,(-)}(r, q)|^2} - \underline{|j_0^C(qr)|^2} \right]$$

Full free

Coulomb w.f.

s-wave w.f.

with Coul.

s-wave

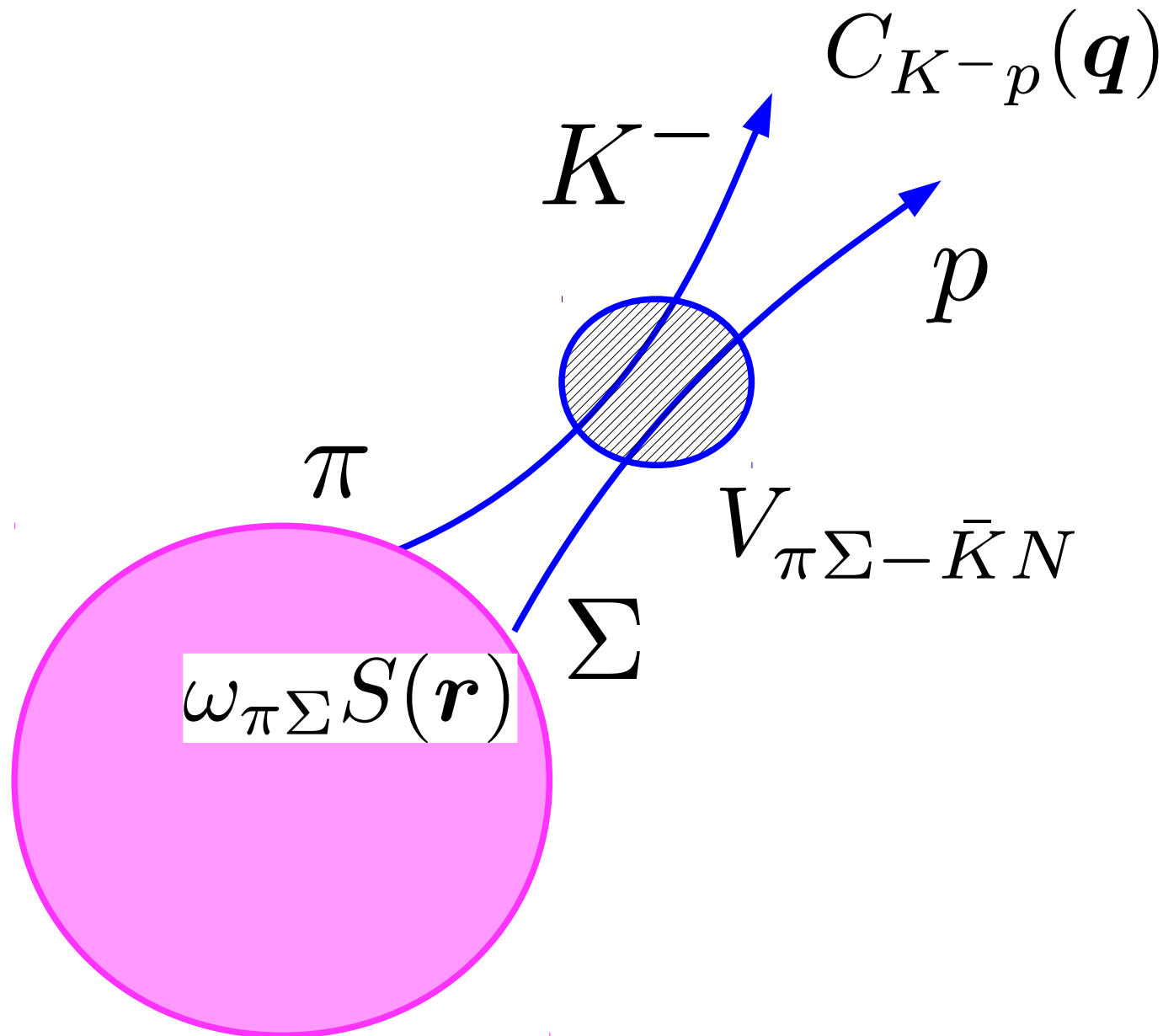
Coul. w.f.

- **Coupled channel, w/ Coulomb**

$$C_i(\mathbf{q}) = \int d\mathbf{r} S_i(\mathbf{r}) \left[\underline{|\varphi^{C,\text{full}}(\mathbf{q}, \mathbf{r})|^2} + \underline{|\chi_i^{C,(-)}(r, q)|^2} - \underline{|j_0^C(qr)|^2} \right] \\ + \sum_{j \neq i} \omega_j \int d\mathbf{r} S_j(\mathbf{r}) \underline{|\chi_j^{C,(-)}(r, q)|^2} \quad \text{s-wave w.f. in j-th channel}$$

Outgoing B.C. in the i-th channel, $\omega_j =$ Source weight ($\omega_j=1$)

Correlation Function with Coupled-Channels Effects



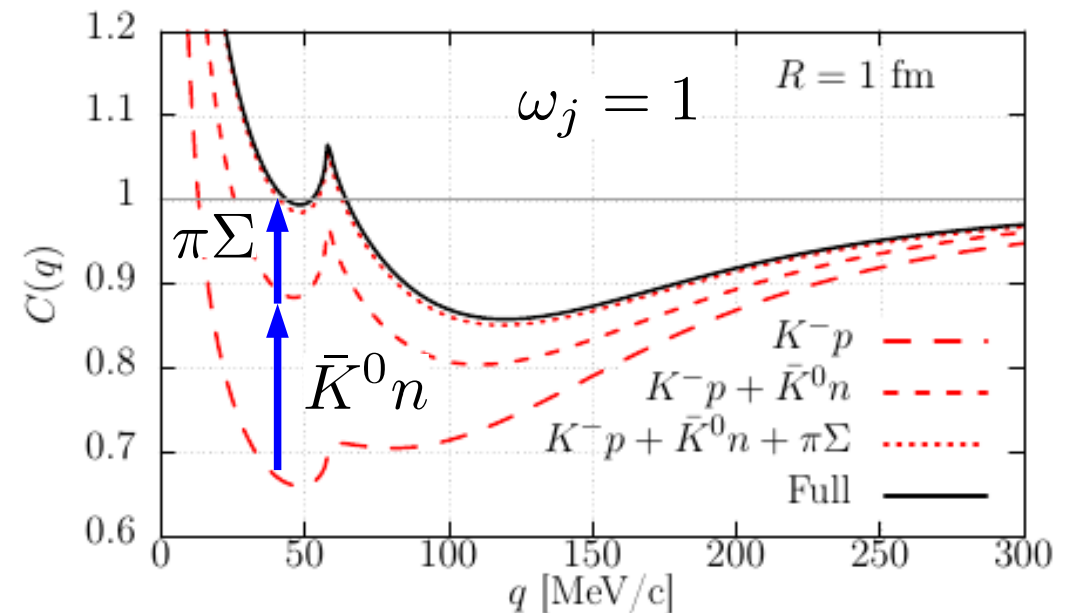
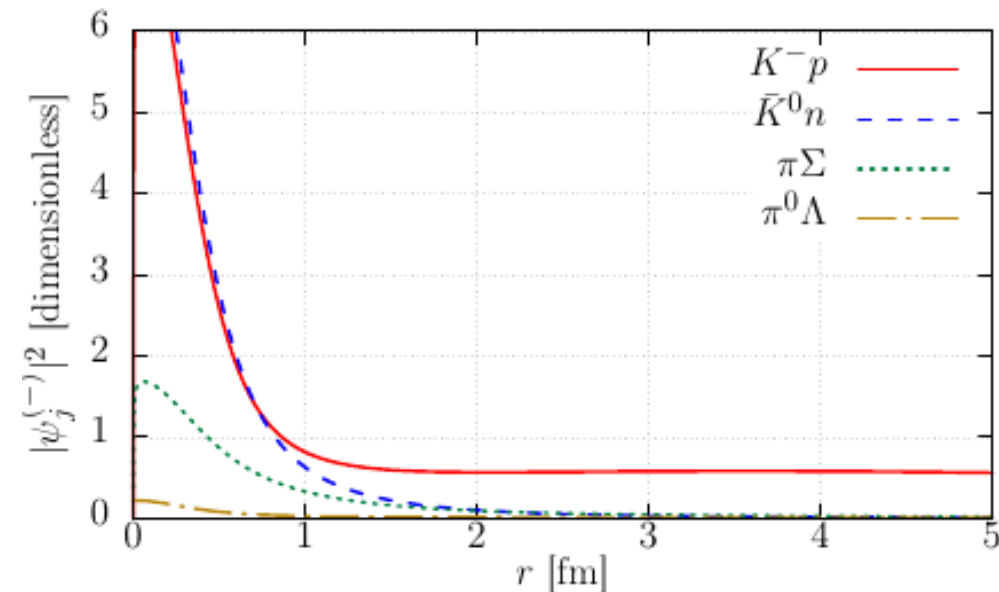
Correlation Function from Chiral $SU(3)$ Potential (1)

