Charmed hadron interactions and correlation functions

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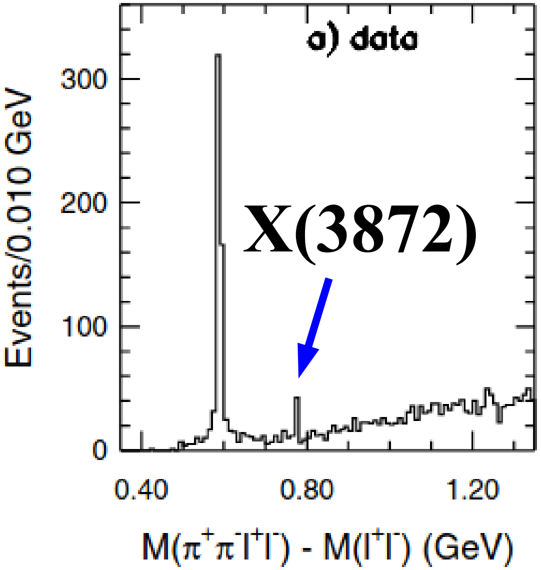
- **Introduction**
- **Charmed hadron interactions –** DD^* and $D\bar{D}^*$
- **Summary**

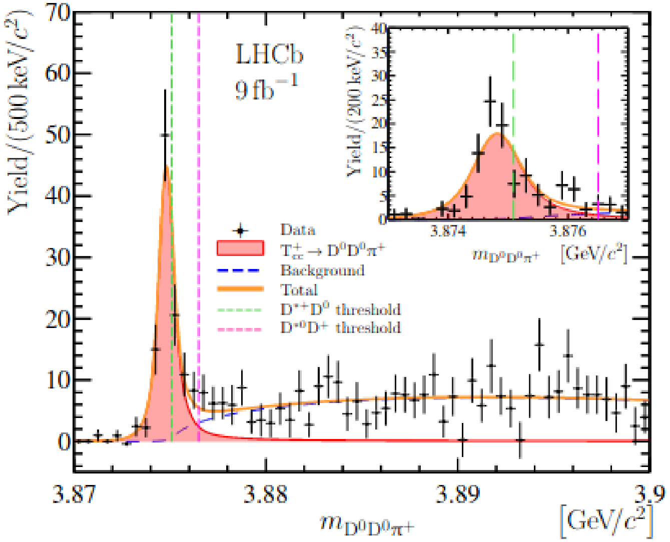
Y. Kamiya, T. Hyodo, A. Ohnishi, arXiv:2203.13814 [hep-ph];

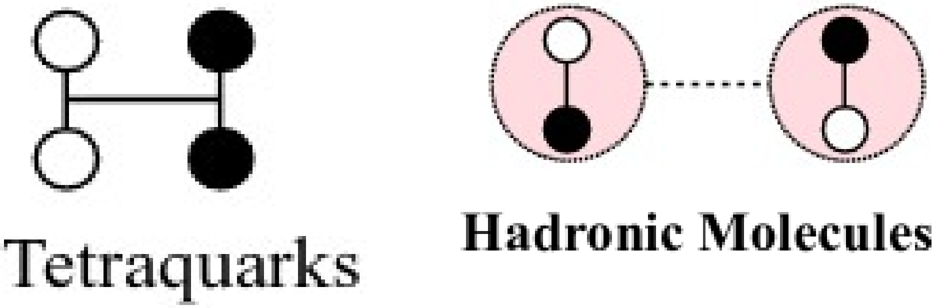
Exotic Hadrons including $c\bar{c}$ / cc / $\bar{c}\bar{c}$

- Main play ground of exotic hadron physics
  - X(3872) *Belle ('03)*
  - Many X,Y,Z states
    Belle, CDF, BaBar, LHCb, CMS, BESIII, ...
  - Charmed pentaquark Pc *LHCb ('15, '19)*
  - Doubly charmed tetraquark state Tcc
    LHCb ('21)
- Structure of exotic hadrons
  - Compact multiquark states
    → "good" [ud] diquark gains energy
  - Hadronic molecules
    → Many exotic states around thresholds
  - Their mixture...



S.K.Choi+[Belle], PRL91, 262001 ('03)

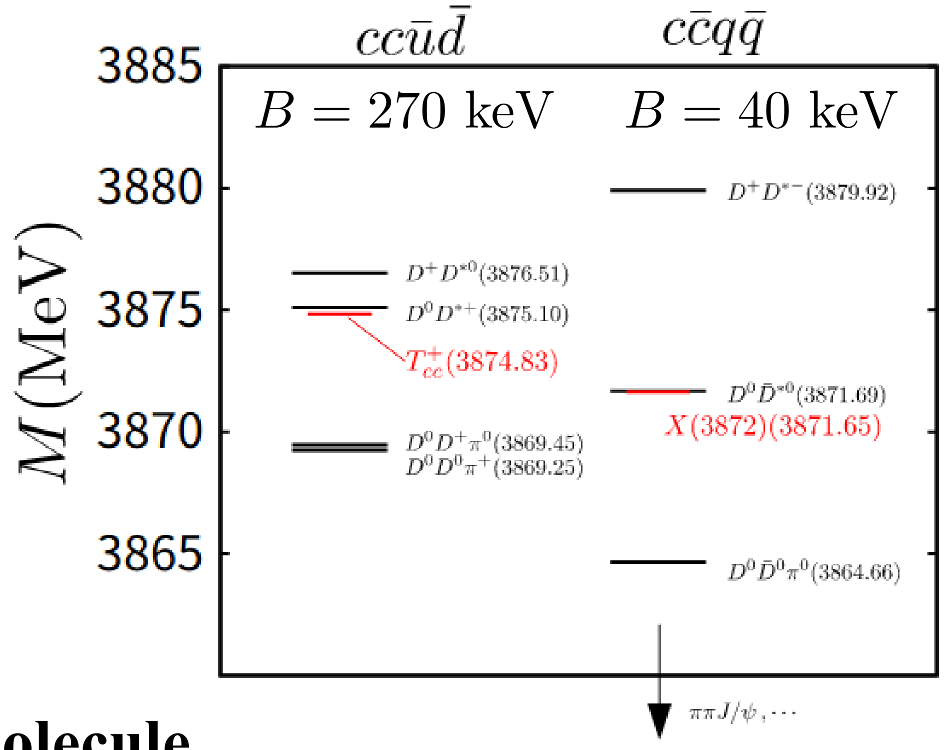




R. Aaji+ [LHCb], 2109.01038, 2109.01056

Hadronic Molecules

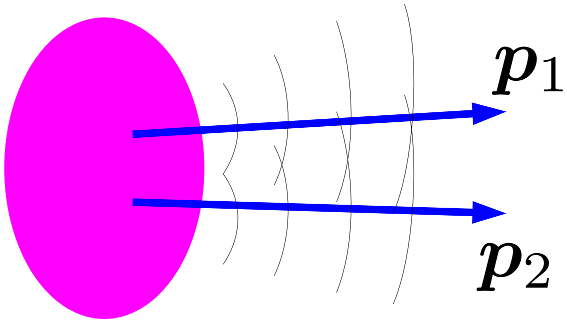
- **Deuteron (np), $\Lambda(1405)(\bar{K}N)$, ….**
- **Tcc = Compact Tetraquark ?**
  S. Zouzou+(‘86), ZPC30,457.
- **X(3872)**
  - **Molecule ? (Many works)**
  - **$c\bar{c}$ component ? production cross section** *Bignamini+ (0906.0882)*
  - **Molecule ! Large yield in Pb+Pb**
    Sirunyan+ [CMS] (2102.13048)
    c.f. Δr/Δp is similar in HIC and molecule.
    ExHIC (‘11,’11,’17)



- **Hadronic Molecule Conditions**
  - **Appears around the threshold (Threshold (Ikeda) rule) → Tcc & X(3872)**
  - **Have large size** $R \simeq 1/\sqrt{2\mu B}$
  - **Described by the *hh* interaction**

How can we access hh int. with charm ? → Femtoscopy

Femtoscopic study of hadron-hadron interaction



- **How can we study interactions between short-lived particles ? → Femtoscopy !**
- **Correlation function (CF)**
  - **Koonin-Pratt formula**

    Koonin('77), Pratt+('86), Lednicky+('82)

$$C(\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r}\, \overset{\text{source fn.}}{\underline{S_{12}(\boldsymbol{r})}}\, \overset{\text{relative w.f.}}{\underline{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})}}|^2$$

