古典場のレプリカ発展における緩和過程

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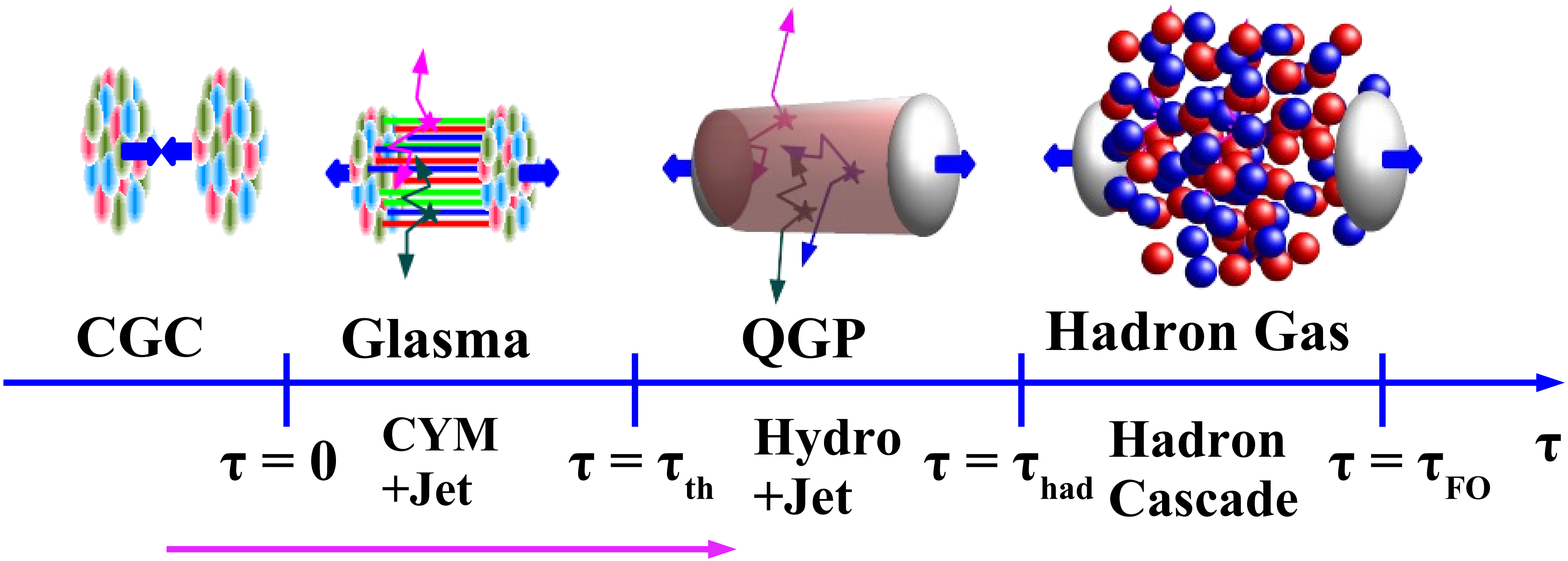
- **Introduction**
  - レプリカ発展法
    AO, H.Matsuda, T.Kunihiro, T.T.Takahashi, PTEP2021('21), 023B09
    doi:10.1093/ptep/ptaa172, arXiv:2008.09556 [hep-lat]
- **レプリカ発展法でのスカラー場 (φ^4 理論) のずり粘性**
  (Work in progress)
  - 時間相関関数、ずり粘性
- **Summary**

格子場の理論で重イオン衝突？

- 高エネルギー重イオン衝突の初期段階
  古典場が支配的な段階 (CGC, Glasma) から熱平衡 (Hydro., QGP) へ
  → 古典的な背景場の中での量子場の緩和過程の記述が必要



古典場から局所熱平衡へ

- しかし量子場の時間発展には強烈な符号問題が ....
  叶わぬ夢か？

$$S_{fi} = \mathcal{N} \int \mathcal{D}\phi \left\langle \Psi(t_f) \right| \exp\left(iS[\phi]\right) \left| \Psi(t_i) \right\rangle$$

古典場のレプリカ発展

- **量子場の時間発展を簡単に (O(L^3) の計算量で) 記述できないか？**
  → 一つ目の目標 = 量子統計性をもつ古典場理論
  Replica evolution of classical fields in 4+1 dimensional spacetime toward real time dynamics of quantum fields, AO, H.Matsuda, T.Kunihiro, T.T.Takahashi, PTEP2021('21), 023B09 [arXiv:2008.09556 [hep-lat]]

  - **虚時間形式での作用 (φ^4 理論)** $(\xi = a/a_\tau = NT)$

$$S[\phi] = \int_0^\beta d\tau \int d\boldsymbol{x} \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + \frac{1}{2}(\boldsymbol{\nabla}\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{24}\phi^4 \right]$$

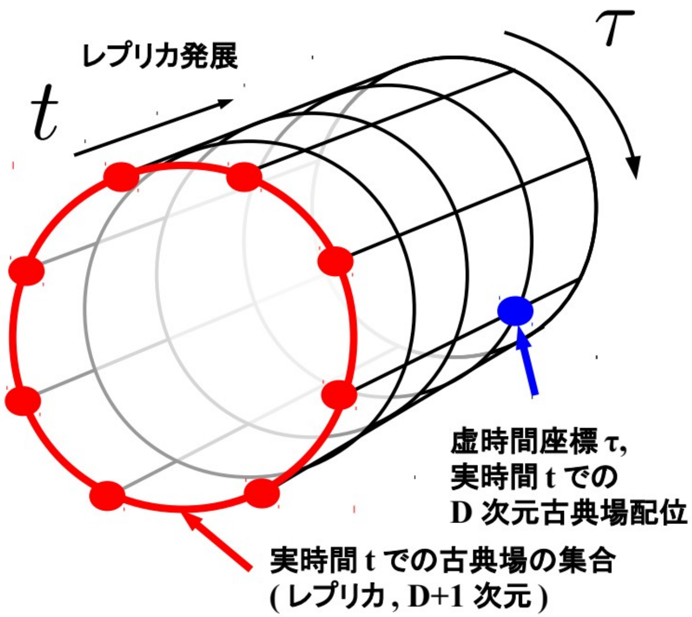
$$\simeq \frac{1}{\xi}\sum_{\tau=0}^{N-1}\sum_{\boldsymbol{x}} \left[\frac{\xi^2}{2}(\phi_{\tau+1,\boldsymbol{x}} - \phi_{\tau,\boldsymbol{x}})^2 + \frac{1}{2}(\boldsymbol{\nabla}\phi_{\tau,\boldsymbol{x}})^2 + \frac{1}{2}m^2\phi_{\tau,\boldsymbol{x}}^2 + \frac{\lambda}{24}\phi_{\tau,\boldsymbol{x}}^4 \right]$$