- **Corr. Fn. from Chiral $SU(3)$ coupled-channels potential + Coulomb + threshold difference (for the first time !)**

Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, arXiv:1911.01041

- **Coupled-channels effect**

- **W.f. of other channels than $K^- p$ decay in $r < 1$ fm.**
- **But they contribute to corr. fn. meaningfully.**



Correlation Function from Chiral $SU(3)$ Potential (2)

- “Free” parameters

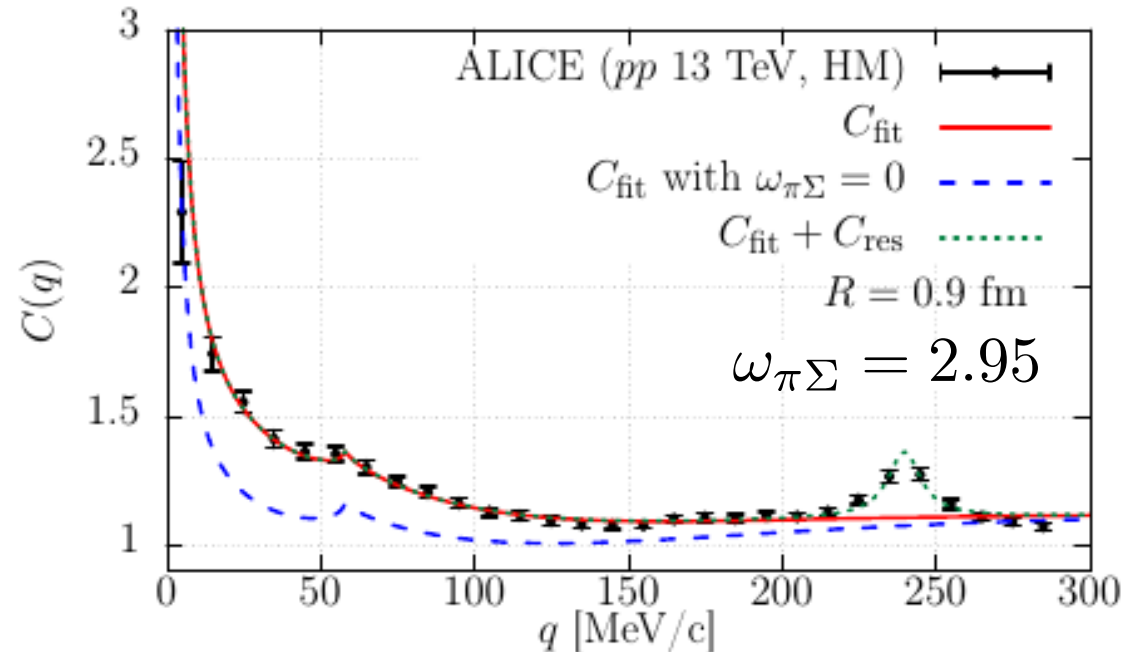
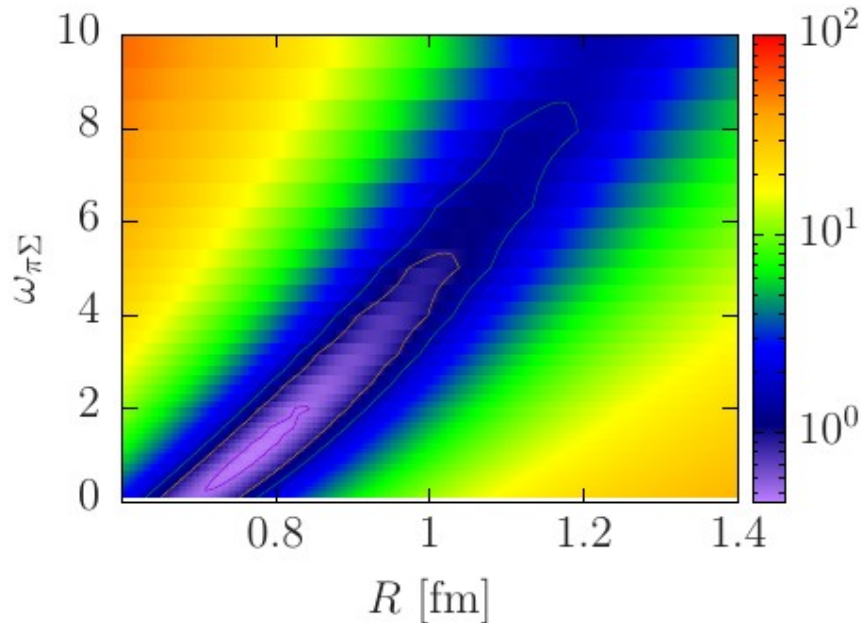
= Source Size R , Source Weight ω_j ← Th+Exp.

+ Normalization + Pair purity (λ) ← Exp.

