Femtoscopy for flavored hadron-hadron interactions

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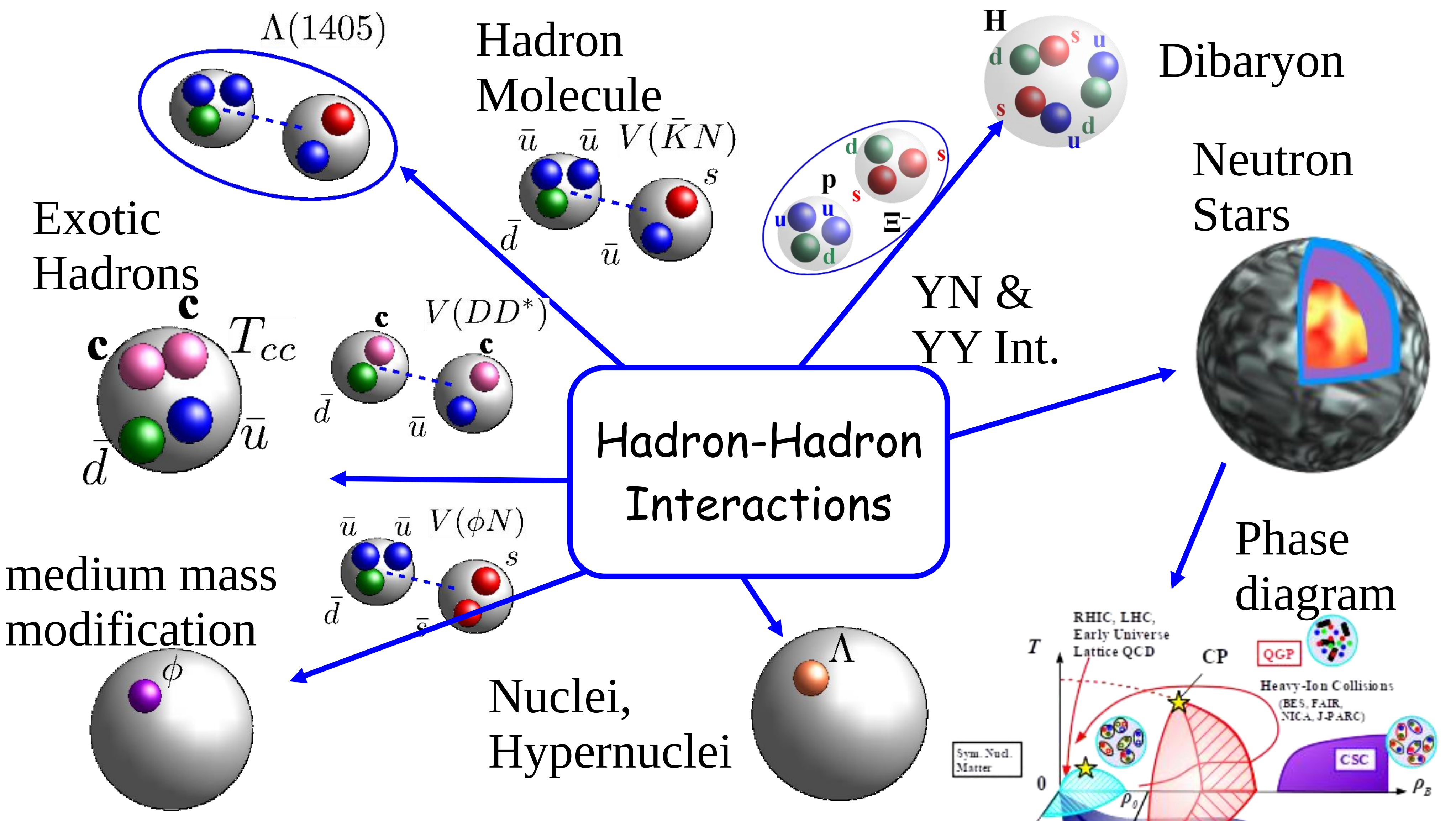
The 59th International Winter Meeting on Nuclear Physics, Bormio, Italy, Jan. 23 to 27, 2023

- **Introduction**
- **Femtoscopy Basics**
  - **Interaction dep. of the correlation function**
- **Hadron-Hadron Interactions and Bound States with Strangeness**
  - **S=-1 ~ -6 BB pairs, Bound state candidates**
- **Homeworks**
- **Summary**

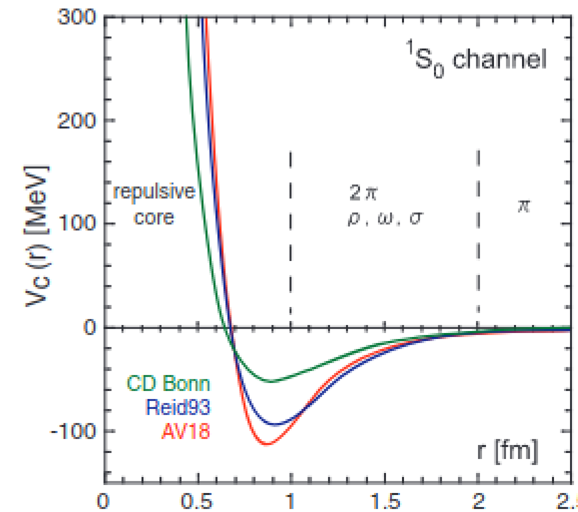
Hadron-Hadron Interactions



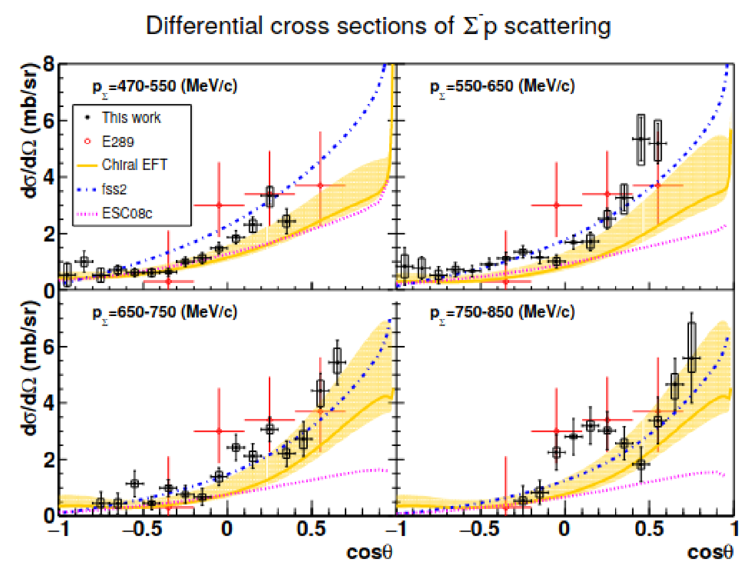
HHIs are the basic inputs
for hadron many-body systems

But info. is limited on flavored hadron int.

- **Nucleon-Nucleon (NN) Interactions**
  - **Based on rich NN phase shift data, high-quality potentials are obtained.**
- **Scattering experiments**
  - **Targets need to be stable (p, nuclei)**
  - **Beam particles need to fly a few cm or more.**
  - **NN, πN, K^+N, K^-N, YN (Hyperon-N)**
    E.g. J-PARC E40 (ΣN scattering)
- **Hypernuclei, Exotic Nuclei**
  - **Interactions from bound states**
    Exotic atoms (π, K^-, Σ^-),
    Hypernuclei (Λ, Σ, Ξ, ΛΛ).
- **Femtoscopy**
  - **Correlation function contains info. on Scattering via Final State Interaction**
  - **Any pair is accessible if PID is possible.**



Ishii, Aoki, Hatsuda ('07)



Miwa+[J-PARC E40]('21)

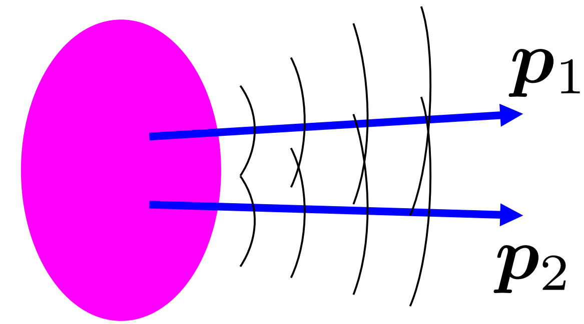
Femtoscopy

Correlation Function

Koonin('77), Pratt+('86), Lednicky+('82)

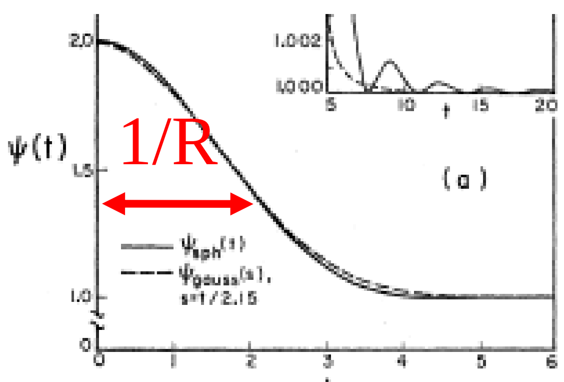
$$C(\boldsymbol{q}) = \int d\boldsymbol{r} S(\boldsymbol{r}) |\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2$$

$S(\boldsymbol{r}) = \text{source function}$, $\varphi_{\boldsymbol{q}}(\boldsymbol{r}) = \text{relative w.f.}$



Source size (HBT)

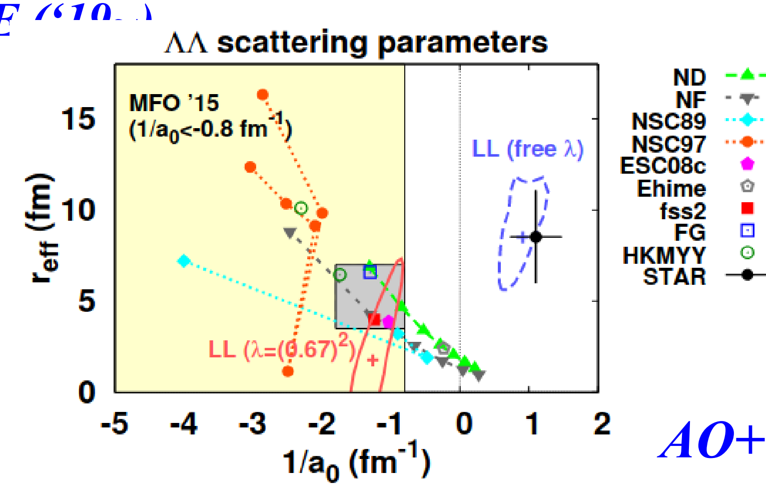
Hanbury Brown & Twiss, Nature 10 (1956), 1047;
Goldhaber, Goldhaber, Lee, Pais, Phys. Rev. 120 (1960), 300.



q (relative momentum)

Hadron-Hadron Interaction

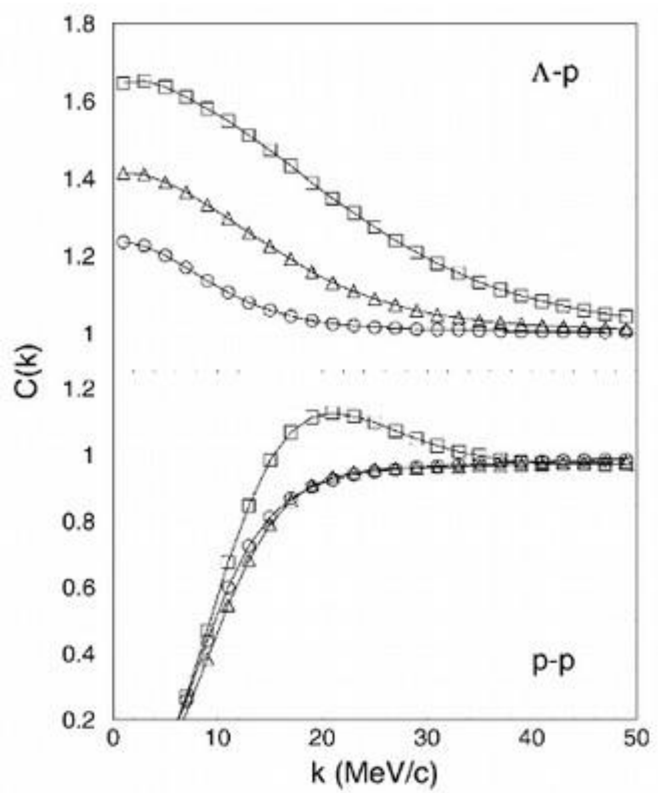
Lednickey, Lyuboshits ('82); Lednicky, Lyuboshits, Lyuboshits ('98); Heidenbauer ('19); C. Greiner, B. Muller, PLB219('89)199; AO+ ('00); Morita+('15~); Kamiya+('20~); STAR ('15~); ALICE ('19~)



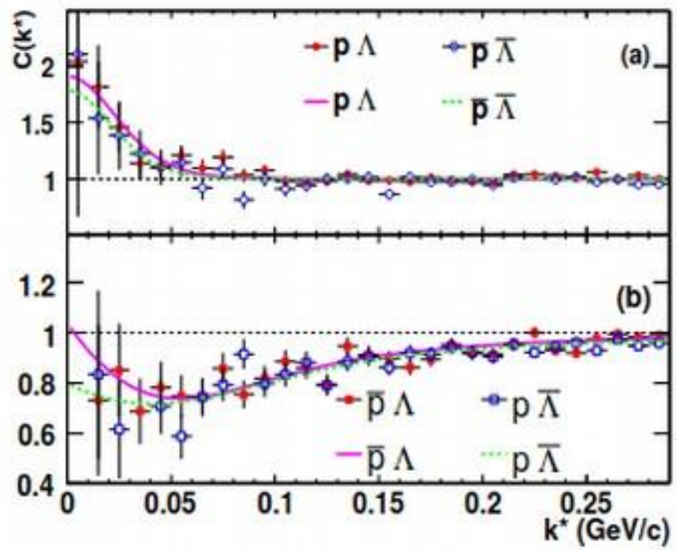
AO+('16)

… and it works

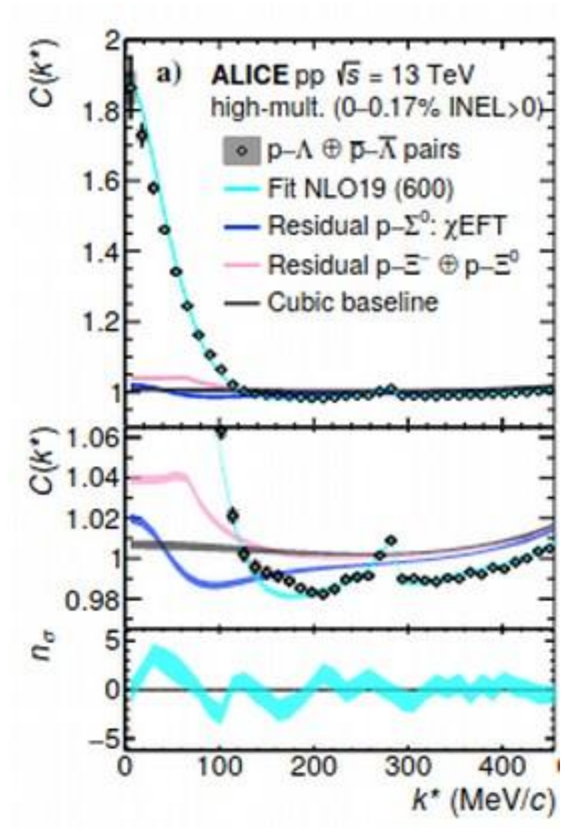
Prediction ⇨ Measurements



Wang, Pratt ('99, PRL, nucl-th/9907019)



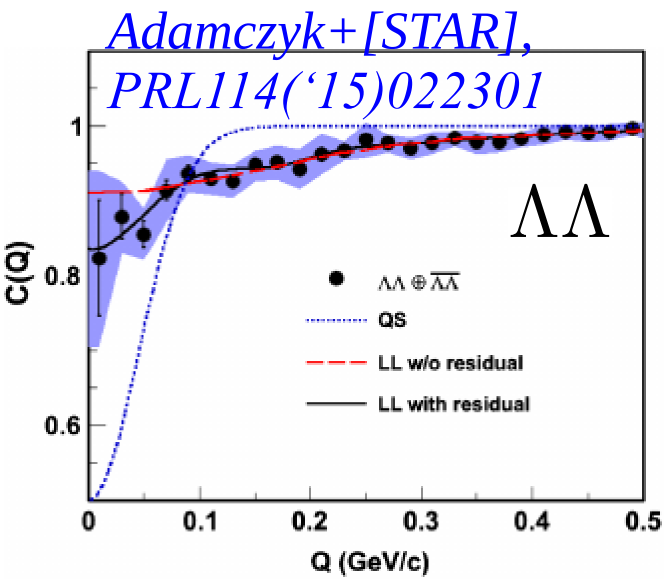
STAR('06) (nucl-ex/0511003)



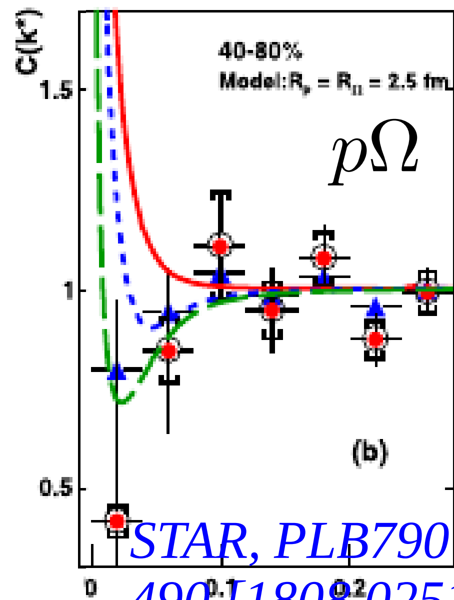
ALICE (2104.04427)

Extracted scattering parameters ~ Known values

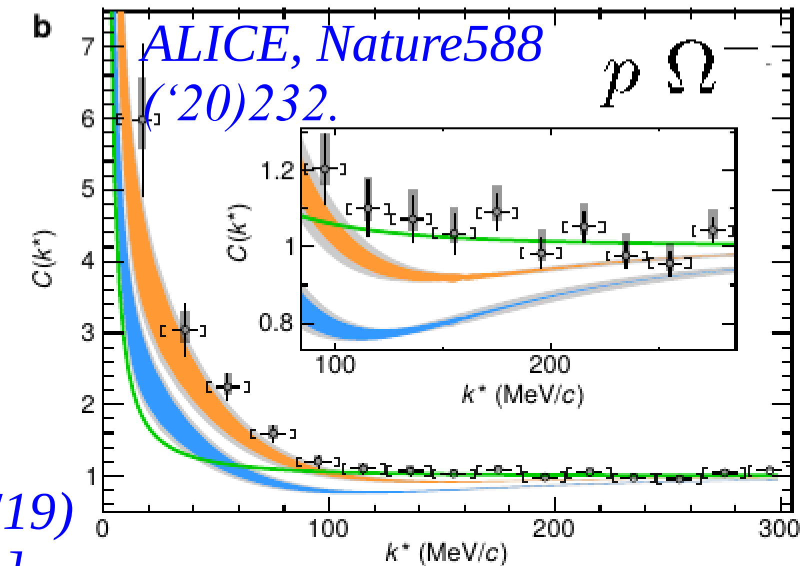
Measured Flavored Hadron CFs (examples)



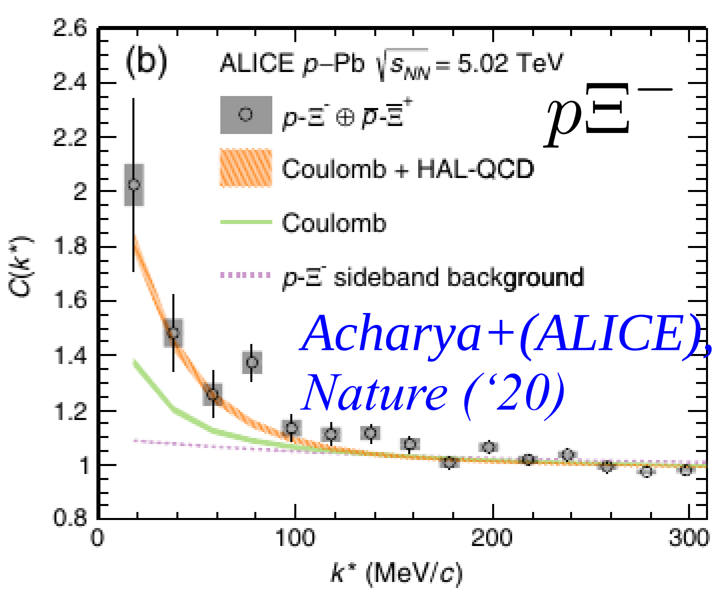
Adamczyk+[STAR], PRL114('15)022301



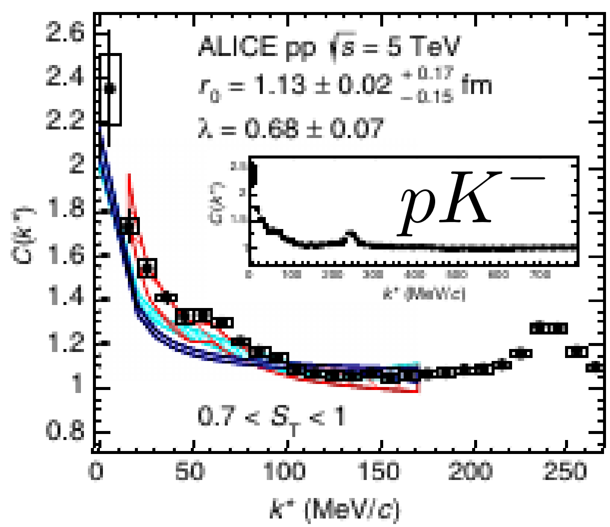
STAR, PLB790 ('19) 490 [1808.02511].



