フェムトスコピーによるハドロン間相互作用の研究

Femtoscopy for hadron-hadron interactions

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第８回クラスター階層領域研究会

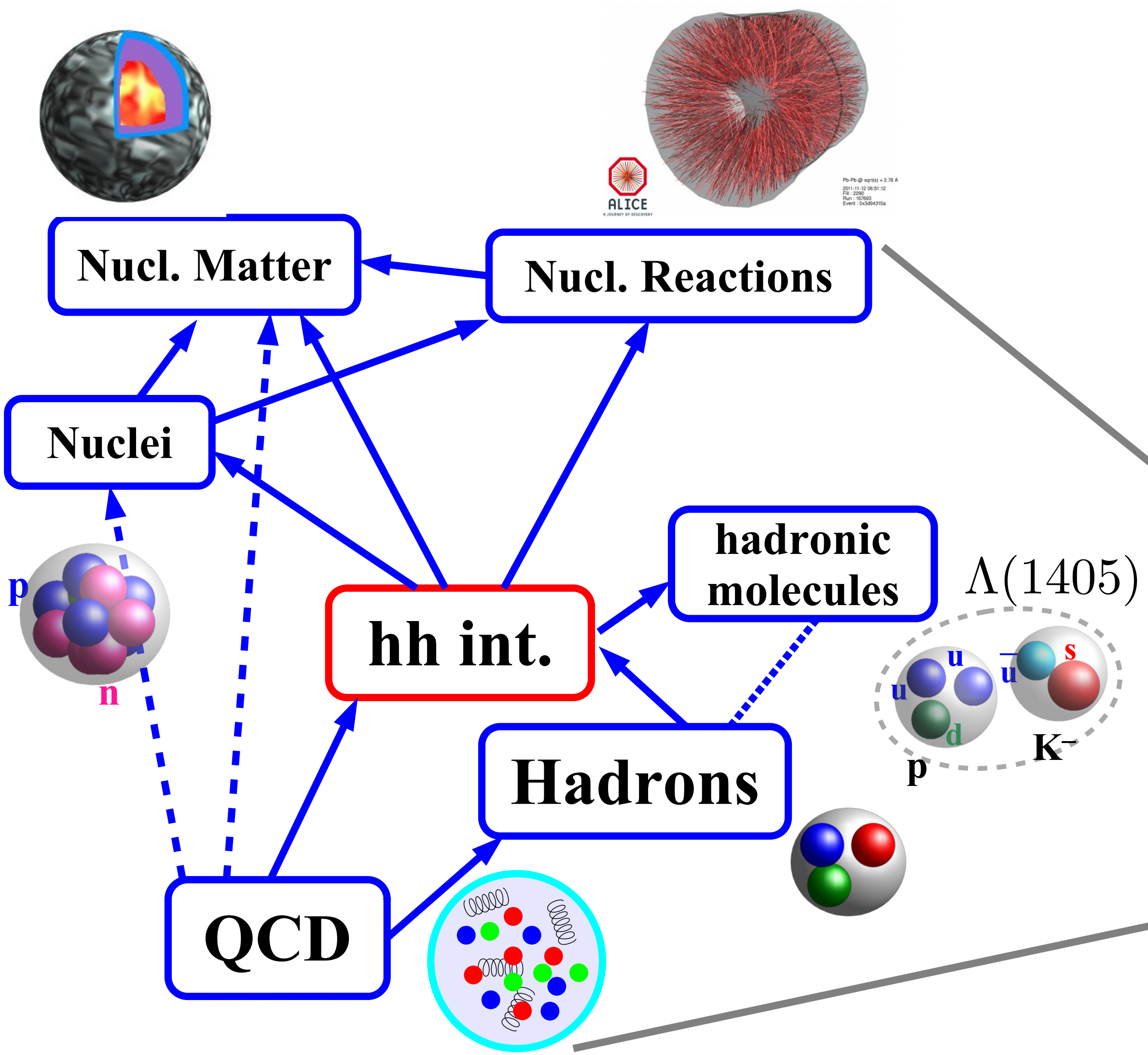
新学術領域「量子クラスターで読み解く物質の階層構造」

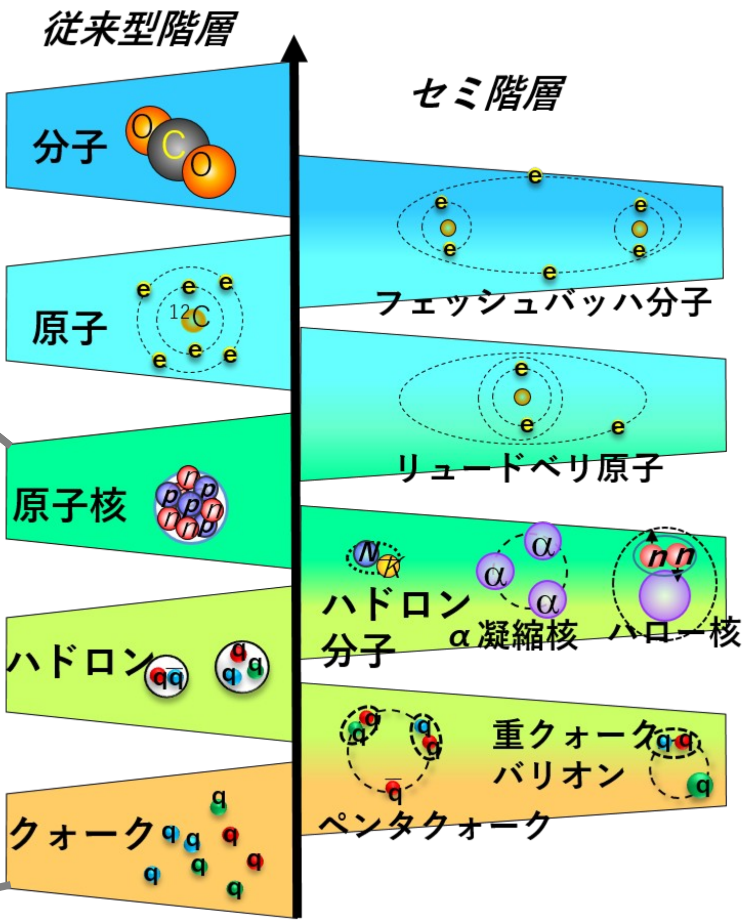
February 9-11, 2023, Arata Hall, Osaka U., Japan / Online (Hybrid)

- Introduction
- Achievements of the grant 21H00121 (JFY2021-2022)
  - D^- p correlation function
  - DD^* and $D\bar{D}^*$ correlation function
  - Ξ^- d correlation function
- Summary

Clusters & Hierarchies

Cluster & Hierarchies





Hadron-Hadron Interactions = basic inputs for hadron many-body systems

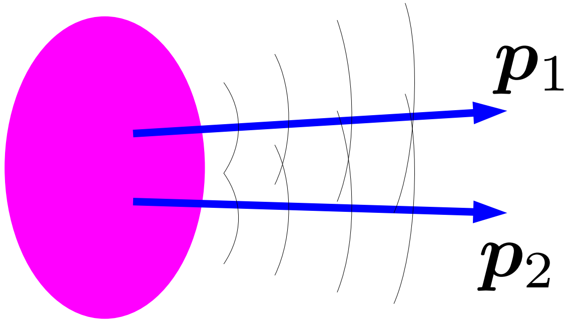
Femtoscopy

Correlation Function

Koonin('77), Pratt+('86), Lednicky+('82)

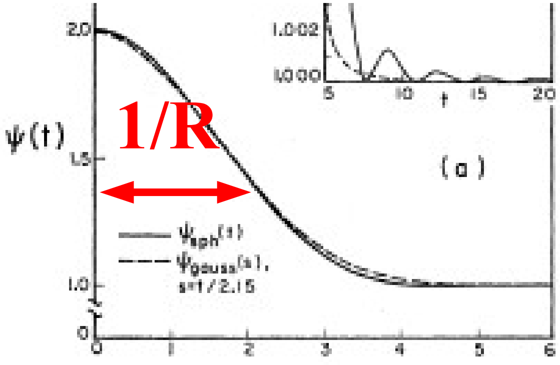
$$C(\boldsymbol{q}) = \int d\boldsymbol{r} S(\boldsymbol{r}) |\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2$$

$S(\boldsymbol{r}) = \text{source function}$, $\varphi_{\boldsymbol{q}}(\boldsymbol{r}) = \text{relative w.f.}$



Source size (HBT)

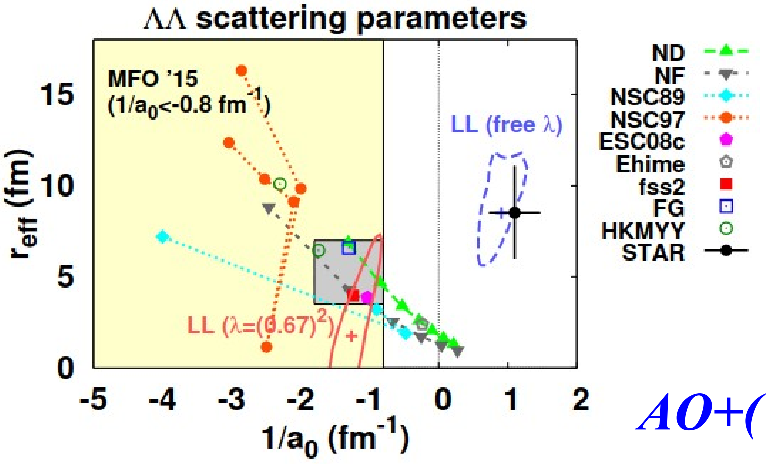
Hanbury Brown & Twiss, Nature 10 (1956), 1047; Goldhaber, Goldhaber, Lee, Pais, Phys. Rev. 120 (1960), 300.



q (relative momentum)

Hadron-Hadron Interaction

Lednickey, Lyuboshits ('82); Lednicky, Lyuboshits, Lyuboshits ('98); Heidenbauer ('19); C. Greiner, B. Muller, PLB219('89)199; AO+ ('00); Morita+ ('15~); Kamiya+('20~); STAR ('15~); ALICE ('19~)



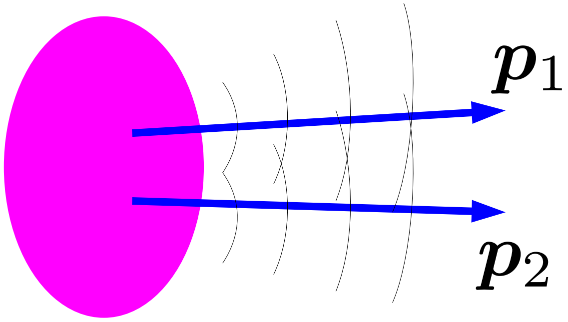
AO+('16)

Two-particle momentum correlation function

- **Koonin-Pratt formula**

  Koonin('77), Pratt('86), Lednicky+('82)

  - **Particles are emitted independently. Correlation is made by final state interaction (or quantum statistics).**



$$C(\boldsymbol{q},\boldsymbol{P}) = \frac{N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)} \simeq \int \underbrace{d\boldsymbol{r}S(\boldsymbol{r})}_{\textbf{source fn.}}\underbrace{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}_{\textbf{relative w.f.}}$$

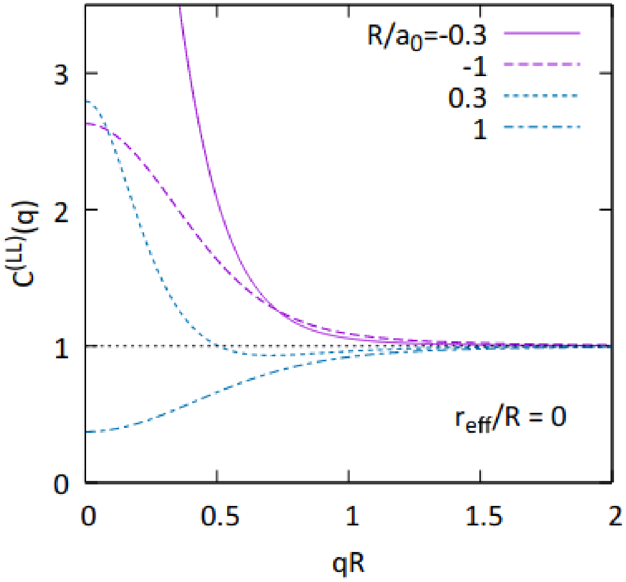
$$\simeq 1 + \int d\boldsymbol{r}S(r)\left\{\underbrace{|\chi_q(r)|^2}_{\textbf{s-wave}} - |j_0(qr)|^2\right\}$$

  - **C(q) shows enhancement of S-wave |w.f.|^2.**

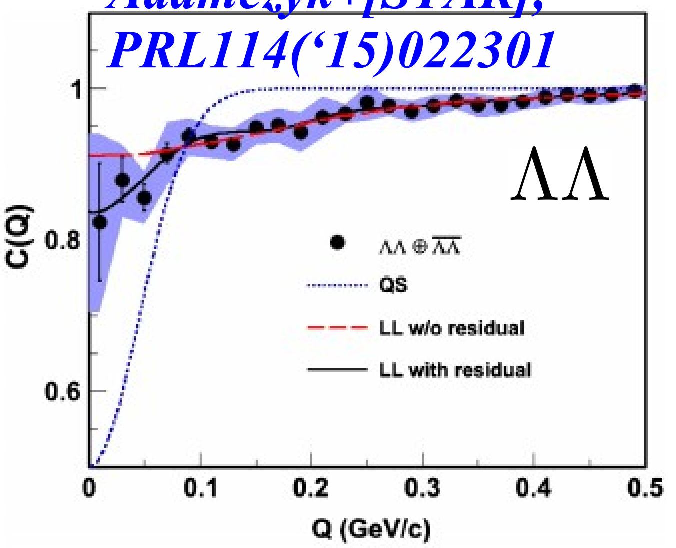
- **C(q) is sensitive to**

  a_0 (scattering length) and R (source size).

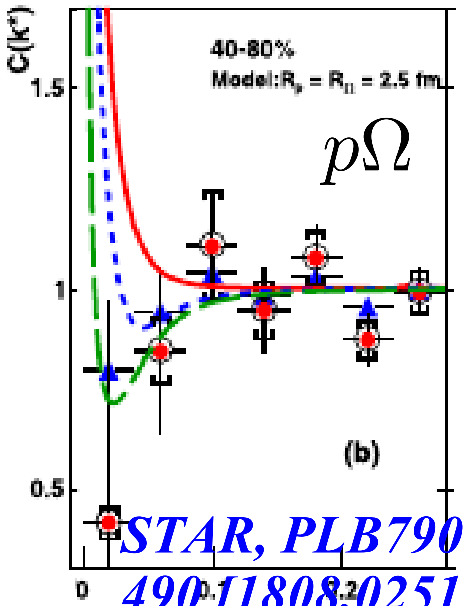
  - **For a0 > 0 (w/ bound state), C(q) at small q is suppressed at large R.**



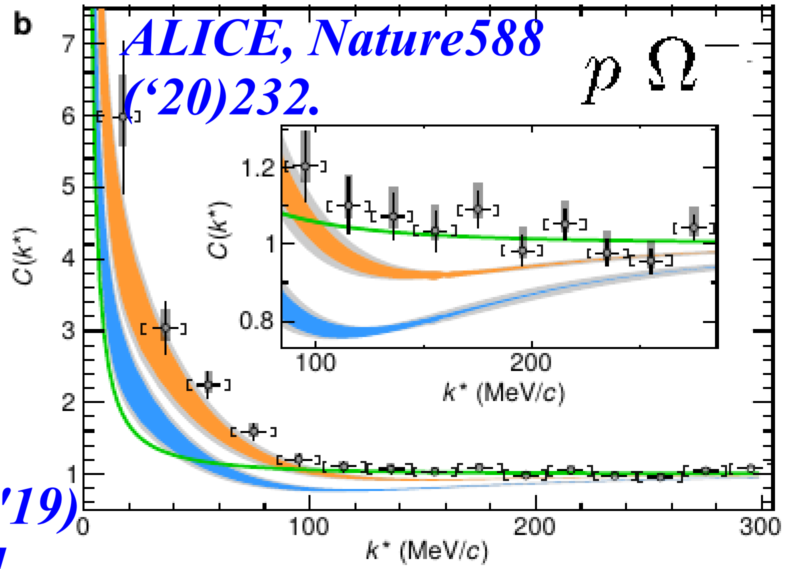
Measured Flavored Hadron CFs (examples)



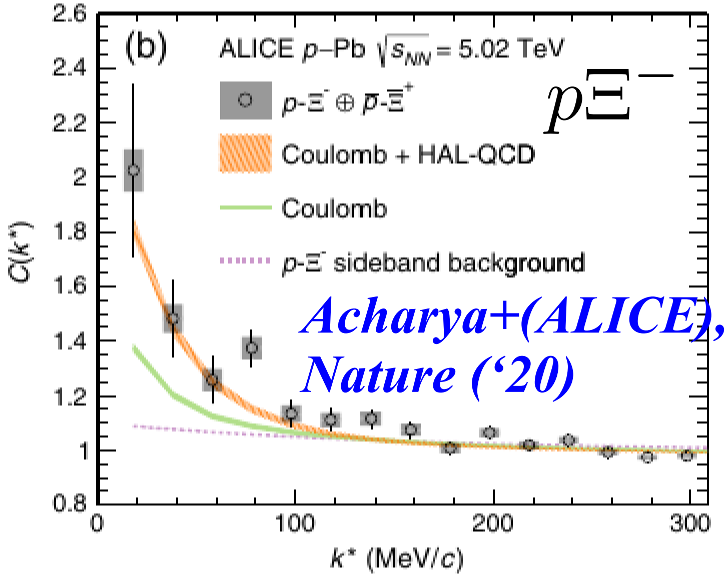
Adamczyk+[STAR], PRL114('15)022301



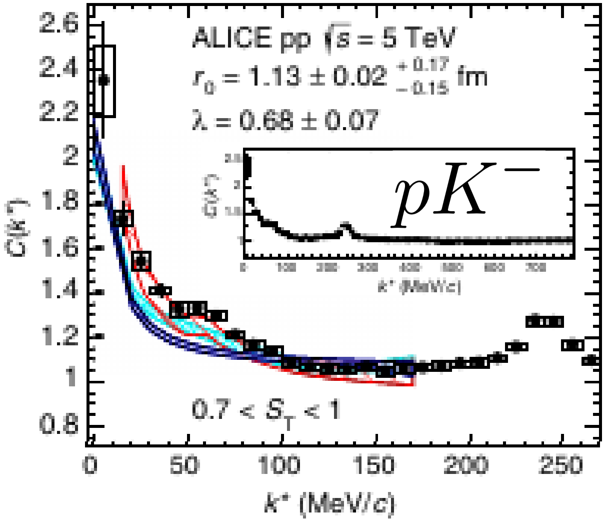
STAR, PLB790 ('19) 490 [1808.02511].