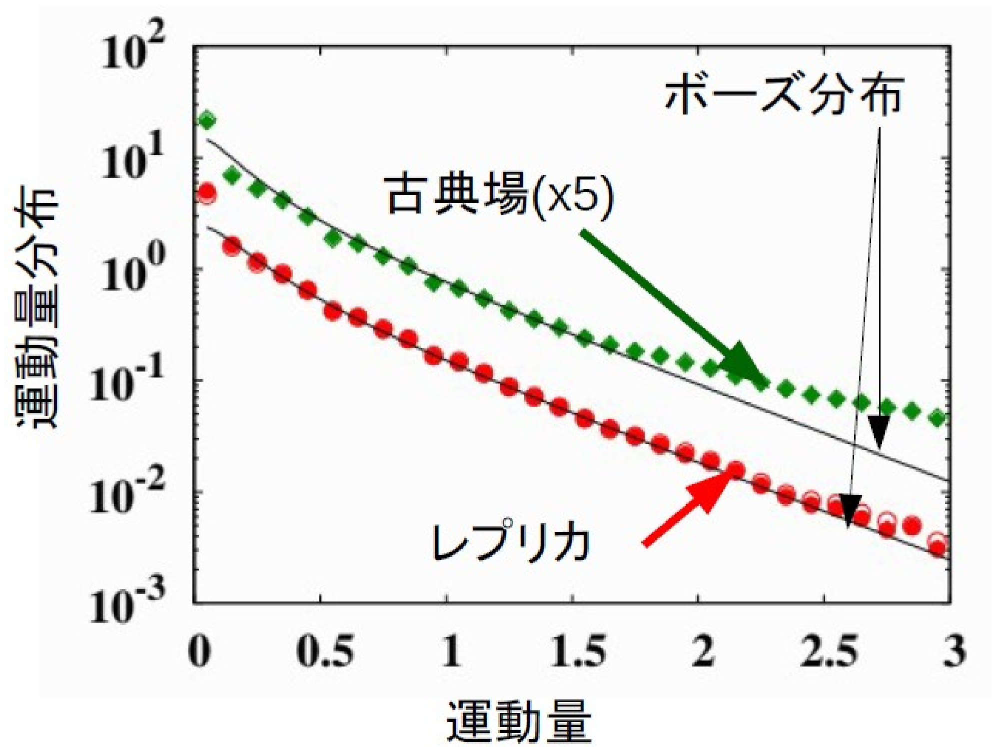
  - **ハミルトニアンと分配関数**

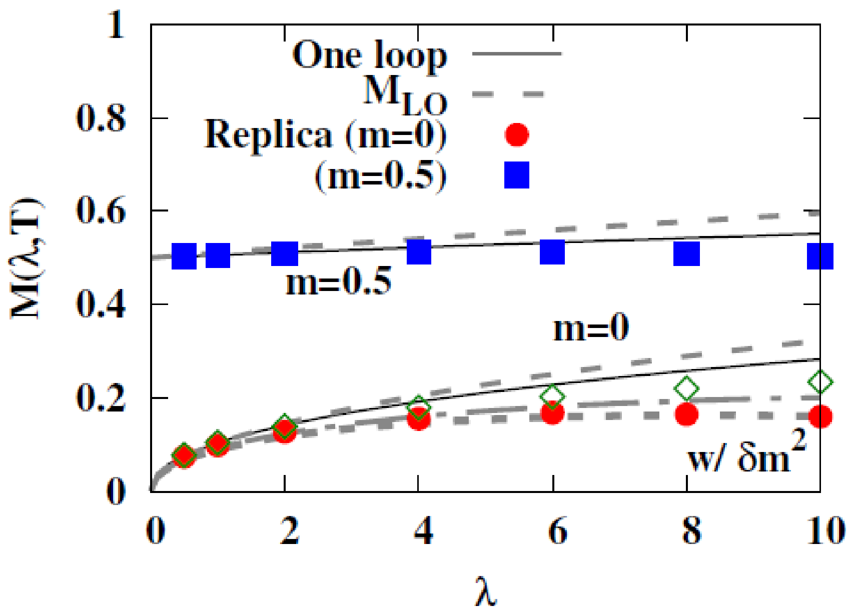
$$\mathcal{H} = \sum_{\tau,\boldsymbol{x}} \frac{1}{2}\pi_{\tau,\boldsymbol{x}}^2 + \xi S[\phi], \quad \mathcal{Z}_T = \int \mathcal{D}\pi\mathcal{D}\phi\ \exp(-\mathcal{H}/\xi) \propto \int \mathcal{D}\phi\ \exp(-S[\phi])$$

カオス性を仮定すれば、τ 方向に拡張した古典場 (レプリカ) の分配関数は虚時間形式の分配関数に比例。

古典場のレプリカ発展







- 古典発展だが統計性は量子的
- ゼロ点振動部分を取り除けば運動量分布は量子論的 *(Bose-Einstein)*
- 時間相関関数から得られる熱質量は *1 loop* の結果よりも *2 loop* の結果に近い

古典場のレプリカ発展

- ユークリッド作用 S[φ] をポテンシャルとする古典力学
  - Hybrid MC の分子動力学部分を実時間発展とみなせるという提案
  - 物理化学で用いられている Path Integral Molecular Dynamics (PIMD) も量子力学多体問題で同様のアイデアが用いられている。
    D. Marx+, J. Chem. Phys. 104, 4077 ('96); M. Shiga, M. Tachikawa, S Miura, Chem. Phys. Lett. 332, 396 ('00); R. Welsh+, J. Chem. Phys. 145, 204118 ('16).
- 取り組むべき課題
  - 非平衡時間発展がどの程度記述できるのか？
    - 厳密に計算できる系での比較 (1+1 次元での 2PI との比較など)
      Aarts, Berges ('13)
    - ボルツマン方程式の導出
      A.H.Müller, Son ('04); P. Copinger+ (work in prog.)
  - 背景場 (古典場) と量子統計性 (粒子) がともに関与する緩和過程
    - 高エネルギー重イオン衝突初期における
      等方化・エントロピー生成・**ずり粘性**・フロー生成・渦度
      古典場での研究 *:Epelbaum, Gelis ('13); Tsukiji+('18); Matsuda+('20); …*
      Replica と gradient flow を用いた EM tensor: Kitazawa+

今回の課題
量子統計性がずり粘性に与える影響は？
(レプリカ発展法によるずり粘性の計算)

レプリカ発展法での
スカラー場 (φ^4 理論) のずり粘性

φ^4 理論での古典場のレプリカ発展とずり粘性

- **Replica Hamiltonian**
  = 虚時間作用をポテンシャルとする古典ハミルトニアン

$$\mathcal{H}=\sum_{\tau=0}^{N-1}\sum_{\boldsymbol{x}}\Bigg[\underbrace{\frac{1}{2}\pi_{\tau,\boldsymbol{x}}^2+\overbrace{\frac{1}{2}(\nabla\phi_{\tau,\boldsymbol{x}})^2+\frac{1}{2}m^2\phi_{\tau,\boldsymbol{x}}^2+\frac{\lambda}{24}\phi_{\tau,\boldsymbol{x}}^4}}_{H(\phi_\tau,\pi_\tau)}\overbrace{+\frac{\xi^2}{2}(\phi_{\tau+1,\boldsymbol{x}}-\phi_{\tau,\boldsymbol{x}})^2}^{\xi S[\phi]}\Bigg]$$

- **線形応答理論 (Green-Nakano-Kubo 関係式) によるずり粘性**

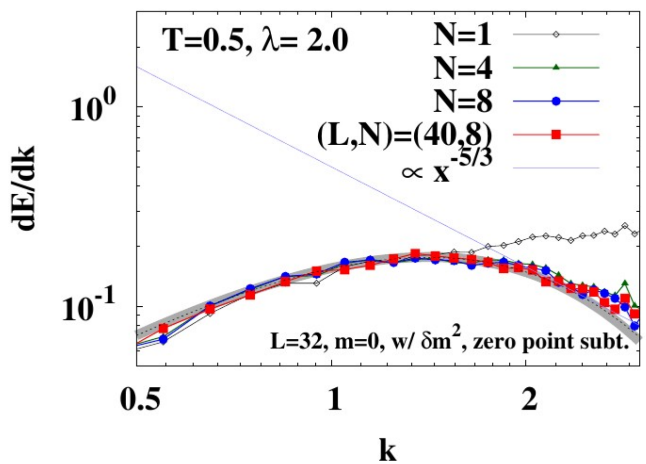
Homor, Jakovac('15); Matsuda+('20)

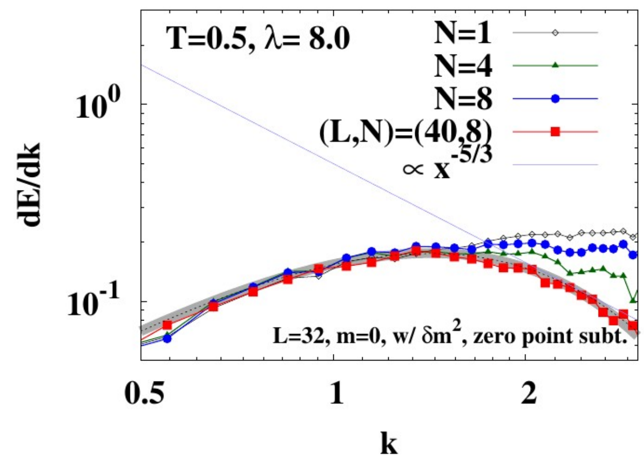
$$\eta=\frac{1}{T}\int_0^\infty C_{12}(t)dt,\ \ C_{12}(t)=\frac{V}{N}\sum_{\tau}\langle\tau_{12,\tau}(t)\,\tau_{12,\tau}(0)\rangle\,,\ \ \tau_{12,\tau}(t)=\frac{1}{V}\sum_{\boldsymbol{x}}T_{12}(\boldsymbol{x},\tau,t)$$