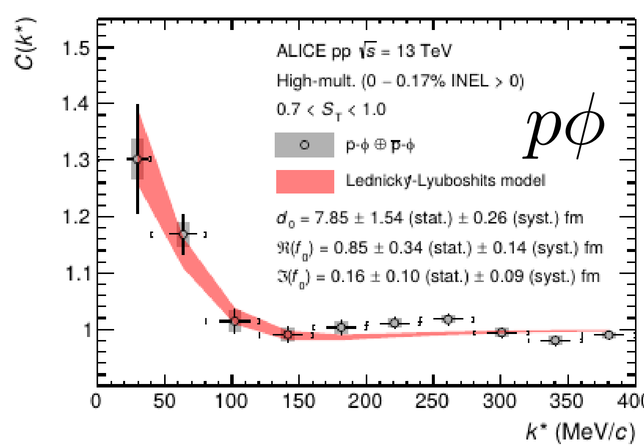
ALICE, Nature588 ('20)232.



Acharya+(ALICE), Nature ('20)



S. Acharya+[ALICE], PRL124('20)092301



ALICE, 2105.05578

Scope of Femtoscopic study of HHI

$p\varphi \longrightarrow a_0$ (Lattice pot. ?)

$pD^{\pm}$
Chamed hadron-nucleon interaction
Bound state ?

pK^-
Chiral CC pot. (examined)
Bound state (favored)

	n	p	K⁻	K⁺	π⁻	π⁺	Λ	Σ	Ξ⁻	Ω⁻	D⁻	D⁺	Ks	φ	+α
n															
p		O	O	O	O	O	O	O	O	O	O	O		O	
K⁻		O	O	O	O	O					O	O	O		
K⁺		O	O	O	O	O					O	O	O		
π⁻		O	O	O	O	O					O	O			
π⁺		O	O	O	O	O					O	O			
Λ		O					O		O						
Σ		O													
Ξ⁻		O					O		O						
Ω⁻		O													
D⁻		O	O	O	O	O									
D⁺		O	O	O	O	O									
Ks			O	O											
φ		O													
+α															

$p\Xi^-$
Lattice QCD CC pot. (examined)
Bound state (disfavored)

$K^{\pm}K^0{}_s$
Tetraquark component in a_0 meson

$p\Omega$
Lattice QCD pot. J=2 (examined)
Bound state (favored)

$\Lambda\Lambda$
Scattering pars. (a_0 , r_{eff})
(constrained)
Bound state (disfavored)

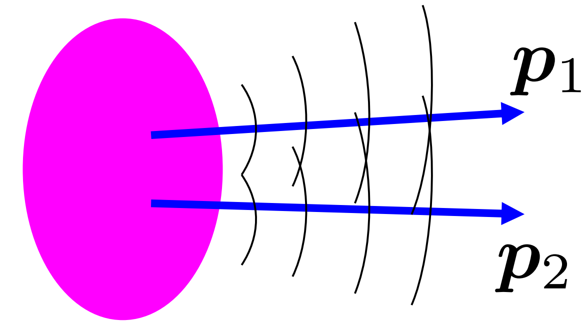
$\Lambda\Xi, DK, D\bar{K}, D\pi,$
$\Xi\Xi, pd, \Lambda d, \ldots$

Contents

- Introduction
- Femtoscopy Basics
  - Koonin-Pratt formula, Lednicky-Lyuboshits formula
  - Interaction dependence of the correlation function
- Hadron-Hadron Interactions and Bound States with Strangeness
  - S=-1, -2, -3, -4, -5, -6 ($p\Lambda$, $p\Xi$, $p\Omega$, $\Xi\Xi$, ??, $\Omega\Omega$) BB pairs
  - Bound state candidates ($N\Omega$, K^-p, D^-p)
- Homeworks
- Summary

Femtoscopy Basics
– Interaction Dependence of Correlation Functions –

Two-particle momentum correlation function



- **Emission function of one particle**

$$N_i(\boldsymbol{p}) = \int d^4x S_i(x, \boldsymbol{p})$$

- **Two-particle momentum distribution**
  - **Assumption: Particles are emitted independently. Correlation is made by final state interaction (or quantum statistics).**

$$N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2) \simeq \int d^4x d^4y \, S_1(x, \boldsymbol{p}_1) \, S_2(y, \boldsymbol{p}_2) |\Psi_{\boldsymbol{p}_1, \boldsymbol{p}_2}(x, y)|^2$$

$$\simeq \int d^4X d\boldsymbol{r} dt \, S_1(x, \boldsymbol{p}_1) \, S_2(y, \boldsymbol{p}_2) |e^{-iPX}|^2 \times \underline{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}$$

Relative w.f.
(pair rest frame)

- **Correlation Function (CF) from the Koonin-Pratt formula**
  - **Integrate over CM coordinates, normalized Source function is used.**

Koonin('77), Pratt('86), Lednicky+('82)

$$C(\boldsymbol{q}, \boldsymbol{P}) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1) N_2(\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r} \underbrace{S(\boldsymbol{r})}_{\text{source fn.}} \underbrace{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}_{\text{relative w.f.}}$$

S-wave contribution to the Correlation Function

- **Effects of Hadron-Hadron Interactions on C(q)**
  - **Only the S-wave w.f. is assumed to be modified.**
    (C(q) deviates from 1 at low q (q<200 MeV/c) → S-wave dominance
  - **Cases with non-identical particle pair and Gaussian source**

$$\varphi_{\boldsymbol{q}}(\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr) + \chi_q(r)$$

$$\rightarrow C(\boldsymbol{q}) = \int d\boldsymbol{r} S(r) |\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2$$

$$= 1 + \int d\boldsymbol{r} S(r) \left\{ |\chi_q(r)|^2 - |j_0(qr)|^2 \right\}$$

$$= 1 + \frac{1}{2\sqrt{\pi}R^3q^2} \int_0^\infty dr e^{-r^2/4R^2} \left\{ |u(r)|^2 - \sin^2 qr \right\} \quad [u(r) = qr\chi_q(r)]$$

$$\langle e^{i\boldsymbol{q}\cdot\boldsymbol{r}} j_0(qr) \rangle_\Omega = j_0^2(qr)$$

$$\langle \underline{[e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr)]} \, \chi_q(r) \rangle_\Omega = 0$$

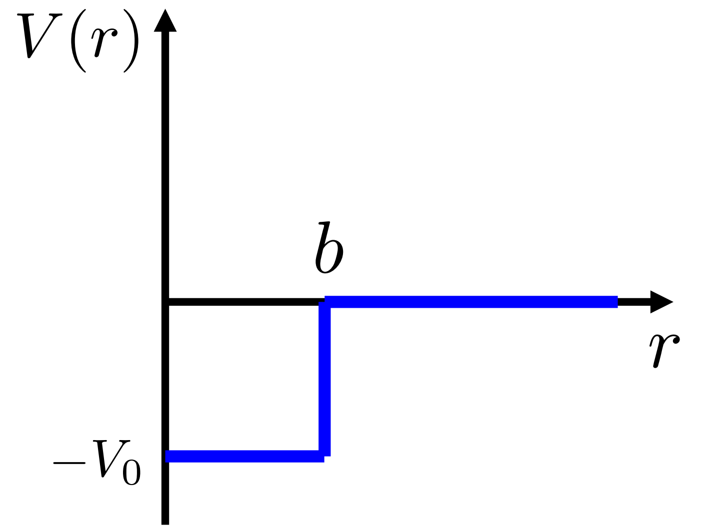
L>0

C(q) shows enhancement of S-wave $|w.f.|^2$ from $|j_0(qr)|^2$.
Solve 1D Schrodinger Eq. and Integrate in 1D.

Square-Well Potential in 3D (1)

- **Square well potential**

$$V(r) = \begin{cases} -V_0 & (r < b) \\ 0 & (r \geq b) \end{cases}$$



- **S-wave wave function**

$$\chi_q(r) = u(r)/qr, \;\; \kappa = \sqrt{q^2 + 2mV_0/\hbar^2}$$

$$u(r) = \begin{cases} A \sin \kappa r & (r < b) \\ \sin(qr + \delta) & (r \geq b) \end{cases}$$

$$\delta = \arctan\left(\frac{qb \tan \kappa b}{\kappa b}\right) - qb \,, \;\; A = \frac{\sin(qb + \delta)}{\sin \kappa b}$$

$$\kappa b \cot \kappa b = qb \cot(qb + \delta)$$ (continuity of logarithmic derivatives)

- **Scattering length and Effective range**

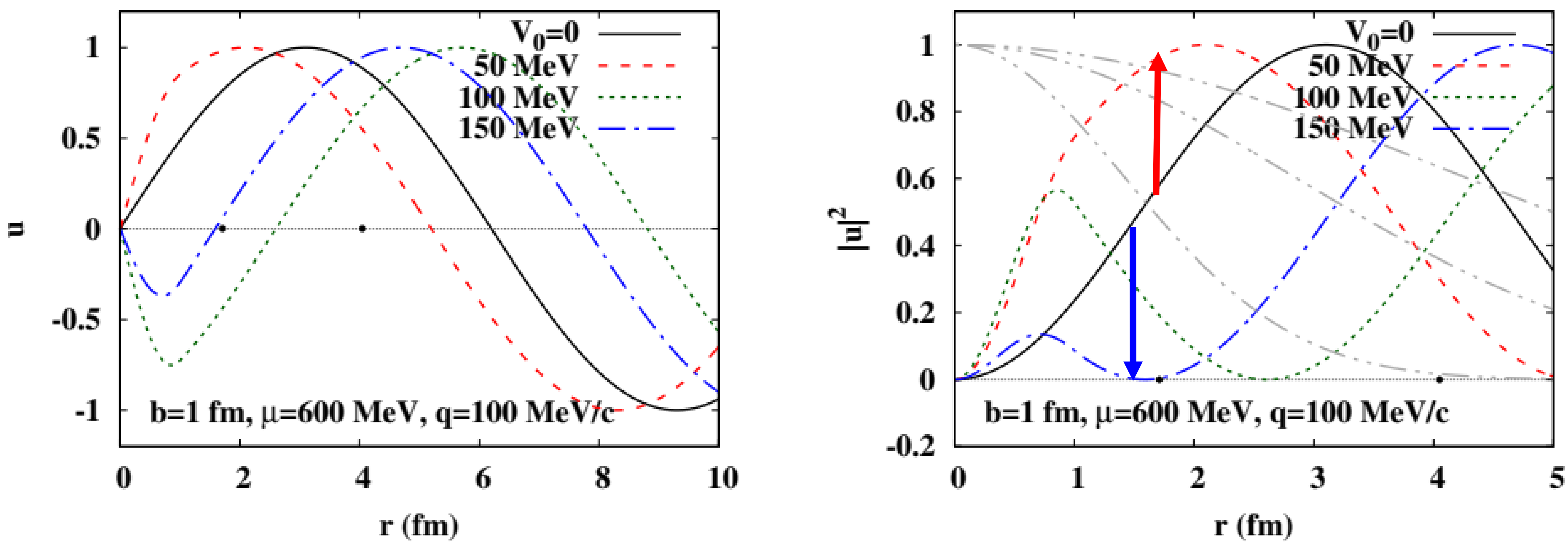
E.Braaten, H.-W.Hammer, Phys.Rept. 428 ('06) 259.

$$a_0 = -bD \,, \;\; r_{\mathrm{eff}} = b \times \frac{3C^2D^2 + 3D - C^2}{3C^2D^2} \;\; \left(C = \sqrt{2mV_0b^2/\hbar^2} \,, \;\; D = \frac{\tan C}{C} - 1\right)$$

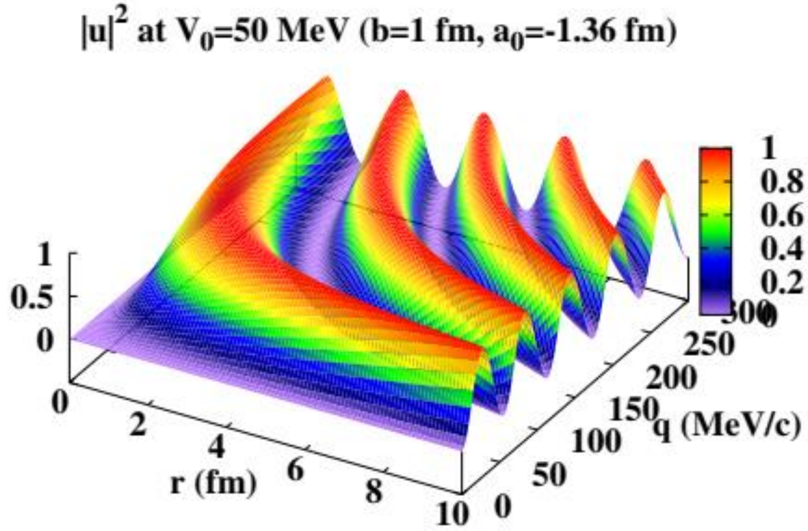
Square-Well Potential in 3D (2)

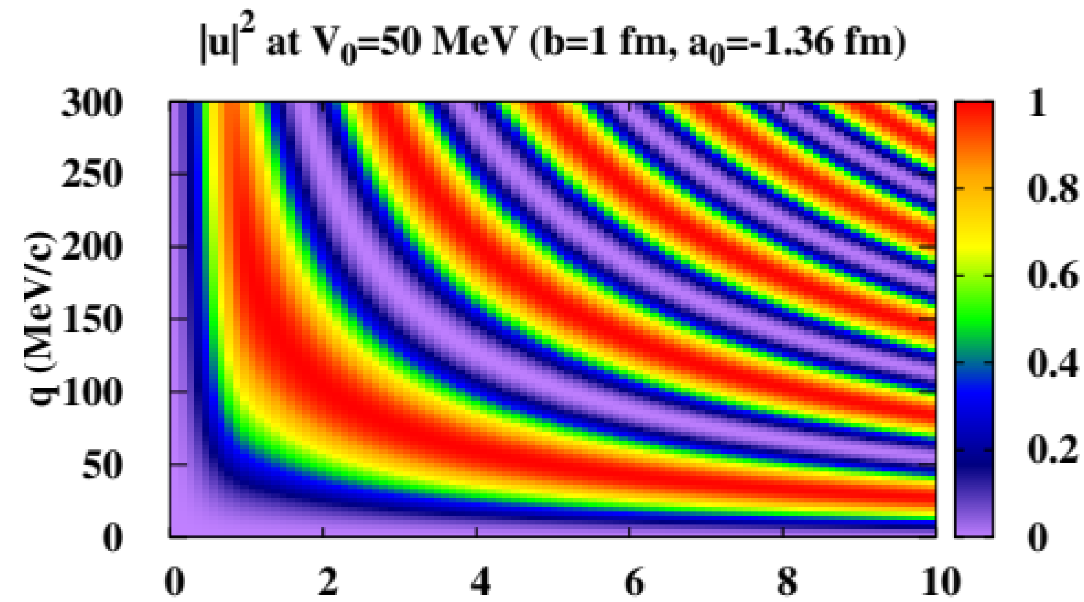
- Weakly attractive pot. (no bound state)
  → |w.f.|^2 is enhanced at small r → C(q) > 1
- Strongly attractive pot. (w/ bound state)
  → |w.f.|^2 is suppressed at small r → C(q) < 1
  - With a bound state (a_0>0), w.f. has a node around r~ a_0

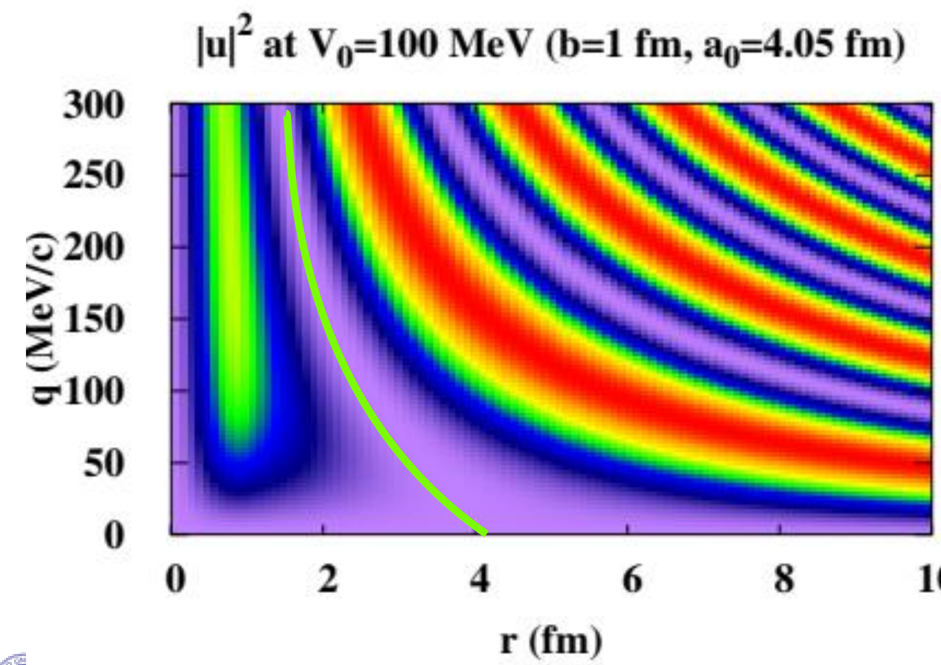
$$u(r, r > b) = \sin(qr + \delta) \simeq \sin(q(r - a_0)) \quad (\delta \simeq -a_0 q)$$

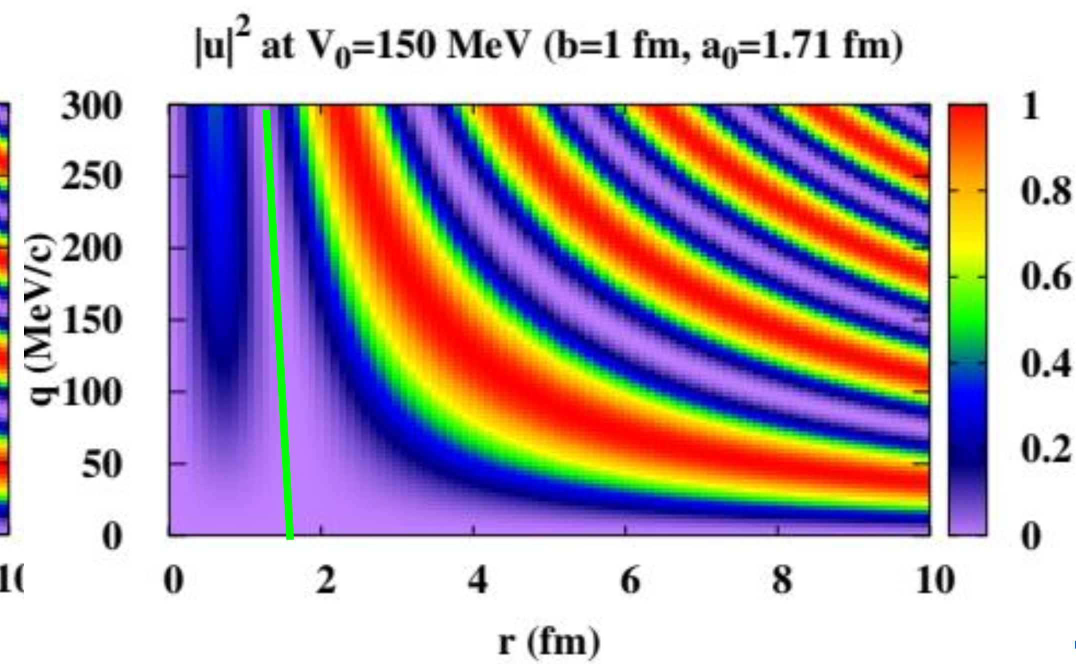


Square-Well Potential in 3D (3)



