# *Exotic hadrons and interactions from HIgh-energy nuclear Collisions (ExHIC)*


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- **Introduction**
- **Exotic Hadron Yields in High-Energy Nuclear Collisions**
- **Exotic Interaction from High-Energy Nuclear Collisions**
- **Summary**

*エキゾチックハドロン研究会*
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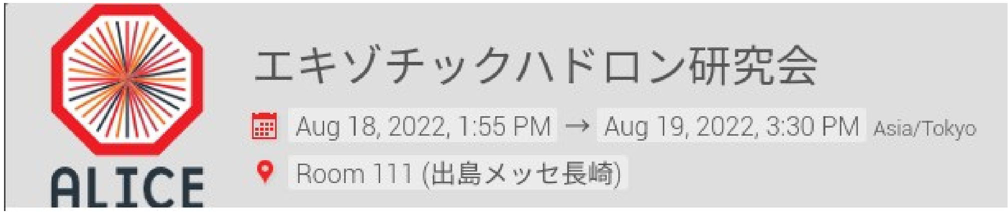
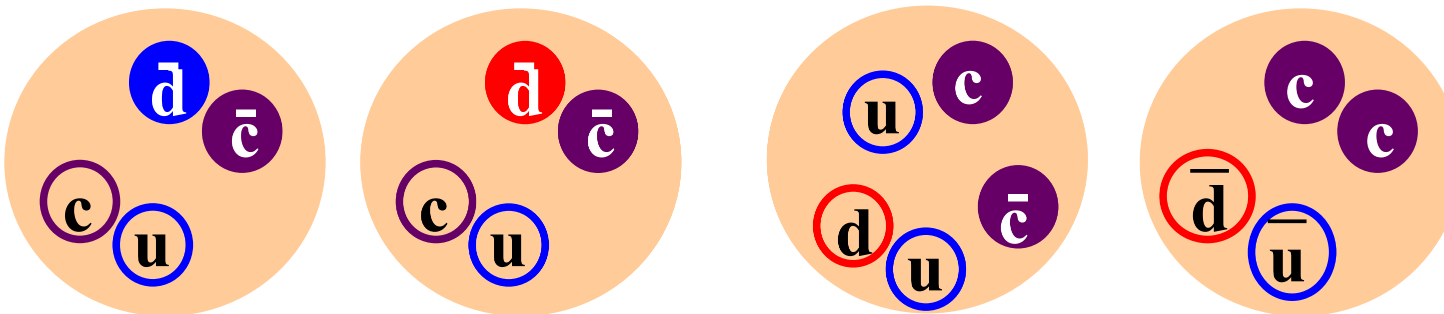
*出町メッセ長崎/Online.*



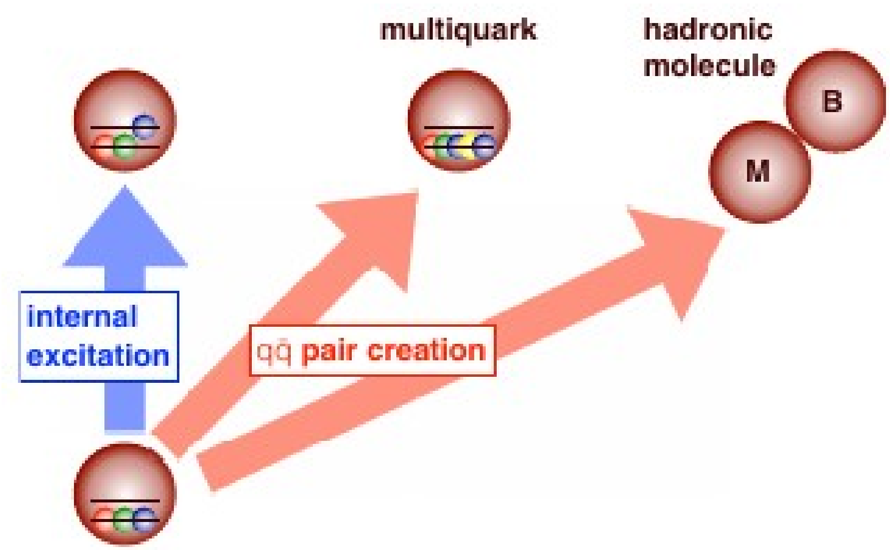
# Exotic Hadrons

- Exotic hadrons ($\Theta^+$, X, Y, Z, Pc ….)
  → Discovered/Proposed at LEPS, Belle, BaBar, BES, LHCb, ...




X(3872) Z(4430) Pc $T_{cc}$

- Various pictures
  - Compact multiquark state with di-quark component
  - Hadronic molecule
  - (Triangle) Singularity
  - $Q\bar{Q}$ couples with $Q\bar{Q}\, q\bar{q}$
  - …

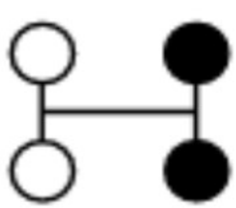



> ***What is the structure of exotic hadrons ?***
> ***Can we access h-h interactions with heavy quarks ?***

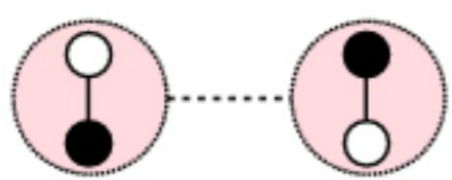
# Multiquark or Molecule ?

- **Multiquark states**
  - **“good” diquark (flavor antisym., spin 0) には $1/m_1 m_2$ に比例する強い引力が働く**
    **→ Tcc では tetraquark 構造 $\{cc\}_{J=1}[\bar{u}\bar{d}]_{J=0}$ が有利**
    *S. Zouzou+(‘86), ZPC30,457.*
  - **閉じ込め力による束縛・共鳴状態 → コンパクト**
- **Hadronic molecule**
  - **ハドロン間 ( カラー1重項間 ) の相互作用による共鳴・束縛状態**
  - **浅い束縛状態では大きなサイズ**
  - **弱束縛関係式 ( 散乱長と束縛エネルギーの関係 )**

$$R_h \simeq 1/\sqrt{2\mu B}\,,\ a_0 \to R_h\ (B \to 0)$$



Tetraquarks

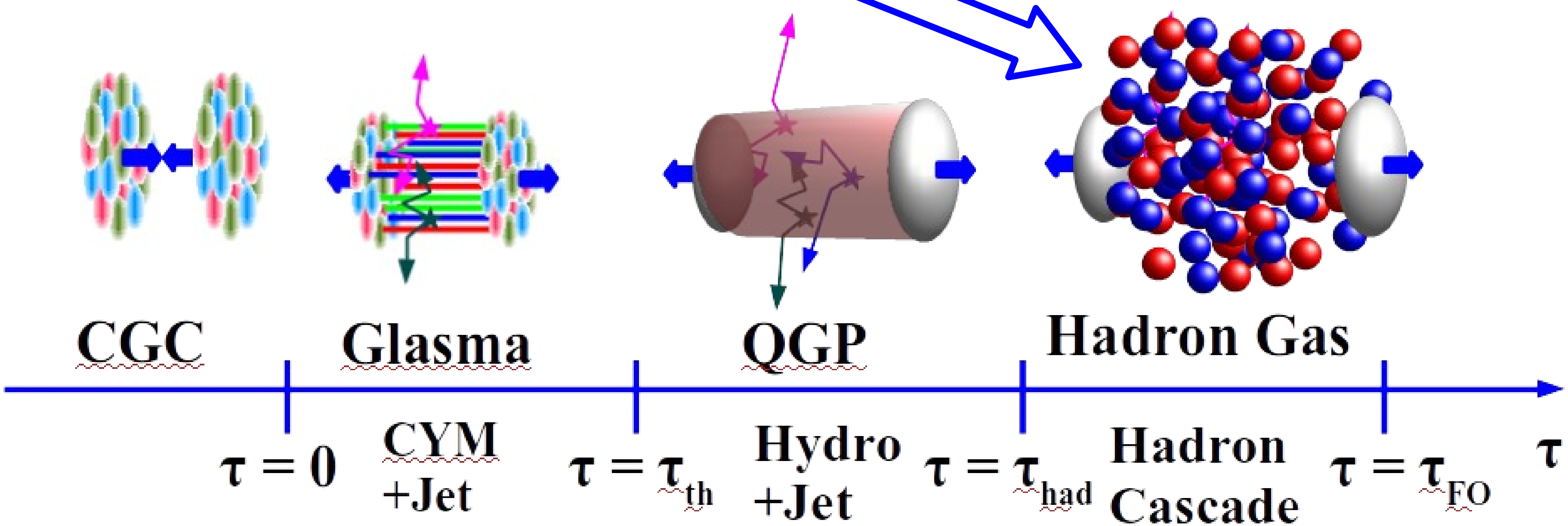


Hadronic Molecules

***Key quantities***
***= Hadron Size***
***& Scattering length***

# High-Energy Heavy-Ion & Nuclear Collisions

- **High-Energy Heavy-Ion Collisions**
  **& High-Energy (and High-Multiplicity) pp and pA collisions**
  - **Too complex → Statistical → Simple and Clean !**
  - **High $T$ & Large volume → Abundant hadrons**
  - **Nearly 4π detector / Vertex detector**
    **→ Let's regard High-Energy Nuclear Collisions as Exotic Hadron Factories**




***We will demonstrate that high-energy nuclear collisions are useful in hadron phyics !***

# Exotic Hadron Yields in High-Energy Nuclear Collisions

- *S.Cho et al. [ExHIC], PRL 106 (‘11), 212001; PRC 84 (2011), 064910; PPNP 95(2017), 279-322.*
- *H.Taya et al. [ExHIC-P], PRC 102 (2020), 021901(R)*

# Statistical Model

- **Statistical model**

$$N_h^{\rm stat} = V_H \frac{g_h}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\gamma_h^{-1} e^{E_h/T_H} \pm 1}$$

**($N_h = dN_h/dy$ (y=rapidity), $V_H$=Chem. freeze-out vol.)**

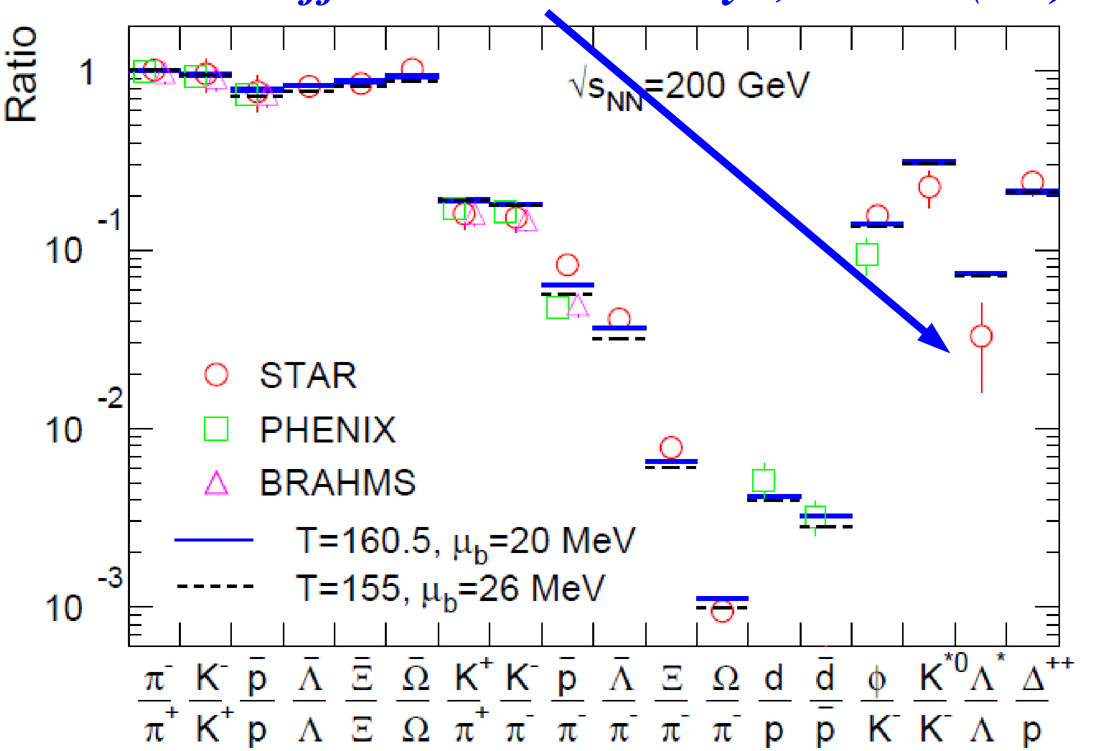
**→ Successful to predict the hadron yield ratio at RHIC**

- **Fugacity factor γ**
  - **u,d,s: chem. equil.**
  - **c,b: enhanced by initial hard processes**

**Fugacities of c and b quarks are set to reproduce expected c and b quark numbers.**

$$\gamma_h = \gamma_c^{n_c + n_{\bar{c}}} \gamma_b^{n_b + n_{\bar{b}}} e^{(\mu_B B + \mu_s S)/T_H}$$

*d-wave effects: Kanada-En'yo, Muller ('06)*




*A. Andronic, P. Braun-Munzinger, J. Stachel, NPA772('06)167.*

# Heavy quarks in statistical models

- **RHIC, LHC の中心衝突では s-quark までは「化学平衡」に達しているが、c, b quarks は jet で作られるため、統計模型で予想されるよりも多く作られる → Fugacity factor**

$$N_h^{stat} = V_H \frac{g_h}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\gamma_h^{-1} e^{E_h/T_H} \pm 1}, \qquad \gamma_h = \gamma_c^{n_c+n_{\bar{c}}} \gamma_b^{n_b+n_{\bar{b}}} e^{(\mu_B B + \mu_s S)/T_H},$$

