Exotic hadrons and interactions from high-energy nuclear collisions

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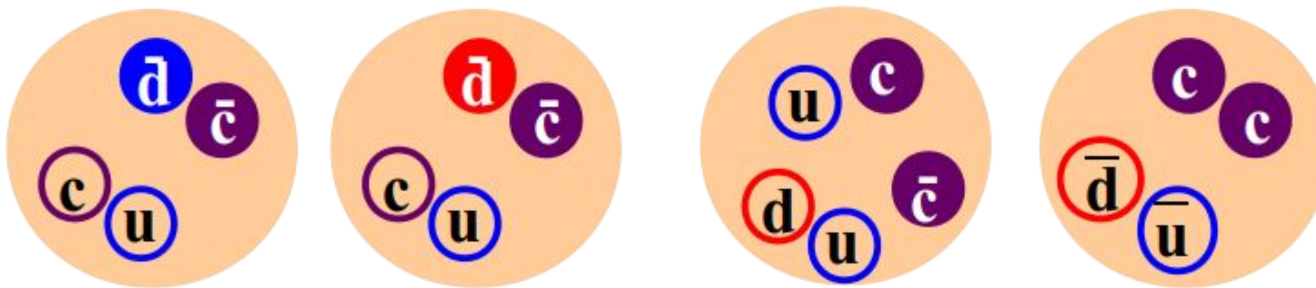
- **Introduction**
- **Exotic Hadron Yields in High-Energy Nuclear Collisions**
- **Exotic Interaction from High-Energy Nuclear Collisions**
- **Summary**

JPS2021.09, Sep. 14-17, 2021,
Online / Kobe, Japan.
"Anatomy of heavy exotic hadrons
in various experimental facilities"
(日本物理学会2021年秋季大会)

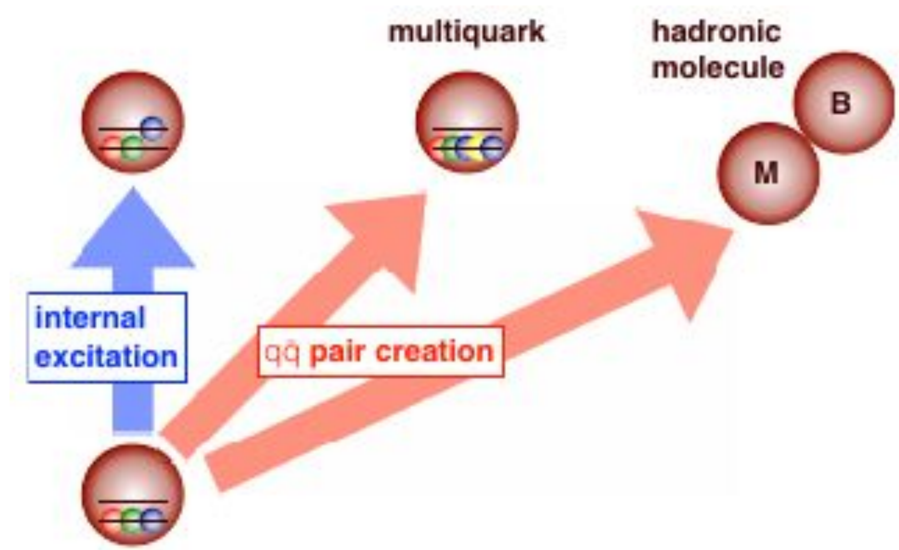
Exotic Hadrons

- Exotic hadrons (Θ^+, X, Y, Z, Pc ….)
  → Discovered/Proposed at LEPS, Belle, BaBar, BES, LHCb, ...



X(3872) Z(4430) Pc T_{cc}

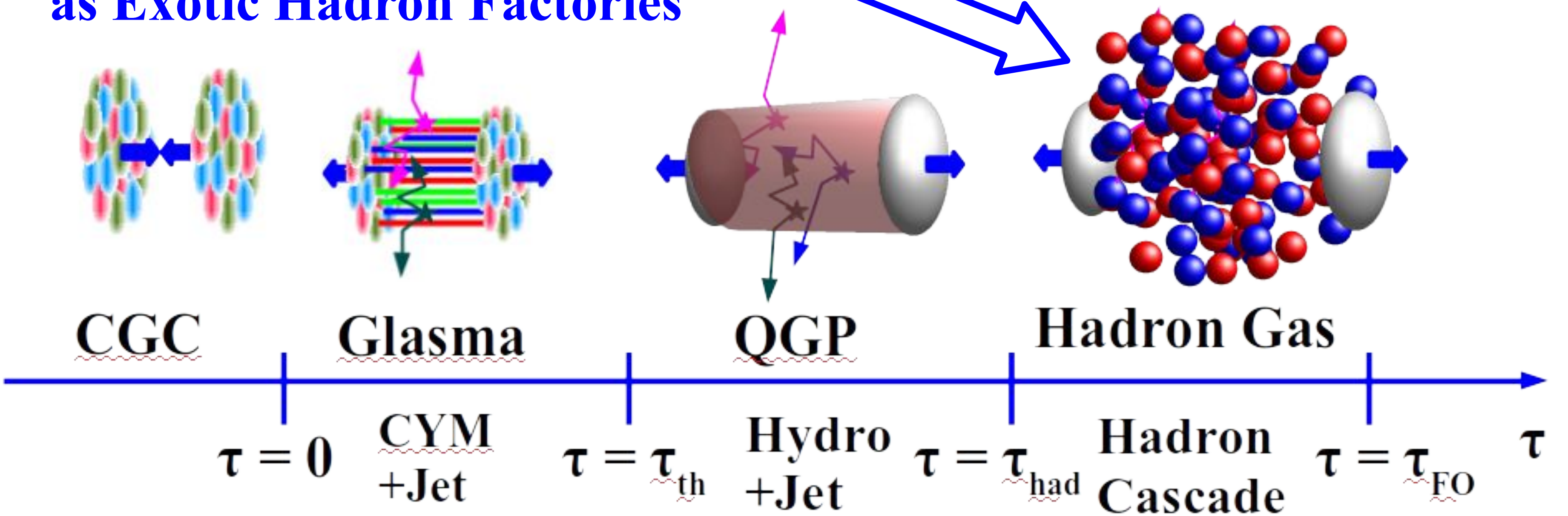
- Various pictures
  - Compact multiquark state with di-quark component
  - Hadronic molecule
  - (Triangle) Singularity
  - $Q\bar{Q}$ couples with $Q\bar{Q}q\bar{q}$
  - …



What is the structure of exotic hadrons ?
Can we access h-h interactions with heavy quarks ?

High-Energy Heavy-Ion & Nuclear Collisions

- **High-Energy Heavy-Ion Collisions**
  & High-Energy (and High-Multiplicity) pp and pA collisions
  - **Too complex → Statistical → Simple and Clean !**
  - **High *T* & Large volume → Abundant hadrons**
  - **Nearly 4π detector / Vertex detector**
    → Let's regard High-Energy Nuclear Collisions as Exotic Hadron Factories



We will demonstrate that high-energy nuclear collisions are useful in hadron phyics !

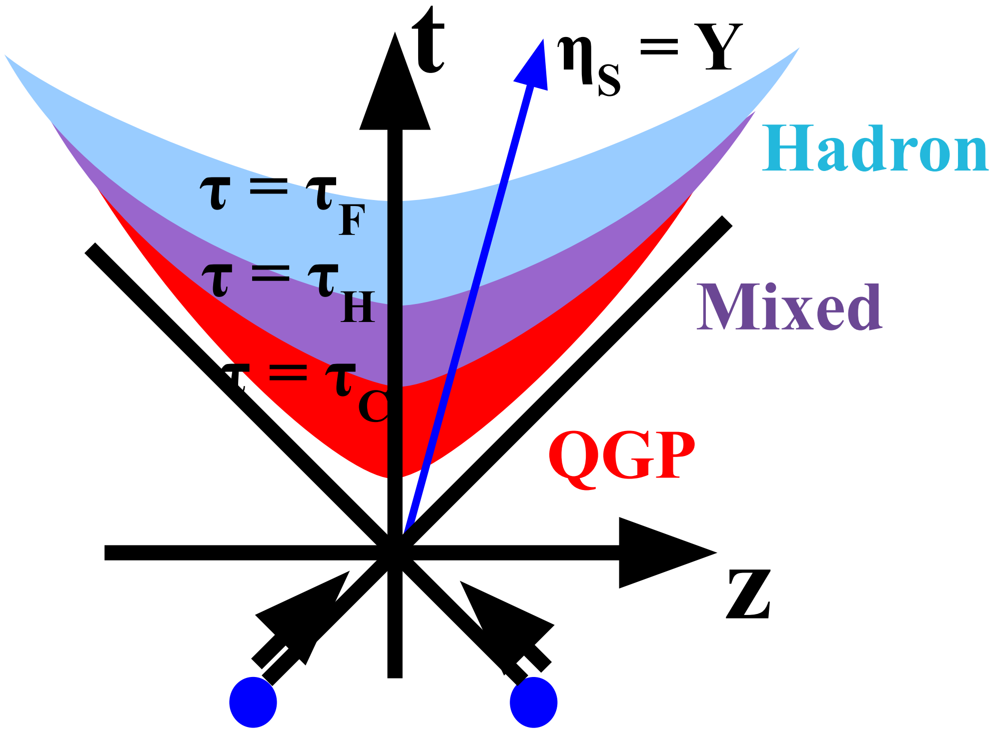
Exotic Hadron Yields in High-Energy Nuclear Collisions

