(Part II) Femtoscopic approach to hadron-hadron interactions

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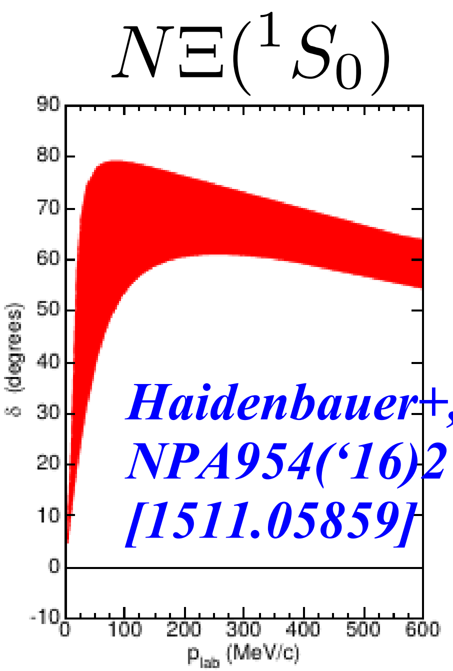
KEK theory seminar [EX], July 1, 2021, Online / KEK

- Introduction
- Correlation function in simple cases
  - Free identical boson / fermion pair (HBT/GGLP effects)
  - Analytic model of correlation function (LL formula)
    (non-identical particle pair, short range interaction, single channel)
- Correlation function in more realistic cases
  - Couple-channel effects, Coulomb potential, Pair purity, Dynamical sources, …
- Recently observed / studied correlation functions
  - Ωp, $K^- p$, $\Xi^- p$, $\Lambda\Lambda$, $\Xi^- d$, $\bar{D}p$, ….
- Summary

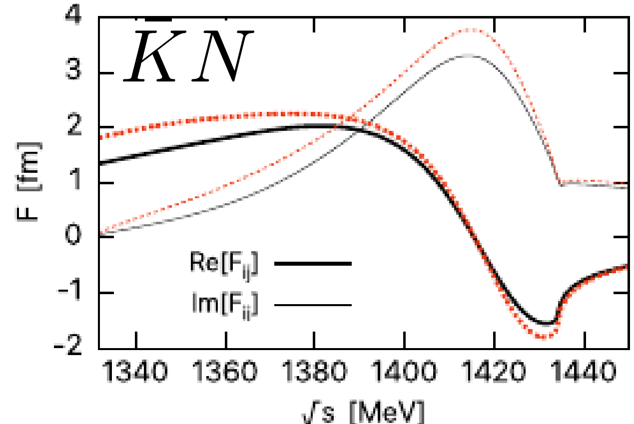
How can we access flavored hh interactions ?

- **Theoretical approaches**
  - **Nuclear force models: meson exch., quark model, … (need data)**
  - ***Ab initio*: chiral EFT (χEFT), lattice QCD (need data or CPU resources)**

Chiral

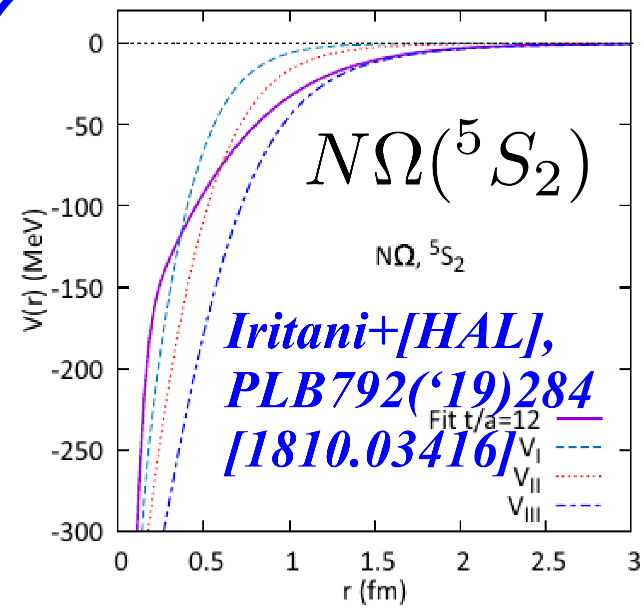


Haidenbauer+, NPA954('16)273 [1511.05859]

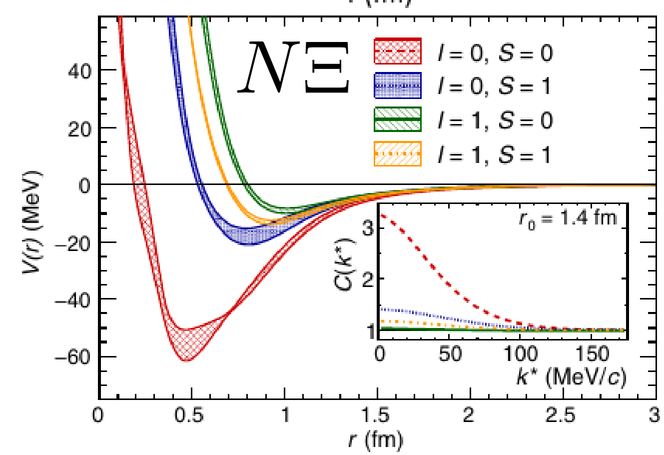


Miyahara, Hyodo, Weise, PRC98('18), 025201 [1804.08269] (Ikeda-Hyodo-Weise amplitude)

Lattice



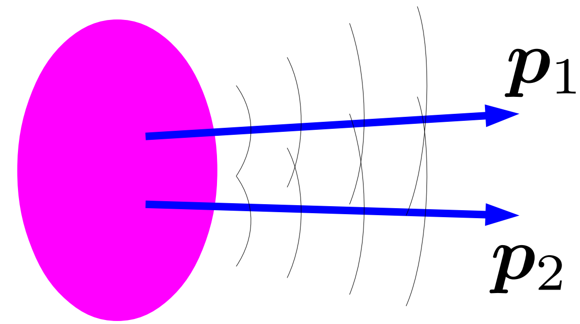
Iritani+[HAL], PLB792('19)284 [1810.03416]



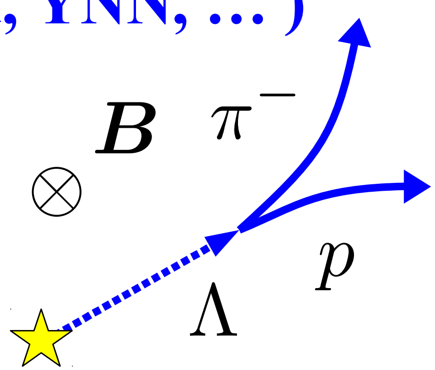
Sasaki+ [HAL], NPA998 ('20)121737 [1912.08630] (taken from ALICE('19))

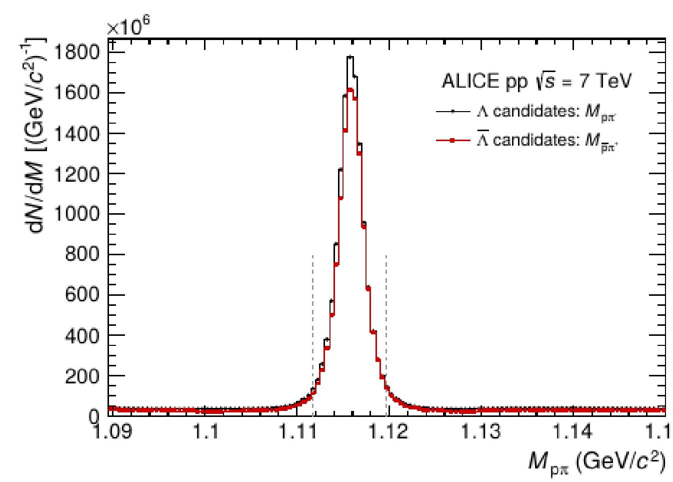
How can we access flavored hh interactions ?

- Experimental approaches
  - hh scattering (NN, YN, πN, KN)
  - Hadronic nuclei (normal nuclei, hypernuclei, kaonic nuclei) and atom (π^-, K^-, Σ^-, Ξ^-, …)
  - Femtoscopy
- Femtoscopic study of hh interactions
  - Correlation function contains information of hh interactions.
  - Koonin-Pratt formula =Valid when the source is chaotic
  - Applicable to various hh pairs (NN, YN, KN, DN, YY, Yd, YNN, …)
  - Weakly decaying particles → Good pair purity
  - Future measurements: Charmed hadron, hNN, ...



$$C(\boldsymbol{q}) = \frac{N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)} = \frac{N_{12}^{\rm same}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{N_{12}^{\rm mixed}(\boldsymbol{p}_1,\boldsymbol{p}_2)} = \int d\boldsymbol{r} S(\boldsymbol{r}) |\varphi(\boldsymbol{r};\boldsymbol{q})|^2$$

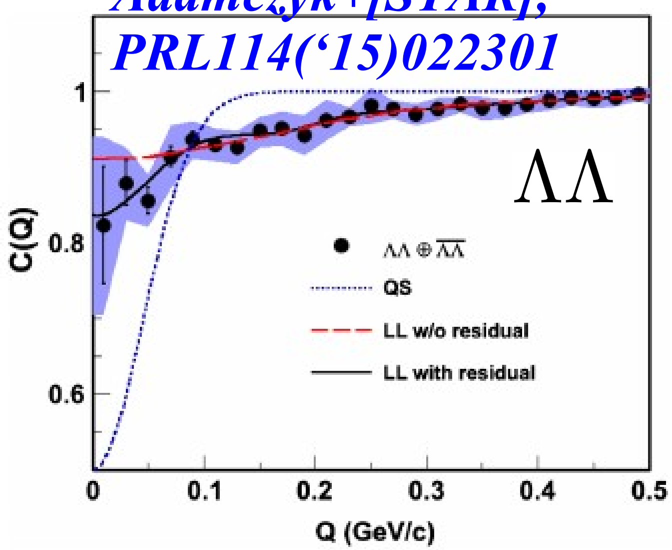


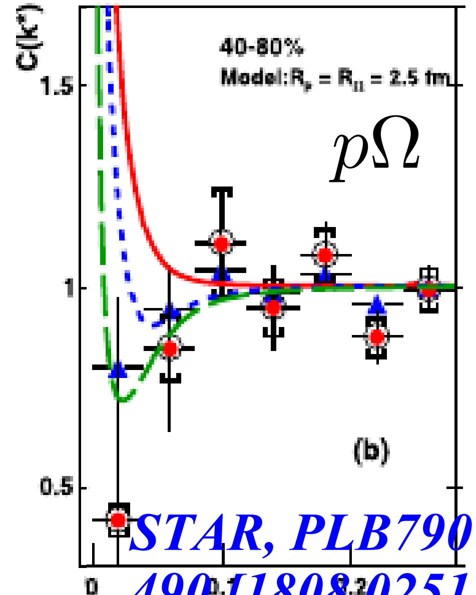


ALICE [1805.12455]

Measured Correlation Functions (examples)

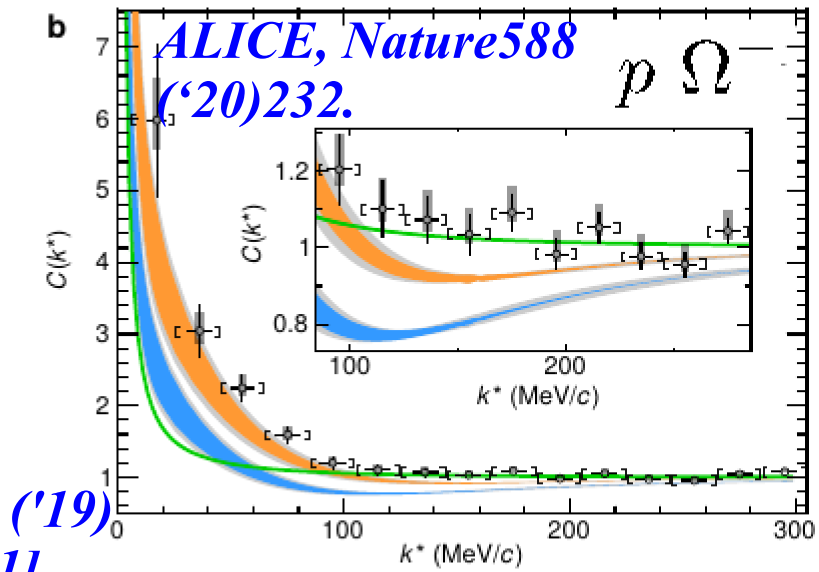
Adamczyk+[STAR], PRL114('15)022301

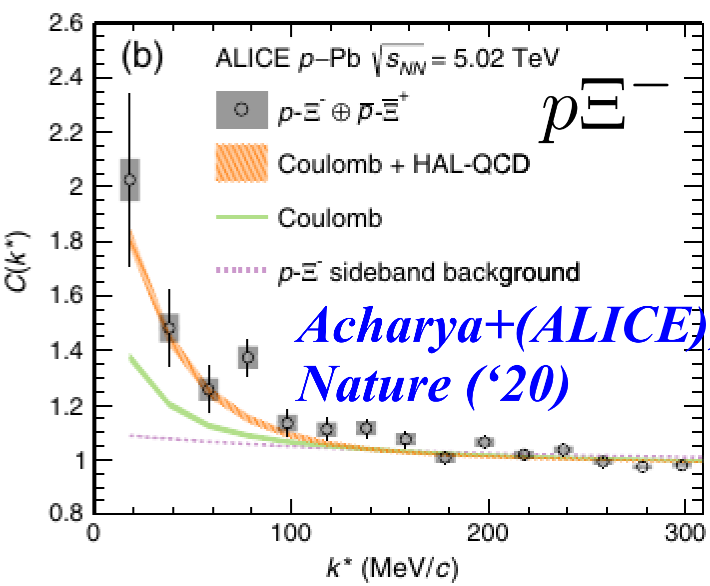




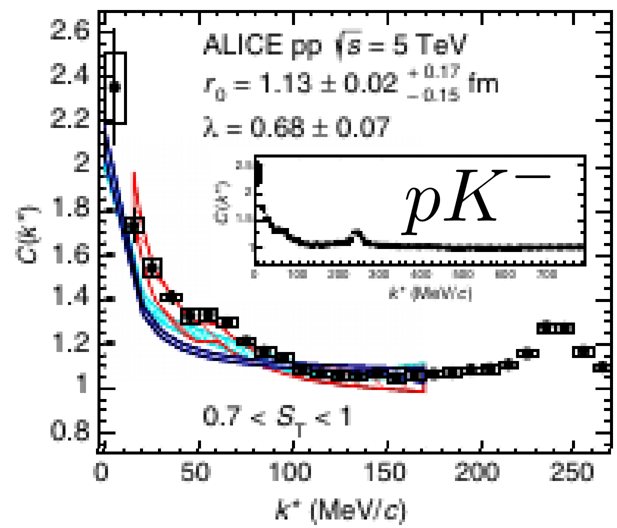
STAR, PLB790 ('19) 490 [1808.02511].

ALICE, Nature588 ('20)232.

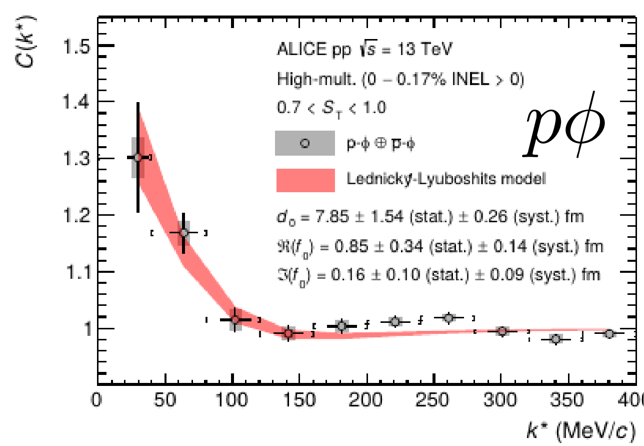




Acharya+(ALICE), Nature ('20)



S. Acharya+[ALICE], PRL124('20)092301



ALICE, 2105.05578

Potentially measurable hh pairs

- **Correlation function is useful to access hadron-hadron interactions as well as to deduce the existence of a bound state.**

	n	p	K^-	K^+	π^-	π^+	Λ	Σ	Ξ^-	Ω^-	D^-	D^+	Ks	d	pp	φ	+α
n																	
p		O	**O**	O	△	△	O	O	**O**	**O**	O	O		O	O	O	
K^-		**O**	O	O	O	O							O				
K^+		O	O	O	O	O							O				
π^-		△	O	O	O	O											
π^+		△	O	O	O	O											
Λ		O					**O**		O						O		
Σ		O						O									
Ξ^-		**O**															
Ω^-		**O**															
D^-		O															
D^+		O															
Ks			O	O													
d		O															
pp		O					O										
φ		O															
+α																	

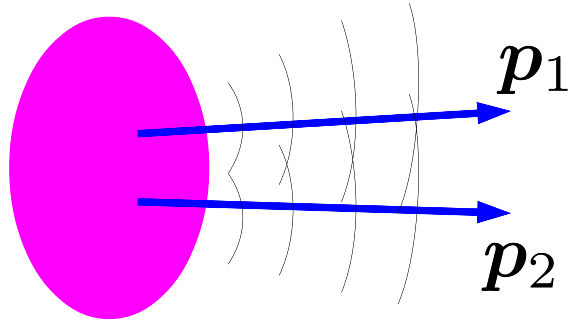
Blue: Pairs we have studied, O: Experimentally measured

Correlation function in simple cases

Two particle momentum correlation function

- **Single particle emission function**

$$N_i(\boldsymbol{p}) = \int d^4x S_i(x, \boldsymbol{p})$$



- **Two particle momentum correlation function**
  - **Two particles are produced independently, and correlation is generated in the final state. (Koonin-Pratt formula)**

Koonin('77), Pratt+('86), Lednicky+('82)

2 body w.f.

$$C(\boldsymbol{q}) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1) N_2(\boldsymbol{p}_2)} \simeq \frac{\int d^4x d^4y S_1(x, \boldsymbol{p}_1) S_2(y, \boldsymbol{p}_2) |\Phi_{\boldsymbol{p}_1, \boldsymbol{p}_2}(x, y)|^2}{\int d^4x d^4y S_1(x, \boldsymbol{p}_1) S_2(x, \boldsymbol{p}_2)}$$

$$= \int d\boldsymbol{r} S(\boldsymbol{r}) |\varphi(\boldsymbol{r}; \boldsymbol{q})|^2 = 1 + \int d\boldsymbol{r} S(r) \left[|\varphi_0(r; q)|^2 - |j_0(qr)|^2 \right]$$

CM var. int. **Source fn.** **relative w.f.** **s-wave**

Spherical static source, non-identical particles, s-wave, No Coulomb

Free identical boson pair: C(q) → Source size R

- **Free identical spin 0 bosons, static Gaussian source function**
  - **Source Function (one-body source size R, temperature T)**

$$S_1(\boldsymbol{x},\boldsymbol{p}) \propto \exp\left[-\frac{\boldsymbol{x}^2}{2R^2} - \frac{\boldsymbol{p}^2}{2MT}\right], \quad S_1(\boldsymbol{x},\boldsymbol{p}_1)S_1(\boldsymbol{y},\boldsymbol{p}_2) \propto \exp\left[-\frac{\boldsymbol{R}_{\rm cm}^2}{R^2} - \frac{\boldsymbol{r}^2}{4R^2} - \frac{\boldsymbol{P}^2}{4MT} - \frac{\boldsymbol{q}^2}{2\mu T}\right]$$

$$S(\boldsymbol{r}) \equiv \frac{\int d\boldsymbol{R}_{\rm cm} S_1(\boldsymbol{x},\boldsymbol{p}_1)S_1(\boldsymbol{y},\boldsymbol{p}_2)}{\int d\boldsymbol{x}\, d\boldsymbol{y} S_1(\boldsymbol{x},\boldsymbol{p}_1)S_1(\boldsymbol{y},\boldsymbol{p}_2)} = \frac{e^{-\boldsymbol{P}^2/4MT-\boldsymbol{q}^2/2\mu T}\mathcal{N}e^{-\boldsymbol{r}^2/4R^2}}{e^{-\boldsymbol{P}^2/4MT-\boldsymbol{q}^2/2\mu T}}$$

$$\rightarrow \boxed{S(\boldsymbol{r}) = \mathcal{N}e^{-\boldsymbol{r}^2/4R^2} \quad \left[\mathcal{N} = (4\pi R^2)^{-3/2}\right]}$$

Non-identical
$R^2 = (R_1^2 + R_2^2)/2$

  - **Two-body wave function**

$$\Phi_{\boldsymbol{p}_1,\boldsymbol{p}_2}(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{\sqrt{2}}\left[e^{i\boldsymbol{p}_1\cdot\boldsymbol{x}+i\boldsymbol{p}_2\cdot\boldsymbol{y}} + e^{i\boldsymbol{p}_1\cdot\boldsymbol{y}+i\boldsymbol{p}_2\cdot\boldsymbol{x}}\right] = e^{i\boldsymbol{P}\cdot\boldsymbol{R}_{\rm cm}} \times \sqrt{2}\cos\boldsymbol{q}\cdot\boldsymbol{r}$$

  - **Correlation function**

$2\cos^2 x = 1 + \cos 2x$

$$C(\boldsymbol{q}) = \int d\boldsymbol{r} S(\boldsymbol{r})\left|\Phi_{\boldsymbol{p}_1,\boldsymbol{p}_2}(\boldsymbol{x},\boldsymbol{y})\right|^2 = \mathcal{N}\int d\boldsymbol{r} e^{-\frac{\boldsymbol{r}^2}{4R^2}} 2\cos^2\boldsymbol{q}\cdot\boldsymbol{r}$$

$$= \mathcal{N}\int d\boldsymbol{r} e^{-\frac{\boldsymbol{r}^2}{4R^2}}\left[1 + \frac{1}{2}(e^{2i\boldsymbol{q}\cdot\boldsymbol{r}} + e^{-2i\boldsymbol{q}\cdot\boldsymbol{r}})\right] \rightarrow \boxed{C(\boldsymbol{q}) = 1 + \exp(-4q^2R^2)}$$

Free identical fermion pair

- **Free identical spin 1/2 fermions, static Gaussian source function**
  - **Source Function (one-body source size R, temperature T)**

$$S(\boldsymbol{r}) = \mathcal{N} e^{-\boldsymbol{r}^2/4R^2} \quad \left[\mathcal{N} = (4\pi R^2)^{-3/2}\right]$$

  - **Two-body wave function**
    - **spin singlet (triplet) → spatially symmetric (anti-symmetric)**

$$\Phi^{\rm singlet}_{\boldsymbol{p}_1,\boldsymbol{p}_2}(\boldsymbol{x},\boldsymbol{y}) = e^{i\boldsymbol{P}\cdot\boldsymbol{R}_{\rm cm}} \times \sqrt{2}\cos\boldsymbol{q}\cdot\boldsymbol{r}$$

$$\Phi^{\rm triplet}_{\boldsymbol{p}_1,\boldsymbol{p}_2}(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{\sqrt{2}}\left[e^{i\boldsymbol{p}_1\cdot\boldsymbol{x}+i\boldsymbol{p}_2\cdot\boldsymbol{y}} - e^{i\boldsymbol{p}_1\cdot\boldsymbol{y}+i\boldsymbol{p}_2\cdot\boldsymbol{x}}\right] = e^{i\boldsymbol{P}\cdot\boldsymbol{R}_{\rm cm}} \times \sqrt{2}i\sin\boldsymbol{q}\cdot\boldsymbol{r}$$

  - **Correlation function**

$$C^{\rm singlet,triplet}(\boldsymbol{q}) = \mathcal{N}\int d\boldsymbol{r} e^{-\frac{\boldsymbol{r}^2}{4R^2}} 2\{\cos,\sin\}^2 \boldsymbol{q}\cdot\boldsymbol{r}$$

$$2\{\cos,\sin\}^2 x = 1 \pm \cos 2x$$

$$= \mathcal{N}\int d\boldsymbol{r} e^{-\frac{\boldsymbol{r}^2}{4R^2}} \left[1 \pm \frac{1}{2}(e^{2i\boldsymbol{q}\cdot\boldsymbol{r}} + e^{-2i\boldsymbol{q}\cdot\boldsymbol{r}})\right] = 1 \pm \exp(-4q^2R^2)$$

    - **Statistical weight of spin singlet:triplet=1:3**

$$C(\boldsymbol{q}) = \frac{1}{4}C^{\rm singlet}(\boldsymbol{q}) + \frac{3}{4}C^{\rm triplet}(\boldsymbol{q}) = 1 - \frac{1}{2}\exp(-4q^2R^2)$$

How can we measure the radius of a star ?