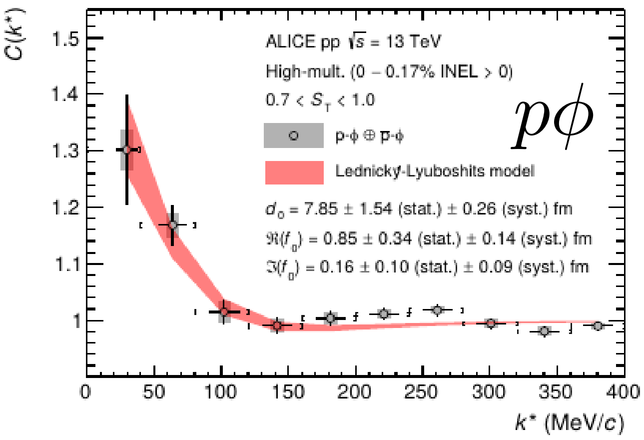
ALICE, Nature588 ('20)232.



Acharya+(ALICE), Nature ('20)

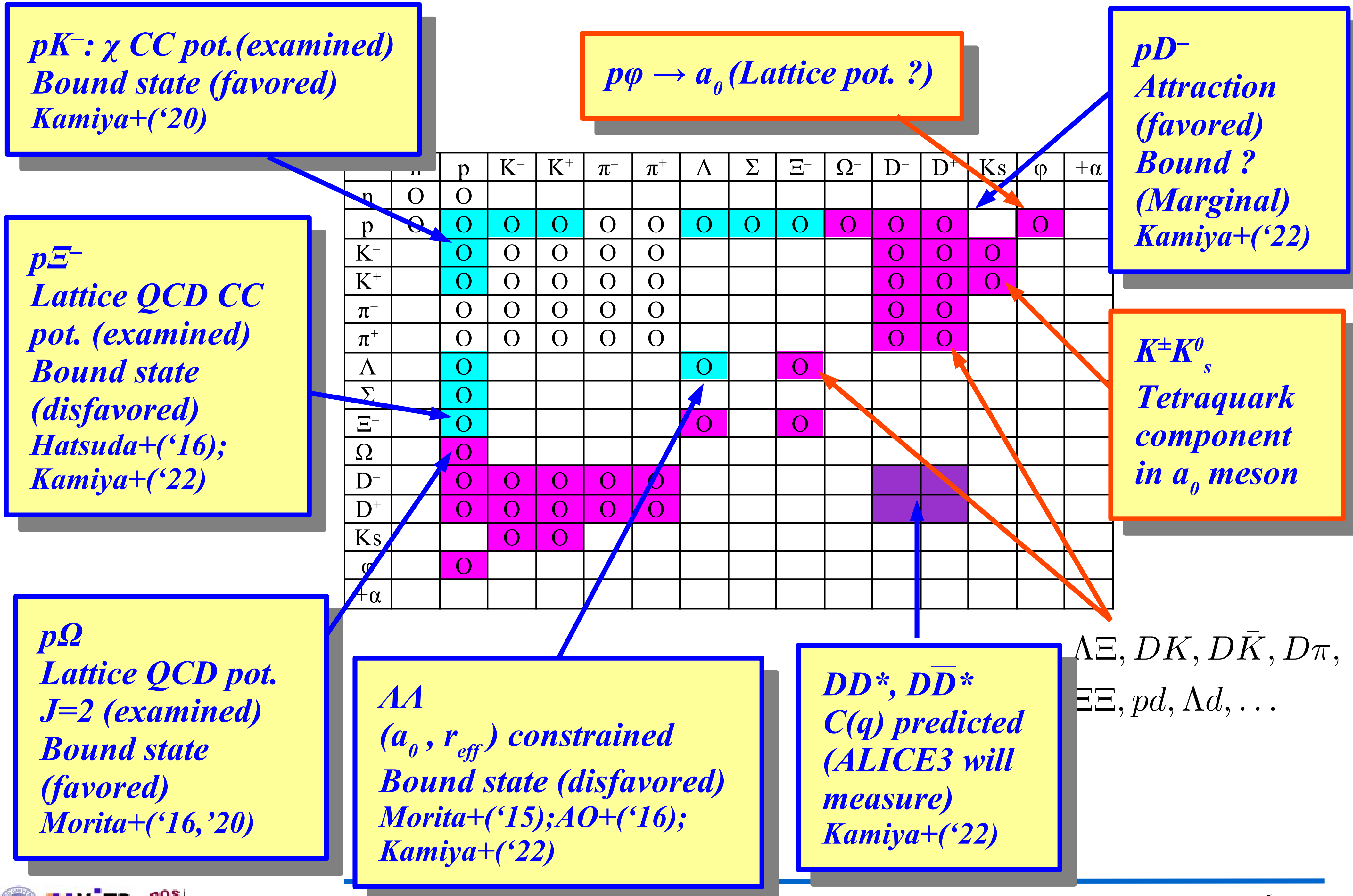


S. Acharya+[ALICE], PRL124('20)092301



ALICE, 2105.05578

Scope of Femtoscopic study of HHI



pK⁻: χ CC pot.(examined)
Bound state (favored)
Kamiya+('20)

pφ → a_0 (Lattice pot. ?)

pD⁻
Attraction (favored)
Bound ? (Marginal)
Kamiya+('22)

pΞ⁻
Lattice QCD CC pot. (examined)
Bound state (disfavored)
Hatsuda+('16); Kamiya+('22)

K±K⁰ₛ
Tetraquark component in a_0 meson

	n	p	K⁻	K⁺	π⁻	π⁺	Λ	Σ	Ξ⁻	Ω⁻	D⁻	D⁺	Ks	φ	+α
n	O	O													
p	O	O	O	O	O	O	O	O	O	O	O	O		O	
K⁻		O	O	O	O	O					O	O	O		
K⁺		O	O	O	O	O					O	O	O		
π⁻		O	O	O	O	O					O	O			
π⁺		O	O	O	O	O					O	O			
Λ		O					O		O						
Σ		O													
Ξ⁻		O					O		O						
Ω⁻		O													
D⁻		O	O	O	O	O									
D⁺		O	O	O	O	O									
Ks			O	O											
φ		O													
+α															

$\Lambda\Xi, DK, D\bar{K}, D\pi,$
$\Xi\Xi, pd, \Lambda d, \ldots$

pΩ
Lattice QCD pot. J=2 (examined)
Bound state (favored)
Morita+('16,'20)

ΛΛ
(a_0 , r_{eff}) constrained
Bound state (disfavored)
Morita+('15);AO+('16); Kamiya+('22)

DD*, D$\bar{D}$*
C(q) predicted (ALICE3 will measure)
Kamiya+('22)

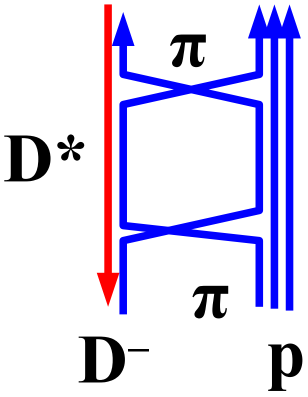
公募研究の成果

- **2 粒子運動量相関から探るハドロン間相互作用としきい値近辺の散乱振幅 (19H05151, 2019-20 年度 , 40+40 万円)**
  - **Probing ΩΩ and pΩ dibaryons with femotoscopic correlations in relativistic heavy-ion collisions, K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC101(‘20), 015201 (Editors’ Suggestion).**
  - **K^-p correlation function from high-energy nuclear collisions and chiral SU(3) dynamics, Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124 (‘20), 132501.**
- **2 粒子・ 3 粒子運動量相関から探るハドロン間相互作用 (21H00121, 2021-22 年度 , 30+30 万円)**
  - **Deuteron breakup effect on deuteron-Ξ correlation function, K. Ogata, T. Fukui, Y. Kamiya, and AO, PRC103 (‘21), 065205 [2103.00100].**
  - **Femtoscopic study of coupled-channel NΞ and ΛΛ interactions, Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, AO, T. Hatsuda, PRC105 (‘22), 014915 [2108.09644].**
  - **Femtoscopic study on DD* and $D\bar{D}^*$ interactions for Tcc and X(3872), Y.Kamiya, T.Hyodo, A.Ohnishi, EPJA58 (‘22), 131 [2203.13814].**
  - **First study of the two-body scattering involving charm hadrons, S. Acharya et al. [ALICE], PRD106 (‘22), 052010 [2201.05352].**
  - **Constraining the $\bar{K}N$ coupled channel dynamics using femtoscopic correlations at the LHC, S. Acharya et al. [ALICE], arXiv:2205.15176**

D^- p correlation function

First study of the two-body scattering involving charm hadrons, S. Acharya et al. [ALICE], PRD106 ('22), 052010 [2201.05352].

Charmed Hadron Interactions



- Extremely important in recent hadron physics.
- $D^-(\bar{c}d)$-p(uud) interaction
  - Probes Θ_c ($\bar{c}$-ud-ud) state (replace $\bar{s}$ in Θ($\bar{s}$-ud-ud) with $\bar{c}$)

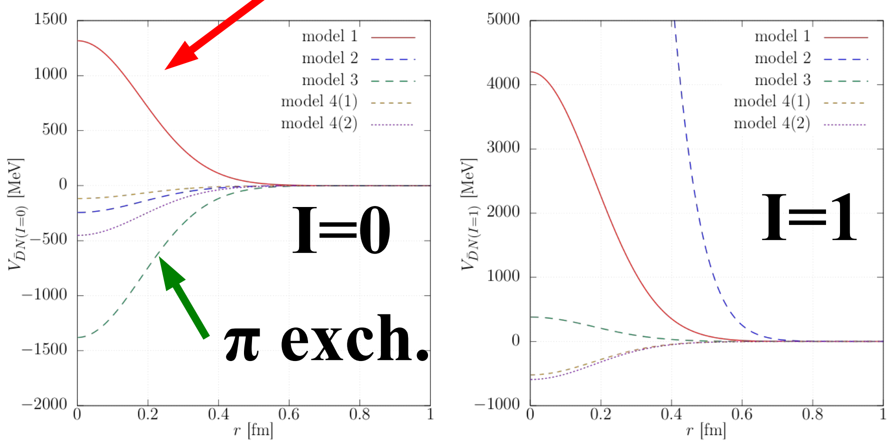
    D. O. Riska, N. N. Scoccola, PLB299('93)338 (pred.);
    A. Aktaset+ [H1], PLB588('04)17 (positive);
    J. M. Linket+ [FOCUS], PLB622('05)229 (negative).
  - Attraction from two pion exchange

    S. Yasui, K. Sudoh, PRD80('09)034008.
  - Easy to calculate the potential in LQCD.

    Y. Ikeda et al. (private communication)
- Proposed potentis of $D^-(\bar{c}d)$-p(uud)

  Hofmann, Lutz ('05) (repulsive);
  Haidenbauer+('07) (repulsive);
  Fontoura+('13) (repulsive);
  Yamaguchi+('11) (att., w/ bs).

SU(4) Contact

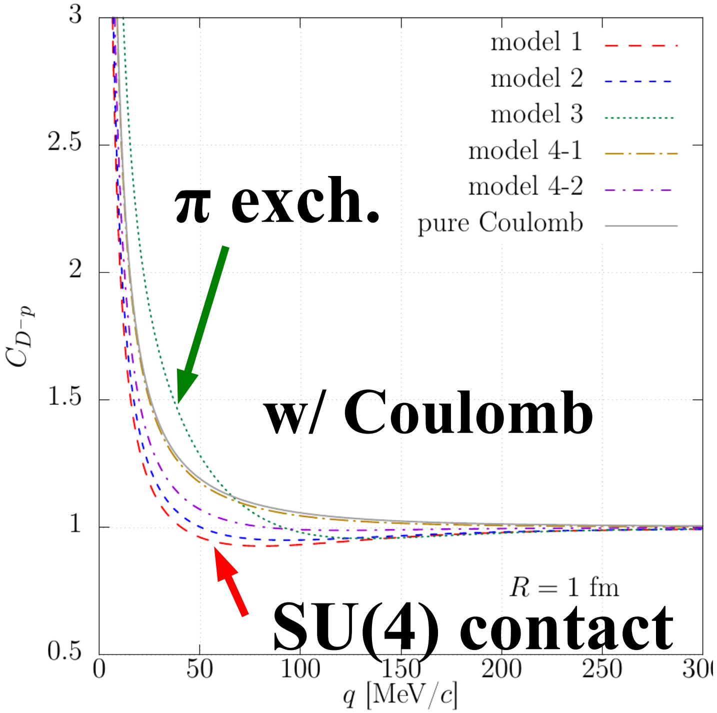


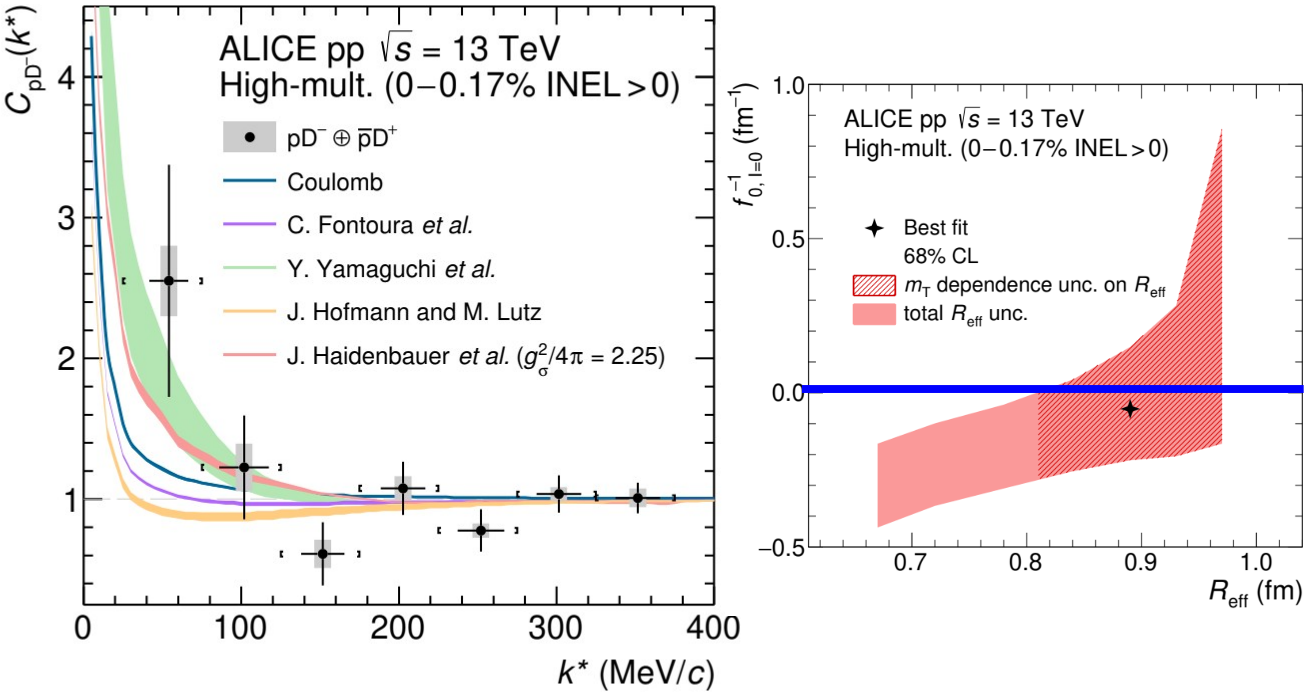
π exch.

Kamiya, Hyodo, AO (in prog.)

C(q) including Charm Hadron

- D^-p CFs from proposed scattering length
  - One-range Gaussian potential strength is fitted to proposed a_0 with the range of ρ meson exchange.
  - Measurable difference is found.





1. Hoffmann, Lutz ('05)
2. Haidenbauer+ ('07)
3. Yamaguchi+('11 → '22)
4. Fontoura+('13)

Kamiya, Hyodo, AO (in prog.)

S. Acharya et al. [ALICE, including KHO] PRD106 ('22), 052010 [2201.05352].

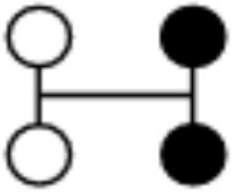
Attractive pot. is favored!