- **Setup (almost the same as *AO+('21)*)**
  - Lattice size = L^3 x N, (L,N)=(32,1), (32,4), (32,8), (40,8)
  - T=0.5 (ξ=NT=2); m=0; λ=0.5, 1, 2, 4, 6, 8, 10
  - One loop mass renormalization (ignored in CF), Init. Cond. from Langevin eq.
  - Solve replica EOM until t=500 with the time step of Δt=0.025.
  - Number of replica configurations =(1000-2000)

運動量分布と時間相関関数

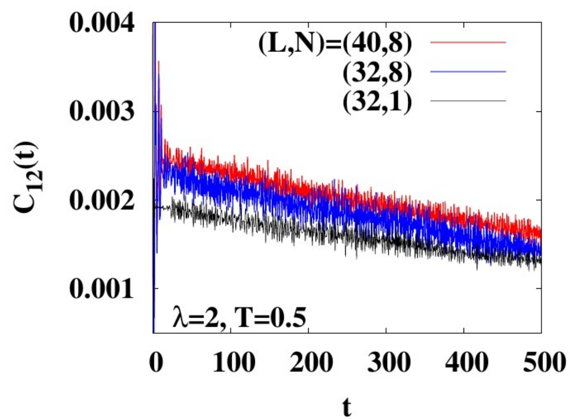
- **運動量分布 (パワースペクトル),** $dE/dk = k^2\omega_{\boldsymbol{k}}^2|\phi(\boldsymbol{k})|^2$
  - **dE/dk は L, N が大きい場合 (連続極限に近い) 場合、Bose-Einstein 分布から期待される分布と無矛盾**

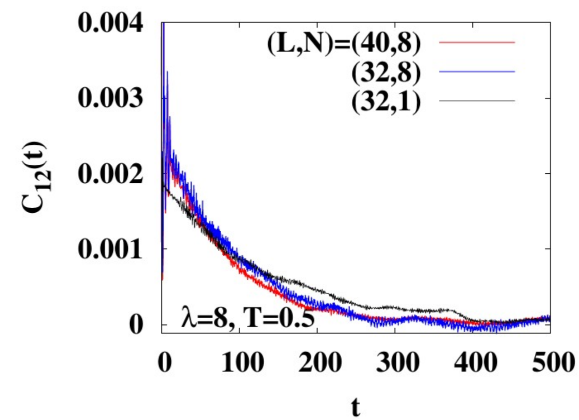




(AO+(‘21) レフェリーの方、λ=8 で乱流が起こるのは多分間違いでした。)

- **時間相関関数**
  - **古典場 (N=1) と比べて、振幅は大きくなる (量子ゆらぎ) が、減衰も速い。**



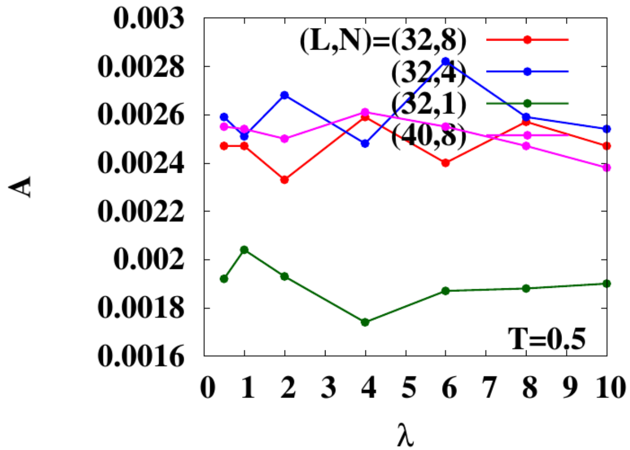


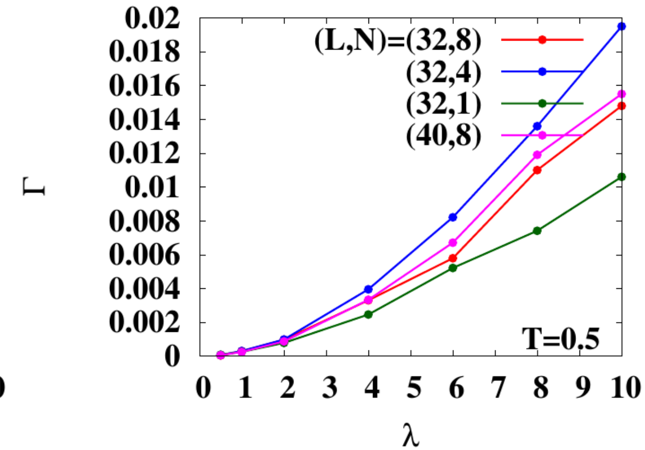
ずり粘性

- **Green-Nakano-Kubo 関係式**

$$C_{12}(t) \simeq A\exp(-\Gamma t)$$

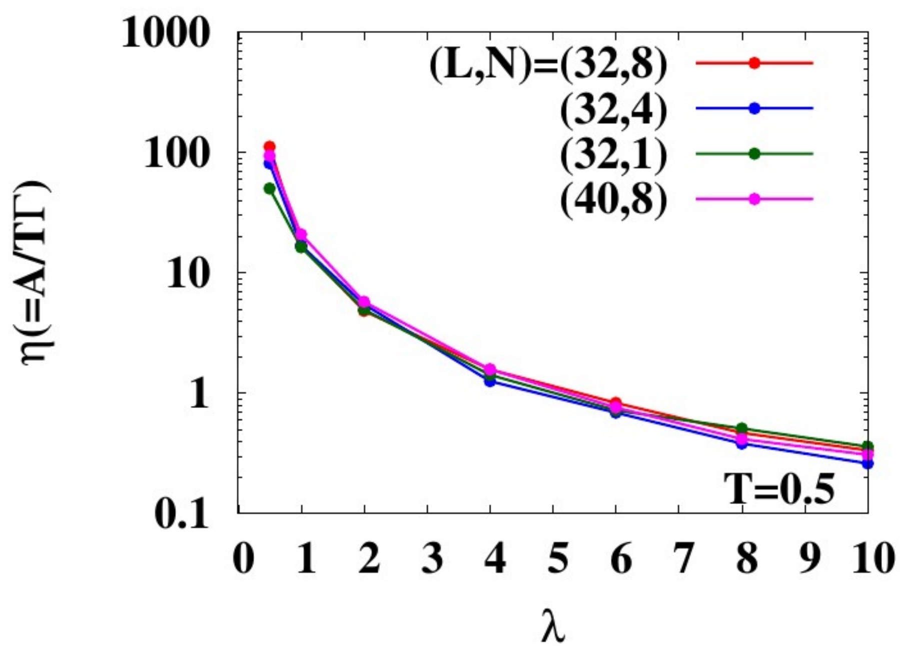
$$\eta = \frac{1}{T}\int_0^\infty C_{12}(t)dt \simeq \frac{A}{T\Gamma}$$





- **古典場に比べて A, Γ ともに大きくなるが、比はあまり変わらない。→ ずり粘性はほぼ同じ (ただし質量くりこみ効果は 小さくない)**

Aarts, Smit ('97); Matsuda, AO, Kunihiro, Takahashi, PTEP2020 ('20), 053D03.