- **Source size from quantum stat. + CF (Femtoscopy)**

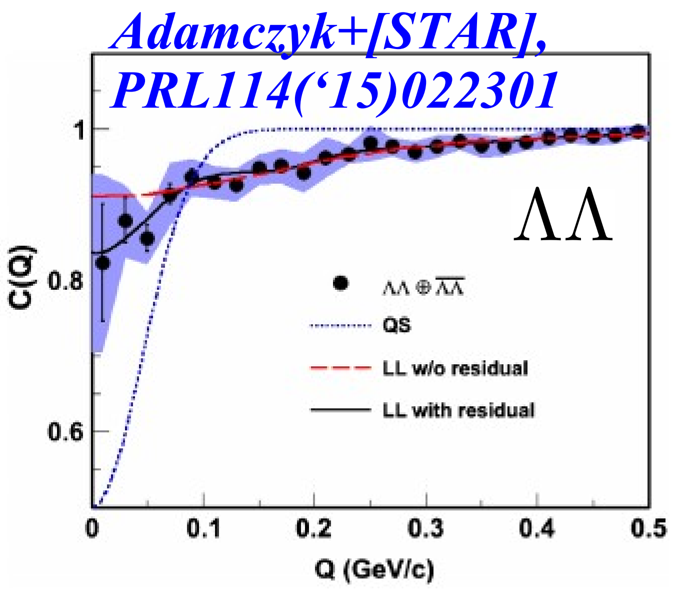
  Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)
- **Hadron-hadron interaction from source size + CF**
  - **CF of non-identical pair from Gaussian source**

    R. Lednicky, V. L. Lyuboshits ('82); K. Morita, T. Furumoto, AO ('15)

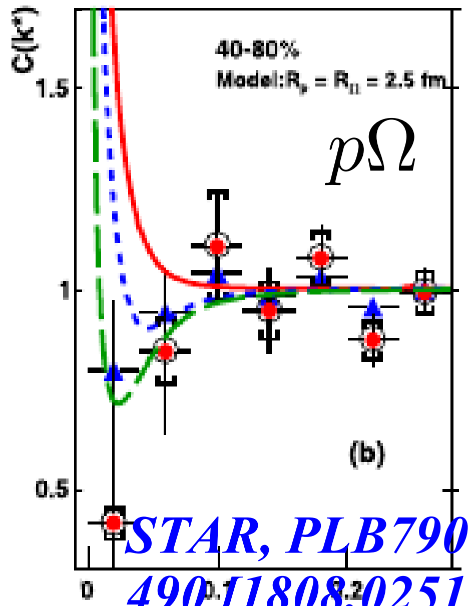
$$C(\boldsymbol{q}) = 1 + \int d\boldsymbol{r}\, S(r) \left\{ |\varphi_0(r)|^2 - |j_0(qr)|^2 \right\} \quad (\varphi_0 = \text{s-wave w.f.})$$

CF shows how much $|\varphi|^2$ is enhanced → V_{hh} effects !

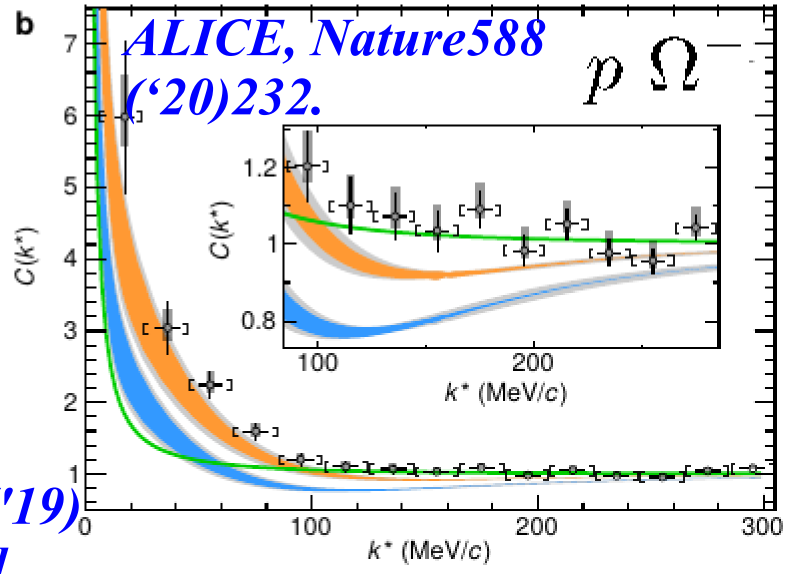
Measured Correlation Functions (examples)



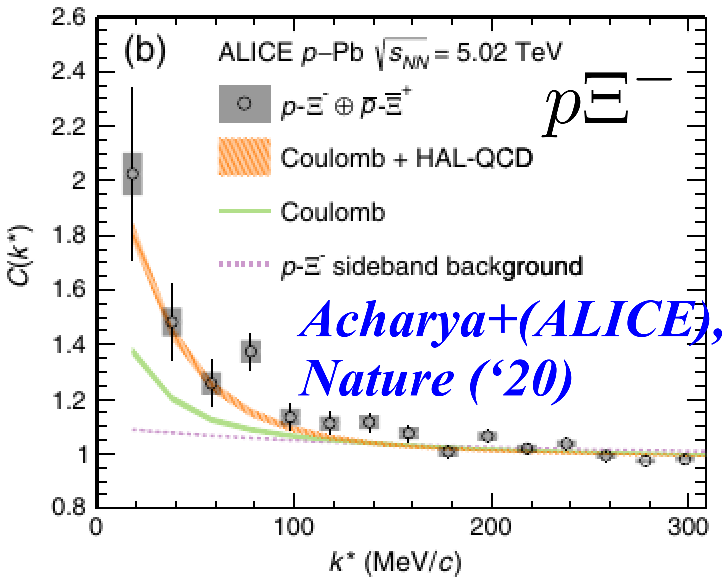
Adamczyk+[STAR], PRL114('15)022301



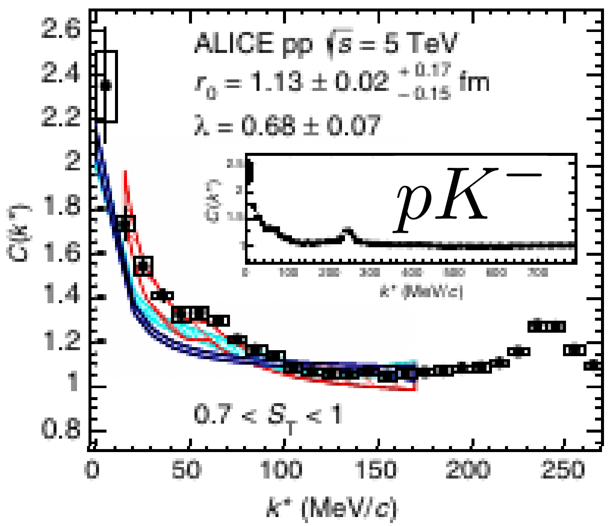
STAR, PLB790 ('19) 490 [1808.02511].



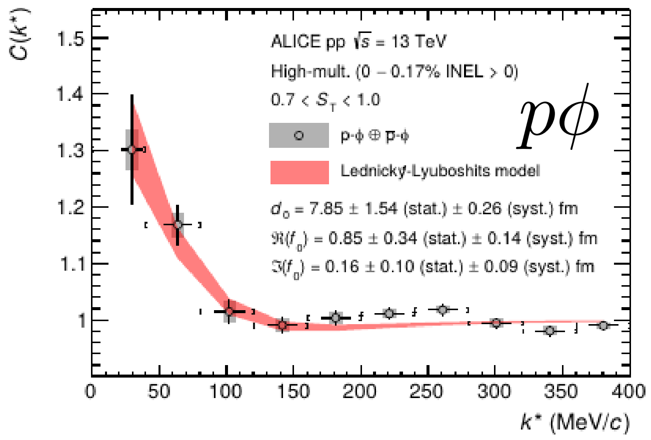
ALICE, Nature588 ('20)232.