DD* and D$\bar{\text{D}}$* correlation function

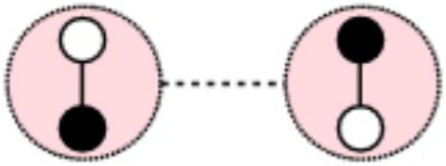
Femtoscopic study on DD* and D$\bar{\text{D}}$* interactions for Tcc and X(3872), Y.Kamiya, T.Hyodo, A.Ohnishi, EPJA58 (‘22), 131 [2203.13814].

Exotic Hadrons including $c\bar{c}$ / cc / $\bar{c}\bar{c}$

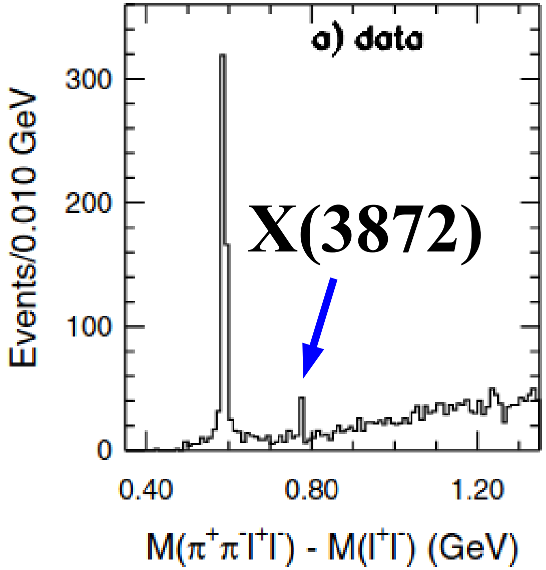
- **Main play ground of exotic hadron physics**
  - **X(3872)** ***Belle ('03)*** $c\bar{c}q\bar{q}$
  - **Many X,Y,Z states**
    Belle, CDF, BaBar, LHCb, CMS, BESIII, ...
  - **Charmed pentaquark Pc** ***LHCb ('15, '19)***
  - **Doubly charmed tetraquark state Tcc**
    LHCb ('21) $cc\bar{q}\bar{q}$
- **Structure of exotic hadrons**
  - **Compact multiquark states**
    → "good" [ud] diquark gains energy
  - **Hadronic molecules**
    → Many exotic states around thresholds
  - **Their mixture...**



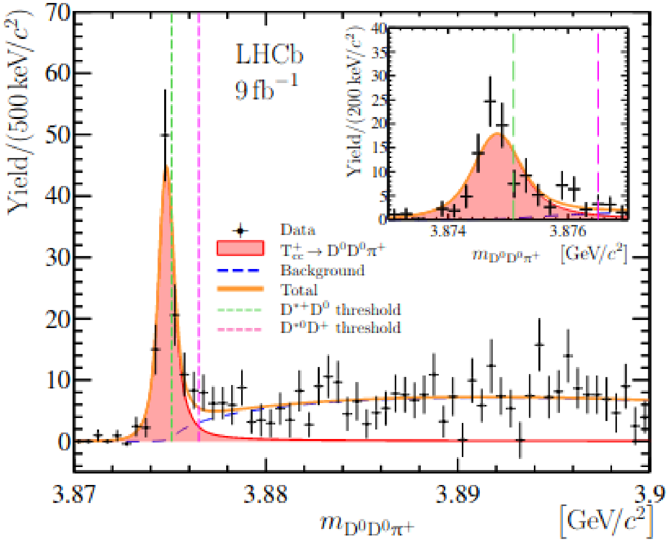
Tetraquarks



Hadronic Molecules



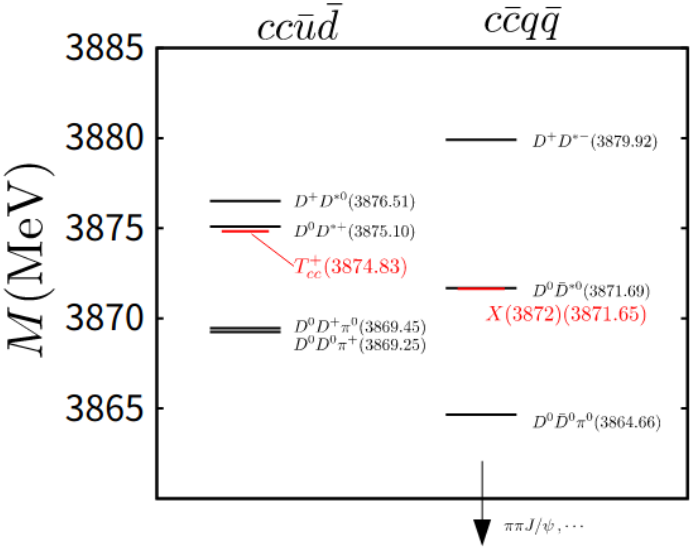
S.K.Choi+[Belle], PRL91, 262001 ('03)

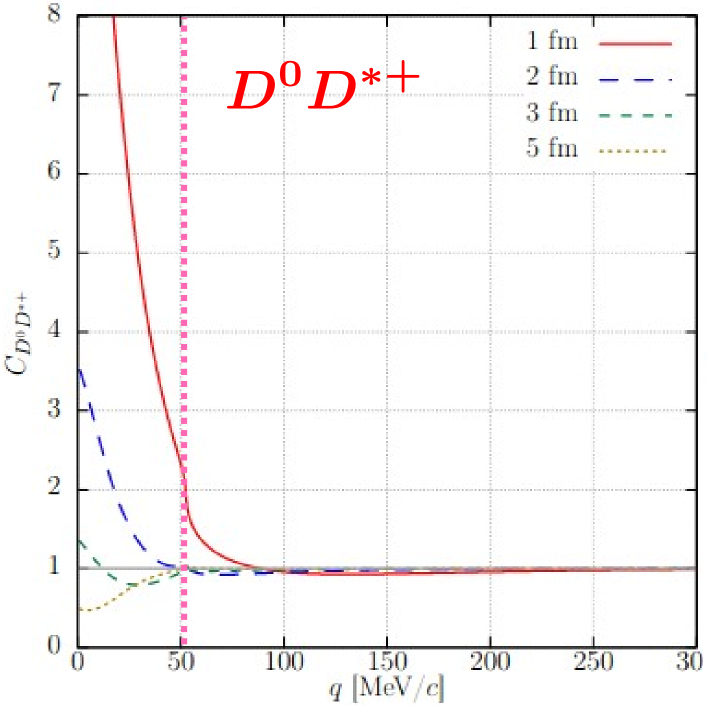


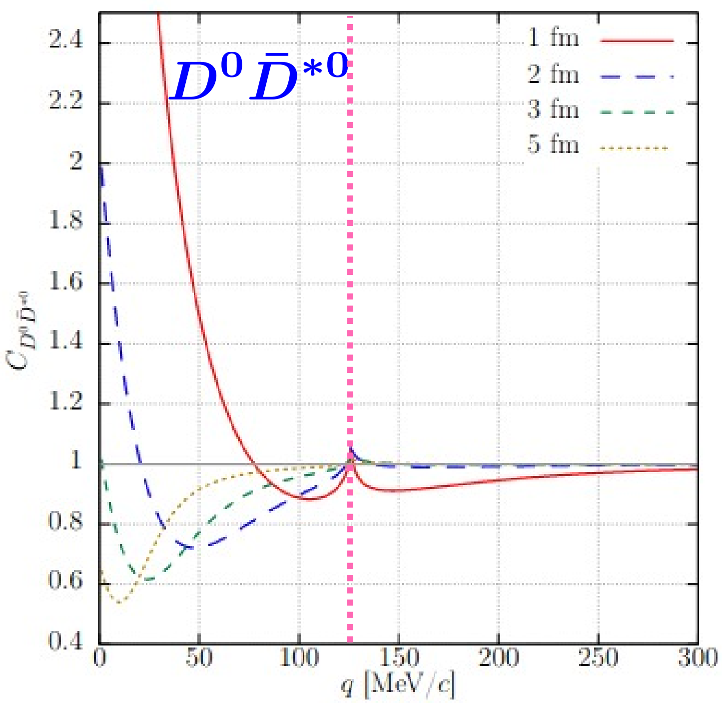
R. Aaji+ [LHCb], 2109.01038, 2109.01056

DD^* and $D\bar{D}^*$ Correlation Functions

- **Gaussian potentials fitted to the pole position of Tcc and X(3872)**
- **Correlation functions show characteristic dependence with a bound state, enhancement for small source (R=1 fm) and dip for large source (R=5 fm).**
- **Compact Multi-quark component will modify CFs.**
- **ALICE3('35~) will measure it!**







Y. Kamiya, T. Hyodo, AO, EPJA 58 ('22), 131 [2203.13814]

Tcc and X(3872) structure

- **Hadronic molecule structure is assumed**
  → Eigenmomentum $k \simeq i/a_0\,,\ a_0 \simeq R = 1/\sqrt{2\mu B}$
- **What happens when multiquark state mixes ?**
  → Deviation from weak binding relation

Weinberg, Phys. Rev. 137, B672 (1965), Hyodo, Jido, Hosaka (1108.5524), Kamiya, Hyodo (1607.01899), Kunigawa, Hyodo (2112.00249)

$$a_0 = R\left[\frac{2X}{1+X}\right] + \mathcal{O}(R_{\rm typ})$$

$$\left[R_{\rm typ} = \max(m_\pi^{-1}, r_{\rm eff}), R = 1/\sqrt{2\mu B}\right]$$

- **Smaller scattering length in DD* may signal the *genuine* tetraquark component in Tcc.**

$$f = \frac{1}{k\cot\delta - ik} \simeq \frac{1}{-1/a_0 - ik}$$

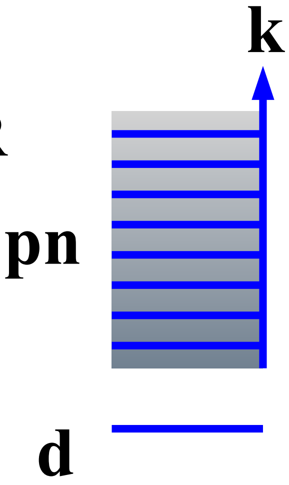
(Nuclear and Atomic phys. convention)

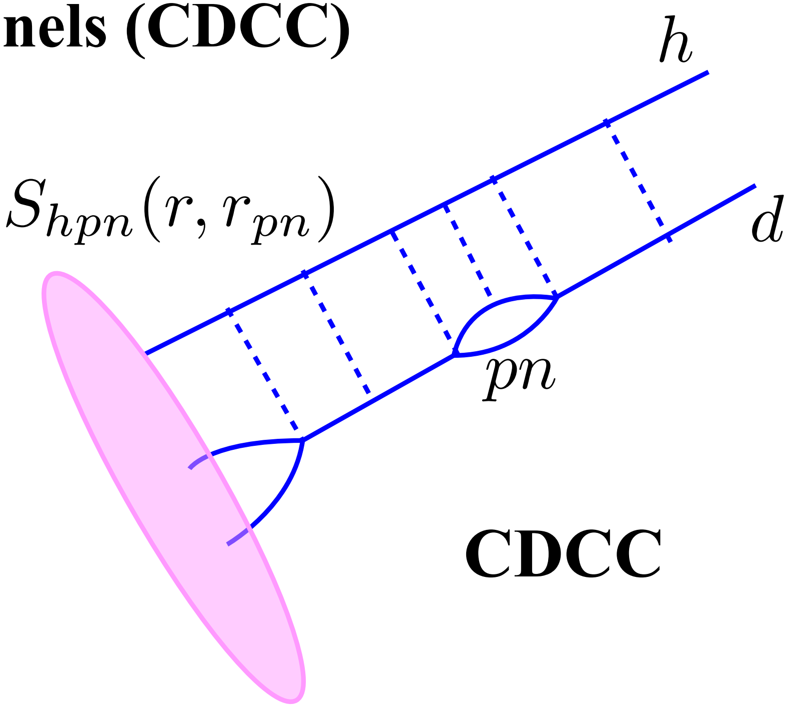
Ξ^- d correlation function

Deuteron breakup effect on deuteron-Ξ correlation function,
K. Ogata, T. Fukui, Y. Kamiya, and AO, PRC103 (‘21), 065205 [2103.00100].

Hadron-Deuteron correlation function

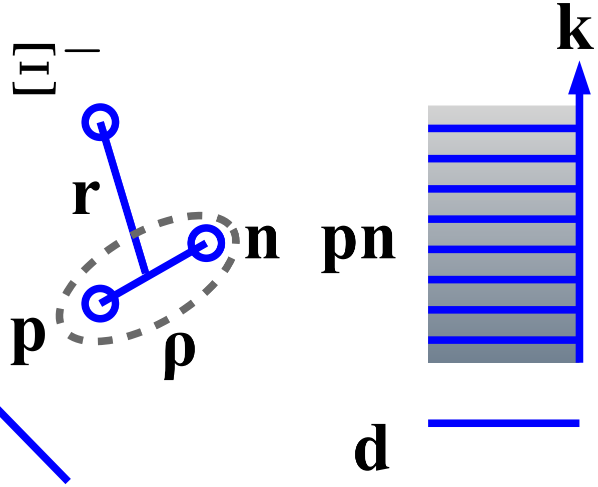
- **Hadron-deuteron correlation (Λd, K^-d, Ξ^-d, Ω^-d, …)**
  S.Mrówczyński, Patrycja Słoń, Acta Phys.Polon.B51('20),1739 [1904.08320](K-d,pd); J.Haidenbauer, PRC102('20)034001[2005.05012](Λd); F.Etminan+[2006.12771](Ωd).
  - **Scattering length data of these are important to evaluate**
    - **binding energy and lifetime of hyper triton (Λd)**
    - **I=1 $\bar{K}$N interaction (K^-d, Ξ^-d)**
    - **and the existence of a bound state.**
  - **Problem: *Breakup and Dynamical Formation of d* (d ↔ pn)**
    → Continuum-discretized coupled-channels (CDCC)
    M.Kamimura+('86); N. Austern+('87);
    M.Yahiro, K.Ogata, T.Matsumoto, K.Minomo, PTEP 2012 (2012) 01A206.
  - **Measurable at LHC-ALICE and (probably) RHIC-STAR**





Ξ^- d C(q) using CDCC

- **Three-body wave functions (s-wave)**



$$\psi^{(-)}(r,\rho;q)=\sum_n\sum_k A_{kn}\underline{\varphi_k(\rho)}\,\underline{\chi_{nk}(r;q_{nk})}$$

J, spin, isospin, ...

intrinsic momentum bin

kinematic factor

normalized pn w.f. in k-th bin

Ξ^--pn w.f.

- **Ξ^- d Correlation function**

$$C(q)=\underline{C^{\mathrm{C}}_{\ell>0}(q)}+\frac{1}{\underline{2\cdot 3}}\int d\boldsymbol{r}\,S(r)\sum_{nk}|\chi_{nk}(r;q_{nk})|^2$$

pure Coulomb

$1/(2J_1+1)/(2J_2+1)$ **"Ξ^- d" source fn.**

- **Potential = HAL QCD potential at almost physical quark masses**

K. Sasaki et al. [HAL QCD Collab.], NPA 998 ('20) 121737 (1912.08630)

(coupling with ΛΛ is ignored).

Ξ^-d correlation function: Result

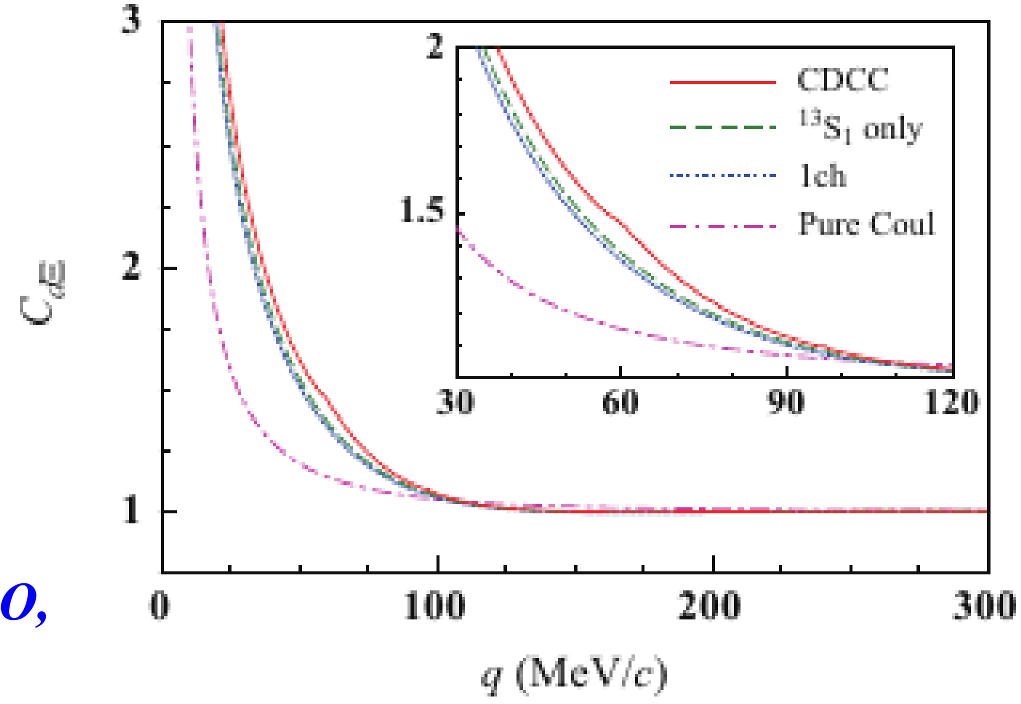
- **CDCC results of Ξ^-d correlation function**
  - **Enhancement from pure Coulomb C(q) by ΞN interaction from HAL QCD potential.**
  - **Breakup & Reformation effects ~ 10 % (Barely measurable)**
  - **Dynamical formation of deuteron is (maximally) included.**

$$\text{Implicit assumption:} \int d\boldsymbol{\rho} S(\rho)|\varphi_k(\rho)|^2 \simeq \text{const.}$$

  - **Threshold cusp at d → pn threshold is seen, but not prominent.**

Single channel description may not be bad.
→ Bound or Unbound in Ξd from Experimental data (if measured).