- **Two photon intensity correlation**
  Hanbury Brown & Twiss, Nature 10 (1956), 1047.
  - **Simultaneous two photon observation probability is enhanced from independent emission cases**
    → angular diameter of Sirius=0.0063 sec

Recent measurement (Wikipedia)
5.936±0.016 msec

A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

By R. HANBURY BROWN
Jodrell Bank Experimental Station, University of Manchester
AND
DR. R. Q. TWISS
Services Electronics Research Laboratory, Baldock

NATURE November 10, 1956 VOL. 178

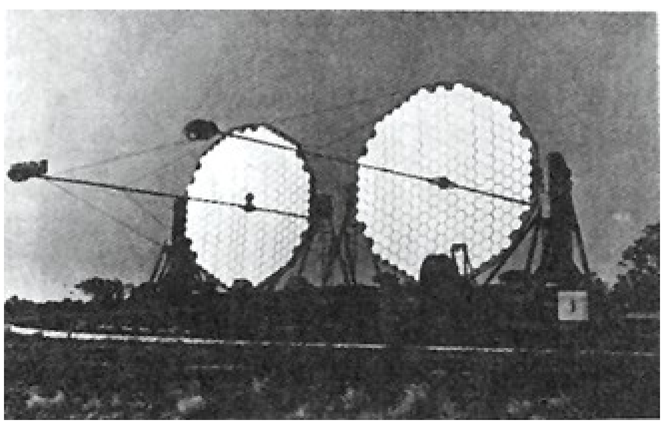
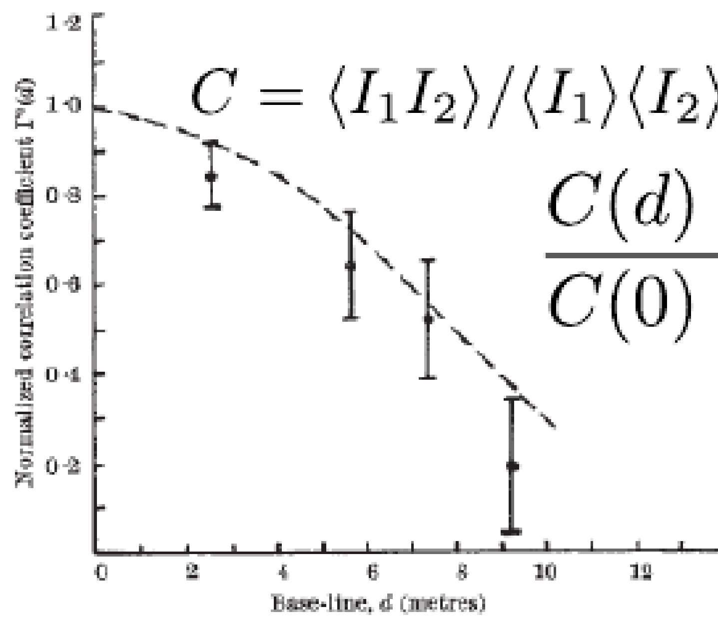


Figure 2. Picture of the two telescopes used in the HBT experiments. The figure was extracted from Ref.[1].

HBP telescope (from Goldhaber, ('91))



$$C = \langle I_1 I_2 \rangle / \langle I_1 \rangle \langle I_2 \rangle$$

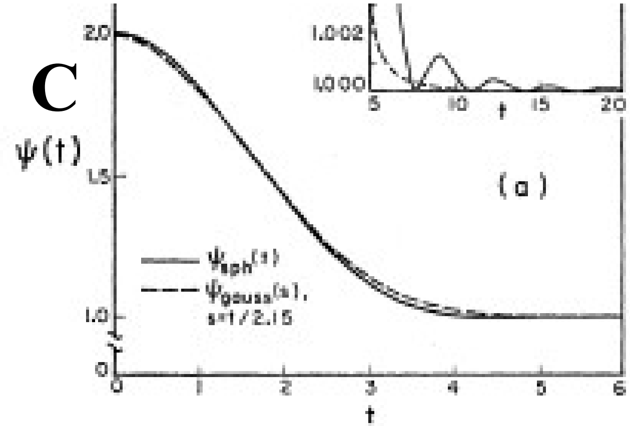
$$\frac{C(d)-1}{C(0)-1}$$

Fig. 2. Comparison between the values of the normalized correlation coefficient Γ²(d) observed from Sirius and the theoretical values for a star of angular diameter 0·0063″. The errors shown are the probable errors of the observations

HBT ('56)

How can we measure source size in nuclear reactions ?

- **Two pion interferometry**
  G. Goldhaber, S. Goldhaber, W. Lee, A. Pais, Phys. Rev. 120 (1960), 300
  - **Two pion emission probability is enhanced at small relative momenta**
    → Pion source size ~ 0.75 ℏ / μc



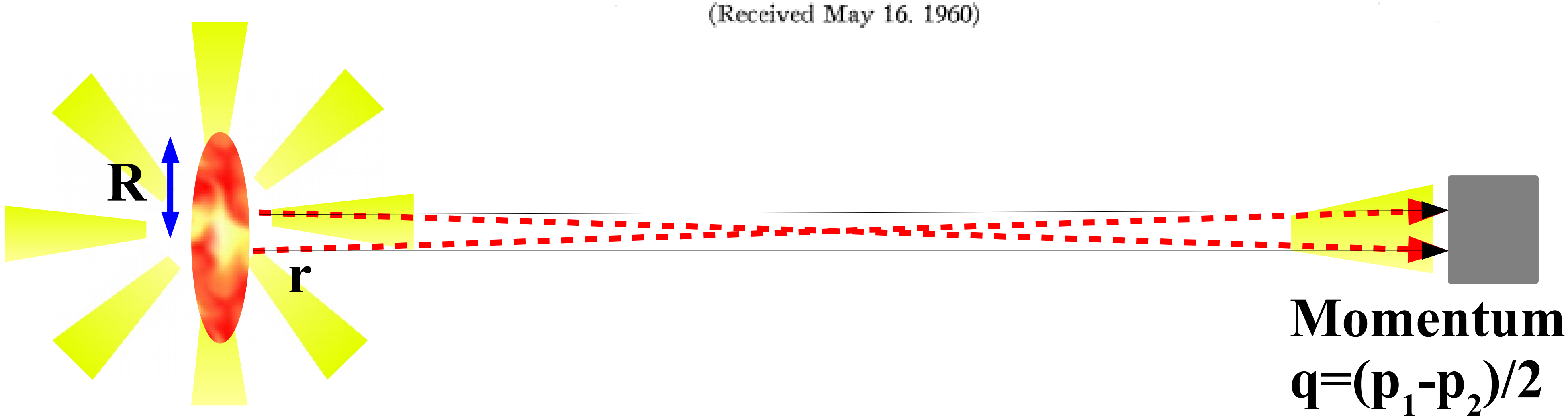
q (relative momentum)

PHYSICAL REVIEW VOLUME 120, NUMBER 1 OCTOBER 1, 1960

Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process*

Gerson Goldhaber, Sulamith Goldhaber, Wonyong Lee, and Abraham Pais†
Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California

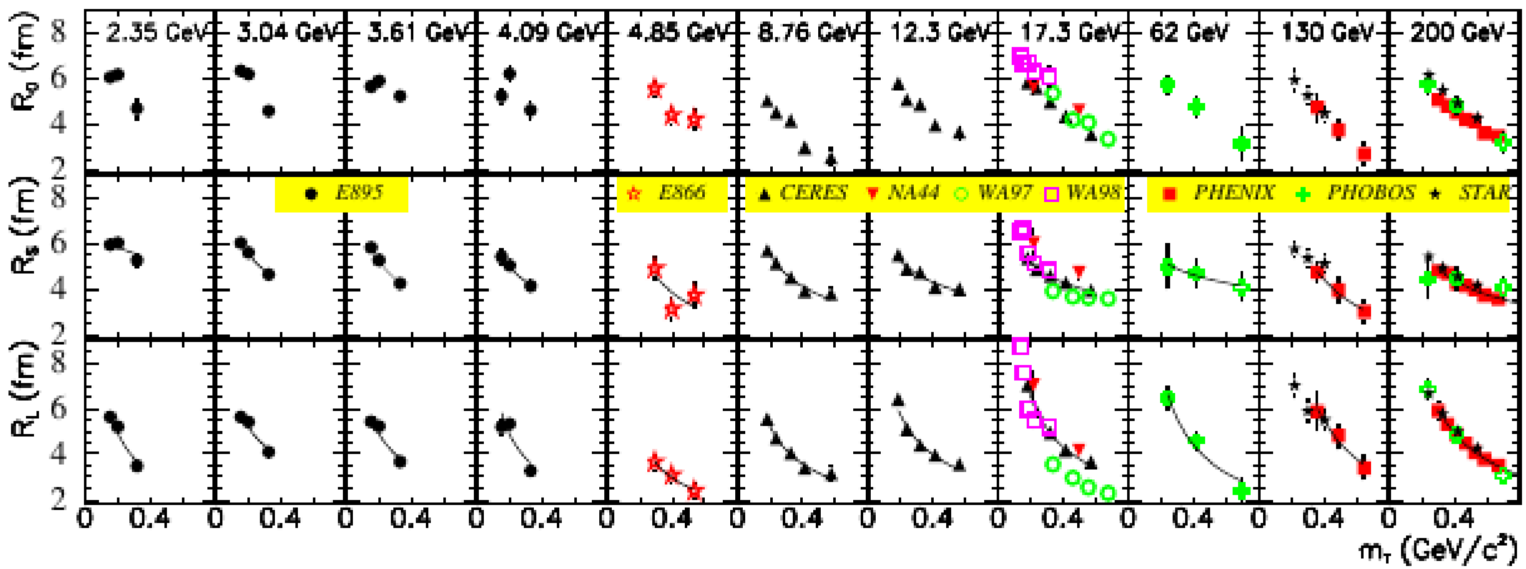
(Received May 16. 1960)



State-of-the-art Femtoscopy of radii

- **Systematic measurement of 3D HBT radii (side, out, long)**

  M. A. Lisa, S. Pratt, R. Soltz, U. Wiedemann, Ann.Rev.Nucl.Part.Sci. 55 (2005) 357-402.



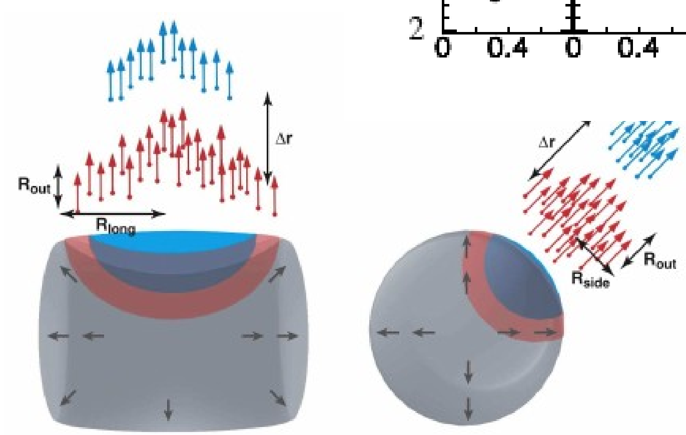
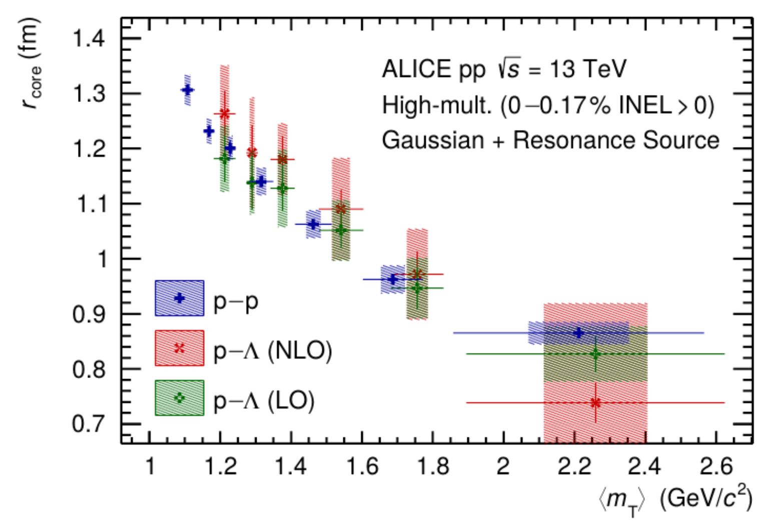


Figure 3: because particles with heavier masses have smaller thermal velocities, their source volumes are more strongly confined by collective flow. For longitudinal flow (*left panel*) this results in smaller values of $R_{\rm long}$ for particles with higher $m_T = \sqrt{m^2 + p_T^2}$. For radial flow (*right panel*) this confines heavier particles toward the surface, which results in both a reduced volume and an offset Δr in the outward direction.



S. Acharya+[ALICE], PLB811('20)135849

Analytic model of correlation function

- **Asymptotic w.f. is described by the scattering amplitude *f(q)***
  (non-identical particle pair, short range int. (only s-wave is modified),
  single channel, no Coulomb pot.)

$$\Phi^{(+)}(\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr) + \varphi_0^{(+)}(r;q)$$

$$\varphi_0^{(+)}(r;q) \to \frac{e^{i\delta}\sin(qr+\delta)}{qr} = \frac{1}{2iqr}(Se^{iqr} - e^{-iqr}) = \frac{\sin qr}{qr} + f(q)\frac{e^{iqr}}{r}$$

$$\varphi_0^{(-)}(r;q) = S^{-1}\varphi^{(+)}(r;q) \quad \left[S = \exp(2i\delta),\ f = (S-1)/2iq = [q\cot\delta - iq]^{-1}\right]$$

- **Correlation function in Lednicky-Lyuboshits (LL) formula**

(with static Gaussian source, real δ) (Lednickey, Lyuboshits ('82))

$$C(q) = \int d\boldsymbol{r} S(r) \left|\Phi^{(-)}(\boldsymbol{r})\right|^2 = 1 + \int d\boldsymbol{r} S(r) \left[\left|\varphi_0^{(-)}(\boldsymbol{r})\right|^2 - (j_0(qr))^2\right]$$

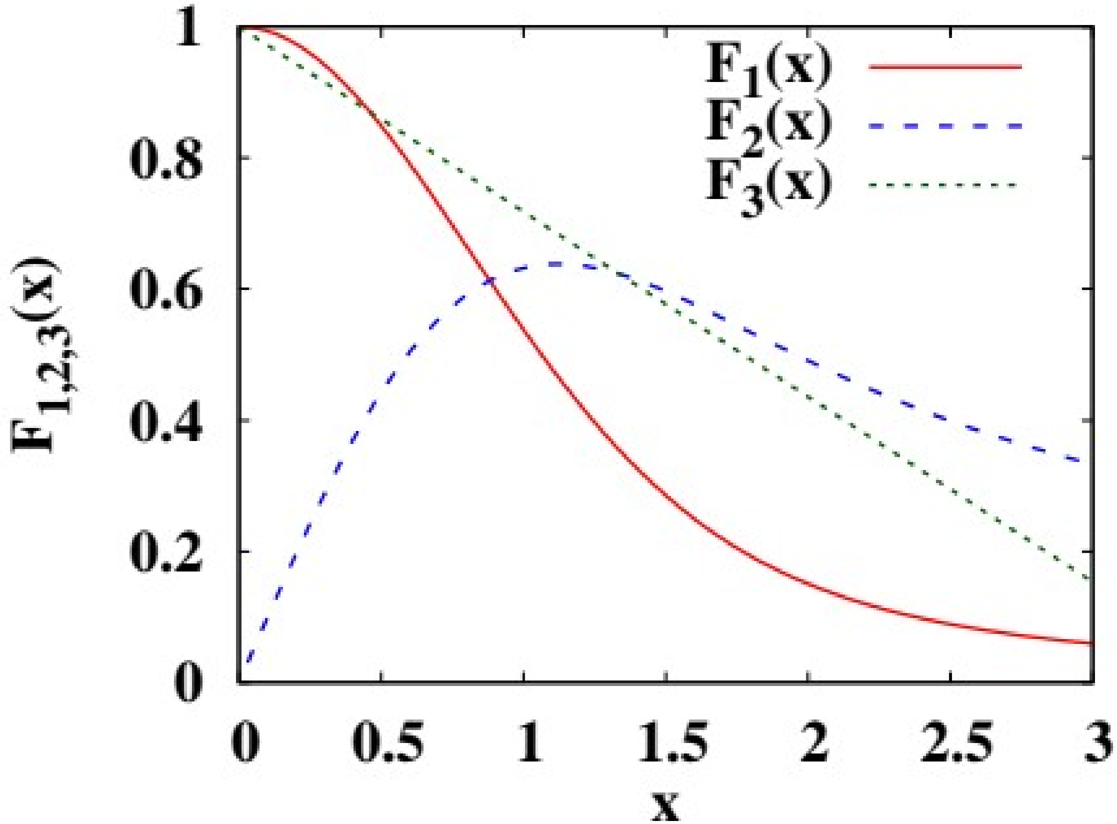
$$\simeq 1 + \int 4\pi dr\, S(r) \left[|f(q)|^2 + \frac{\sin qr}{q}\left\{f(q)e^{iqr} + f^*(q)e^{-iqr}\right\}\right]$$

$$C_{\rm LL}(q) = 1 + \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\rm eff}}{R}\right) + \frac{2{\rm Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{{\rm Im}\, f(q)}{R} F_2(2qR)$$

$$\left[f(q) = (q\cot\delta - iq)^{-1},\ F_1(x) = \frac{1}{x}\int_0^x dt e^{t^2-x^2},\ F_2(x) = (1-e^{-x^2})/x,\ F_3(x) = 1 - \frac{x}{2\sqrt{\pi}}\right]$$

Lednicky-Lyuboshits functions

$$F_1(x) = \frac{1}{x}\int_0^x dt e^{t^2 - x^2}, \;\; F_2(x) = (1 - e^{-x^2})/x, \;\; F_3(x) = 1 - \frac{x}{2\sqrt{\pi}}$$



$$F_1(x) \simeq \frac{1 + c_1 x^2 + c_2 x^4 + c_3 x^6}{1 + (c_1 + 2/3)x^2 + c_4 x^4 + c_5 x^6 + c_3 x^8} \;\; (0 \leq x < 20)$$

$$(c_1, c_2, c_3, c_4, c_5) = (0.123, 0.0376, 0.0107, 0.304, 0.0617)$$

AO,Morita,Mihayara,Hyodo, NPA 954 ('16)294.

Bird's-eye view of C(q)

- **Zero eff. range pot. → C(q)=F(R/a$_0$, qR)**

$$r_{\rm eff}=0 \to q\cot\delta = -1/a_0 \to f(q) = (q\cot\delta - iq)^{-1} = -\frac{R}{R/a_0 + iqR}$$

$$C(x,y) = 1 + \frac{1}{x^2+y^2}\left[\frac{1}{2} - \frac{2y}{\sqrt{\pi}}F_1(2x) - xF_2(2x)\right] \quad (x=qR, y=R/a_0)$$

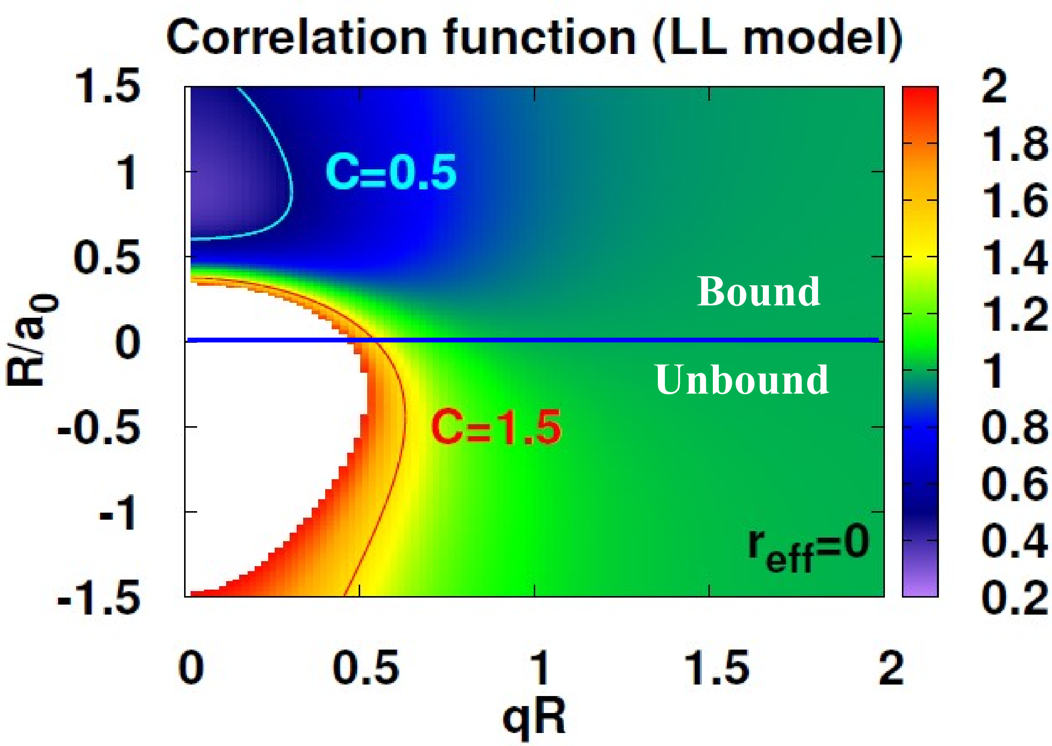
- **Low momentum limit**

$$C(x,y) \to \frac{1}{2}\left(\frac{1}{y}-\frac{2}{\sqrt{\pi}}\right)^2 + 1 - \frac{2}{\pi} \quad (F_1\to 1, F_2 \to 0 \text{ at } x\to 0)$$

  - **Enhanced C(q) at small q with $a_0<0$**

$$C_{\rm LL}(0) = 1 - \frac{2}{\sqrt{\pi}}\left(\frac{a_0}{R}\right) + \frac{1}{2}\left(\frac{a_0}{R}\right)^2$$