Table 3.1: Statistical and coalescence model parameters for Scenario 1 and 2 at RHIC (200 GeV), LHC (2.76 TeV) and LHC (5.02 TeV), and those given in Refs. [14, 15]. Quark masses are taken to be $m_q$ = 350 MeV, $m_s$ = 500 MeV, $m_c$ = 1500 MeV and $m_b$ = 4700 MeV. In Refs. [14, 15], light quark masses were taken to be $m_q$ = 300 MeV.

| | RHIC | | LHC (2.76 TeV) | | LHC (5.02 TeV) | | RHIC | LHC (5 TeV) |
|---|---|---|---|---|---|---|---|---|
| | Sc. 1 | Sc. 2 | Sc. 1 | Sc. 2 | Sc. 1 | Sc. 2 | Refs [14, 15] | |
| $T_H$ (MeV) | 162 | | 156 | | | | 175 | |
| $V_H$ (fm$^3$) | 2100 | | 5380 | | | | 1908 | 5152 |
| $\mu_B$ (MeV) | 24 | | 0 | | | | 20 | 0 |
| $\mu_s$ (MeV) | 10 | | 0 | | | | 10 | 0 |
| $\gamma_c$ | 22 | | 39 | | 50 | | 6.40 | 15.8 |
| $\gamma_b$ | $4.0 \times 10^7$ | | $8.6 \times 10^8$ | | $1.4 \times 10^9$ | | $2.2 \times 10^6$ | $3.3 \times 10^7$ |
| $T_C$ (MeV) | 162 | 166 | 156 | 166 | 156 | 166 | 175 | |
| $V_C$ (fm$^3$) | 2100 | 1791 | 5380 | 3533 | 5380 | 3533 | 1000 | 2700 |
| $\omega$(MeV) | 590 | 608 | 564 | 609 | 564 | 609 | 550 | |
| $\omega_s$(MeV) | 431 | 462 | 426 | 502 | 426 | 502 | 519 | |
| $\omega_c$(MeV) | 222 | 244 | 219 | 278 | 220 | 279 | 385 | |
| $\omega_b$(MeV) | 183 | 202 | 181 | 232 | 182 | 234 | 338 | |
| $N_u = N_d$ | 320 | 302 | 700 | 593 | 700 | 593 | 245 | 662 |
| $N_s = N_{\bar{s}}$ | 183 | 176 | 386 | 347 | 386 | 347 | 150 | 405 |
| $N_c = N_{\bar{c}}$ | 4.1 | | 11 | | 14 | | 3 | 20 |
| $N_b = N_{\bar{b}}$ | 0.03 | | 0.44 | | 0.71 | | 0.02 | 0.8 |

*Cho+[ExHIC]('17)*

*AGS, SPS での重イオン衝突では strange quark について使われた「由緒正しい」統計模型*

# Coalescence model

- **Yield = Overlap of const. dist. & Hadron intrinsic Wigner func. (Sudden approximation)**
  *Sato, Yazaki (1984), Hwa, Yang (2003), Greco, Ko, Levai (2003), Fries, Muller, Nonaka, Bass (2003), Chen, Ko, Lee (2003)*

$$N_h^{\rm coal} = g_h \int \underbrace{\left[\prod_{i=1}^{n} \frac{1}{g_i} \frac{p_i \cdot d\sigma_i}{(2\pi)^3} \frac{{\rm d}^3{\rm p}_i}{E_i} f(x_i, p_i)\right]}_{\text{Dist. of constituents}} \times \underbrace{f^W(x_1, \cdots, x_n : p_1, \cdots, p_n)}_{\text{Intrinsic Wigner func.}}$$

- **Yield in HIC**
  - **Quark & hadron dist. = Transverse Boltzmann + Bjorken**
    *Chen, Ko, Liu, Nielsen (2007)*
  - **Hadron intr. Wigner func. = s-wave and p-wave HO w.f.**
    *Kanada-En'yo, Muller (2006)*

$$N_h^{\rm coal} \simeq g_h \prod_{j=1}^{n} \frac{N_j}{g_j} \prod_{i=1}^{n-1} \frac{(4\pi\sigma_i^2)^{3/2}}{V(1+2\mu_i T\sigma_i^2)} \left[\frac{4\mu_i T\sigma_i^2}{3(1+2\mu_i T\sigma_i^2)}\right]^{l_i}$$

  **σ = Gaussian width, μ=reduced mass, N = constituent yield**
  - **Available structure information → σ (or ℏω)**

# *Which hadrons are enhanced in coalescence ?*

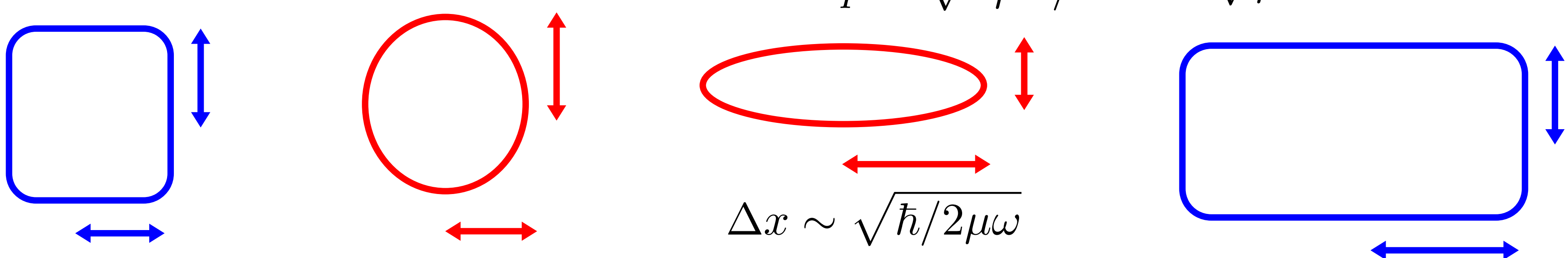
- **Simple estimate: 2-body, Gaussian w.f. + Thermal dist. of constiuents**
  **→ Yield is large when $f_w$ shape is similar to $f_{th}$ in phase space.**

$$N_h \propto \int \frac{d^Dx d^Dp}{(2\pi\hbar)^D} \underline{f_W(x,p)}\ \underline{f_{\rm th}(x,p)} = \left[\left(\frac{4}{\hbar^2}\right)\left((\Delta p)^2+\mu T\right)\left((\Delta x)^2+2R^2\right)\right]^{-D/2}$$

**Intrinsic** **Constituents (thermal)**

$$f_W(x,p) = \left(\frac{\hbar}{\Delta x \Delta p}\right)^D \exp\left(-\frac{x^2}{2(\Delta x)^2} - \frac{p^2}{2(\Delta p)^2}\right)$$

$$f_{\rm th}(x,p) = \left(\frac{\hbar^2}{2\mu T R^2}\right)^{D/2} \exp\left(-\frac{x^2}{4R^2} - \frac{p^2}{2\mu T}\right)$$



$\sqrt{\mu T} \sim 400$ MeV, $R \sim 1$ fm — **pp collisions**

$\Delta p \sim 300$ MeV, $\Delta x \sim 0.6$ fm — **normal hadrons**

$\Delta p \sim \sqrt{\hbar\mu\omega/2}$, $\Delta x \sim \sqrt{\hbar/2\mu\omega}$ — **hadronic molecule**

$(\hbar\omega \simeq 6 \times \text{B.E.} \simeq 3\hbar^2/2\mu\langle r^2\rangle)$

$\sqrt{\mu T} \sim 400$ MeV, $R \sim 5$ fm — **HIC**

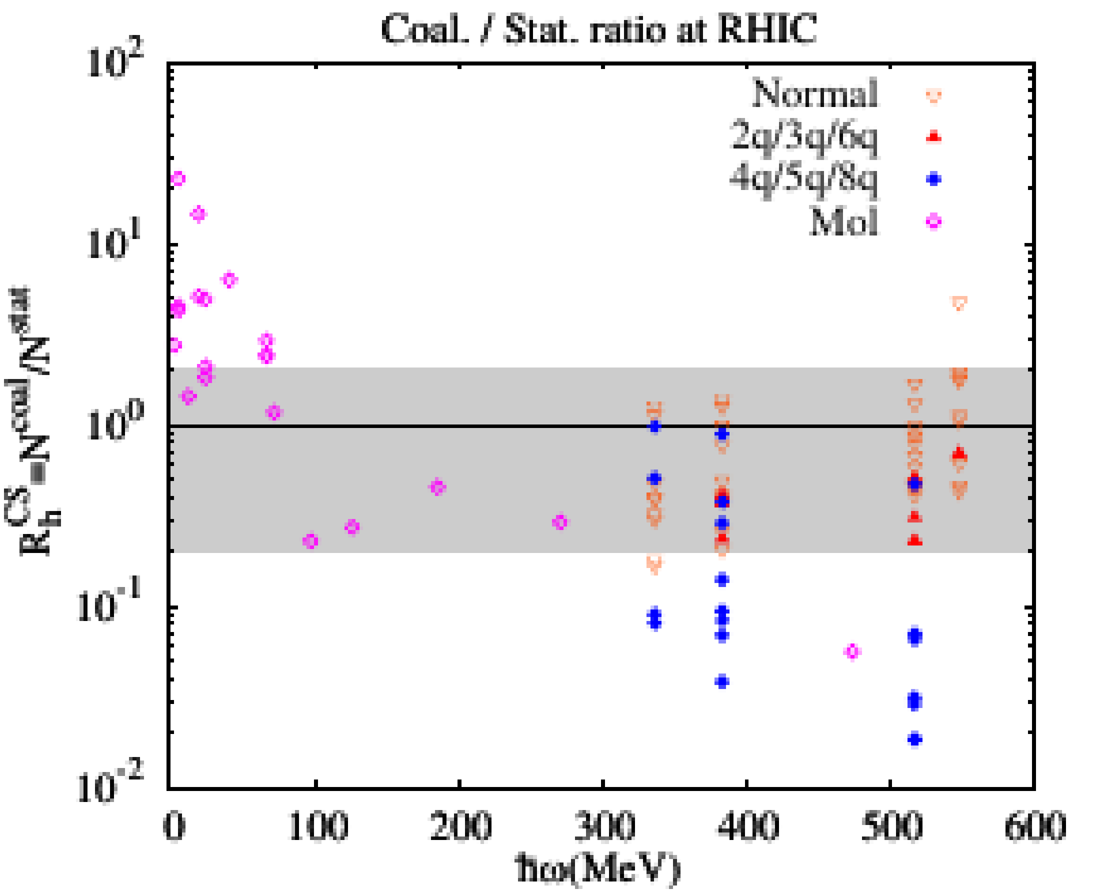
# *Which hadrons are enhanced in coalescence ?*

- **"Optimal" size of hadrons**
  - **The shapes of $f_w$ and $f_{th}$ are similar in phase space.**

$$(\Delta p/\Delta x)^2 = \mu T/2R^2 \rightarrow \hbar\omega = \sqrt{\hbar^2 T/2\mu R^2}$$

  - **hadrons with heavy-quarks (μ ~ 1 GeV),**
    **T ~ Tc ~ 160 MeV,**
    **HIC (R ~ 5 fm)**
    **→ ħω = 11 MeV**
    **(B.E. ~ 2 MeV)**