まとめ

- 量子統計性をもつ古典場理論であるレプリカ発展法を用いて緩和過程の研究を開始。
- 今回はスカラー場理論におけるずり粘性の量子統計性が与える影響を調べた。
  - 時間相関関数 $C_{12}(t)$ は指数関数的に減少する成分の周りでの振動を示す。(古典場の場合と同様)
  - $C_{12}(t)$ の振幅 (A) は量子統計性により古典場 (N=1) の場合に比べて大きくなる。指数関数部分の減衰率 (Γ) も大きくなるため、ずり粘性 $\eta \sim A / T\,\Gamma$ は量子統計性を導入しても大きく変化しない。(preliminary)
  - 摂動論と比較すると小さなずり粘性が現れる。異常ずり粘性か？
    Jeon('95), Jeon, Yaffe ('96), Asakawa,Bass, Muller ('06,'06), Matsuda, Kunihiro, AO, Takahashi ('20)
- 今後の課題
  小さな依存性の理解、エネルギー運動量テンソルの繰りこみ、やんミルズ理論への適用、厳密解との比較、ボルツマン方程式の導出 ,

Thank you for your attention !




AO

Hidefumi Matsuda

Teiji Kunihiro

Toru T. Takahashi

"Replica evolution of classical field in 4+1 dimensional spacetime toward real time dynamics of quantum field", A. Ohnishi, H. Matsuda, T. Kunihiro, T. T. Takahashi, PTEP2021('21), 023B09 arXiv:2008.09556 [hep-lat].
Shear Viscosity, work in progress.

Replica Evolution (Quantum Mechanics)

- **Replica Equation of Motion**

$$\frac{dx_\tau}{dt} = \frac{\partial \mathcal{H}}{\partial p_\tau} = p_\tau , \qquad \frac{dp_\tau}{dt} = -\frac{\partial \mathcal{H}}{\partial x_\tau} = -\frac{\partial U(x_\tau)}{\partial x_\tau} + \xi^2(x_{\tau+1} + x_{\tau-1} - 2x_\tau)$$

Force from τ-derivative term

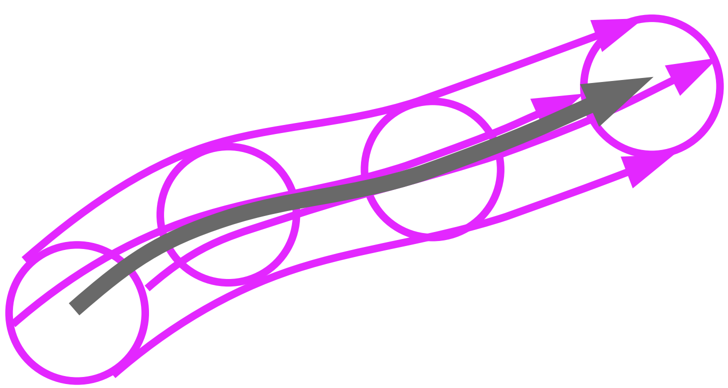
- **Replica index average**

$$\widetilde{x} \equiv \frac{1}{N}\sum_\tau x_\tau , \quad \widetilde{p} \equiv \frac{1}{N}\sum_\tau p_\tau$$

$$\frac{d\widetilde{x}}{dt} = \widetilde{p}, \qquad \frac{d\widetilde{p}}{dt} = -\frac{1}{N}\sum_\tau \frac{\partial U(x_\tau)}{\partial x_\tau} + 0 = -\frac{\partial U(\widetilde{x})}{\partial \widetilde{x}} + \mathcal{O}((\delta x)^2)$$

Ehrenfest's theorem

Replica index average obeys classical EOM (when flucuations are small).



Harmonic Oscillator

- **Replica Hamiltonian = N free HO Hamiltonian**

$$\mathcal{H} = \sum_\tau \left[\frac{p_\tau^2}{2} + \frac{\omega^2 x_\tau^2}{2} + \underbrace{\frac{\xi^2}{2}(x_{\tau+1} - x_\tau)^2}_{\tau\text{-deriv. term}}\right] = \sum_n \left[\frac{\bar{p}_n^2}{2} + \frac{M_n^2 \bar{x}_n^2}{2}\right]$$

$$M_n^2 = \omega^2 + 4\xi^2 \sin^2(\pi n/N)$$

Fourier transf.

- **Expectation value of x^2 in Replica**

Matsubara freq. sum

$$\langle x^2 \rangle = \frac{1}{N}\sum_\tau \langle x_\tau^2 \rangle = \frac{1}{N}\sum_n \langle \bar{x}_n^2 \rangle \overset{\exp(-\mathcal{H}/\xi)}{=} \frac{1}{N}\sum_n \frac{\xi}{M_n^2} = \frac{\coth(\Omega/2T)}{2\omega\sqrt{1+\omega^2/4\xi^2}}$$

$$\Omega = 2\xi \mathrm{arcsinh}(\omega/2\xi)$$

zero point

thermal

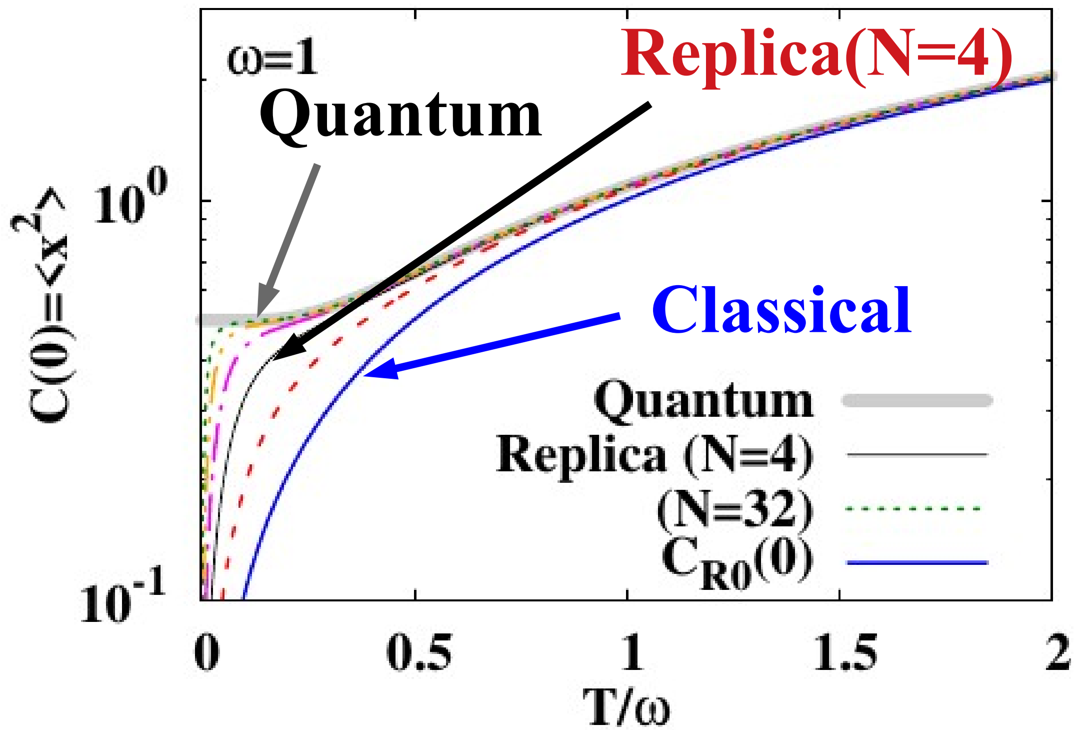
$$\frac{T}{\omega^2} (N = 1, \mathrm{Classical})$$

$$\to \frac{\coth(\omega/2T)}{2\omega} = \frac{1}{\omega}\left[\frac{1}{2} + \frac{1}{e^{\omega/T} - 1}\right] (N \to \infty, \mathrm{Quantum})$$