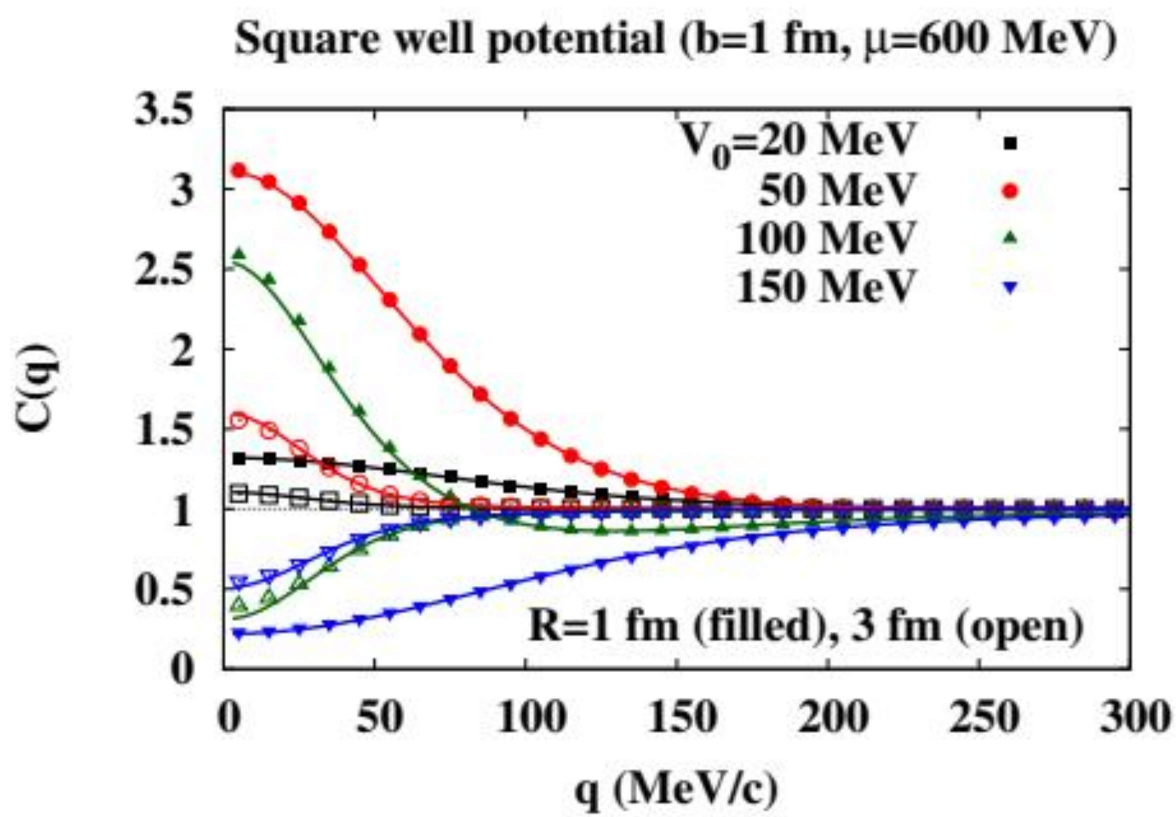






Correlation function from square-well potential

- **One dimensional integral (Combination of complex error fn.)**
- **Non-monotonic dep. on the potential strength**
  - **V_0 < 80 MeV (no bound state) → C(q) increases with V_0**
  - **V_0 > 80 MeV (with bound state) → C(q) is suppressed with large R**
- **Agrees with Lednicky-Lyuboshits analytical model results**



Analytic model of correlation function

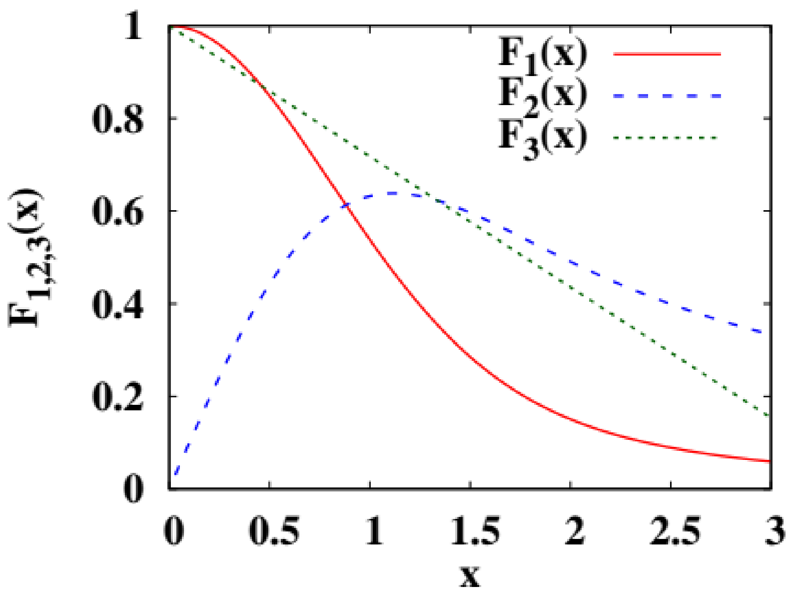
Correlation function in Lednicky-Lyuboshits (LL) formula

(asymptotic w.f., non-identical particle pair, short range int. (only s-wave is modified), single channel, no Coulomb pot., static Gaussian source, real δ) *(Lednickey, Lyuboshits ('82))*

$$\varphi_0^{(-)}(r;q) \simeq \frac{e^{-i\delta}\sin(qr+\delta)}{qr}\,,\quad f(q)=\frac{e^{i\delta}\sin\delta}{q}=\frac{1}{q\cot\delta - iq}\,,\quad q\cot\delta = -\frac{1}{a_0}+\frac{1}{2}r_{\mathrm{eff}}q^2+\mathcal{O}(q^4)$$

$$C_{\mathrm{LL}}(q) = 1 + \frac{2\mathrm{Re}\,f(q)}{\sqrt{\pi}R}F_1(2qR) - \frac{\mathrm{Im}\,f(q)}{R}F_2(2qR) + \frac{|f(q)|^2}{2R^2}F_3\left(\frac{r_{\mathrm{eff}}}{R}\right)$$

$$\left[f(q)=(q\cot\delta - iq)^{-1},\ F_1(x)=\frac{1}{x}\int_0^x dt e^{t^2-x^2},\ F_2(x)=(1-e^{-x^2})/x,\ F_3(x)=1-\frac{x}{2\sqrt{\pi}}\right]$$



If you have a_0, r_{eff} and R, you can draw C(q) !

$$F_1(x) \simeq \frac{1+c_1x^2+c_2x^4+c_3x^6}{1+(c_1+2/3)x^2+c_4x^4+c_5x^6+c_3x^8}\quad (0\le x<20)$$

$$(c_1,c_2,c_3,c_4,c_5)=(0.123, 0.0376, 0.0107, 0.304, 0.0617)$$

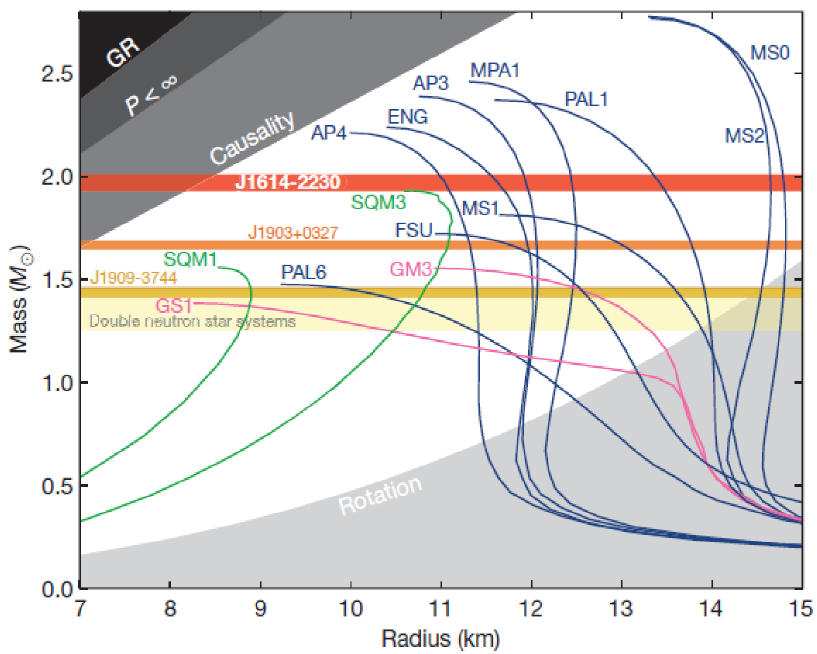
AO,Morita,Mihayara,Hyodo, NPA 954 ('16)294.

Hadron-Hadron Interactions and Bound States with Strangeness using Femtoscopy

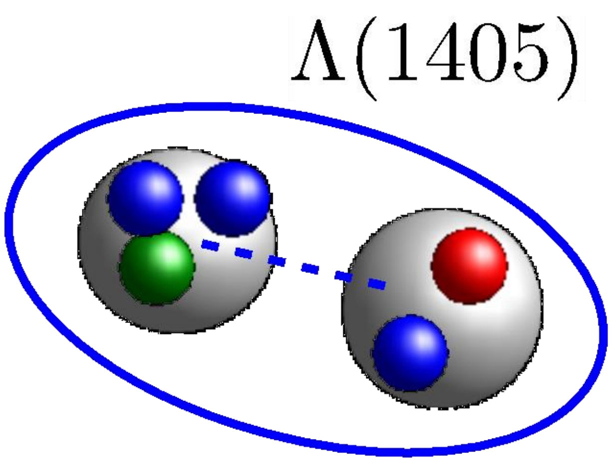
c.f. Talk by Harald Appelshauser (Mon)

Hadron-Hadron Interactions with Strangeness

- **Hyperonic matter EOS with empirical Hyperon-Nucleon (YN) interactions cannot support 2 solar mass neutron stars (Hyperon Puzzle).**
  - **YN interaction details ? YNN 3-body int. ? Many-body theories ? Gradual transition to quark matter ? Modified gravity ?**
- **Anti-kaon Nucleon interaction predicts a bound state Λ(1405), which is the starting point of "Hadronic Molecule".**



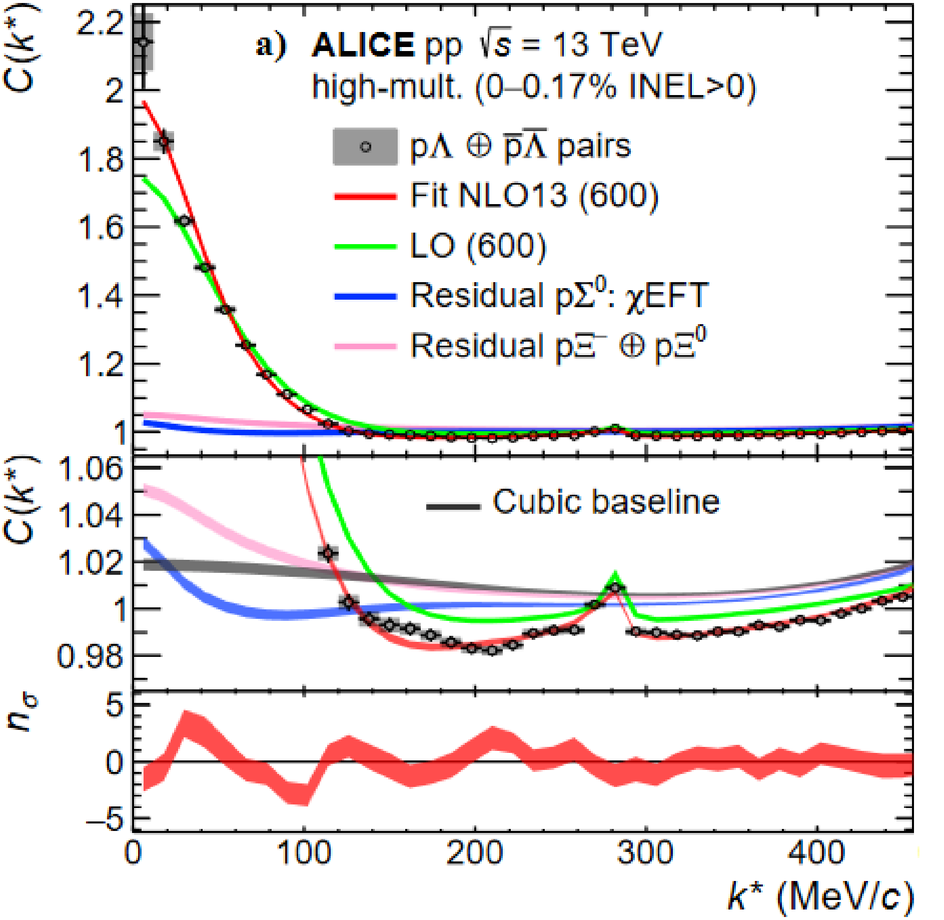
Demorest et al., Nature 467 (2010) 1081 (Oct.28, 2010).



S=-1: ΛN interaction

- **Scattering data exist, but only at high momenta (q>200 MeV/c).**
- **Constrained by hypernuclear spectroscopy, but indirect.**
- **Femtoscopy**
  - **High precision data including low momentum region.**
  - **NΣ cusp is clearly seen.**
  - **NLO chiral EFT better explains the data than LO.**

S. Acharya+[ALICE], PLB 833('22)137272

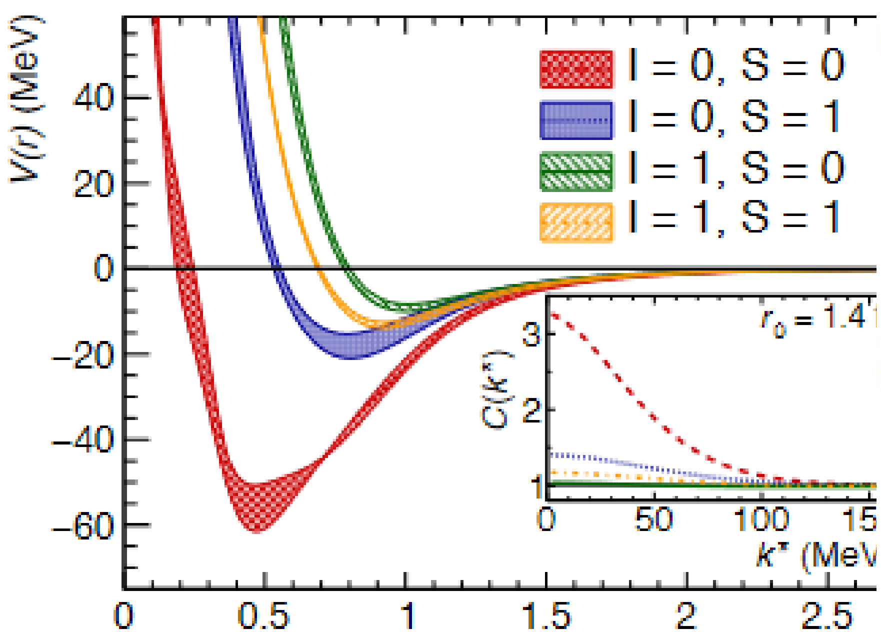


Femtoscopy would be valid alternative to scattering experiments (at least at low energies).
– Laura Fabbietti, ExHIC2016 @ YITP –

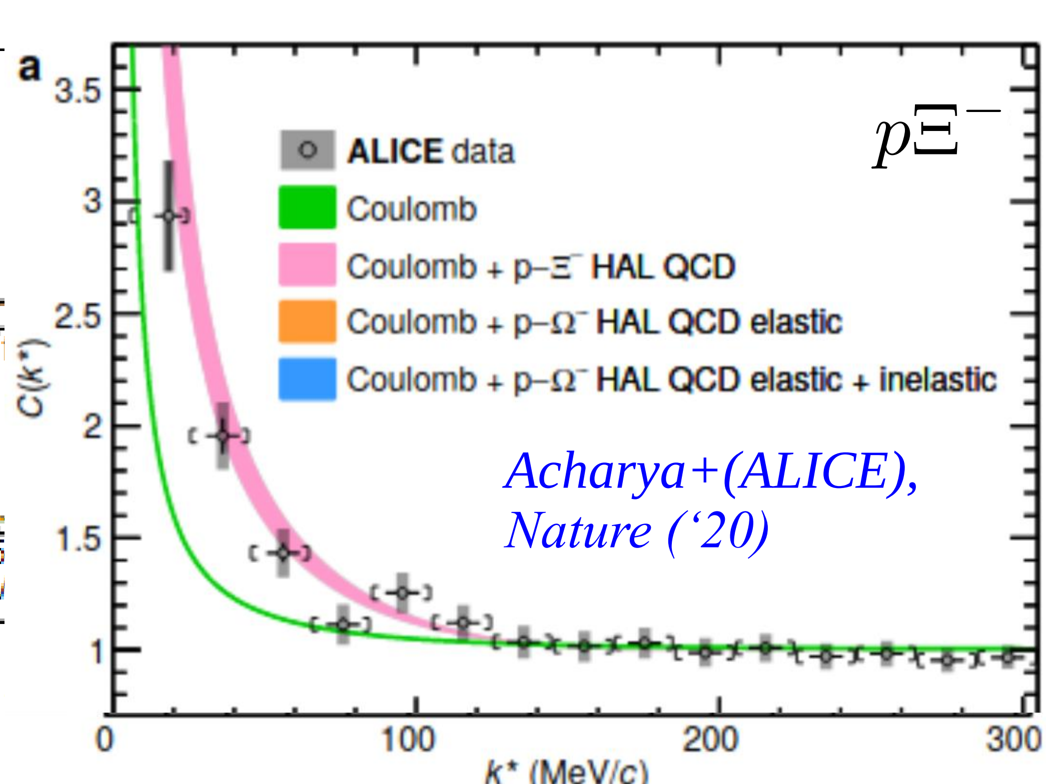
S=-2: ΞN interaction

- **Lattice S=-2 BB interaction** *K. Sasaki+[HAL QCD], NPA 998 ('20) 121737.*
- **Corr. Fn. data (from pp collisions)** *S. Acharya+ [ALICE] (Nature, '20)*

Lattice QCD S= –2 BB potential is examined!

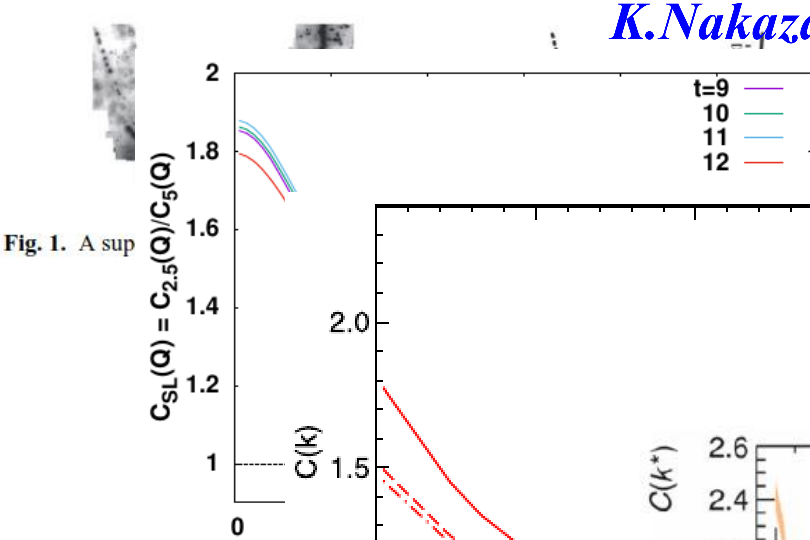


K. Sasaki+ [HAL QCD]('20)
Taken from L. Fabbietti+,
Ann. Rev. Nucl. Part. Sci. 71 ('21) 377.



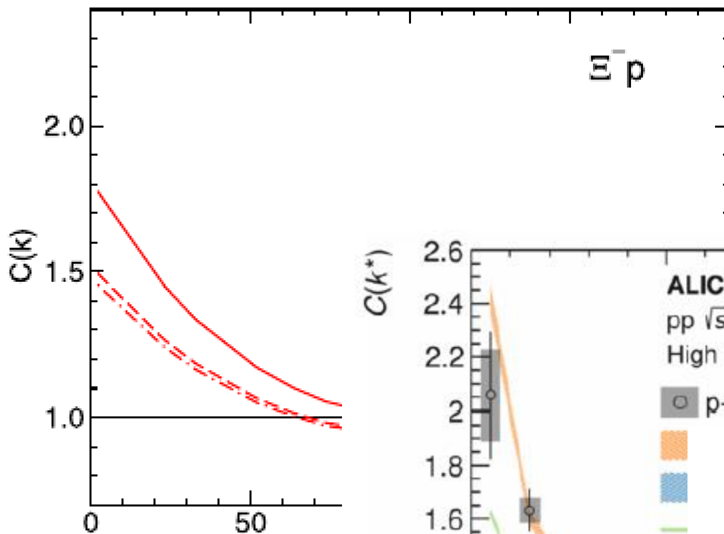
Acharya+(ALICE), Nature ('20)

Comparison with other results

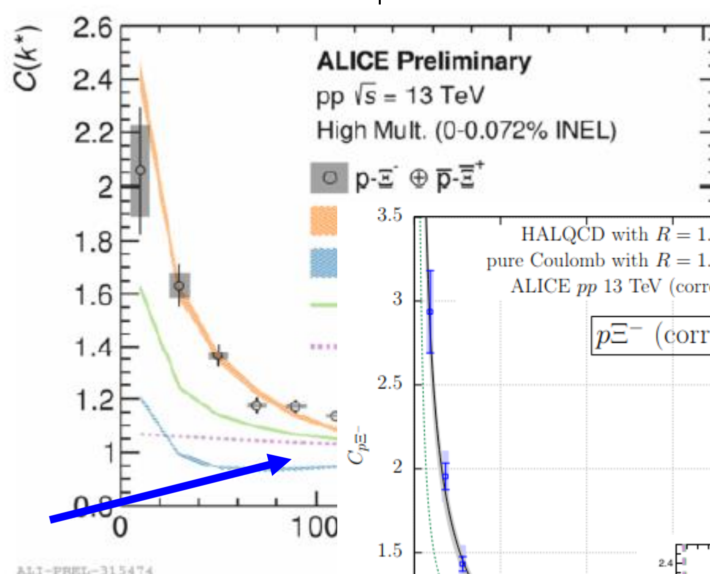


K.Nakazawa+, PTEP2015('15)033D02. Ξ hypernucleus (2015)

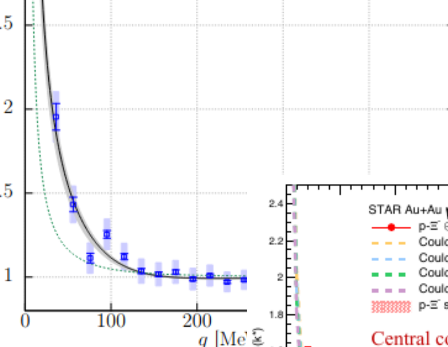
T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17)856. (heavier quark mass)



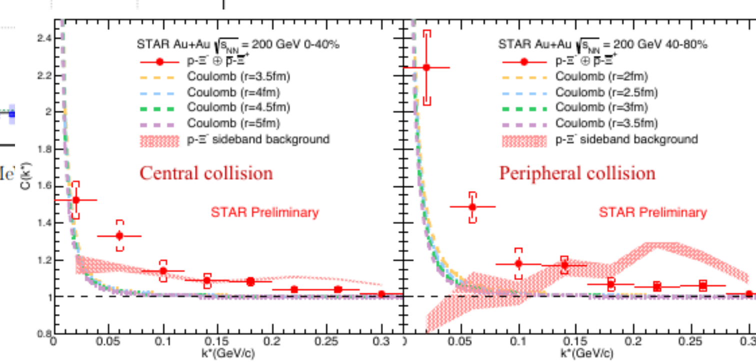
J. Haidenbauer, NPA981('19)1. (NLO(600), w/ CC effects, w/o Coulomb) (w/ Coulomb, it will be comparable with data.)