***Loosely bound hadronic molecules are favored in HIC, with coalescence !***

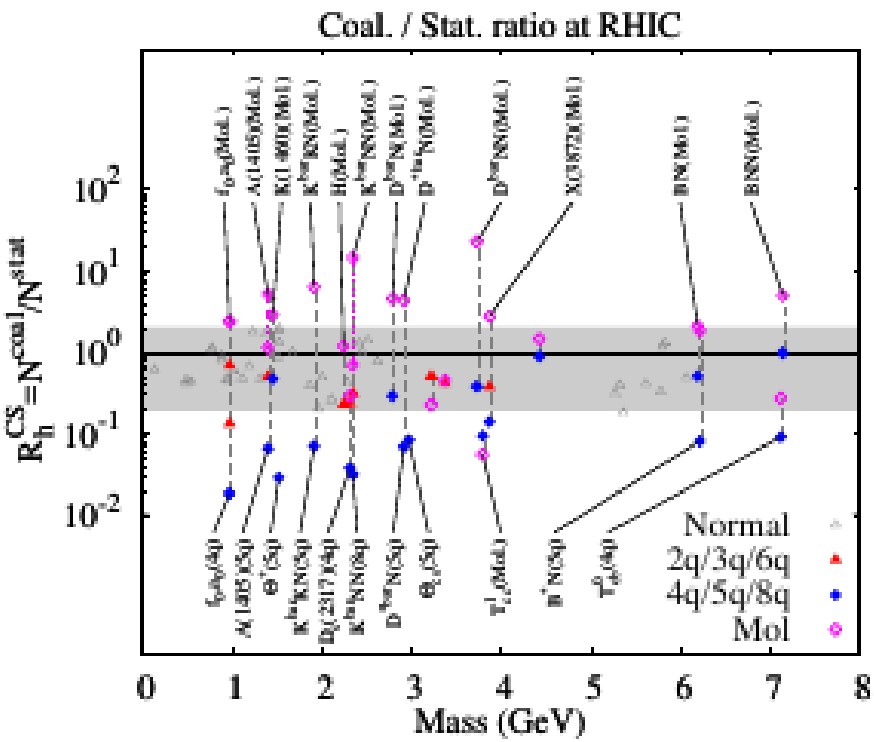



*AO+('13) [Hyp2012 proc.]*

# Coalescence / Statistical Ratio

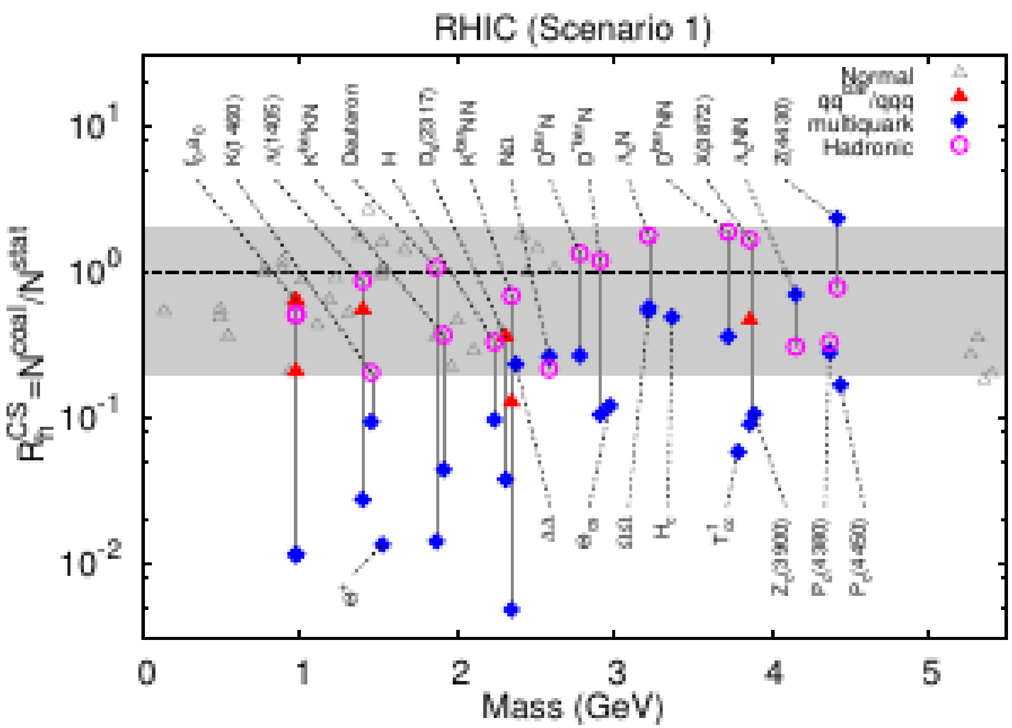
- If the coalescence is the underlying hadronization mechanism, hadron yields will deviate from statistical model estimate depending on the number of constituents, spin, and size.

**ExHIC (2011)**




Coalescence deuteron yield is larger than stat. model. ($R^{CS}_d \sim 1$ in data.)

**ExHIC (2017)**




Freeze-out T is carefully chosen to give $R^{CS}_d \sim 1$.

## Conclusion by ExHIC collaboration (2017)

Provided that coalescence is the underlying hadron production mechanism,
loosely bound hadronic molecules will be produced as frequently as normal hadrons,
while compact multiquark states
would be suppressed (by coal. penalty factor)
in heavy-ion collisions.

# A New Insight from CMS: Exotic/Normal Ratio

- **ExHIC index = Coalescence / Statistical Ratio**

$$R_h^{\rm CS} = \frac{\text{Yields in Coalescence}}{\text{Yields in Statistical model}}$$

- **CMS index = Exotic / Normal Ratio**

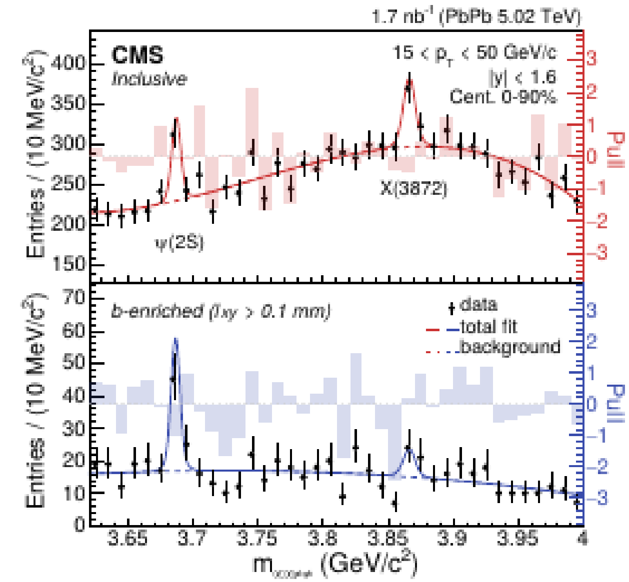
*Sirunyan+ [CMS], arXiv:2102.13048*

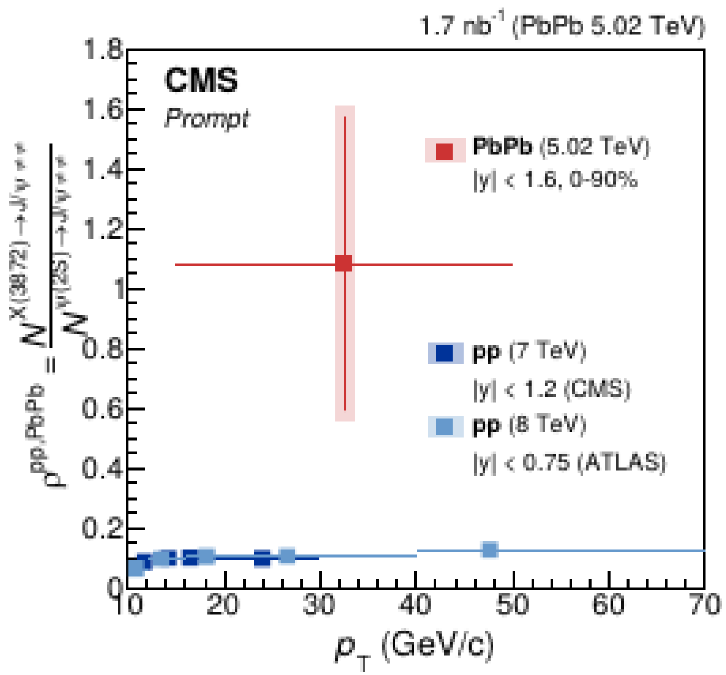
$$\rho_{\rm exo/nor} = \frac{N(\text{Exotic hadron candidate})}{N(\text{Normal hadron})}$$

- **X(3872) / ψ(2S) ratio in pp and PbPb collisions.**

$$\rho_{X/\psi}(\text{PbPb}) = 1.08 \pm 0.49(\text{stat.}) \pm 0.52(\text{syst.})$$
$$\rho_{X/\psi}(pp) \simeq 0.1$$

***ExHIC prediction is found to be (qualitatively) true !***



# Toward Quantitative Understanding...

- **With the mass difference from M(ψ(2S)=3686 MeV), $R^{CS}_X$ from data would be larger than ExHIC prediction (R~2). [ExHIC(2011) precition: R ~ 2.8]**

$$R^{\rm CS}_{X(3872)}({\rm HIC}) = 1.08 \times \exp((3872/3686)/T) \simeq 3.1$$

**→ Earlier hadronic coalescence with heavy quarks ?**

- **AMPT prediction** ***Zhang+(PRL 126, 012301 (2021))***

$$R^{\rm molecule}_{AA} \gg R^{\rm tetraquark}_{AA} \quad (R_{AA} = N_h(AA)/N_h(pp))$$

- **TAMU transport model** ***Wu+ (EPJA 57, 122 (2021))***

$$R^{\rm molecule}_{AA} \sim R^{\rm tetraquark}_{AA}/2$$

- **Suppression of ψ(2S) ($R^{\psi(2S)}_{AA}$ ~ 0.14) causes the apparent enhancement ?**

***Theorists have to work harder on production of exotic hadrons with heavy quarks toward a few 10 % level of accuarcy !***

# Another usage of hadron yields: Vorticity

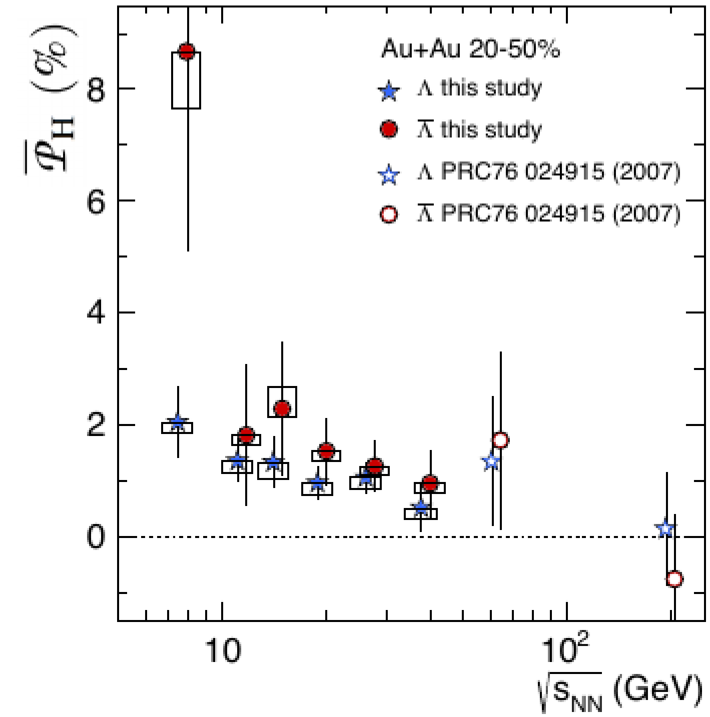
- **Global rotation in HIC was measured via Λ polarization.**
  *L. Adamczyk+(‘17)[STAR], Nature 548 (2017) 62-65.*
- **How about the “local” vorticity?**
  *H.Taya, A.Park, S.Cho, P.Gubler, K.Hattori, J.Hong, X.-G.Huang, S.H.Lee, A.Monnai, A.Ohnishi, M.Oka, D.-L.Yang [ExHIC-P Collaboration], (‘20).*
  - **Stat./Coal. model with vorticity**

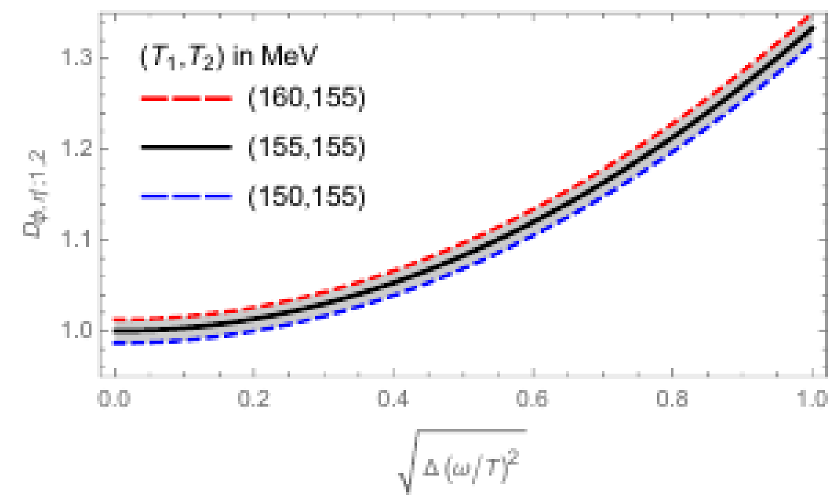
$$E(\omega) = E(\omega=0) - \boldsymbol{\omega}\cdot\boldsymbol{J}$$

$$\rightarrow \frac{N(\omega)}{N(\omega=0)} \simeq 1 + \frac{s(s+1)}{6}\frac{\langle\omega^2\rangle}{T^2}$$

**(Almost model independent.)**

  - **φ(1030)/η’(958) double ratio will clearly show the existence of local vorticity.**



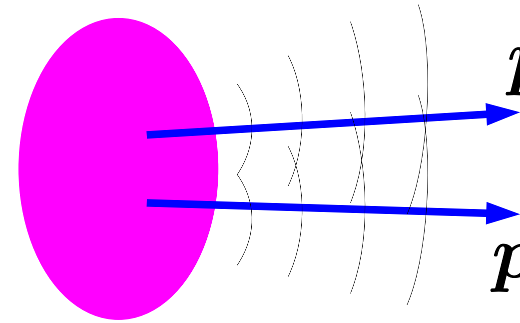
## Hadron-Hadron Interaction from High-Energy Nuclear Collisions

c.f. 神谷さんの講演

# Two particle momentum correlation function

- **Single particle emission function**

$$N_i(\boldsymbol{p}) = \int d^4x S_i(x, \boldsymbol{p})$$




- **Two particle momentum correlation function**
  - **Two particles are produced independently, and correlation is generated in the final state. (Koonin-Pratt formula)**

*Koonin('77), Pratt+('86), Lednicky+('82)*

**2 body w.f.**

$$C(\boldsymbol{q}) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1) N_2(\boldsymbol{p}_2)} \simeq \frac{\int d^4x d^4y S_1(x, \boldsymbol{p}_1) S_2(y, \boldsymbol{p}_2) |\Phi_{\boldsymbol{p}_1, \boldsymbol{p}_2}(x, y)|^2}{\int d^4x d^4y S_1(x, \boldsymbol{p}_1) S_2(x, \boldsymbol{p}_2)}$$

$$= \int d\boldsymbol{r} S(\boldsymbol{r}) |\varphi(\boldsymbol{r}; \boldsymbol{q})|^2 = 1 + \int d\boldsymbol{r} S(r) \left[ |\varphi_0(r; q)|^2 - |j_0(qr)|^2 \right]$$