- *Identifying Multiquark hadrons from Heavy Ion Collisions,*
  S.Cho, T.Furumoto, T.Hyodo, D.Jido, C.M.Ko, S.H.Lee, M.Nielsen, A.Ohnishi, T.Sekihara, S.Yasui, K.Yazaki (ExHIC Collaboration),
  Phys. Rev. Lett. 106 (2011), 212001 (1-4).
- *Exotic Hadrons in Heavy Ion Collisions,*
  S.Cho et al. (ExHIC Collaboration), Phys. Rev. C 84 (2011), 064910 (1-17).
- *Exotic hadrons from heavy ion collisions,*
  S.Cho, T.Hyodo, D.Jido, C.M.Ko, S.H.Lee, S.Maeda, K.Miyahara, K.Morita, M.Nielsen, A.Ohnishi, T.Sekihara, T.Song, S.Yasui, K.Yazaki
  (ExHIC Collaboration), Prog. Part. Nucl. Phys. 95(2017), 279-322.
- *Signatures of the vortical quark-gluon plasma in hadron yields,*
  H.Taya, A.Park, S.Cho, P.Gubler, K.Hattori, J.Hong, X.-G.Huang, S.H.Lee, A.Monnai, A.Ohnishi, M.Oka, D.-L.Yang (ExHIC-P Collaboration),
  Phys. Rev. C 102 (2020), 021901(R)(1-6)

Schematic picture of HIC

- HIC picture based on the (approx.) 1st order phase transition
  - $\tau = \tau_C$, $T=T_C$, $V=V_C$ → QGP start to hadronize (quark coal.)
  - $\tau = \tau_H$, $T=T_H=T_C$, $V=V_H$ → Hadronization is over (stat. model)
  - $\tau = \tau_F$, $T=T_F$, $V=V_F$ → Hadronic Freeze-out (hadron coal.)

	RHIC	LHC
$N_u = N_d$	245	662
$N_s = N_{\bar{s}}$	150	405
$N_c = N_{\bar{c}}$	3	20
$N_b = N_{\bar{b}}$	0.02	0.8
V_C	1000 fm^3	2700 fm^3
$T_C = T_H$	175 MeV	175 MeV
V_H	1908 fm^3	5152 fm^3
μ_B	20 MeV	20 MeV
μ_s	10 MeV	10 MeV
V_F	11322 fm^3	30569 fm^3
T_F	125 MeV	125 MeV



L.W.Chen, V.Greco, C.M.Ko, S.H.Lee, W.Liu, PLB 601('04)34.

Statistical Model

- **Statistical model**

$$N_h^{\rm stat} = V_H \frac{g_h}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\gamma_h^{-1} e^{E_h/T_H} \pm 1}$$

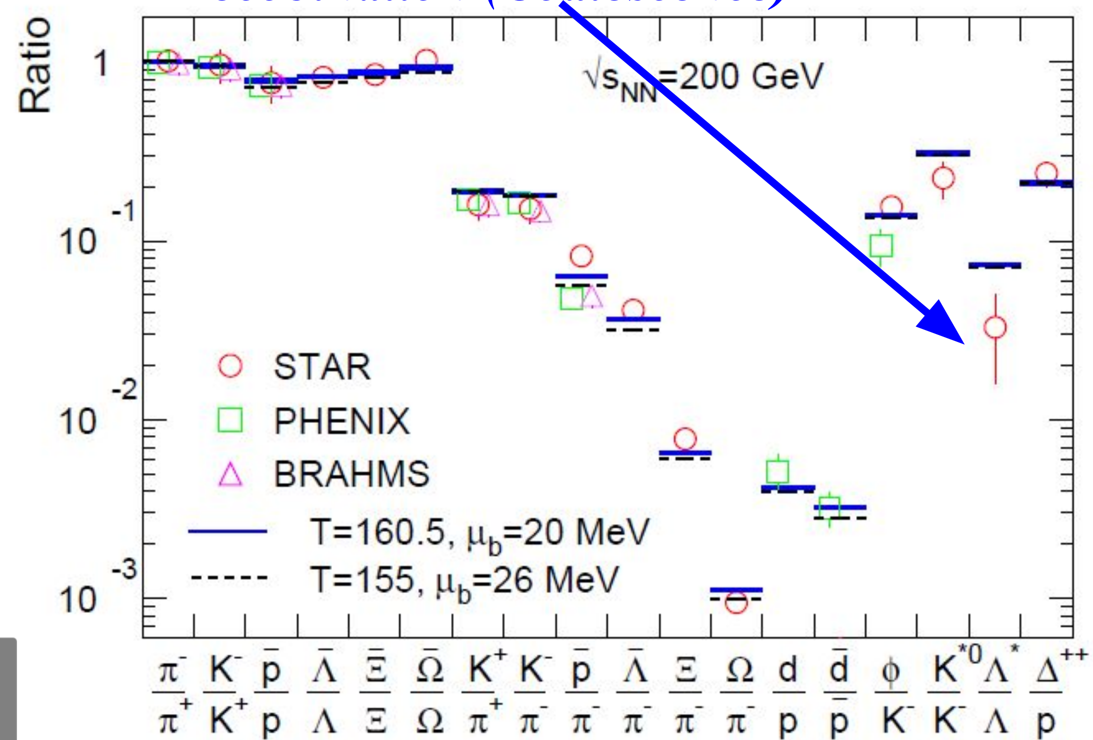
(N_h = dN_h/dy (y=rapidity), V_H=Chem. freeze-out vol.)
→ Successful to predict the hadron yield ratio at RHIC

- **Fugacity factor γ**
  - **u,d,s: chem. equil.**
  - **c,b: enhanced by initial hard processes**
- **Fugacities of c and b quarks are set to reproduce expected c and b quark numbers.**

$$\gamma_h = \gamma_c^{n_c+n_{\bar c}} \gamma_b^{n_b+n_{\bar b}} e^{(\mu_B B+\mu_s S)/T_H}$$

Underlying mechanism ?

L effects: Kanada-En'yo, Muller ('06)
→ Recobination (Coalescence)



A. Andronic, P. Braun-Munzinger, J. Stachel, NPA772('06)167.

Coalescence model

- **Yield = Overlap of const. dist. & Hadron intrinsic Wigner func. (Sudden approximation)**
  Sato, Yazaki (1984), Hwa, Yang (2003), Greco, Ko, Levai (2003), Fries, Muller, Nonaka, Bass (2003), Chen, Ko, Lee (2003)

$$N_h^{\rm coal} = g_h \int \left[\prod_{i=1}^{n} \frac{1}{g_i} \frac{p_i \cdot d\sigma_i}{(2\pi)^3} \frac{d^3 p_i}{E_i} f(x_i, p_i) \right] \times f^W(x_1, \cdots, x_n : p_1, \cdots, p_n)$$

Dist. of constituents | **Intrinsic Wigner func.**

- **Yield in HIC**
  - **Quark & hadron dist. = Transverse Boltzmann + Bjorken**
    Chen, Ko, Liu, Nielsen (2007)
  - **Hadron intr. Wigner func. = s-wave and p-wave HO w.f.**
    Kanada-En'yo, Muller (2006)

$$N_h^{\rm coal} \simeq g_h \prod_{j=1}^{n} \frac{N_j}{g_j} \prod_{i=1}^{n-1} \frac{(4\pi\sigma_i^2)^{3/2}}{V(1+2\mu_i T\sigma_i^2)} \left[\frac{4\mu_i T \sigma_i^2}{3(1+2\mu_i T\sigma_i^2)} \right]^{l_i}$$

σ = Gaussian width, μ=reduced mass, N = constituent yield

  - **Available structure information → σ (or ℏω)**

Which hadrons are enhanced in coalescence ?