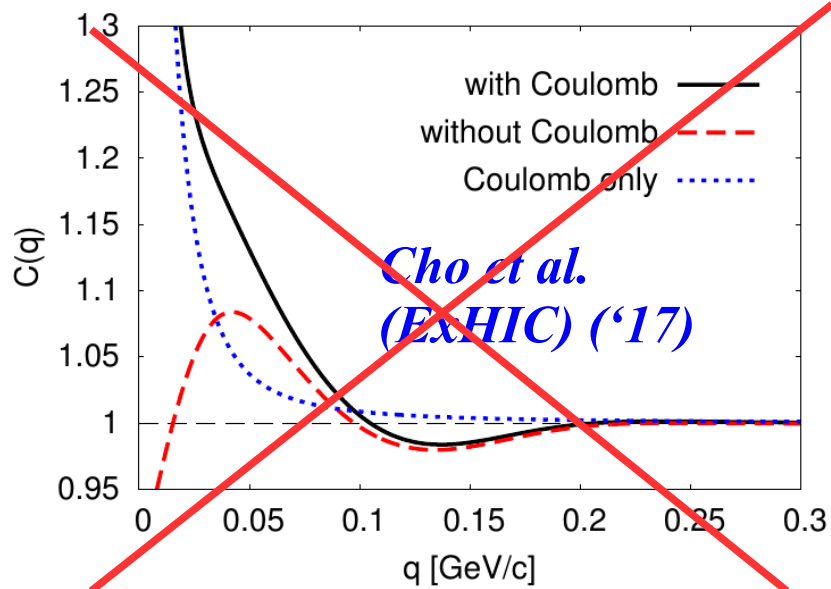
- Larger $R \rightarrow$ Smaller couple-channels effect from $\pi\Sigma$ (Favorable values of R and ω_j are correlated)

- Simple statistical model estimate

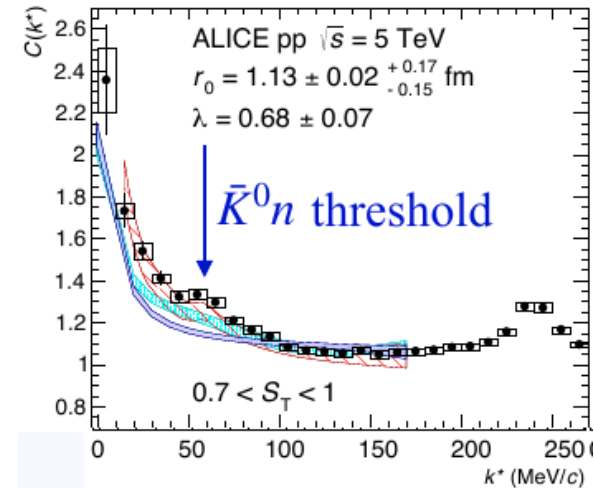
$$\omega_{\pi\Sigma} \sim \exp[(m_K + m_N - m_\pi - m_\Sigma)/T] \sim 2.$$



Comparison with other estimates

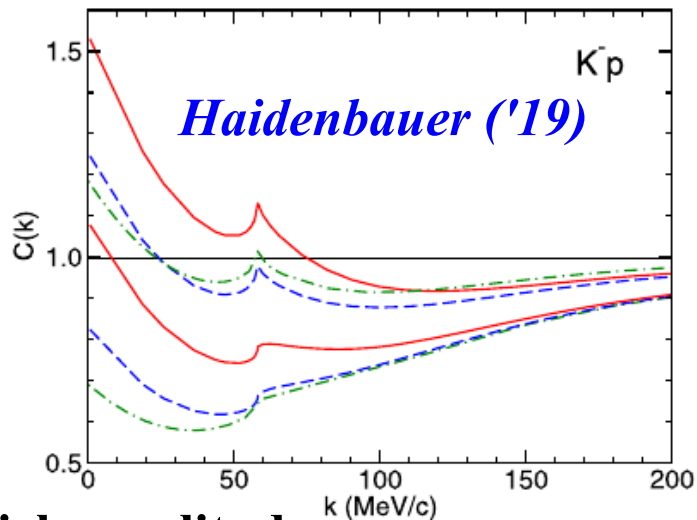


No $\bar{K}^0n, \pi\Sigma$ source
No threshold diff.

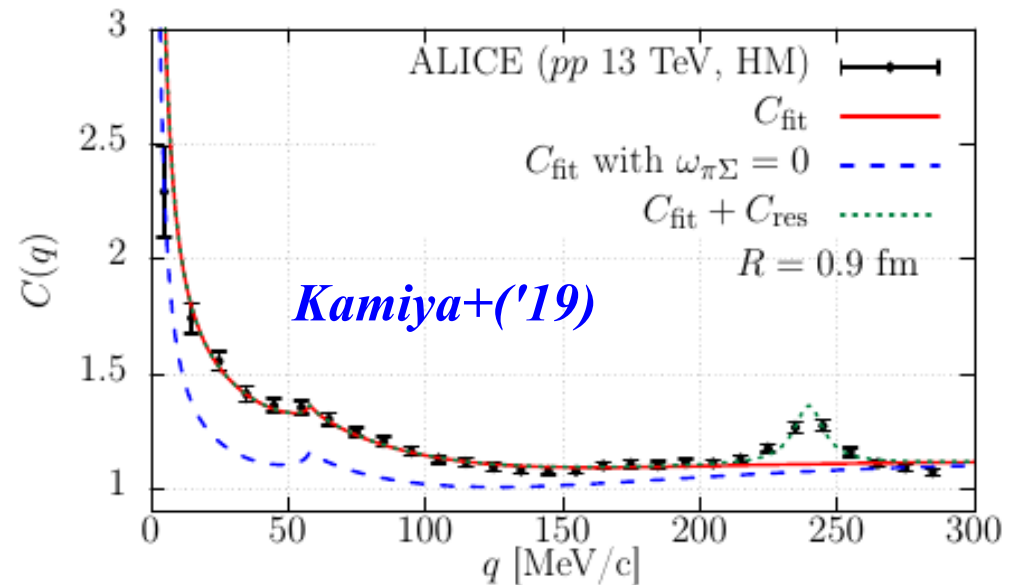


ALICE

Kyoto model (\sim Cho+('17)) \rightarrow accident
Julich model (Blue) Gamow corr. for Coulomb