K. Ogata, T. Fukui, Y. Kamiya, and AO, PRC103 ('21), 065205 [2103.00100].



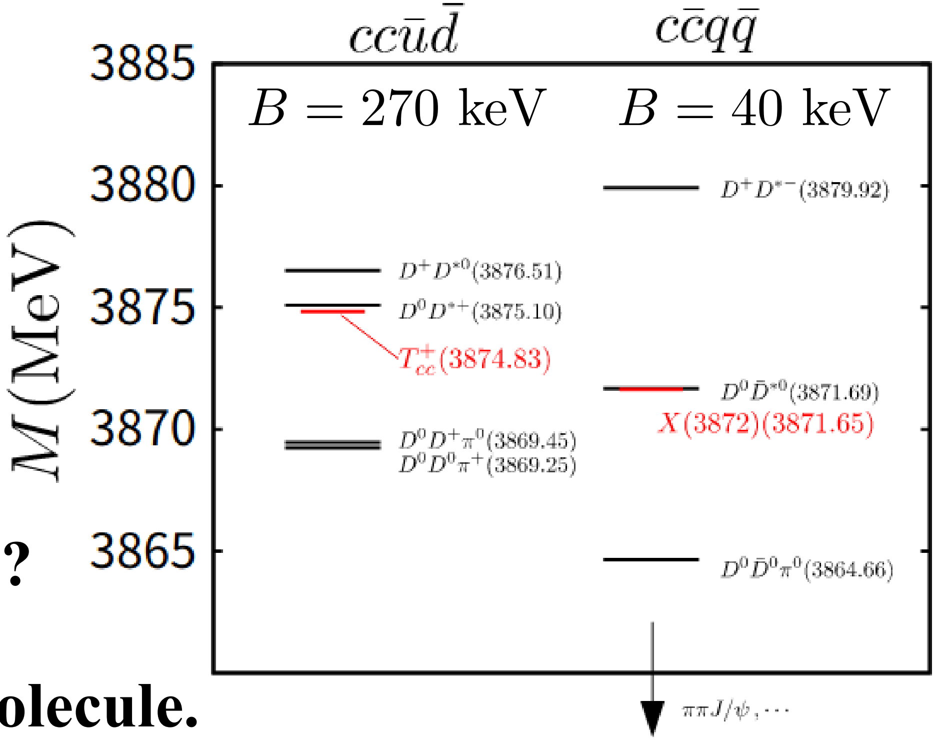
Summary

- Femtoscopy has doubled the number of experimentally accessible hadron-hadron interactions in 8 years (2015–2023).
  - The number is still growing rapidly.
- New trends: Heavy-quark and Three-body
  - Charm hadron interactions seem to have unexpected features, such as attraction from two-pion exchange.
    ”Compositeness” of some exotic hadrons may be obtained in the (near) future.
  - Few-body techniques such as CDCC are useful also in femtoscopy.
    (CDCC should be also applicable to CFs of K^+d and Λd.)
- In 21H00121, we have published 2 papers on charm hadron interactions, and 1 paper on hadron-deuteron correlation function.
  - Actual 3-body correlation functions (ppp, ppΛ, ppK, …) are still difficult to calculate.
    (Continuum w.f. (with Coulomb) from Faddeev equation!)

Thank you for your attention !

Compact Tetraquarks or Hadronic Molecules

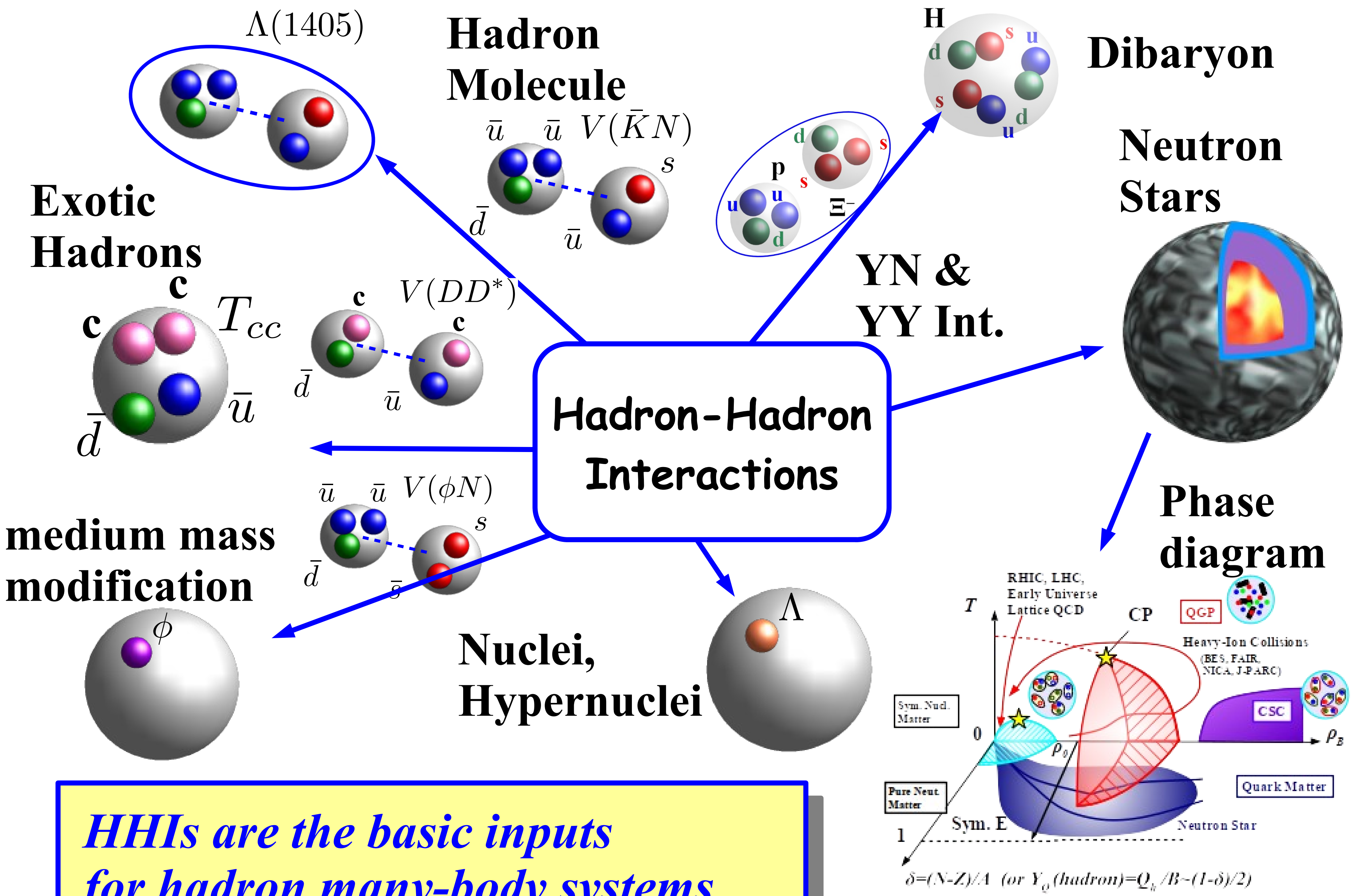
- **Tcc = Compact Tetraquark ?**
  Good $[\bar{u}\bar{d}]$ diquark gains energy
  S. Zouzou+('86), ZPC30,457.
- **X(3872)**
  - $c\bar{c}$ **component ? production cross section** *Bignamini+ (0906.0882)*
  - **Large yield in Pb+Pb → Molecule?**
    Sirunyan+ [CMS] (2102.13048)
    c.f. Δr/Δp is similar in HIC and molecule.
    ExHIC ('11,'11,'17)



- **Hadronic Molecule Conditions**
  - **Appears around the threshold → OK**
  - **Have large size** $R \simeq 1/\sqrt{2\mu B}$ **→ Yield**
  - **Described by the *hh* interaction**

How can we access hh int. with charm ?
→ Femtoscopy

Hadron-Hadron Interactions



Hadron-Hadron Interactions

Hadron Molecule

Dibaryon

Exotic Hadrons

YN & YY Int.

Neutron Stars

Phase diagram

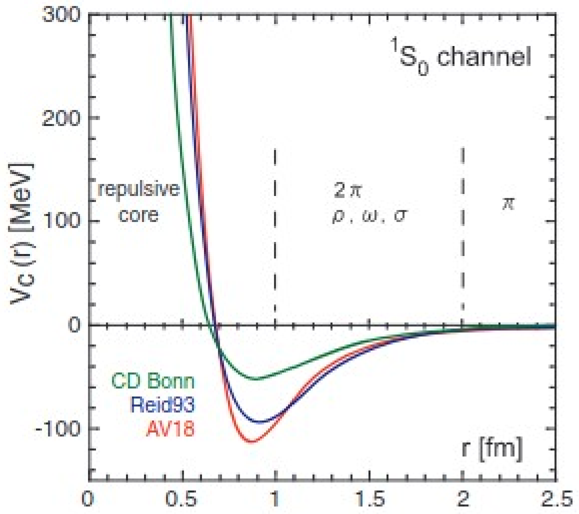
medium mass modification

Nuclei, Hypernuclei

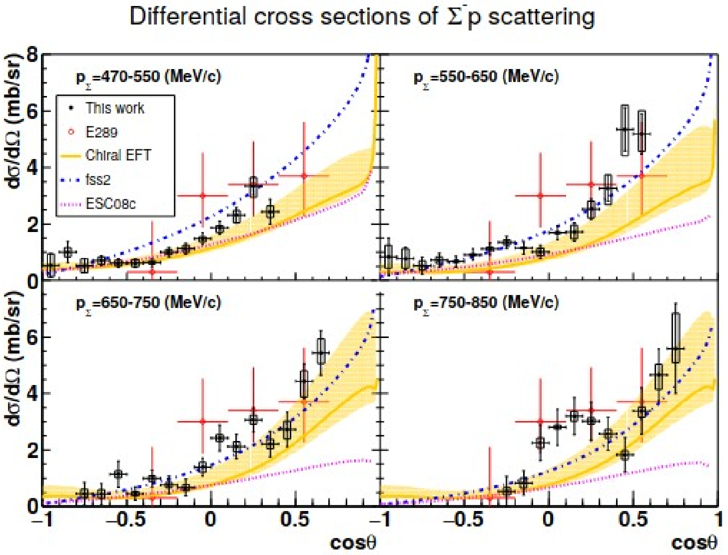
HHIs are the basic inputs
for hadron many-body systems

Experimental info. on Hadron-Hadron Interactions

- **Nucleon-Nucleon (NN) Interactions**
  - **Based on rich NN phase shift data, high-quality potentials are obtained.**
- **Scattering experiments**
  - **Targets need to be stable (p, nuclei)**
  - **Beam particles need to fly a few cm or more.**
  - **NN, πN, K^+N, K^-N, YN (Hyperon-N)**
    E.g. J-PARC E40 (ΣN scattering)
- **Hypernuclei, Exotic Nuclei**
  - **Interactions from bound states**
    Exotic atoms (π, K^-, Σ^-),
    Hypernuclei (Λ, Σ, Ξ, ΛΛ).
- **Femtoscopy**
  - **Correlation function contains info. on Scattering via Final State Interaction**
  - **Any pair is accessible if PID is possible.**



Ishii, Aoki, Hatsuda ('07)



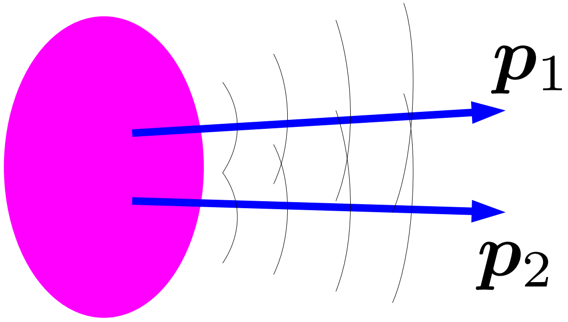
Miwa+[J-PARC E40]('21)

Femtoscopy Basics

Two-particle momentum correlation function

- **Emission function of one particle**

$$N_i(\boldsymbol{p}) = \int d^4x S_i(x,\boldsymbol{p})$$



- **Two-particle momentum distribution**
  - **Assumption: Particles are emitted independently. Correlation is made by final state interaction (or quantum statistics).**

$$\begin{aligned} N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2) &\simeq \int d^4x d^4y\, S_1(x,\boldsymbol{p}_1)\, S_2(y,\boldsymbol{p}_2) |\Psi_{\boldsymbol{p}_1,\boldsymbol{p}_2}(x,y)|^2 \\ &\simeq \int d^4X d\boldsymbol{r} dt\, S_1(x,\boldsymbol{p}_1)\, S_2(y,\boldsymbol{p}_2) |e^{-iPX}|^2 \times \underline{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2} \end{aligned}$$

Relative w.f.

(pair rest frame)

- **Correlation Function (CF) from the Koonin-Pratt formula**
  - **Integrate over CM coordinates, normalized Source function is used.**

Koonin('77), Pratt('86), Lednicky+('82)

$$C(\boldsymbol{q},\boldsymbol{P}) = \frac{N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r} \underbrace{S(\boldsymbol{r})}_{\text{source fn.}} \underbrace{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}_{\text{relative w.f.}}$$

S-wave contribution to the Correlation Function

- **Effects of Hadron-Hadron Interactions on C(q)**
  - **Only the S-wave w.f. is assumed to be modified.**
    (C(q) deviates from 1 at low q (q<200 MeV/c) → S-wave dominance
  - **Cases with non-identical particle pair and Gaussian source**

$$
\begin{aligned}
\varphi_{\boldsymbol{q}}(\boldsymbol{r}) =& e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr) + \chi_q(r) \\
\to C(\boldsymbol{q}) =& \int d\boldsymbol{r} S(r) |\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2 \\
=& 1 + \int d\boldsymbol{r} S(r) \left\{ |\chi_q(r)|^2 - |j_0(qr)|^2 \right\} \\
=& 1 + \frac{1}{2\sqrt{\pi}R^3q^2} \int_0^\infty dr e^{-r^2/4R^2} \left\{ |u(r)|^2 - \sin^2 qr \right\} \ \ [u(r) = qr\chi_q(r)]
\end{aligned}
$$

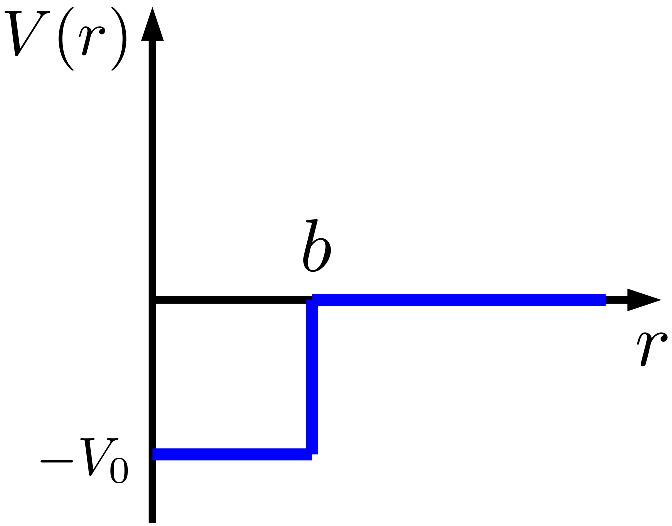
$$
\begin{aligned}
&\langle e^{i\boldsymbol{q}\cdot\boldsymbol{r}} j_0(qr) \rangle_\Omega = j_0^2(qr) \\
&\langle \underset{\textbf{L>0}}{\underline{[e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr)]}} \, \chi_q(r) \rangle_\Omega = 0
\end{aligned}
$$

C(q) shows enhancement of S-wave |w.f.|² from | j₀(qr)|².
Solve 1D Schrodinger Eq. and Integrate in 1D.