- **Simple estimate: 2-body, Gaussian w.f. + Thermal dist. of constiuents**

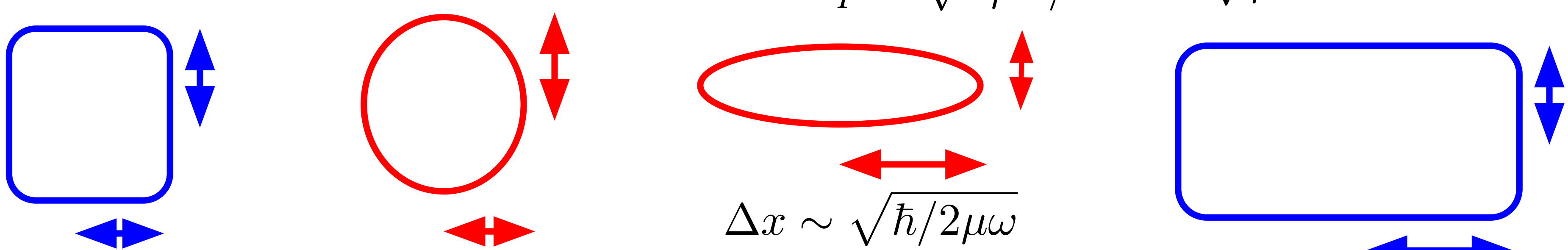
  → Yield is large when f_w shape is similar to f_{th} in phase space.

$$N_h \propto \int \frac{d^Dx d^Dp}{(2\pi\hbar)^D} \underline{f_W(x,p)}\, \underline{f_{\rm th}(x,p)} = \left[\left(\frac{4}{\hbar^2}\right)\left((\Delta p)^2 + \mu T\right)\left((\Delta x)^2 + 2R^2\right)\right]^{-D/2}$$

Intrinsic | **Constituents (thermal)**

$$f_W(x,p) = \left(\frac{\hbar}{\Delta x \Delta p}\right)^D \exp\left(-\frac{x^2}{2(\Delta x)^2} - \frac{p^2}{2(\Delta p)^2}\right)$$

$$f_{\rm th}(x,p) = \left(\frac{\hbar^2}{2\mu T R^2}\right)^{D/2} \exp\left(-\frac{x^2}{4R^2} - \frac{p^2}{2\mu T}\right)$$



pp collisions	normal hadrons	hadronic molecule	HIC
$\sqrt{\mu T} \sim 400$ MeV	$\Delta p \sim 300$ MeV	$\Delta p \sim \sqrt{\hbar\mu\omega/2}$	$\sqrt{\mu T} \sim 400$ MeV
$R \sim 1$ fm	$\Delta x \sim 0.6$ fm	$\Delta x \sim \sqrt{\hbar/2\mu\omega}$	$R \sim 5$ fm

$(\hbar\omega \simeq 6 \times \text{B.E.} \simeq 3\hbar^2/2\mu\langle r^2\rangle)$

Which hadrons are enhanced in coalescence ?

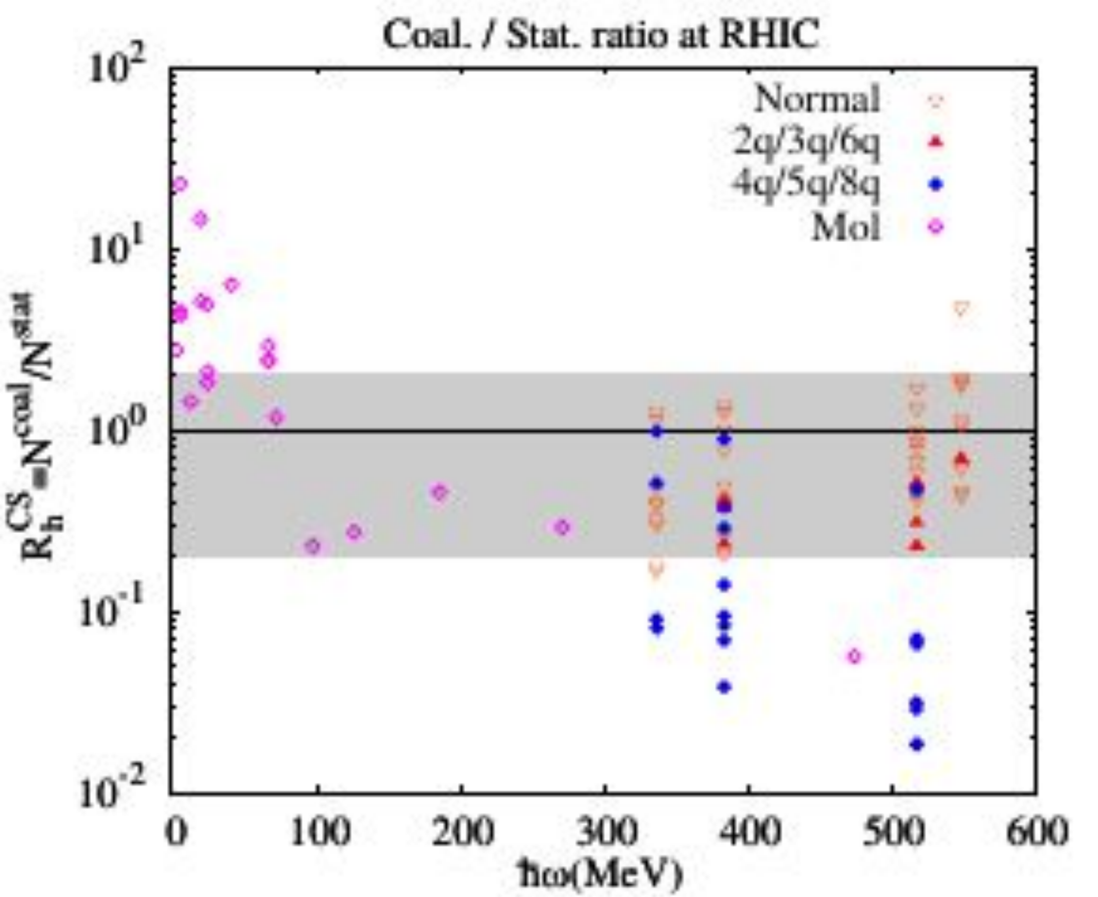
- **"Optimal" size of hadrons**
  - **The shapes of f_w and f_{th} are similar in phase space.**

$$(\Delta p/\Delta x)^2 = \mu T/2R^2 \to \hbar\omega = \sqrt{\hbar^2 T/2\mu R^2}$$

  - **hadrons with heavy-quarks (μ ~ 1 GeV),**
    T ~ Tc ~ 160 MeV,
    HIC (R ~ 5 fm)
    → ħω = 11 MeV
    (B.E. ~ 2 MeV)

Loosely bound hadronic molecules are favored in HIC, with coalescence !

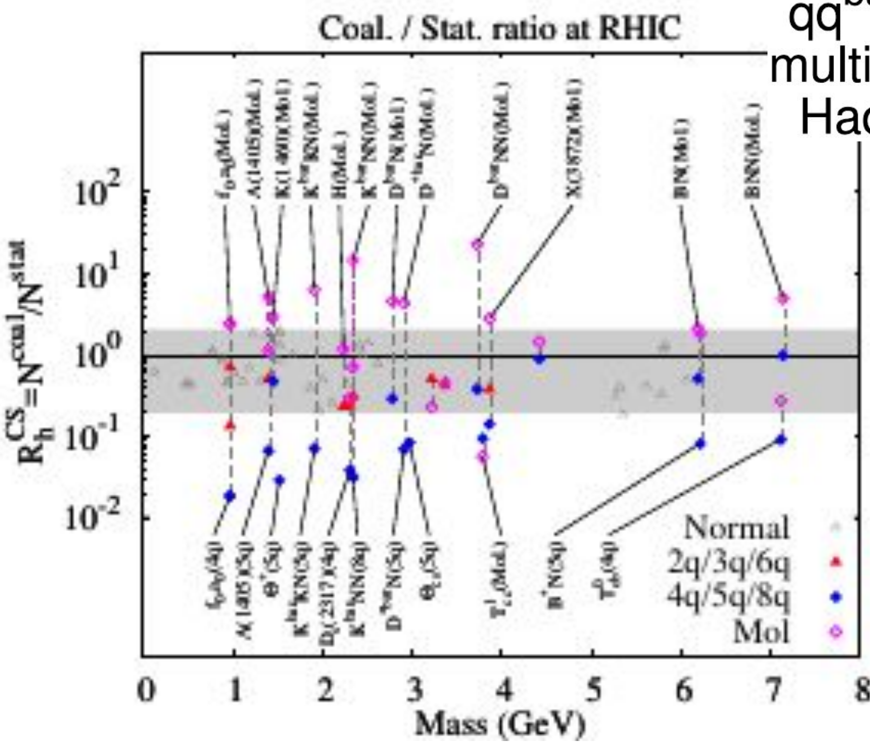
AO+('13) [Hyp2012 proc.]