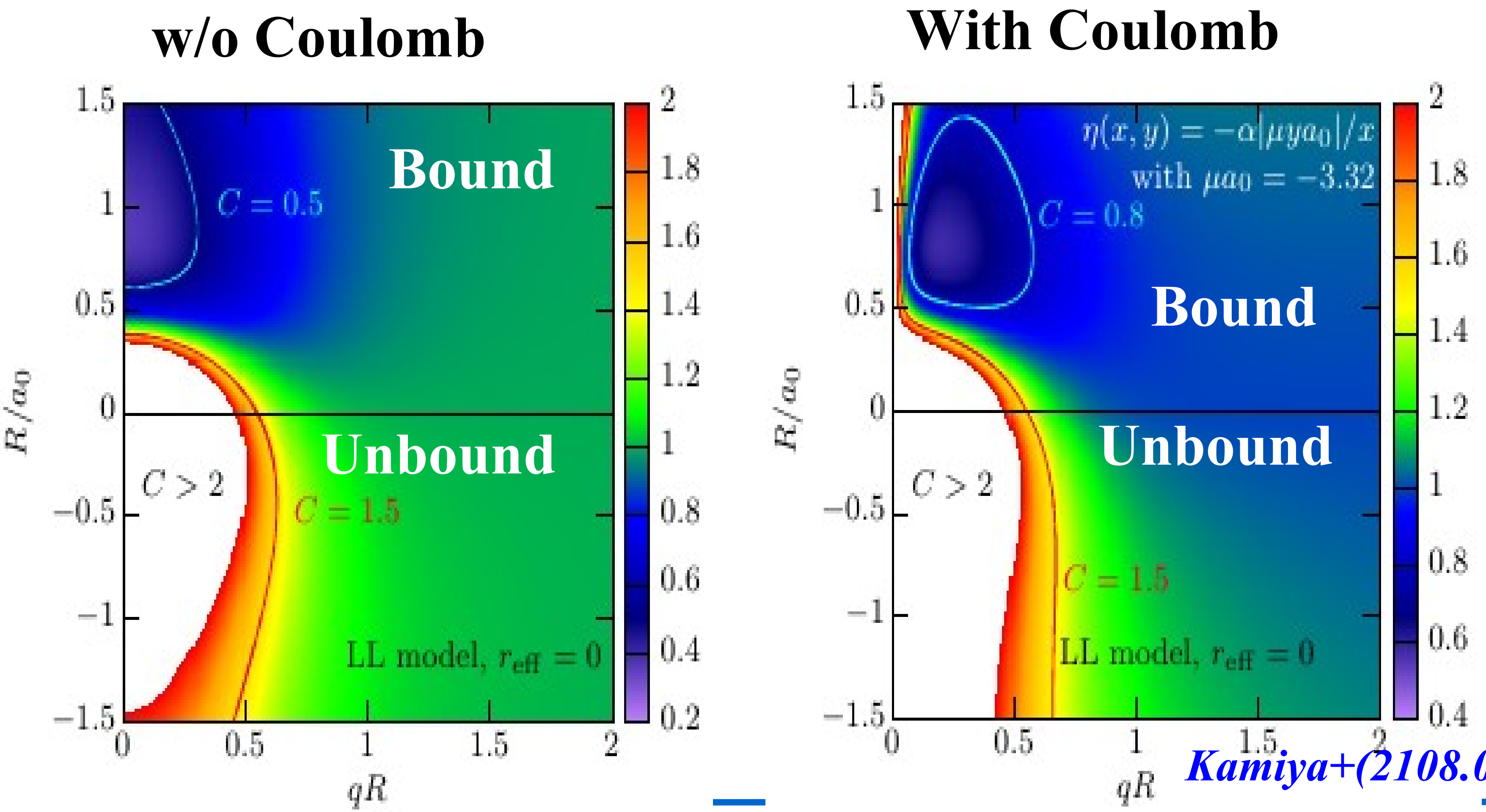
**CM var. int.** **Source fn.** **relative w.f.** **s-wave**

*Spherical static source, non-identical particles, s-wave, No Coulomb*

# Bird-eye view of correlation function

- 相関関数は Source size (R) と散乱長 ($a_0$) に敏感
  - 絶対値が大きい散乱長 → 小さな q で大きな C(q) ($R / |a_0| << 1$)
  - 束縛状態の有無によってサイズ依存性が変化

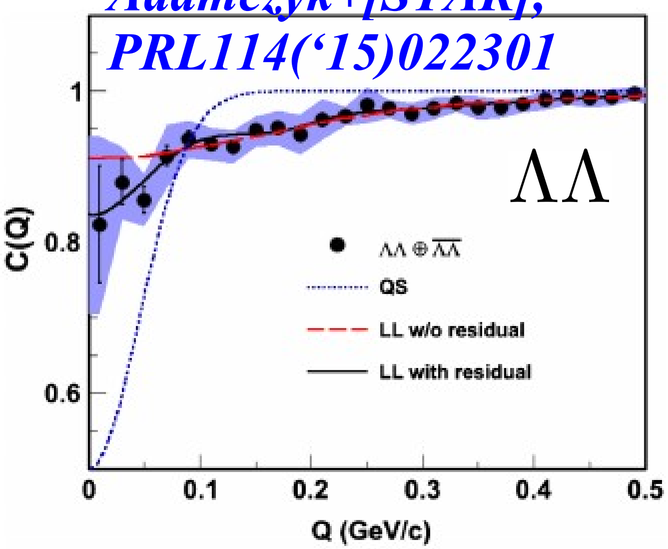
*Morita+('16, '20), Kamiya+('20), Kamiya+(2108.09644)*

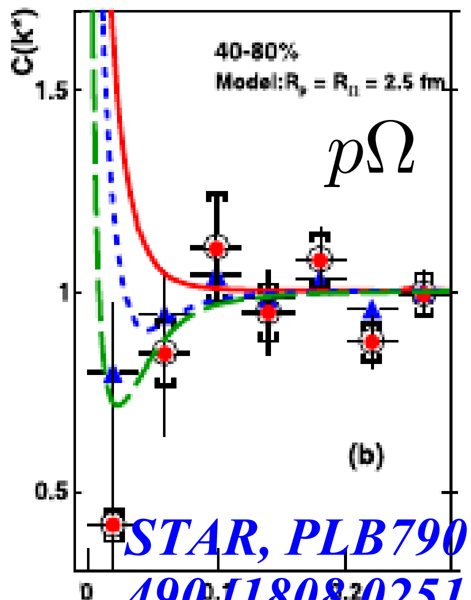



*Kamiya+(2108.09644)*

# Measured Correlation Functions (examples)

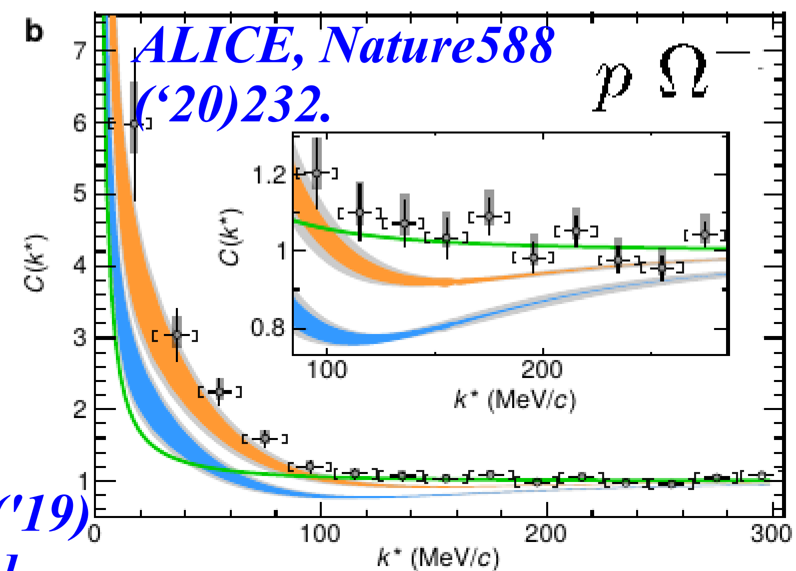
*Adamczyk+[STAR], PRL114('15)022301*

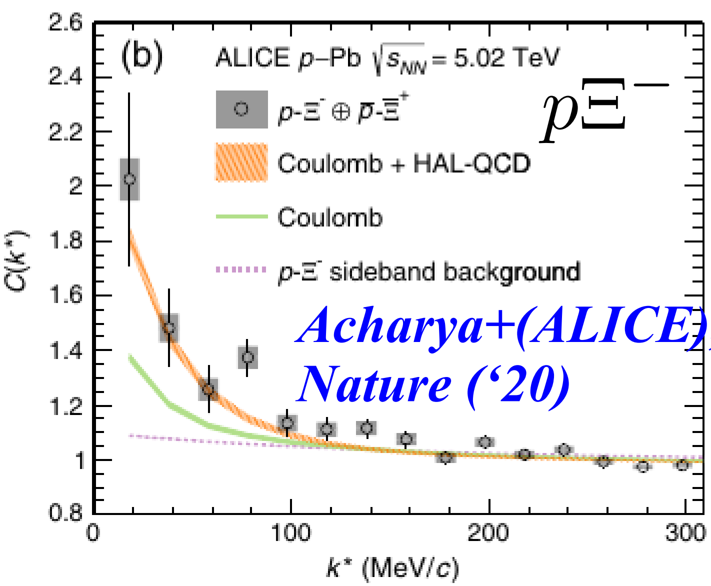


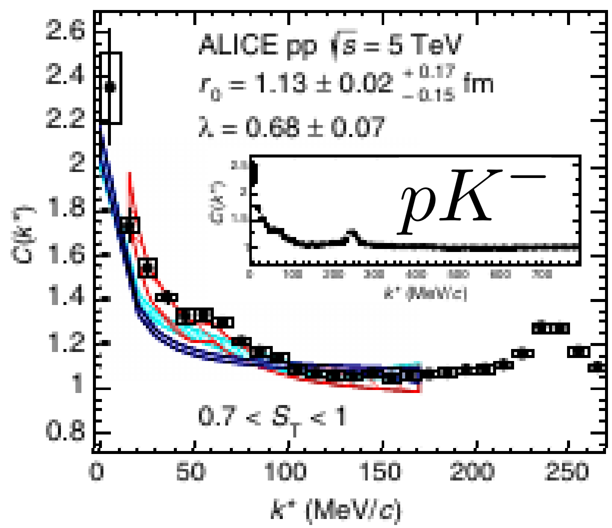

*STAR, PLB790 ('19) 490 [1808.02511].*

*ALICE, Nature588 ('20)232.*

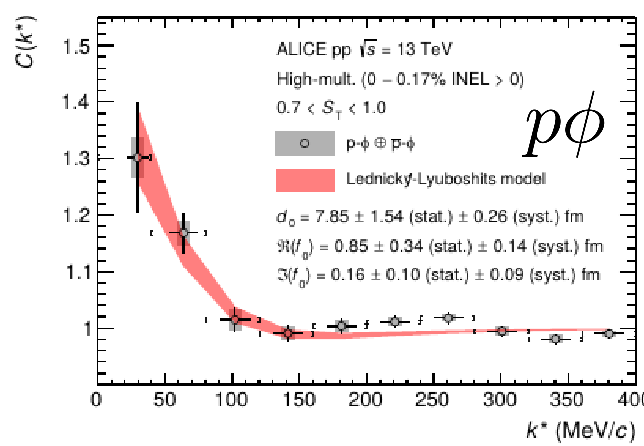



*Acharya+(ALICE), Nature ('20)*




*S. Acharya+[ALICE], PRL124('20)092301*



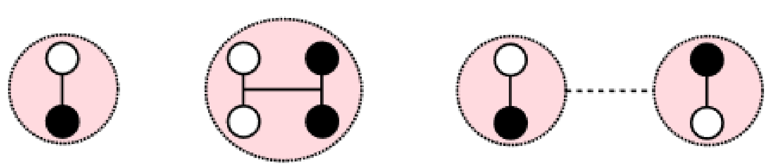

*ALICE, 2105.05578*

# From $a_0$ to Exotic Hadron Structure

- **Loosely bound hadronic molecule**
  **→ Eigenmomentum** $k \simeq i/a_0\,,\ a_0 \simeq R_h = 1/\sqrt{2\mu B}$
- **What happens when multiquark state mixes ?**
  **→ Deviation from weak binding relation**
  *Weinberg, Phys. Rev. 137, B672 (1965), Hyodo, Jido, Hosaka (1108.5524),*
  *Kamiya, Hyodo, (1607.01899), Kunigawa, Hyodo (2112.00249)*

$$a_0 = R_h \left[\frac{2X}{1+X}\right] + \mathcal{O}(R_{\rm typ}) \qquad \left[R_{\rm typ} = \max(m_\pi^{-1}, r_{\rm eff})\right]$$

- **Smaller scattering length than $R_h$ may signal the admixture of compact multiquark (and normal) component**
  *Bignamini+('09)*



**quark core + hadronic halo**

**(w.f. は直交しませんが ...)**

$$f = \frac{1}{k\cot\delta - ik} \simeq \frac{1}{-1/a_0 - ik}$$

**(Nucl. phys. convention)**

重イオン衝突(or 高エネルギーpp,pA 衝突)を用いてエキゾチックハドロンの構造(e.g. compositeness)を調べられる可能性あり。

ただしO($R_{typ}$)の不定性があるので、実験データから直接compositeness を決めるのは難しいのではないでしょうか？

Multiquark 成分とMolecule 成分を重ね合わせた具体的な模型を構成して(e.g. Yamaguchi,Takizawa,Takeuchi,Yasui,...)、様々な観測量を説明することが必要と思います。