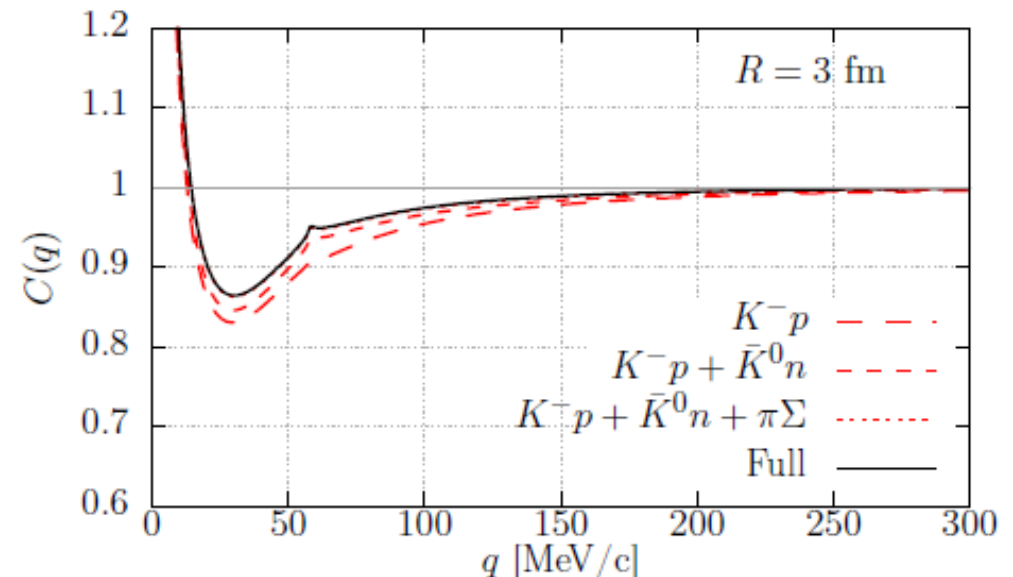
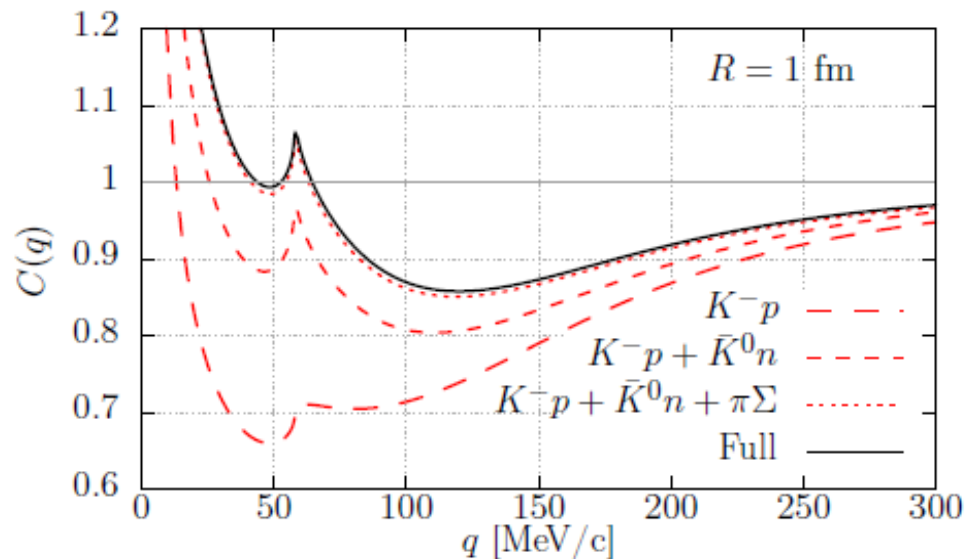


Julich amplitude
Consistent with Kamiya+('19) w/o Coulomb



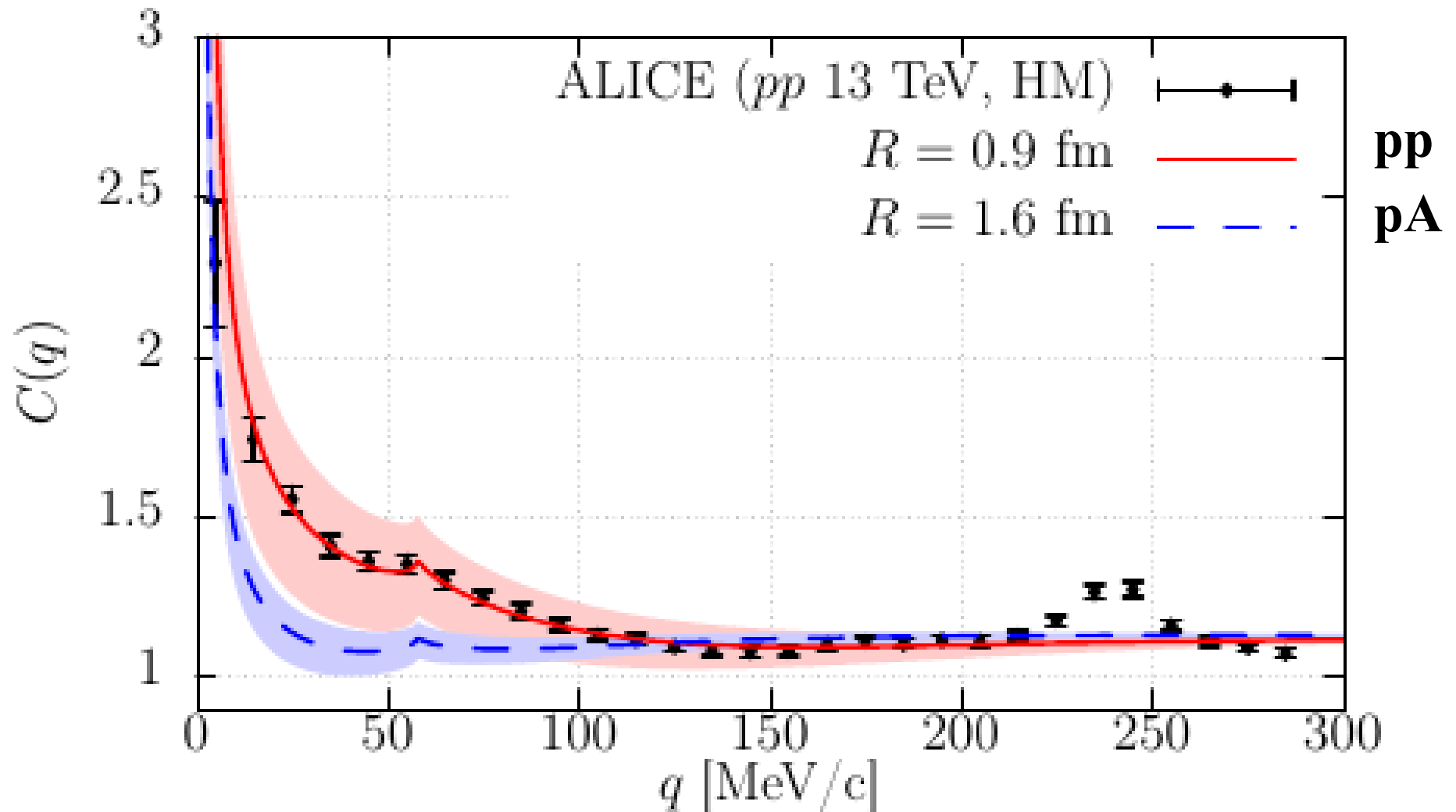
Source Size Dependence (2)

- Experimental confirmation of coupled-channels contribution
→ Source size dependence
 - Channel w.f. other than $K^- p$ are localized at around $r=0$. (Outgoing boundary condition for $K^- p$)
 - Contribution of $\pi\Sigma$ source is suppressed for larger R .



Source Size Dependence (2)

- Corr. Fn. from pA & AA collisions will elucidate the role of $\pi\Sigma$
 - $R \sim 1.6$ fm \rightarrow $\pi\Sigma$ effects are suppressed.