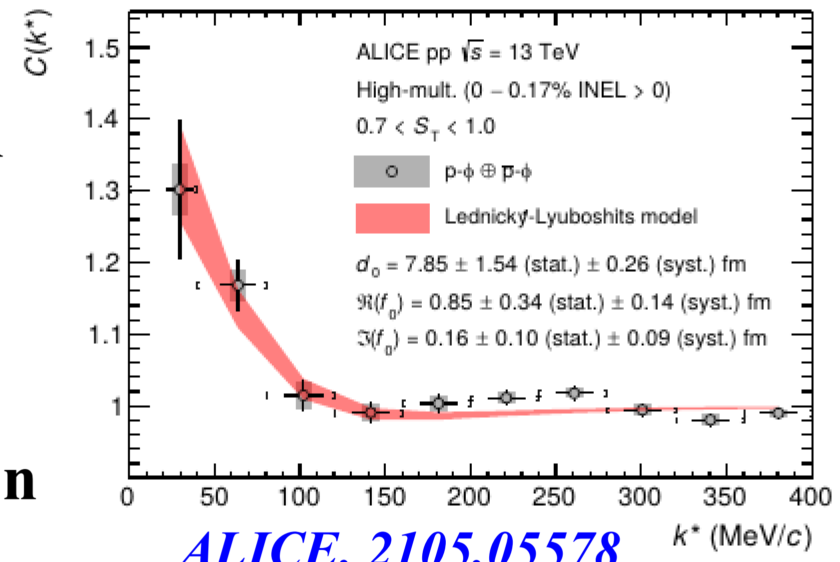
  - **$a_0>0$ → Size dependent C(q)**
    - **C(q) > 1 at small R**
    - **C(q) < 1 at R ~ a_0 (w.f. node at r ~ a_0)**



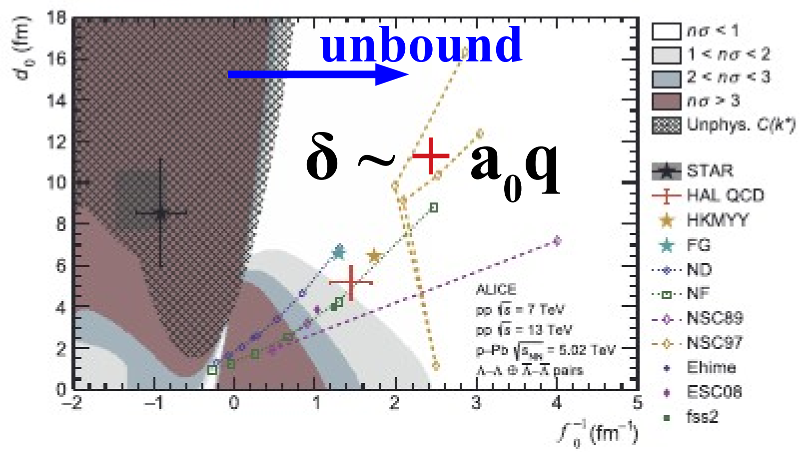
Lednicky-Lyuboshits formula application examples

- **pϕ correlation function**
  - $\mathrm{Re}(a_0)$=0.85±0.34 (stat.)±0.14(syst.) fm ($q \cot \delta \sim 1/a_0$, high-energy physics convention)
- **ΛΛ correlation function**
  - Quantum statistics + strong interaction
  - Weakly attractive potential

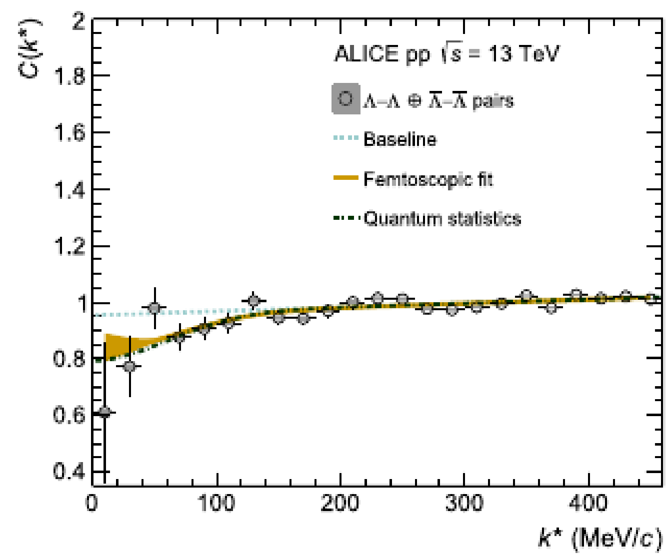
$$C(\boldsymbol{q}) = 1 - \frac{\lambda}{2} e^{-4q^2R^2} + \frac{\lambda}{2} \int d\boldsymbol{r} S(r) \left\{ |\varphi_0(r)|^2 - |j_0(qr)|^2 \right\}$$



ALICE, 2105.05578



S. Acharya+[ALICE], PLB797('19)134822



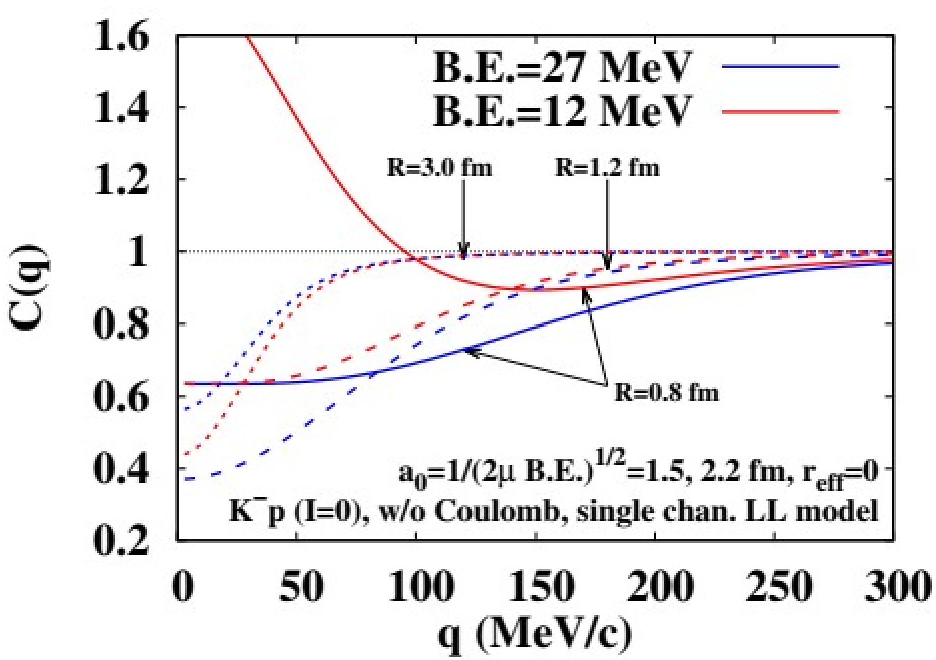
ALICE, PLB797 ('19) 134822 [1905.07209]

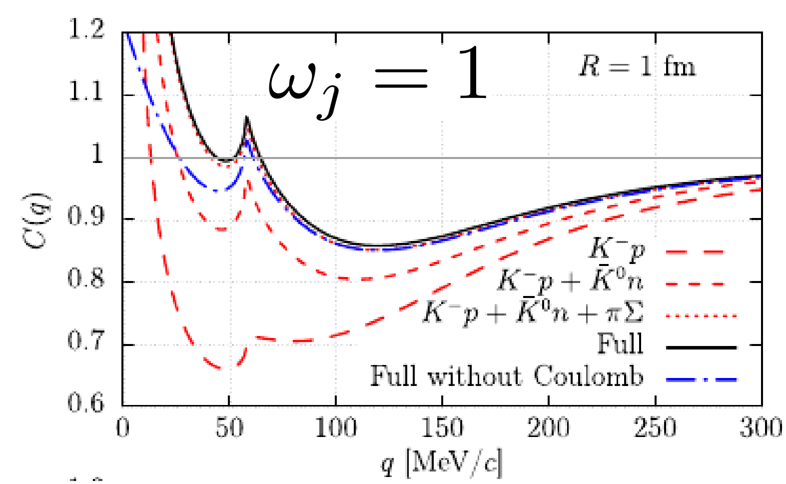
Another example: binding energy dependence of C(q)

- **A frequently asked question**
  Can we guess the binding energy from C(q) ?
- **Λ(1405) ~ $\bar{\mathrm{K}}$N (I=0) bound state**
  - **M=1405 MeV (B.E.=27 MeV) or 1420 MeV (B.E.=12 MeV) ?**
  - **A toy model: zero r_{eff}, single channel LL model w/o Coulomb, I=0**

$$a_0 = \hbar/\sqrt{2\mu \times \mathrm{B.E.}}$$

  - **C(q) depends on B.E. at small R. (Do not be serious!)**



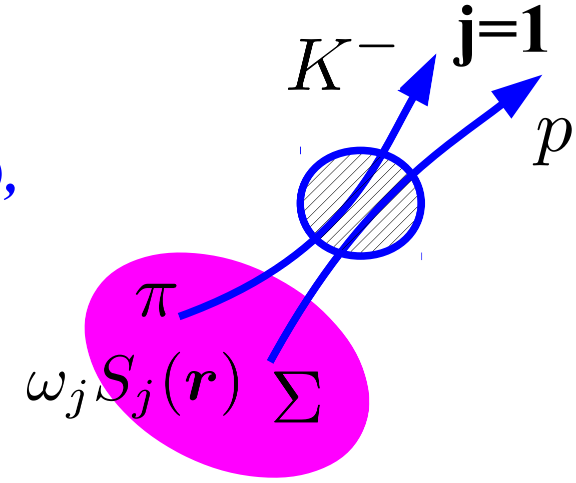


More serious calculation
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.

Correlation function in more realistic cases

Correlation function with coupled-channel effects

- **KPLLL formula = CC Schrodinger eq. under $\Psi^{(-)}$ boundary cond. + channel source**
  Koonin('77), Pratt+('86), Lednicky-Lyuboshits-Lyuboshits ('98), Heidenbauer ('19), Kamiya, Hyodo, Morita, AO, Weise ('20).



$$\Psi^{(-)}(\boldsymbol{q};\boldsymbol{r}) = [\phi(\boldsymbol{q};\boldsymbol{r}) - \phi_0(q;r)]\,\delta_{1j} + \psi^{(-)}(q;r)$$

$$\psi_j^{(-)}(q;r) \to \frac{1}{2iq_j}\left[\frac{u_j^{(+)}(q_j r)}{r}\delta_{1j} - A_j(q)\frac{u_j^{(-)}(q_j r)}{r}\right]$$

$$C(q) = \int d\boldsymbol{r} S_1(r)\left[|\phi(\boldsymbol{q};\boldsymbol{r})|^2 - |\phi_0(q;r)|^2\right] + \sum_j \int d\boldsymbol{r}\,\omega_j S_j(r)|\psi_j^{(-)}(q;r)|^2$$

- **No Coulomb** $\phi(\boldsymbol{q};\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}}, \phi_0(q;r) = j_0(qr), u_j^{(\pm)}(qr) = e^{\pm iqr},$

$$A_j(q) = \sqrt{(\mu_j q_j)/(\mu_1 q_1)}S_{1j}^\dagger(q_1)\ \ (S_{ji} = i \to j \text{ S-matrix})$$

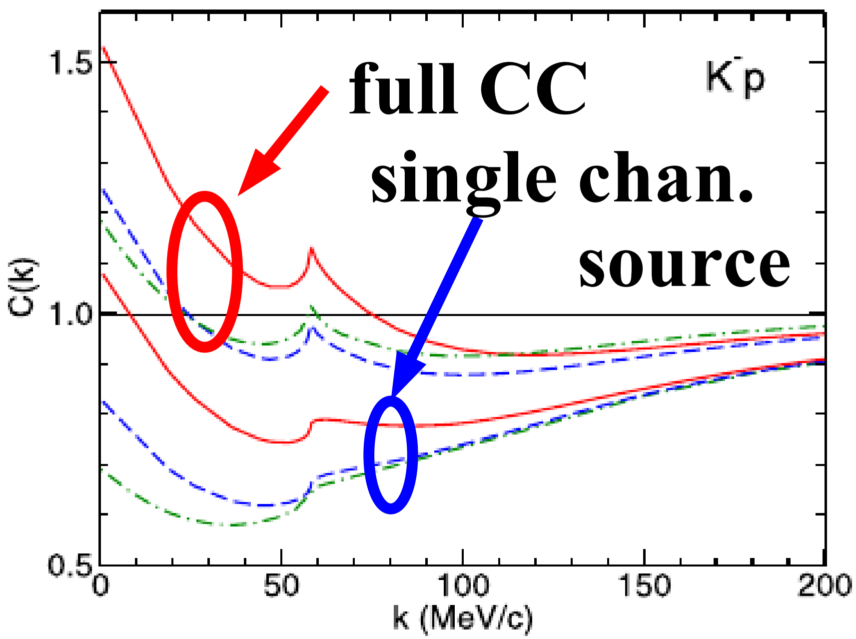
- **With Coulomb**

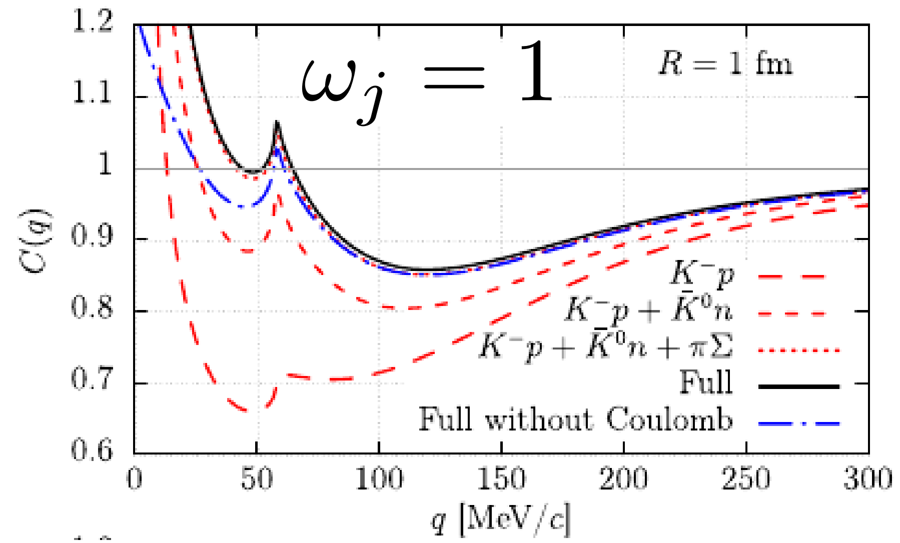
$$\phi(\boldsymbol{q};\boldsymbol{r}) = \text{Full Coulomb w.f.}, \phi_0(q;r) = \text{s-wave Coulomb w.f.},$$

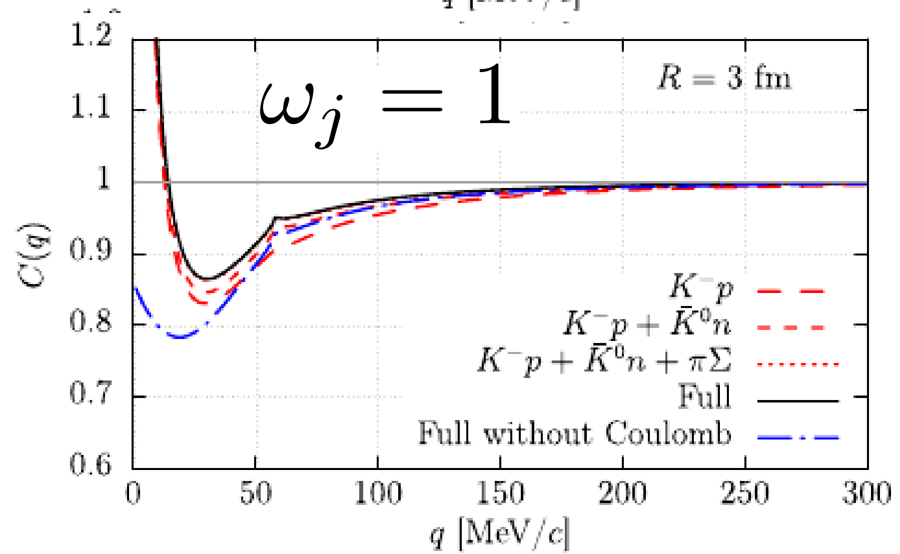
$$u_j^{(\pm)}(qr) = \pm e^{\mp i\sigma_j}[iF(qr) \pm G(qr)]\,(F, G = \text{regular (irregular) Coulomb fn.})$$

Coupled-channel effects in K⁻ p correlation function

- $K^- p - \bar{K}^0 n$ ($\bar{K}N$-$\pi\Sigma$) coupling is decisive (visible) at R=1 fm.
- Source effects of $\bar{K}^0 n$ and $\pi\Sigma$ are not large at R=3 fm. (Solving CC Eq. is still important.)







J. Haidenbauer, NPA981('19)1.
(Julich, NLO30, w/ CC effects, w/o Coulomb)

Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.

Parameters in correlation function data

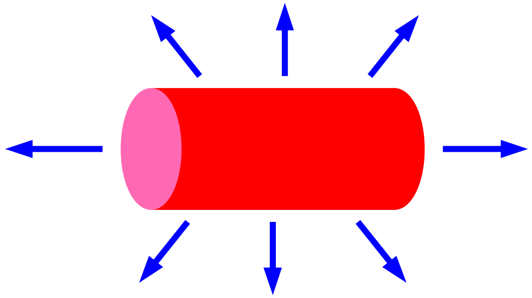
- **Actual data contains non-primary and misidentified particles, particles from jets, and the source size and weights are not fully known.**

$$C_{\rm exp}(q; R, \lambda, N, \omega) = N(q)\left[1 + \lambda(C_{\rm theory}(q; R, \omega) - 1)\right]$$

- **R = Source size (length of homogeneity)**
  - **Guess based on systematics (m_T scaling) or dynamical models.**
  - **Flow and source shape are also important for identical pairs.**
- **λ = chaoticity parameter → pair purity**
  - **λ = (“primary” pair) / (accepted pair)**
  - **In the best case of ΛΛ → $\lambda = [(\text{primary }\Lambda) / (\text{primary }\Lambda + \Sigma^0)]^2$**
- **N(q)=a + bq, Normalization + Jet effects**
- **ω_j = Source weight**
  - **$\omega_j \propto$ product of particle number at around the emission time.**
  - **Statistical model, blast-wave, MC simulation, …**

Semi-Realistic Source Function

- **"Cylindrical shape + blast wave" model**
  S. Chapman, P. Scotto, U. Heinz, Heavy Ion Phys.1,1('95);
  K. Morita, T. Furumoto, AO, PRC91, 024916 ('15);
  K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC101('20), 015201.



$$S_{\rm cyl}(x,\boldsymbol{k}) = \frac{2J+1}{(2\pi)^3} m_T \cosh(y-\eta_s)\, \underline{n_F(u\cdot k/T)}\ \underline{e^{-\boldsymbol{r}_T^2/2R_T^2}}\ f_\tau(\tau)$$

Fermi dist.
(Gaussian in r_T)

$$f_\tau(\tau) = e^{-(\tau-\tau_0)^2/2(\Delta\tau)^2}/\sqrt{2\pi(\Delta\tau)^2} \to \delta(\tau-\tau_0)$$

$$u^\mu = (\cosh y_T \cosh\eta_s, \sinh y_T \cos\phi, \sinh y_T \sin\phi, \cosh y_T \sinh\eta_s)$$

$$y_T = \alpha\rho^\beta \ \ (\rho = r_T/R_T)$$

Bjorken+radial flow

$$E\frac{dN}{d\boldsymbol{k}} = \int d^4x S_{\rm cyl}(x,\boldsymbol{k})$$

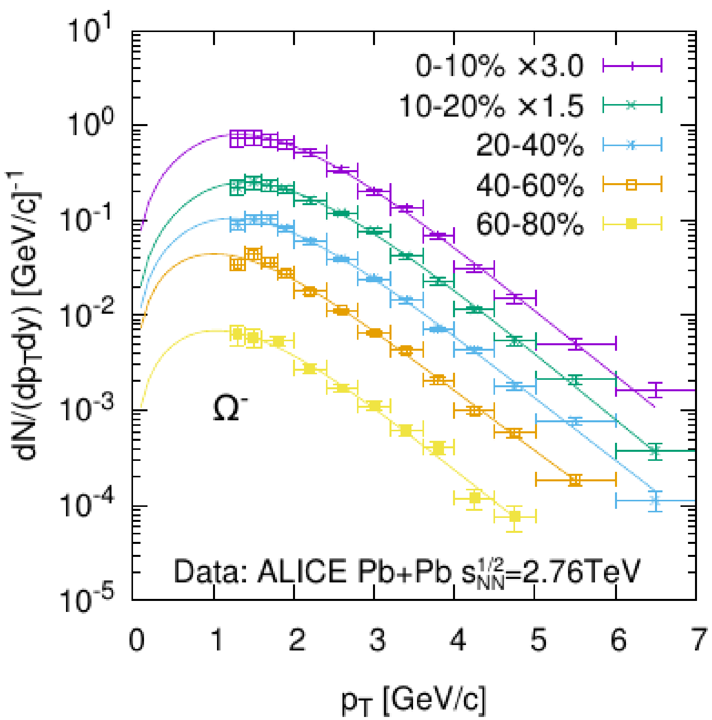
$$= \frac{2J+1}{(2\pi)^3} 2m_T V \int_0^\infty \rho d\rho e^{-\rho^2/2} I_0\left(\frac{p_T}{T}\sinh y_T\right) K_1\left(\frac{m_T}{T}\cosh y_T\right)$$

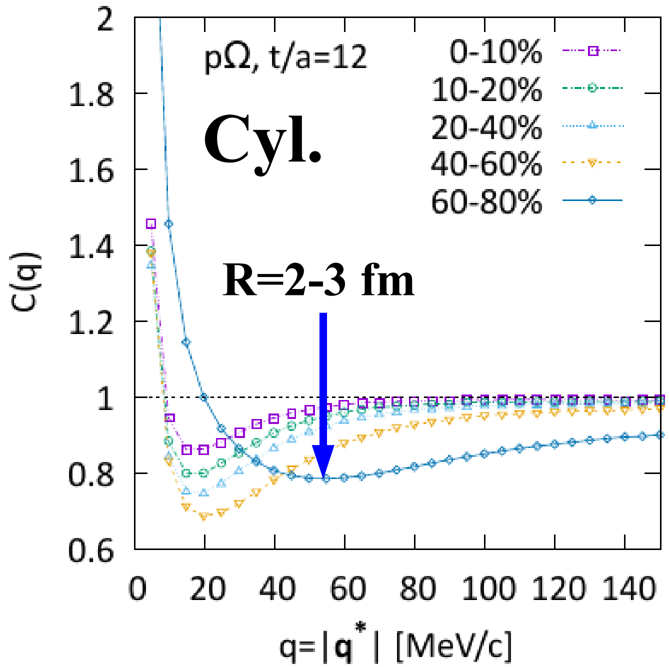
$$\text{modified Bessel } I_0(z) = \frac{1}{2\pi}\int_0^{2\pi} e^{z\cos\theta}\,d\theta,\ K_1(z) = \frac{1}{2}\int_{-\infty}^{\infty} e^{-z\cosh\eta}\cosh\eta\,d\eta$$

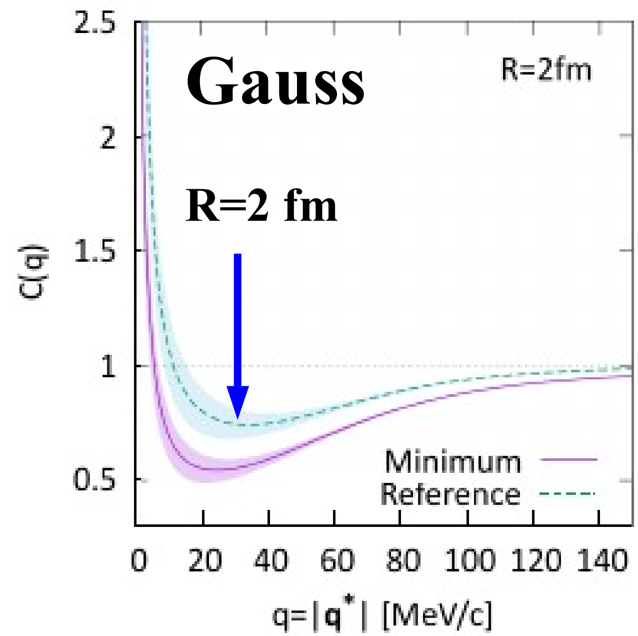
Semi-Realistic Source Function

- Correlation function from cylindrical source
  - Production spectra are well described.
  - Dip momentum at the similar size is shifted upwards by the flow.
  - Problem: 9D integral
  - R= homogeneity length
    ≠ actual source size
    Correction factor ?