Acharya+(ALICE), Nature ('20)



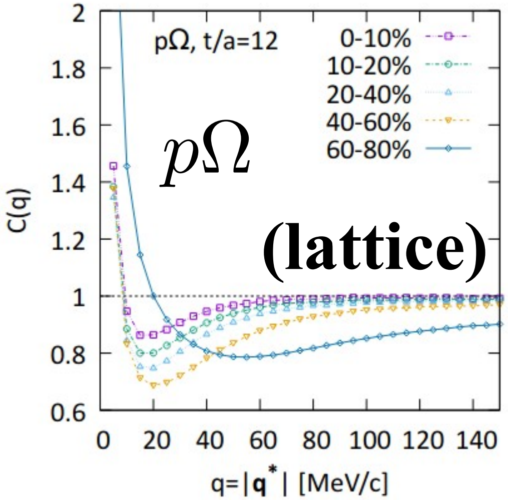
S. Acharya+[ALICE], PRL124('20)092301



ALICE, 2105.05578

c.f. talk by
V. Mantovani Sarti,
F. Grosa, N. Agrawal

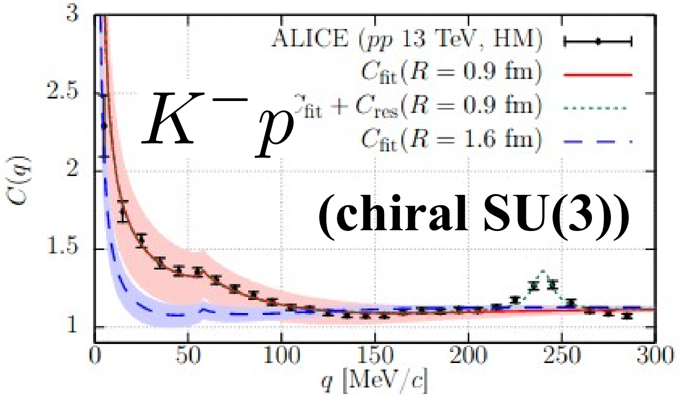
Theoretical femtoscopic study of hh int. (examples)



$p\Omega$

(lattice)

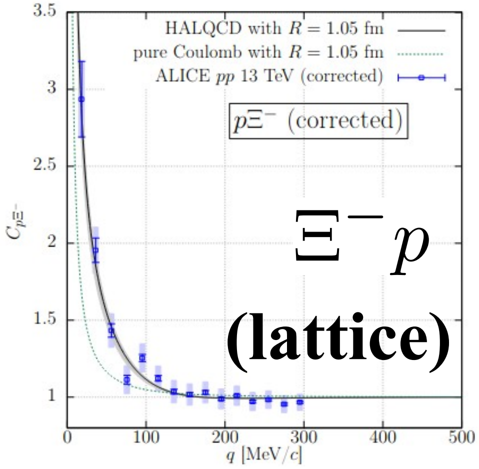
Morita, Gongyo et al., (1908.05414),
Morita, AO, Etminan, Hatsuda (1605.06765)



$K^- p$

(chiral SU(3))

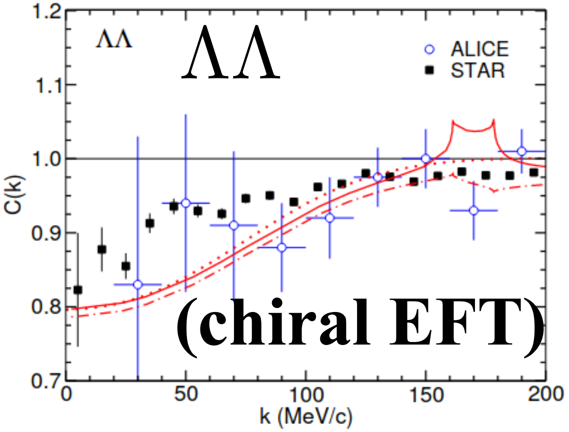
Kamiya+(1911.01041)



$\Xi^- p$

(lattice)

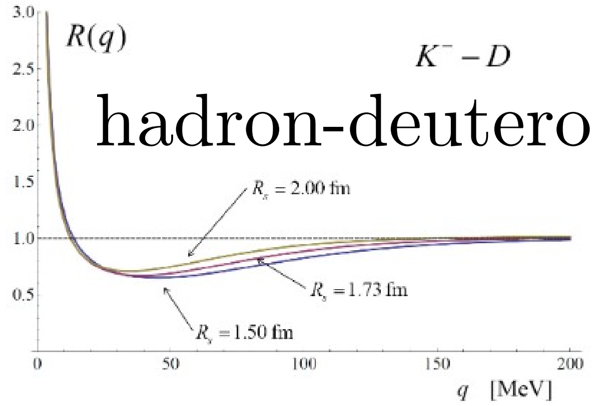
Y.Kamiya, K.Sasaki, et al., (2108.09644)



$\Lambda\Lambda$

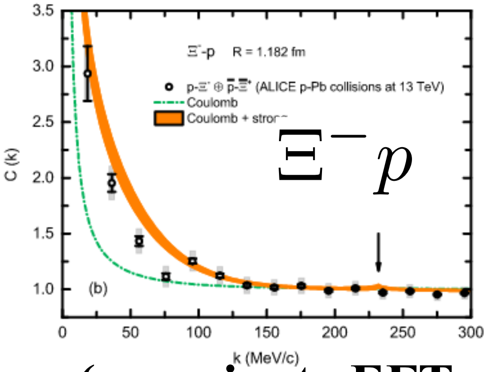
(chiral EFT)

Haidenbauer(1808.05049),
Morita+(1408.6682)



hadron-deuteron

Mrówczyński, Słón (1904.08320, K⁻d),
Haidenbauer (2005.05012, Λd),
Etminan, Firoozabadi (1908.11484, Ωd),
K.Ogata+ (Ξ⁻ d,2103.00100)



$\Xi^- p$

(covariant χEFT, S=-2)

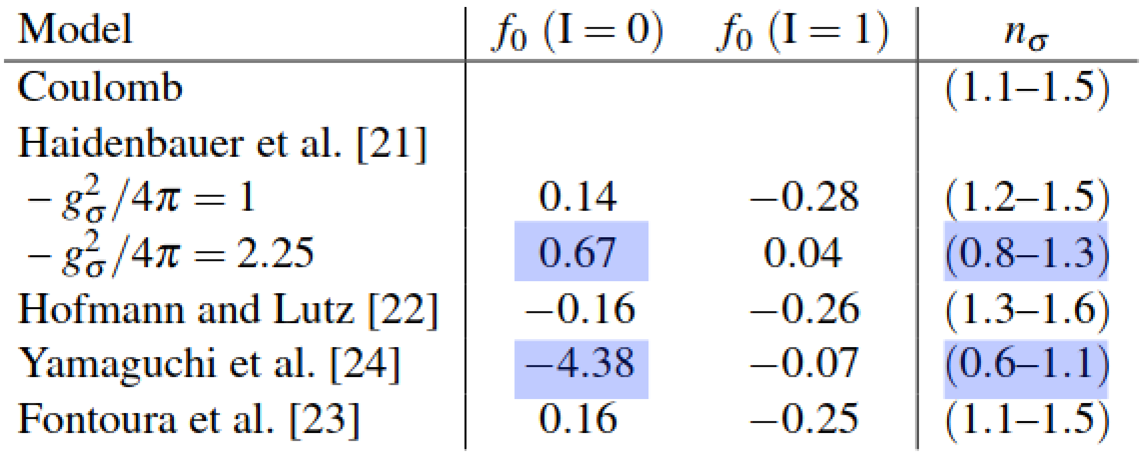
Z.-W. Liu, K.-W. Li, L.-S. Geng (2201.04997)

Femtoscopic study of charmed hadron int.