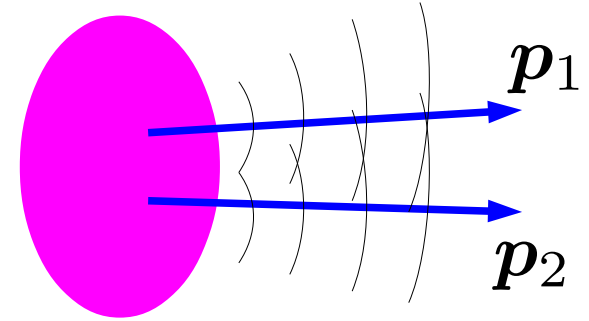


Correlation functions

Correlation Function: Standard and Non-Std usage

■ Correlation function

- Correlation from the quantum statistics and the final state int. under indep. particle production assumption lead KP formula, *Koonin('77), Pratt+('86), Lednicky+('82)*



$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{N_{12}(\mathbf{p}_1, \mathbf{p}_2)}{N_1(\mathbf{p}_1)N_2(\mathbf{p}_2)} \simeq \int d\mathbf{r} \underbrace{S_{12}(\mathbf{r})}_{\text{source fn.}} \underbrace{|\varphi_q(\mathbf{r})|^2}_{\text{relative w.f.}}$$

■ Standard: Source size from CF (HBT-GGLP effects)

Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)

- CF of free identical scalar bosons from spherical Gaussian source
 $\phi(\mathbf{r}) = \sqrt{2} \cos \mathbf{q} \cdot \mathbf{r} \rightarrow C(q) = 1 + \exp(-4R^2 q^2)$

■ Non-standard: hadron-hadron interaction from CF

- CF of non-identical pair from Gaussian source

R. Lednicky, V. L. Lyuboshits ('82); K. Morita, T. Furumoto, AO ('15)

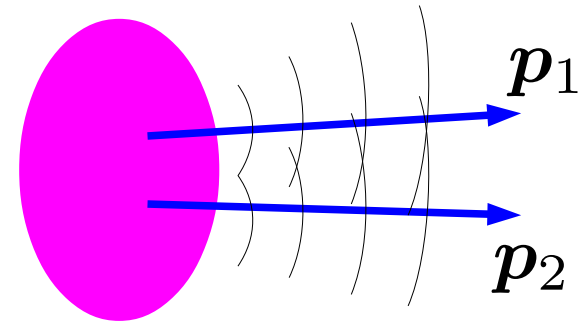
$$C(\mathbf{q}) = 1 + \int d\mathbf{r} S(r) \{ |\varphi_0(r)|^2 - |j_0(qr)|^2 \} \quad (\varphi_0 = \text{s-wave w.f.})$$

CF shows how much $|\varphi|^2$ is enhanced $\rightarrow V_{hh}$ effects !

2粒子運動量相関関数

■ 粒子の放出点分布関数

$$N_i(\mathbf{p}) = \int d^4x S_i(x, \mathbf{p})$$



■ 2粒子運動量分布

- 2粒子が独立に作られ、終状態の波動関数で相関が作られるとする。

$$\begin{aligned} N_{12}(\mathbf{p}_1, \mathbf{p}_2) &\simeq \int d^4x d^4y S_1(x, \mathbf{p}_1) S_2(y, \mathbf{p}_2) \underbrace{|\Psi_{\mathbf{p}_1, \mathbf{p}_2}(x, y)|^2}_{\text{2 粒子 w.f.}} \\ &\simeq \int d^4x d^4y S_1(x, \mathbf{p}_1) S_2(y, \mathbf{p}_2) \underbrace{|\varphi_q(\mathbf{r})|^2}_{\text{相対波動関数}} \end{aligned}$$

■ 相関関数

$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{N_{12}(\mathbf{p}_1, \mathbf{p}_2)}{N_1(\mathbf{p}_1)N_2(\mathbf{p}_2)} \simeq \int d\mathbf{r} S_{12}(\mathbf{r}) |\varphi_q(\mathbf{r})|^2$$

2粒子運動量相関関数

- 例：同種自由ボソン ($J=0$)、ガウス型放出関数、同時刻、非相対論
 - 重心・相対座標を分離して考える。

$$S(\mathbf{x}, \mathbf{p}) \propto \exp \left[-\frac{\mathbf{x}^2}{2R^2} - \frac{\mathbf{p}^2}{2MT} \right]$$

$$S(\mathbf{x}, \mathbf{p}_1) S(\mathbf{y}, \mathbf{p}_2) \propto \exp \left[-\frac{R_{\text{cm}}^2}{R^2} - \frac{\mathbf{r}^2}{4R^2} - \frac{\mathbf{P}^2}{4MT} - \frac{\mathbf{q}^2}{2\mu T} \right]$$

$$\begin{aligned} \Psi_{\mathbf{p}_1, \mathbf{p}_2}(\mathbf{x}, \mathbf{y}) &\propto \frac{1}{\sqrt{2}} \left[e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} + e^{i\mathbf{p}_1 \cdot \mathbf{y} + i\mathbf{p}_2 \cdot \mathbf{x}} \right] \\ &= e^{i\mathbf{P} \cdot \mathbf{R}_{\text{cm}}} \times \sqrt{2} \cos \mathbf{q} \cdot \mathbf{r} \end{aligned}$$

- 相関関数

$$\begin{aligned} C(\mathbf{q}) &= (4\pi R^2)^{-3/2} \int d\mathbf{r} \exp \left[-\frac{\mathbf{r}^2}{4R^2} \right] 2 \cos^2 \mathbf{q} \cdot \mathbf{r} \\ &= 1 + \exp(-4q^2 R^2) \end{aligned}$$

相関関数から粒子放出源のサイズが分かる！

How can we measure the radius of a star ?

Two photon intensity correlation

Hanbury Brown & Twiss, Nature 10 (1956), 1047.

- Simultaneous two photon observation probability is enhanced from independent emission cases
→ angular diameter of Sirius=0.0063”

最近の測定

(Wikipedia)

5.936±0.016 ミリ秒

A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

By R. HANBURY BROWN

Jodrell Bank Experimental Station, University of Manchester

AND

DR. R. Q. TWISS

Services Electronics Research Laboratory, Baldock

NATURE November 10, 1956 VOL. 178

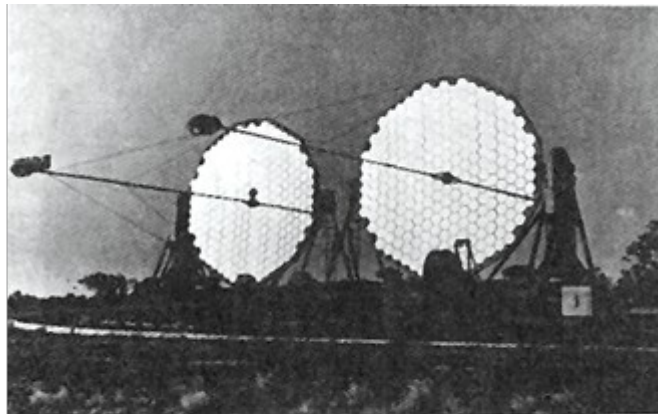


Figure 2. Picture of the two telescopes used in the HBT experiments. The figure was extracted from Ref.[1].