Centrality	τ_0 [fm/c]	R_T^Ω [fm]	R_T^p	α^Ω	β^Ω	α^p	β^p
0 – 10%	10.0	8.0	6.8	0.584	0.628	0.759	0.421
10 – 20%	9.085	6.75	6.23	0.618	0.579	0.750	0.425
20 – 40%	7.5	5.88	5.2	0.546	0.692	0.707	0.466
40 – 60%	5.5	4.38	3.92	0.444	0.858	0.604	0.6
60 – 80%	3.62	2.12	2.66	0.456	0.812	0.456	0.82





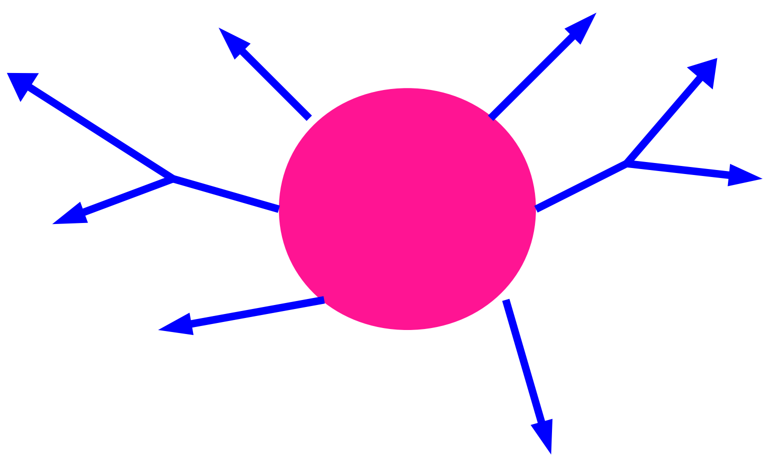


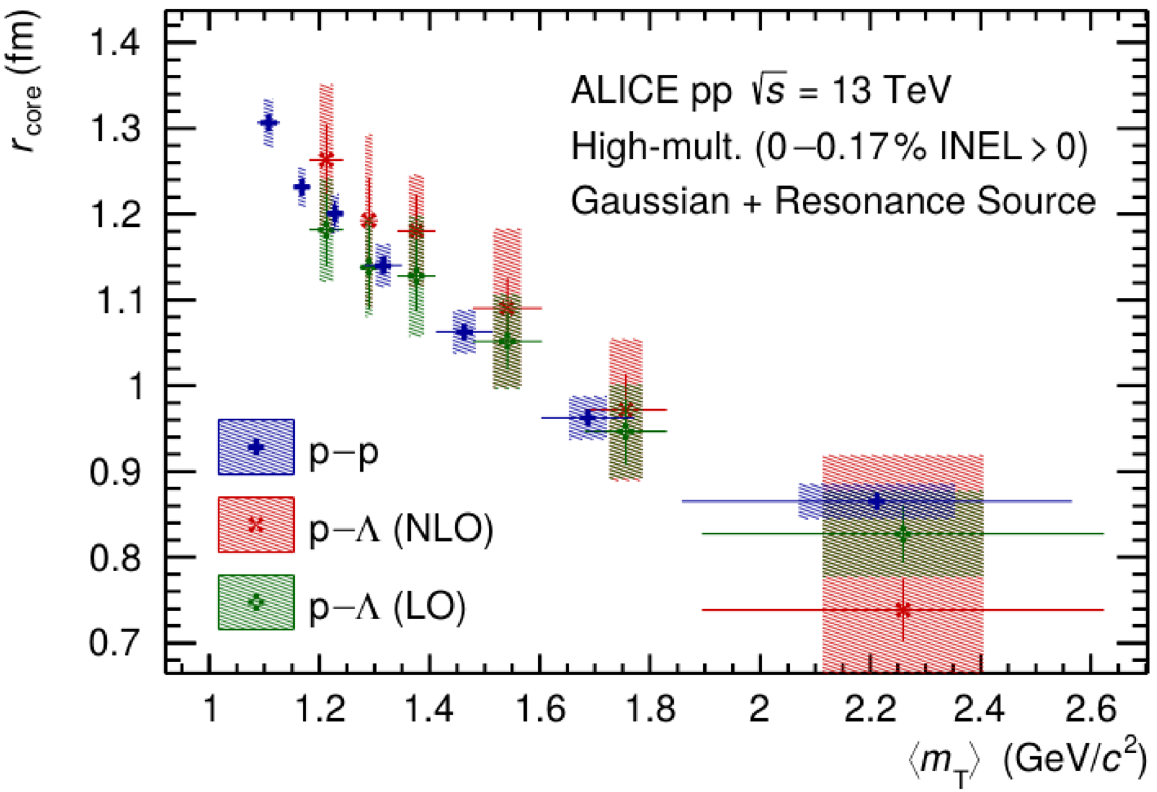
Source Size

- "Universal" source model

  S. Acharya+[ALICE], PLB811('20)135849

  - Fit pp and pΛ correlation function with Gaussian source (core) and decay of resonances.
  - Then the core size (r_{core}) seems to be universal as a function of m_T
  - Universal core + decay gives effective size
  - Good as the first guess.
  - (We need to allow 20-30 % uncertainty.)





S. Acharya+[ALICE], PLB811('20)135849

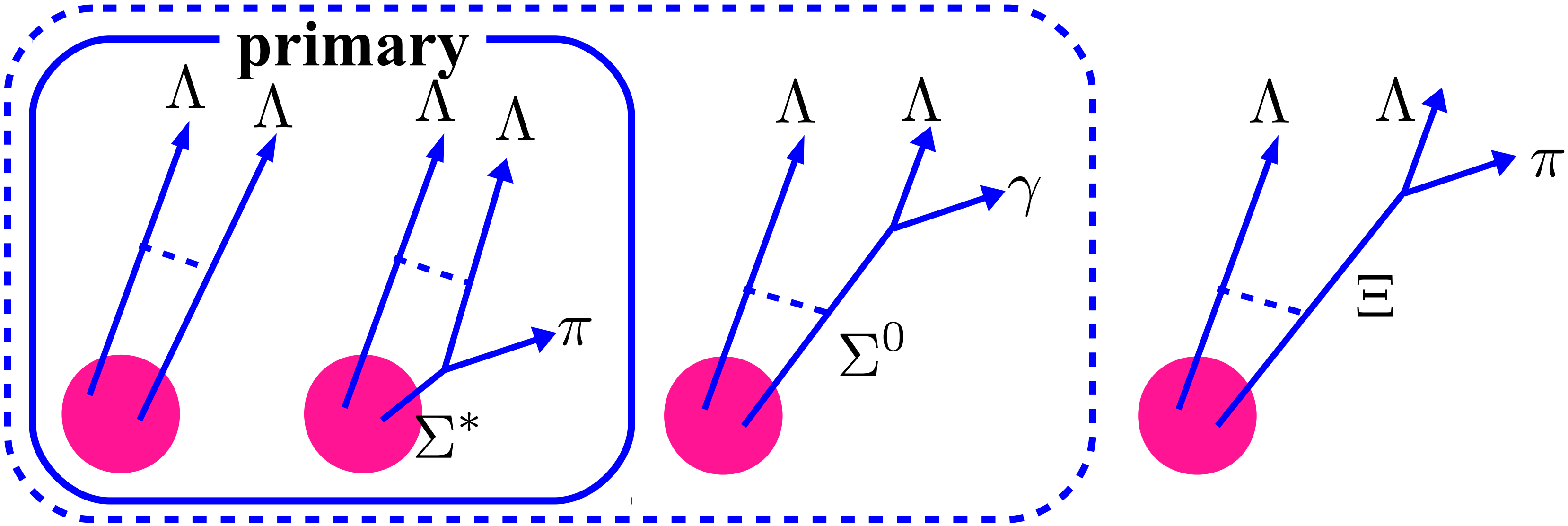
λ (chaoticity parameter → pair purity)

- **λ = chaoticity parameter → pair purity**
  - **λ = ("primary" pair) / (accepted pair)** $C_{\rm exp}(q) = N\left[1 + \lambda(C_{\rm theory}(q) - 1)\right]$
  - **In the best case of ΛΛ → λ = [(primary Λ) / (primary Λ + Σ^0)]^2**
  - **MC simulations seem to be useful.**

Table 1
The weight parameters (Eq. (4)) $\lambda_i^{\rm pp}$ and $\lambda_i^{\rm p-Pb}$ of the individual components of the p–p, p–Λ, p–Ξ^- and Λ–Λ correlation functions. The sub-indexes are used to indicate the mother particle in case of feed-down. Only the non-flat feed-down (residual) contributions are listed individually, while all other contributions are listed as "flat residuals (res.)". All misidentified (fake) pairs are assumed to be uncorrelated, thus resulting in a flat correlation signal.

ALICE ('20)

p–p			p–Λ			p–Ξ^-			Λ–Λ		
Pair	$\lambda_i^{\rm pp}$ (%)	$\lambda_i^{\rm p-Pb}$ (%)	Pair	$\lambda_i^{\rm pp}$ (%)	$\lambda_i^{\rm p-Pb}$ (%)	Pair	$\lambda_i^{\rm pp}$ (%)	$\lambda_i^{\rm p-Pb}$ (%)	Pair	$\lambda_i^{\rm pp}$ (%)	$\lambda_i^{\rm p-Pb}$ (%)
pp	74.8	72.8	pΛ	50.3	41.5	pΞ^-	55.5	50.8	ΛΛ	33.8	23.9
$\rm pp_\Lambda$	15.1	16.1	$\rm p\Lambda_{\Sigma^0}$	16.8	13.8	$\rm p\Xi^-_{\Xi(1530)^-}$	8.8	8.1			
			$\rm p\Lambda_{\Xi^-}$	8.3	12.1						
flat res.	8.1	8.0	flat res.	20.4	24.9	flat res.	30.3	28.3	flat res.	59.8	64.0
fakes	2.0	3.1	fakes	4.2	7.7	fakes	5.4	12.8	fakes	6.4	12.1



Lorentz invariant representation of C(q)

- **d^3p is not Lorentz invariant, but d^3p/E is invariant.**

$$C(\boldsymbol{q},\boldsymbol{P}) = \frac{E_1 E_2 dN_{12}/d\boldsymbol{p}_1 d\boldsymbol{p}_2}{(E_1 dN_1/d\boldsymbol{p}_1)(E_2 dN_2/d\boldsymbol{p}_2)}$$

$$P \equiv p_1 + p_2, q^\mu \equiv \frac{1}{2}\left[(p_1-p_2)^\mu - \frac{(p_1-p_2)\cdot P}{p^2}P^\mu\right] = \frac{E_2' p_1^\mu - E_1' p_2^\mu}{M_{\rm inv}}$$

$(E_i' = E_i$ in the pair rest frame$)$

- **Free two-body wave function**

$$\exp(-ip_1x_1 - ip_2x_2) = \exp(-iPX - iq(x_1-x_2)) = \exp(-iPX + i\boldsymbol{q}\cdot\boldsymbol{r})$$

$$X = \frac{E_1' x_1 + E_2' x_2}{M_{\rm inv}}, \boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2 - \boldsymbol{v}(t_1-t_2), \boldsymbol{v} = \boldsymbol{P}/\sqrt{M_{\rm inv}^2 + \boldsymbol{P}^2}$$

$(p_1 = E_1' P/M_{\rm inv} + q, p_2 = E_2' P/M_{\rm inv} - q)$

- **Correlation function (w.f. is defined in the pair rest frame)**

$$C(\boldsymbol{q},\boldsymbol{P}) = \frac{\int d^4x_1 d^4x_2 S_1(x_1,\boldsymbol{p}_1) S_2(x_2,\boldsymbol{p}_2)|\varphi^{(-)}(\boldsymbol{r},\boldsymbol{q})|^2}{\int d^4x_1 S_1(x_1,\boldsymbol{p}_1)\int d^4x_2 S_2(x_2,\boldsymbol{p}_2)} = \int d\boldsymbol{r} S(\boldsymbol{r};\boldsymbol{q},\boldsymbol{P})|\varphi^{(-)}(\boldsymbol{r},\boldsymbol{q})|^2$$

$$S(\boldsymbol{r};\boldsymbol{q},\boldsymbol{P}) = \frac{\int dt d^4X S_1(X + E_2' x/M_{\rm inv},\boldsymbol{p}_1) S_2(X + E_1' x/M_{\rm inv},\boldsymbol{p}_2)}{\int d^4x_1 S_1(x_1,\boldsymbol{p}_1)\int d^4x_2 S_2(x_2,\boldsymbol{p}_2)} \quad [x = x_1 - x_2 = (t,\boldsymbol{r})]$$

(Source function can depend on q and P.)

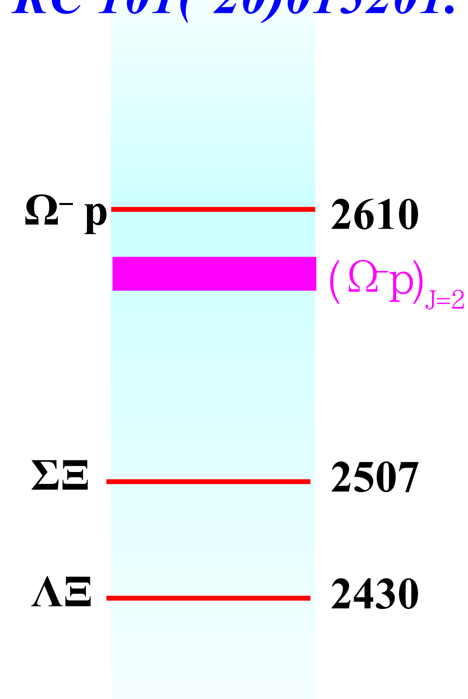
Recently observed (studied) correlation functions

Ωp correlation function

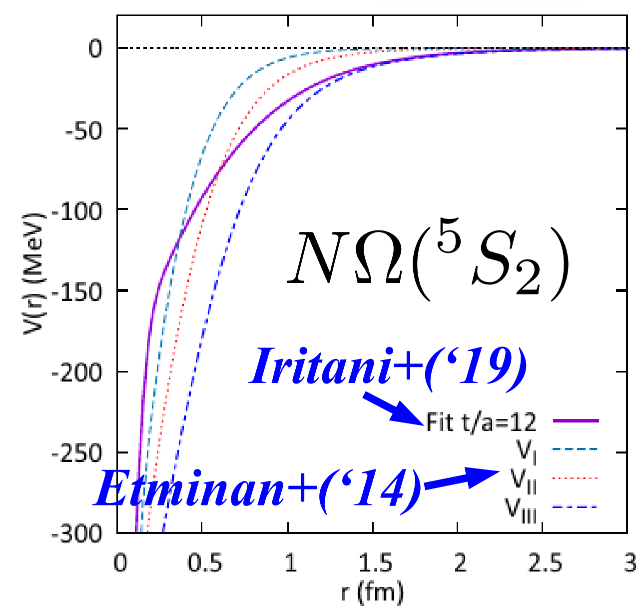
$N\Omega$ interaction and $N\Omega$ bound state

K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC 101('20)015201.

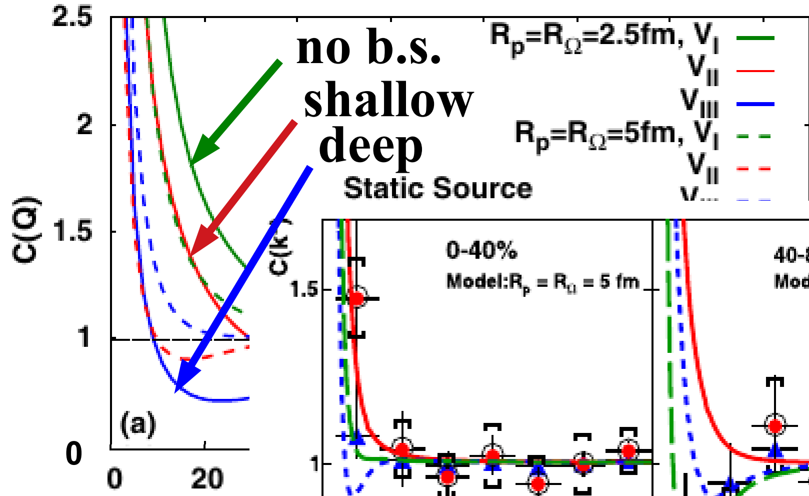
- **Ω^- (sss): J^π=3/2+, M=1672 MeV**
- **Ω^- p bound state as a S= –3 dibaryon ?**
  - **No quark Pauli blocking in ΩN, H=uuddss, and d*=ΔΔ channels.**
    Oka ('88), Gal ('16)
  - **J=2 state ($^5S_2$) couples to Octet-Octet baryon pair only with L ≥ 2 → Small width is expected.**
    T. Goldman+, PRL59('87),627;
    F. Etminan+[HAL], NPA928('14)89;
    Iritani+[HAL], PLB792('19)284;
    Sekihara,Kamiya,Hyodo, PRC98('18)015205.
  - **Correlation has been measured at RHIC & LHC !** *STAR ('19); ALICE ('20)*



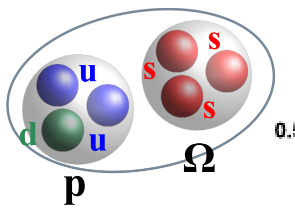
Let us try to discover the first S<0 dibaryon !

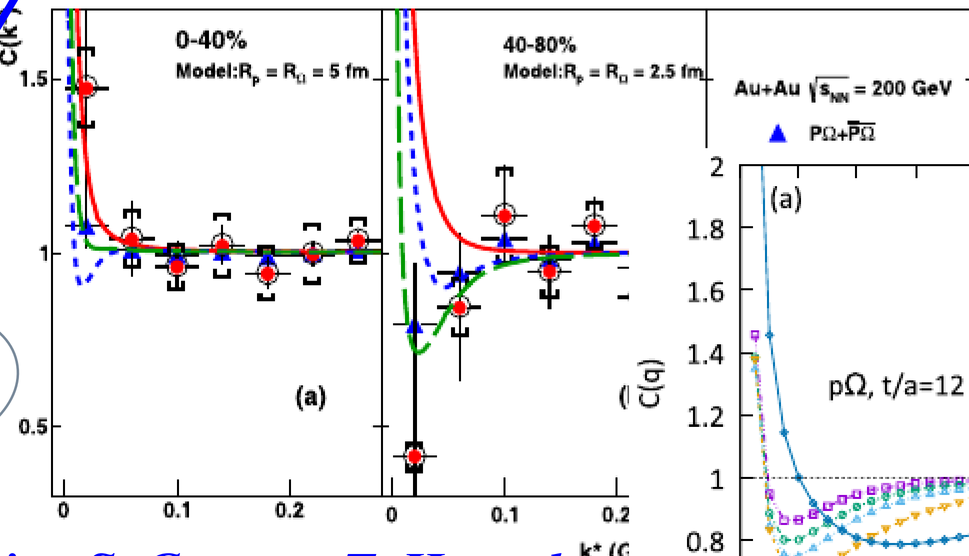


pΩ⁻ correlation function

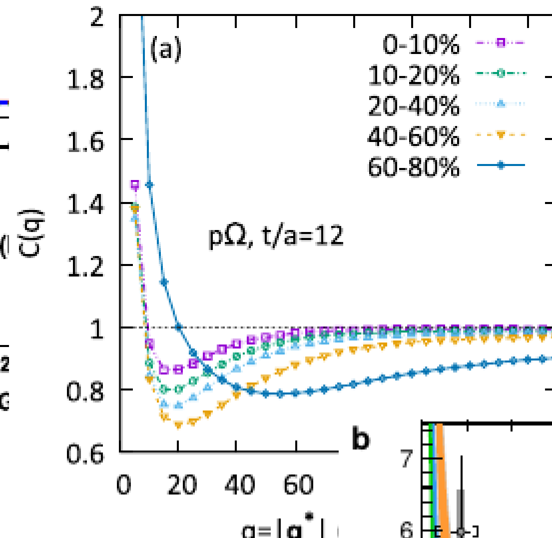


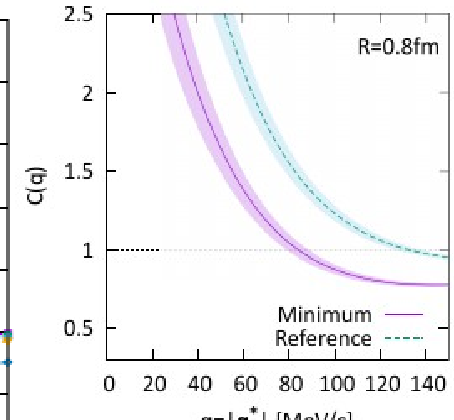
K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R) (w/ Lattice potential with heavier quark mass)



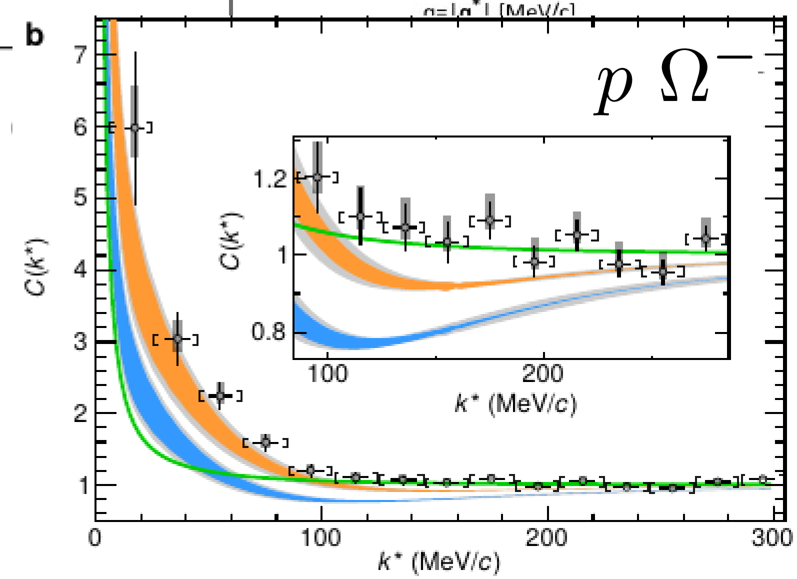


J. Adam+[STAR], PLB790('19)490.





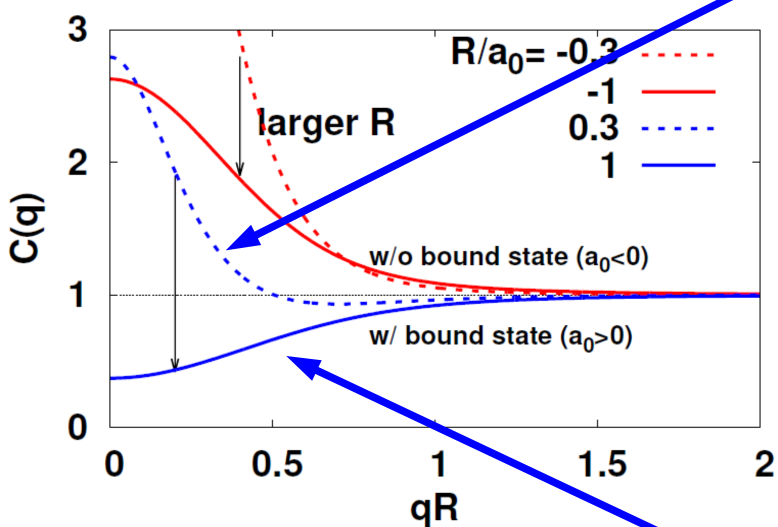
K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC 101('20)015201. (w/ Lattice potential at physical quark mass, a_0 ~ 3.4 fm, expanding source, Gauss source (R=0.8 fm))