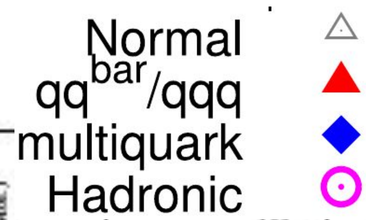


Coalescence / Statistical Ratio

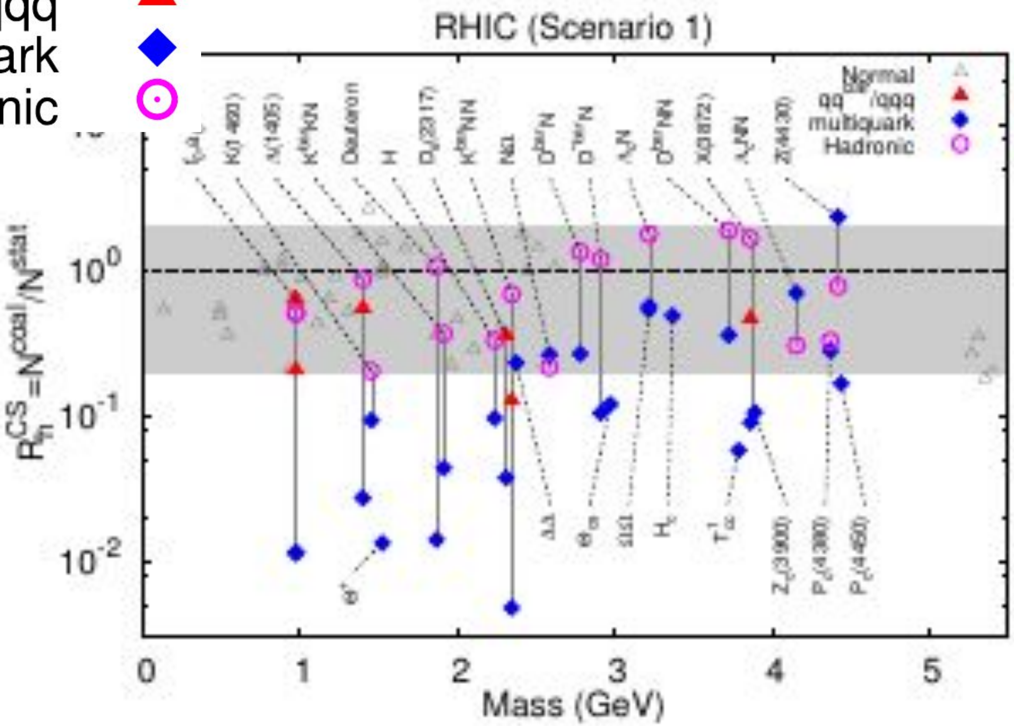
- **If the coalescence is the underlying hadronization mechanism, hadron yields will deviate from statistical model estimate depending on the number of constituents, spin, and size.**

ExHIC (2011)





ExHIC (2017)

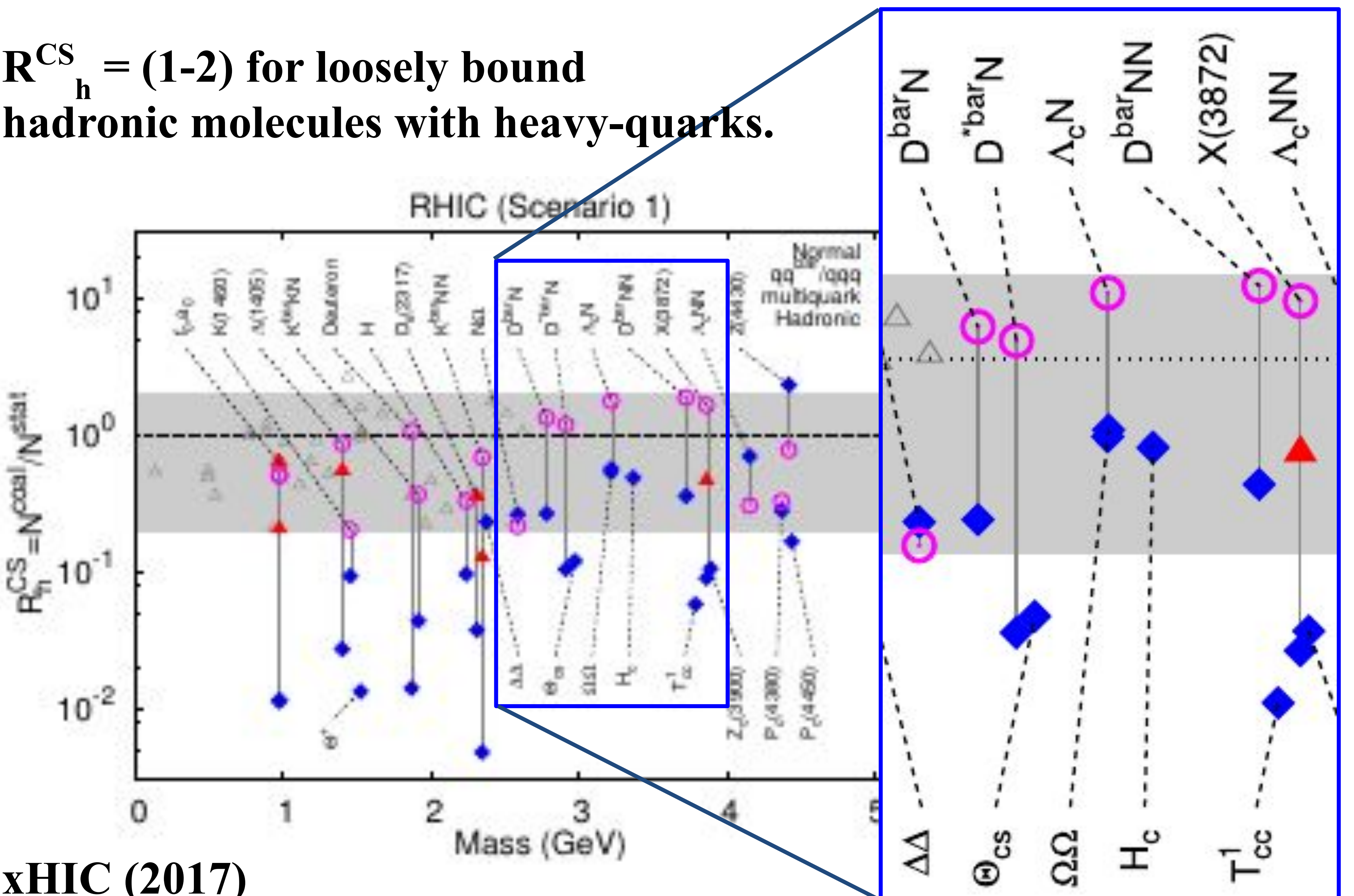


Coalescence deuteron yield is larger than stat. model.
($R^{CS}_d \sim 1$ in data.)

Freeze-out T is carefully chosen to give $R^{CS}_d \sim 1$.

Coalescence / Statistical Ratio

R^{CS}_h = (1-2) for loosely bound hadronic molecules with heavy-quarks.



ExHIC (2017)

Conclusion by ExHIC(2017) collaboration

Provided that coalescence is the underlying hadron production mechanism,
loosely bound hadronic molecules will be produced as frequently as normal hadrons,
while compact multiquark states would be suppressed (by coal. penalty factor) in heavy-ion collisions.

A New Insight from CMS: Exotic/Normal Ratio

- **ExHIC index = Coalescence / Statistical Ratio**

$$R_h^{\rm CS} = \frac{\text{Yields in Coalescence}}{\text{Yields in Statistical model}}$$

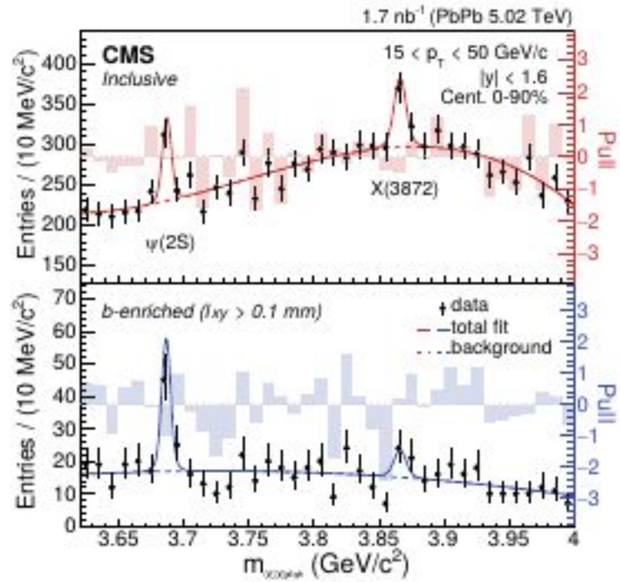
- **CMS index = Exotic / Normal Ratio**

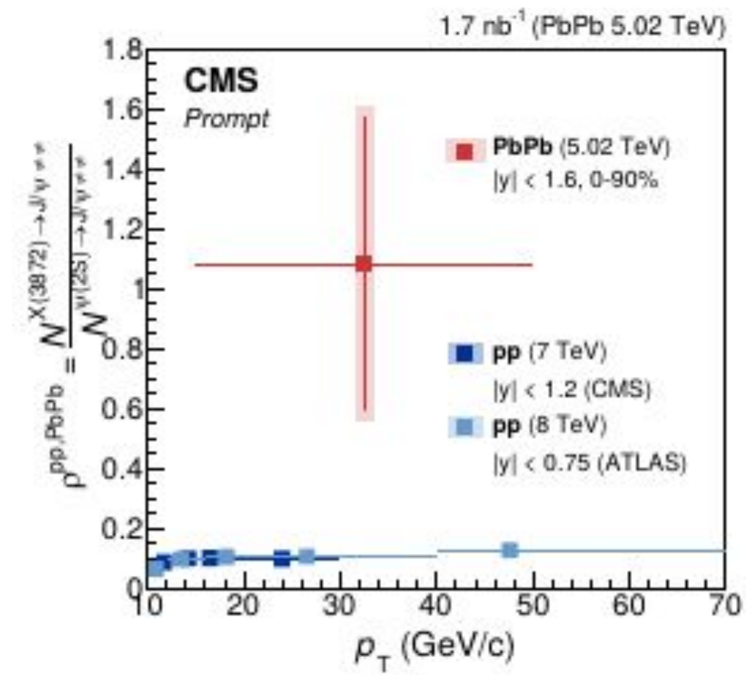
Sirunyan+ [CMS], arXiv:2102.13048 → J. Wang's talk

$$\rho_{\rm exo/nor} = \frac{N(\text{Exotic hadron candidate})}{N(\text{Normal hadron})}$$

- **X(3872) / ψ(2S) ratio in pp and PbPb collisions.**

$$\rho_{X/\psi}(\text{PbPb}) = 1.08 \pm 0.49(\text{stat.}) \pm 0.52(\text{syst.})$$
$$\rho_{X/\psi}(pp) \simeq 0.1$$





ExHIC prediction is found to be (qualitatively) realized !