# Summary

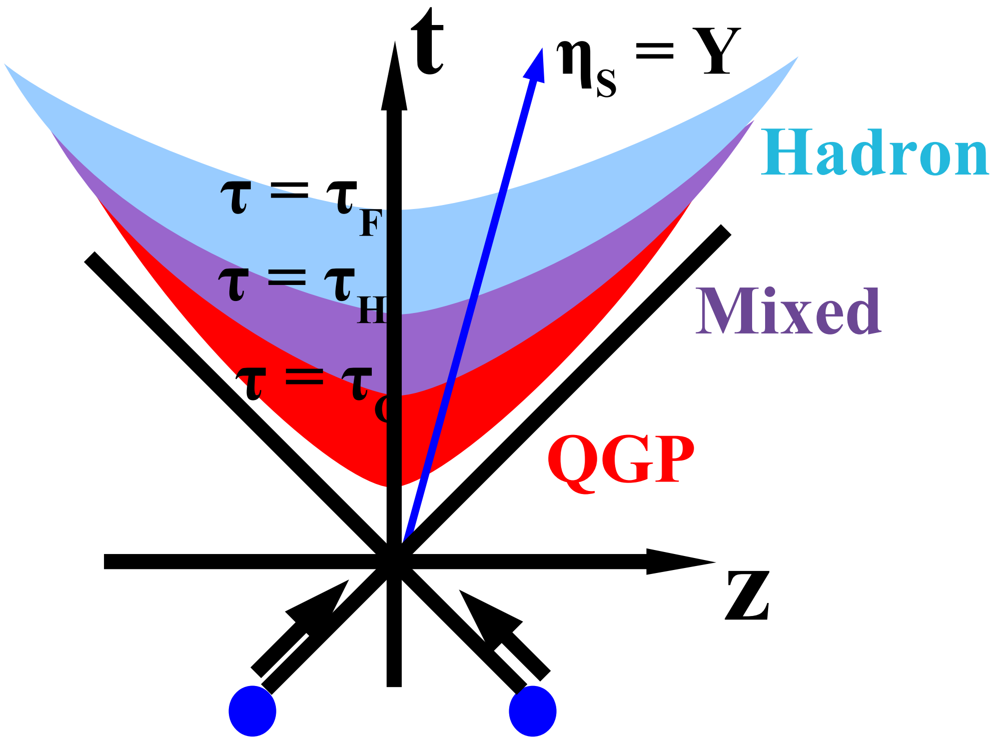
- Multiquark 状態・ハドロン分子などへの分類は、エキゾチック・ハドロン物理の大きな課題であり、重イオン衝突は、( いくつかの仮定のもとで ) この課題に大きく寄与できる可能性がある。
  - Coalescence によるハドロン生成では、重イオン衝突でゆるく束縛したハドロン分子が通常のハドロンと同程度生成される
  - 高エネルギー原子核衝突からの相関関数はソースサイズ (R) と散乱長 ($a_0$) に敏感
  - 束縛エネルギーと散乱長がともに分かれば、compositeness が推定できるかも。
- To do ( 理論 )
  - pp 、pA (e+e-?) 衝突でのエキゾチックハドロン生成率
  - Flavor hadron interaction の散乱長を系統的に予言しておくこと
  - hadron-deuteron 相関関数、3 体相関関数、…

**Thank you for attention !**

# Schematic picture of HIC

- HIC picture based on the (approximate) first order phase transition
  - $\tau = \tau_C$ , $T=T_C$ , $V=V_C$ → QGP start to hadronize (quark coal.)
  - $\tau = \tau_H$ , $T=T_H=T_C$ , $V=V_H$ → Hadronization is over (stat. model)
  - $\tau = \tau_F$ , $T=T_F$ , $V=V_F$ → Hadronic Freeze-out (hadron coal.)

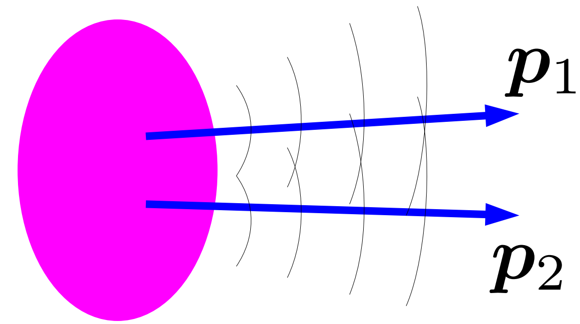
| | RHIC | LHC |
|---|---|---|
| $N_u = N_d$ | 245 | 662 |
| $N_s = N_{\bar{s}}$ | 150 | 405 |
| $N_c = N_{\bar{c}}$ | 3 | 20 |
| $N_b = N_{\bar{b}}$ | 0.02 | 0.8 |
| $V_C$ | 1000 fm$^3$ | 2700 fm$^3$ |
| $T_C = T_H$ | 175 MeV | 175 MeV |
| $V_H$ | 1908 fm$^3$ | 5152 fm$^3$ |
| $\mu_B$ | 20 MeV | 20 MeV |
| $\mu_s$ | 10 MeV | 10 MeV |
| $V_F$ | 11322 fm$^3$ | 30569 fm$^3$ |
| $T_F$ | 125 MeV | 125 MeV |



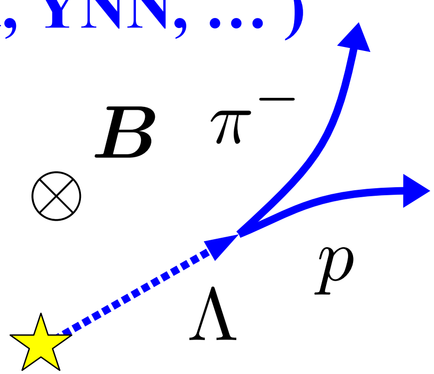

*L.W.Chen, V.Greco, C.M.Ko, S.H.Lee, W.Liu, PLB 601('04)34.*

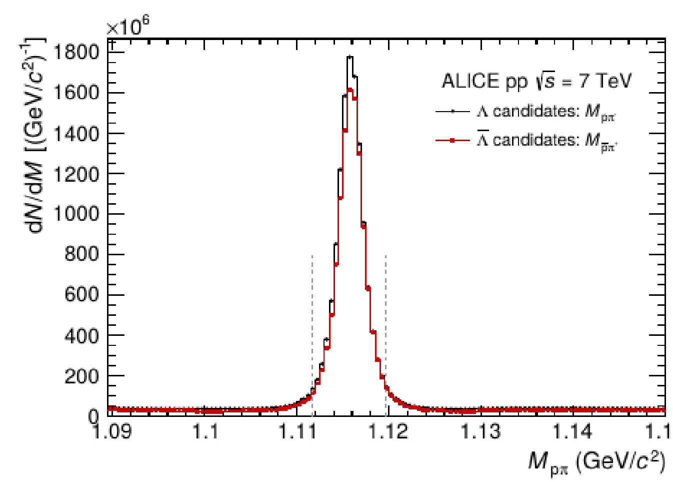
# How can we access flavored hh interactions ?

- **Experimental approaches**
  - **hh scattering (NN, YN, πN, KN)**
  - **Hadronic nuclei (normal nuclei, hypernuclei, kaonic nuclei) and atom ($\pi^-$, $K^-$, $\Sigma^-$, $\Xi^-$, …)**
  - **Femtoscopy**
- **Femtoscopic study of hh interactions**
  - **Correlation function contains information of hh interactions.**
  - **Koonin-Pratt formula =Valid when the source is chaotic**
  - **Applicable to various hh pairs (NN, YN, KN, DN, YY, Yd, YNN, … )**
  - **Weakly decaying particles → Good pair purity**
  - **Future measurements: Charmed hadron, hNN, ...**




$$C(\boldsymbol{q}) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)} = \frac{N_{12}^{\text{same}}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_{12}^{\text{mixed}}(\boldsymbol{p}_1, \boldsymbol{p}_2)} = \int d\boldsymbol{r} S(\boldsymbol{r}) |\varphi(\boldsymbol{r}; \boldsymbol{q})|^2$$



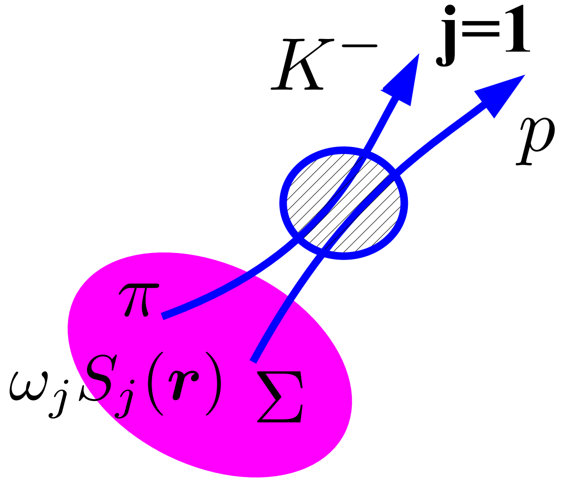




ALICE [1805.12455]

# Correlation function with coupled-channel effects

- **KPLLL formula = CC Schrodinger eq. under $\Psi^{(-)}$ boundary cond. + channel source**

  *Koonin('77), Pratt+('86), Lednicky-Lyuboshits-Lyuboshits ('98), Heidenbauer ('19), Kamiya, Hyodo, Morita, AO, Weise ('20).*




$$\Psi^{(-)}(\boldsymbol{q};\boldsymbol{r}) = [\phi(\boldsymbol{q};\boldsymbol{r}) - \phi_0(q;r)]\,\delta_{1j} + \psi^{(-)}(q;r)$$

$$\psi_j^{(-)}(q;r) \to \frac{1}{2iq_j}\left[\frac{u_j^{(+)}(q_j r)}{r}\delta_{1j} - A_j(q)\frac{u_j^{(-)}(q_j r)}{r}\right]$$

$$C(q) = \int d\boldsymbol{r} S_1(r)\left[|\phi(\boldsymbol{q};\boldsymbol{r})|^2 - |\phi_0(q;r)|^2\right] + \sum_j \int d\boldsymbol{r}\,\omega_j S_j(r)|\psi_j^{(-)}(q;r)|^2$$

- **No Coulomb**

$$\phi(\boldsymbol{q};\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}},\ \phi_0(q;r) = j_0(qr),\ u_j^{(\pm)}(qr) = e^{\pm iqr},$$

$$A_j(q) = \sqrt{(\mu_j q_j)/(\mu_1 q_1)}S_{1j}^{\dagger}(q_1)\ \ (S_{ji} = i \to j \text{ S-matrix})$$

- **With Coulomb**

$$\phi(\boldsymbol{q};\boldsymbol{r}) = \text{Full Coulomb w.f.},\ \phi_0(q;r) = \text{s-wave Coulomb w.f.},$$

$$u_j^{(\pm)}(qr) = \pm e^{\mp i\sigma_j}\left[iF(qr) \pm G(qr)\right]\ (F, G = \text{regular (irregular) Coulomb fn.})$$

# Wave function around threshold (S-wave, attraction)

- **Low energy w.f. and phase shift**

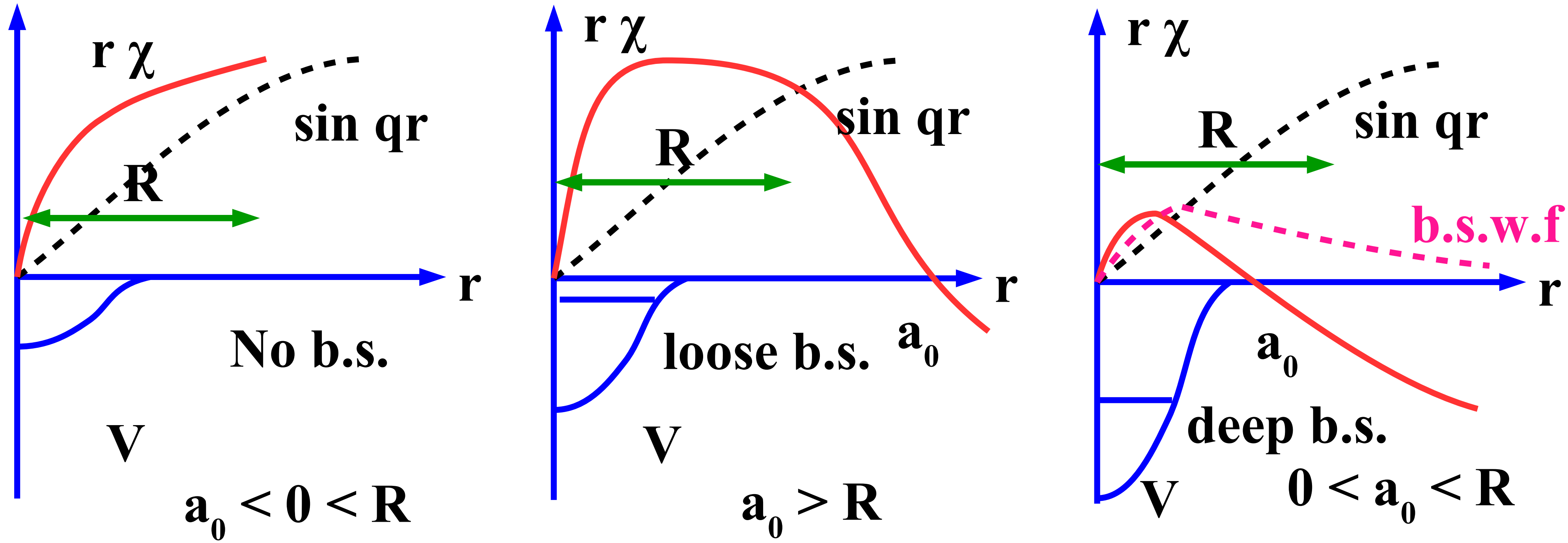
$$u(r) = qr\chi_q(r) \to \sin(qr + \delta(q)) \sim \sin(q(r - a_0))$$

$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} q^2 + \mathcal{O}(q^4)\ (\delta \sim -a_0 q)$$

*$a_0$ =scatt. length*
*$r_{eff}$=eff. range*

**Nucl. and Atomic Phys. convention**

- **Wave function grows rapidly at small r with attraction.**
- **With a bound state ($a_0$>0), a node appears around r=$a_0$**
  **→ Suppressed |w.f.|$^2$ on average**