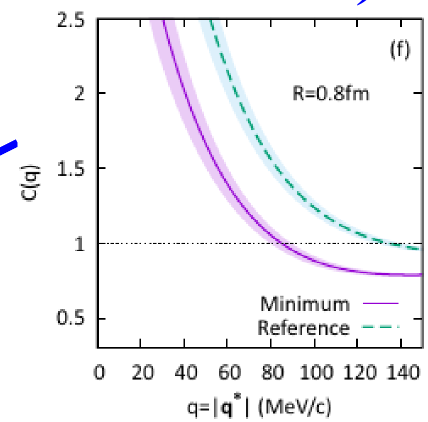


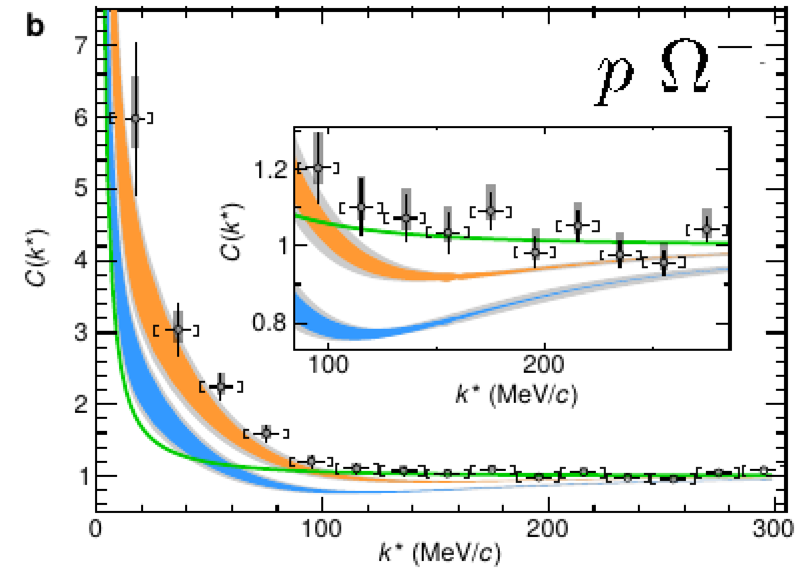
S. Acharya+[ALICE], Nature 588 ('20), 232 [2005.11495] (pp 13 TeV)

STAR+ALICE suggests a NΩ dibaryon state

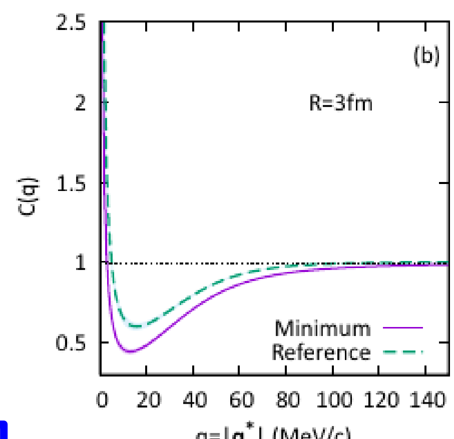
Morita+, PRC101('20)015201 [1908.0414] (Gaussian source)

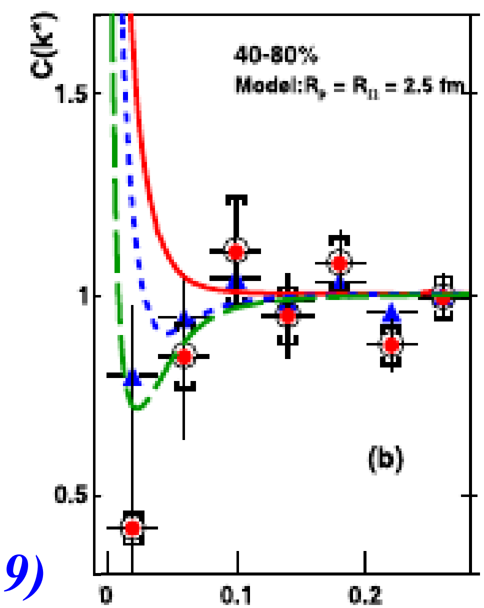






ALICE, Nature588('20)232 [2005.11495]





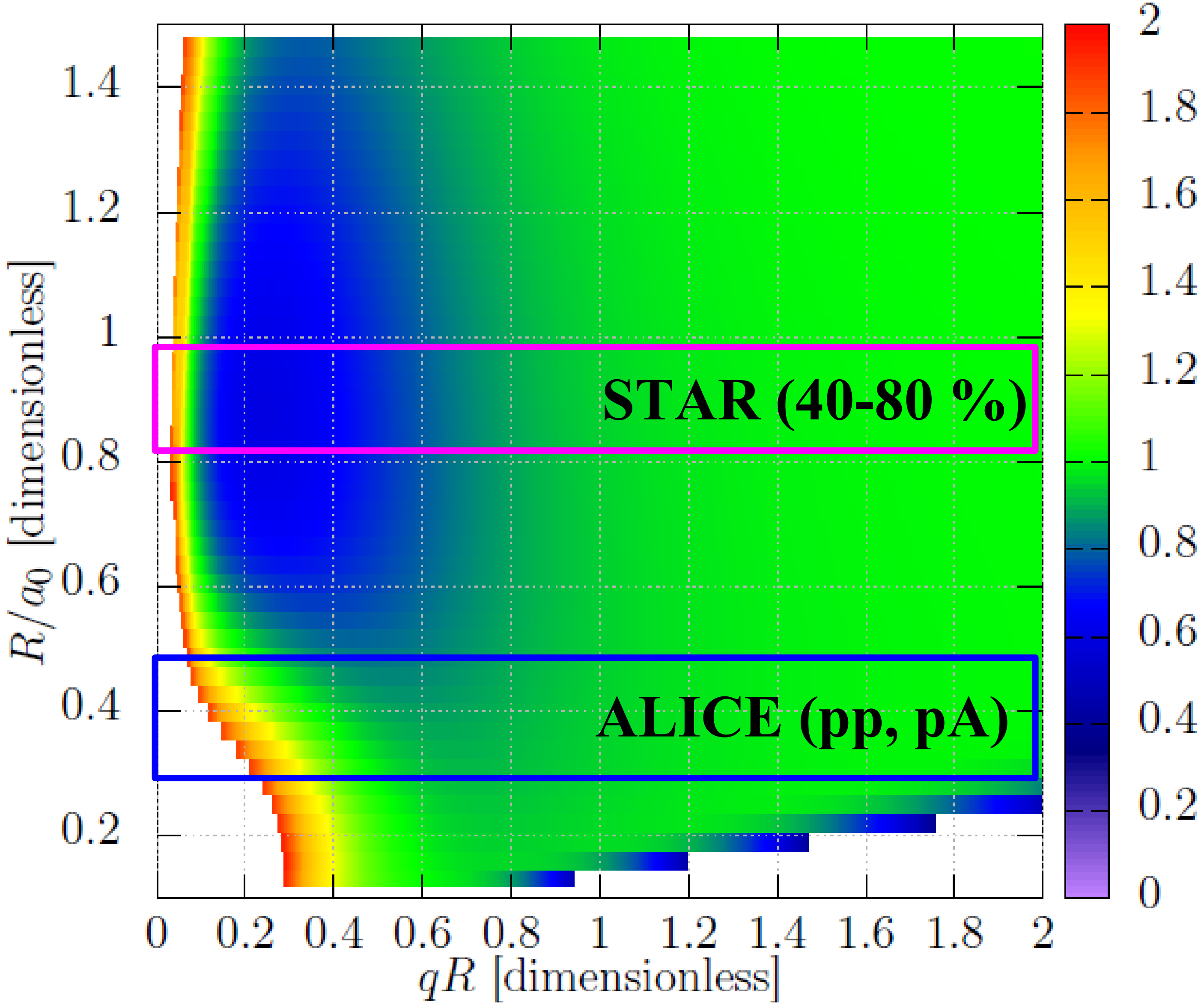
STAR, PLB790 ('19) 490 [1808.02511].

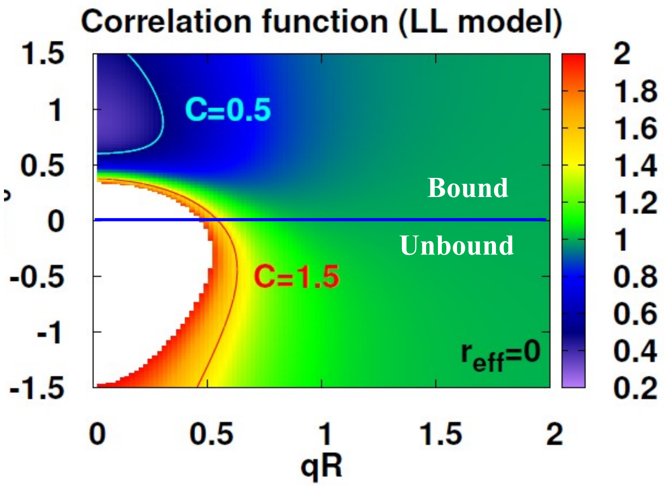
Reference: $V_{J=1}=V_{J=2}$

Minimum: $\varphi_{J=1}=0$

Dip from a bound state survives Coulomb.

Ωp Correlation Function with Gaussian source



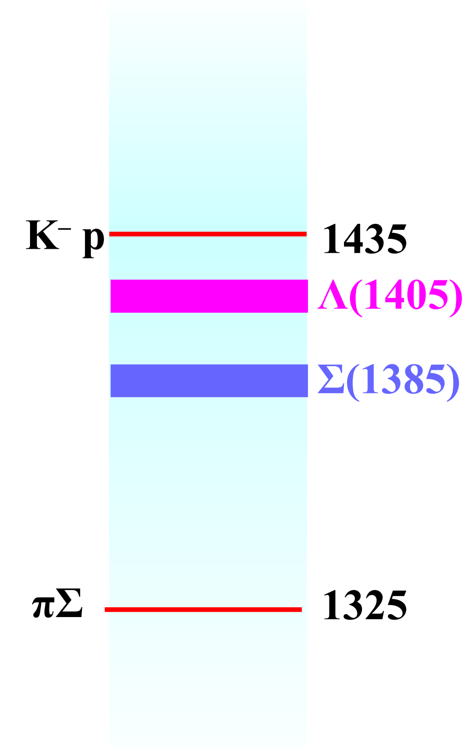


NΩ potential (J=2, HAL QCD, a_0=3.4 fm) + Coulomb

$K^{-}p$ correlation function

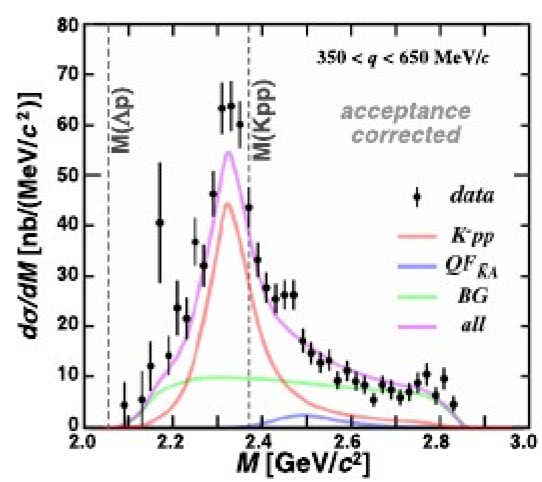
$\bar{K}N$ interaction and pK^- correlation function

- **$\Lambda(1405)$= $\bar{K}N$ quasi-bound state**
  Dalitz, Tuan ('60); Koch ('94); Kaiser, Siegel, Weise ('95); AO, Nara, Koch ('97)
  - **Positive scattering length in K^- atoms**
    M.Iwasaki et al. PRL78('97)3067; M.Bazzi et al. [SIDDHARTA Collab.], PLB704('11)113.



- **Kaonic nuclei ?**
  Nogami ('63); Akaishi, Yamazaki ('02); Shevchenko, Gal, Mares ('07); Ikeda, Sato ('07); Dote, Hyodo, Weise ('09); S.Ajimura+ [J-PARC E15], PLB 789 (2019) 620.
  → **Needs precise info. on $\bar{K}N$ int.**

- **Scattering amplitude and Potential fitting scattering and SIDDARTA data in chiral approach**
  Ikeda, Hyodo, Weise ('11,'12); A. Cieplý, J. Smejkal ('12, NLO30); Miyahara, Hyodo, Weise ('18, CC $N\bar{K}$-$\pi\Sigma$-$\pi\Lambda$ potential)

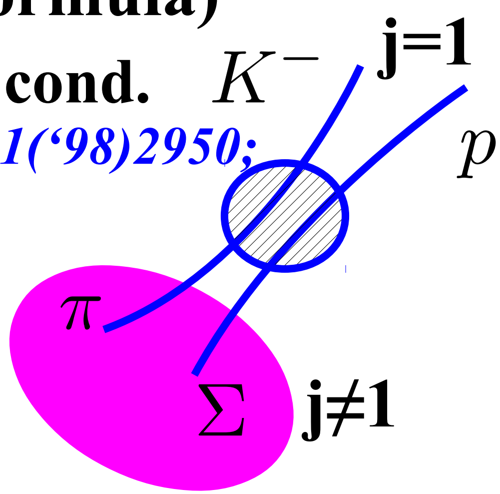


J-PARC E15 ('19)

How about K^-p correlation ?

Correlation Function with Coupled-Channel Effects

- **To evaluate pK^- correlation function, we need to take account of coupled-channel effects of NK-πΣ !**
- **Correlation function formula with CC (KPLLL formula)**
  - **Coupled-channel contributions with $\psi^{(-)}$ boundary cond.**
    R.Lednicky,V.V.Lyuboshits,V.L.Lyuboshits, Phys.Atom.Nucl.61('98)2950; J. Haudenbauer, NPA981('19)1 [1808.05049].



$$C(\boldsymbol{q}) = \int d\boldsymbol{r} \sum_j \omega_j S_j(\boldsymbol{r}) |\Psi_j^{(-)}(\boldsymbol{r})|^2$$

$$= 1 - \int d\boldsymbol{r} S_1(r) |j_0(qr)|^2 + \int d\boldsymbol{r} \sum_j \omega_j S_j(r) |\psi_j^{(-)}(q;r)|^2$$

$$\psi_{j=1}(r) \to [e^{iqr} + A_1(q) e^{-iqr}]/2iqr \quad (\omega_1 = 1)$$

$$\psi_{j\neq 1}(r) \to A_j(q) e^{-iqr}/2iqr \quad [\Psi^{(-)}\ \text{boundary condition}]$$

(No Coulomb case)

$$\omega_j S_j(\boldsymbol{r}) |\psi_j^{(-)}(q;r)|^2$$

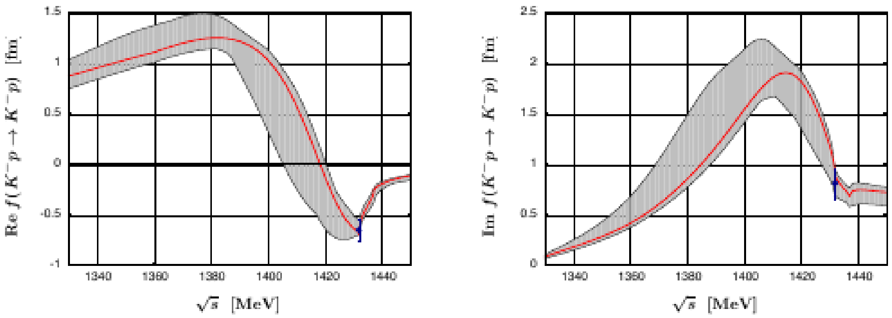
Source weight (ω_j) — **Normalized Source fn.** ($S_j(\boldsymbol{r})$)

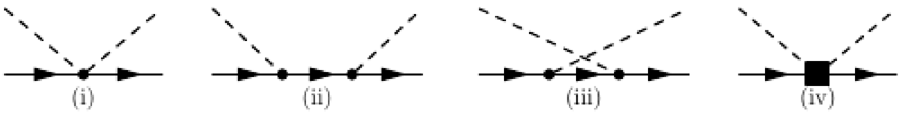
  - **Effects of coupled-channel, strong & Coulomb pot., and threshold difference are taken into account in the charge base.**
    Y. Kamiya+, PRL('20)
  - **Source size R and weight ω_j (j≠1) are taken as the parameter.**

- **Chiral SU(3) $\bar{K}$N scattering amplitude**

  Y. Ikeda, T. Hyodo, W. Weise, NPA881('12)98.

  - **Tomozawa-Weinberg + Born (w/ Exchange) + NLO**
  - **Fit to SIDDHARTA data of $\bar{K}$N diagonal scattering amplitude at threshold.**
  - **$\bar{K}$N-πΣ-πΛ-ηΛ-ηΣ-KΞ**

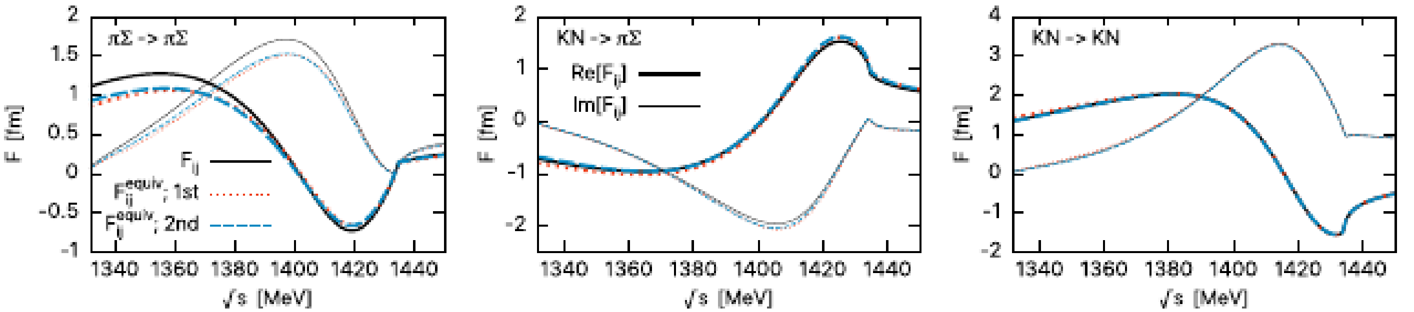


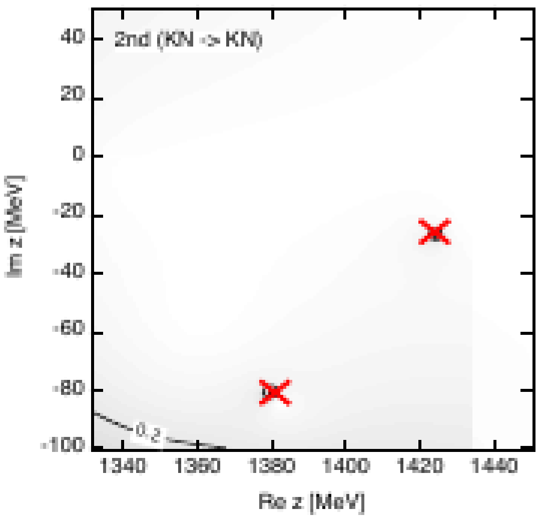


- **Coupled-channel $\bar{K}$N-πΣ-πΛ potential based on IHW amplitude**

  K. Miyahara, T. Hyodo, W. Weise, PRC98('18)025201.

  - **Fit to IHW amplitude and pole positions of**





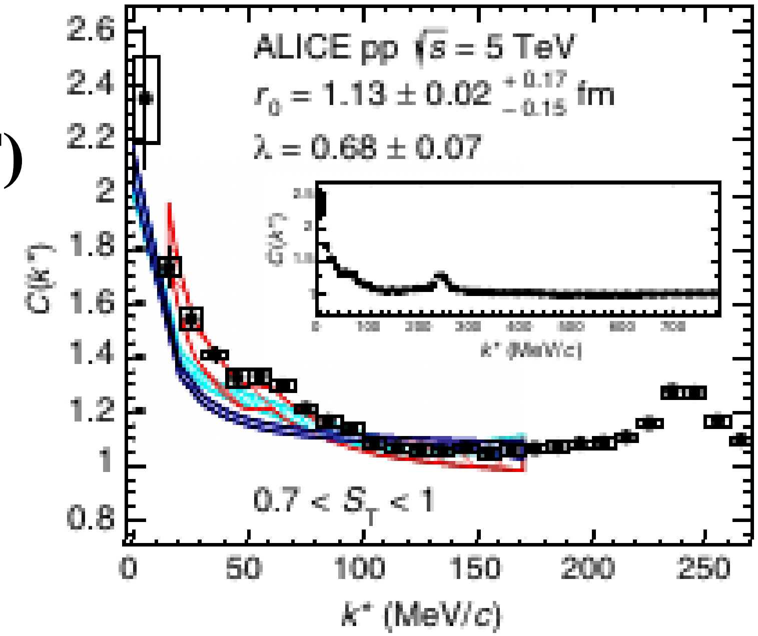
Comparison with ALICE data

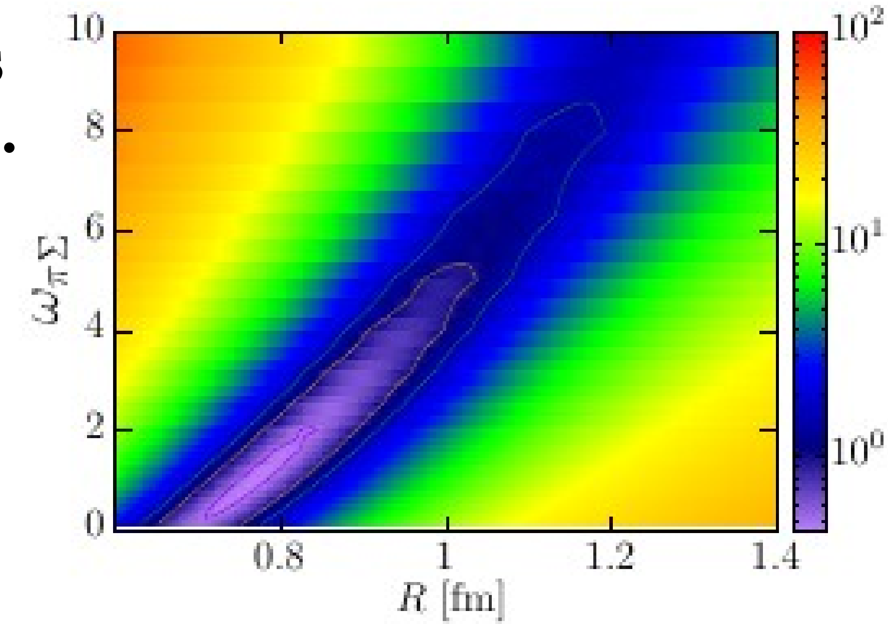
- **Physics parameters = R and $\omega_{\pi\Sigma}$**
  - **ALICE value (single channel) R=1.13 fm (Deterimined by K^+p(Jülich+Gamow) CF)**
  - **Kamiya+('20) fits (R, $\omega_{\pi\Sigma}$) to C(q) data**
- **Observationn parameters = N and λ**

$$C_{\rm fit}(q) = \mathcal{N}\left[1 + \lambda(C(q) - 1)\right]$$

  - **Normalization (N) and pair purity (λ) depend on the measurement.**
    → Use values from experimentalists
    or fit them to data for each (R, $\omega_{\pi\Sigma}$).