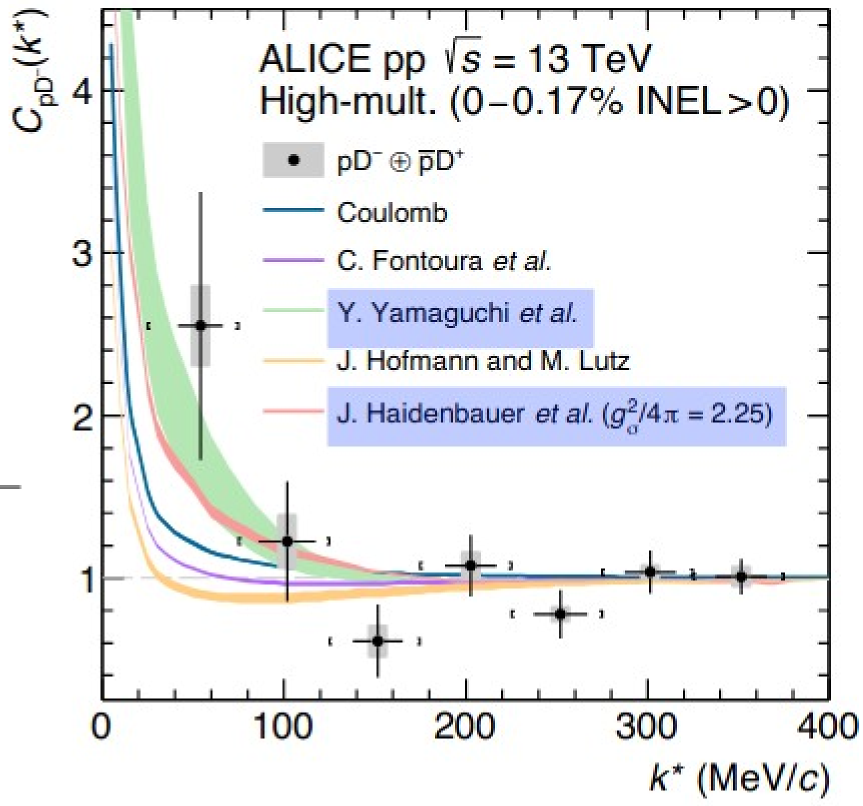
- **“First study of the two-body scattering involving charm hadrons”**

  Acharya+[ALICE] (2201.05352)

  - **D^- p corr. func. is measured.**
  - **Enhanced CF from Coulomb.**
  - **One range gaussian potential with strength fitted to the I=0 scattering length of the model → attractive potentials are favored**



Model	f_0 (I = 0)	f_0 (I = 1)	n_σ
Coulomb			(1.1–1.5)
Haidenbauer et al. [21]			
$- g_\sigma^2/4\pi = 1$	0.14	−0.28	(1.2–1.5)
$- g_\sigma^2/4\pi = 2.25$	0.67	0.04	(0.8–1.3)
Hofmann and Lutz [22]	−0.16	−0.26	(1.3–1.6)
Yamaguchi et al. [24]	−4.38	−0.07	(0.6–1.1)
Fontoura et al. [23]	0.16	−0.25	(1.1–1.5)



[21] Haidenbauer+(0704.3668) (weakly / mildly attractive (I=0))
[22] Hofmann, Lutz (hep-ph/0507071) (repulsive (I=0))
[23] Fontoura+(1208.4058) (weakly attractive (I=0))

[24] Yamaguchi, Ohkoda, Yasui, Hosaka (1105.0734) (att., w/ bound state (I=0))

To be bound or not to be bound

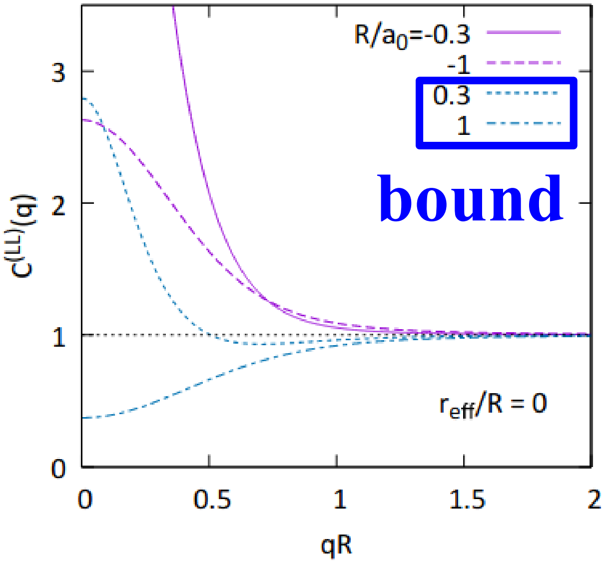
- When there is a bound state, CF shows interesting dependence on the source size and relative momentum.
- D⁻ p corr. func. shows the behavior with a bound state, and the best fit parameter set (R, a_0) is in the bound region. (If bound, it is the first weakly decaying pentaquark state.)

$$k\ \cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} k^2 + \mathcal{O}(k^3)$$

(Nuclear and atomic phys. convention.)

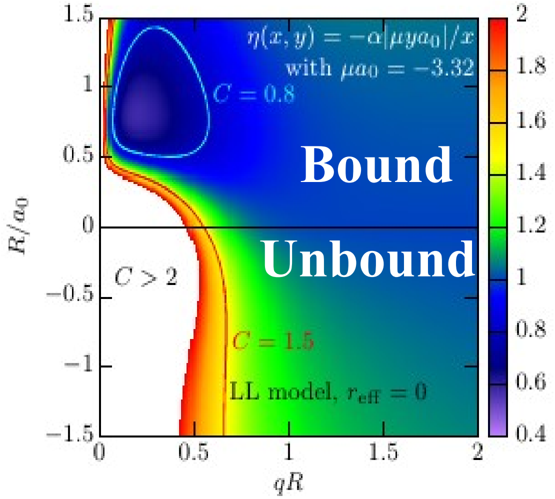
$$k\ \cot\delta = +\frac{1}{f_0} + \frac{1}{2} r_{\rm eff} k^2 + \mathcal{O}(k^3)$$

(High-E. phys. convention.)

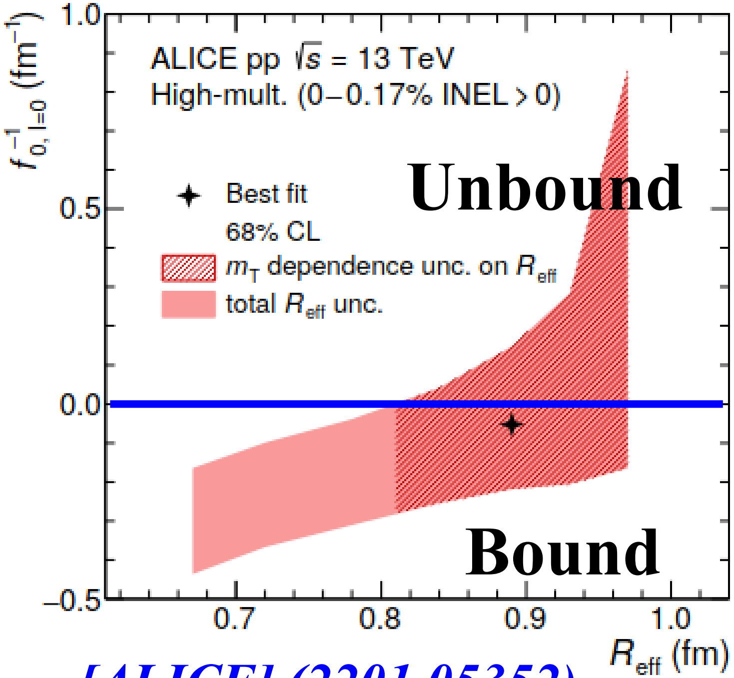


Morita+(1908.05414)

With Coulomb



Kamiya+(2108.09644)



[ALICE] (2201.05352)

Femtoscopic study of charm hadron interactions

- DD^* and $D\bar{D}^*$ correlation func. -

Femtoscopic study of charmed hadron int. (2)

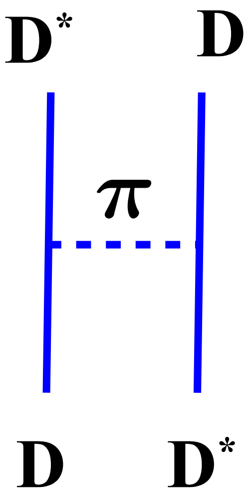
- **DD^* and $D\bar{D}^*$ correlation functions.** *Kamiya, Hyodo, AO, in prep.*
  - **Related with Tcc and X(3872)**
  - **DD^* and $D\bar{D}^*$ interactions**

$$V = \frac{1}{2}\begin{pmatrix} V_{I=0}+V_{I=1} & V_{I=0}-V_{I=1} \\ V_{I=0}-V_{I=1} & V_{I=0}+V_{I=1} \end{pmatrix}$$