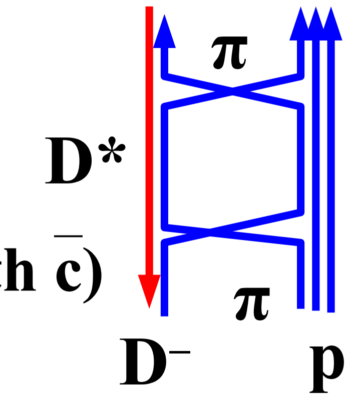
# *Charmed Hadron Interactions*

- **C(q) including a charmed hadron**
  - **Extremely important in recent hadron physics.**
  - **$D^-(\bar{c}d)$-p(uud) correlation**
    - **Probes $\Theta_c(\bar{c}$-ud-ud) state (replace $\bar{s}$ in $\Theta(\bar{s}$-ud-ud) with $\bar{c}$)**

      *D. O. Riska, N. N. Scoccola, PLB299('93)338 (pred.);*
      *A. Aktaset+ [H1], PLB588('04)17 (positive);*
      *J. M. Linket+ [FOCUS], PLB622('05)229 (negative).*
    - **Attraction from two pion exchange**

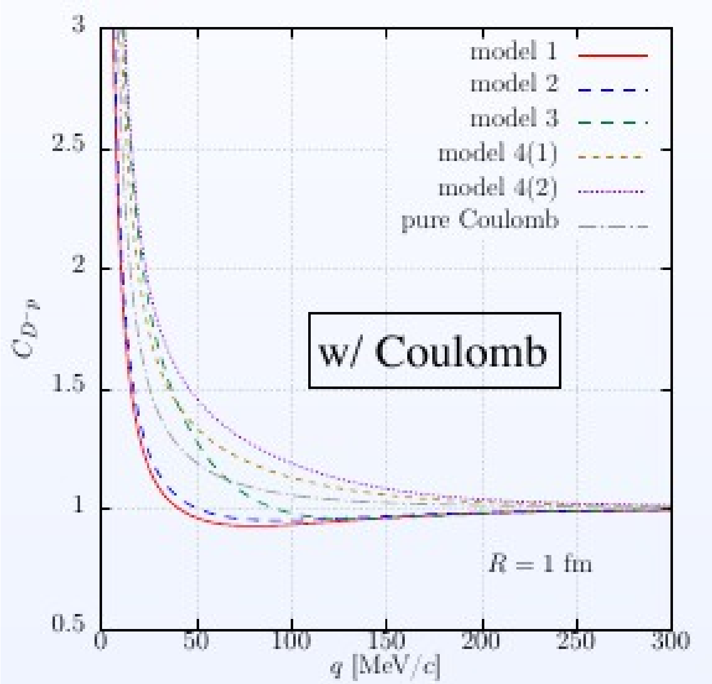
      *S. Yasui, K. Sudoh, PRD80('09)034008.*
    - **Easy to calculate the potential in LQCD.**

      *Y. Ikeda et al. (private communication)*




- **$D^-(\bar{c}d)$-p(uud) CFs from proposed potentials**

  *Hofmann, Lutz ('05) (repulsive);*
  *Haidenbauer+('07) (repulsive);*
  *Yamaguchi+('11) (att., w/ bs); Fontoura+('13) (repulsive)*

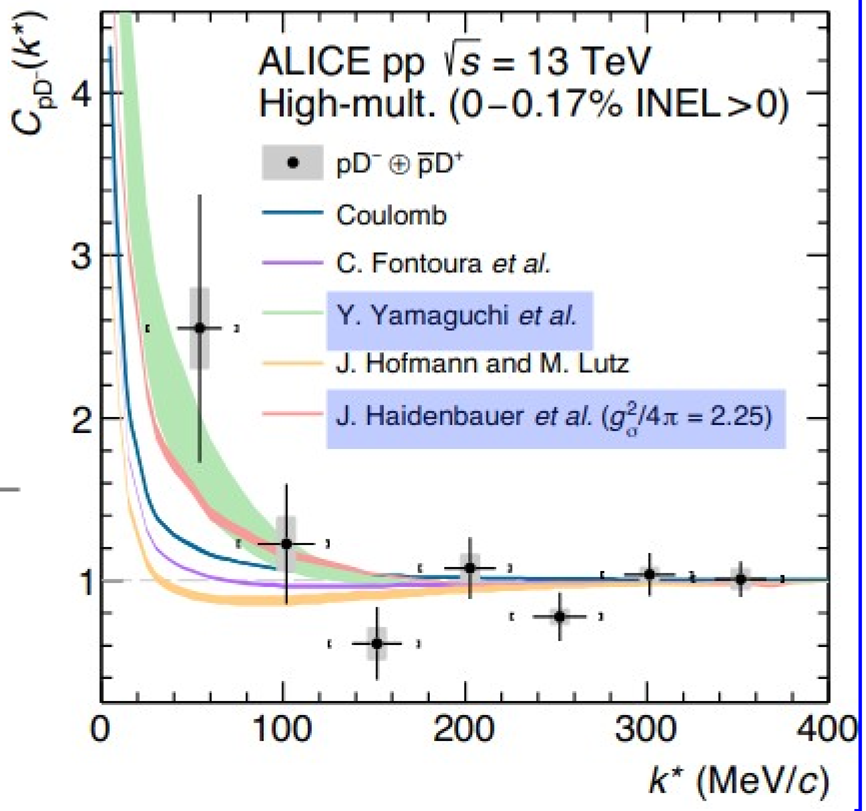



*Kamiya, Hyodo, AO (in prog.)*

***Data will discriminate these potentials !***

# *Marginal case: $D^-p$ correlation function*

- **"First study of the two-body scattering involving charm hadrons"**
  *Acharya+[ALICE] (2201.05352)*
  - **$D^-p$ corr. func. is measured.**
  - **Enhanced CF from Coulomb.**
  - **One range gaussian potential with strength fitted to the I=0 scattering length of the model → attractive potentials are favored**




| Model | $f_0$ (I = 0) | $f_0$ (I = 1) | $n_\sigma$ |
|---|---|---|---|
| Coulomb | | | (1.1–1.5) |
| Haidenbauer et al. [21] | | | |
| $- g_\sigma^2/4\pi = 1$ | 0.14 | −0.28 | (1.2–1.5) |
| $- g_\sigma^2/4\pi = 2.25$ | 0.67 | 0.04 | (0.8–1.3) |
| Hofmann and Lutz [22] | −0.16 | −0.26 | (1.3–1.6) |
| Yamaguchi et al. [24] | −4.38 | −0.07 | (0.6–1.1) |
| Fontoura et al. [23] | 0.16 | −0.25 | (1.1–1.5) |


*[21] Haidenbauer+(0704.3668) (weakly / mildly attractive (I=0))*
*[22] Hofmann, Lutz (hep-ph/0507071) (repulsive (I=0))*
*[23] Fontoura+(1208.4058) (weakly attractive (I=0))*
*[24] Yamaguchi, Ohkoda, Yasui, Hosaka (1105.0734) (att., w/ bound state (I=0))*

# To be bound or not to be bound

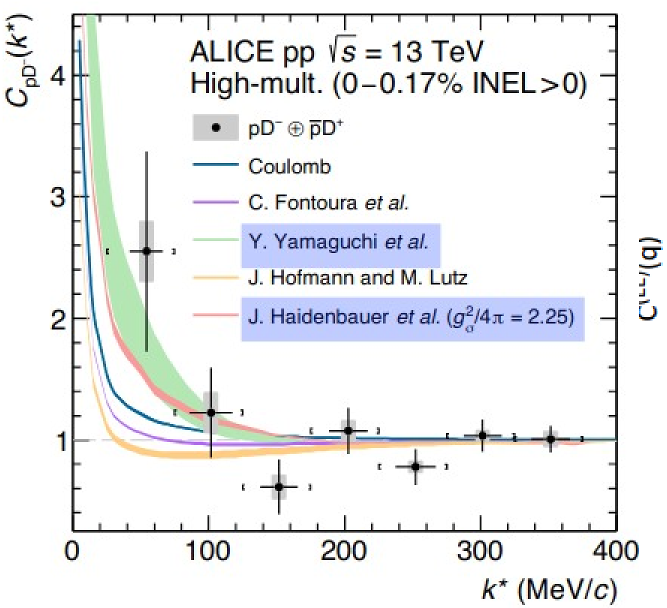
- **When there is a bound state, CF shows interesting dependence on the source size and relative momentum.**
- **D⁻ p corr. func. shows the behavior with a bound state, and the best fit parameter set (R, $a_0$) is in the bound region. (If bound, it is the first weakly decaying pentaquark state.)**

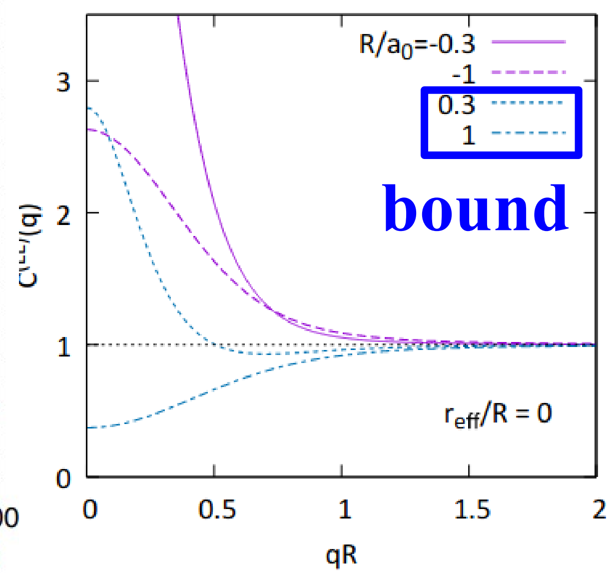
$$k\cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} k^2 + \mathcal{O}(k^3)$$

(Nuclear and atomic phys. convention.)

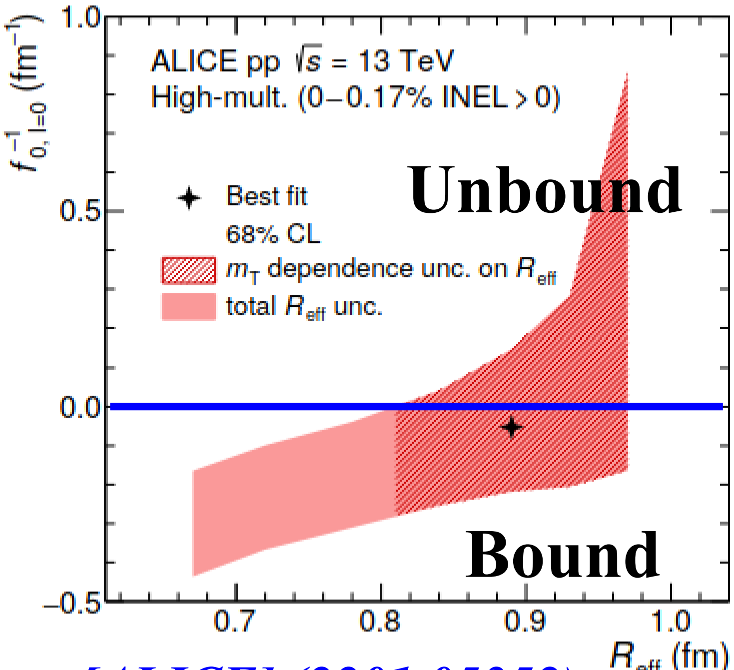
$$k\cot\delta = +\frac{1}{f_0} + \frac{1}{2} r_{\rm eff} k^2 + \mathcal{O}(k^3)$$

(High-E. phys. convention.)




Morita+(1908.05414)

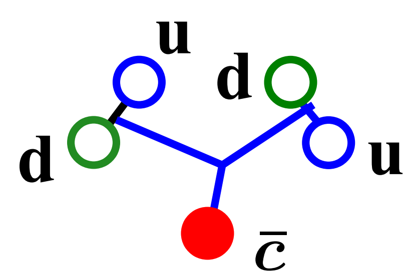


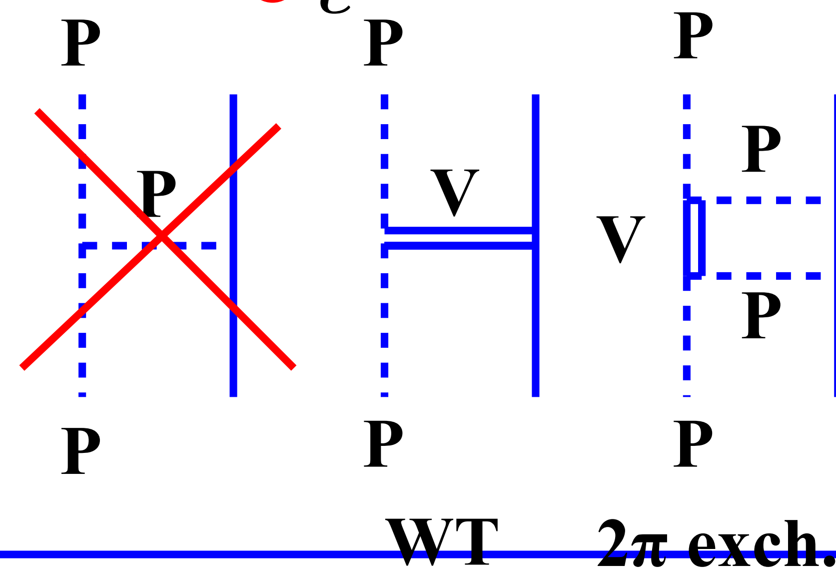

[ALICE] (2201.05352)

# Is it interesting ? Yes !

- **Chiral symmetry → PS-B int. is dominated by vector exch.**
  - **Weinberg-Tomozawa interaction (vector coupling) appears in the leading order in the chiral quark model.**
  - **WT int. is generally repulsive in exotic channels.**
- **With heavy-quarks, PS and V meson masses becomes closer (heavy-quark sym.), then (two) PS meson exch. can be important. (higer-order in chiral perturbation)**
  *Yasui, Sudoh (0906.1452)*
- **Charmed pentaquark ($\Theta_c$) may exist.**