Equal time observables of x are reproduced at N → ∞

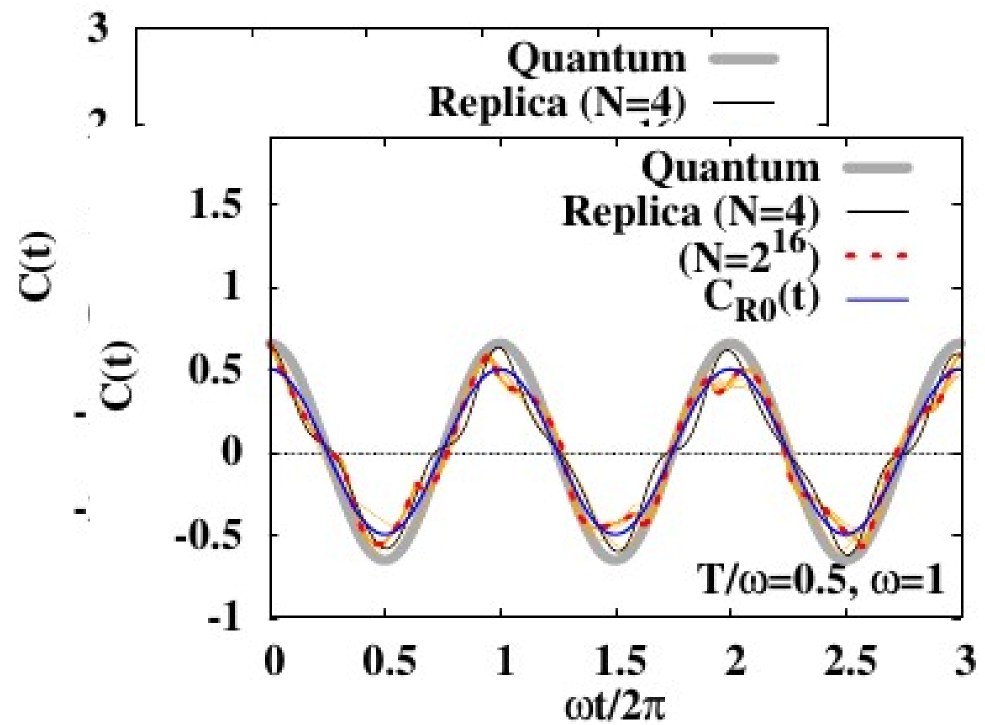
Time Correlation Function in HO

$$C(t) = \langle x(t)x(0) \rangle$$



Equal time observables
→ Exact at N → ∞

Unequal time corr. fn.
→ Not exact,
but good for T/ω > 0.5

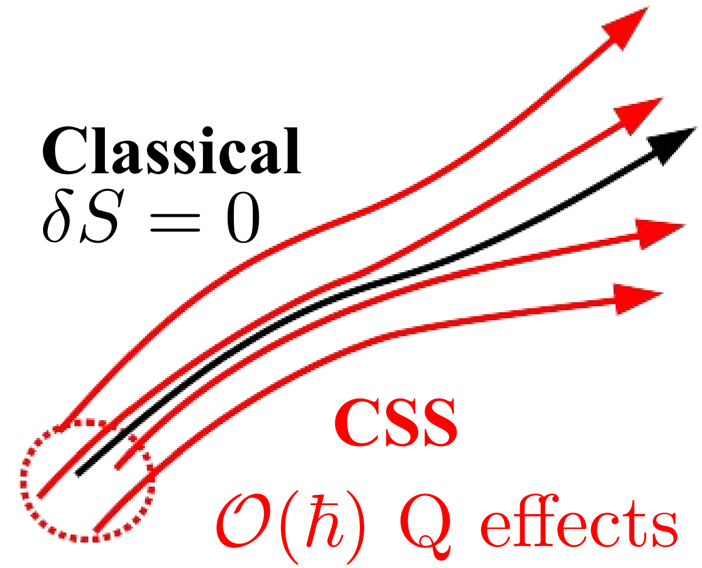


Sounds nice. How about field theory ?

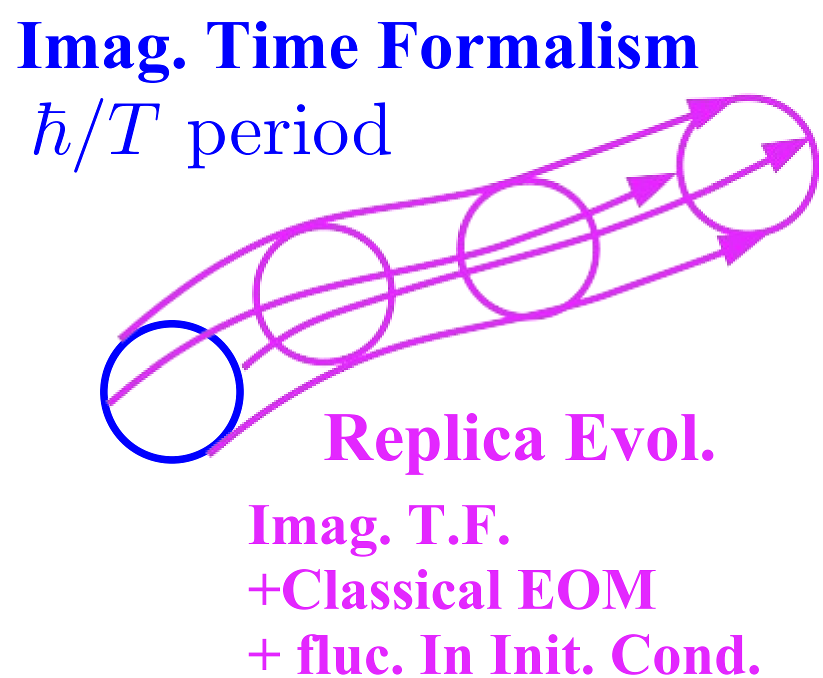
Application to Scalar Field Theory

From Classical Field to Replica Evolution

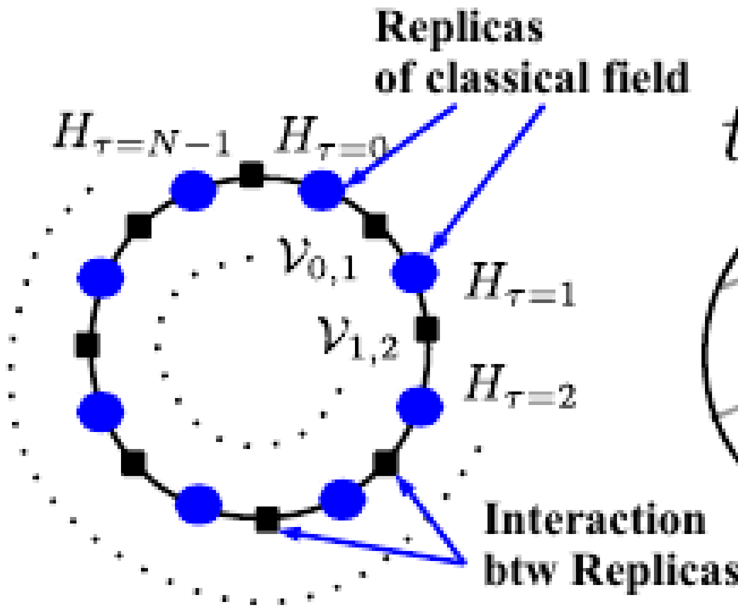
- **Classical Statistical Simulation**
  - **Classical Field equation of motion has $\mathcal{O}(\hbar^2)$ precision**
  - **Instability/Chaoticity**
  - $\mathcal{O}(\hbar)$ **effect is included in init. cond.**
  - **Classical equilibrium**

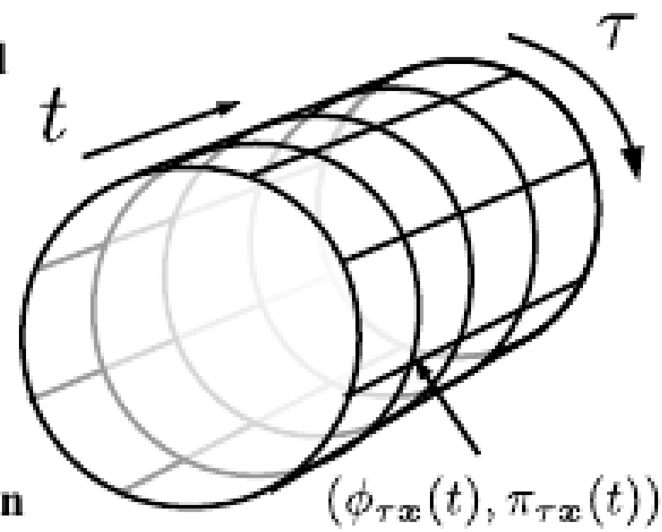


- **Imaginary Time Formalism**
  - **Enlarged "Classical" field configs. 3D → 4(=3+1)D**
  - **Quantum stat. equilibrium**
- **Replica evolution**
  - **Real time evolution of enlarged 4D "Classical" field configs.**
  - **Classical feature + Quant. Stat. Equil.**