  - **I=0: One range gaussian, strength fitted to the mass**
  - **I=1: ignored**
  - **Range = one pion exchange** *Yasui, Sudoh (0906.1452)*
  - **Strength is fitted to the pole mass.**

$$\{D^0\bar{D}^{*0}\} = (D^0\bar{D}^{*0} + \bar{D}^0 D^{*0})/\sqrt{2}\ (C=+1)$$

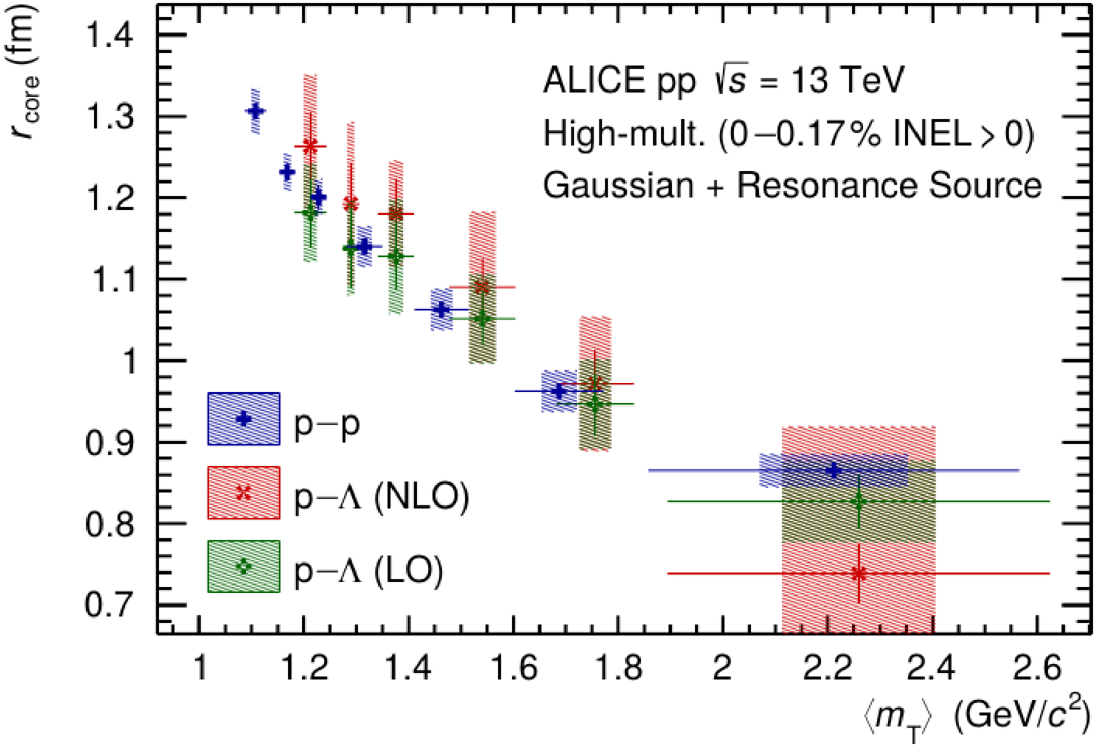
$$\{D^+D^{*-}\} = (D^+D^{*-} + D^-D^{*+})/\sqrt{2}\ (C=+1)$$



DD^*	V_0 [MeV]	$a_0^{D^0D^{*+}}$ [fm]	$a_0^{D^+D^{*0}}$ [fm]
	$-36.569 - i1.243$	$-7.16 + i1.85$	$-1.75 + i1.82$
$\{D\bar{D}^*\}$	V_0 [MeV]	$a_0^{\{D^0\bar{D}^{*0}\}}$ [fm]	$a_0^{\{D^+D^{*-}\}}$ [fm]
	$-43.265 - i6.091$	$-4.23 + i3.95$	$-0.41 + i1.47$

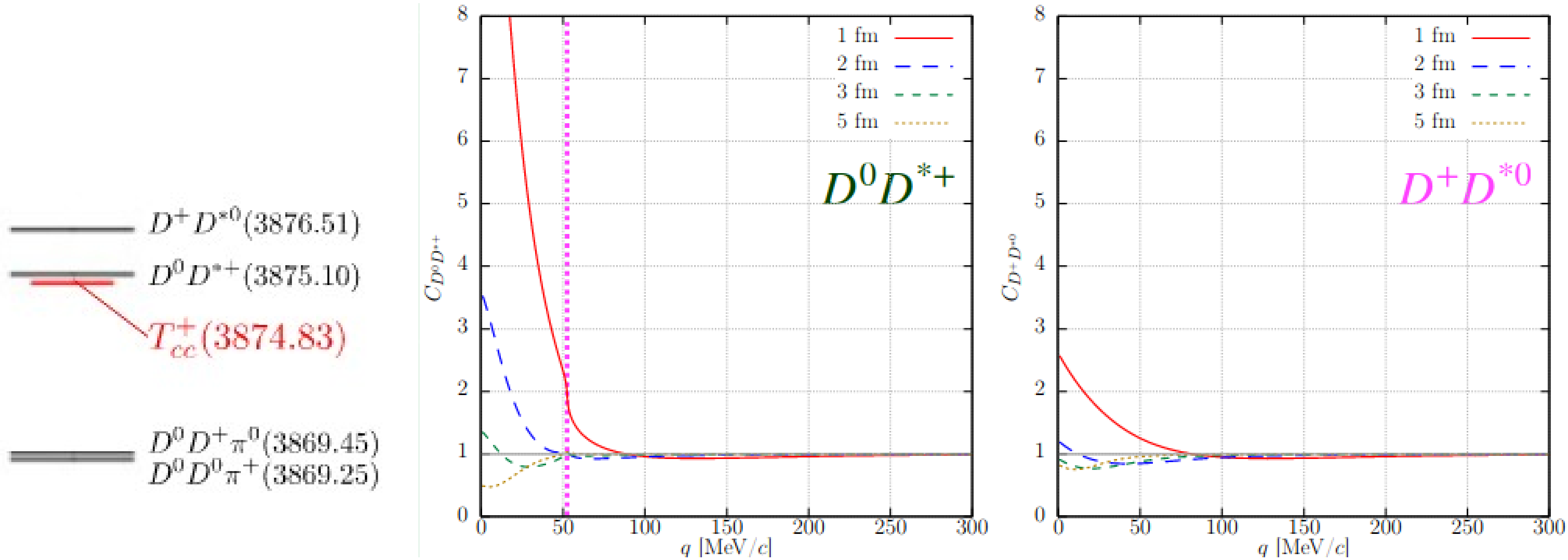
We are sorry, but we use a Gaussian Source !

- **Calculating HBT radius in dynamical models is not easy**
  M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014]; S. Pratt, PRL102('09)232301 [0811.3363].
- **and a Gaussian source seems to work at the current precision of hh interaction studies.**
  S. Acharya+[ALICE], PLB811('20)135849.
  - **primary (universal ?)+ decay of short-lived resonances ~ eff. Gaussian**
  - **Flow and source geometry effects are seen in CF, but the uncertainty of hh int. is the largest.**



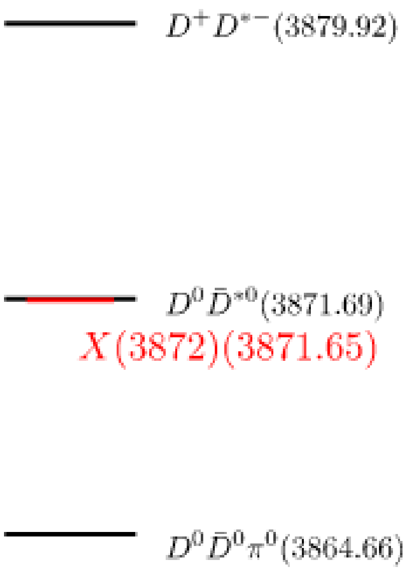
$D^0 D^{*+}$ and $D^+ D^{*0}$ Correlation Functions

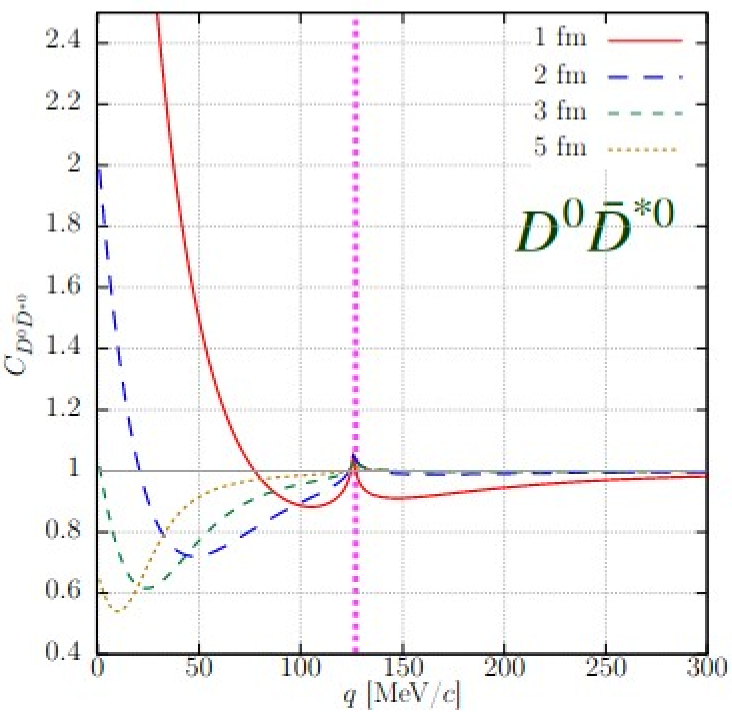
- **For small source (R= 1 fm)**
  C(q) > 8 for the lower channel (D^0D^{*+}) (Very strong)
  C(q) ~ 2.5 for upper channel (D^+D^{*0}) (strong)
- **For large source (R=5 fm), CF show a dip**
- **Strong enhancement for small source, dip for large source**
  → Characteristic dependence with a bound state (Tcc)
- **Cusp is not significant**