D. L. Mihairov+[ALICE], NPA1005('21)121760 (QM2019). (Nijmegen pot. does not explain the data.)



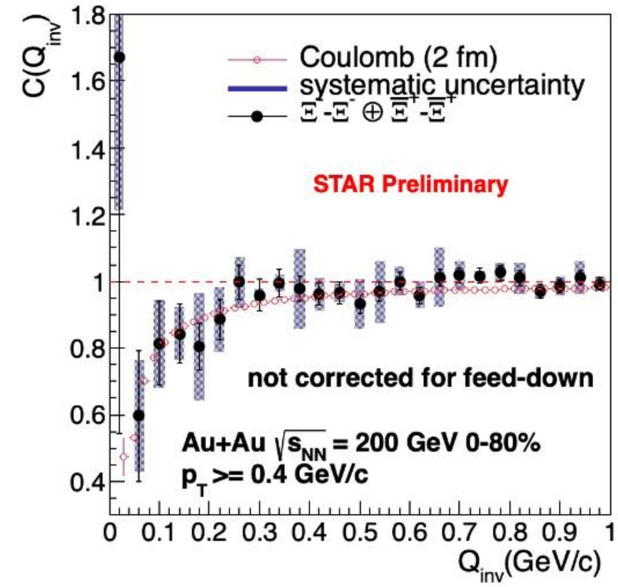
Kamiya+('22) (w/ Lattice BB pot. at phys. m_q and CC effects with ΛΛ)



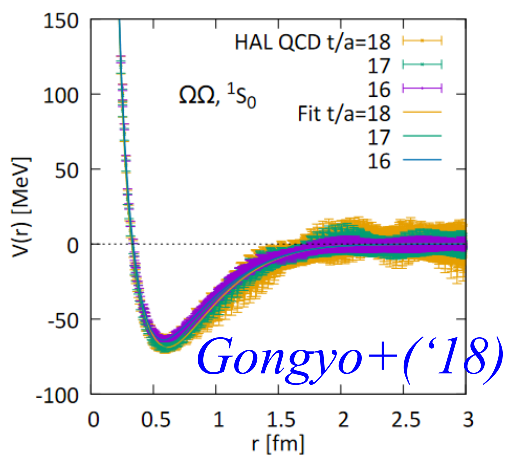
K. Mi+(STAR, preliminary), Au+Au 200 AGeV, APS2021.
(No Dip at larger R)

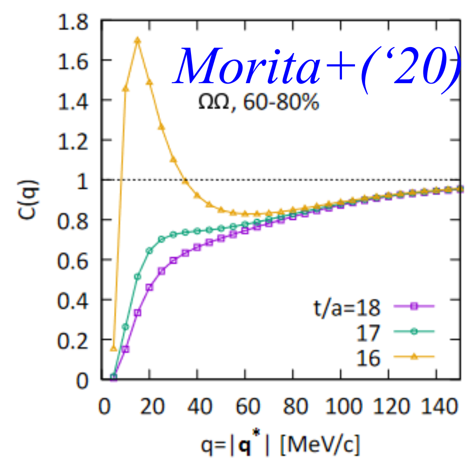
S ≤ -4 BB interactions: How far can we go?

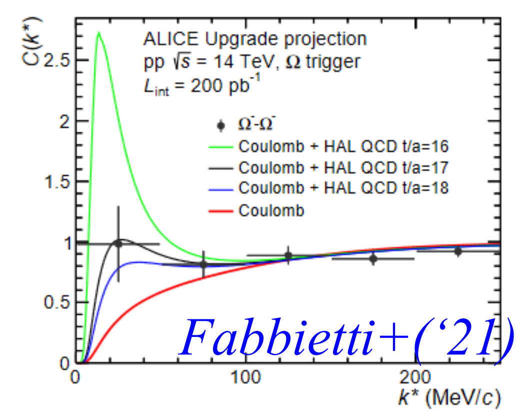
- **S=–4 (ΞΞ)**
  - **Chiral EFT and Lattice QCD predict attraction (no bound state)**
    J. Haidenbauer+('15); T. Doi+('18)
  - **STAR preliminary data seems to agree with pure Coulomb results.**
    (Attraction compensate quantum statistics)



- **S=-6 (ΩΩ, most strange BB pair)**
  - **Lattice QCD predicts a bound state (J=0)** *S. Gongyo+[HAL QCD]('18)*
  - **Prediction of the correlation function** *Morita+('20)*
  - **To be measured in LHC Run3** *L.Fabbietti+('21)*

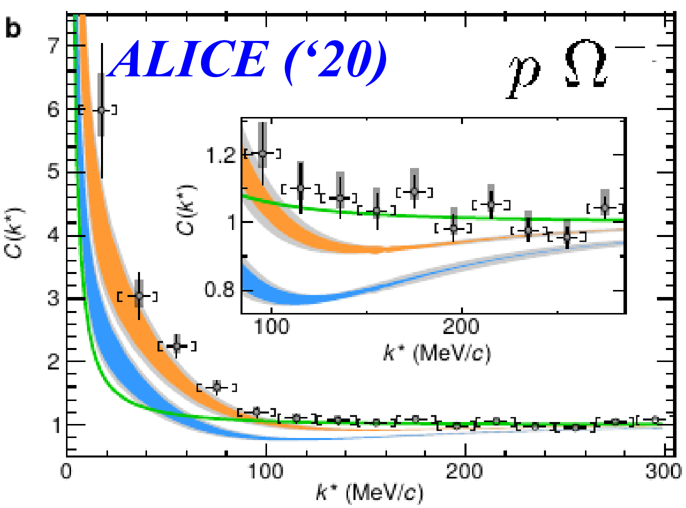


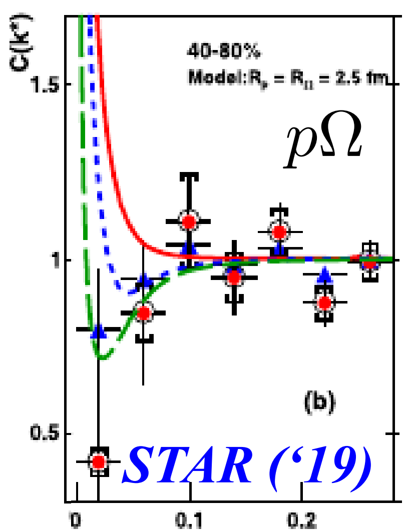


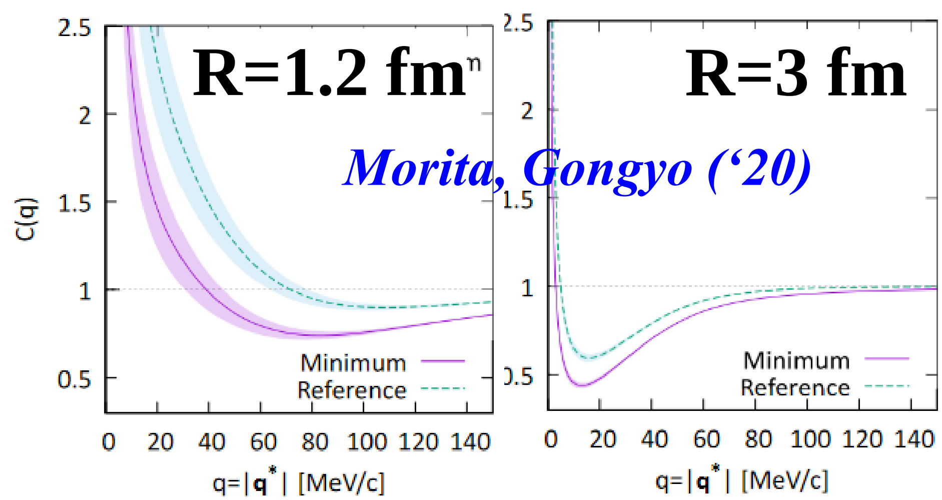


S=-3: ΩN interactions

- Ω : quark content=sss, J^π=3/2+, M=1672 MeV
- ΩN bound state as a S= –3 dibaryon ? *T.Goldman+('87); M.Oka('88).*
  (No Pauli blocking, Attractive one gluon exch. pot)
- Lattice QCD potential (HAL QCD, J=2) *F.Etminan+('14);T.Iritani+('19)*
- pΩ correlation functions *STAR('19); ALICE('20)*
  - Prediction using lattice potential *Morita+('16, '20)*
  - Lattice NΩ potential (J=2) seems reasonable.
  - J=1 component will fill the dip.
  - Source size dependence → Bound state?



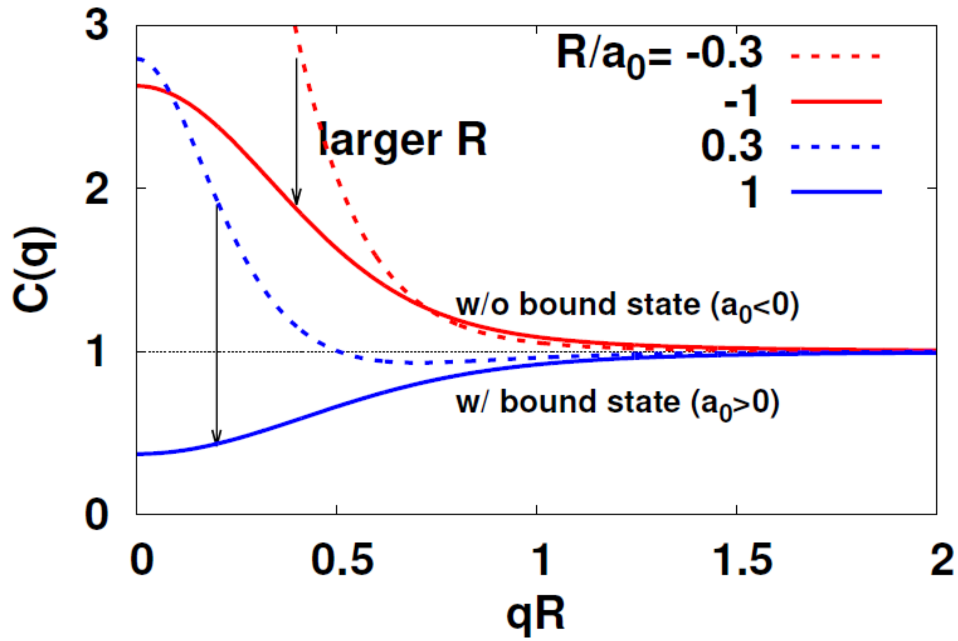




R Dependence of Correlation Function

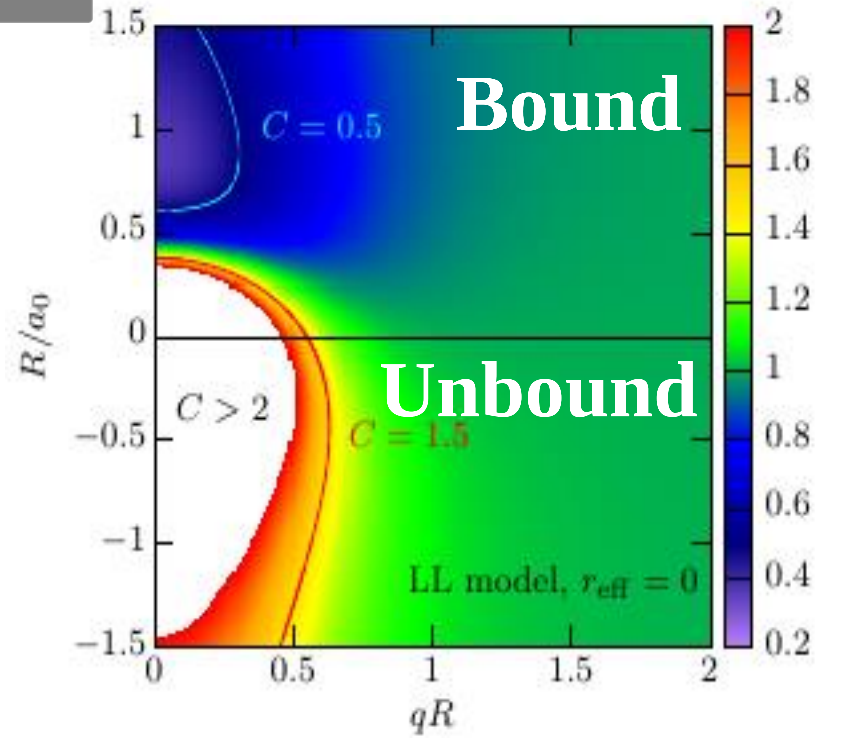
- **With a bound state, |w.f.|^2 is suppressed at r ~ |a_0|**
  → C(q) is suppressed at small q when R ~ |a_0|
  Morita+('16, '20), Kamiya+('20, '22)

One can guess the existence of a bound state from R dep. of C(q)



K.Morita+('20)

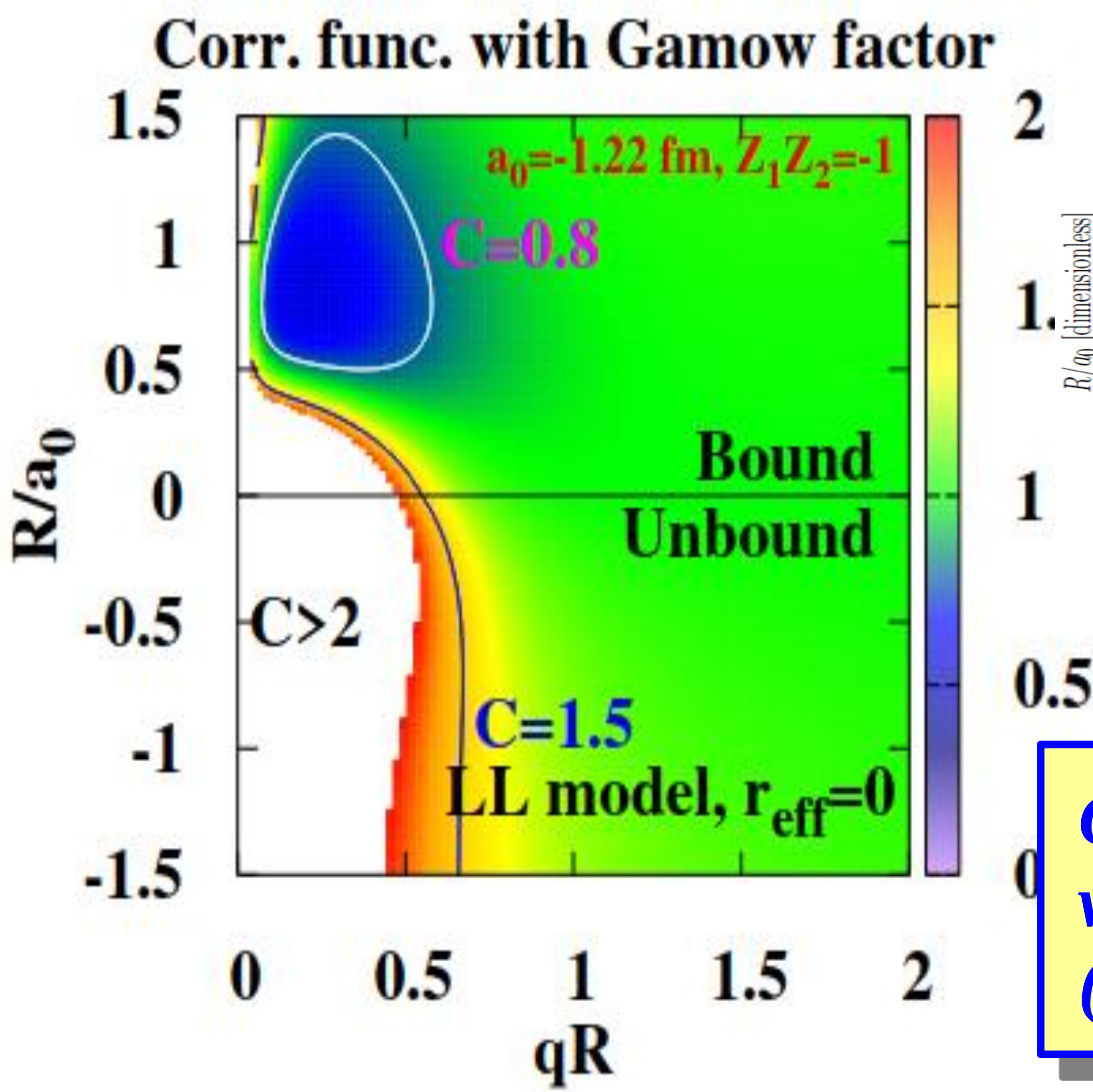
w/o Coulomb



Y.Kamiya+('22)

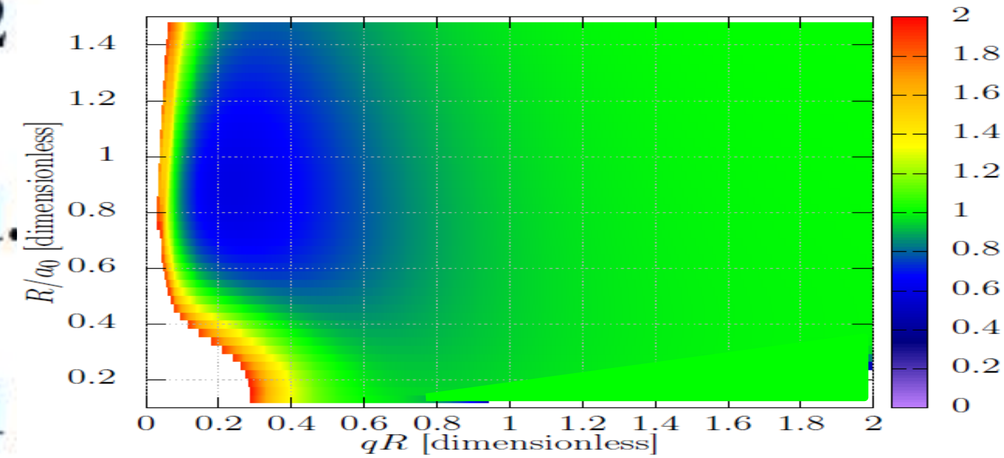
R Dependence of Correlation Function

LL model with Coulomb
(r_{eff}=0)