S. Acharya+[ALICE],PRL124('20)092301.

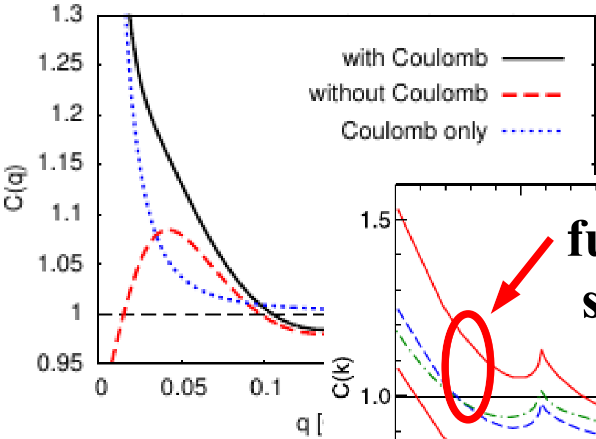


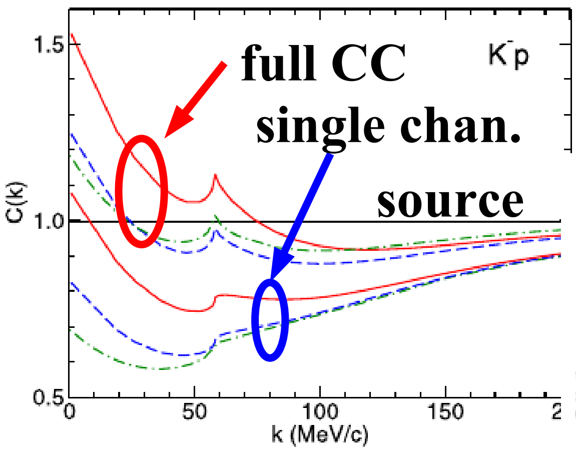


Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.

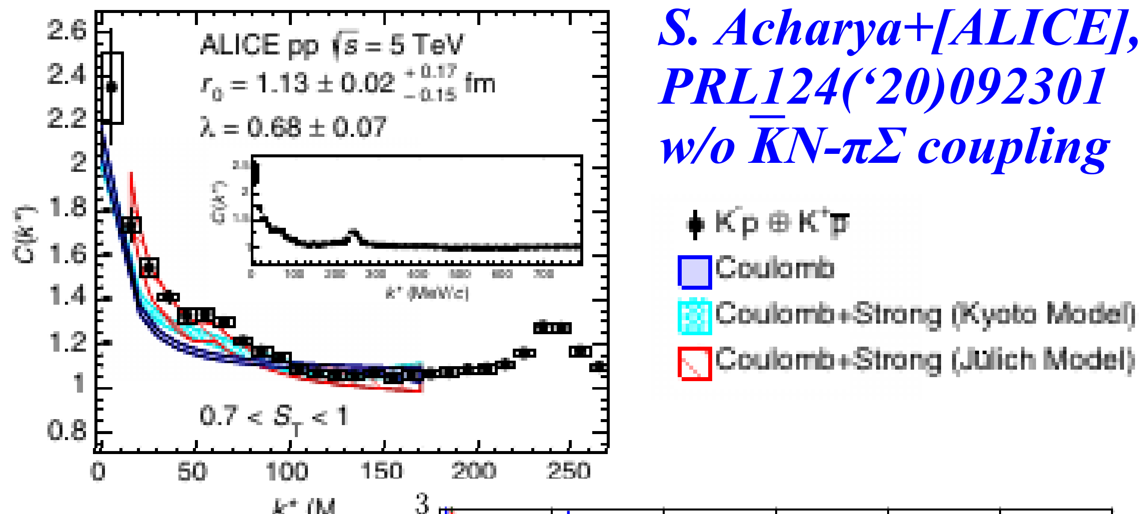
pK^- correlation

S. Cho+ [ExHIC], PPNP95('17)279.
(Insufficient coupled-channel effects)



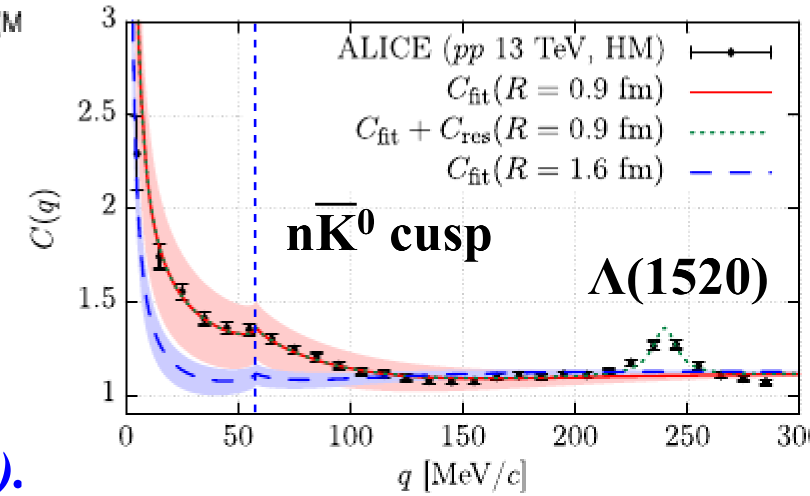


J. Haidenbauer, NPA981('19)1.
(Julich, NLO30, w/ CC effects,
w/o Coulomb)



S. Acharya+[ALICE],
PRL124('20)092301
w/o $\bar{K}N$-$\pi\Sigma$ coupling

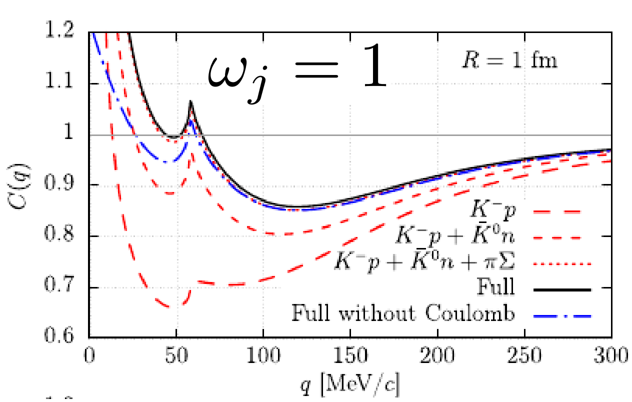
CF with small source is explained ! Source size dep. may clarify bound state nature of Λ(1405).

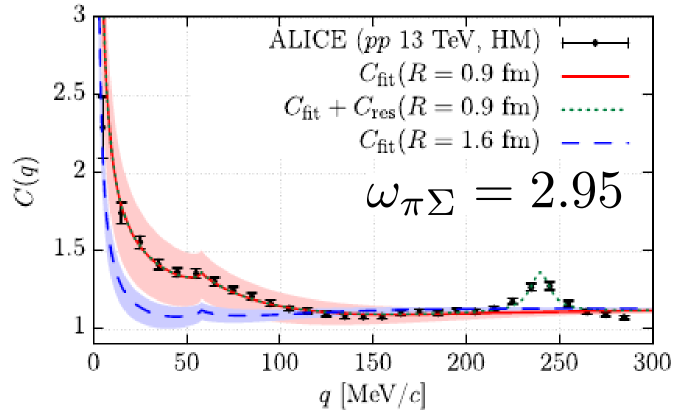


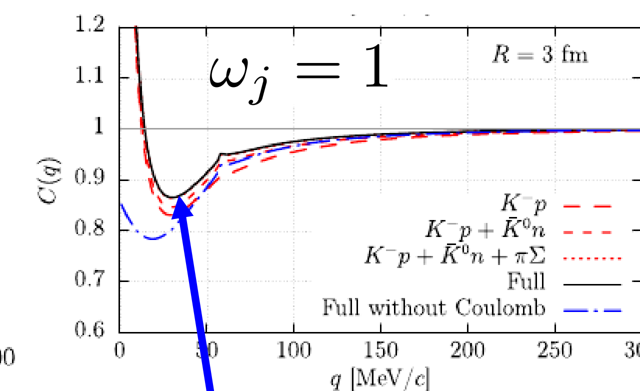
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501 [1911.01041] (Chiral SU(3) dynamics).

Source Size Dependence of C(pK⁻)

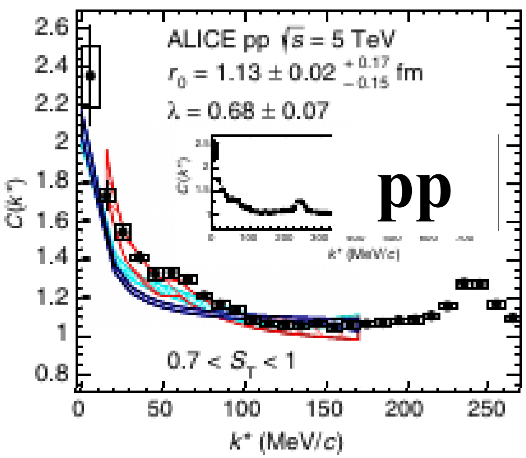
- **Coupled-channel effects are suppressed when R is large, and "pure" pK⁻ wave function may be observed in HIC.**



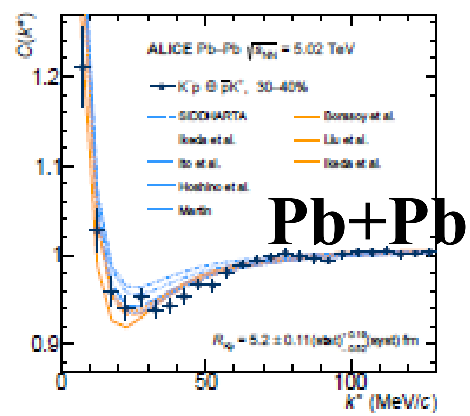


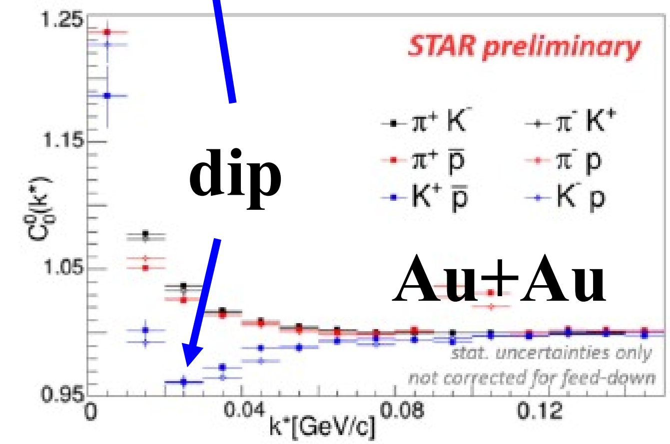


Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.



pA





S. Acharya+[ALICE], PRL124('20)092301

S. Acharya+[ALICE], 2105.05683

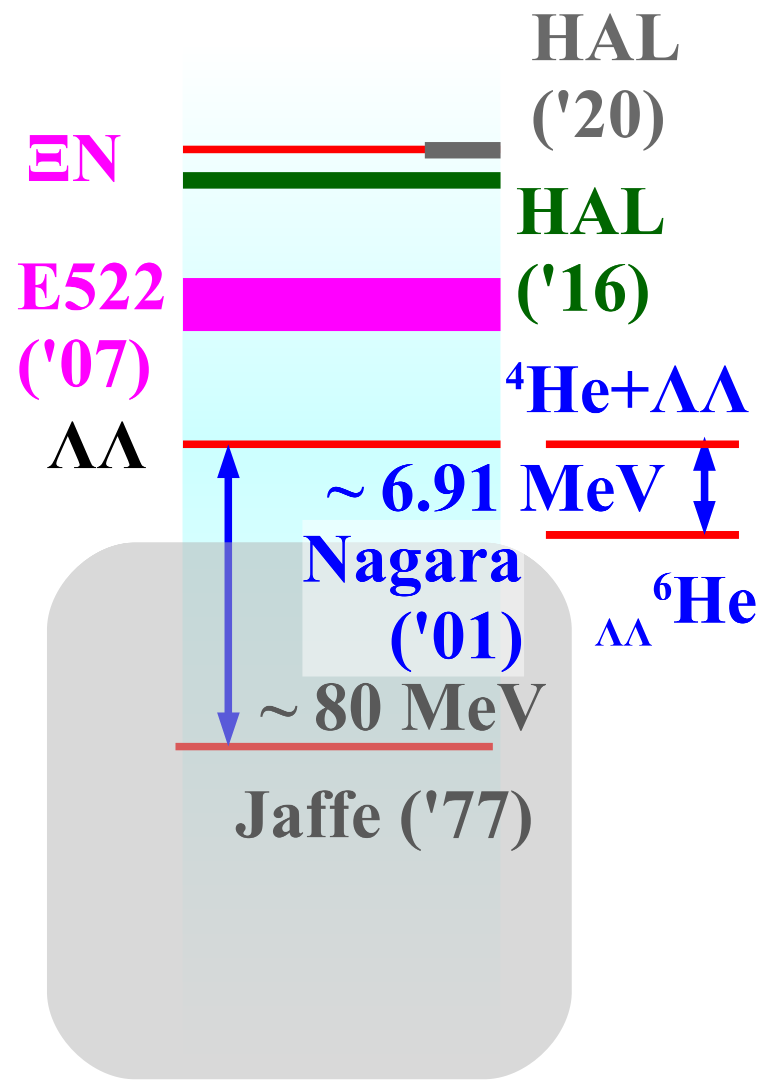
Siejka+[STAR, preliminary], NPA982 ('19)359.

STAR(prel.) & new ALICE data show dip at small q.

Ξ^-p and ΛΛ correlation function

H dibaryon state, to be bound or not to be bound ?

- **H-dibaryon: 6-quark state (uudd ss)**
  - **Prediction:** *R.L.Jaffe, PRL38(1977)195*
  - **Ruled-out by double Λ hypernucleus**
    Takahashi et al.,PRL87('01) 212502
  - **Resonance or Bound "H" ?**
    Yoon et al.(KEK-E522)+AO ('07)
  - **Discovery of Ξ^- nucleus**
    Nakazawaet al. PTEP2015('15),033D02
- **Lattice QCD results**
  - **Bound (below ΛΛ threshold):**
    HALQCD('11), NPLQCD('11,'13), Mainz('19)
    (heavier quark mass or SU(3) limit)
  - **Resonance (Bound state of NΞ):**
    HAL QCD ('16,18) **(HAL preliminary)**
  - **Virtual Pole (around NΞ threshold)**
    HAL QCD ('20) **(almost physical m_q)**



We examine LQCD NΞ-ΛΛ potential and discuss H using CF !

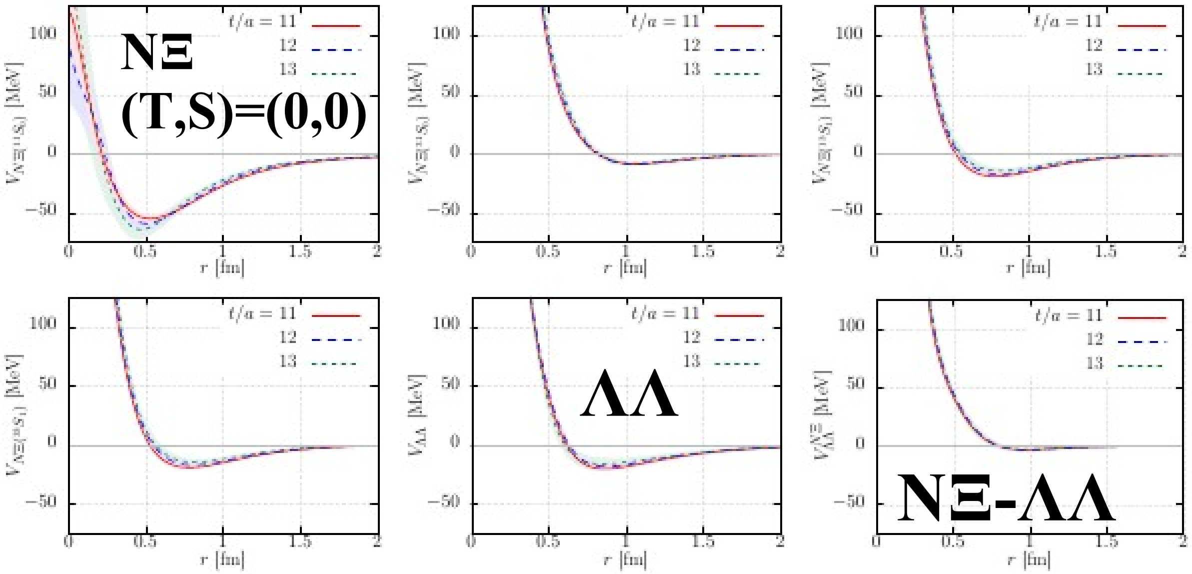
NΞ-ΛΛ potential from Lattice QCD

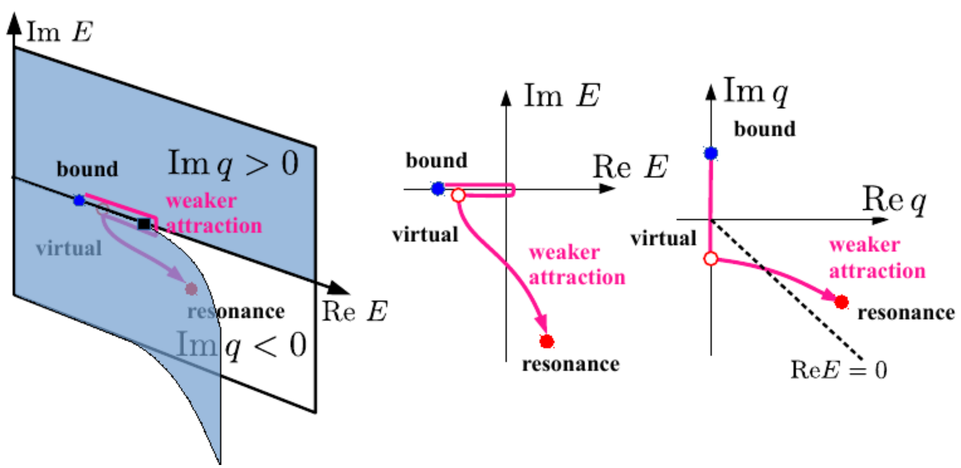
- **NΞ-ΛΛ potential at almost physical quark mass (m_π=146 MeV) by HAL QCD Collaboration**

  K. Sasaki et al. [HAL QCD Collab.], NPA 998 ('20) 121737 (1912.08630)

  - **Strong attraction in (T,S)=(0,0) of NΞ**
  - **Weak attraction in ΛΛ (Coupling with NΞ causes ΛΛ attraction)**
  - **There is no bound state in NΞ-ΛΛ system (except for Ξ^- atom), but there is a virtual pole around the NΞ threshold (3.93 MeV below $n\Xi^0$ threshold) on the irrelevant Riemann sheet, (+, –, +) [relevant=(–,+,+)]**

sign of Im(eignen momentum)

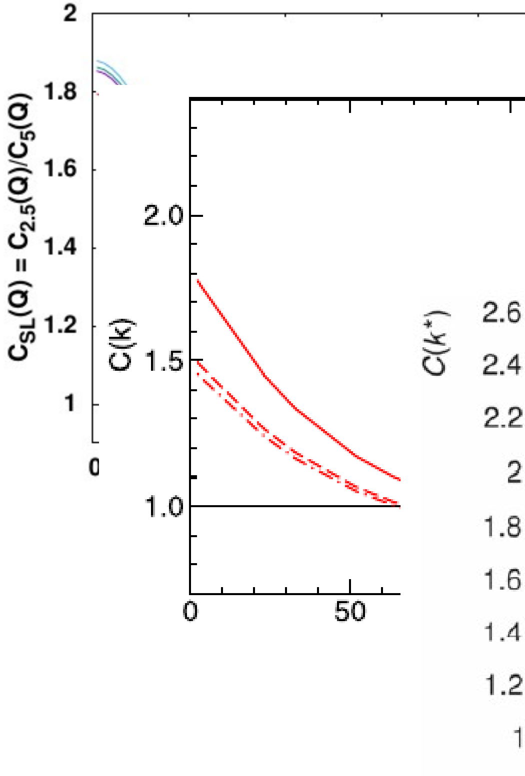




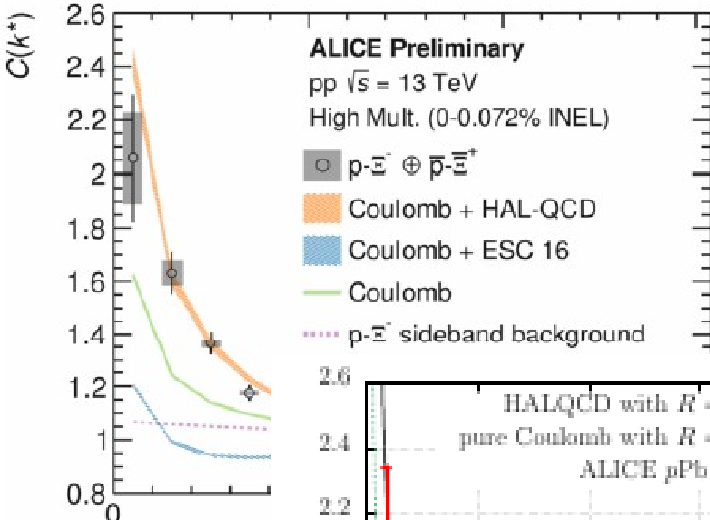
$p\Xi^-$ correlation function

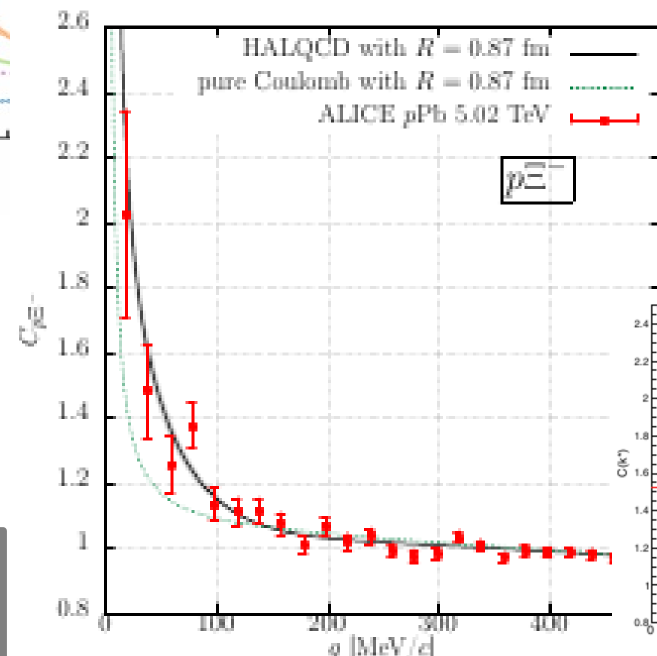
T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17)856.
(heavier quark mass, I=0 only, w/o CC effects)

J. Haidenbauer, NPA981('19)1.
(NLO(600), w/ CC effects, w/o Coulomb)
(w/ Coulomb, it will be comparable with data.)



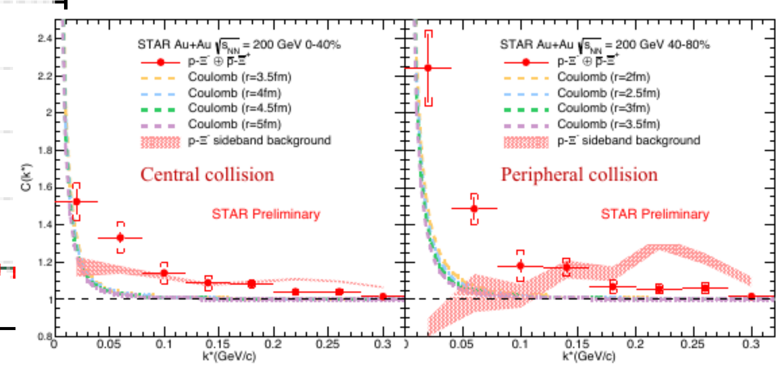
D. L. Mihairov+[ALICE], NPA 1005 ('21)121760 (QM2019). (Nijmegen pot. does not explain the data. w/o CC)
Acharya+(ALICE), Nature ('20)





Kamiya, Sasaki, Fukui, Hatsuda, Hyodo, Morita, Ogata, AO (in prep.),
w/ Lattice BB pot. at phys. m_q
CC effects with ΛΛ.