  $D^-(\bar{c}d) - p(uud) \to \bar{c} - ud - ud$ (pentaquark)
- **Attraction btw PS-B suggests importance of higher-order term(s) in chiral perturbation theory.**

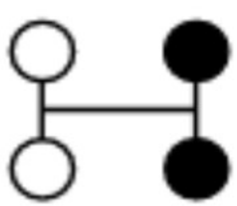


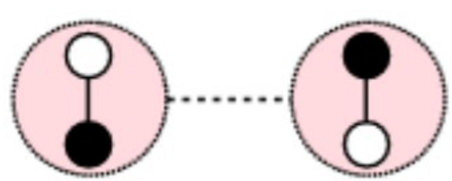

***Femtoscopy may cause change of paradigm in hadron physics !***

**Recently observed / studied correlation functions, Homeworks, and perspectives**

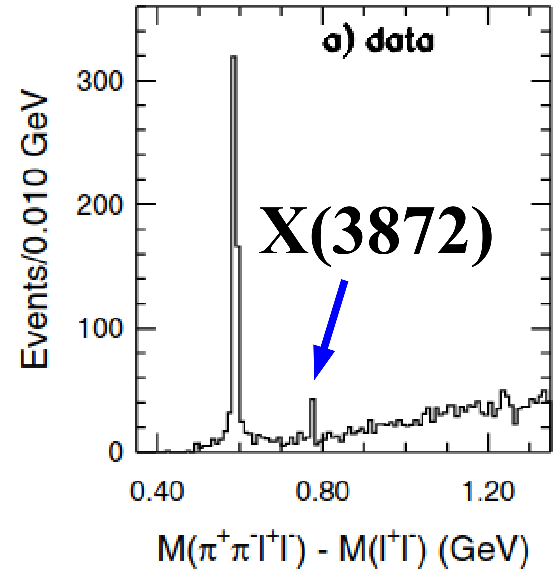
- **Main play ground of exotic hadron physics**
  - X(3872) *Belle ('03)* $c\bar{c}q\bar{q}$
  - Many X,Y,Z states
    *Belle, CDF, BaBar, LHCb, CMS, BESIII, ...* (Beijing Spectrometer)
  - Charmed pentaquark Pc *LHCb ('15, '19)*
  - Doubly charmed tetraquark state Tcc
    *LHCb ('21)* $cc\bar{q}\bar{q}$
- **Structure of exotic hadrons**
  - Compact multiquark states
    → "good" [ud] diquark gains energy
  - Hadronic molecules
    → Many exotic states around thresholds
  - Their mixture...



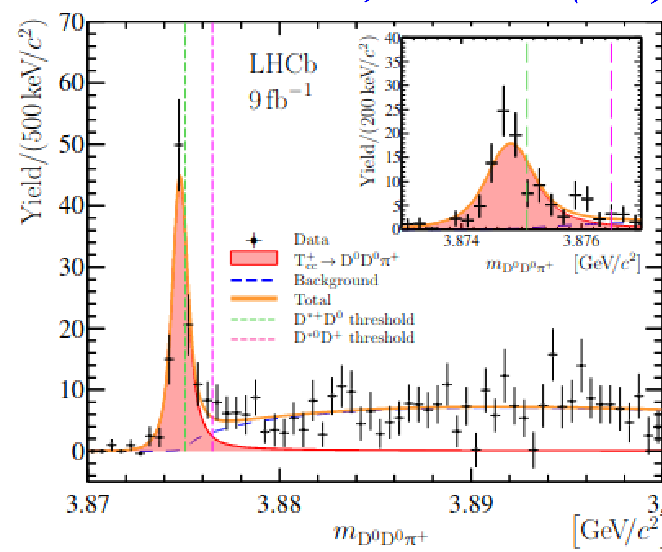
Tetraquarks



Hadronic Molecules



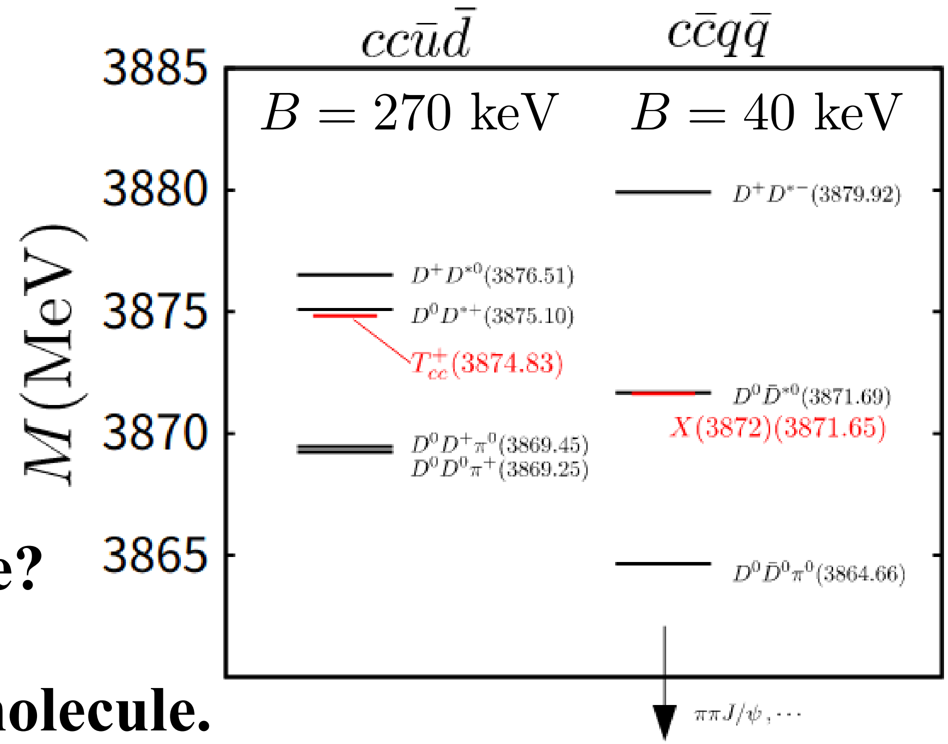

*S.K.Choi+[Belle], PRL91, 262001 ('03)*




*R. Aaji+ [LHCb], 2109.01038, 2109.01056*

# Compact Tetraquarks or Hadronic Molecules

- **Tcc = Compact Tetraquark ?**
  **Good $[\bar{u}\bar{d}]$ diquark gains energy**
  *S. Zouzou+('86), ZPC30,457.*




- **X(3872)**
  - **$c\bar{c}$ component ? production cross section** *Bignamini+ (0906.0882)*
  - **Large yield in Pb+Pb → Molecule?**
    *Sirunyan+ [CMS] (2102.13048)*
    **c.f. Δr/Δp is similar in HIC and molecule.**
    *ExHIC ('11,'11,'17)*

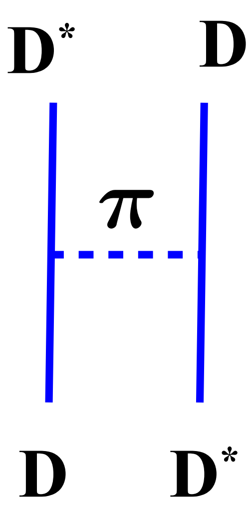
- **Hadronic Molecule Conditions**
  - **Appears around the threshold → OK**
  - **Have large size $R \simeq 1/\sqrt{2\mu B}$ → Yield**
  - **Described by the *hh* interaction**

***How can we access hh int. with charm ? → Femtoscopy***

# Femtoscopic study of charmed hadron int.

- $DD^*$ and $D\bar{D}^*$ correlation functions. *Kamiya, Hyodo, AO (2203.13814)*
  - Related with Tcc and X(3872)
  - *ALICE run3 can measure the correlation functions.*
- Model interaction
  - Range = one pion exchange *Yasui, Sudoh (0906.1452)*
  - Strength is fitted to the pole mass.
  - Isospin dep.
    - I=0: One range gaussian, strength fitted to the mass
    - I=1: ignored

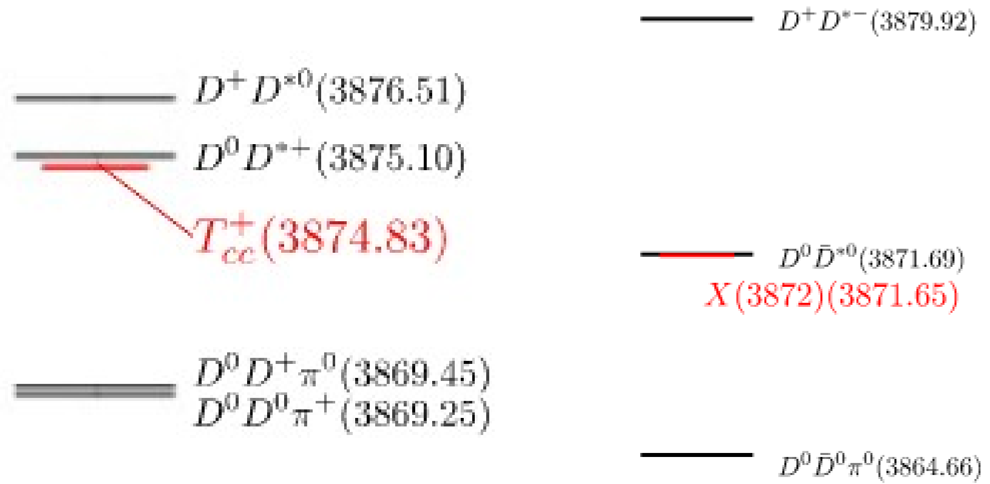
$$\{D^0\bar{D}^{*0}\} = (D^0\bar{D}^{*0} + \bar{D}^0 D^{*0})/\sqrt{2}\ (C=+1)$$
$$\{D^+D^{*-}\} = (D^+D^{*-} + D^-D^{*+})/\sqrt{2}\ (C=+1)$$

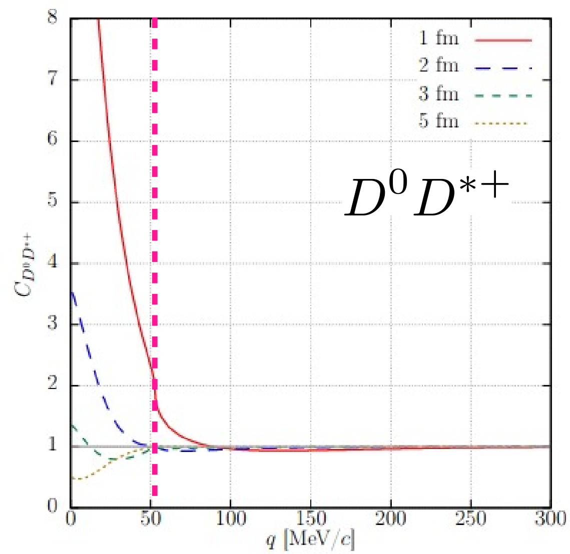


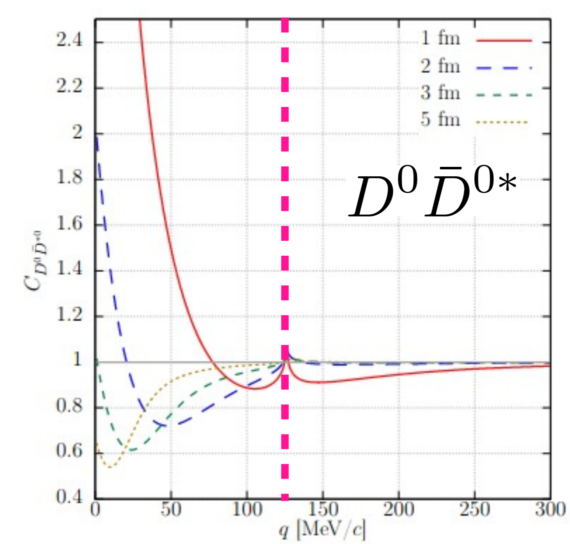

| $DD^*$ | $V_0$ [MeV] | $a_0^{D^0D^{*+}}$ [fm] | $a_0^{D^+D^{*0}}$ [fm] |
|---|---|---|---|
| | $-36.569 - i1.243$ | $-7.16 + i1.85$ | $-1.75 + i1.82$ |
| $\{D\bar{D}^*\}$ | $V_0$ [MeV] | $a_0^{\{D^0\bar{D}^{*0}\}}$ [fm] | $a_0^{\{D^+D^{*-}\}}$ [fm] |
| | $-43.265 - i6.091$ | $-4.23 + i3.95$ | $-0.41 + i1.47$ |

# $D^0 D^{*+}$ and $D^+ \bar{D}^{*0}$ Correlation Functions

- **Features of C(q) with a bound state**
  - **Enhancement at small source, Dip at large source.**
  - **Modification of potential (Changing the range, V(I=1)=0 or ± V(I=0)/3) does not change C(q) significantly. (dominated by the pole)**
  - **Measurement in Run3 is awaited.**



# Tcc and X(3872) structure

- **Hadronic molecule structure is assumed**
  **→ Eigenmomentum** $k \simeq -i/a_0\,,\ a_0 \simeq R = 1/\sqrt{2\mu B}$
- **What happens when multiquark state mixes ?**
  **→ Deviation from weak binding relation (X=compositeness)**
  *Weinberg, Phys. Rev. 137, B672 (1965), Hyodo, Jido, Hosaka (1108.5524), Kunigawa, Hyodo (2112.00249)*