Replica Evolution





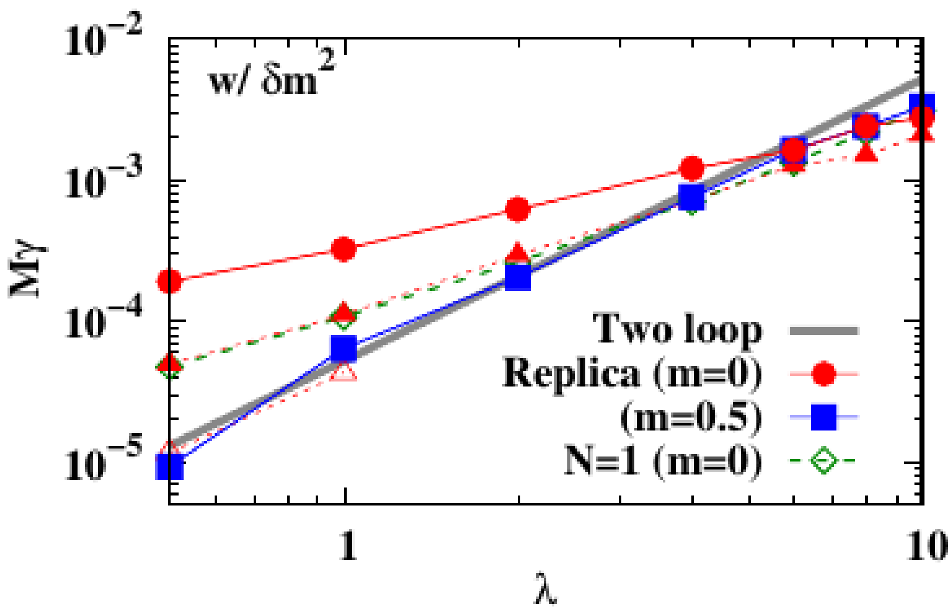
$$\mathcal{H} = \sum_{\tau} H_{\tau} + \sum_{\tau} \mathcal{V}_{\tau,\tau+1} = \frac{1}{2}\sum_{\tau,\boldsymbol{x}} \pi^2_{\tau,\boldsymbol{x}} + \xi S[\phi]$$

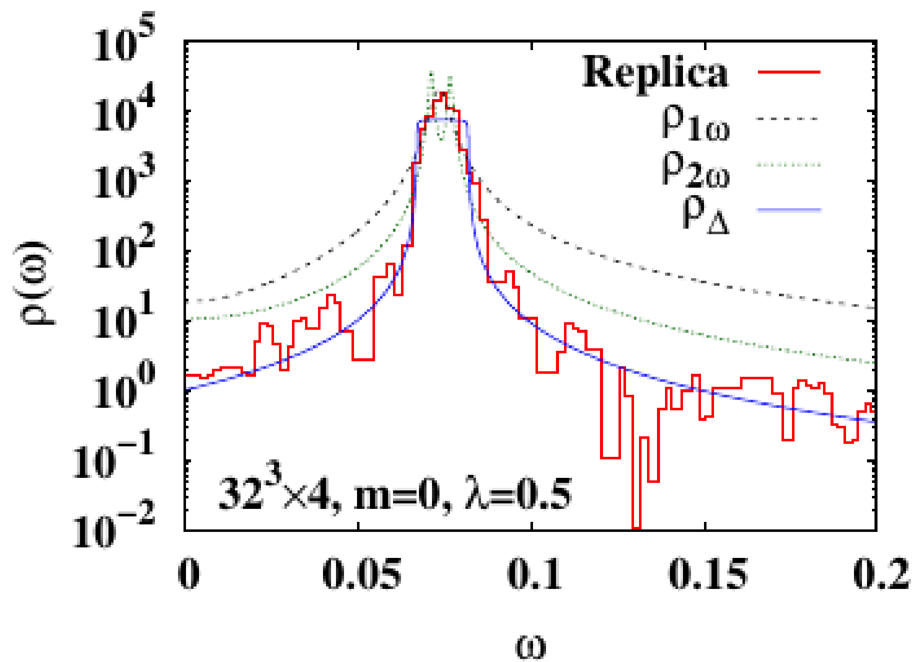
$$\mathcal{Z}_R = \int \mathcal{D}\pi\,\mathcal{D}\phi\,\exp(-\mathcal{H}/\xi) \propto \int \mathcal{D}\phi\,\exp(-S[\phi])$$

Replica Evolution
= Classical Dynamics
with Quantum Statistics in Equilibrium

Damping Rate

- **Apparent damping rate in replica evolution is larger than 2-loop results at small coupling. Why ?**
  - **Classical results (N=1) better agrees with 2-loop results.**
    Aarts ('01)
  - **Power spectrum shows wide spread of the mass, but falls off quickly. Fragmentation of zero-momentum single particle mode ?**





Commutator in Classical Dynamics

- **Classical-Quantum Correspondence**

$$[A,B] \to i\hbar\{A,B\}_{\mathrm{PB}} + \mathcal{O}(\hbar^3)$$

- **Unequal-time Poisson bracket** *Aarts ('01)*

$$\langle \frac{1}{2}[\hat{x}_H(t),\hat{x}_H(0)]\rangle \simeq \left\langle \frac{i}{2}\{x(t),x(0)\}_{\mathrm{PB}} \right\rangle$$

$$= \frac{i}{2}\langle \sum_{n,n'} \left[\frac{\partial \bar{x}_n(t)}{\partial \bar{x}_{n'}(t_0)} \frac{\partial \bar{x}_n(0)}{\partial \bar{p}_{n'}(t_0)} - \frac{\partial \bar{x}_n(t)}{\partial \bar{p}_{n'}(t_0)} \frac{\partial \bar{x}_n(0)}{\partial \bar{x}_{n'}(t_0)} \right] \rangle$$

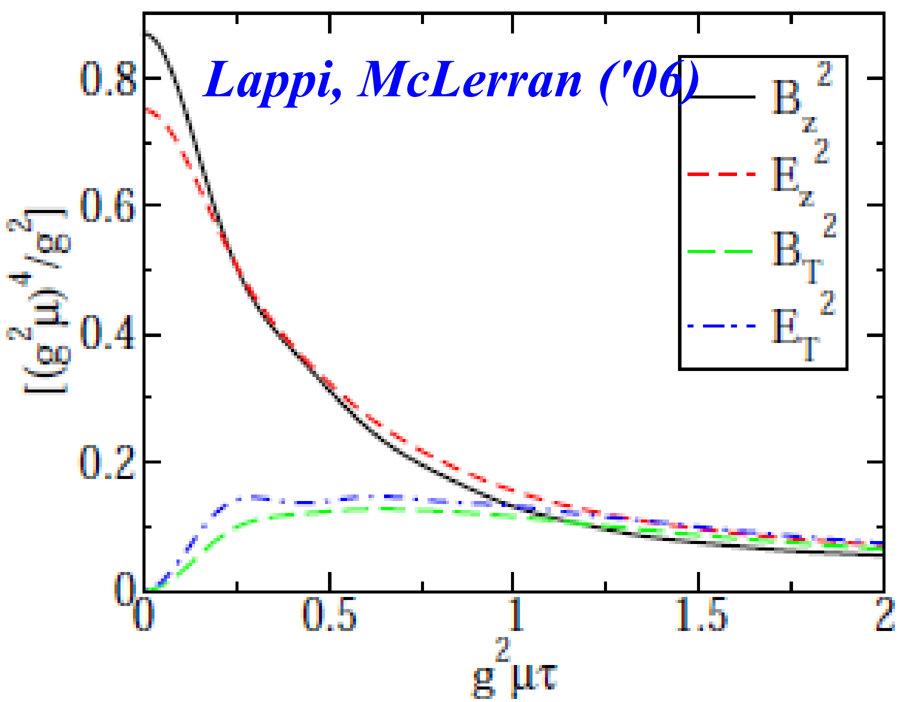
$$\xrightarrow[\text{Free}]{} -\frac{i}{2}\sum_n \frac{1}{M_n}\sin M_n t$$