Toward Qunatitative Understanding...

- **With the mass difference from M(ψ(2S)=3686 MeV), R^{CS}_{X} from data would be larger than ExHIC prediction (R~2). [ExHIC(2011) precition: R ~ 2.8 (T is higher)]**

  $$R^{CS}_{X(3872)}(\mathrm{HIC}) = 1.08 \times \exp((3872/3686)/T) \simeq 3.1$$

  → Earlier hadronic coalescence with heavy quarks ?

- **AMPT prediction** *Zhang+(PRL 126, 012301 (2021))*

  $$R^{\mathrm{molecule}}_{AA} \gg R^{\mathrm{tetraquark}}_{AA} \quad (R_{AA} = N_h(AA)/N_h(pp))$$

- **TAMU transport model** *Wu+ (EPJA 57, 122 (2021))*

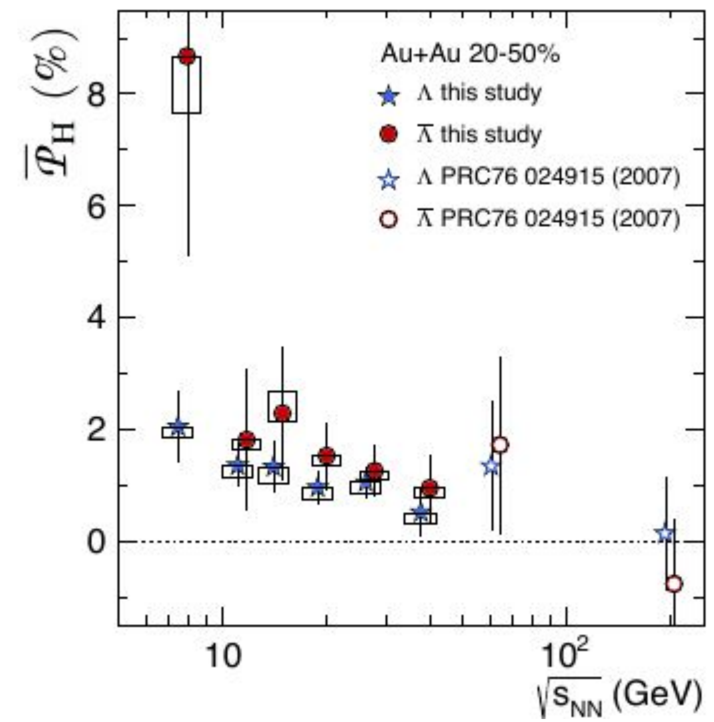
  $$R^{\mathrm{molecule}}_{AA} \sim R^{\mathrm{tetraquark}}_{AA}/2$$

- **Suppression of ψ(2S) ($R^{\psi(2S)}_{AA}$ ~ 0.14) caused the apparent enhancement of X/ψ ?**

Theorists have to work harder on production of exotic hadrons with heavy quarks toward a few 10 % level of accuarcy !

Another usage of hadron yields: Vorticity

- **Global rotation in HIC was measured via Λ polarization.**
  L. Adamczyk+('17)[STAR], Nature 548 (2017) 62-65.

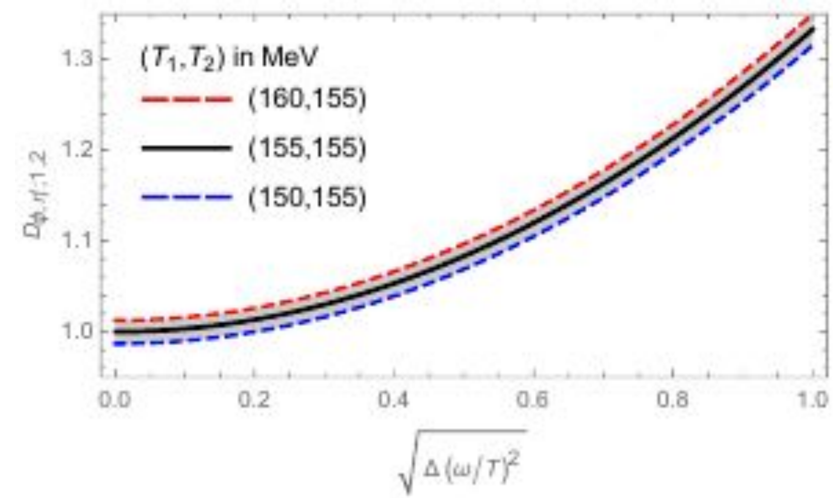


- **How about the "local" vorticity?**
  H.Taya, A.Park, S.Cho, P.Gubler, K.Hattori, J.Hong, X.-G.Huang, S.H.Lee, A.Monnai, A.Ohnishi, M.Oka, D.-L.Yang [ExHIC-P Collaboration], ('20).
  - **Stat./Coal. model with vorticity**

$$E(\omega) = E(\omega = 0) - \boldsymbol{\omega} \cdot \boldsymbol{J}$$

$$\to \frac{N(\omega)}{N(\omega = 0)} \simeq 1 + \frac{s(s+1)}{6} \frac{\langle \omega^2 \rangle}{T^2}$$

  (Almost model independent.)

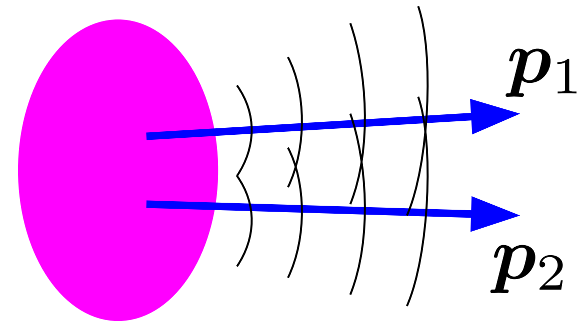


  - **φ(1030)/η'(958) double ratio will clearly show the existence of local vorticity, ω/T = O(0.1).**

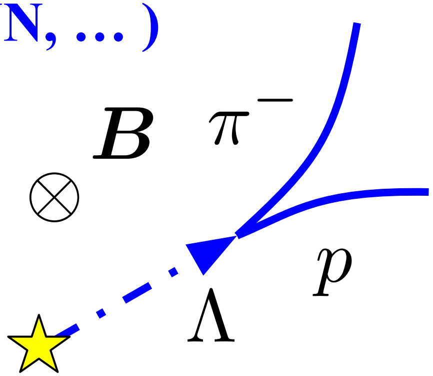
Exotic Interaction from High-Energy Nuclear Collisions

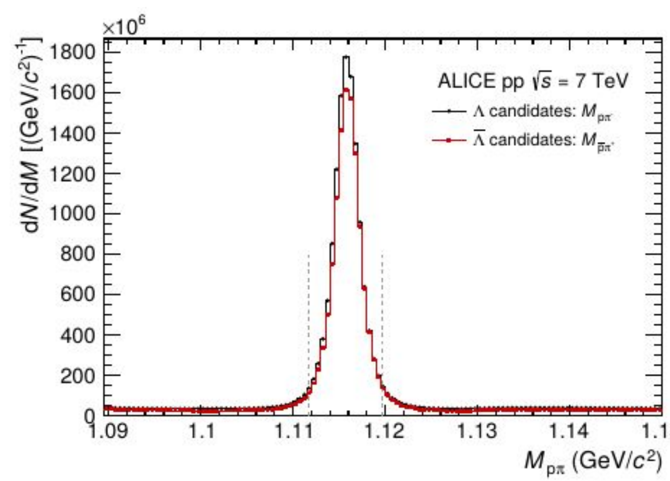
How can we access flavored hh interactions ?