K. Mi+(STAR, preliminary), Au+Au 200 AGeV, APS2021.
(No Dip at larger R)



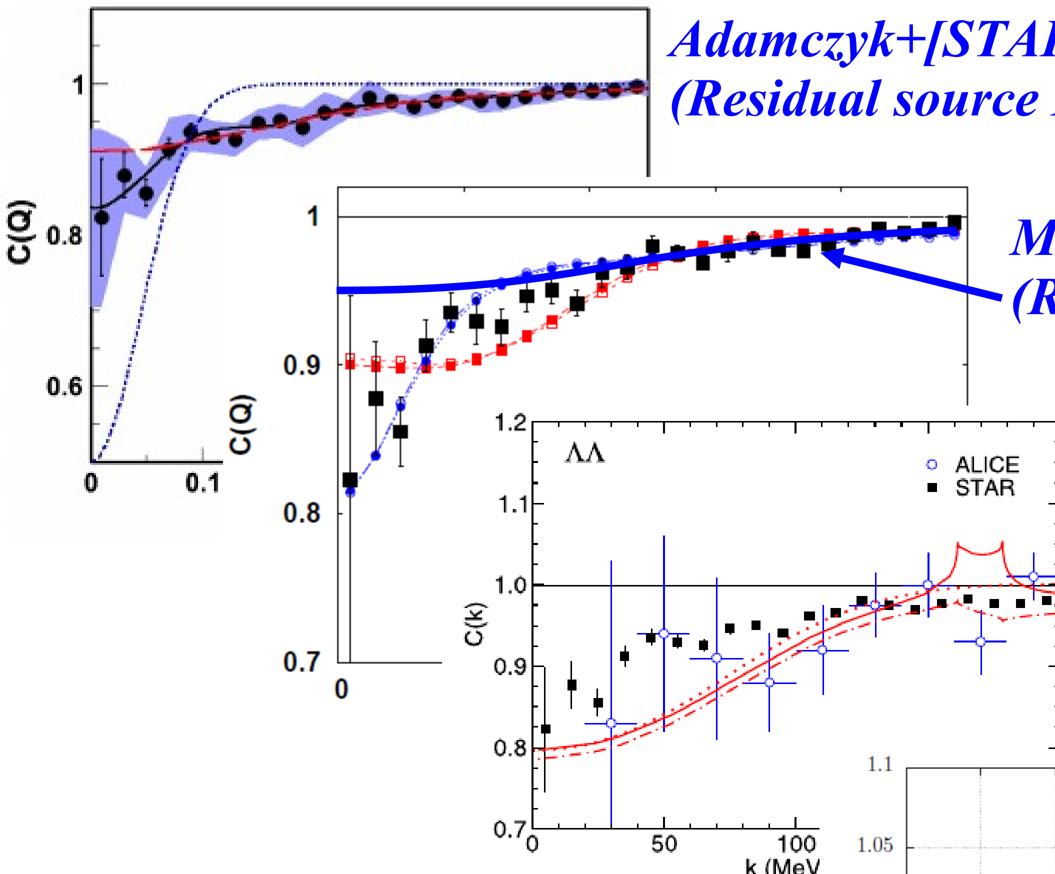
There is no signal of bound state.

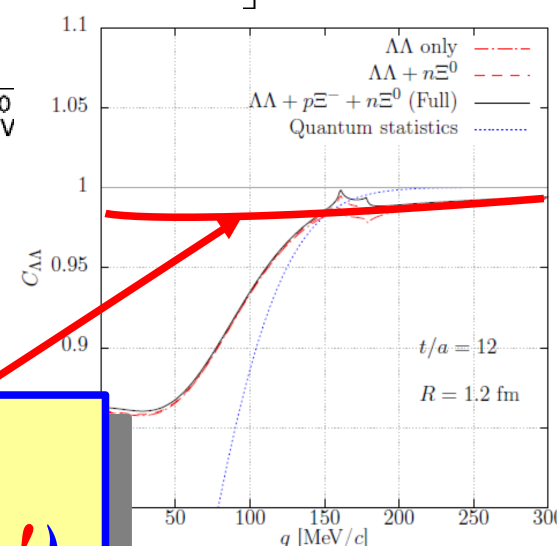
ΛΛ correlation function

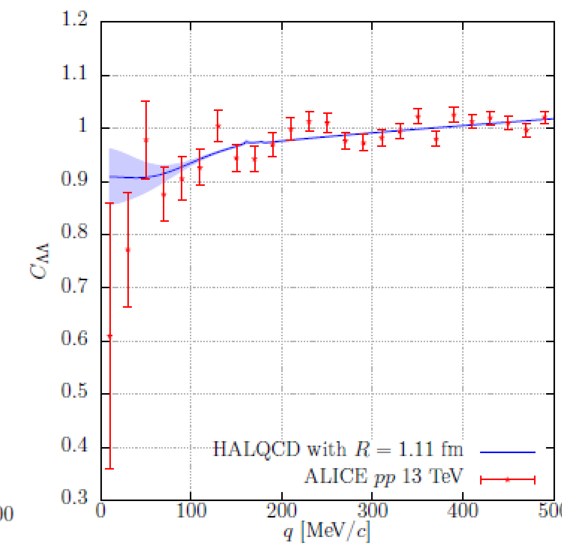
Adamczyk+[STAR], PRL114('15)022301
(Residual source R ~ 0.5 fm was assumed.)

Morita, Furumoto, AO, PRC91('15) 024916.
(Res.Source (R~0.5 fm) + flow)

J. Haidenbauer, NPA981('19)1.
(NLO600)





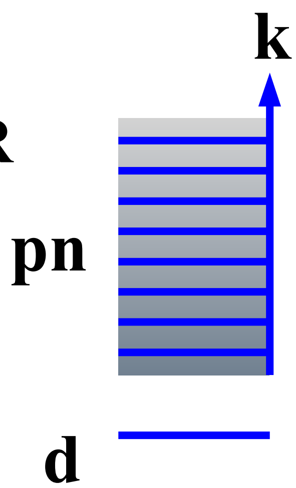


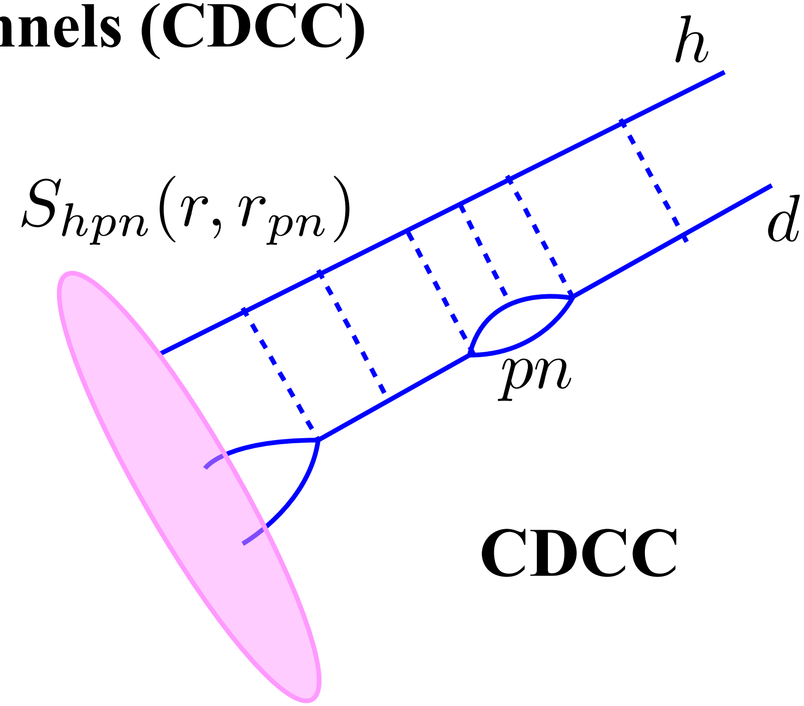
Kamiya+(in prep.).
(CC simulates res. source !)

Ξ^-d correlation function

Hadron-Deuteron correlation function

- **Hadron-deuteron correlation (Λd, K$^-$d, Ξ^-d, Ω^-d, …)**
  S.Mrówczyński, Patrycja Słoń, Acta Phys.Polon.B51('20),1739 [1904.08320](K-d,pd); J.Haidenbauer, PRC102('20)034001[2005.05012](Λd); F.Etminan+[2006.12771](Ωd).
  - **Scattering length data of these are important to evaluate**
    - **binding energy and lifetime of hyper triton (Λd)**
    - **I=1 $\bar{K}$N interaction (K$^-$d, Ξ^-d)**
    - **and the existence of a bound state.**
  - **Problem: *Breakup and Dynamical Formation of d* (d ↔ pn)**
    → Continuum-discretized coupled-channels (CDCC)
    M.Kamimura+('86); N. Austern+('87); M.Yahiro, K.Ogata, T.Matsumoto, K.Minomo, PTEP 2012 (2012) 01A206.
  - **Measurable at LHC-ALICE and (probably) RHIC-STAR**



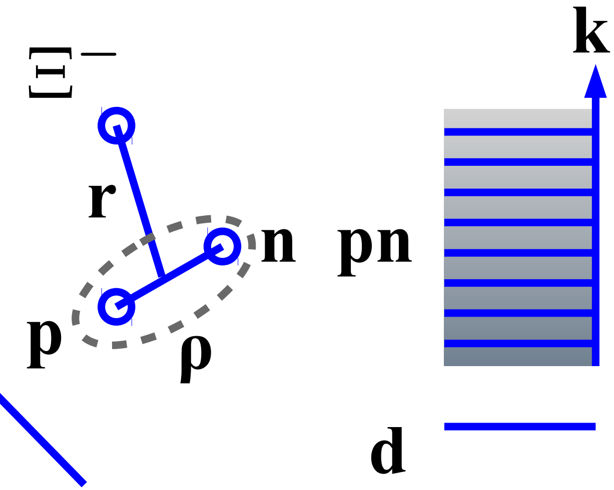


$\Xi^{-}d$ C(q) using CDCC

- **Three-body wave functions (s-wave)**

$$\psi^{(-)}(r,\rho;q) = \sum_n \sum_k A_{kn}\varphi_k(\rho)\chi_{nk}(r;q_{nk})$$

- n, k: J, spin, isospin, ...
- k: intrinsic momentum bin
- A_{kn}: kinematic factor
- $\varphi_k(\rho)$: normalized pn w.f. in k-th bin
- $\chi_{nk}(r;q_{nk})$: Ξ^--pn w.f.



- **Ξ^{-}d Correlation function**

$$C(q) = C^{\mathrm{C}}_{\ell>0}(q) + \frac{1}{2\cdot 3}\int d\boldsymbol{r}\, S(r) \sum_{nk} |\chi_{nk}(r;q_{nk})|^2$$

- $C^{\mathrm{C}}_{\ell>0}(q)$: pure Coulomb
- $\frac{1}{2\cdot 3}$: $1/(2J_1+1)/(2J_2+1)$
- $S(r)$: "Ξ^{-}d" source fn.

- **Potential = HAL QCD potential at almost physical quark masses**

K. Sasaki et al. [HAL QCD Collab.], NPA 998 ('20) 121737 (1912.08630)

(coupling with ΛΛ is ignored).

Ξ⁻d correlation function: Result

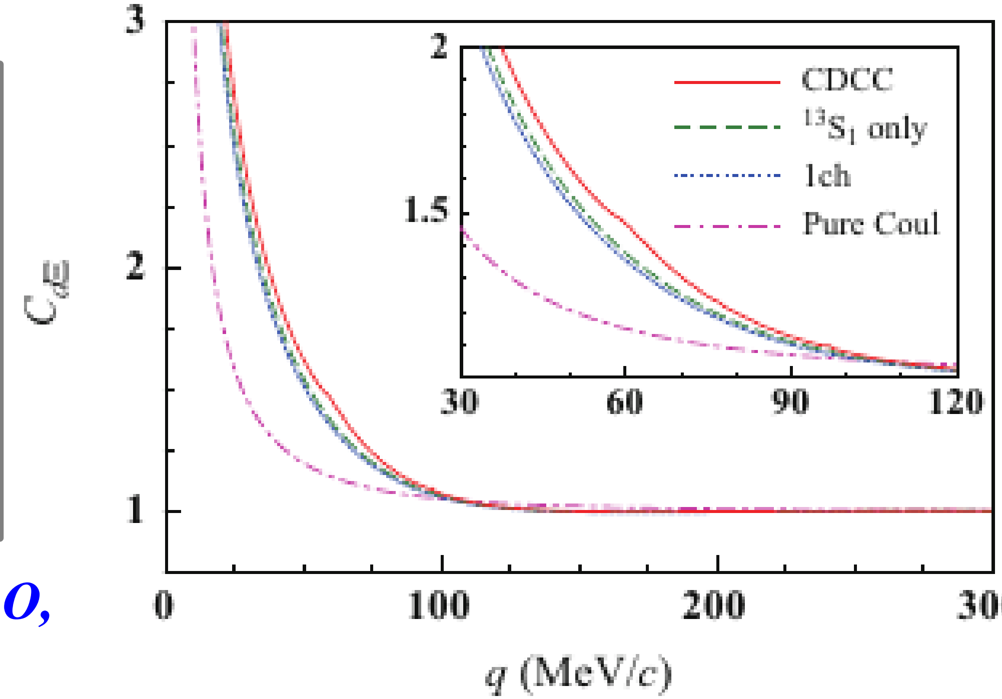
- **CDCC results of Ξ^-d correlation function**
  - **Enhancement from pure Coulomb C(q) by ΞN interaction from HAL QCD potential.**
  - **Breakup & Reformation effects ~ 10 % (Barely measurable)**
  - **Dynamical formation of deuteron is (maximally) included.**

$$\text{Implicit assumption: } \int d\boldsymbol{\rho} S(\rho)|\varphi_k(\rho)|^2 \simeq \text{const.}$$

  - **Threshold cusp at d → pn threshold is seen, but not prominent.**

Single channel description may not be bad.
→ Bound or Unbound in Ξd from Experimental data (if measured).

K. Ogata, T. Fukui, Y. Kamiya, and AO, PRC, to appear (arXiv:2103.00100).

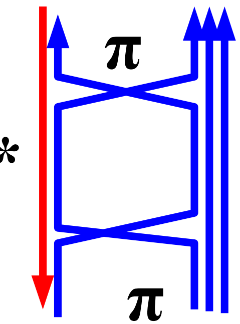


New type of correlation functions

Correlation functions of Charmed Hadron and Nucleon

- C(q) including a charmed hadron
  - Extremely important in recent hadron physics.
  - $D^-(\bar{c}d)$-p(uud) correlation
    - Probes $\Theta_c(\bar{c}$-ud-ud) state (replace $\bar{s}$ in $\Theta(\bar{s}$-ud-ud) with $\bar{c}$)

      D. O. Riska, N. N. Scoccola, PLB299('93)338 (pred.);
      A. Aktaset+ [H1], PLB588('04)17 (positive);
      J. M. Linket+ [FOCUS], PLB622('05)229 (negative).



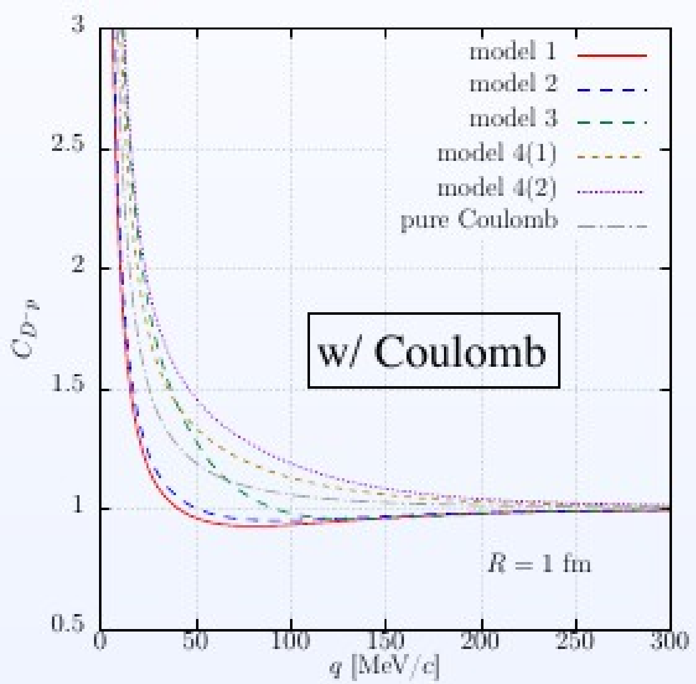
    - Attraction from two pion exchange

      S. Yasui, K. Sudoh, PRD80('09)034008.
    - Easy to calculate the potential in LQCD.

      Y. Ikeda et al. (private communication)
- $D^-(\bar{c}d)$-p(uud) CFs from proposed potentials

  Hofmann, Lutz ('05) (repulsive);
  Haidenbauer+('07) (repulsive);
  Yamaguchi+('11) (att., w/ bs); Fontoura+('13) (repulsive)

Data will discriminate these potentials !



Kamiya, Hyodo, AO (in prog.)

Three-body correlation functions

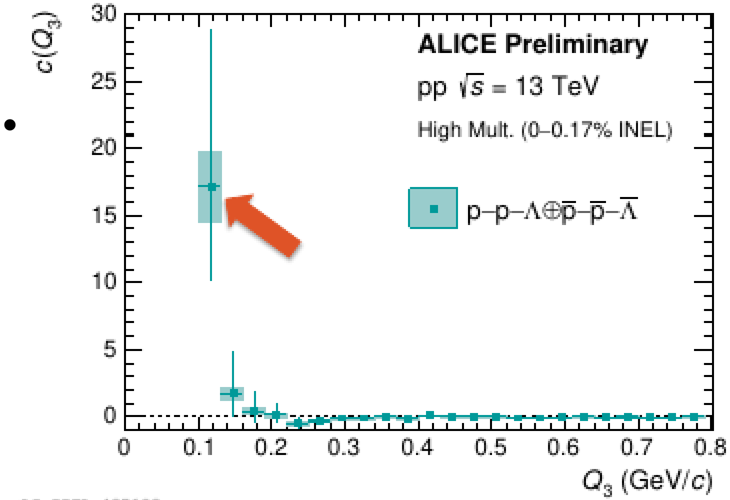
- Three-body correlation functions are measured and discussed; ppp, Λpp, pd, …
- Continuum Three-body w.f. at various momenta with Coulomb.
  - Riverside approximation (3π)
    E.g. E. O. Alt, T. Csorgo, B. Lorstad, J. Schmidt-Sorensen, PLB458 ('99)407.

$$\Psi_{123} = \psi(\boldsymbol{q}_{12})\psi(\boldsymbol{q}_{23})\psi(\boldsymbol{q}_{31})$$

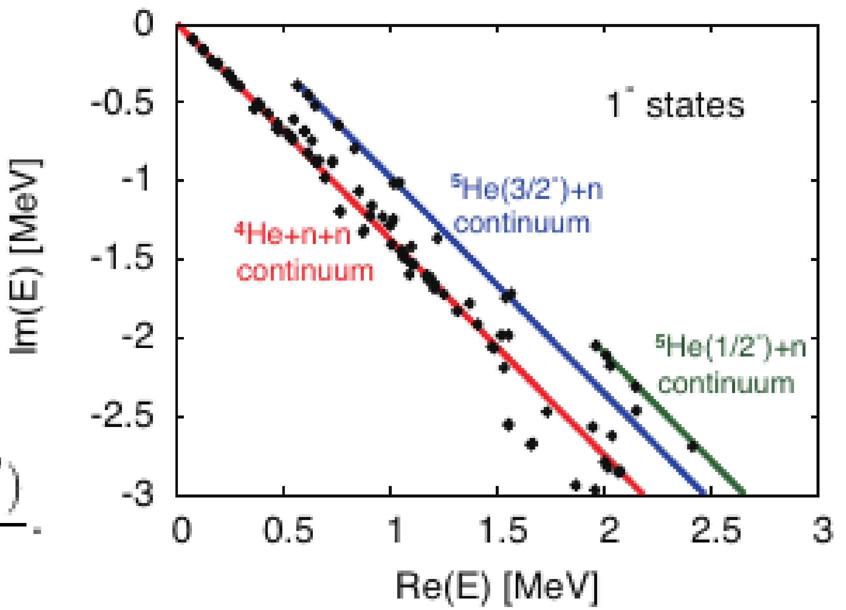
    → Does not give free correct w.f.
  - Complex Scaling ?
    Y. Kikuchi, T. Myo, M. Takashina, K. Kato, K. Ikeda, PTP122 ('09) 499;
    T.Myo, AO, K.Kato, PTP99('98)801.

$$\mathcal{G}^\theta(E;\xi,\xi') = \left\langle \xi \left| \frac{1}{E-\hat{H}^\theta} \right| \xi' \right\rangle = \sum\!\!\!\!\!\!\int_n \frac{\chi_n^\theta(\xi)\tilde{\chi}_n^\theta(\xi')}{E-E_n^\theta}.$$

  - Other idea ?



V. Mantovani Sarti @SQM2021



Summary

- **Femtoscopy (study using correlation functions) is useful to explore various hadron-hadron interactions in the s-wave.**
  - **Multi-strangeness pairs (S=-1,-2,-3, … -6(?)),**
    K^- p "scattering" at low-energy (e.g., q < 200 MeV/c),
    Charmed hadron interactions (D^- p, D^+ p, …),
    Three-body correlation, …
  - **An analytic model (Lednicky-Lyuboshits) is useful to discuss qualitative features.**
  - **Coupled-channel framework including Coulomb and threshold difference has been developed and is ready to use for the two-body correlation functions (Yuki Kamiya).**
  - **For more realistic estimate, reliable interactions and reliable source models are desired.**
- **Is the same technique useful for other reactions such as hadron production from e^+e^- ? Did someone try ?**

Correlation function from e+e- ?

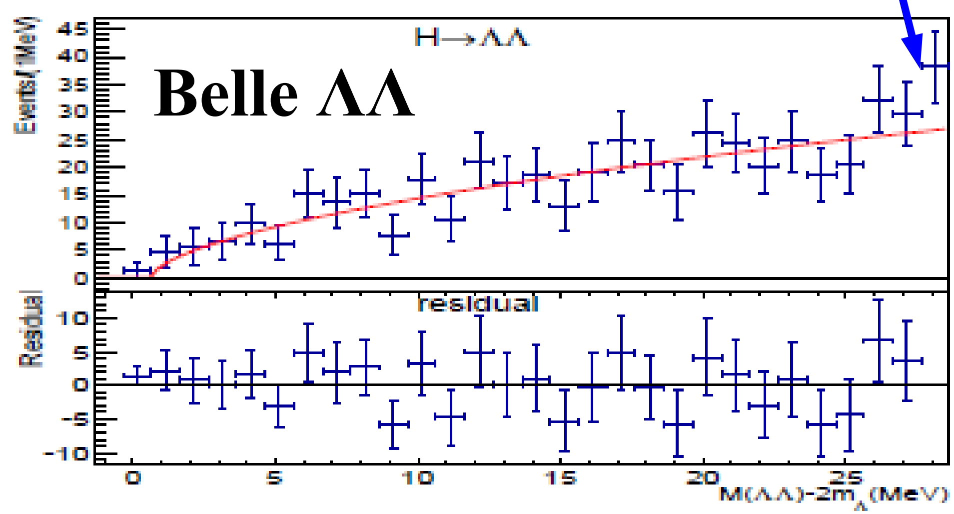
- C(q) can be obtained from the invariant mass spectrum

$$\frac{dN_{12}}{dM_{\mathrm{inv}}} = \frac{dN_{12}}{dq}\left[\frac{dM_{\mathrm{inv}}}{dq}\right]^{-1} \simeq \frac{dN_{12}}{dq}\frac{\mu}{q} \propto \frac{\mu}{q}C(q)$$

$$[M_{\mathrm{inv}} \simeq m_1 + m_2 + q^2/2\mu]$$

- e^+e^- reaction is not complex enough, but it may be valuable (?) to see C(q), since KP formula gives rough (average) pair yield. (I thank Prof. Olsen.)
- What is the difference between the cusp height at NΞ threshold in e^+e^- and pp collisions ?
- How about $A(K^-, K^+\Lambda\Lambda)$ at J-PARC and $\gamma A \to \eta' pX$ at ELPH ?