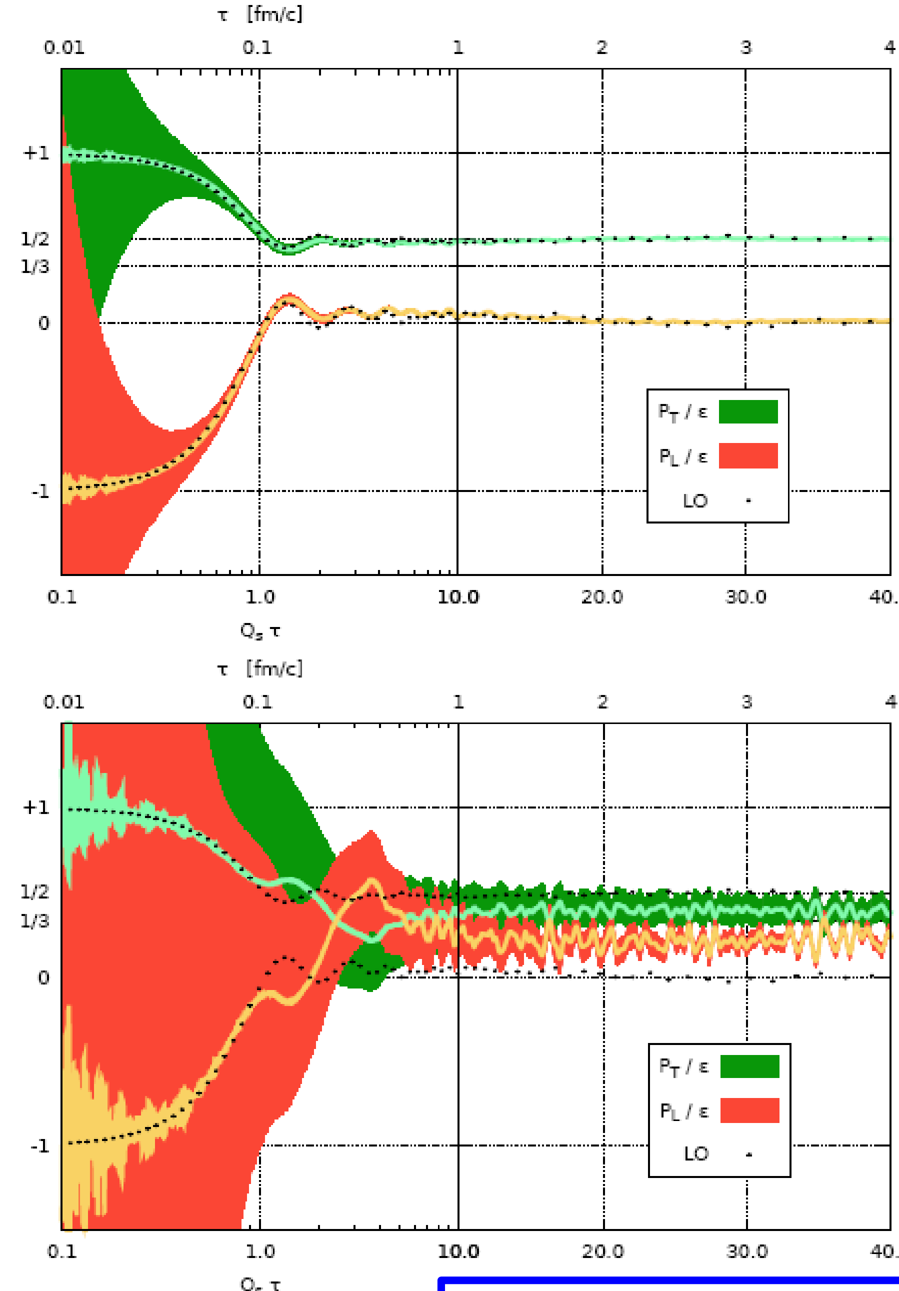
  - **n=0 term reproduces quantum mechanical result in a HO.**
  - **Unequal-time derivative can be obtained by using the Trotter formula together with Hessian matrix.**
    Kunihiro, Muller, AO, Schafer, Takahashi, Yamamoto ('10)

Real-time evolution of Classical Yang-Mills field

- **Classical Statistical Simulation**

  McLerran, Venugopalan ('94), Romatschke, Venugopalan ('06), Lappi, McLerran ('06), Berges, Scheffler, Sexty ('08), Fukushima ('11), Fukushima, Gelis ('12), Epelbaum, Gelis ('13)





Epelbaum, Gelis ('13)

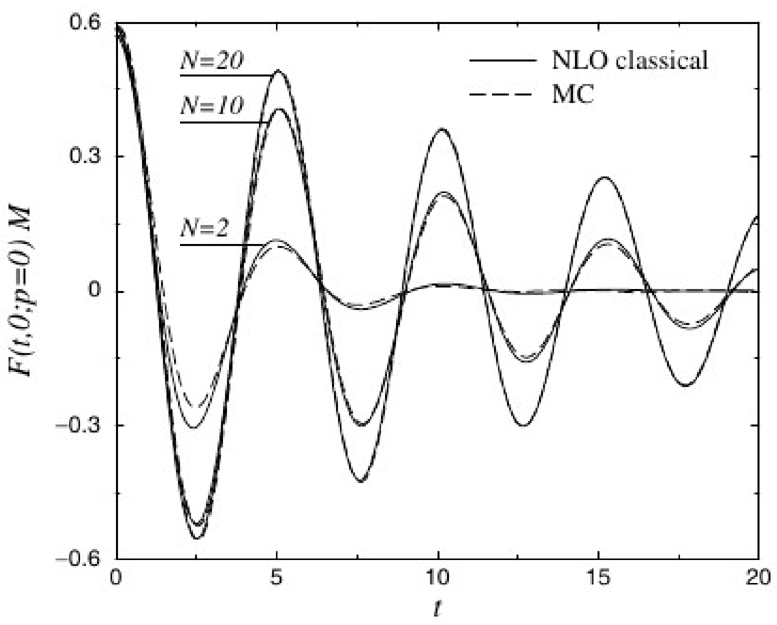
Approaches using 2PI effective action

VOLUME 88, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JANUARY 2002

Classical Aspects of Quantum Fields Far from Equilibrium

Gert Aarts and Jürgen Berges
Institut für Theoretische Physik, Philosophenweg 16, 69120 Heidelberg, Germany
(Received 16 July 2001; published 15 January 2002)

We consider the time evolution of nonequilibrium quantum scalar fields in the $O(N)$ model, using the next-to-leading order $1/N$ expansion of the two-particle irreducible effective action. A comparison with exact numerical simulations in 1 + 1 dimensions in the classical limit shows that the $1/N$ expansion gives quantitatively precise results already for moderate values of N. For sufficiently high initial occupation numbers the time evolution of quantum fields is shown to be accurately described by classical physics. Eventually the correspondence breaks down due to the difference between classical and quantum thermal equilibrium.



Time-correlation function is reasonably described by classical field, but statistics in equilibrium is problematic.