- **Experimental approaches**
  - **hh scattering (NN, YN, πN, KN)**
  - **Hadronic nuclei (normal nuclei, hypernuclei, kaonic nuclei) and atom (π^-, K^-, Σ^-, Ξ^-, …)**
  - **Femtoscopy**
- **Femtoscopic study of hh interactions**
  - **Correlation function contains information of hh interactions.**
  - **Koonin-Pratt formula =Valid when the source is chaotic**
  - **Applicable to various hh pairs (NN, YN, KN, DN, YY, Yd, YNN, …)**
  - **Weakly decaying particles → Good pair purity**
  - **Future measurements: Charmed hadron, hNN, ...**



$$C(\boldsymbol{q}) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1) N_2(\boldsymbol{p}_2)} = \frac{N_{12}^{\text{same}}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_{12}^{\text{mixed}}(\boldsymbol{p}_1, \boldsymbol{p}_2)} = \int d\boldsymbol{r} S(\boldsymbol{r}) |\varphi(\boldsymbol{r}; \boldsymbol{q})|^2$$

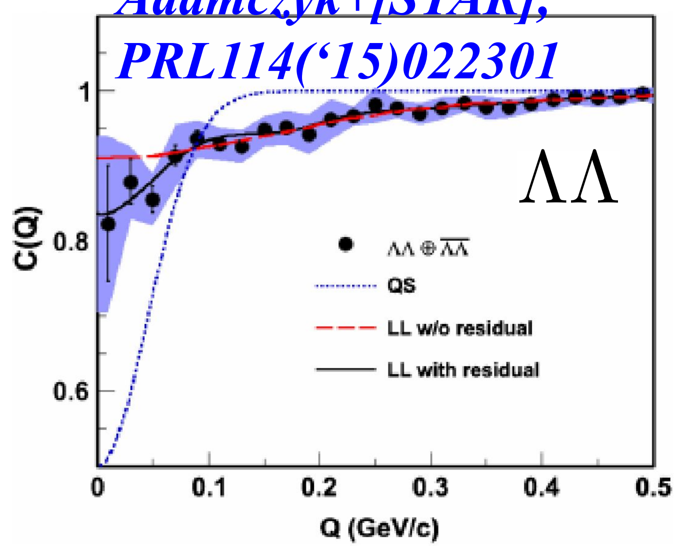


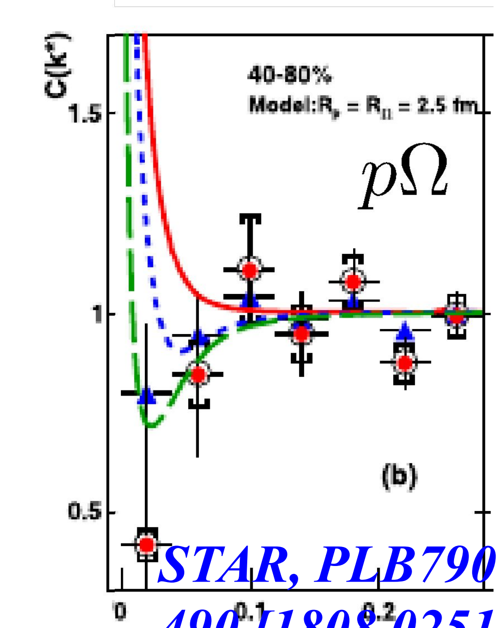


ALICE [1805.12455]

Measured Correlation Functions (examples)

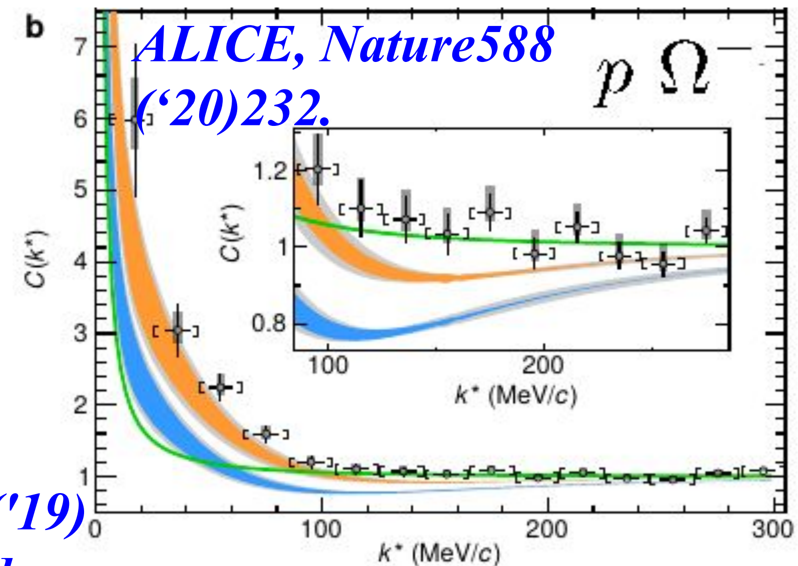
Adamczyk+[STAR], PRL114('15)022301

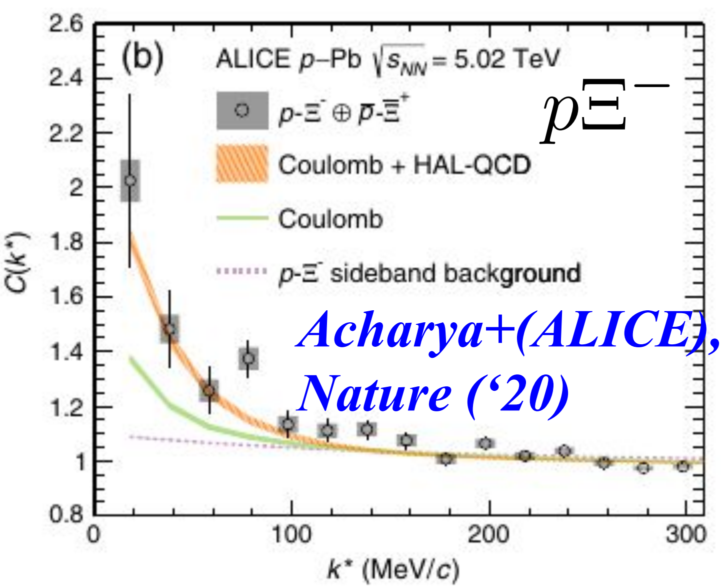




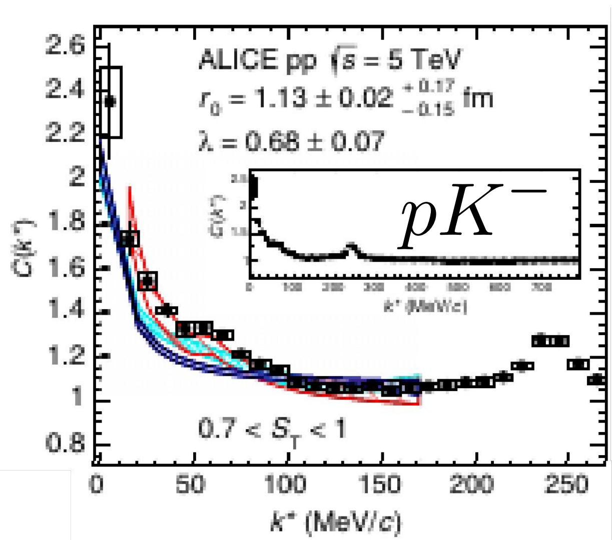
STAR, PLB790 ('19) 490 [1808.02511].

ALICE, Nature588 ('20)232.

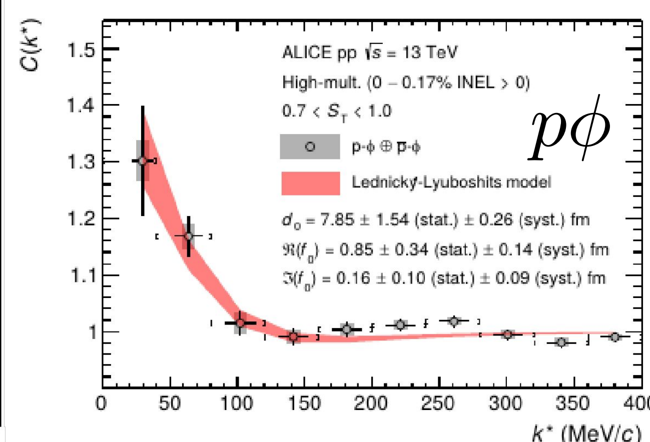




Acharya+(ALICE), Nature ('20)

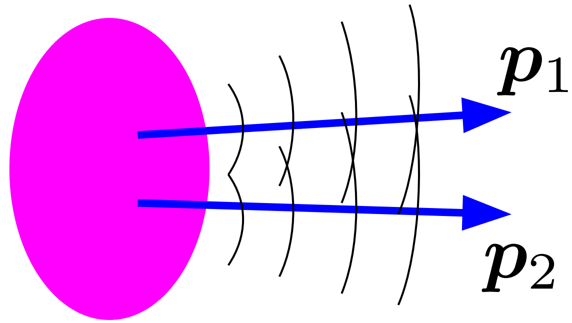


S. Acharya+[ALICE], PRL124('20)092301



ALICE, 2105.05578

Two particle momentum correlation function



- **Single particle emission function**

$$N_i(\boldsymbol{p}) = \int d^4x S_i(x,\boldsymbol{p})$$

- **Two particle momentum correlation function**
  - **Two particles are produced independently, and correlation is generated in the final state. (Koonin-Pratt formula)**

Koonin('77), Pratt+('86), Lednicky+('82)

2 body w.f.

$$C(\boldsymbol{q}) = \frac{N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)} \simeq \frac{\int d^4x d^4y S_1(x,\boldsymbol{p}_1) S_2(y,\boldsymbol{p}_2) |\Phi_{\boldsymbol{p}_1,\boldsymbol{p}_2}(x,y)|^2}{\int d^4x d^4y S_1(x,\boldsymbol{p}_1) S_2(x,\boldsymbol{p}_2)}$$

$$= \int d\boldsymbol{r} S(\boldsymbol{r}) |\varphi(\boldsymbol{r};\boldsymbol{q})|^2 = 1 + \int d\boldsymbol{r} S(r) \left[|\varphi_0(r;q)|^2 - |j_0(qr)|^2 \right]$$