HBP telescope (from Goldhaber, ('91))

$$= \langle I_1 I_2 \rangle / \langle I_1 \rangle \langle I_2 \rangle$$

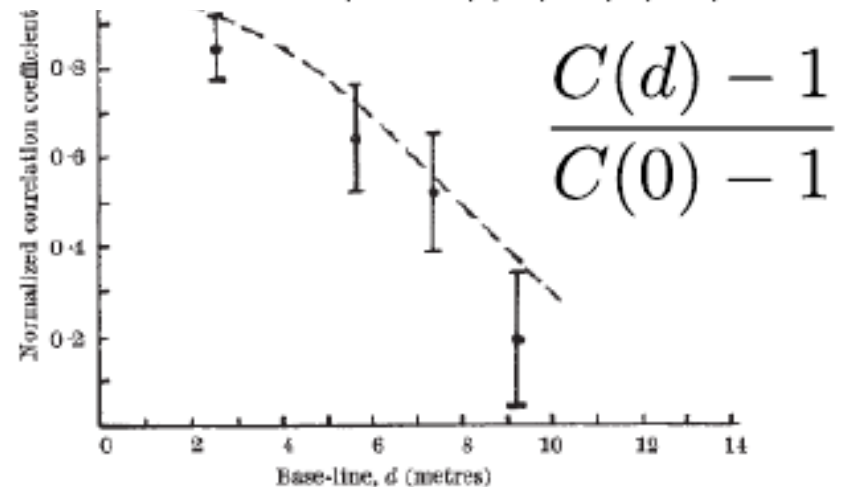


Fig. 2. Comparison between the values of the normalized correlation coefficient $I^N(d)$ observed from Sirius and the theoretical values for a star of angular diameter 0.0063". The errors shown are the probable errors of the observations

HBT ('56)

Two particle intensity correlation

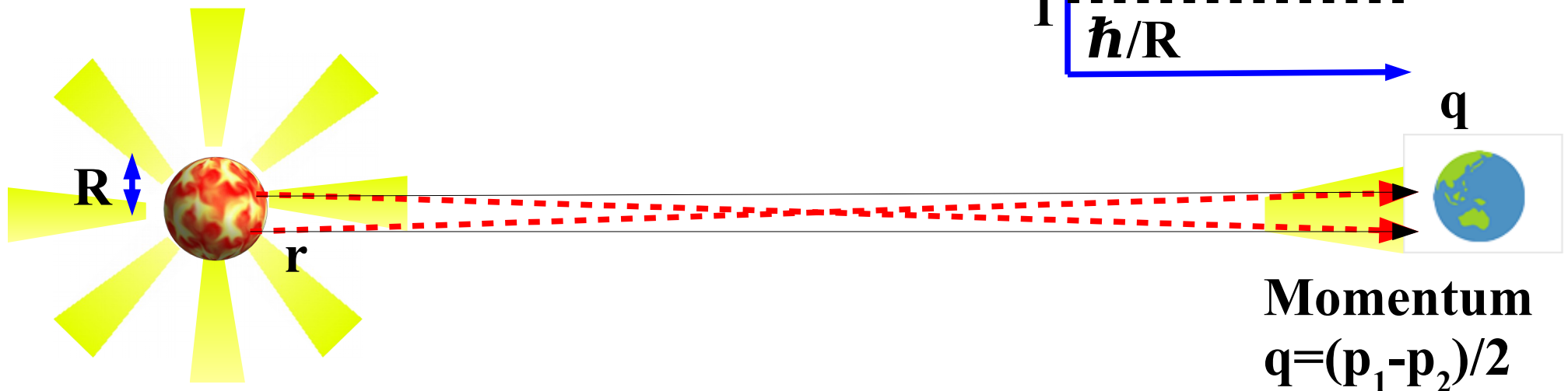
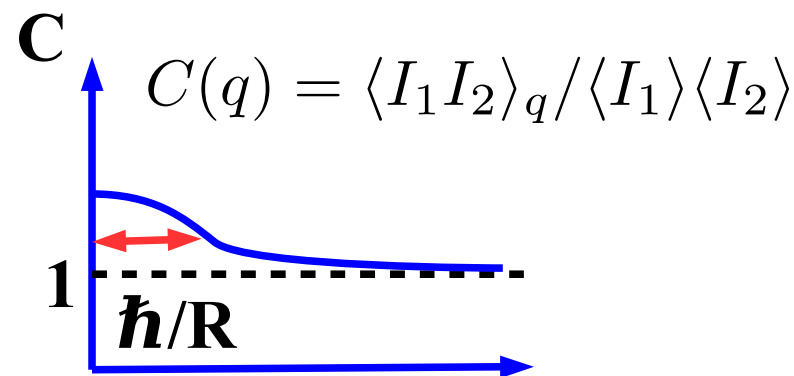
- Wave function symmetrization from quantum statistics

$$C(\mathbf{q}) = \int d^3r S(\mathbf{q}, \mathbf{r}) \left| \frac{1}{\sqrt{2}} (e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}}) \right|^2 \simeq \underline{1 + \exp(-4q^2 R^2)}$$

Source fn.
(\mathbf{r} =relative coordinate)
(symmetrized w.f.)²

Static spherical source case

→ Small relative momenta are favored due to symmetrization of the relative wave function.



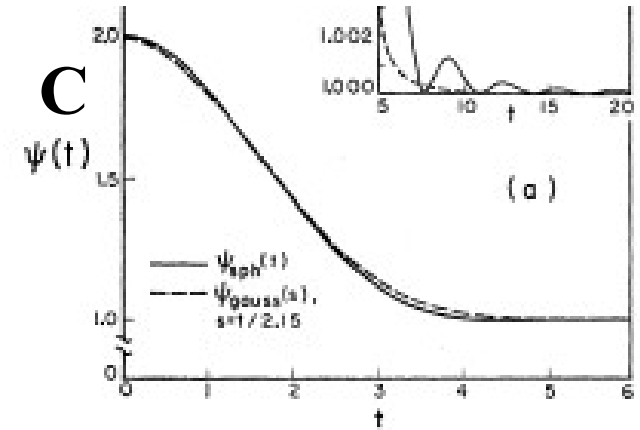
How can we measure source size in nuclear reactions ?

■ Two pion interferometry

G. Goldhaber, S. Goldhaber, W. Lee, A. Pais, Phys. Rev. 120 (1960), 300

- Two pion emission probability is enhanced at small relative momenta

→ Pion source size $\sim 0.75 \hbar / \mu c$



q (relative momentum)

PHYSICAL REVIEW

VOLUME 120, NUMBER 1

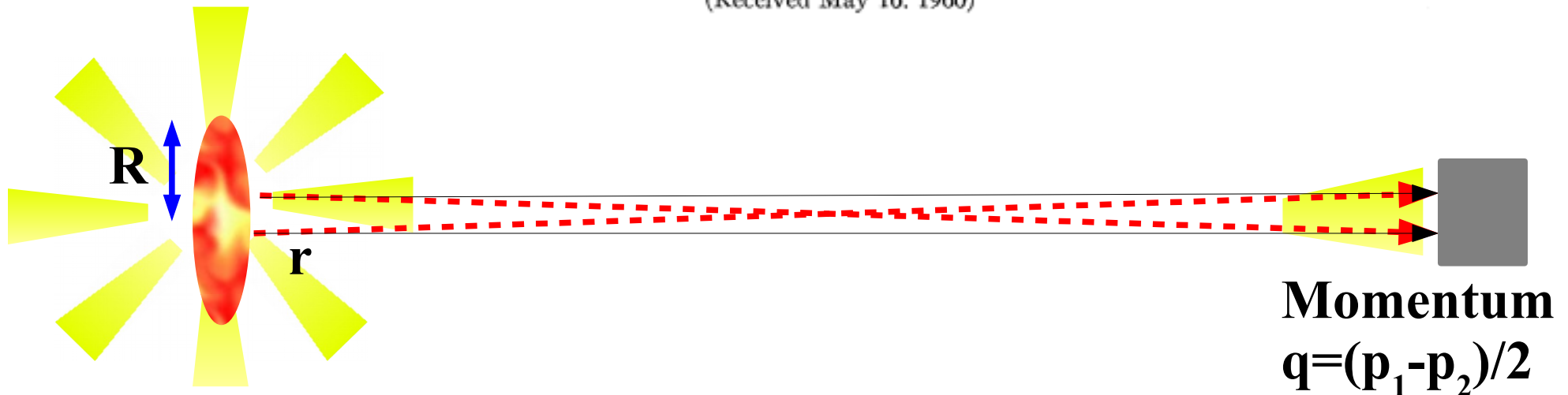
OCTOBER 1, 1960

Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process*

GERSON GOLDHABER, SULAMITH GOLDHABER, WONYONG LEE, AND ABRAHAM PAIS†

Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California

(Received May 16, 1960)



Other bound states ?