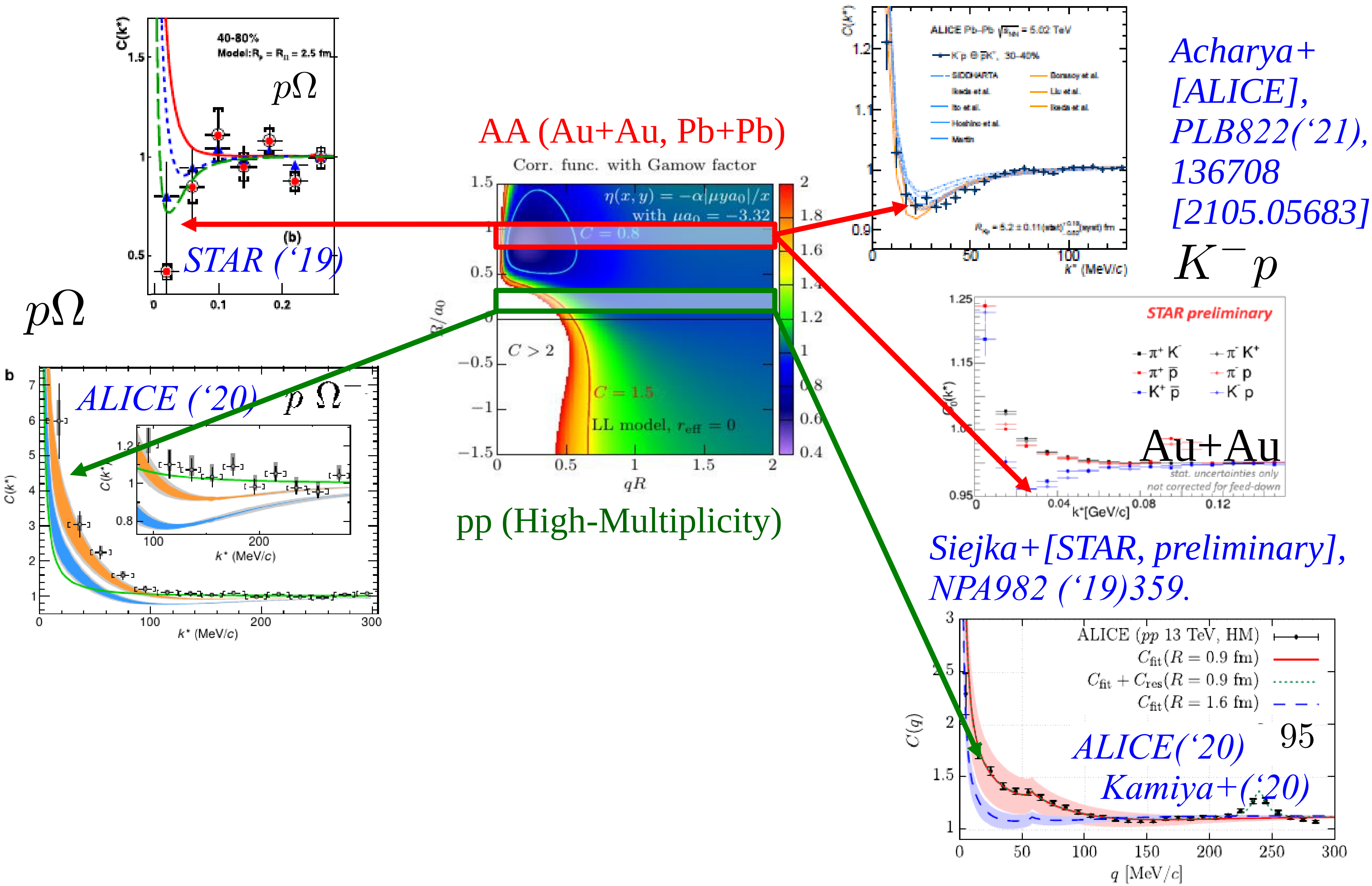
Realistic NΩ potential
(J=2, HAL QCD, a_0=3.4 fm)
+ Coulomb, Coupled-channel

Courtesy of Y. Kamiya



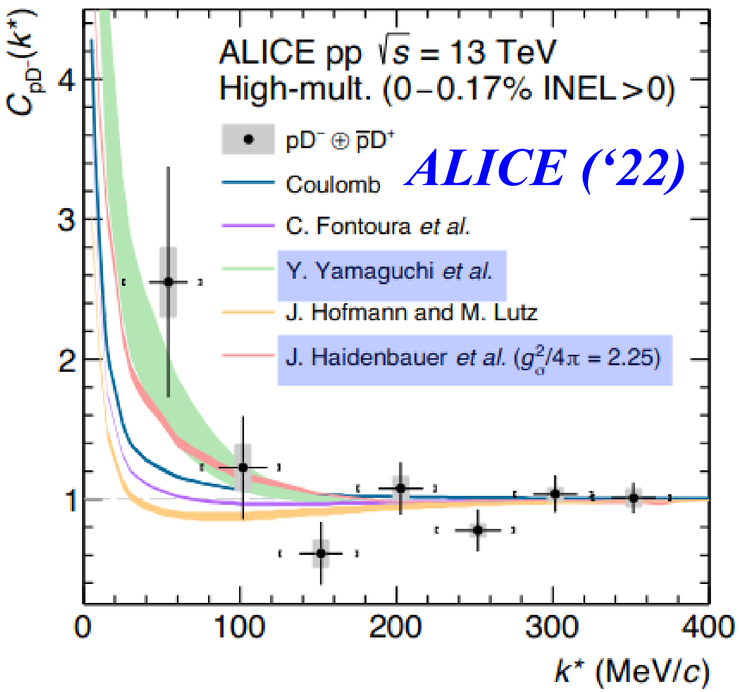
Qualitative feature remains with realistic interactions (and coupled-channel effects)

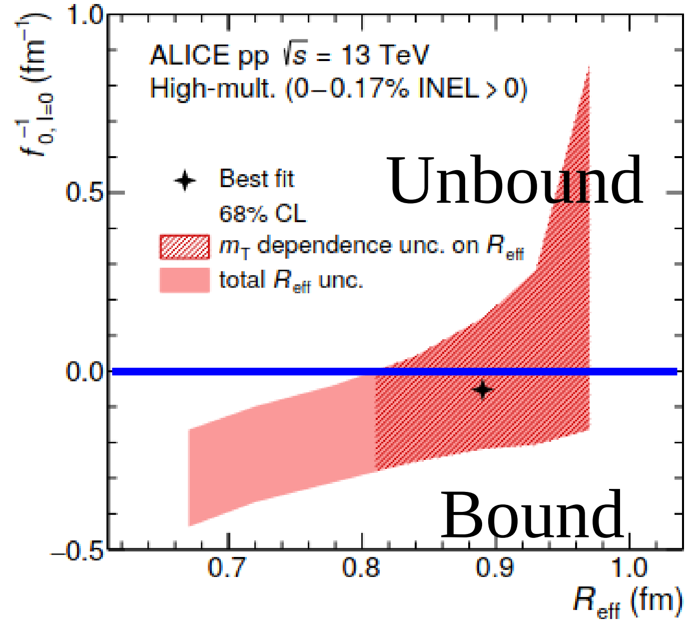
Bound State Dip



How about D⁻ p ?

- **"First study of the two-body scattering involving charm hadrons"**
  Acharya+[ALICE] (2201.05352)
  - **Enhanced CF from Coulomb → attractive potentials are favored**
  - **If bound, $\Theta_c(=\bar{c}uudd)$ is a bound hadronic molecule**



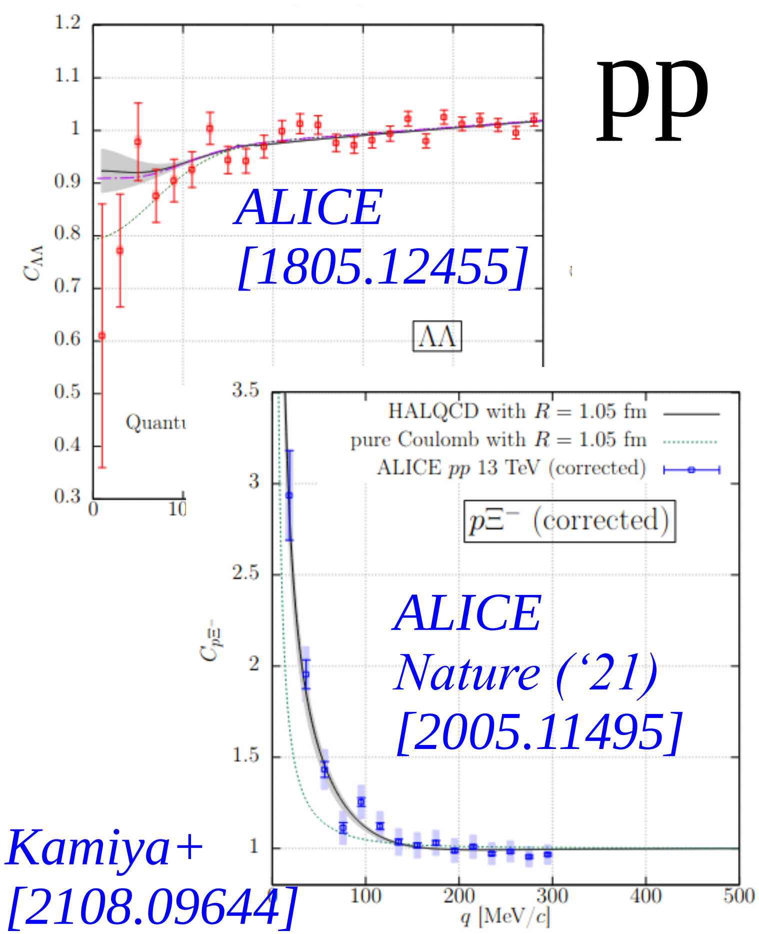


Fontoura+(1208.4058) (weakly attractive (I=0)) n_σ=(1.1-1.5)
Yamaguchi+ (1105.0734) (att., w/ bound state (I=0))n_σ=(0.6-1.1)
Hofmann, Lutz (hep-ph/0507071) (repulsive (I=0)) n_σ=(1.3-1.6)
Haidenbauer+(0704.3668) (weakly / mildly attractive (I=0)) n_σ=(1.2-1.5 / 0.8-1.3)

Case without a bound state

- **ΛΛ and NΞ seem to be unbound from lattice QCD calculation !**
  Sasaki+ [HAL], NPA998 ('20)121737 [1912.08630]
- **Source size dependence of ΛΛ and pΞ⁻ correlation functions**
  → No dip or suppressed behavior in AA collisions.

pp

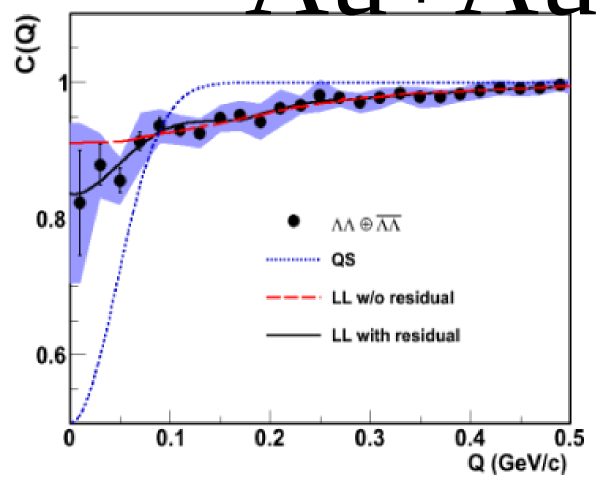


ALICE [1805.12455]

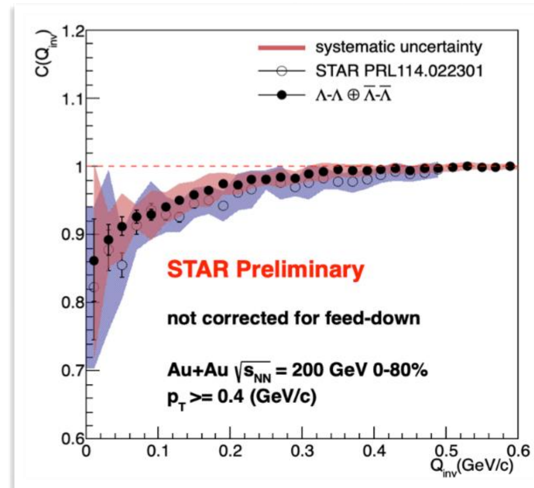
ALICE Nature ('21) [2005.11495]

Kamiya+ [2108.09644]

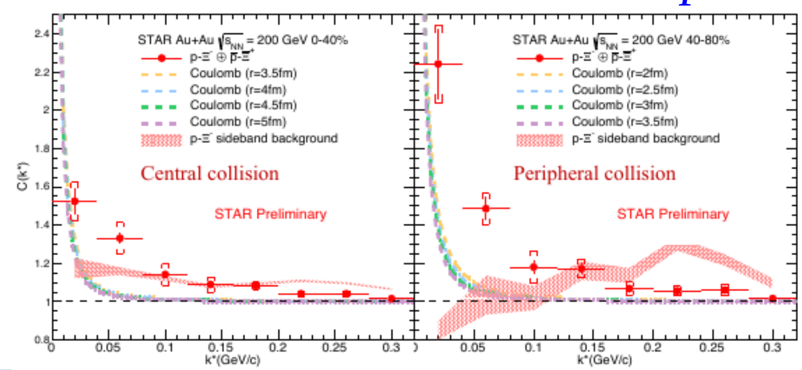
Au+Au



STAR, PRL('15) [1408.4360]



Moe Isshiki+ (STAR, prelim., 2109.10953).

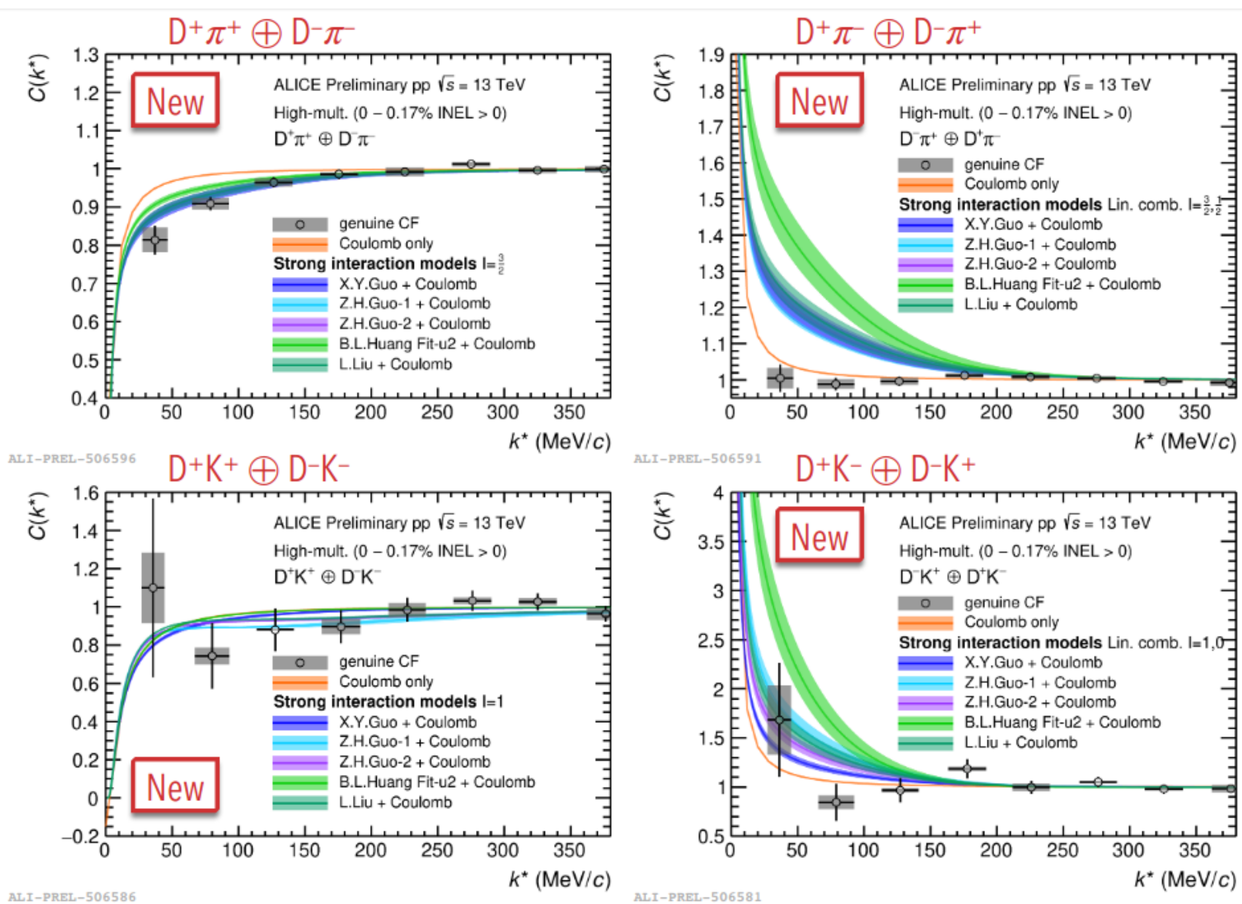


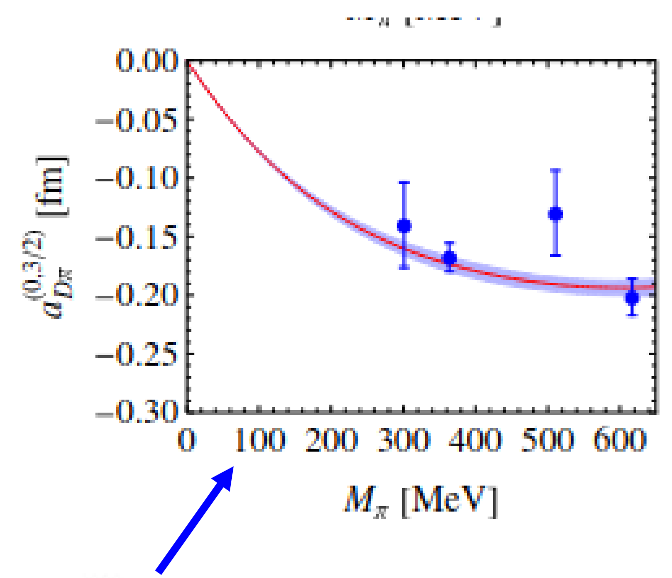
K. Mi+ (STAR, preliminary), Au+Au, APS2021.

Homeworks to hadron physicists from femtoscopy

Homework to Hadron Physics (1)

- **Present chiral models do not explain $D\pi$ and $D\bar{K}$ correlation.**
  - **Overestimate C(D$^+\pi^-$) → Mystery ? Extrapolation to phys. mass ?**
    Leading order = Weinberg-Tomozawa (vector exch., repulsive)
    Further repulsive interaction ?
  - **Overestimate C(D$^+$K$^-$) → Further repulsion or bound state ?**





- L. Liu et al, Phys. Rev. D87 (2013) 014508
- X.-Y. Guo et al, Phys. Rev. D 98 (2018) 014510
- B.-L. Huang et al, Phys. Rev. D 105 (2022) 036016
- Z.-H. Guo et al Eur. Phys. J. C 79 (2019) 13

Fabrizio Grosa@QM2022

Homework to Hadron (Nuclear) Physics (2)

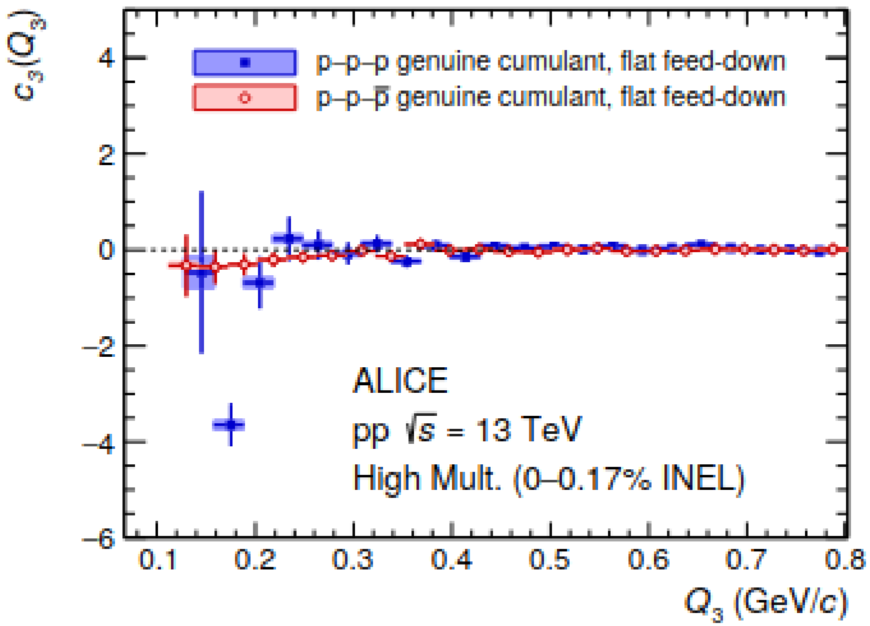
- **Proton-Deuteron Correlation function** *c.f. Bhawani Singh (Fri)*
  - **K^+d C(q) is well explained with f_0 ~ -0.5 fm** *Haidenbauer, Hyodo*
  - **Other hd corr. fn. are also described approximately by two-body potential** *S.Mrowczynski, P.Słon, Acta Phys.Pol.B51('20),1739; J.Haidenbauer, PRC102('20)034001; F.Etminan, M.M.Firoozabadi, (1908.11484); K.Ogata, TFukui, Y.Kamiya, AO, PRC103('21),065205.*
  - **pd correlation function cannot be explained by using LL formula or potential model with known f_0. One needs to solve Faddeev Eq.**
    Michael Viviani's talk (Wed)
  - **Rearrangement?**
    Anti-sym. of
    two protons?

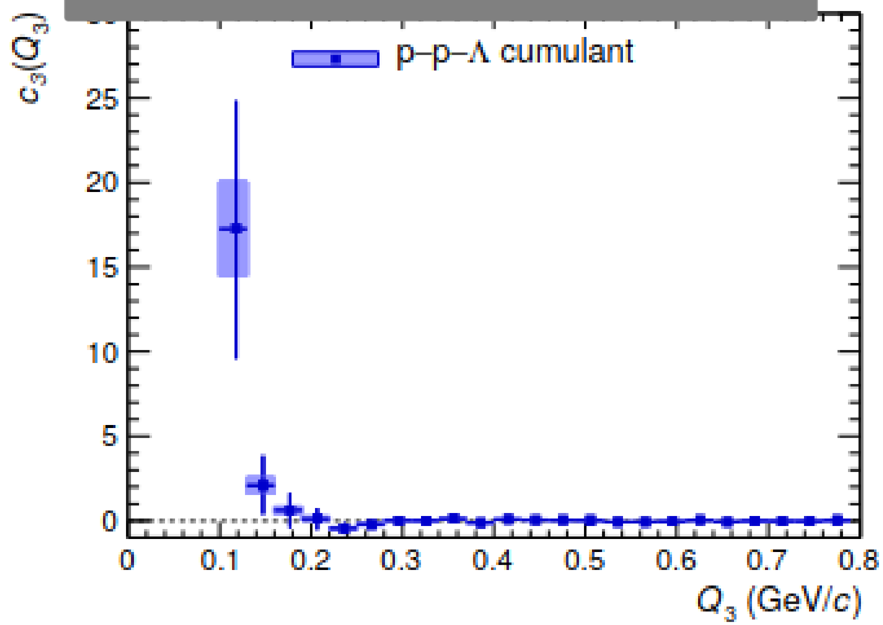
K^+d and pd correlation function from ALICE.
To be reported.