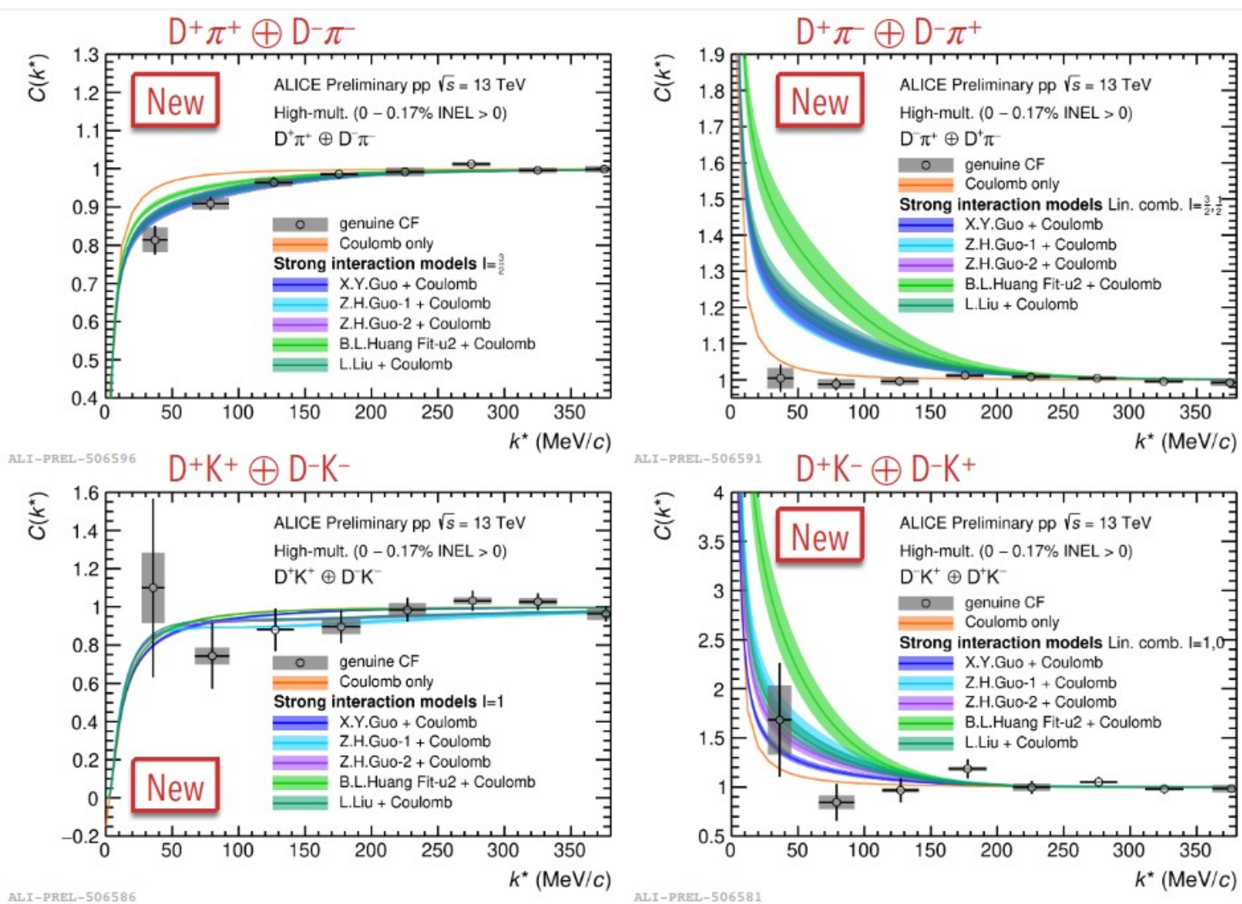
$$a_0 = R\left[\frac{2X}{1+X}\right] + \mathcal{O}(R_{\rm typ})$$

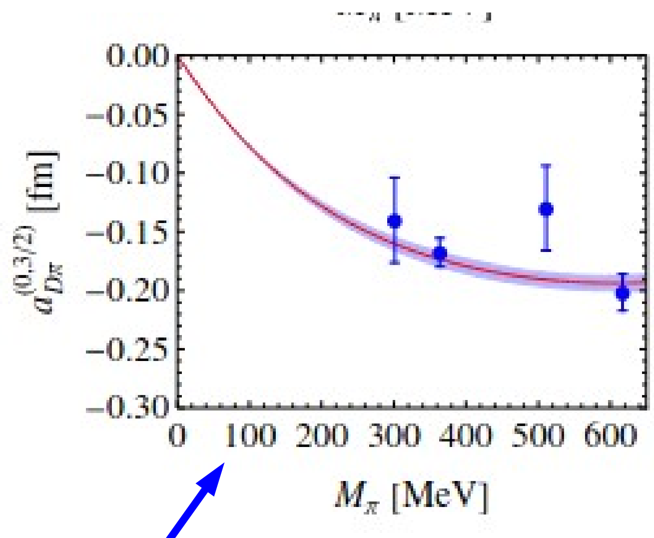
$$\left[R_{\rm typ} = \max(m_\pi^{-1}, r_{\rm eff}), R = 1/\sqrt{2\mu B}\right]$$

  - **Hadronic molecule assumption → X=1**
    **Pure multiquark state → X=0**
  - **Smaller scattering length in DD* may signal the *genuine* tetraquark nature of Tcc.**

# Homework to Hadron Physics (1)

- **Present chiral models do not explain $D\pi$ and $D\bar{K}$ correlation.**
  - **Overestimate C(D$^+\pi^-$) → Mystery ? Extrapolation to phys. mass ?**
    **Leading order = Weinberg-Tomozawa (vector exch., repulsive)**
    **Further repulsive interaction ?**
  - **Overestimate C(D$^+$K$^-$) → Further repulsion or bound state ?**



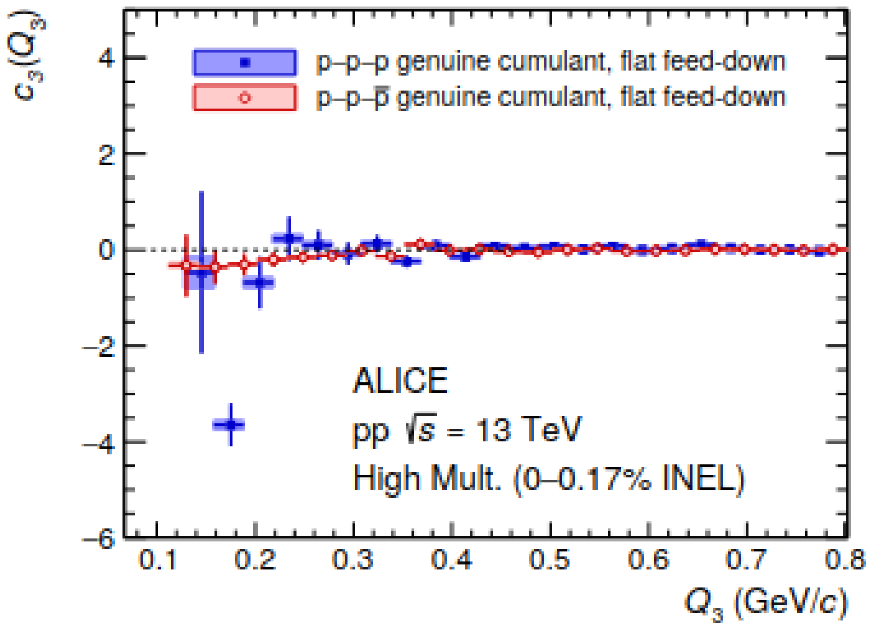

L. Liu et al, Phys. Rev. D87 (2013) 014508
X.-Y. Guo et al, Phys. Rev. D 98 (2018) 014510
B.-L. Huang et al, Phys. Rev. D 105 (2022) 036016
Z.-H. Guo et al Eur. Phys. J. C 79 (2019) 13

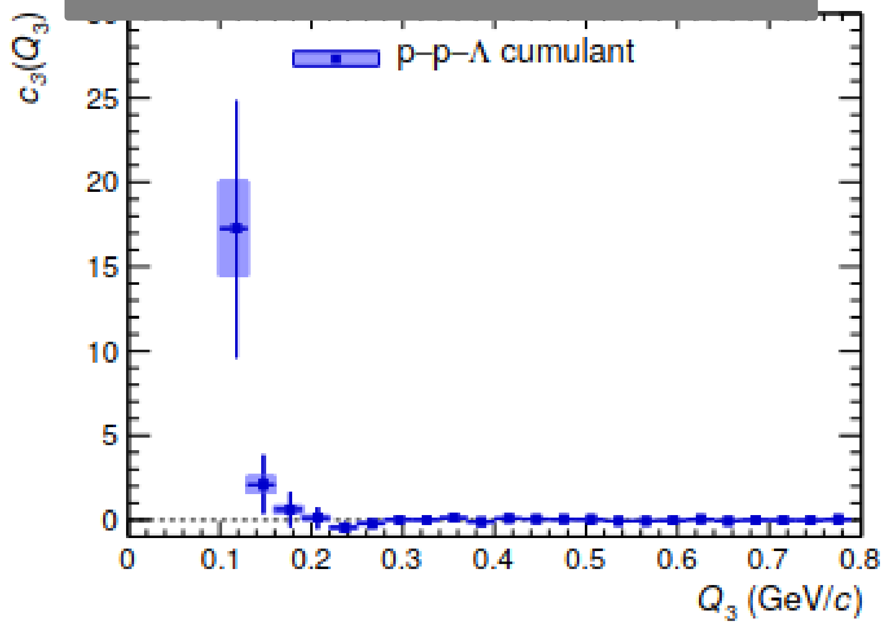
*Fabrizio Grosa@QM2022*

# Homework to Hadron (Nuclear) Physics (2)

- **Three-body correlation function (ppp, ppΛ)**
  - **Cumulant** $$c_3 = C_{123} - C_{12} - C_{23} - C_{31} + 2$$
  - **Can we extract three-baryon repulsion ?**
    **(important to solve the hyperon puzzle)**
    **→ One needs to solve continuum three-body w.f.**
    **with Coulmb potential.**

***Theoretical challenge***




*ALICE [2206.03344] (Raffaele Del Grande @QM2022)*

# Homework to Hadron (Nuclear) Physics (3)

- **Correlation function including vector mesons**
  - **Femtoscopy** *ALICE (PRL, 2105.05578)*

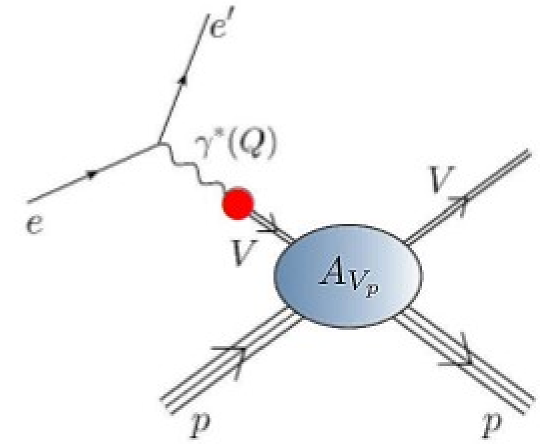
$$a_0(\phi p) = 0.85 + i0.16 \text{ fm}$$

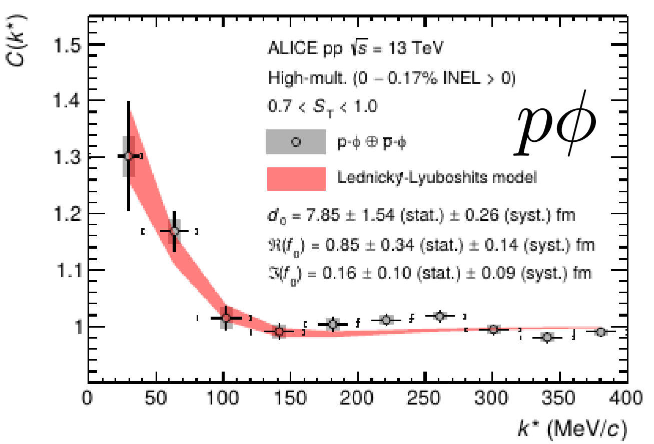
  - **Contradiction with the photo production ?**
    **scattering length is O(0.1 fm)**
    *E.g. Strakovsky, Pentchev, Titov (2001.08851)*

$$|a_0(\phi p)| = (0.063 \pm 0.010) \text{ fm}$$

  - **Smaller than lattice QCD result (J=3/2) ?**
    *Lyu, Doi, Hatsuda, Ikeda (2205.10544)*

$$a_0(\phi p, J = 3/2) = 1.43 \text{ fm}$$




*ALICE, 2105.05578*

***What's wrong ?***

# Toward dynamical source

- **Calculating HBT radius in dynamical models is not easy (HBT puzzle).**
  *M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014];*

  choices then tends to exceed the number of experimental constraints. In fact, all the model results that we review in the current subsection remain unsatisfactory with this respect: They either deviate significantly from femtoscopic data, or they reproduce these data at the price of missing other important experimental information. In particular, there is so far no dynamically consistent model that reproduces quantitatively both the systematic trends discussed in Section 4 and the corresponding single inclusive spectra. In this situation, the scope of this subsection is

- **But carefully constructed hydrodynamic model may answer.**
  *S. Pratt, PRL102('09)232301 [0811.3363].*

  Two particle correlation data from the BNL Relativistic Heavy Ion Collider have provided detailed femtoscopic information describing pion emission. In contrast with the success of hydrodynamics in reproducing other classes of observables, these data had avoided description with hydrodynamic-based approaches. This failure has inspired the term "HBT puzzle," where HBT refers to femtoscopic studies which were originally based on Hanbury Brown–Twiss interferometry. Here, the puzzle is shown to originate not from a single shortcoming of hydrodynamic models, but the combination of several effects: mainly prethermalized acceleration, using a stiffer equation of state, and adding viscosity.

- **How about afterburner effects ?**