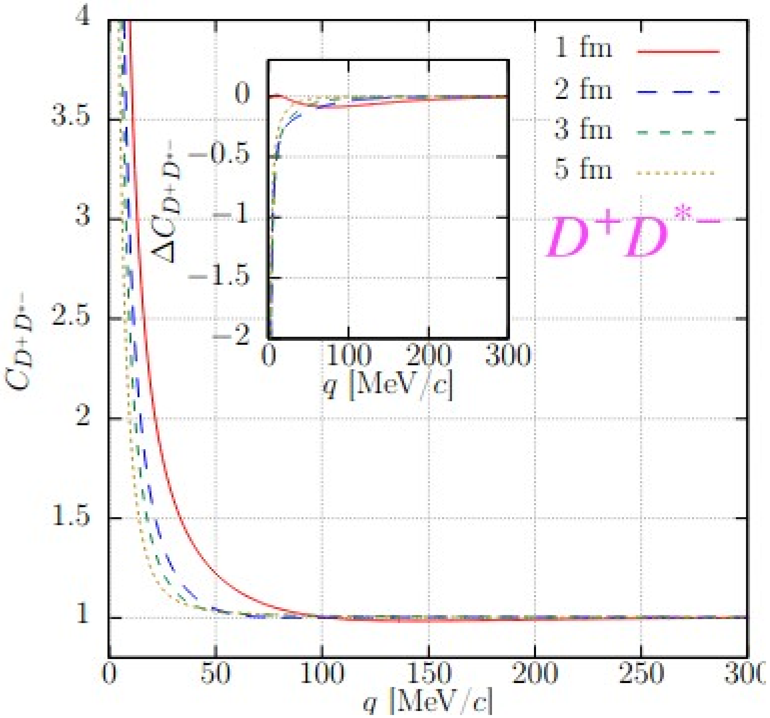


$D^0\bar{D}^{*0}$ and D^+D^{*-} Correlation Functions

- $C(D^0\bar{D}^{*0})$: **Strong enh. for small source, dip for large source** → **Characteristic dependence with a bound state (X(3872))**
- $C(D^+D^{*-})$: **Coulomb dominant**
- **Cusp may be observed for small size**







Tcc and X(3872) structure

- **Hadronic molecule structure is assumed**
  → Eigenmomentum $k \simeq -i/a_0\,,\ a_0 \simeq 1/\sqrt{2\mu B}$
- **What happens when multiquark state mixes ?**
  → Deviation from weak binding relation
  Weinberg, Phys. Rev. 137, B672 (1965), Hyodo, Jido, Hosaka (1108.5524), Kunigawa, Hyodo (2112.00249)

$$a_0 = R\left[\frac{2X}{1+X}\right] + \mathcal{O}(R_{\rm typ}) \ \ \left[R_{\rm typ} = \max(m_\pi^{-1}, r_{\rm eff})\right]$$

  - **Smaller scattering length in DD* may signal the *genuine* tetraquark nature of Tcc.**

$$f = \frac{1}{k\cot\delta - ik} \simeq \frac{1}{1/a_0 - ik}$$

(high-energy phys. convention)

Summary

- **Two-particle correlation functions are useful to deduce**
  - **Scattering length**
  - **Existence of a bound state**
  - **and hopefully the compositeness**
- **Charm hadron interactions are within the reach.**
  - **D^- p correlation function has been measured, and the data favor attractive interaction.** ***ALICE (2201.05352)***
  - DD^* and $D\bar{D}^*$ **correlation functions are predicted to reflect the existence of bound states (Tcc and X(3872)) by using simple potentials fitting to the mass and width.**
    Y. Kamiya, T. Hyodo, AO (2203.13814)
  - **Precise measurement of the correlation function will constrain the scattering length, which may tell us the structure of exotic hadrons.**

Thank you for attention !

Y. Kamiya

T. Hyodo

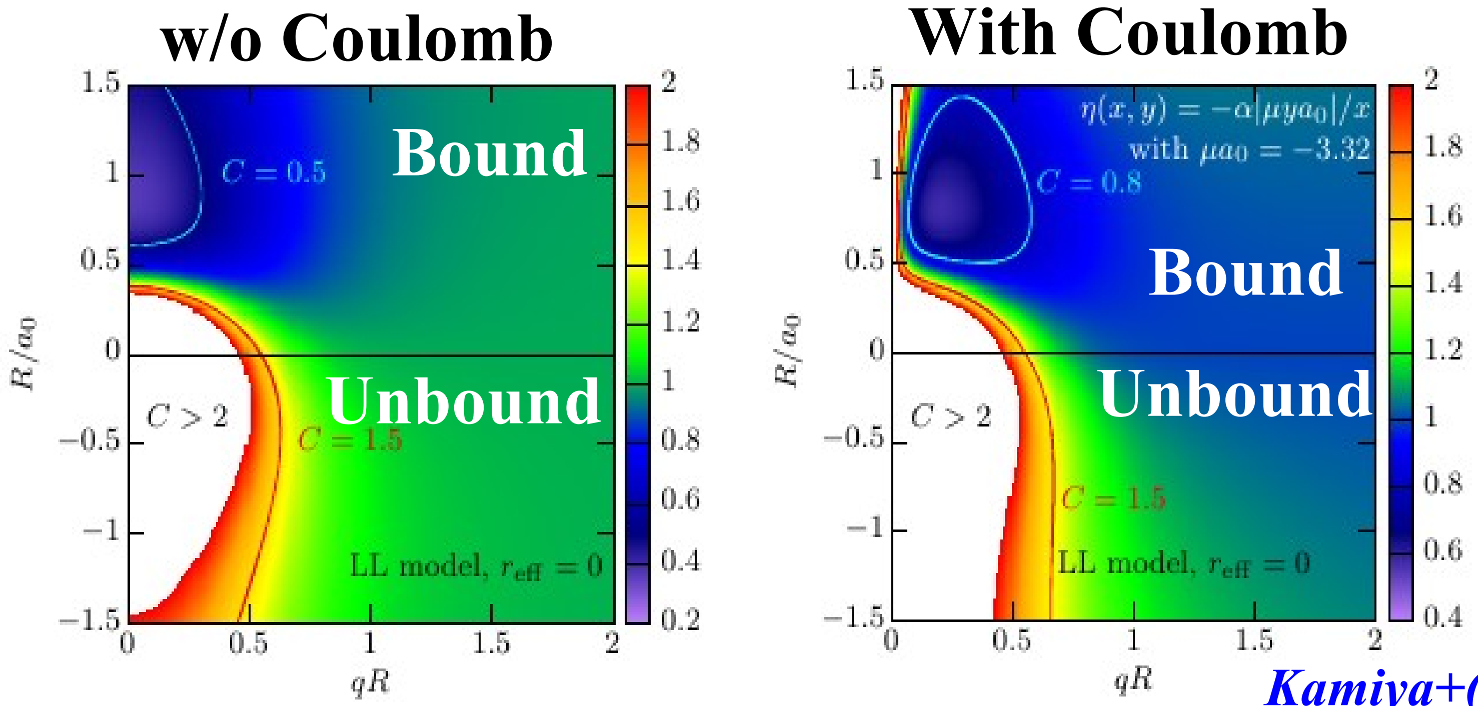
AO

R Dependence of Correlation Function

- **Source size (R) dependence of C(q) is helpful to deduce the existence of a bound state.**

  Morita+('16, '20), Kamiya+('20), Kamiya+(2108.09644)