Isn't it enough to use perturbative and lattice field theory

- **Hydrodynamics with EOS and transport coeff. from pQCD and/or LQCD**
  R. Baier, A.H. Mueller, D. Schiff, D.T. Son ('01, pQCD, τ_{th}), P.Arnold, D.G.Moore, L.G.Yaffe ('03, pQCD, η); A. Nakamura, S. Sakai ('05, LQCD, η); A. Bazavov et al. [HotQCD] ('14, LQCD, EOS); S. Borsanyi et al. ('14, LQCD, EOS)
  - **Not enough: early thermalization puzzle, large η (pQCD), large uncertainty in η (LQCD)**
- **Why ? Background field effect ?**
  - **Anomalous viscosity under strong disordered field**
    M. Asakawa, S. A. Bass, B. Müller ('06)
    Under disordered background field, momentum transfer is promoted more than perturbation predicts → Small η
  - **Classical field evolution also predict small η**
    H. Matsuda, T. Kunihiro, AO, T.T.Takahashi ('20)

$$\eta \propto (g^4 \log(1/g))^{-1}\ (\text{pQCD}) \to \eta \propto g^{-3/2} (\text{ABM, CYM})$$

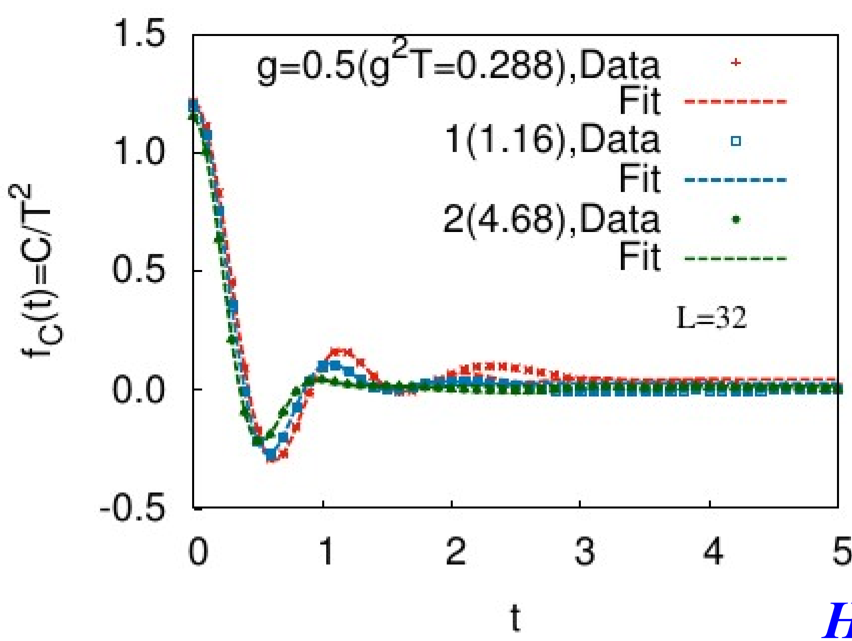
We need evolution of quantum field under inhomogeneous And non-equilibrium background field.

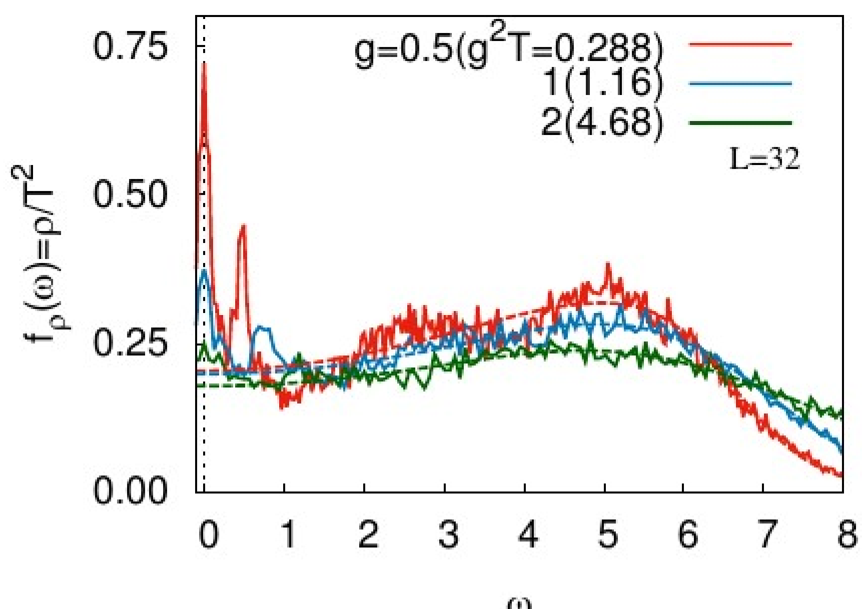
Shear Viscosity of Classical Yang-Mills field

- **Green-Kubo formula**

$$\eta = \frac{1}{T}\int_0^\infty dt\, C(t), \quad C(t) = \frac{V}{3}\sum_{i<j}\tau_{ij}(t)\tau_{ij}(0)$$

  - **Numerical integration of the time-correlation function of the energy-momentum tensor of classical field.**
  - **This should be simulating η of IP-glasma model.**





H. Matsuda, T. Kunihiro, AO, T.T.Takahashi, arXiv:2007.06886 [hep-ph]

Anomalous Shear Viscosity

- **Anomalous viscosity under strong disordered field**
  M. Asakawa, S. A. Bass, B. Müller, PRL96 ('06)252301; PTP116 ('07) 725.
  Disordered background field promotes momentum transfer.

$$\eta_{\mathrm{A}} = \left(\frac{2(N_c^2-1)\nu_4\zeta(4)T\tau}{25 b_0 N_c \nu_2'\zeta(2)}\right)^{1/2} \frac{s}{g^{3/2}}.$$

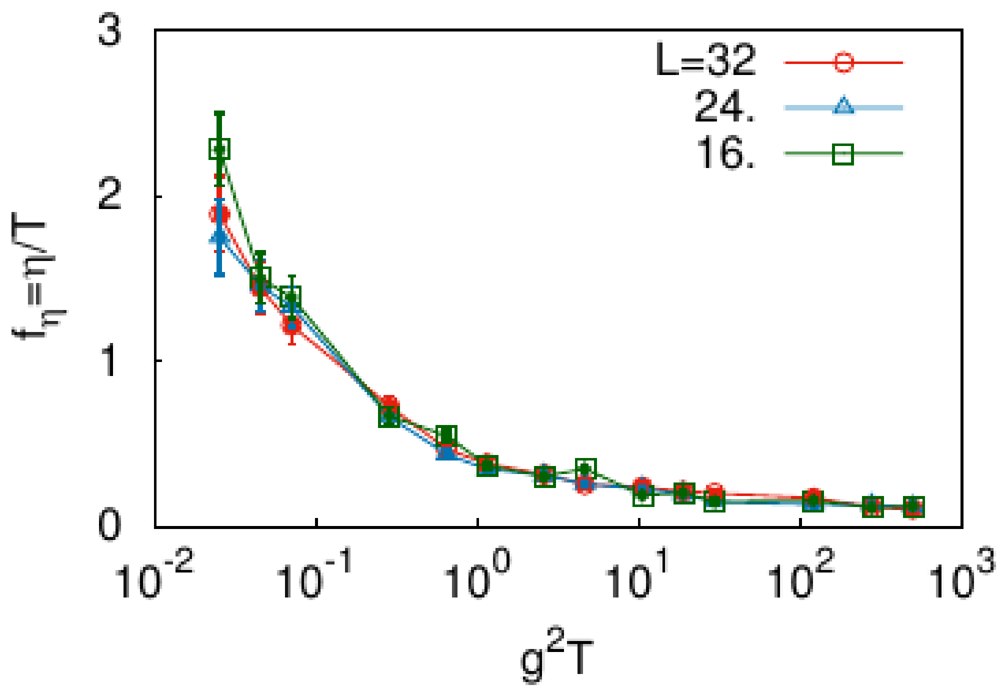
$$\eta \propto (g^4 \log(1/g))^{-1} \quad \text{(pQCD)}$$

- **Classical Field simulation supports this idea.**
  H. Matsuda, T. Kunihiro, AO, T.T.Takahashi (arXiv:2007.06886)

$$\alpha x^{-\beta/2} + \gamma x^{-\delta/2}$$

$$\alpha = 0.09 \pm 0.07, \ \boxed{\beta = 1.49 \pm 0.39,}$$
$$\gamma = 0.33 \pm 0.06, \ \delta = 0.35 \pm 0.07.$$

x=g²T



Application to Gauge theories and Fermion Systems

- **Gauge theory**
  - **Temporal component of the gauge field is Wick-rotated in the imaginary time formalism, and replica evolution cannot be applied as it is, except for the case in the temporal gauge (A_0=0).**
- **Fermions**
  - **We do not know (yet) how to handle Grassman number in replica.**
  - **Time-dependent Hartree-Fock theory may help.**