Square-Well Potential in 3D (1)

- **Square well potential**

$$V(r) = \begin{cases} -V_0 & (r < b) \\ 0 & (r \geq b) \end{cases}$$



- **S-wave wave function**

$$\chi_q(r) = u(r)/qr, \ \kappa = \sqrt{q^2 + 2mV_0/\hbar^2}$$

$$u(r) = \begin{cases} A\sin\kappa r & (r < b) \\ \sin(qr + \delta) & (r \geq b) \end{cases}$$

$$\delta = \arctan\left(\frac{qb\tan\kappa b}{\kappa b}\right) - qb\,, \ A = \frac{\sin(qb+\delta)}{\sin\kappa b}$$

$$\kappa b\cot\kappa b = qb\cot(qb+\delta)$$ **(continuity of logarithmic derivatives)**

- **Scattering length and Effective range**

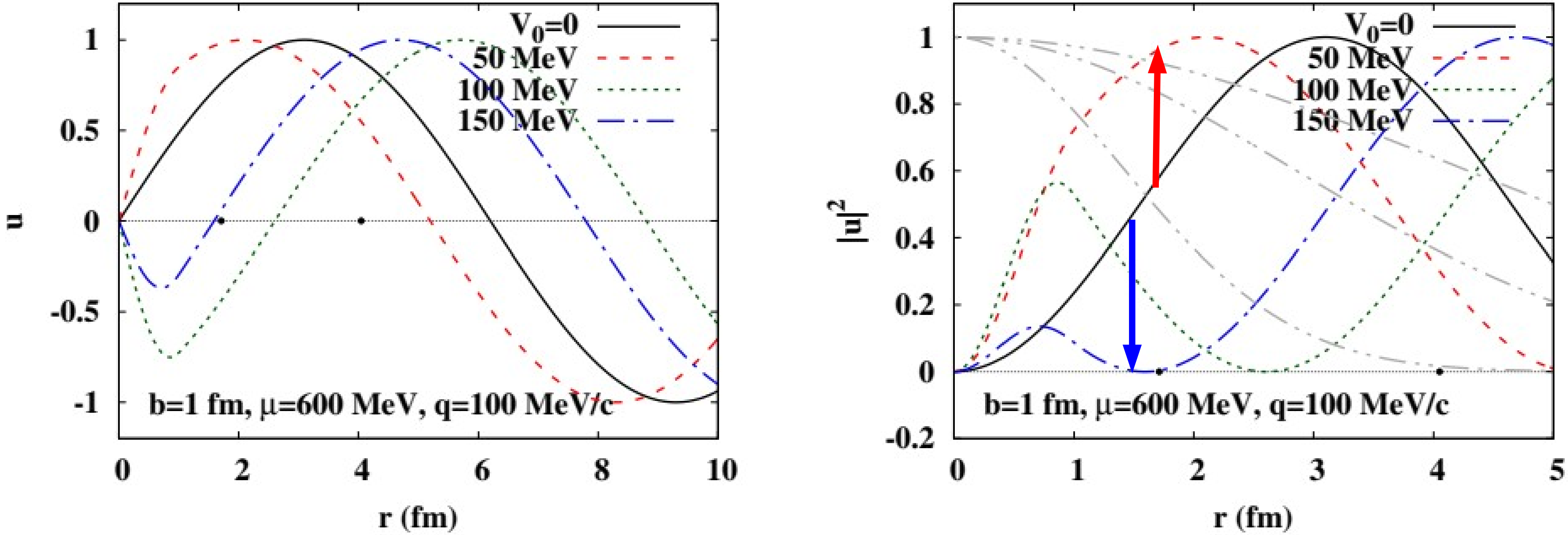
E.Braaten, H.-W.Hammer, Phys.Rept. 428 ('06) 259.

$$a_0 = -bD\,, \ r_{\mathrm{eff}} = b \times \frac{3C^2D^2 + 3D - C^2}{3C^2D^2} \ \left(C = \sqrt{2mV_0b^2/\hbar^2}\,, \ D = \frac{\tan C}{C} - 1\right)$$

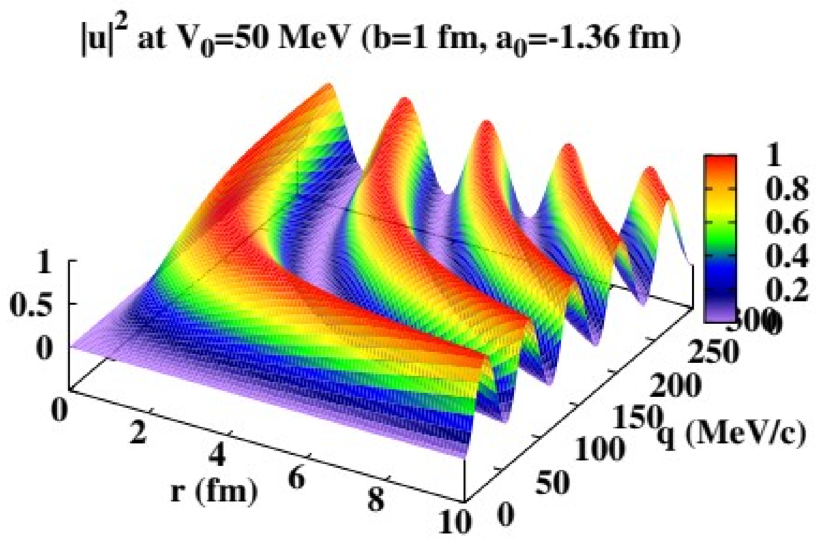
Square-Well Potential in 3D (2)

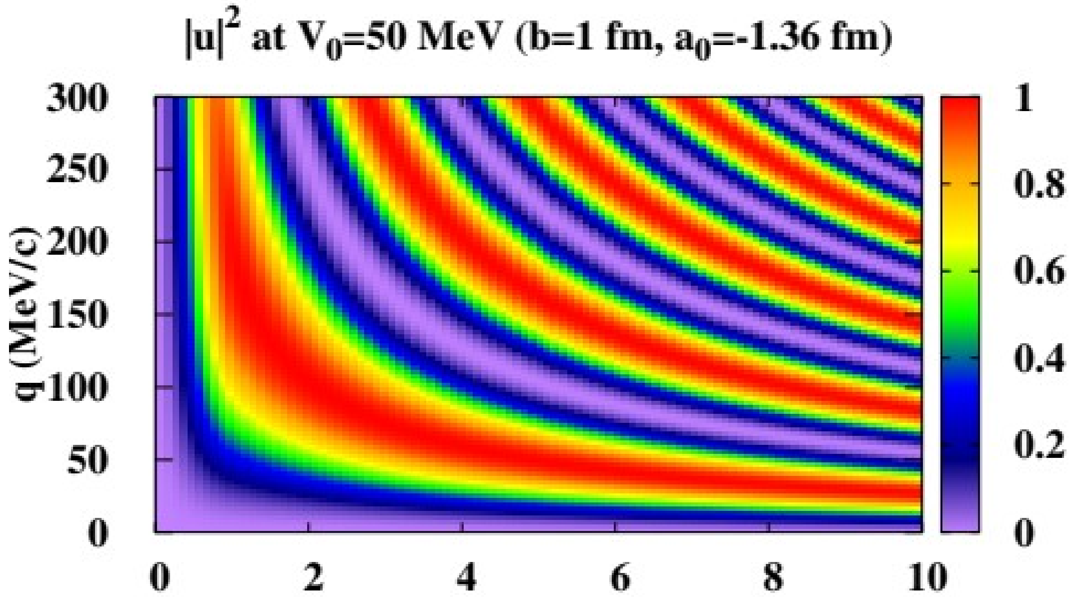
- **Weakly attractive pot. (no bound state)**
  → |w.f.|^2 is enhanced at small r → C(q) > 1
- **Strongly attractive pot. (w/ bound state)**
  → |w.f.|^2 is suppressed at small r → C(q) < 1
  - **With a bound state (a_0>0), w.f. has a node around r~ a_0**

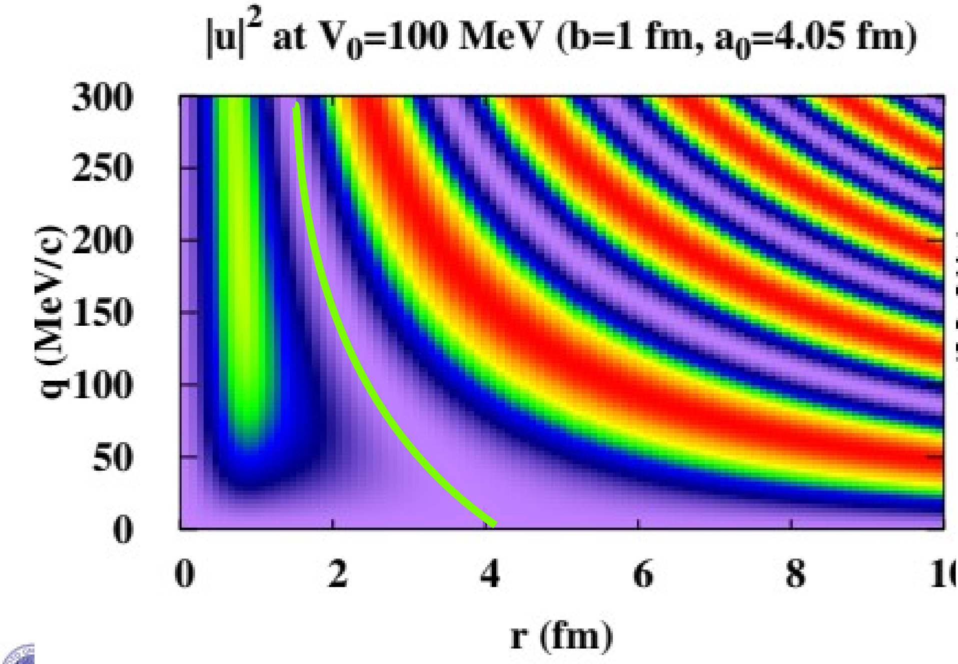
$$u(r, r > b) = \sin(qr + \delta) \simeq \sin(q(r - a_0)) \quad (\delta \simeq -a_0 q)$$

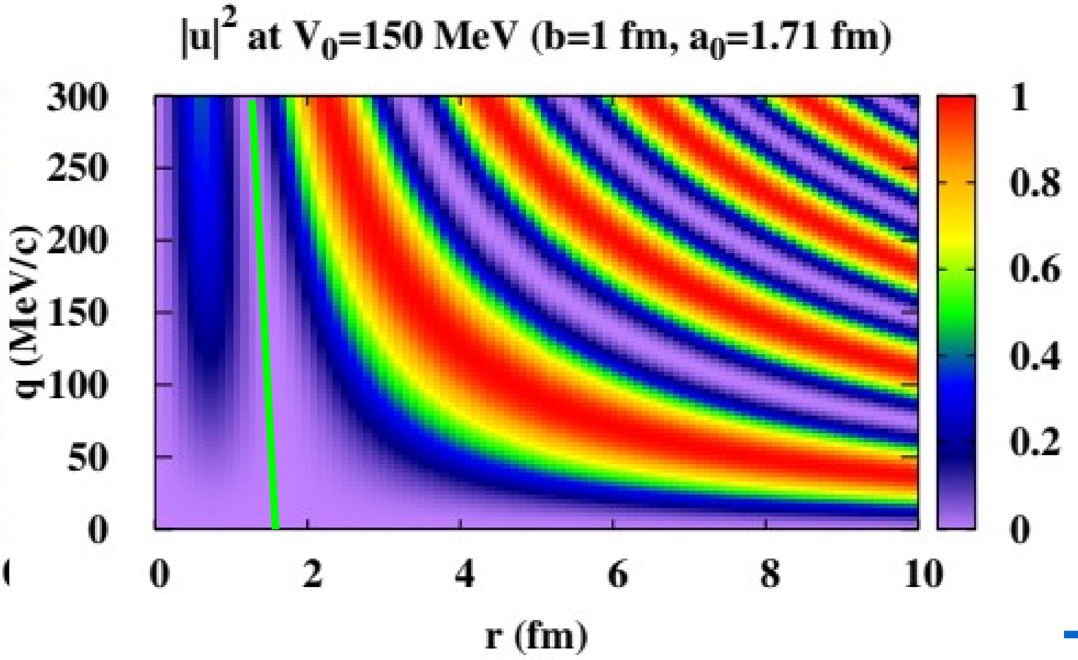


Square-Well Potential in 3D (3)


$|u|^2$ at V_0=50 MeV (b=1 fm, a_0=-1.36 fm)

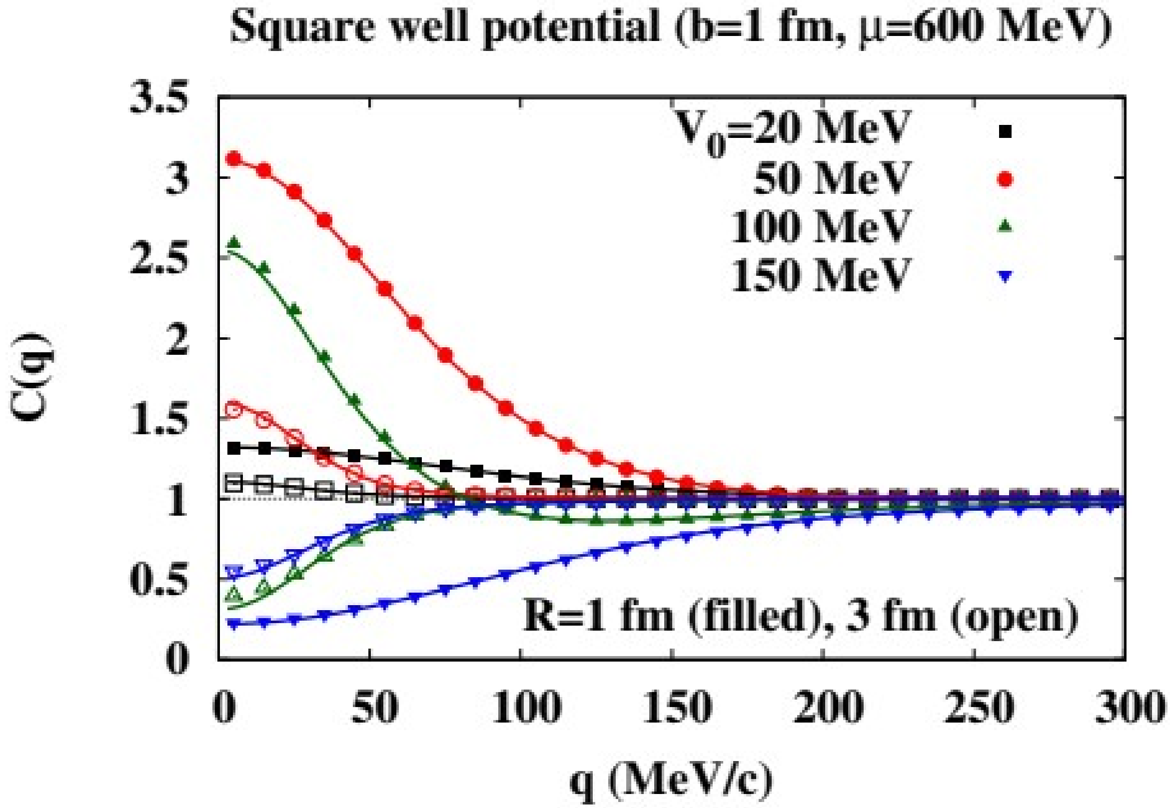

$|u|^2$ at V_0=50 MeV (b=1 fm, a_0=-1.36 fm)


$|u|^2$ at V_0=100 MeV (b=1 fm, a_0=4.05 fm)


$|u|^2$ at V_0=150 MeV (b=1 fm, a_0=1.71 fm)

Correlation function from square-well potential

- **One dimensional integral (Combination of complex error fn.)**
- **Non-monotonic dep. on the potential strength**
  - **V_0 < 80 MeV (no bound state) → C(q) increases with V_0**
  - **V_0 > 80 MeV (with bound state) → C(q) is suppressed with large R**
- **Agrees with Lednicky-Lyuboshits analytical model results**



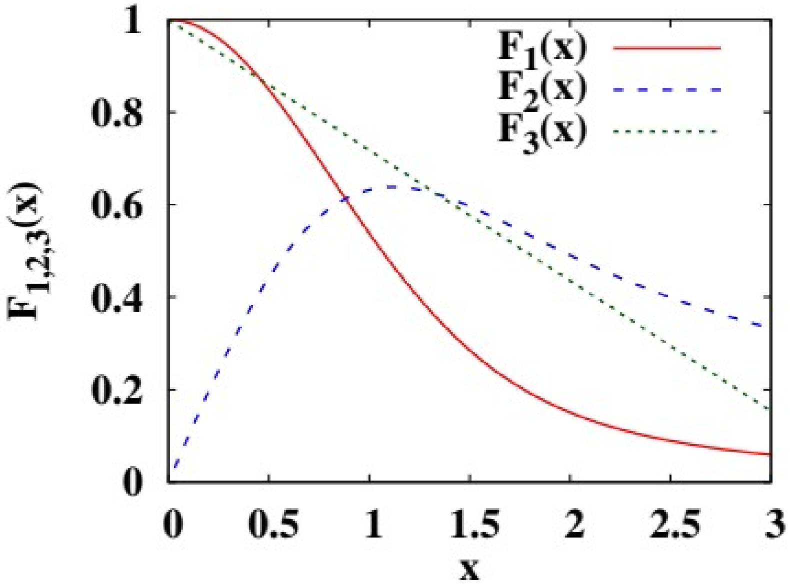
Analytic model of correlation function

- **Correlation function in Lednicky-Lyuboshits (LL) formula**
  (asymptotic w.f., non-identical particle pair, short range int. (only s-wave is modified), single channel, no Coulomb pot., static Gaussian source, real δ) *(Lednickey, Lyuboshits ('82))*

$$\varphi_0^{(-)}(r;q) \simeq \frac{e^{-i\delta}\sin(qr+\delta)}{qr}, \quad f(q) = \frac{e^{i\delta}\sin\delta}{q} = \frac{1}{q\cot\delta - iq}, \quad q\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}q^2 + \mathcal{O}(q^4)$$

$$C_{\rm LL}(q) = 1 + \frac{2{\rm Re}\,f(q)}{\sqrt{\pi}R}F_1(2qR) - \frac{{\rm Im}\,f(q)}{R}F_2(2qR) + \frac{|f(q)|^2}{2R^2}F_3\left(\frac{r_{\rm eff}}{R}\right)$$

$$\left[f(q) = (q\cot\delta - iq)^{-1},\ F_1(x) = \frac{1}{x}\int_0^x dt e^{t^2-x^2},\ F_2(x) = (1-e^{-x^2})/x,\ F_3(x) = 1 - \frac{x}{2\sqrt{\pi}}\right]$$



If you have a_0, r_{eff} and R, you can draw C(q) !

$$F_1(x) \simeq \frac{1+c_1x^2+c_2x^4+c_3x^6}{1+(c_1+2/3)x^2+c_4x^4+c_5x^6+c_3x^8} \quad (0 \le x < 20)$$

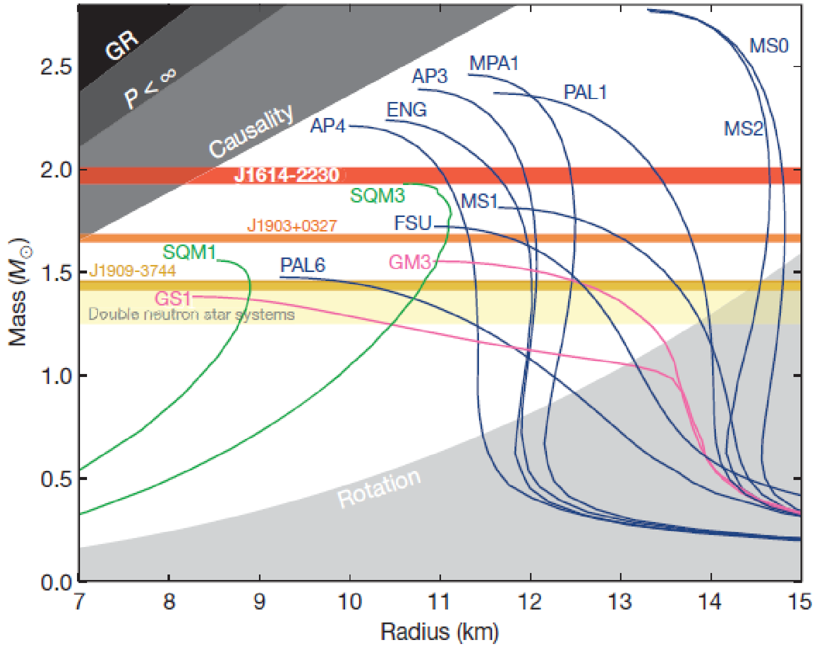
$$(c_1, c_2, c_3, c_4, c_5) = (0.123, 0.0376, 0.0107, 0.304, 0.0617)$$

AO,Morita,Mihayara,Hyodo, NPA 954 ('16)294.