Homework to Hadron (Nuclear) Physics (3)

- **Three-body correlation function (c.f. Bhawani Singh (Fri))**
  - **Cumulant** $$c_3 = C_{123} - C_{12} - C_{23} - C_{31} + 2$$
  - **Can we extract three-baryon repulsion ?**
    (important to solve the hyperon puzzle)
    $\rightarrow$ **One needs to solve continuum three-body w.f. with Coulmb potential.**

Theoretical challenge



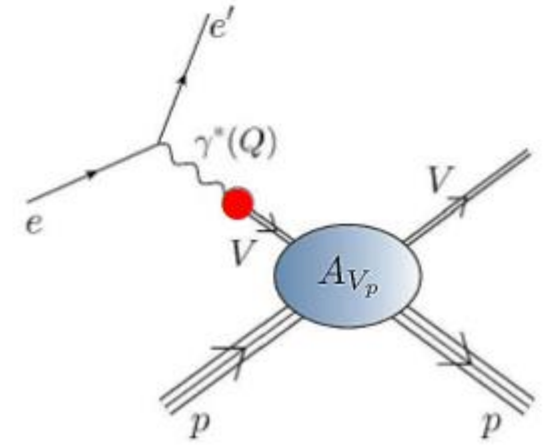


ALICE [2206.03344] (Raffaele Del Grande @QM2022)

Homework to Hadron Physics (4)

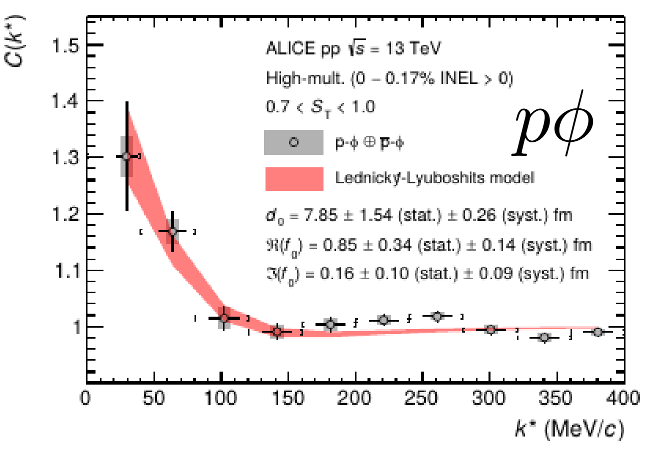
- **Correlation function including vector mesons**
  - **Femtoscopy** ***ALICE (PRL, 2105.05578)***

$$a_0(\phi p) = 0.85 + i0.16 \text{ fm}$$



  - **Contradiction with the photo production ? scattering length is O(0.1 fm)**
    E.g. Strakovsky, Pentchev, Titov (2001.08851)

$$|a_0(\phi p)| = (0.063 \pm 0.010) \text{ fm}$$



  - **Smaller than lattice QCD result (J=3/2) ?**
    Lyu, Doi, Hatsuda, Ikeda (2205.10544)

$$a_0(\phi p, J = 3/2) = 1.43 \text{ fm}$$

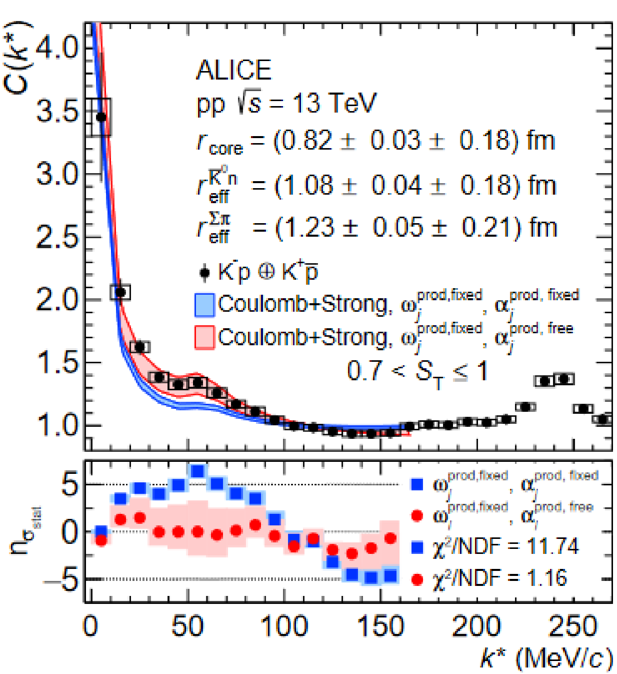
  - **Bound state in J=1/2 channel**
    E. Chizzali, Y. Kamiya, R. Del Grande, T. Doi, L. Fabbietti, T. Hatsuda, Y.Lyu (2212.12690).

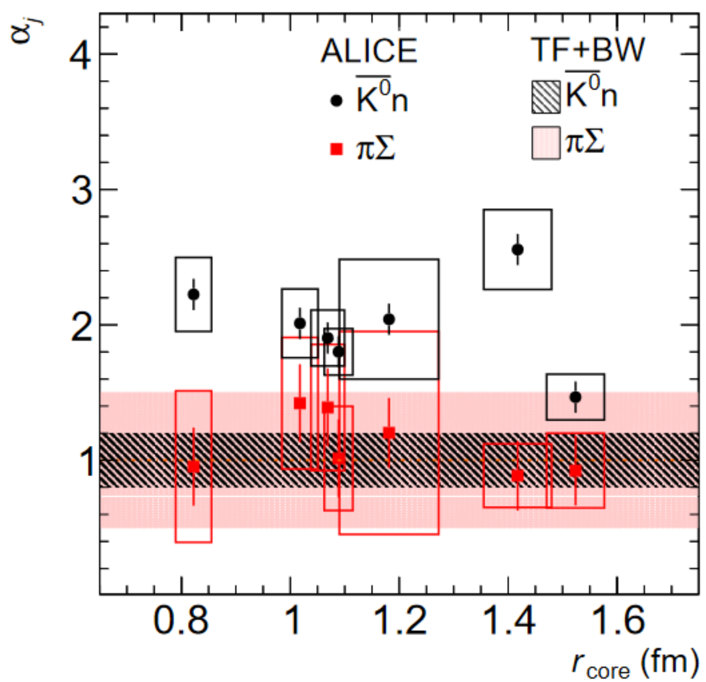
ALICE, 2105.05578

Homework to Hadron Physics (5)

- $K^- p$ Correlation function
  - AA collisions → consistent with atomic data and the potential from chiral SU(3) dynamics *ALICE('21)*
  - pp collisions → C(q) can be explained with the potential from chiral SU(3) dynamics by tuning the source weight of $\pi\Sigma$ *ALICE('20), Kamiya+('20)*
  - pA collisions → Source weight of $K^0 n$ needs to be doubled! *ALICE (2205.15176)*

KN potential needs update. (Transition pot. should be 1.4 times larger.)





Summary

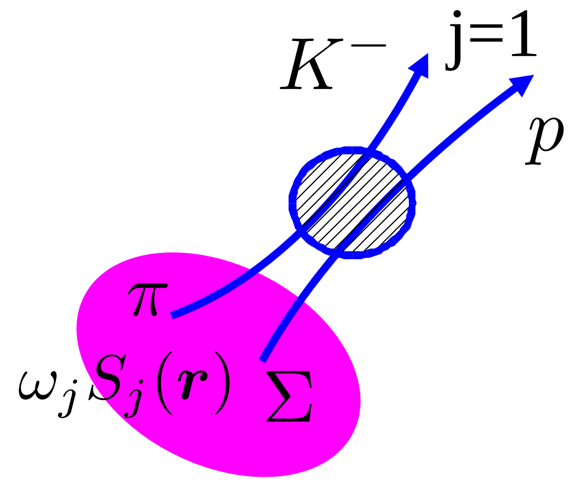
- Femtoscopy has doubled the number of experimentally accessible hadron-hadron interactions in 8 years (2015–2023).
  - The number is still growing rapidly.
- Some of the hadron-hadron interactions have been constrained.
  - First-principles methods in QCD, chiral EFT and lattice QCD (HAL QCD), have been found to be successful in femtoscopy.
- Source size dependence of the correlation function is helpful to guess the existence of a bound state.
  - $\bar{K}$N (→ Λ(1405)), ΩN (=sssqqq), D⁻p (= $\bar{c}uud$)(marginal), $N\phi$(J=1/2) may have a bound state.

- High-precision data now requires higher quality theoretical results and/or updates.
- It may be the time (for me) to consider (go back to) dynamical sources (hydro / transport) in femtoscopy.
  - Scott Pratt and STAR are working on it!

Thank you for your attention !

Correlation function with coupled-channel effects

- **KPLLL formula = CC Schrodinger eq. under $\Psi^{(-)}$ boundary cond. + channel source**
  Koonin('77), Pratt+('86), Lednicky-Lyuboshits-Lyuboshits ('98), Heidenbauer ('19), Kamiya, Hyodo, Morita, AO, Weise ('20).



$$\Psi^{(-)}(\boldsymbol{q};\boldsymbol{r}) = [\phi(\boldsymbol{q};\boldsymbol{r}) - \phi_0(q;r)]\,\delta_{1j} + \psi^{(-)}(q;r)$$

$$\psi_j^{(-)}(q;r) \to \frac{1}{2iq_j}\left[\frac{u_j^{(+)}(q_j r)}{r}\delta_{1j} - A_j(q)\frac{u_j^{(-)}(q_j r)}{r}\right]$$

$$\boxed{C(q) = \int d\boldsymbol{r} S_1(r)\left[|\phi(\boldsymbol{q};\boldsymbol{r})|^2 - |\phi_0(q;r)|^2\right] + \sum_j \int d\boldsymbol{r}\,\omega_j S_j(r) |\psi_j^{(-)}(q;r)|^2}$$

- **No Coulomb** $\phi(\boldsymbol{q};\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}}, \phi_0(q;r) = j_0(qr), u_j^{(\pm)}(qr) = e^{\pm iqr},$

$$A_j(q) = \sqrt{(\mu_j q_j)/(\mu_1 q_1)} S_{1j}^\dagger(q_1)\ \ (S_{ji} = i \to j \text{ S-matrix})$$

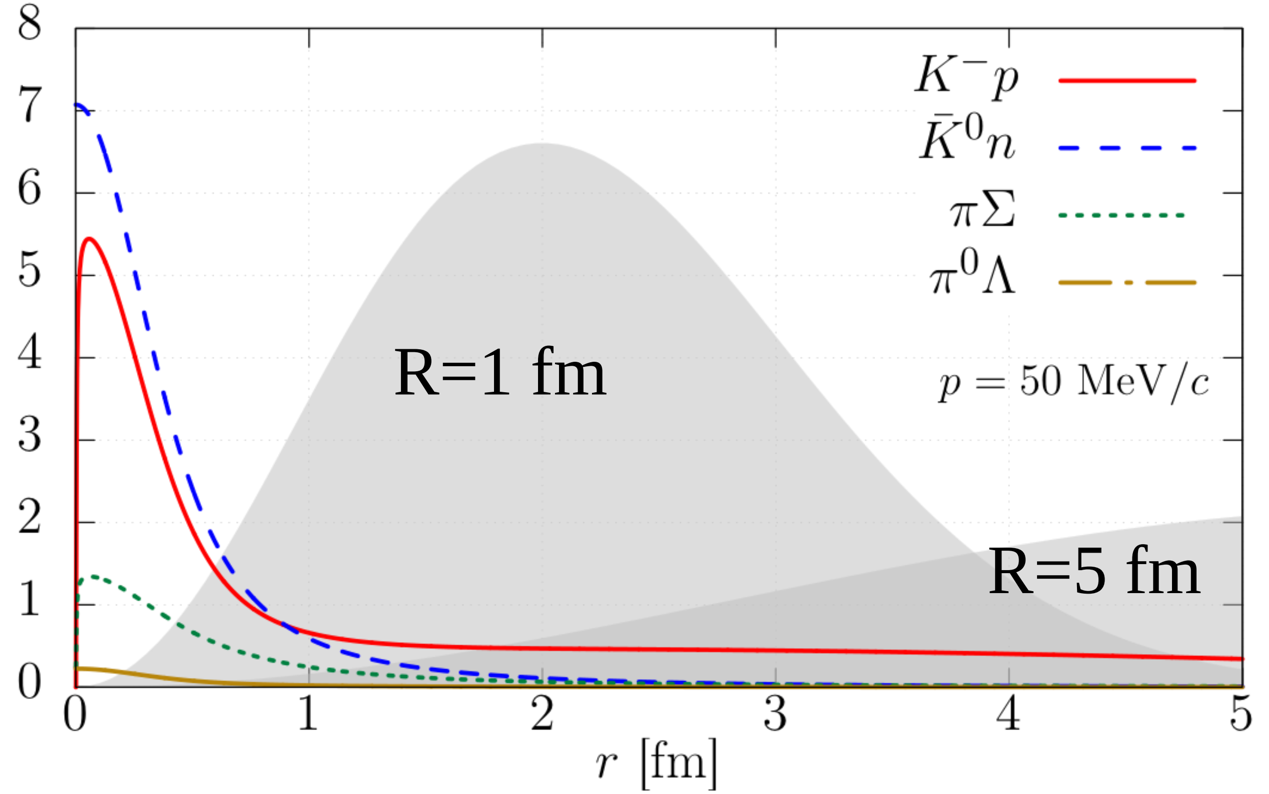
- **With Coulomb**

$$\phi(\boldsymbol{q};\boldsymbol{r}) = \text{Full Coulomb w.f.}, \phi_0(q;r) = \text{s-wave Coulomb w.f.},$$

$$u_j^{(\pm)}(qr) = \pm e^{\mp i\sigma_j}\left[iF(qr) \pm G(qr)\right] (F, G = \text{regular (irregular) Coulomb fn.})$$

Discriminating Coupled-Channel Effects

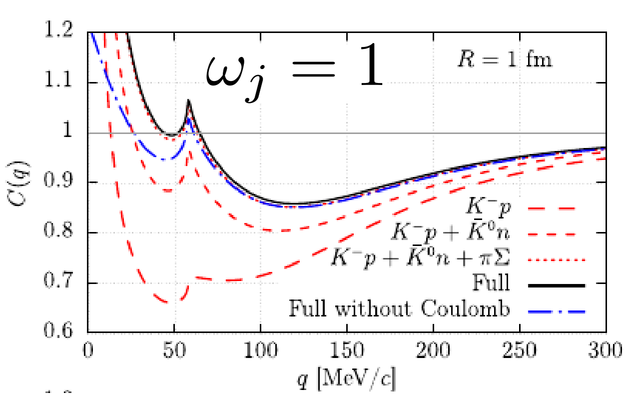
- **Source size dependence again !**
  - **Unmeasured coupled-channel wave functions disappear soon.**
    → CFs with large source is dominated
    by the measured channel wave function !
  - **Scattering parameters from CFs with large source**
    Coupled-channel effects from CFs with small source.

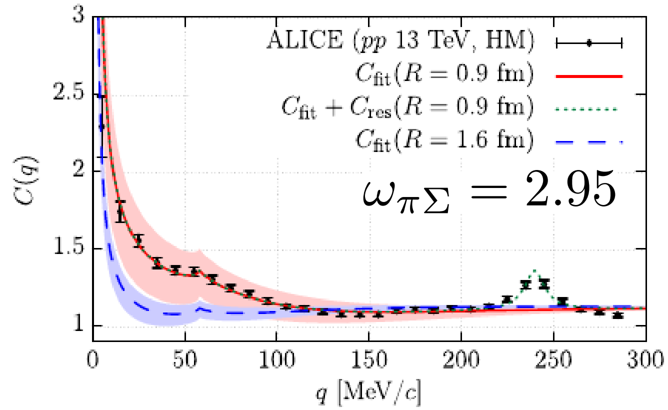


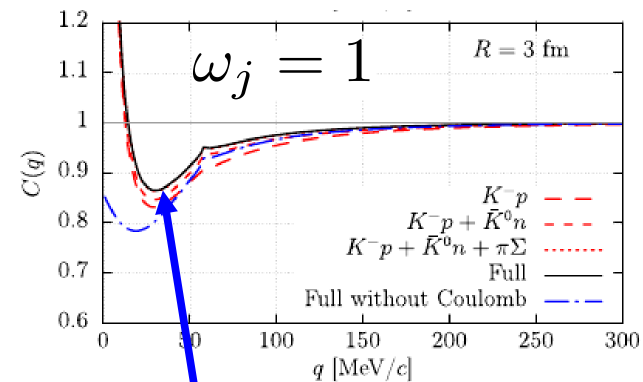
w.f. Kamiya+,arXiv:1911.01041v1

Source Size Dependence of C(pK $^-$)

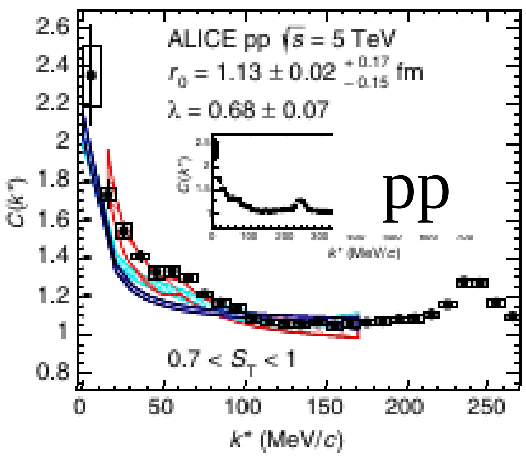
- **Coupled-channel effects are suppressed when R is large, and "pure" pK$^-$ wave function may be observed in HIC.**

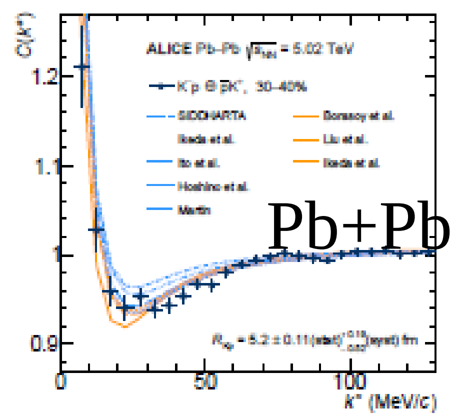


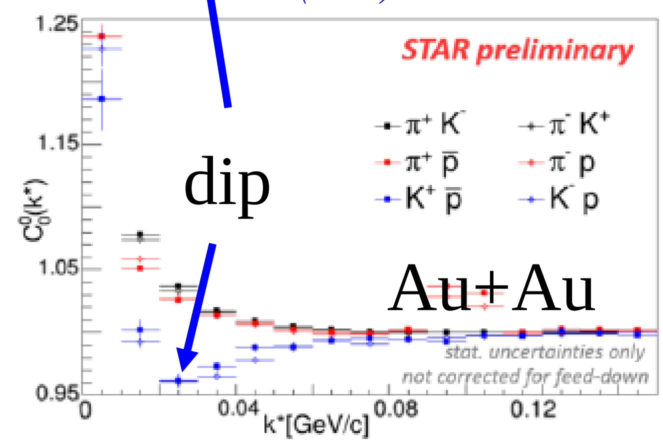




Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.







S. Acharya+[ALICE], PRL124('20)092301

S. Acharya+[ALICE], 2105.05683

Siejka+[STAR, preliminary], NPA982 ('19)359.