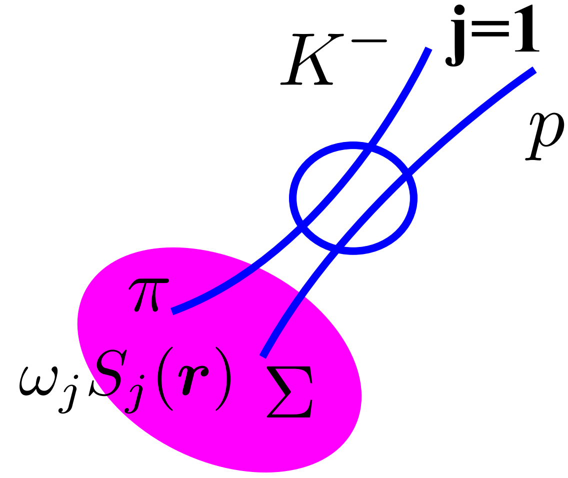
CM var. int. **Source fn.** **relative w.f.** **s-wave**

Spherical static source, non-identical particles, s-wave, No Coulomb

Correlation function with coupled-channel effects

- **KPLLL formula = CC Schrodinger eq. under $\Psi^{(-)}$ boundary cond. + channel source**

  Koonin('77), Pratt+('86), Lednicky-Lyuboshits-Lyuboshits ('98), Heidenbauer ('19), Kamiya, Hyodo, Morita, AO, Weise ('20).



$$\Psi^{(-)}(\boldsymbol{q};\boldsymbol{r}) = [\phi(\boldsymbol{q};\boldsymbol{r}) - \phi_0(q;r)]\,\delta_{1j} + \psi^{(-)}(q;r)$$

$$\psi_j^{(-)}(q;r) \to \frac{1}{2iq_j}\left[\frac{u_j^{(+)}(q_j r)}{r}\delta_{1j} - A_j(q)\frac{u_j^{(-)}(q_j r)}{r}\right]$$

$$C(q) = \int d\boldsymbol{r} S_1(r)\left[|\phi(\boldsymbol{q};\boldsymbol{r})|^2 - |\phi_0(q;r)|^2\right] + \sum_j \int d\boldsymbol{r}\,\omega_j S_j(r)|\psi_j^{(-)}(q;r)|^2$$

- **No Coulomb**

$$\phi(\boldsymbol{q};\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}},\ \phi_0(q;r) = j_0(qr),\ u_j^{(\pm)}(qr) = e^{\pm iqr},$$

$$A_j(q) = \sqrt{(\mu_j q_j)/(\mu_1 q_1)}\,S_{1j}^{\dagger}(q_1)\ \ (S_{ji} = i \to j \text{ S-matrix})$$

- **With Coulomb**

$$\phi(\boldsymbol{q};\boldsymbol{r}) = \text{Full Coulomb w.f.},\ \phi_0(q;r) = \text{s-wave Coulomb w.f.},$$

$$u_j^{(\pm)}(qr) = \pm e^{\mp i\sigma_j}\,[iF(qr) \pm G(qr)]\ (F, G = \text{regular (irregular) Coulomb fn.})$$

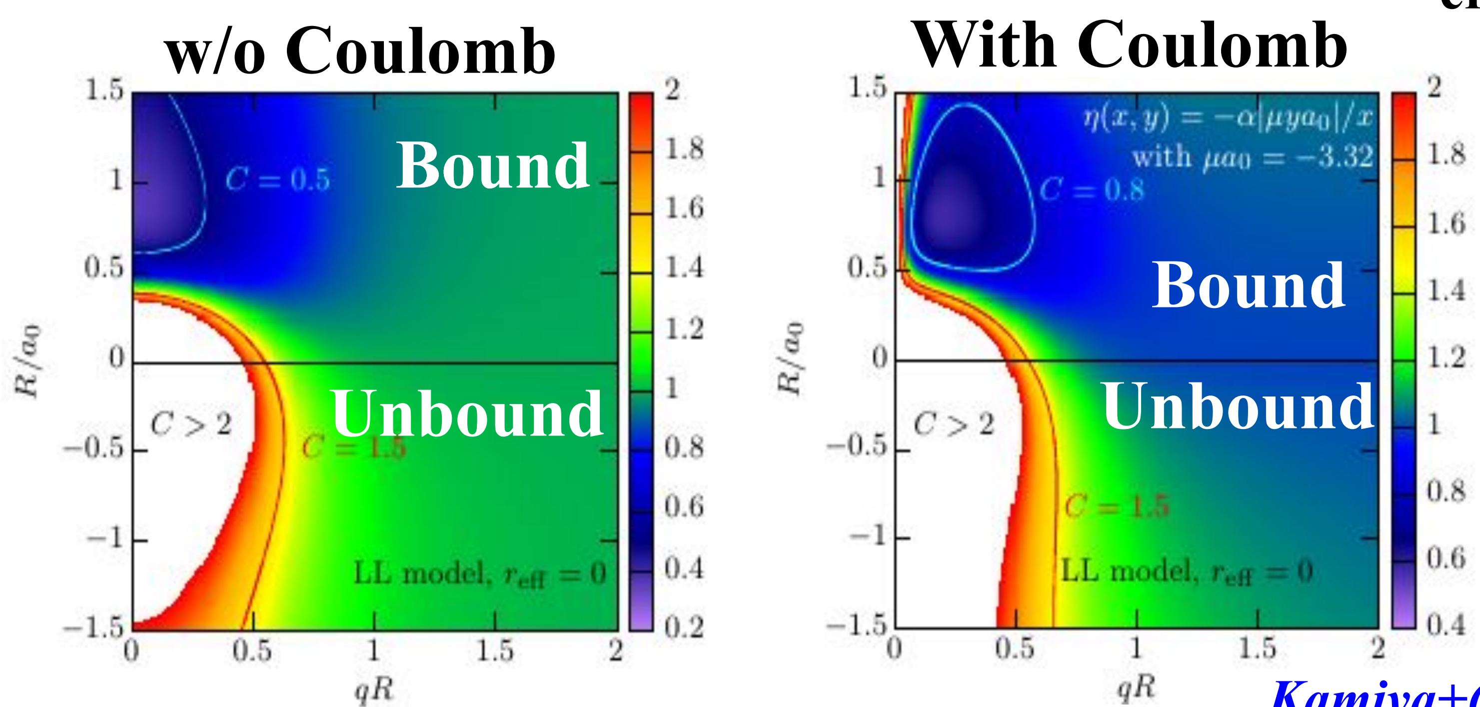
R Dependence of Correlation Function

- **Source size (R) dependence of C(q) is helpful to deduce the existence of a bound hadronic molecule state.**

  Morita+('16, '20), Kamiya+('20), Kamiya+(2108.09644)

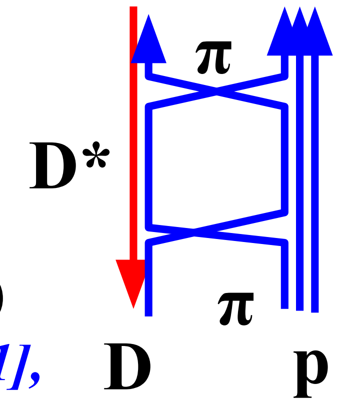
  - **With a bound state, C(q) is suppressed at small q when R ~ $|a_0|$. (w.f. has a node at r ~ $|a_0|$ with a bound state.)**
  - **Qualitative understanding by the analytic model (LL formula)**

    [Lednickey, Lyuboshits ('82)] **with the zero range approx. (r_{eff}=0)**



Kamiya+(2108.09644)

Charmed Hadron Interactions



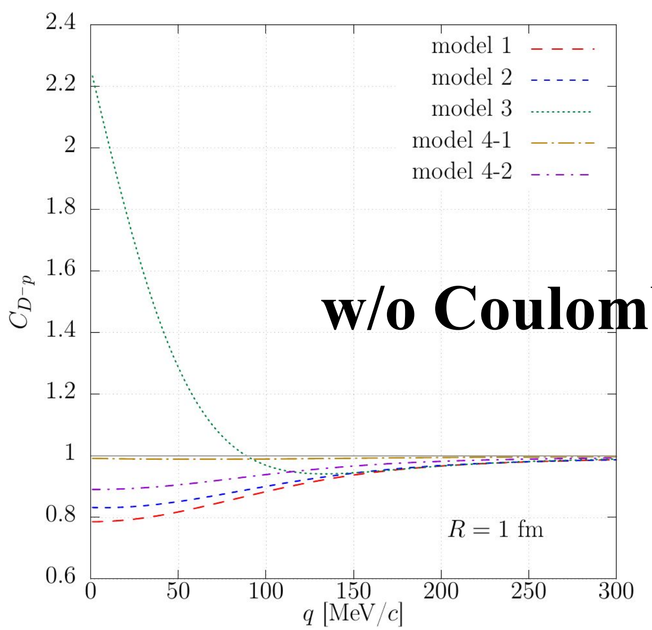
- C(q) including a charmed hadron
  - Extremely important in recent hadron physics.
- $D^-(c\bar{d})$-p(uud) correlation
  - Probes Θ_c(c-ud-ud) state (replace s in Θ(s-ud-ud) with c)
    D. O. Riska, N. N. Scoccola, PLB299('93)338 (pred.); A. Aktaset+ [H1], PLB588('04)17 (positive); J. M. Linket+ [FOCUS], PLB622('05)229 (negative).
  - Proposed potentials generally predict weak or repulsive interaction.
    Hofmann, Lutz ('05) (repulsive); Haidenbauer+('07) (repulsive); Yamaguchi+('11) (att., w/ bs); Fontoura+('13) (repulsive)
  - Attraction from two pion exchange *S. Yasui, K. Sudoh, PRD80('09)034008.*
  - Easy to calculate the potential in LQCD. *Y. Ikeda et al. (private comm.)*