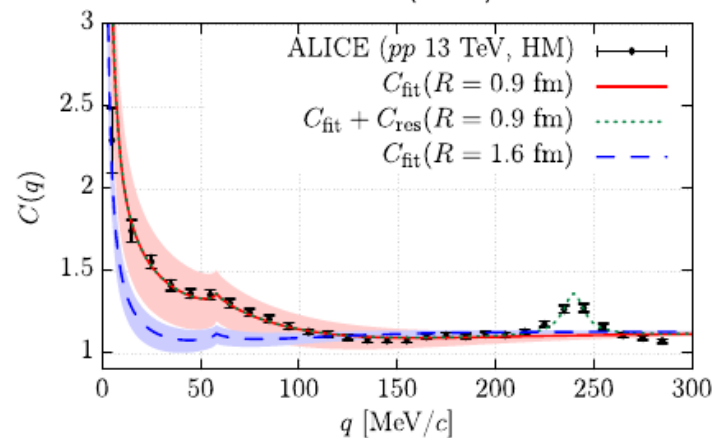
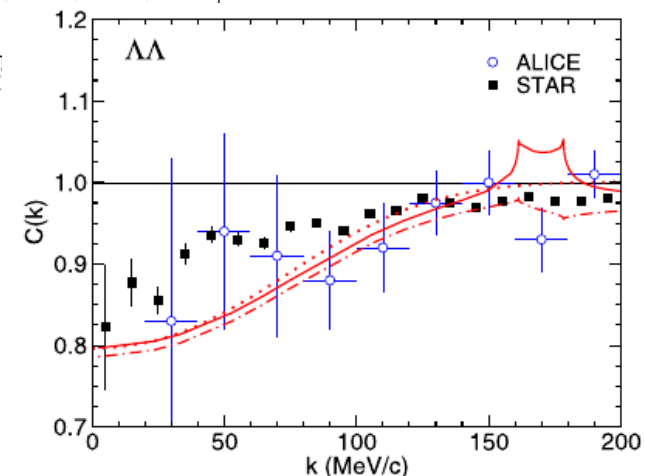
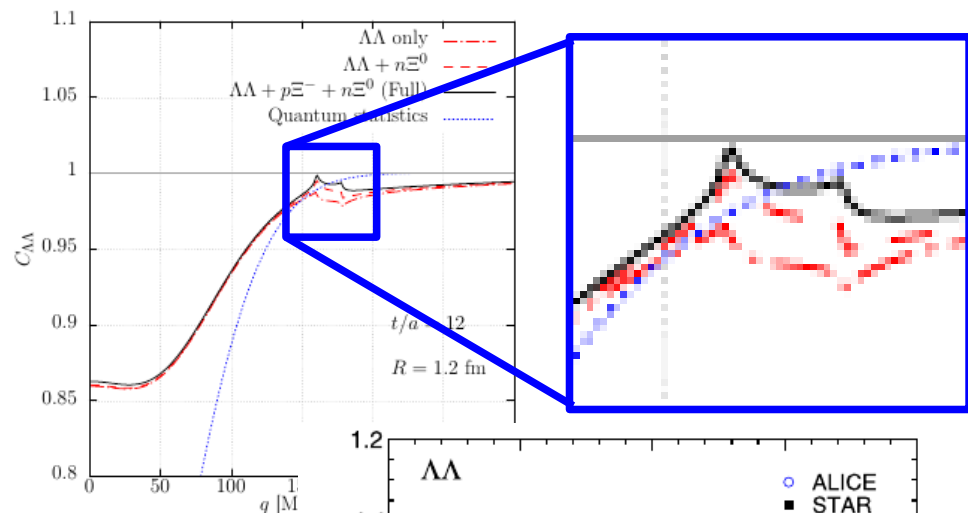
■ $\Lambda\Lambda$ - $N\Xi$

- $C_{\Lambda\Lambda}(q)$ in AA(RHIC) and pp(LHC) are similar (No b.s. below $\Lambda\Lambda$).
- LQCD predicts a virtual pole near $N\Xi$ threshold, which can be detected as the cusp in $C_{\Lambda\Lambda}(q)$.
- NLO(600) potential predicts the same. (The fate of H particle)

*K. Sasaki+[HAL QCD], NPA998('20)121737;
Y. Kamiya+, in prep.; Haidenbauer('19).*

■ $\overline{K}N$

- $\Lambda(1405)$ is believed to be the bound state of $\overline{K}N$, and “dip” is expected at $R \sim a_0$.
 - However, Coulomb and coupled-channel effects modify the dip-like behavior.
- Kamiya+ ('20).*



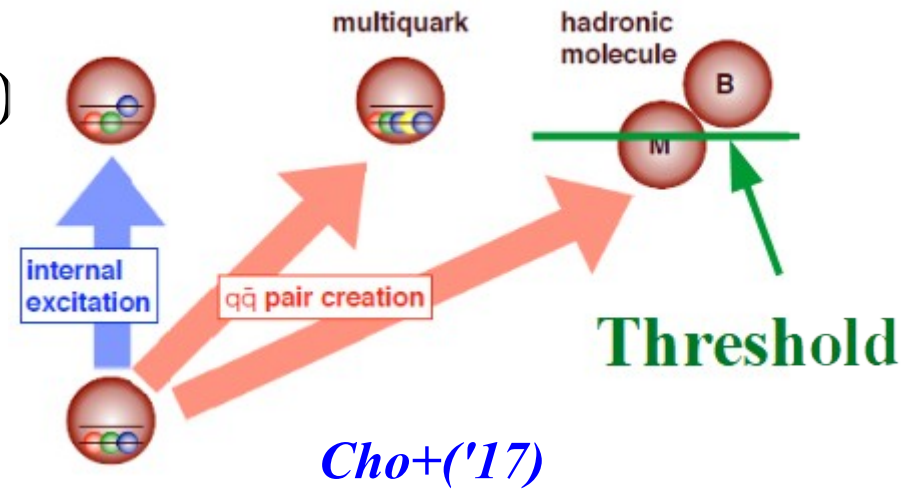
Trends in Hadron Physics

■ Hadron-Hadron interaction is closely related with ...

- Quark-gluon structure of hadrons (Multi-quark or Hadronic molecule)

*To be bound or not to be,
That is the problem.*

- Hadrons with heavy-quarks
- Hadrons in nuclear matter and EOS of nuclear matter



■ High-Energy Nuclear Collisions ($\sqrt{s_{NN}}=40 \text{ GeV} - 14 \text{ TeV}$) are favorable as a Hadron Factory !