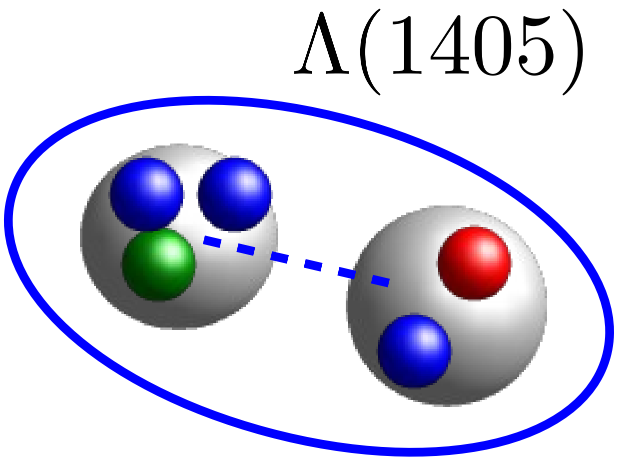
Hadron-Hadron Interactions and Bound States with Strangeness using Femtoscopy

Hadron-Hadron Interactions with Strangeness

- **Hyperonic matter EOS with empirical Hyperon-Nucleon (YN) interactions cannot support 2 solar mass neutron stars (Hyperon Puzzle).**
  - **YN interaction details ? YNN 3-body int. ? Many-body theories ? Gradual transition to quark matter ? Modified gravity ?**
- **Anti-kaon Nucleon interaction predicts a bound state Λ(1405), which is the starting point of "Hadronic Molecule".**



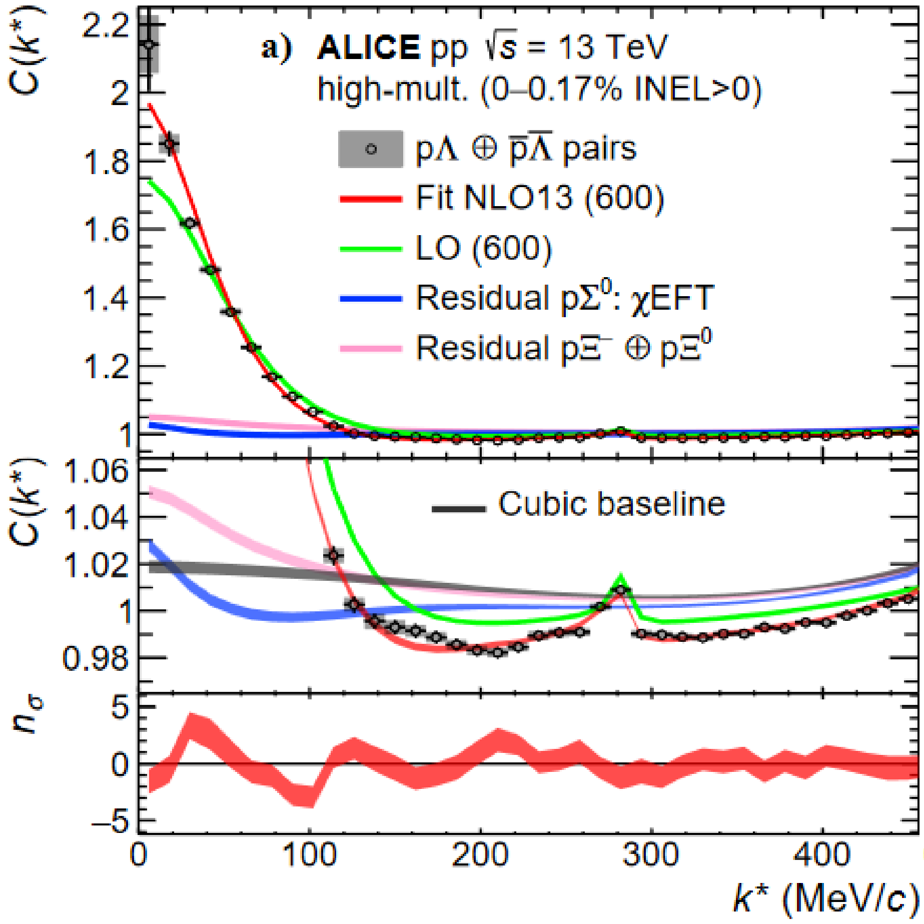
Demorest et al., Nature 467 (2010) 1081 (Oct.28, 2010).



S=-1: ΛN interaction

- Scattering data exist, but only at high momenta (q>200 MeV/c).
- Constrained by hypernuclear spectroscopy, but indirect.
- Femtoscopy
  - High precision data including low momentum region.
  - NΣ cusp is clearly seen.
  - NLO chiral EFT better explains the data than LO.

S. Acharya+[ALICE], PLB 833('22)137272

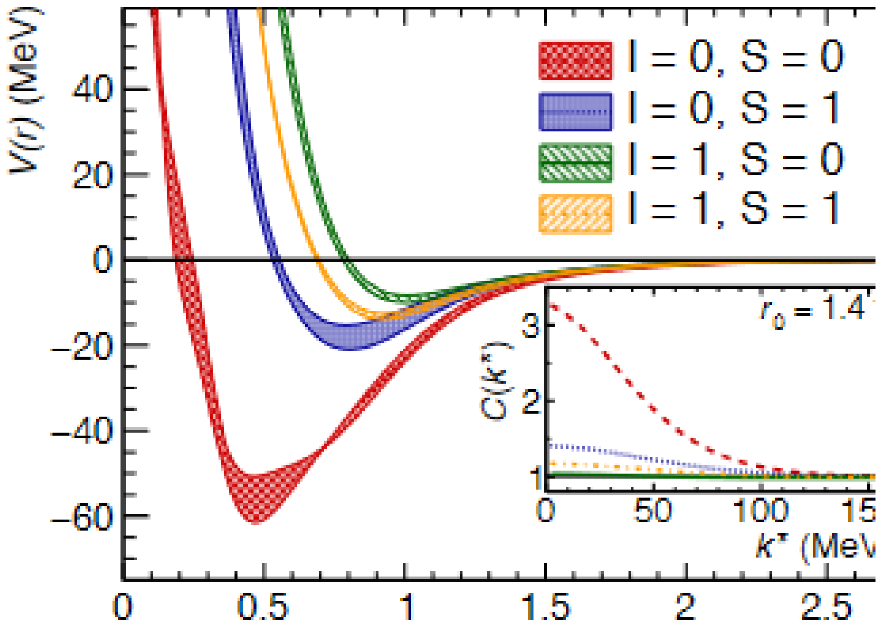


Femtoscopy would be valid alternative to scattering experiments (at least at low energies).
– Laura Fabbietti, ExHIC2016 @ YITP –

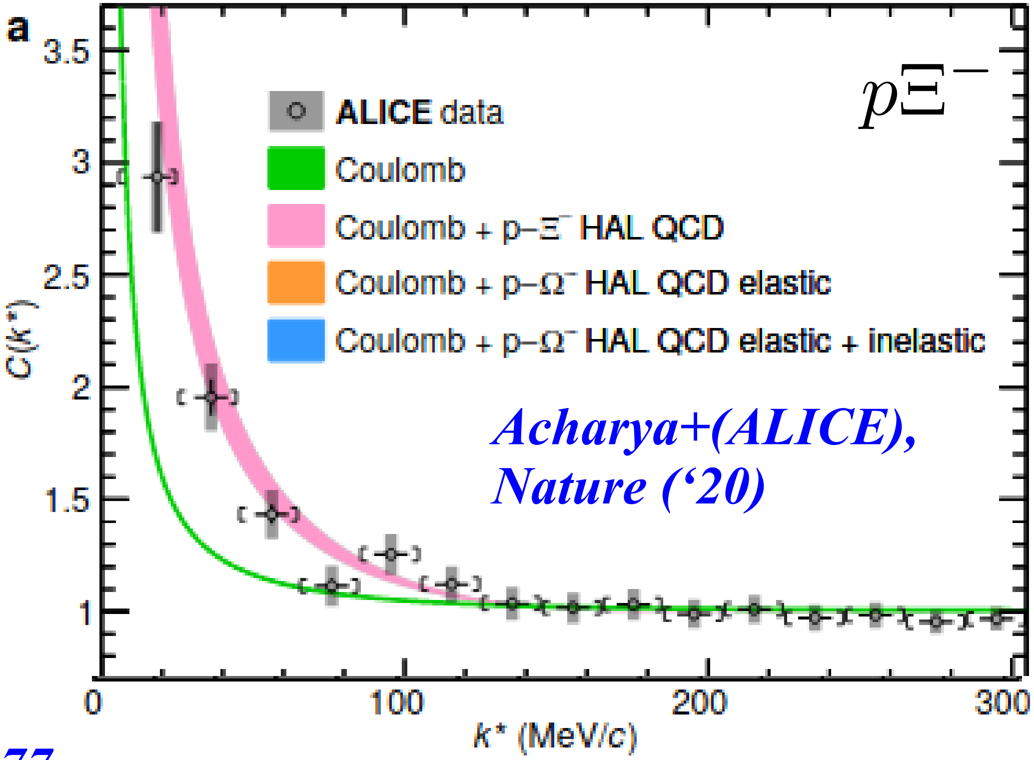
S=-2: ΞN interaction

- **Lattice S=-2 BB interaction** *K. Sasaki+[HAL QCD], NPA 998 (‘20) 121737.*
- **Corr. Fn. data (from pp collisions)** *S. Acharya+ [ALICE] (Nature, ‘20)*

Lattice QCD S= –2 BB potential is examined!

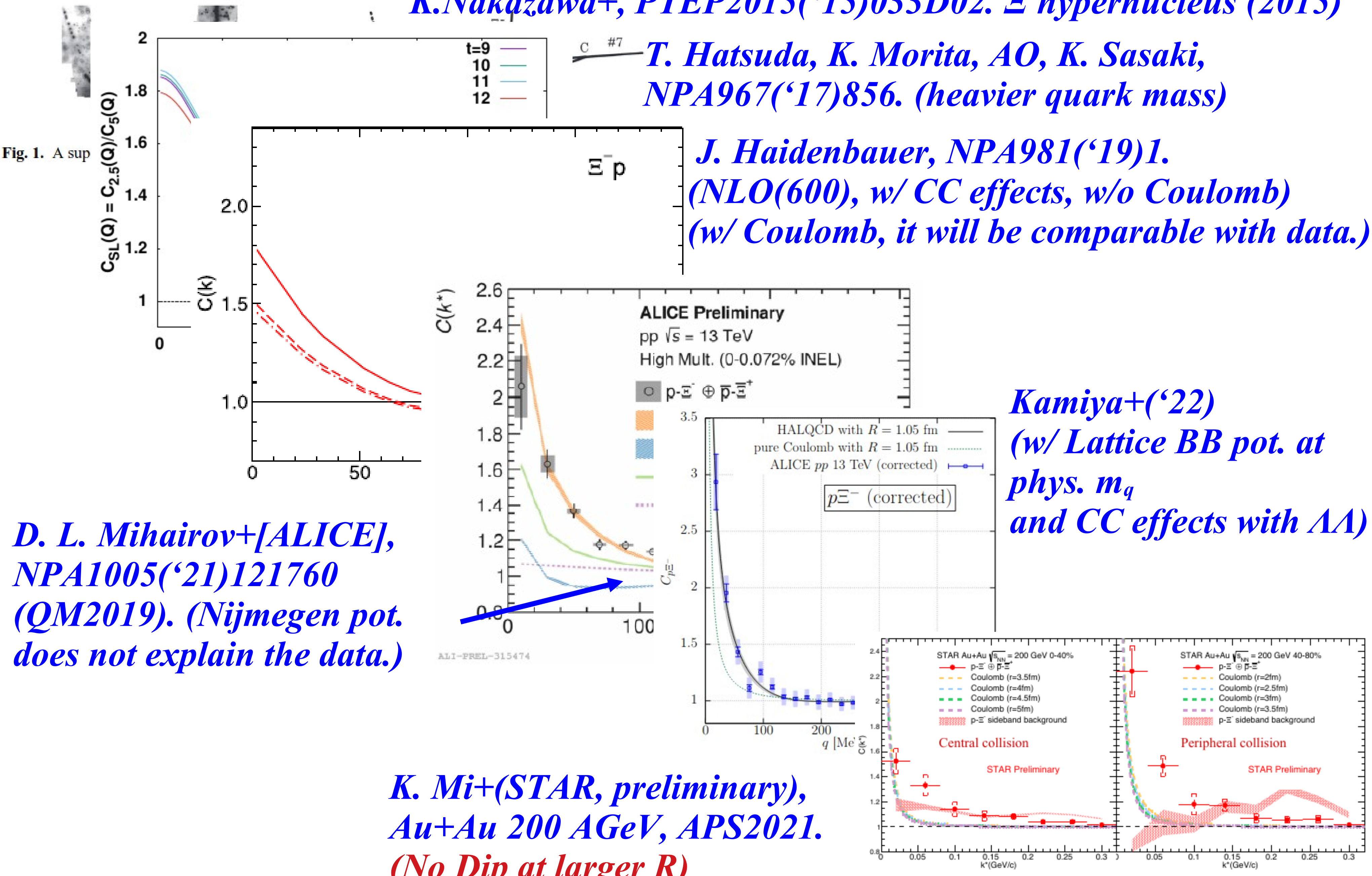


K. Sasaki+ [HAL QCD](‘20)
Taken from L. Fabbietti+,
Ann. Rev. Nucl. Part. Sci. 71 (‘21) 377.



Acharya+(ALICE),
Nature (‘20)

Comparison with other results



K.Nakazawa+, PTEP2015('15)033D02. Ξ hypernucleus (2015)

T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17)856. (heavier quark mass)

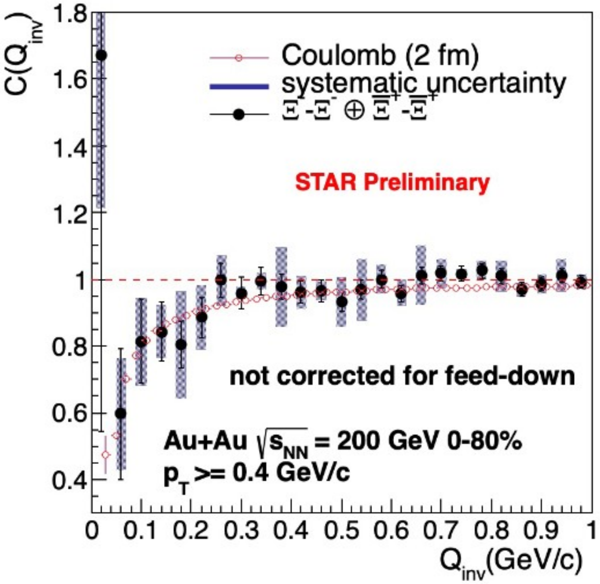
J. Haidenbauer, NPA981('19)1. (NLO(600), w/ CC effects, w/o Coulomb) (w/ Coulomb, it will be comparable with data.)

Kamiya+('22) (w/ Lattice BB pot. at phys. m_q and CC effects with ΛΛ)

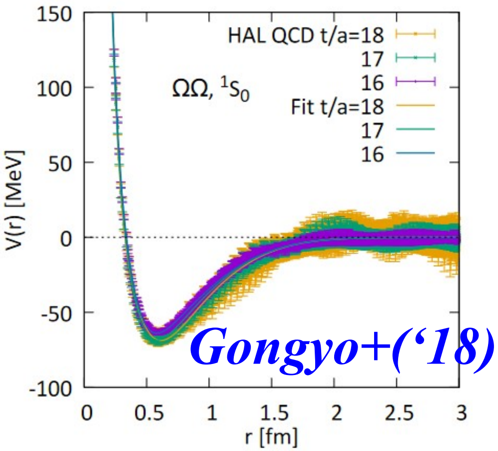
D. L. Mihairov+[ALICE], NPA1005('21)121760 (QM2019). (Nijmegen pot. does not explain the data.)