  - **With a bound state, C(q) is suppressed at small q when R ~ $|a_0|$. (w.f. has a node at r ~ $|a_0|$ with a bound state.)**
  - **Qualitative understanding by the analytic model (LL formula)** *[Lednickey, Lyuboshits ('82)]* **with the zero range approx. ($r_{\rm eff}$=0)**


w/o Coulomb

With Coulomb

Kamiya+(2108.09644)

Coupled-Channel Correlation Function

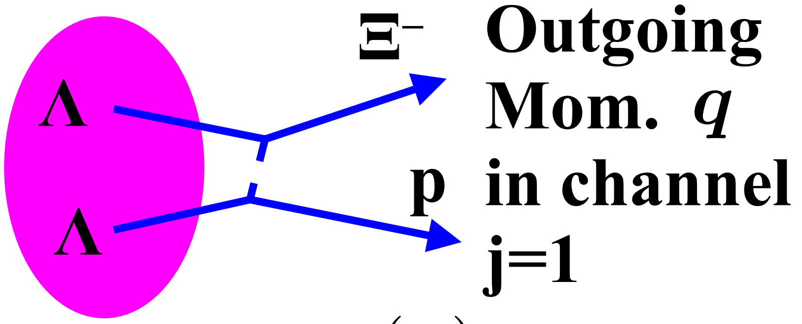
- **Correlation function with CC effects (KPLLL formula)**
  → sum of j-th channel contributions leading to j=1 with outgoing momentum *q*
  Lednicky, Lyuboshits, Lyuboshits ('98); Haudenbauer ('19)

$$C(\boldsymbol{q}) = \sum_j \omega_j \int d\boldsymbol{r} S_j(\boldsymbol{r}) |\Psi_j^{(-)}(\boldsymbol{r})|^2$$

$$\Psi_j^{(-)}(\boldsymbol{r}) = [e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr)]\delta_{1j} + \psi_j^{(-)}(r)$$

$$\psi_j^{(-)}(q) \propto e^{-iqr}/r \text{ or } e^{-\kappa r}/r \ (r \to \infty)$$

(No Coulomb case)



$$\underline{\omega_j}\,\underline{S_j(\boldsymbol{r})}|\Psi_j^{(-)}(\boldsymbol{r})|^2$$

Source weight

Normalized Source fn.

- **Effects of coupled-channel, strong & Coulomb pot., and threshold difference are taken into account in the charge base, $p\Xi^-$, $n\Xi^0$, ΛΛ.**
  Y. Kamiya+, PRL('20, $K^- p$)
- **Source size (R) and source weight (ω_j) need to be determined.**

A New Insight from CMS: Exotic/Normal Ratio

- **ExHIC index = Coalescence / Statistical Ratio**

$$R_h^{\mathrm{CS}} = \frac{\text{Yields in Coalescence}}{\text{Yields in Statistical model}}$$

- **CMS index = Exotic / Normal Ratio**

Sirunyan+ [CMS], arXiv:2102.13048

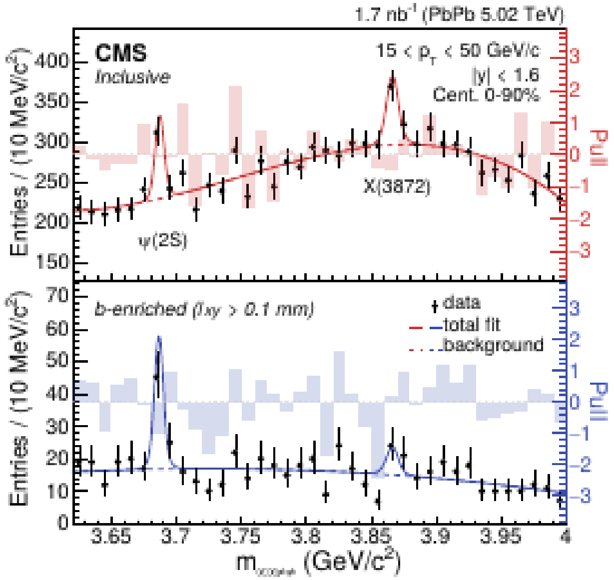
$$\rho_{\mathrm{exo/nor}} = \frac{N(\text{Exotic hadron candidate})}{N(\text{Normal hadron})}$$

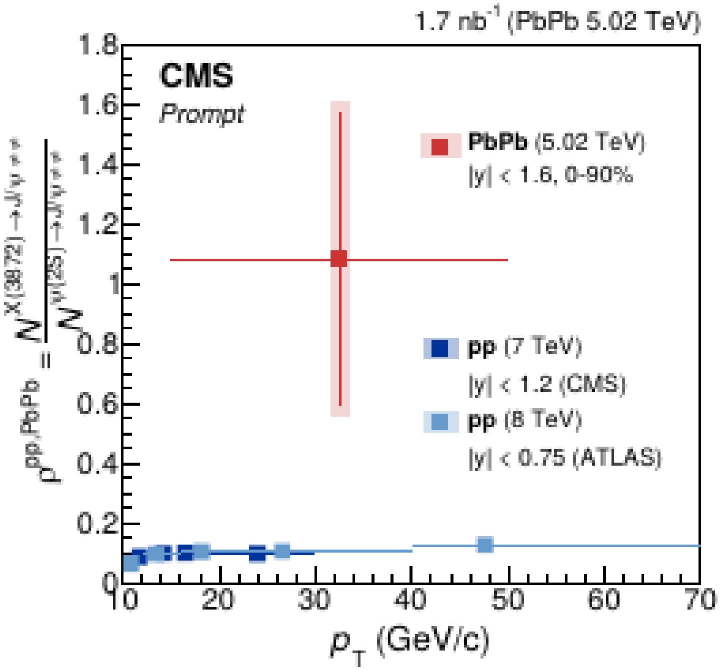
  - **X(3872) / ψ(2S) ratio in pp and PbPb collisions.**

$$\rho_{X/\psi}(\mathrm{PbPb}) = 1.08 \pm 0.49(\mathrm{stat.}) \pm 0.52(\mathrm{syst.})$$

$$\rho_{X/\psi}(pp) \simeq 0.1$$

ExHIC prediction is found to be (qualitatively) true !