> *STAR(prel.) & new ALICE data show a dip at small q.*

Scattering length from K^-p correlation function

- **LL model fit (w/ Coulomb) to the correlation function data**

  S. Acharya+[ALICE], PLB 822 ('21) 136708 [2105.05683] ($\delta \sim +a_0 q$, HEP convention)

  $$a_0 = -0.91 \pm 0.03(\text{stat})^{+0.17}_{-0.03}(\text{syst}) + i[0.92 \pm 0.05(\text{stat})^{+0.12}_{-0.33}(\text{syst})]\ \text{fm}$$

- **Consistent with SIDDHARTA (kaonic atom) data, and errors are comparable to previous dedicated experiments.**

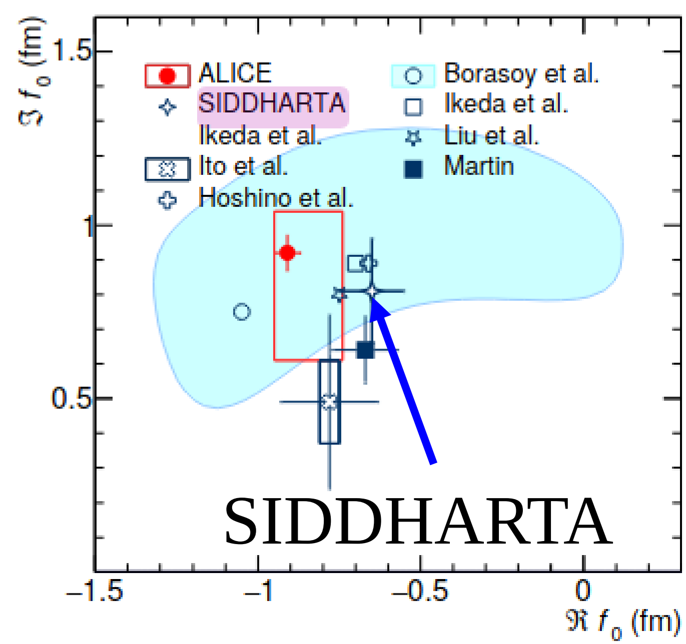
  M. Bassi et al. [SIDDHARTA], NPA 881 ('12) 88 [1201.4635]

  $$a_0 = -0.65 \pm 0.10 + i[0.81 \pm 0.15]\ \text{fm}$$

- **Femtoscopy reconfirmed $\bar{K}N$ bound state nature of $\Lambda(1405)$**

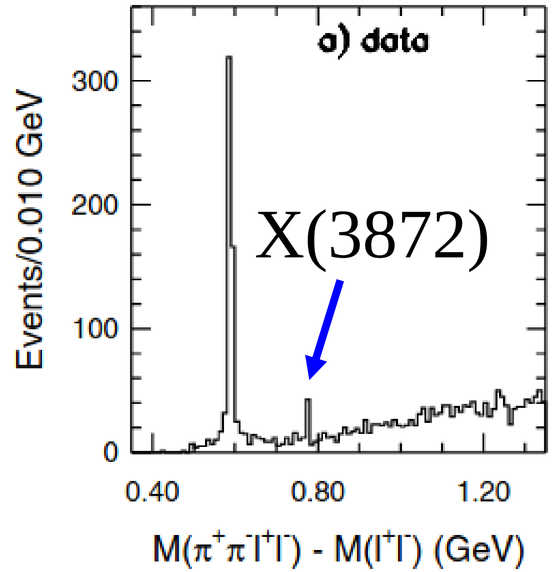
Table 4: Values of the scattering parameters and the χ^2/ndf for the deviation between the ALICE data and available model calculations and previous measurements for K^-p pairs at low relative momentum.

Model calculation:	$\Re f_0$ (fm)	$\Im f_0$ (fm)	χ^2/ndf
Lednický–Lyuboshitz fit to data	$-0.91 \pm 0.03(\text{stat})^{+0.17}_{-0.03}(\text{syst})$	$0.92 \pm 0.05(\text{stat})^{+0.12}_{-0.33}(\text{syst})$	1.4
Kyoto [39, 80]	–	–	2.8
Lednický–Lyuboshitz with fixed parameters from:			
Kaonic deuterium (Hoshino et al.) [78]	−0.66	0.89	2.0
Scattering experiments (Martin) [75]	−0.67±0.1	0.64±0.1	3.3
Chiral SU(3) (Ikeda et al.) [17, 18]	−0.7	0.89	1.9
SIDDHARTA chiral SU(3) [17, 18]	−0.65±0.1	0.81±0.15	2.3
Hamiltonian EFT (Liu et al.) [77]	−0.75	0.80	1.9
Kaonic hydrogen (Ito et al.) [76]	−0.78±0.15	0.49±0.25	4.2
Chiral SU(3) (Borasoy et al.) [79]	−1.05±0.5	0.75±0.4	1.6



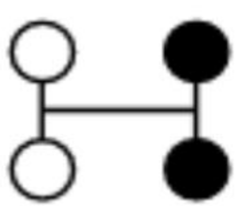
Exotic Hadrons including $c\bar{c}$ / cc / $\bar{c}\bar{c}$

- **Main play ground of exotic hadron physics**
  - **X(3872)** ***Belle ('03)*** $c\bar{c}q\bar{q}$ Beijing Spectrometer
  - **Many X,Y,Z states**
    Belle, CDF, BaBar, LHCb, CMS, BESIII, ...
  - **Charmed pentaquark Pc** ***LHCb ('15, '19)***
  - **Doubly charmed tetraquark state Tcc**
    LHCb ('21) $cc\bar{q}\bar{q}$

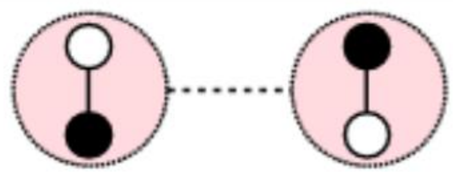


S.K.Choi+[Belle], PRL91, 262001 ('03)

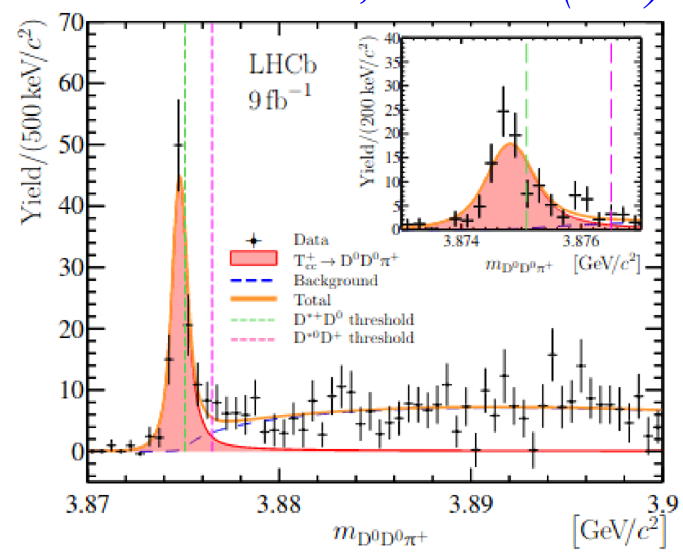
- **Structure of exotic hadrons**
  - **Compact multiquark states**
    → "good" [ud] diquark gains energy
  - **Hadronic molecules**
    → Many exotic states around thresholds
  - **Their mixture...**



Tetraquarks



Hadronic Molecules

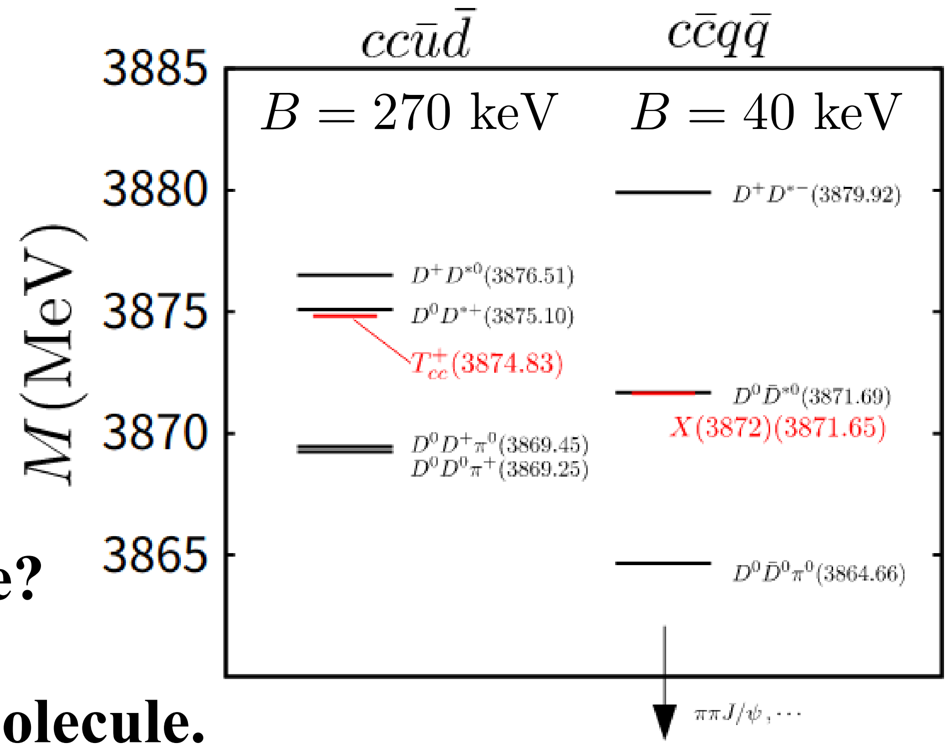


R. Aaji+ [LHCb], 2109.01038, 2109.01056

Compact Tetraquarks or Hadronic Molecules

- **Tcc = Compact Tetraquark ?**
  Good $[\bar{u}\bar{d}]$ diquark gains energy
  S. Zouzou+('86), ZPC30,457.

- **X(3872)**
  - **$c\bar{c}$ component ? production cross section** *Bignamini+ (0906.0882)*
  - **Large yield in Pb+Pb → Molecule?**
    Sirunyan+ [CMS] (2102.13048)
    c.f. Δr/Δp is similar in HIC and molecule.
    ExHIC ('11,'11,'17)



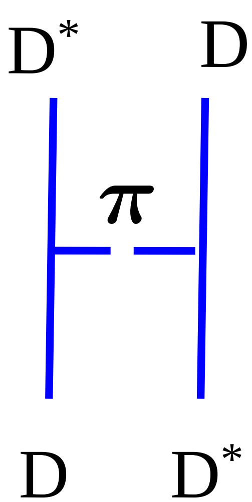
- **Hadronic Molecule Conditions**
  - **Appears around the threshold → OK**
  - **Have large size $R \simeq 1/\sqrt{2\mu B}$ → Yield**
  - **Described by the *hh* interaction**

How can we access hh int. with charm ?
→ Femtoscopy

Femtoscopic study of charmed hadron int.

- DD^* and $D\bar{D}^*$ correlation functions. *Kamiya, Hyodo, AO (2203.13814)*
  - Related with Tcc and X(3872)
  - *ALICE run3 can measure the correlation functions.*
- Model interaction
  - Range = one pion exchange *Yasui, Sudoh (0906.1452)*
  - Strength is fitted to the pole mass.
  - Isospin dep.
    - I=0: One range gaussian, strength fitted to the mass
    - I=1: ignored

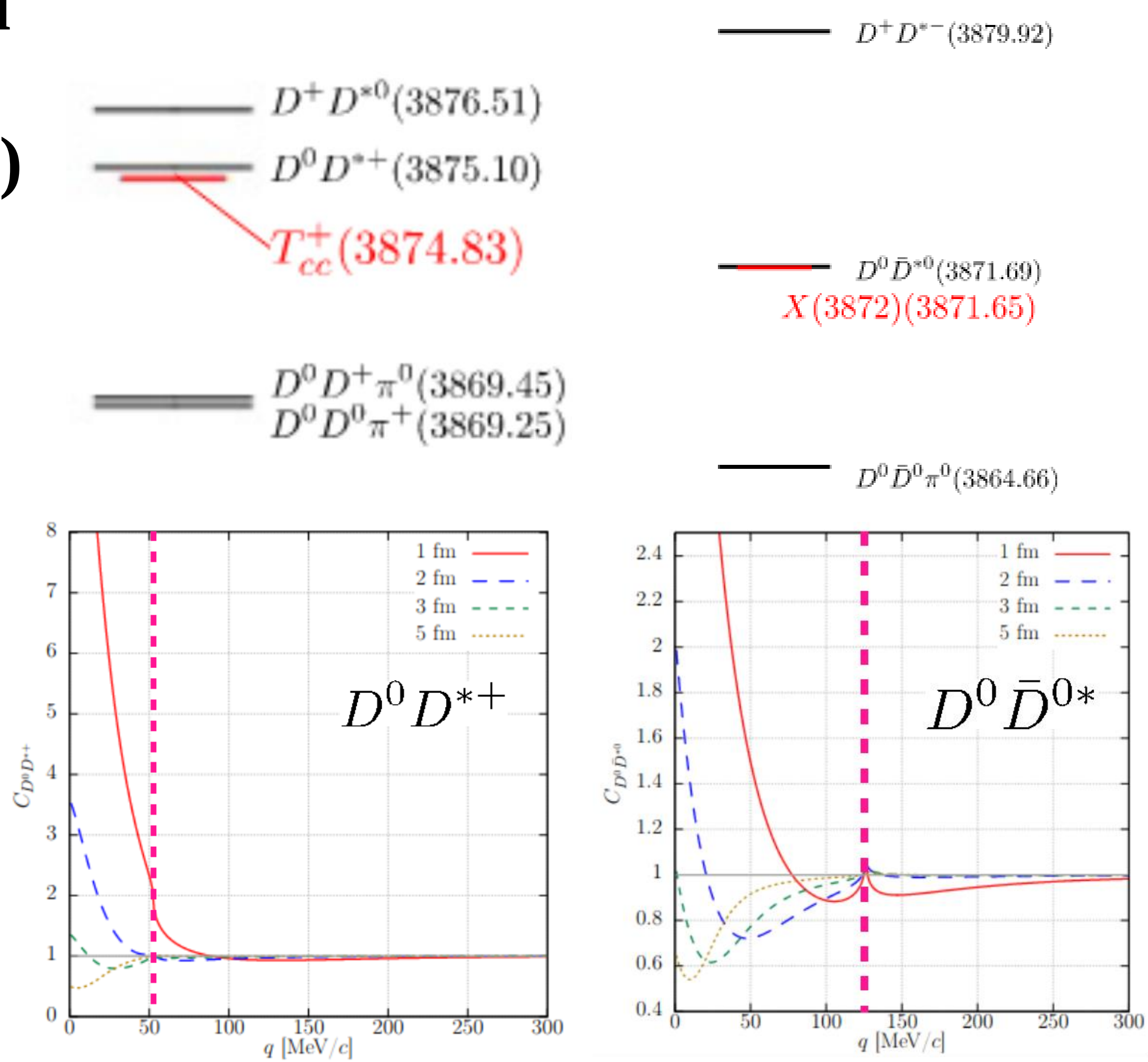
$$\{D^0\bar{D}^{*0}\} = (D^0\bar{D}^{*0} + \bar{D}^0 D^{*0})/\sqrt{2}\ (C=+1)$$
$$\{D^+D^{*-}\} = (D^+D^{*-} + D^- D^{*+})/\sqrt{2}\ (C=+1)$$



DD^*	V_0 [MeV]	$a_0^{D^0D^{*+}}$ [fm]	$a_0^{D^+D^{*0}}$ [fm]
	$-36.569 - i1.243$	$-7.16 + i1.85$	$-1.75 + i1.82$
$\{D\bar{D}^*\}$	V_0 [MeV]	$a_0^{\{D^0\bar{D}^{*0}\}}$ [fm]	$a_0^{\{D^+D^{*-}\}}$ [fm]
	$-43.265 - i6.091$	$-4.23 + i3.95$	$-0.41 + i1.47$

$D^0 D^{*+}$ and $D^+ \bar{D}^{*0}$ Correlation Functions

- **Features of C(q) with a bound state**
  - **Enhancement at small source, Dip at large source.**
  - **Modification of potential (Changing the range, V(I=1)=0 or ± V(I=0)/3) does not change C(q) significantly. (dominated by the pole)**
  - **Measurement in Run3 is awaited.**



Tcc and X(3872) structure

- **Hadronic molecule structure is assumed**
  → Eigenmomentum $k \simeq -i/a_0\,,\ a_0 \simeq R = 1/\sqrt{2\mu B}$
- **What happens when multiquark state mixes ?**
  → Deviation from weak binding relation (X=compositeness)
  Weinberg, Phys. Rev. 137, B672 (1965), Hyodo, Jido, Hosaka (1108.5524), Kunigawa, Hyodo (2112.00249)

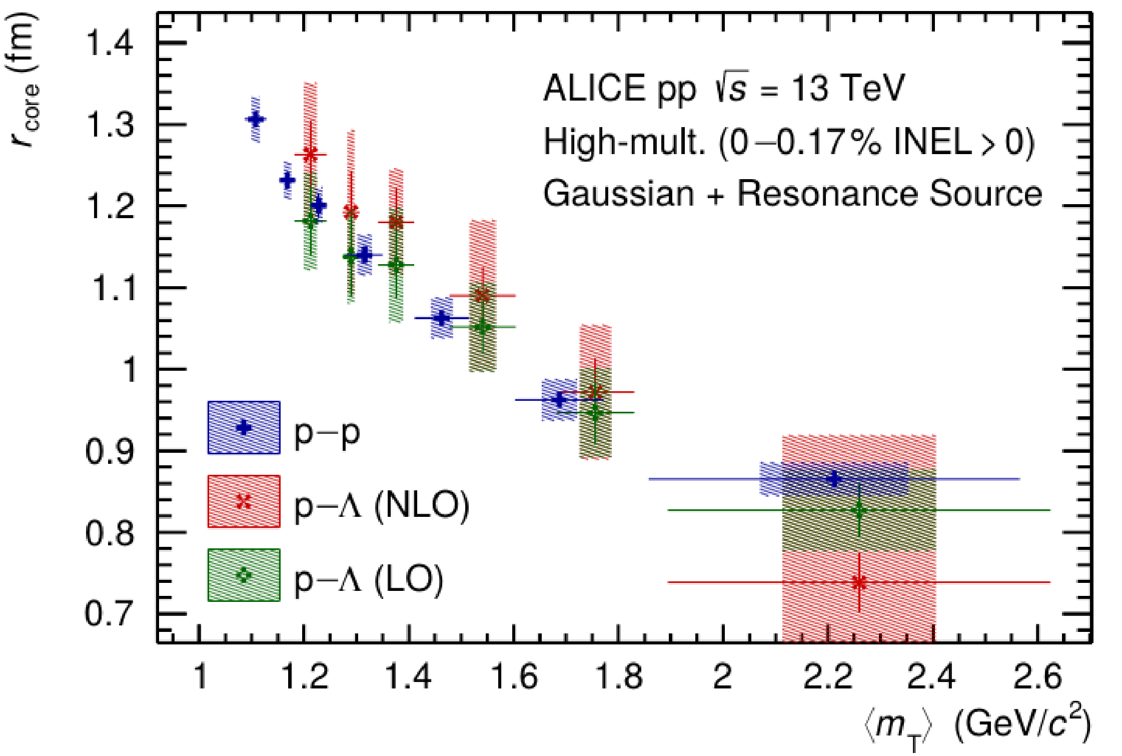
$$a_0 = R\left[\frac{2X}{1+X}\right] + \mathcal{O}(R_{\rm typ})$$

$$\left[R_{\rm typ} = \max(m_\pi^{-1}, r_{\rm eff}), R = 1/\sqrt{2\mu B}\right]$$

  - **Hadronic molecule assumption → X=1**
    Pure multiquark state → X=0
  - **Smaller scattering length in DD* may signal the *genuine* tetraquark nature of Tcc.**

We are sorry, but we use a Gaussian Source !

- **Calculating HBT radius in dynamical models is not easy**
  M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014]; S. Pratt, PRL102('09)232301 [0811.3363].
- **and a Gaussian source seems to work at the current precision of hh interaction studies.**
  S. Acharya+[ALICE], PLB811('20)135849.
  - **primary (universal ?)+ decay of short-lived resonances ~ eff. Gaussian**
  - **Flow and source geometry effects are seen in CF, but the uncertainty of hh int. is the largest.**



Toward dynamical source

- **Calculating HBT radius in dynamical models is not easy (HBT puzzle).**
  M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014];

  choices then tends to exceed the number of experimental constraints. In fact, all the model results that we review in the current subsection remain unsatisfactory with this respect: They either deviate significantly from femtoscopic data, or they reproduce these data at the price of missing other important experimental information. In particular, there is so far no dynamically consistent model that reproduces quantitatively both the systematic trends discussed in Section 4 and the corresponding single inclusive spectra. In this situation, the scope of this subsection is

- **But carefully constructed hydrodynamic model may answer.**
  S. Pratt, PRL102('09)232301 [0811.3363].

  Two particle correlation data from the BNL Relativistic Heavy Ion Collider have provided detailed femtoscopic information describing pion emission. In contrast with the success of hydrodynamics in reproducing other classes of observables, these data had avoided description with hydrodynamic-based approaches. This failure has inspired the term "HBT puzzle," where HBT refers to femtoscopic studies which were originally based on Hanbury Brown–Twiss interferometry. Here, the puzzle is shown to originate not from a single shortcoming of hydrodynamic models, but the combination of several effects: mainly prethermalized acceleration, using a stiffer equation of state, and adding viscosity.