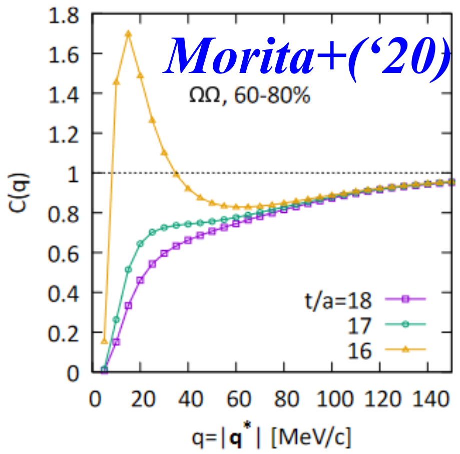
K. Mi+(STAR, preliminary), Au+Au 200 AGeV, APS2021.
(No Dip at larger R)

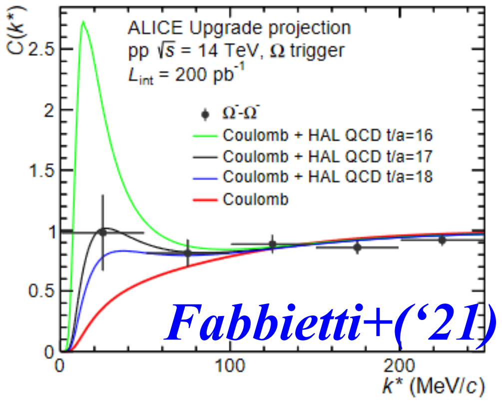
S ≤ -4 BB interactions: How far can we go?



- **S=–4 (ΞΞ)**
  - **Chiral EFT and Lattice QCD predict attraction (no bound state)**
    J. Haidenbauer+('15); T. Doi+('18)
  - **STAR preliminary data seems to agree with pure Coulomb results. (Attraction compensate quantum statistics)**
- **S=-6 (ΩΩ, most strange BB pair)**
  - **Lattice QCD predicts a bound state (J=0)** *S. Gongyo+[HAL QCD]('18)*
  - **Prediction of the correlation function** *Morita+('20)*
  - **To be measured in LHC Run3** *L.Fabbietti+('21)*

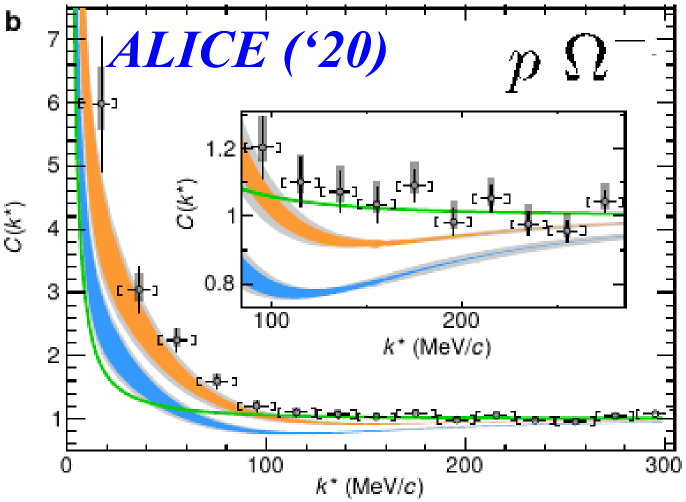


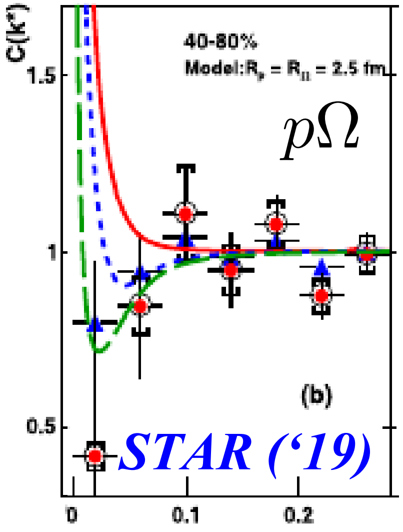


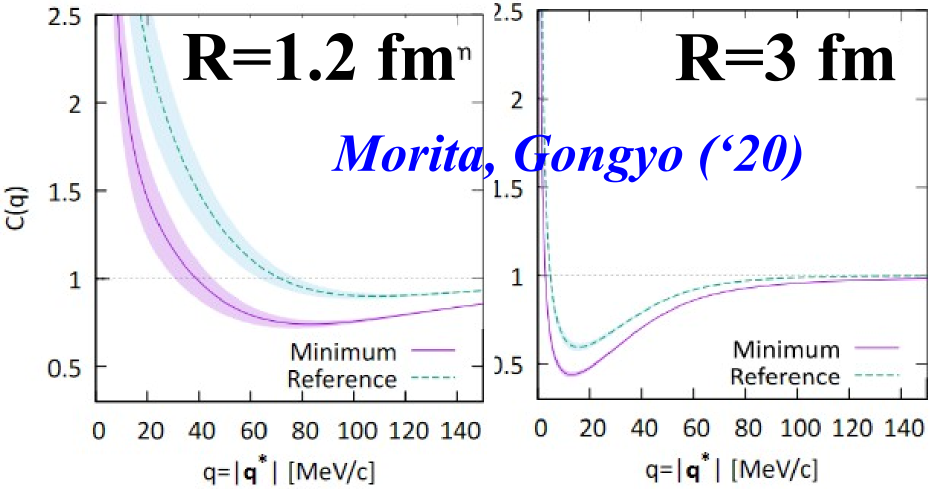


S=-3: ΩN interactions

- **Ω : quark content=sss, J^π=3/2+, M=1672 MeV**
- **ΩN bound state as a S= –3 dibaryon ?** *T.Goldman+('87); M.Oka('88).*
  (No Pauli blocking, Attractive one gluon exch. pot)
- **Lattice QCD potential (HAL QCD, J=2)** *F.Etminan+('14);T.Iritani+('19)*
- **pΩ correlation functions** *STAR('19); ALICE('20)*
  - **Prediction using lattice potential** *Morita+('16, '20)*
  - **Lattice NΩ potential (J=2) seems reasonable.**
  - **J=1 component will fill the dip.**
  - **Source size dependence → Bound state?**





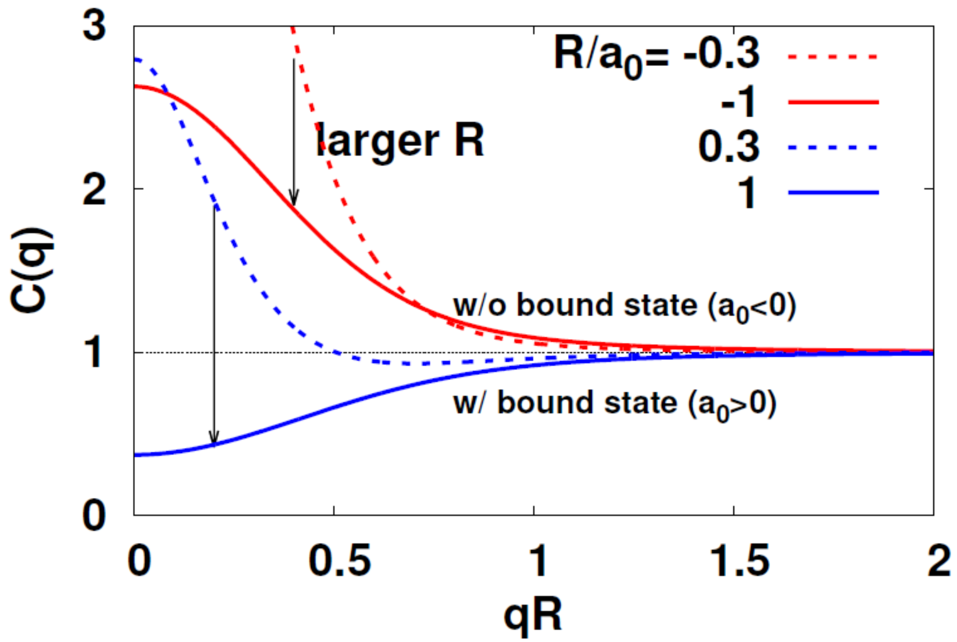


R Dependence of Correlation Function

- **With a bound state, |w.f.|^2 is suppressed at r ~ |a_0|**
  → C(q) is suppressed at small q when R ~ |a_0|

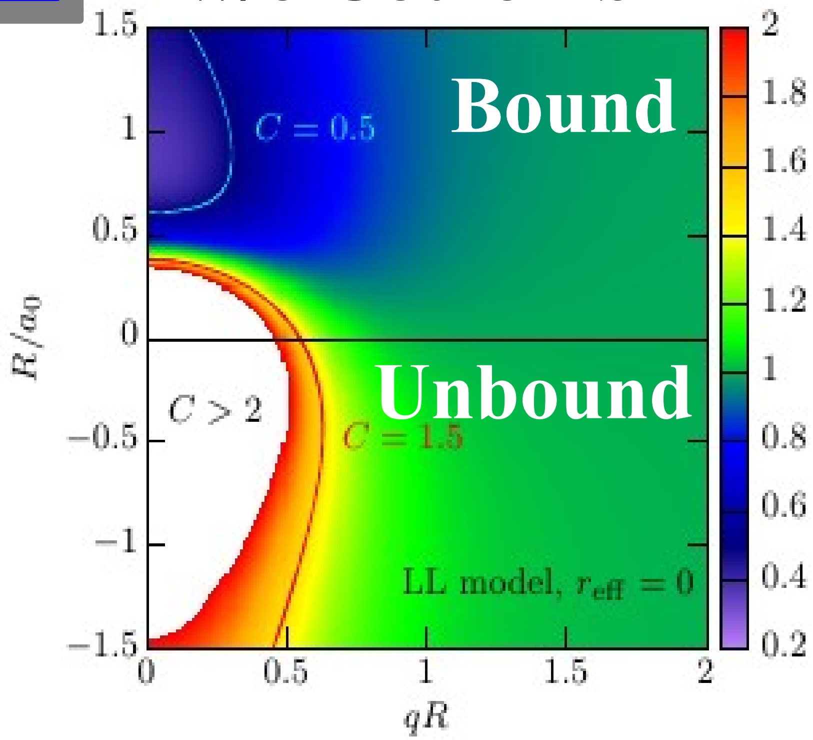
Morita+('16, '20), Kamiya+('20, '22)

One can guess the existence of a bound state from R dep. of C(q)



K.Morita+('20)

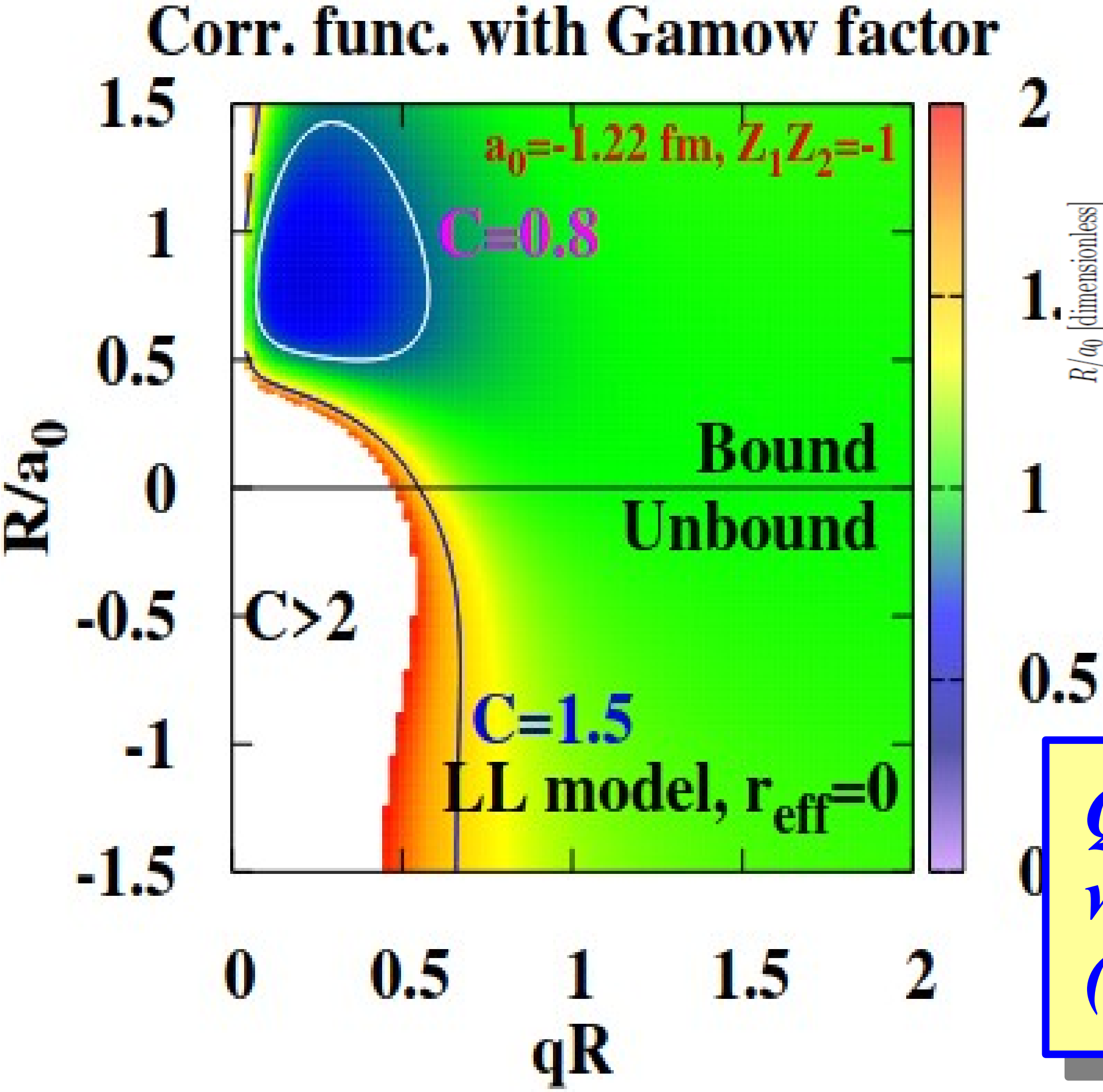
w/o Coulomb



Y.Kamiya+('22)

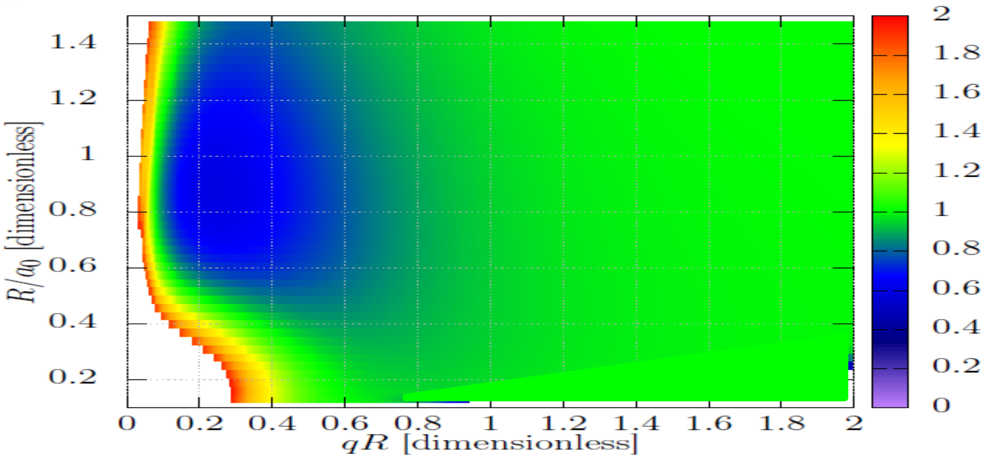
R Dependence of Correlation Function

LL model with Coulomb
(r_{eff}=0)