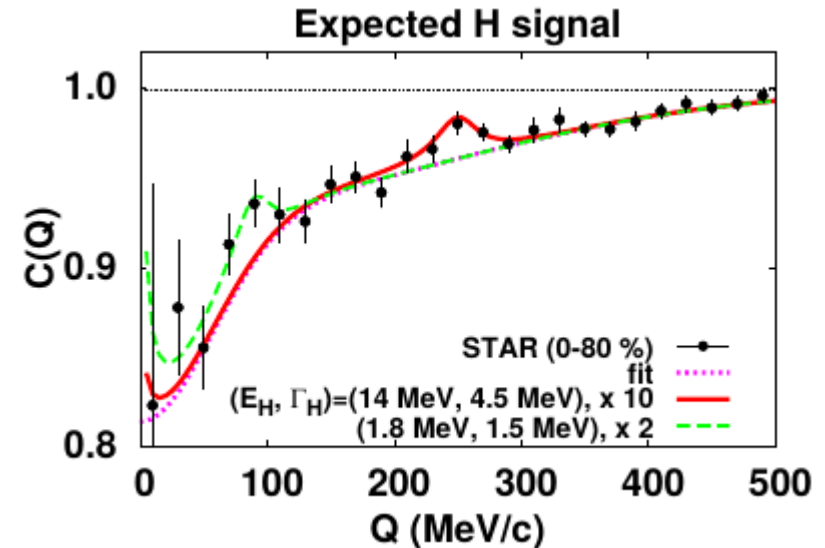
- $dN/dy \sim 1000$ (RHIC, Au+Au) $\rightarrow 10^3$ - 10^5 hadrons in one event
- Various hadrons, nuclei ($A \leq 4$) and anti-nuclei are formed.
- Yield \sim Stat. Model calc.
(Formation processes are too complicated to be out of statistical.)

*Correlation functions
in the near future*

CF from ALICE in the near future

■ S=-3 baryon-baryon correlation (e.g. $\Lambda\Xi^-$)

- Important to confirm $N\Omega$ bound state as a peak in $C_{\Lambda\Xi}(q)$.
- Statistically challenging. In $C_{\Lambda\Lambda}$ data from STAR, statistical fluc. is 10 times larger than expected signal from statistical model estimate.



■ Three-body correlation (e.g. Λpp)

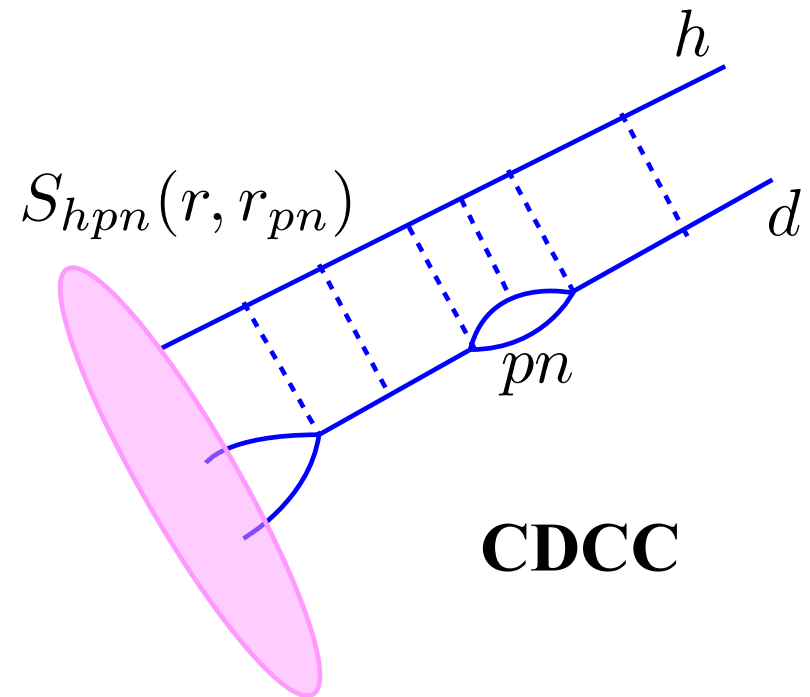
- Extremely important to neutron star matter EOS, if we can extract three-body force.
- We may need to develop a framework beyond the “Riverside approximation” to include hh and hhh interaction.

E. O. Alt, T. Csorgo, B. Lorstad, J. Schmidt-Sorensen, PLB458 ('99)407 for 3π .

(I got the info. mainly from Laura, Valentina and Oton, but I never told it to people other than CF collaborators of mine.)

CF from ALICE in the near future (cont.)

- **Hadron-deuteron correlation (Λ d, K^- d, Ξ^- d, Ω^- d, ...)**
 - Scattering length data of these are important to evaluate binding energy and lifetime of hyper triton (Λ d), $I=1$ \overline{KN} interaction (K^- d), and the existence of a bound state. [Etminan+ \(2006.12771\)](#); [J. Haidenbauer, PRC102\('20\)034001](#).
 - For serious estimate, deuteron breakup effects ($d \leftrightarrow pn$) need to be accounted for. I asked two low-energy few-body nuclear physicists (K. Ogata, T. Fukui) to apply the few-body reaction framework (Continuum-discretized coupled-channels (CDCC)) to hadron-deuteron correlation.



(I got the info. mainly from Laura, Valentina and Oton, but I never told it to people other than CF collaborators of mine.)

CF from ALICE in the near future (cont.)

■ CF including charmed hadron

- Extremely important in recent hadron physics.
- $D^+(\bar{c}d)$ - $p(uud)$ correlation
qq can annihilate, and D^+p couples with many other channels. (LQCD calc. is difficult.)

- $D^-(\bar{c}d)$ - $p(uud)$ correlation
Probes $\Theta_c(\bar{c}-ud-ud)$ state (replace \bar{s} in $\Theta(\bar{s}-ud-ud)$ with \bar{c})

Two pion exchange can induce attraction.

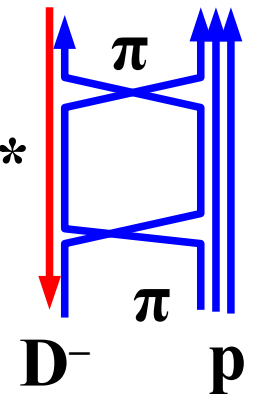
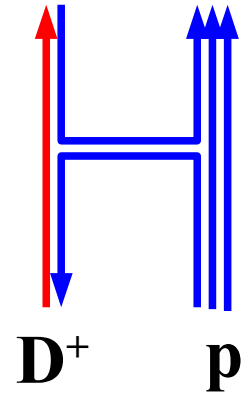
Provides production-mechanism-free (thermal) result. D^*

Easy to calculate the potential in LQCD.

D. O. Riska, N. N. Scoccola, PLB299('93)338 (pred.);

A. Aktaset+ [H1], PLB588('04)17 (positive);

J. M. Linket+ [FOCUS], PLB622('05)229 (negative).



(I got the info. mainly from Laura, Valentina and Oton, but I never told it to people other than CF collaborators of mine.)