NΞ threshold cusp



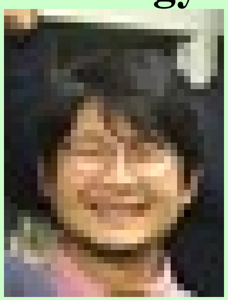
Belle Collaboration (Kim, B.H. et al.), PRL110('13)222002.

Thank you for your attention !

Coauthors of arXiv:1908.05414 (pΩ, ΩΩ) and arXiv:1911.01041 (pK⁻),
arXiv:2103.00100 (dΞ⁻),
and next paper on pΞ⁻-ΛΛ (Y. Kamiya, K. Sasaki, T. Fukui, T. Hatsuda, T. Hyodo, K. Morita, K. Ogata, AO, in prep.)

K. Morita

S. Gongyo

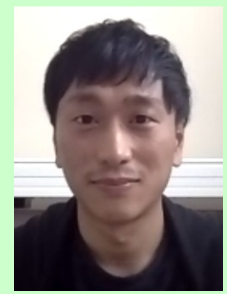


T. Hatsuda

T. Hyodo

K. Ogata

T. Fukui



(J. Haidenbauer)

K.Sasaki Y. Kamiya (ALICE)

Correlation function from T-matrix

- **s-wave w.f. using the half-off-shell T-matrix (T_0)**

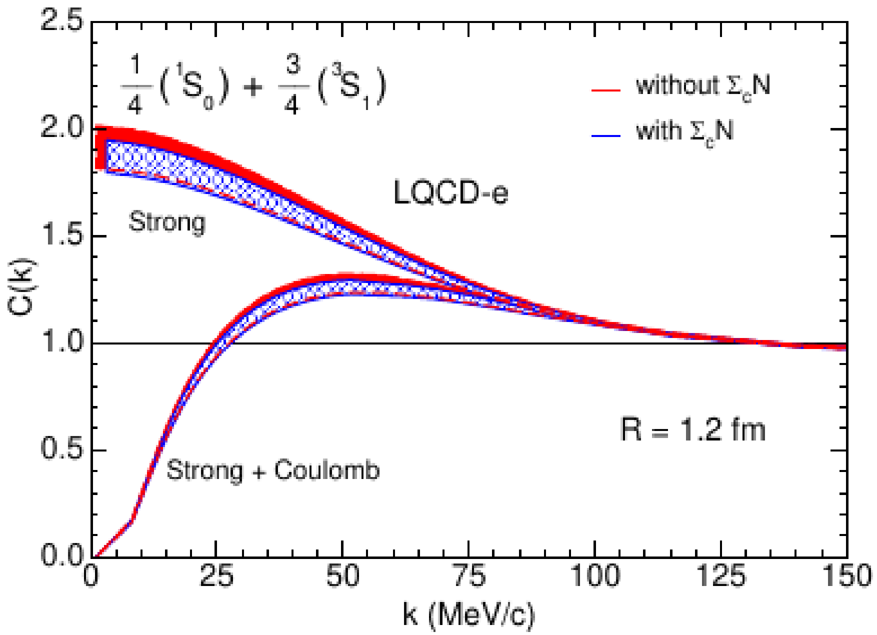
J. Haidenbauer, NPA 981('19)1.

$$\widetilde{\psi}_0(k,r) = j_0(kr) + \frac{1}{\pi}\int dq\, q^2 j_0(qr)\,\frac{1}{E - E_1(q) - E_2(q) + i\varepsilon} T_0(q,k;E)$$

$$\psi_0^{(-)}(k,r) = e^{-2i\delta_0}\widetilde{\psi}_0(k,r) \to \frac{e^{-i\delta_0}}{kr}\sin(kr+\delta_0) = \frac{1}{2ikr}\left(e^{ikr} - e^{-2i\delta_0}e^{-ikr}\right)$$

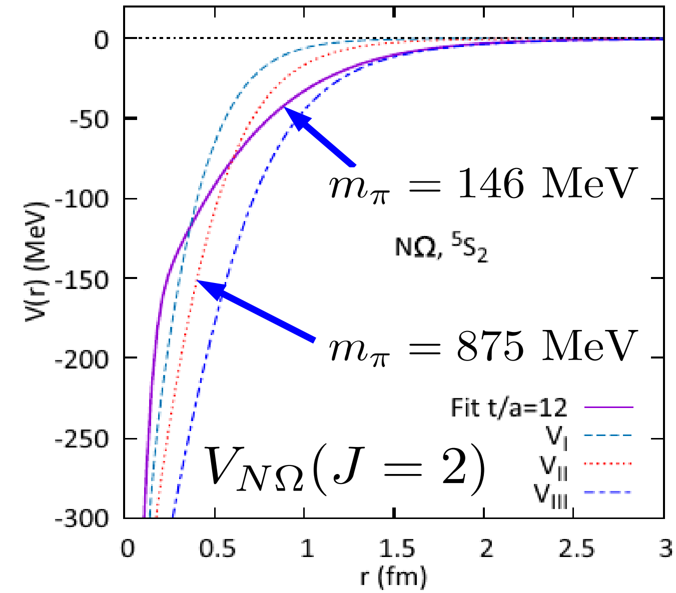
- **Strong T-matrix + Coulomb potential**

J. Haidenbauer, G. Krein, and T. C. Peixoto, EPJA 56 ('20)184;
using the Vincent-Phatak method
[C.M. Vincent and S.C. Phatak, PRC10('74)391;
B. Holzenkamp, K. Holinde and J. Speth,
NPA 500('89)485 (1989)]

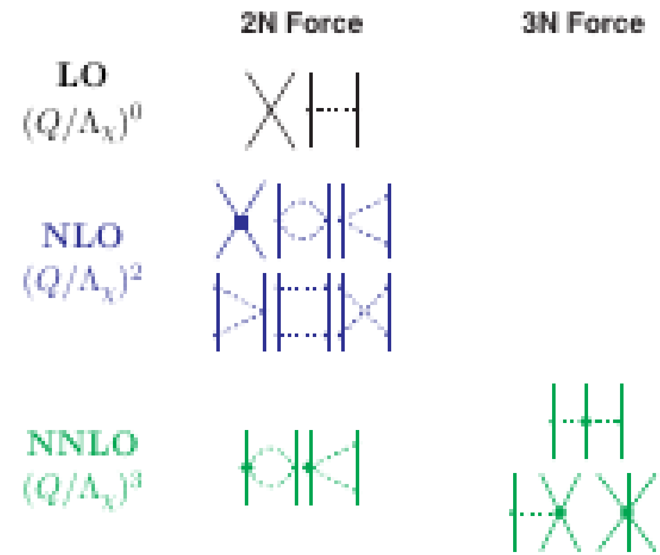


Modern Hadron-Hadron Interactions

- Lattice QCD *hh* potential
  - V_{hh} is obtained from the Schrödinger eq. for the Nambu-Bethe-Salpeter (NBS) amplitude.
    N. Ishii, S. Aoki, T. Hatsuda, PRL99('07)022001.
    → ΩΩ, NΩ, ΛΛ-NΞ potentials
    at phys. quark mass are published



- Chiral EFT / Chiral SU(3) dynamics
  - V_{hh} at low E. can be expanded systematically in powers of Q/Λ.
    S. Weinberg ('79);R. Machleidt, F. Sammarruca ('16); Y. Ikeda, T. Hyodo, W. Weise ('12).
    → NN, NY, YY, $\bar{K}$N-πΣ-πΛ, ...



- Quark cluster models,
  Meson exchange models,
  More phenomenological models, …

Let us examine modern hh interactions !

C(q) in the low momentum limit

- **Correlation function at small q (and r_{eff}=0) → F_1=1, F_2=0, F_3=1**

$$\Delta C_{\text{LL}}(q) \to \frac{|f(0)|^2}{2R^2} + \frac{2\text{Re}f(0)}{\sqrt{\pi}R} \quad (q \to 0)$$

$$f(q) = (q\cot\delta - iq)^{-1} \simeq \left(-\frac{1}{a_0} + \frac{1}{2}r_{\text{eff}}q^2 - iq\right)^{-1} \to -a_0$$

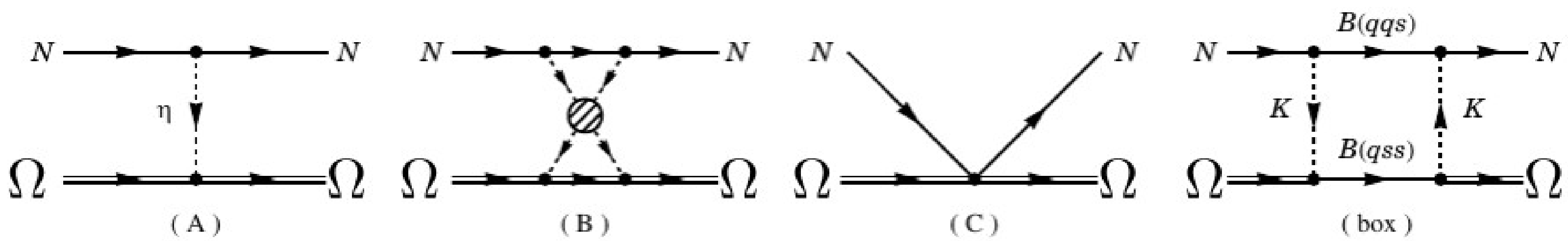
$$C_{\text{LL}}(q \to 0) = 1 + \frac{a_0^2}{2R^2} - \frac{2a_0}{\sqrt{\pi}R} = 1 - \frac{2}{\pi} + \frac{1}{2}\left(\frac{a_0}{R} - \frac{2}{\sqrt{\pi}}\right)^2$$

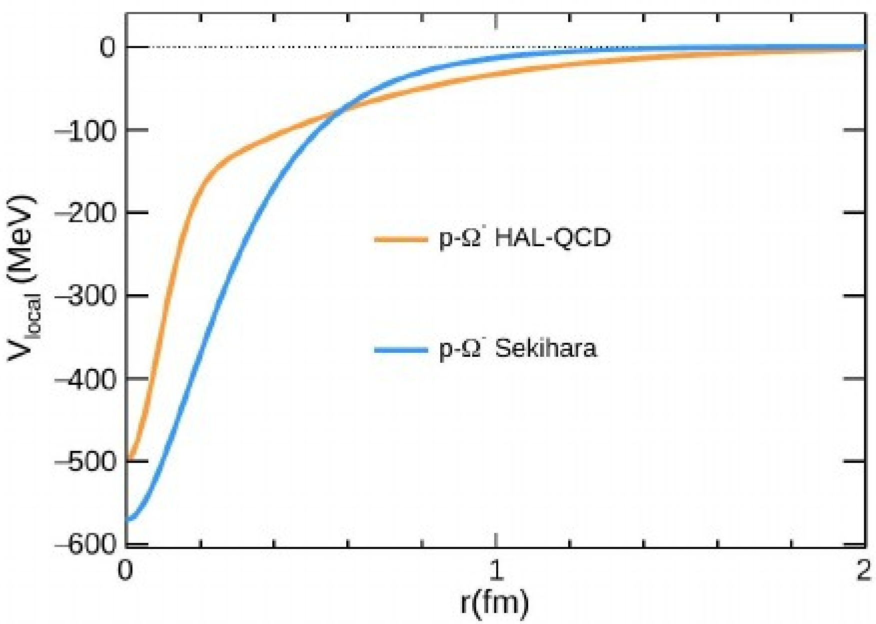
$$1 - 2/\pi \simeq 0.36, \ \sqrt{\pi}/2 \simeq 0.89$$

C(q → 0) takes a minimum of 0.36 at R/a_0 = 0.89 in the LL model.

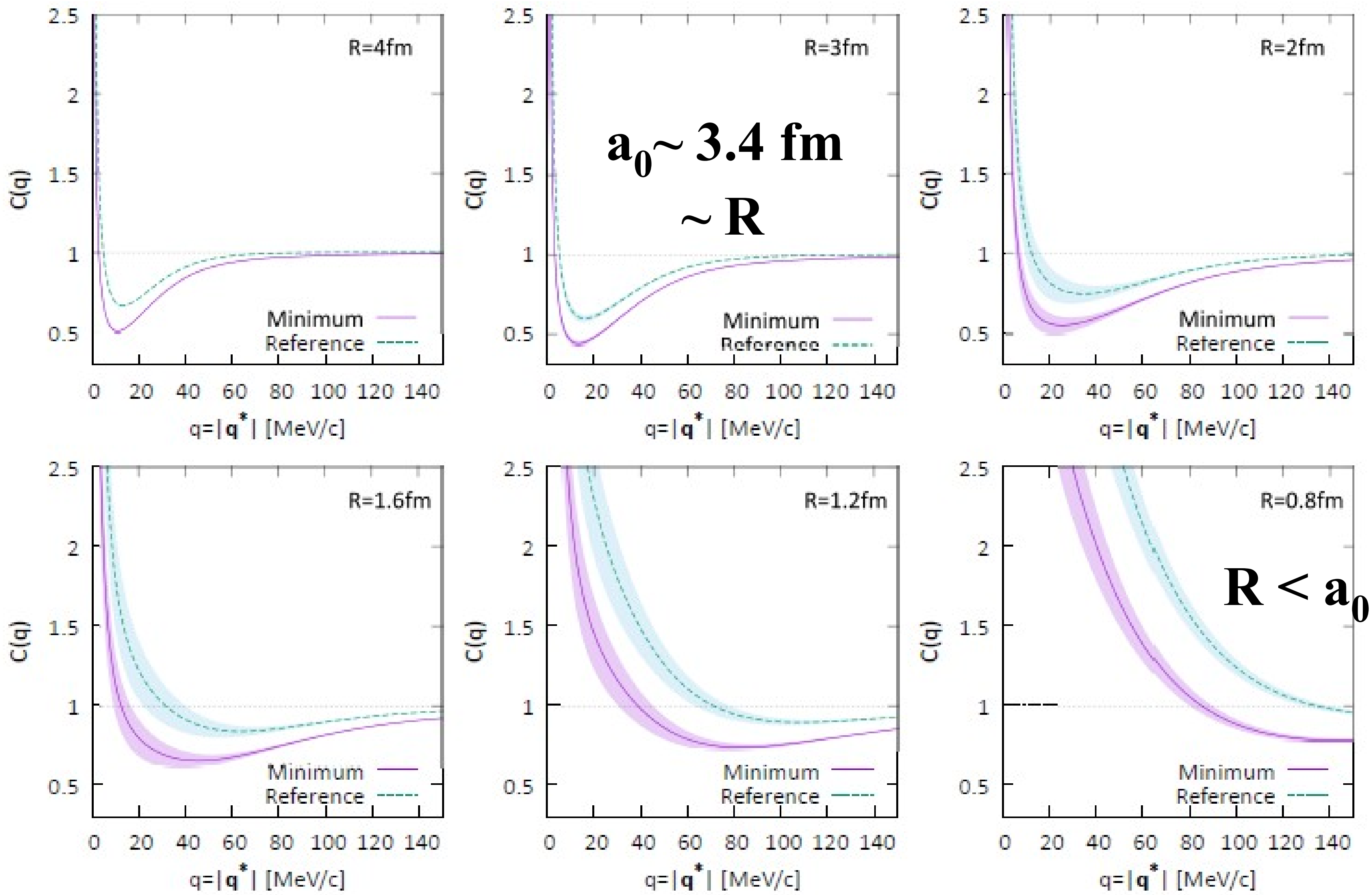
NΩ Potential in a Meson Exchange model

- **Meson exchange NΩ potential**
  T. Sekihara, Y. Kamiya, T. Hyodo, PRC98 ('18) 015205
  - **η meson exchange, σ exchange, contact term, box diagram.**
  - **Contact term is fitted to the scatt. length of HAL QCD potential.**





Source Size Dependence of Correlation Function



Gaussian Source

K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO ('20)

Scattering Length

- **pΩ (a_0 in nuclear physics convention)**
  K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC101('20)015201 [1908.05414]

TABLE III. S-wave scattering length a_0, effective range $r_{\rm eff}$, and binding energy of the $p\Omega$ pair with the lattice QCD potential for different t/a and the Coulomb attraction.

t/a	a_0 [fm]	$r_{\rm eff}$ [fm]	E_B [MeV]
11	3.45	1.33	2.15
12	3.38	1.31	2.27
13	3.49	1.31	2.08
14	3.40	1.33	2.24

- **K^-N (a_0 in high-energy physics convention)**
  Y. Ikeda, T. Hyodo, W. Weise, NPA881('12) 98 [1201.6549]

$$a(K^-p) = -0.93 + i\,0.82 \text{ fm} \quad (\text{TW}) ,$$
$$a(K^-p) = -0.94 + i\,0.85 \text{ fm} \quad (\text{TWB})$$
$$a(K^-p) = -0.70 + i\,0.89 \text{ fm} \quad (\text{NLO})$$

$$a(K^-n) = 0.29 + i\,0.76 \text{ fm} \quad (\text{TW})$$
$$a(K^-n) = 0.27 + i\,0.74 \text{ fm} \quad (\text{TWB})$$
$$a(K^-n) = 0.57 + i\,0.73 \text{ fm} \quad (\text{NLO})$$

Wave function around threshold (S-wave, attraction)

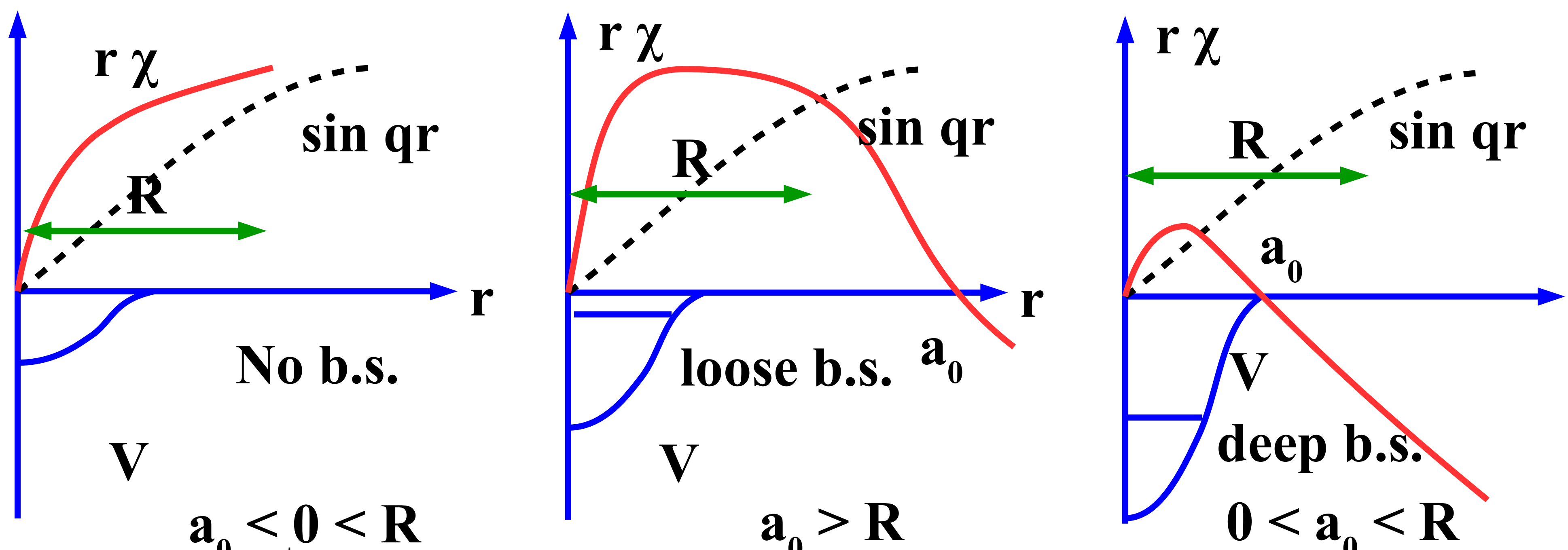
- **Low energy w.f. and phase shift**

$$u(r) = qr\chi_q(r) \to \sin(qr + \delta(q)) \sim \sin(q(r - a_0))$$

$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} q^2 + \mathcal{O}(q^4)\ (\delta \sim -a_0 q)$$

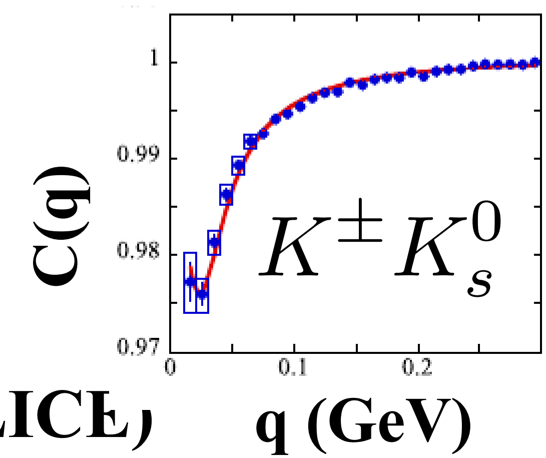
a_0 =scatt. length
r_{eff}=eff. range

- **Wave function grows rapidly at small r with attraction.**
- **With a bound state (a_0>0), a node appears around r=a_0**



Recent & Near-Future Correlation Functions

- $\bar{p}p$, $p\bar{\Lambda}$ *E.g. A. Kisiel [ALICE], Acta Phys.Polon.Supp. 6 ('13)519*
- $K^{\pm}K^0_s$ *S.Acharya+ [ALICE], PLB774 ('17)64 [1705.04929]*
  → **Slightly suppressed at low q**
  Tetraquark component of a_0 meson



- **pΛ [2104.04427], pϕ [2105.05578],**
  $p\bar{\Lambda}$, $\Lambda\bar{\Lambda}$ [2105.05190], $p\Sigma^0$ ['20 [1910.14407]] (ALICE)
- **$pD^{\pm}$ (in prog.) Scatt. length is strongly model dependent.**
  → **To be discriminated by experiment !**

$\bar{D}p$

model	$a_0^{DN(I=0)}$ [fm]	$a_0^{DN(I=1)}$ [fm]	boutnd state (I=0)	bound state (I=1)	
1 [1]	-0.16	-0.26	None	None	*Hofmann+('05)*
2 [2]	0.07	-0.45	None	None	*Haidenbauer+('07)*
3 [3]	-4.38	-0.07	2804	None	*Yamaguchi+('11)*
4 [4]	0.03-0.16	0.20-0.25	None	None	*Fontoura+('13)*

- **deuteron-hadron CF**
  S. Mrówczyński and P. Słoń, Acta Phys.Polon.B51('20)1739 [1904.08320]; F. Etminan, M. M. Firoozabadi, [1908.11484]; J. Haidenbauer, PRC102('20)034001 [2005.05012]; K.Ogata, T.Fukui, Y.Kamiya, AO [2103.00100].