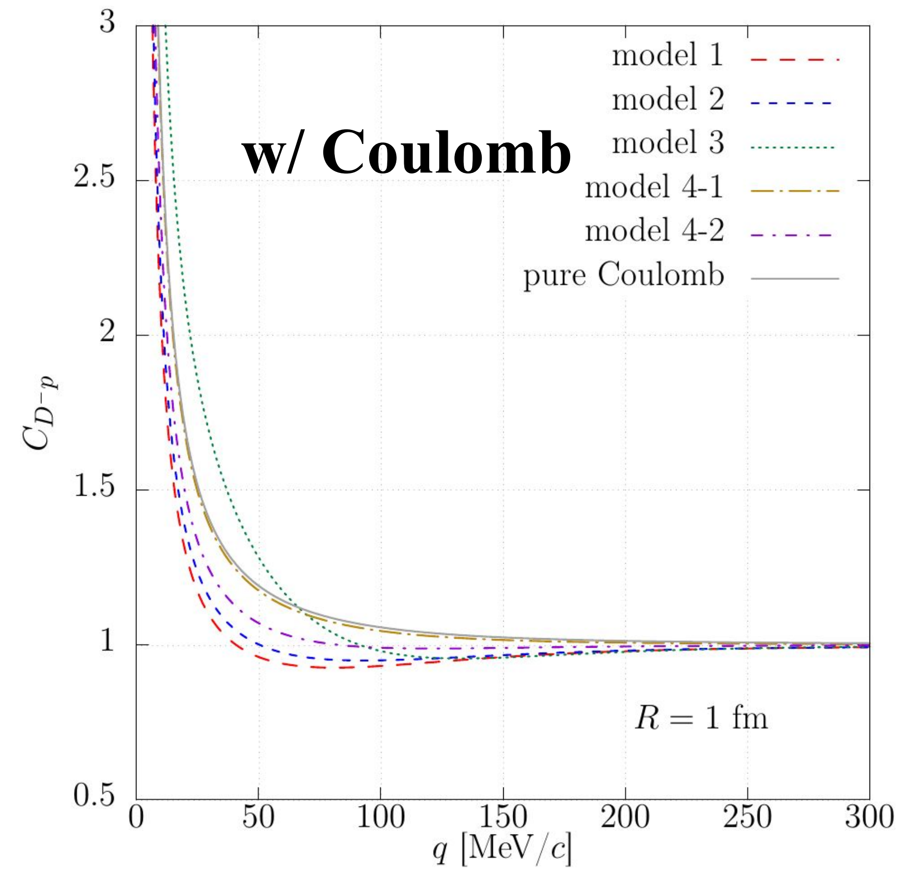
Model	$a^{I=0}_{\bar{D}N}$	$a^{I=1}_{\bar{D}N}$	$a_{\bar{D}}$	
SU(4) contact [185]	−0.16	−0.26	−0.24	*Hoffmann, Lutz ('05)*
Meson exchange [194]	0.07	−0.45	−0.32	*Haidenbauer+ ('07)*
Pion exchange [192]	−4.38	−0.07	−1.15	*Yamaguchi+('11)*
Chiral quark model [219]	0.03–0.16	-(0.20-0.25)	-(0.14-0.15)	*Fontoura+('13)*

Taken from Hosaka, Hyodo, Sudoh, Yamaguchi, Yasui, PPNP96('17)88

Charmed Hadron Interactions

- **D⁻(cd)-p(uud) CFs from proposed scattering length**
  Kamiya, Hyodo, AO (in prog.)
  - **One-range Gaussian potential strength is fitted to proposed a_0 with the range of ρ meson exchange.**
  - **Measurable difference is found.**





Data will discriminate these potentials !

c.f. Talk by T. Hyodo and K. Ogata (Sep. 14 (Tue)) .

Summary

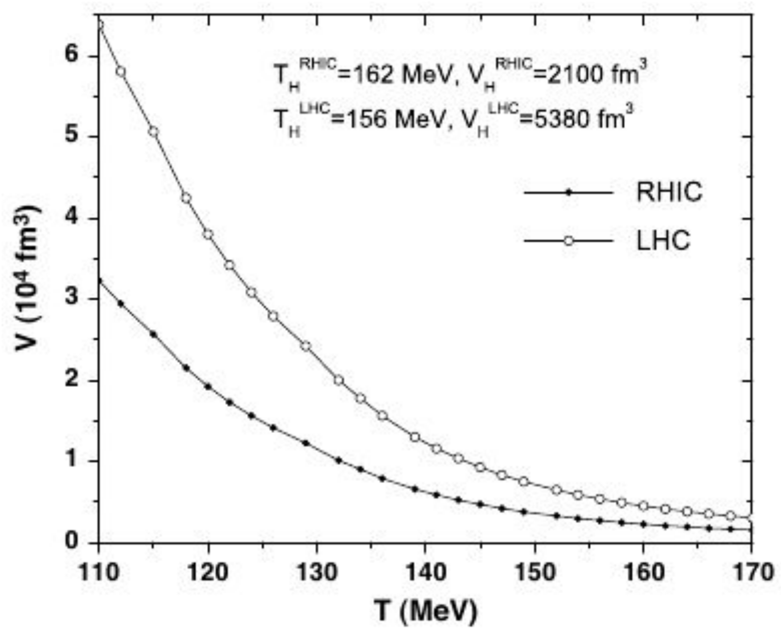
- **High-energy nuclear collisions (pp(high-multiplicity), pA, AA) are helpful to study hadron physics.**
  - **Many hadrons are produced in one event. (Hadron factory)**
  - **Reaction processes are so complicated that statistical (chaotic) source would be applicable, then one may access the generic feature of hadron production mechanism.**
  - **If the coalescence (of quarks and hadrons) is the underlying hadron production mechanism, hadronic molecules will be formed as frequently as normal hadrons in HIC.**
  - **Hadron-hadron correlation functions tells us information on hh interactions as well as the possibility of existence of a bound state.**
  - **Many types of reactions are necessary to identify the structure.**
- **There are many works to do.**
  - **Quantitative estimate of hadron production yields, especially of those recently observed exotic hadrons (X(3872), Tcc).**
  - **Estimate of hadron yields from e^+e^- and hA collisions.**
  - **Classification of exotic hadrons, multiquark, molecule, singularity, ...**

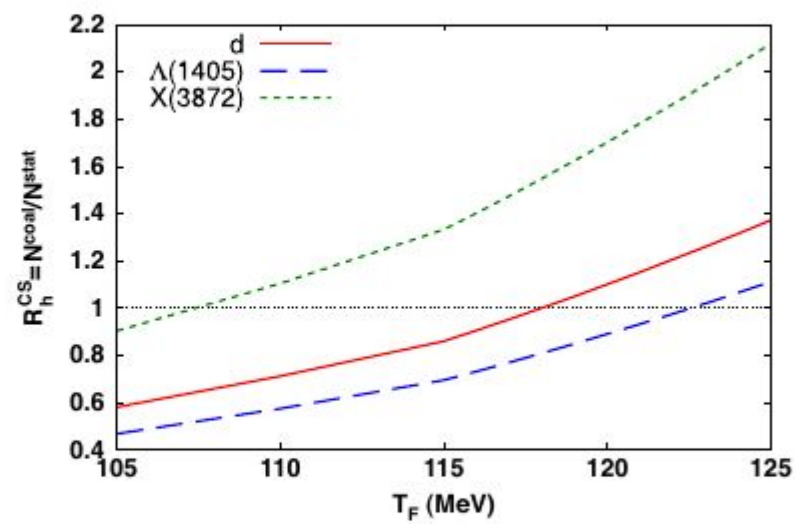
Thank you for attention !

Freeze-out T dependence of yields

- **Deuteron production yield depends on the hadronic coalescence temperature.**

	RHIC Sc. 1	RHIC Sc. 2	LHC (2.76 TeV) Sc. 1	LHC (2.76 TeV) Sc. 2	LHC (5.02 TeV) Sc. 1	LHC (5.02 TeV) Sc. 2	RHIC Refs [14,15]	LHC (5 TeV) Refs [14,15]
T_H (MeV)	162		156				175	
V_H (fm^3)	2100		5380				1908	5152
T_C (MeV)	162	166	156	166	156	166	175	
V_C (fm^3)	2100	1791	5380	3533	5380	3533	1000	2700
T_F (MeV)	119		115				125	
V_F (fm^3)	20355		50646				11322	30569





S. Cho et al. [ExHIC Collaboration] ('17)