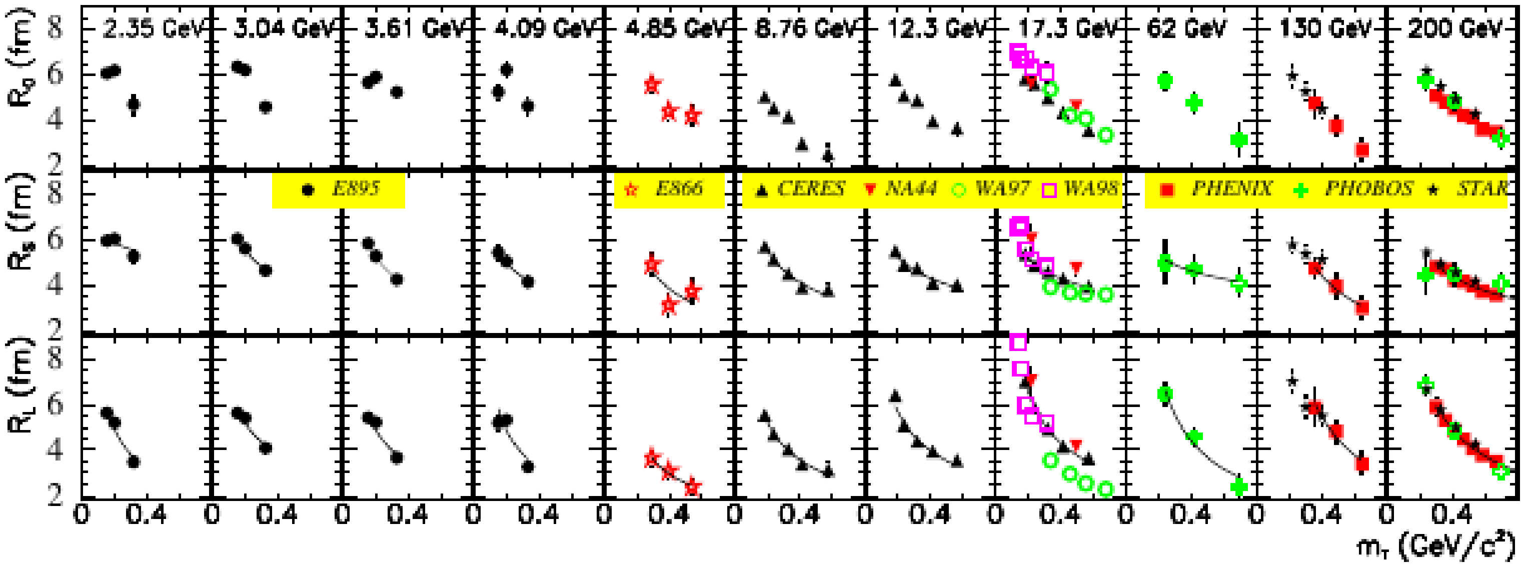




State-of-the-art Femtoscopy of radii

- **Systematic measurement of 3D HBT radii (side, out, long)**

  M. A. Lisa, S. Pratt, R. Soltz, U. Wiedemann, Ann.Rev.Nucl.Part.Sci. 55 (2005) 357-402.



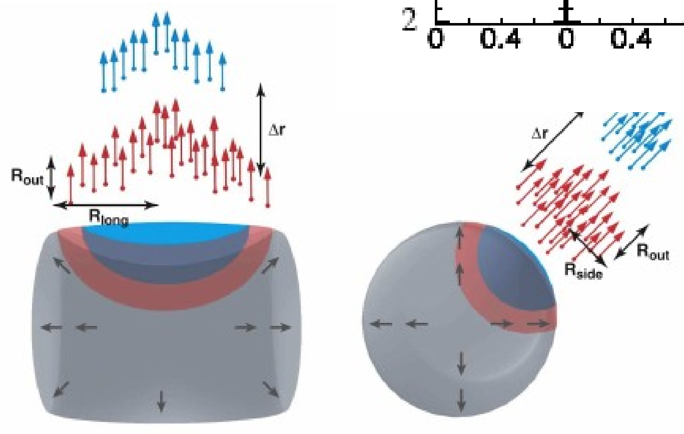
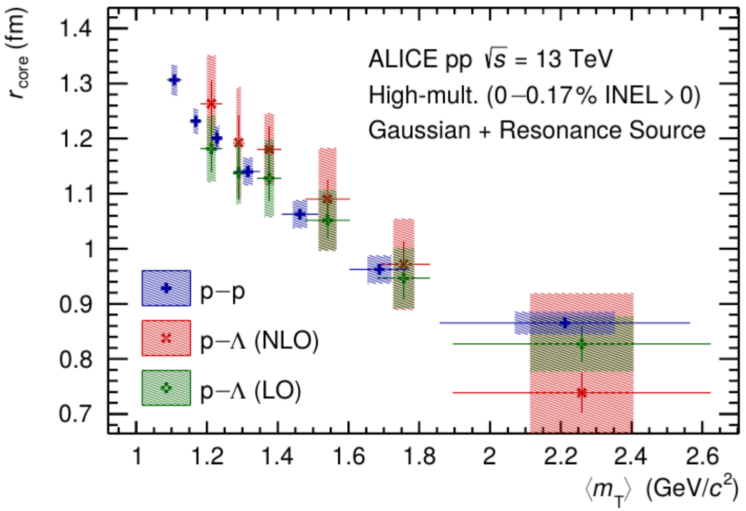


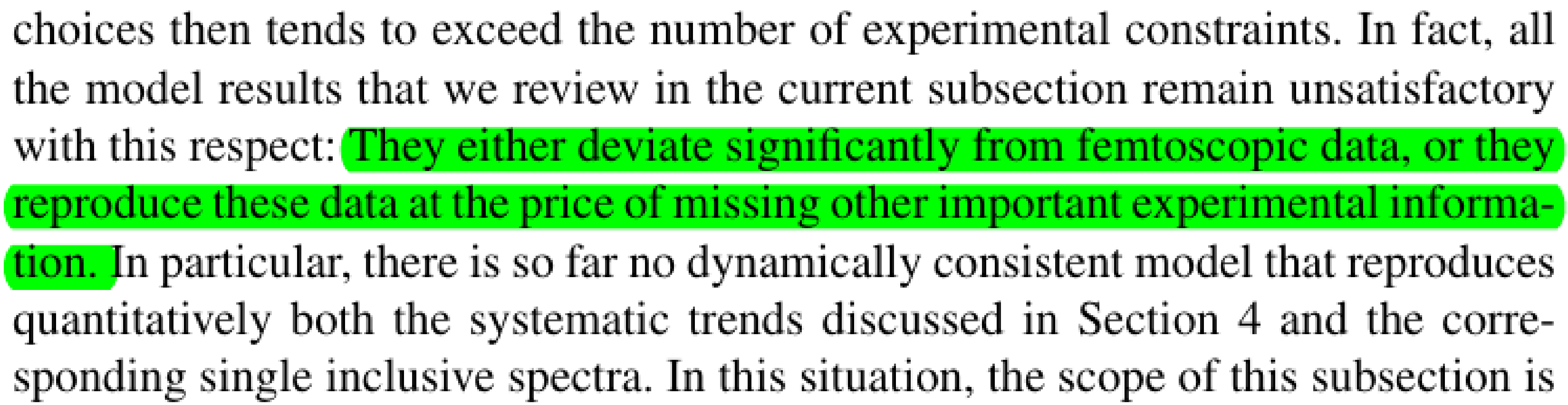
Figure 3: because particles with heavier masses have smaller thermal velocities, their source volumes are more strongly confined by collective flow. For longitudinal flow (*left panel*) this results in smaller values of $R_{\rm long}$ for particles with higher $m_T = \sqrt{m^2 + p_T^2}$. For radial flow (*right panel*) this confines heavier particles toward the surface, which results in both a reduced volume and an offset Δr in the outward direction.



S. Acharya+[ALICE], PLB811('20)135849

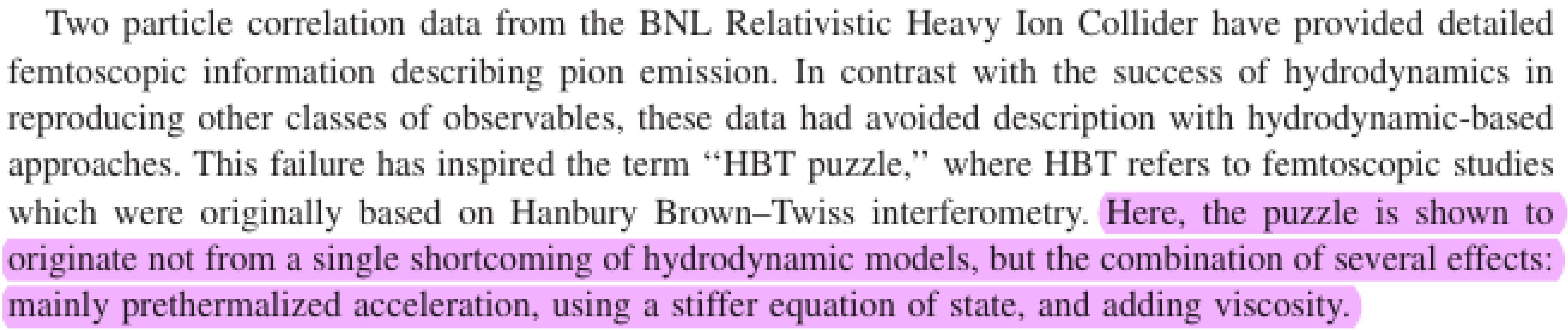
We are sorry for using a Gaussian Source !

- **Calculating HBT radius in dynamical models is not easy (HBT puzzle).**
  - *M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014];*



choices then tends to exceed the number of experimental constraints. In fact, all the model results that we review in the current subsection remain unsatisfactory with this respect: They either deviate significantly from femtoscopic data, or they reproduce these data at the price of missing other important experimental information. In particular, there is so far no dynamically consistent model that reproduces quantitatively both the systematic trends discussed in Section 4 and the corresponding single inclusive spectra. In this situation, the scope of this subsection is

  - *S. Pratt, PRL102('09)232301 [0811.3363].*



Two particle correlation data from the BNL Relativistic Heavy Ion Collider have provided detailed femtoscopic information describing pion emission. In contrast with the success of hydrodynamics in reproducing other classes of observables, these data had avoided description with hydrodynamic-based approaches. This failure has inspired the term "HBT puzzle," where HBT refers to femtoscopic studies which were originally based on Hanbury Brown–Twiss interferometry. Here, the puzzle is shown to originate not from a single shortcoming of hydrodynamic models, but the combination of several effects: mainly prethermalized acceleration, using a stiffer equation of state, and adding viscosity.

How about afterburner effects ?