- **How about afterburner effects ?**

Wave function around threshold (S-wave, attraction)

- **Low energy w.f. and phase shift**

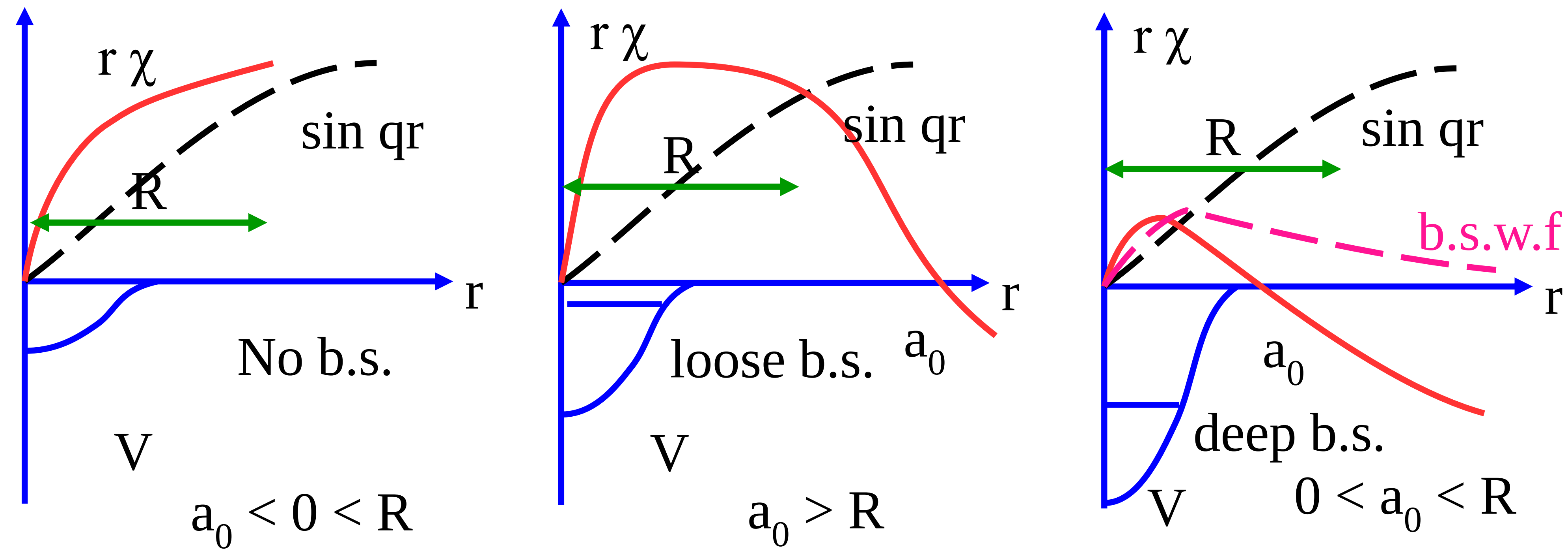
$$u(r) = qr\chi_q(r) \to \sin(qr + \delta(q)) \sim \sin(q(r - a_0))$$

$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} q^2 + \mathcal{O}(q^4) \ \ (\delta \sim -a_0 q)$$

a_0 =scatt. length
r_{eff} =eff. range

Nucl. and Atomic Phys. convention

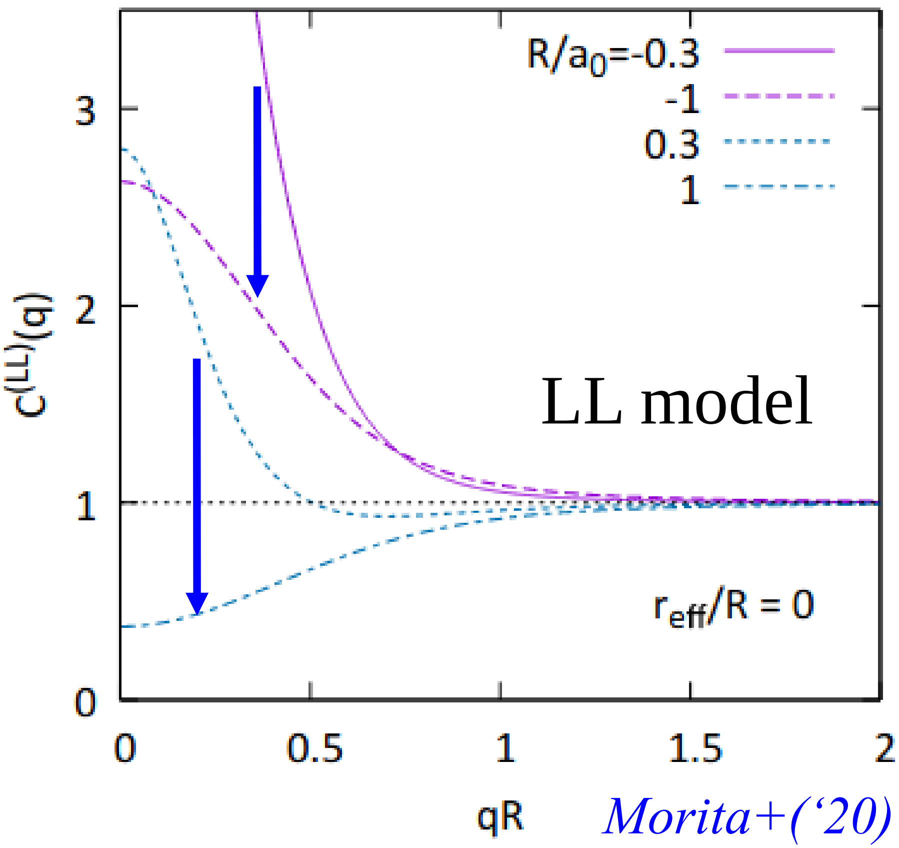
  - **Wave function grows rapidly at small r with attraction.**
  - **With a bound state (a_0>0), a node appears around r=a_0**
    → Suppressed |w.f.|^2 on average



Interaction Dependence of C(q)

- **Repulsive interaction → C(q) is suppressed.**
- **Attractive interaction**
  - **Wave function grows rapidly at small r with attraction.**
    → C(q) is enhanced for small source.
  - **Without a bound state ($a_0 < 0$)**
    → C(q) > 1
  - **With a bound state ($a_0 > 0$)**
    → Region with C(q) < 1 appears

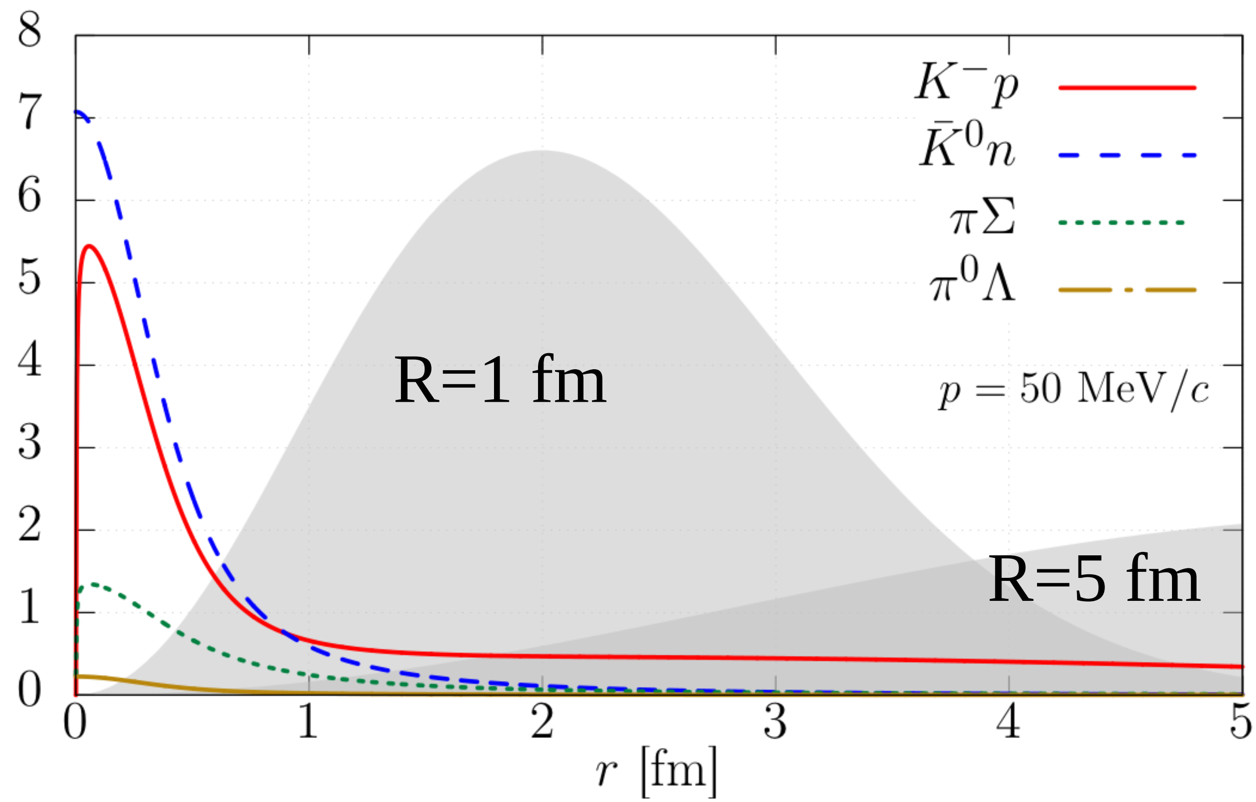
Why is C(q) suppressed
when there is a bound state ?
Do we really see
enhanced C(q) for small R
and suppressed C(q) for large R
when there is a bound state ?



Morita+('20)

Discriminating Coupled-Channel Effects

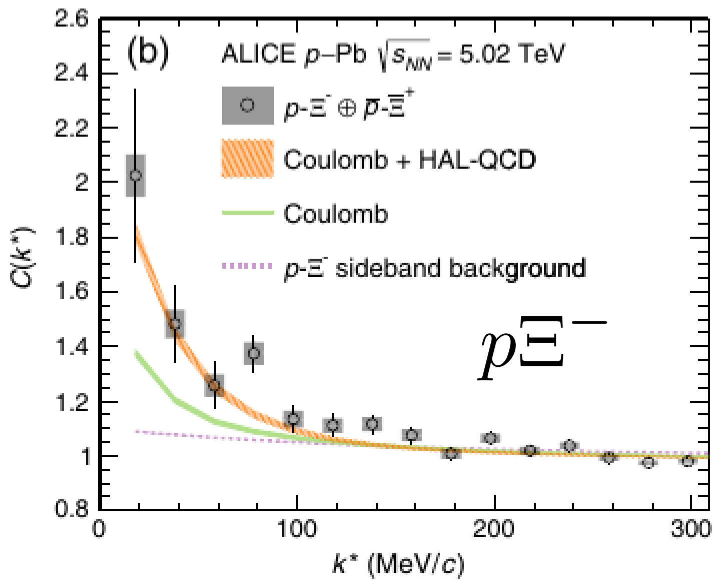
- **Source size dependence again !**
  - **Unmeasured coupled-channel wave functions disappear soon.**
    → CFs with large source is dominated
    by the measured channel wave function !
  - **Scattering parameters from CFs with large source**
    Coupled-channel effects from CFs with small source.



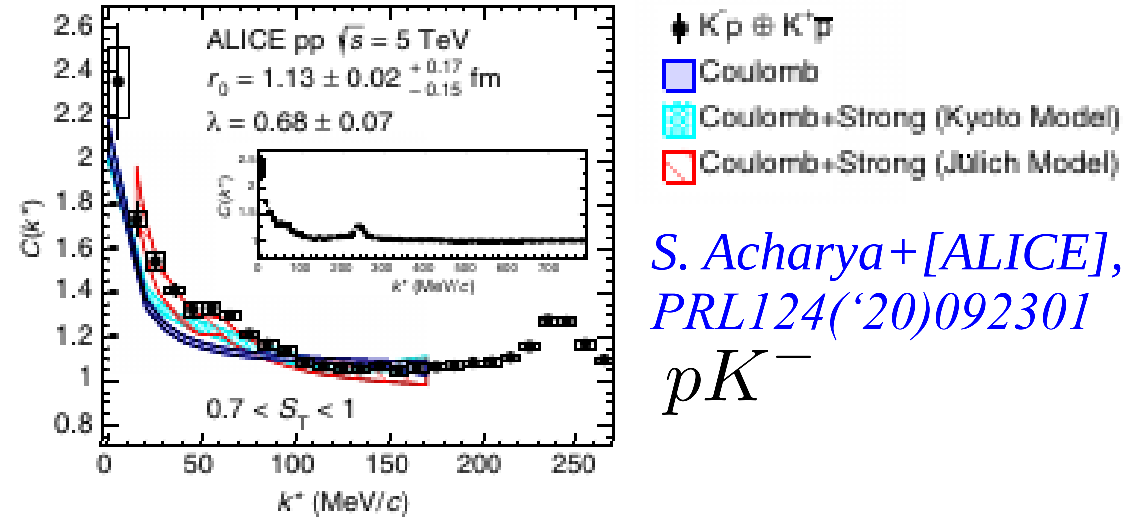
w.f. Kamiya+,arXiv:1911.01041v1

Examples of Enhanced C(q) from small source

High-Multiplicity events from pp

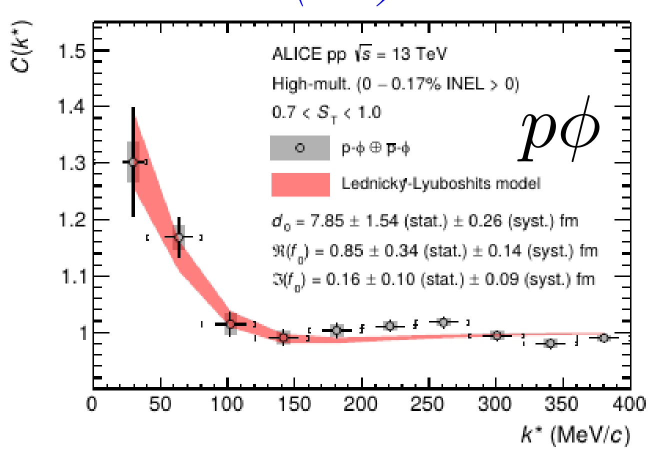


S. Acharya+[ALICE], PRL123('19)112002.

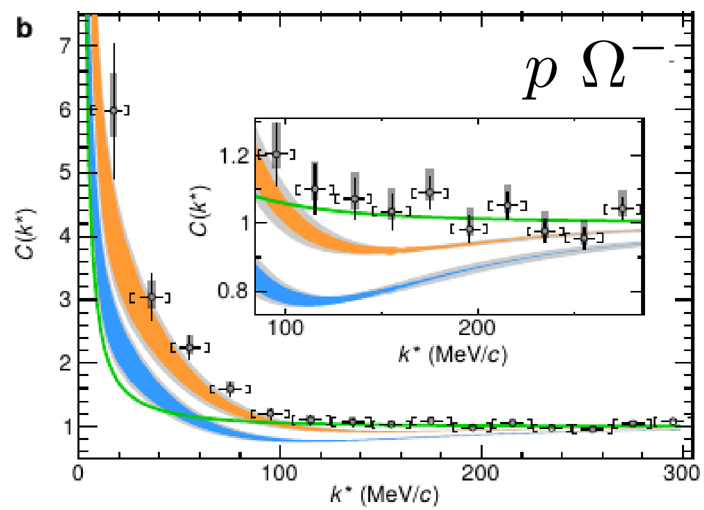


S. Acharya+[ALICE], PRL124('20)092301

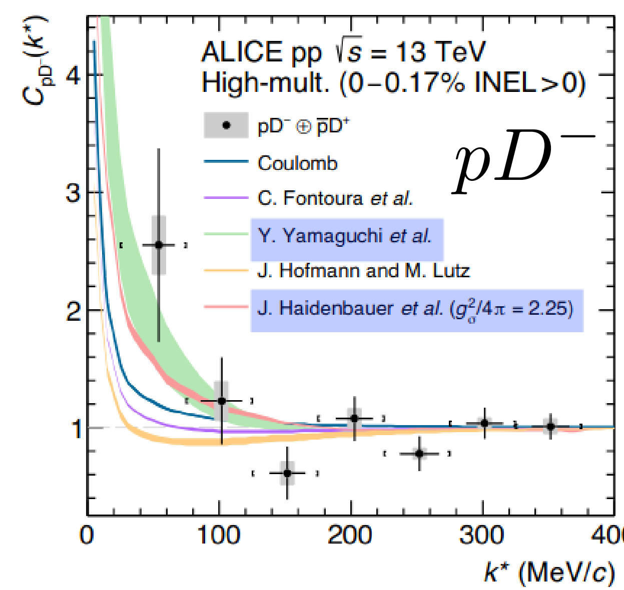
pK^-



ALICE, 2105.05578

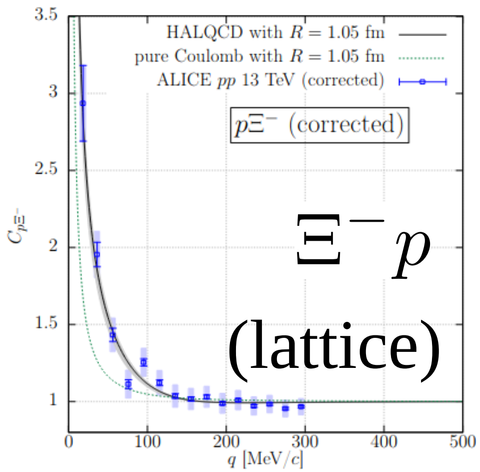


S. Acharya+[ALICE], 2005.11495 [nucl-ex] (pp 13 TeV)

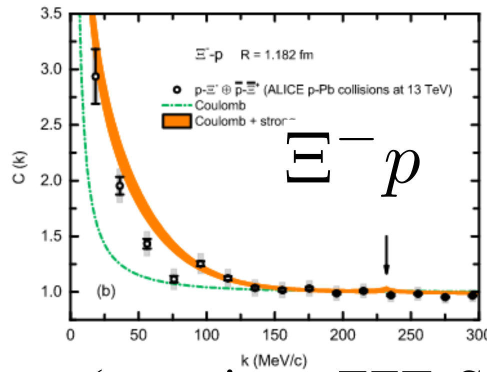


Acharya+[ALICE] (2201.05352)

Theoretical femtoscopic study of hh int. (examples)

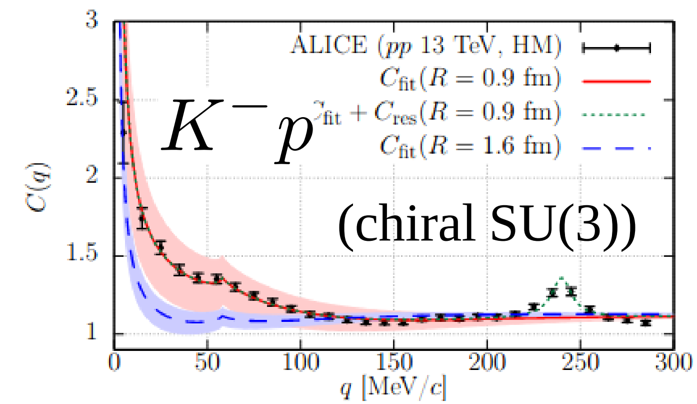


Y.Kamiya, K.Sasaki, et al., (2108.09644)

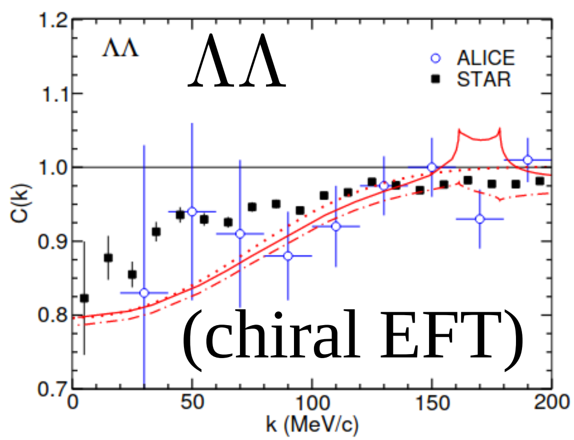


(covariant χEFT, S=-2)

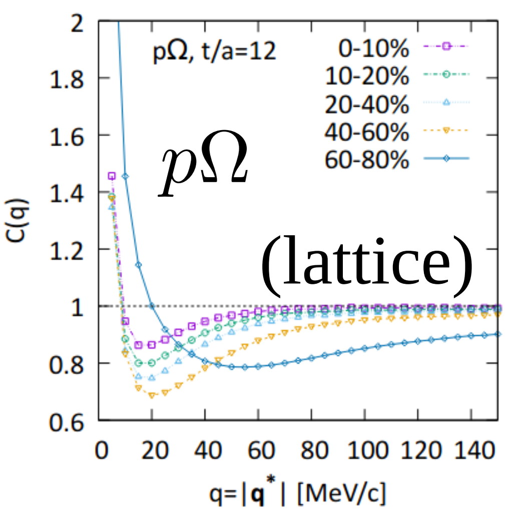
Z.-W. Liu, K.-W. Li, L.-S. Geng (2201.04997)



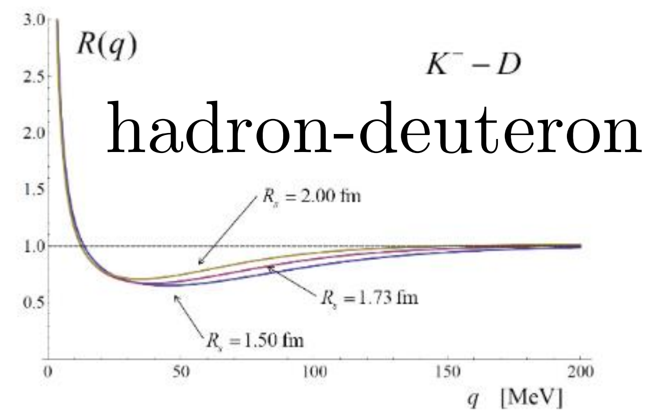
Kamiya+(1911.01041)



Haidenbauer(1808.05049), Morita+(1408.6682)



Morita, Gongyo et al.,(1908.05414), Morita, AO, Etminan, Hatsuda (1605.06765)



Mrówczyński, Słón (1904.08320, K⁻d), Haidenbauer (2005.05012, Λd), Etminan, Firoozabadi (1908.11484, Ωd), K.Ogata+ (Ξ⁻ d,2103.00100)

High-Energy Heavy-Ion Collisions

- **Main Goal of HIC physics = Discovery and Properties of QGP**
- **HIC as a playground / tool**
  - **Development of dynamical models**
  - **Physics of extreme conditions and/or strong field**
  - **Hadron physics**

Hadron Physics using HIC as Hadron Factories
- *Simultaneous Prod. of many hadrons statistically*
- *Nearly 4π detectors & Vertex detectors*