Corr. func. with Gamow factor

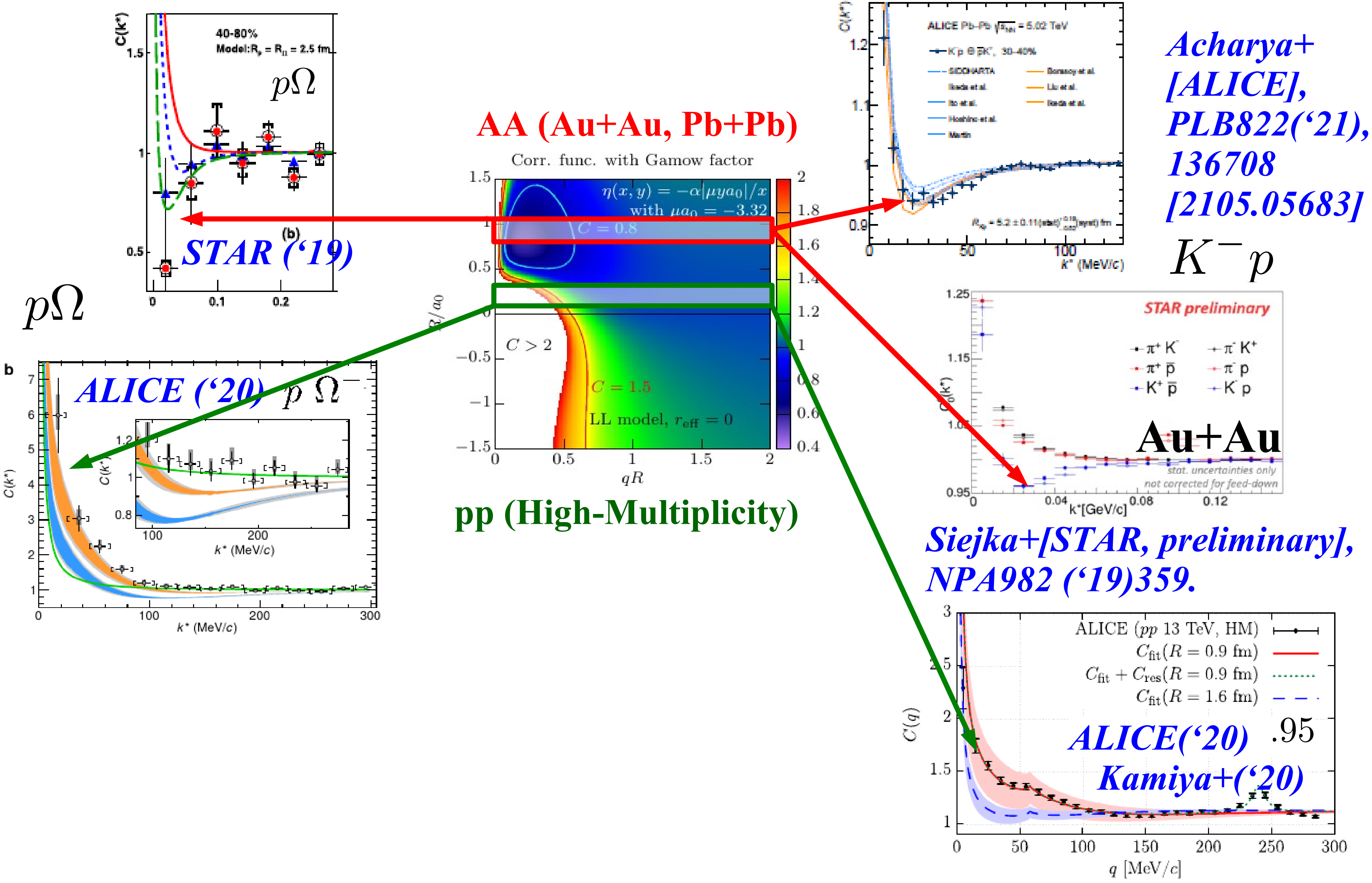
Realistic NΩ potential
(J=2, HAL QCD, a_0=3.4 fm)
+ Coulomb, Coupled-channel

Courtesy of Y. Kamiya



Qualitative feature remains with realistic interactions (and coupled-channel effects)

Bound State Dip



$p\Omega$

STAR ('19)

$p\Omega$

AA (Au+Au, Pb+Pb)

pp (High-Multiplicity)

ALICE ('20) $p\,\Omega^-$

Acharya+ [ALICE], PLB822('21), 136708 [2105.05683]

K^-p

Au+Au

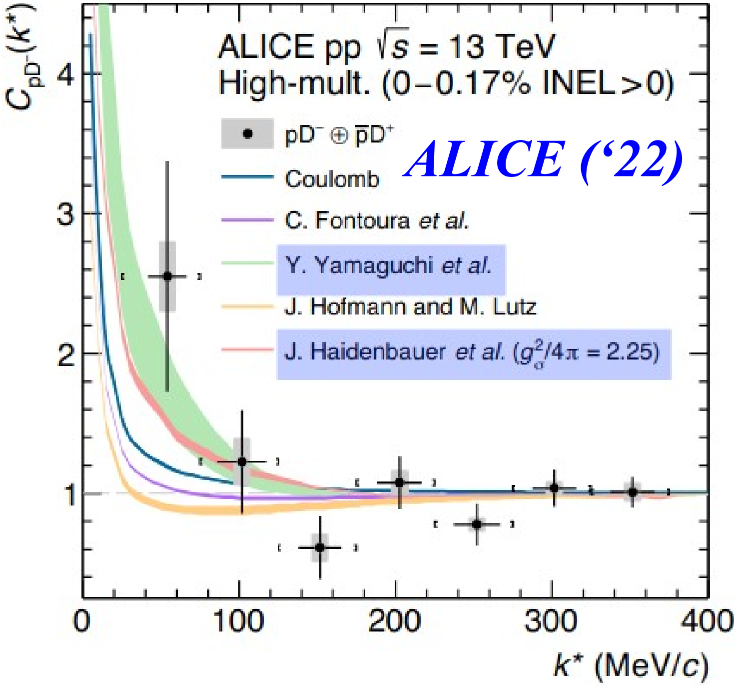
Siejka+[STAR, preliminary], NPA982 ('19)359.

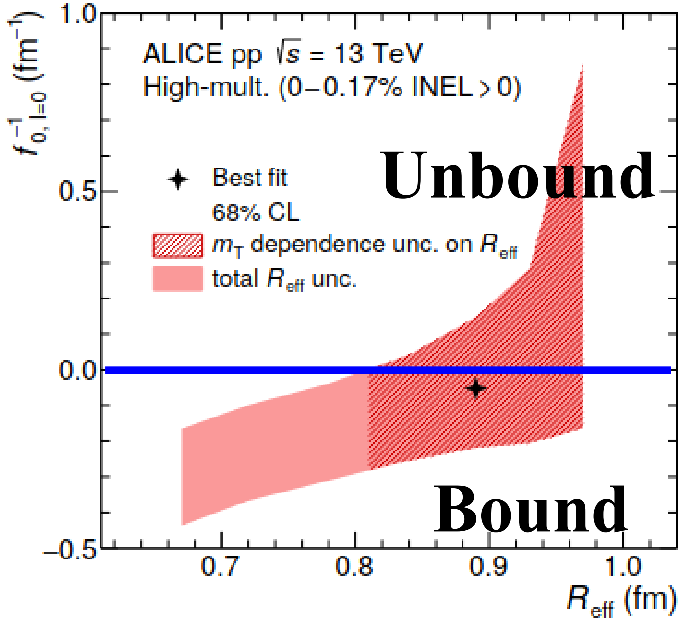
ALICE('20) .95

Kamiya+('20)

How about D^- p ?

- **“First study of the two-body scattering involving charm hadrons”**
  Acharya+[ALICE] (2201.05352)
  - **Enhanced CF from Coulomb → attractive potentials are favored**
  - **If bound, Θc is a bound hadronic molecule**





Fontoura+(1208.4058) (weakly attractive (I=0)) n_σ*=(1.1-1.5)*

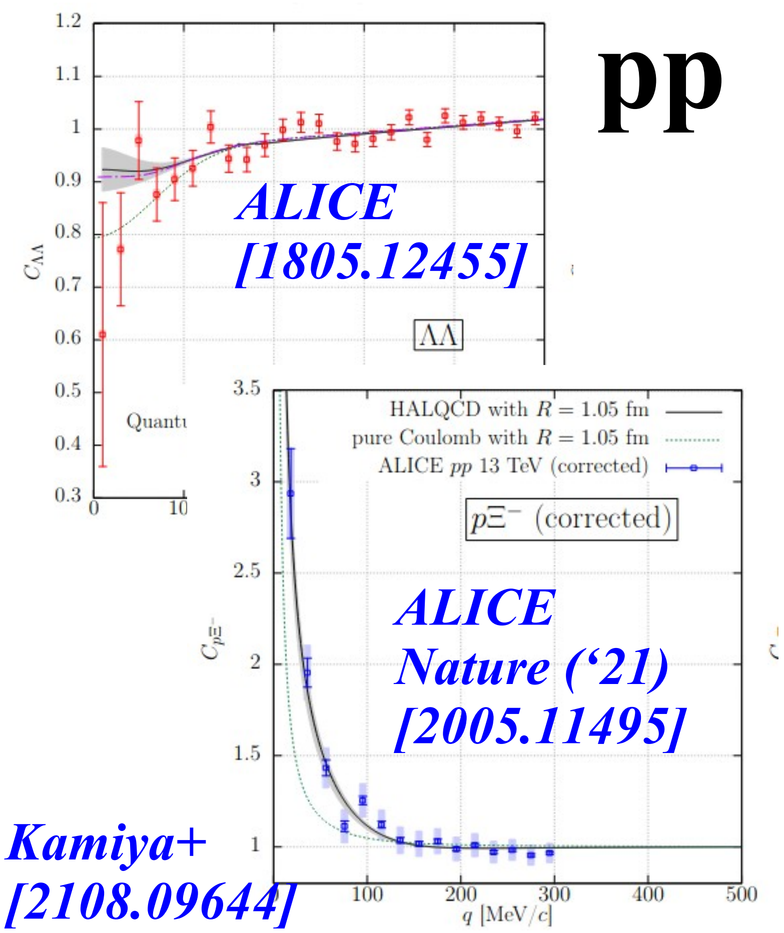
Yamaguchi+ (1105.0734) (att., w/ bound state (I=0))n_σ*=(0.6-1.1)*

Hofmann, Lutz (hep-ph/0507071) (repulsive (I=0)) n_σ*=(1.3-1.6)*

Haidenbauer+(0704.3668) (weakly / mildly attractive (I=0)) n_σ*=(1.2-1.5 / 0.8-1.3)*

Case without a bound state

- **ΛΛ and NΞ seem to be unbound from lattice QCD calculation !**
  Sasaki+ [HAL], NPA998 ('20)121737 [1912.08630]
- **Source size dependence of ΛΛ and pΞ⁻ correlation functions**
  → No dip or suppressed behavior in AA collisions.



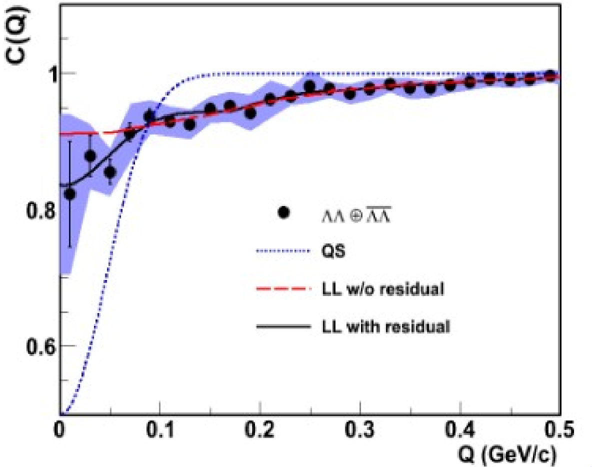
pp

ALICE [1805.12455]

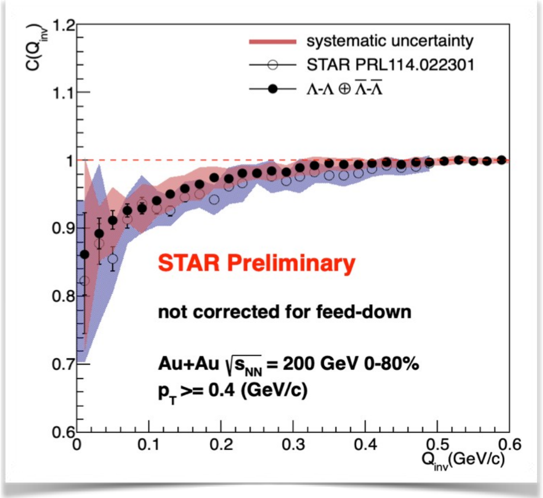
ALICE Nature ('21) [2005.11495]

Kamiya+ [2108.09644]

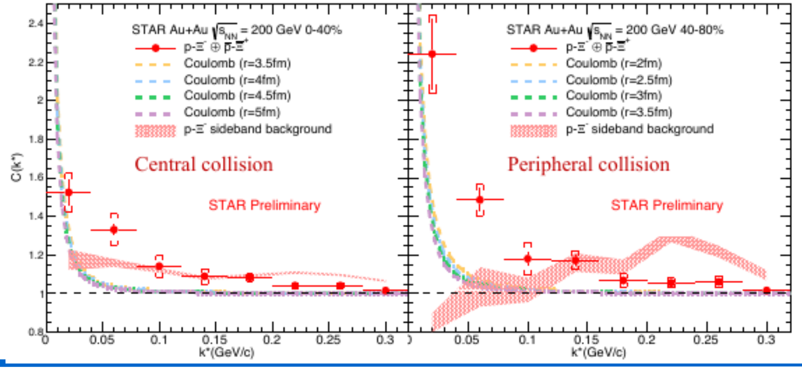
Au+Au



STAR, PRL('15) [1408.4360]



Moe Isshiki+ (STAR, prelim., 2109.10953).

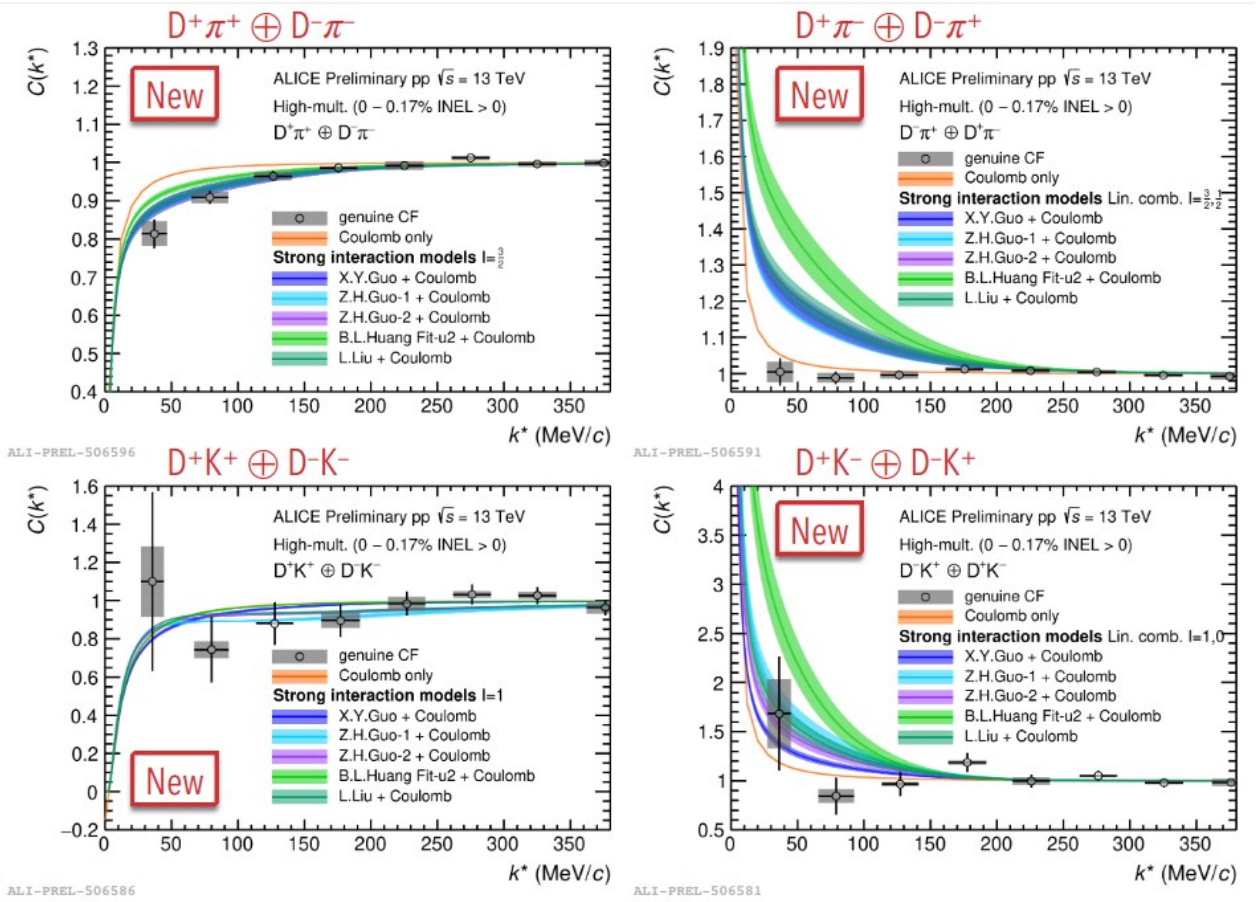


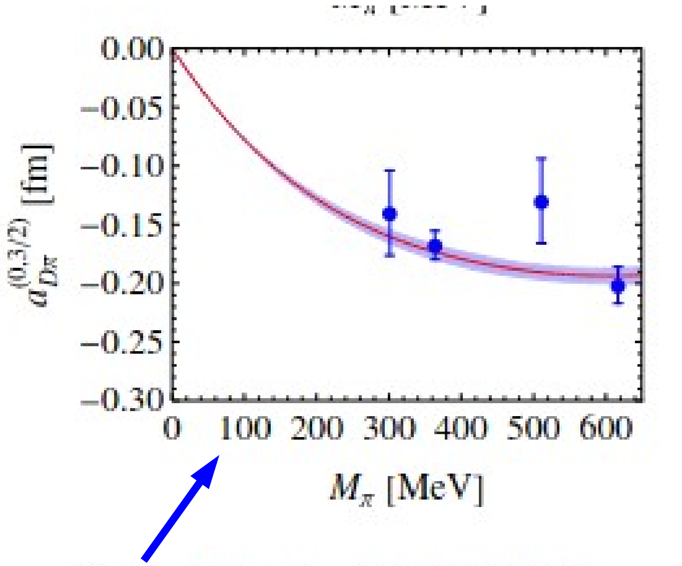
K. Mi+ (STAR, preliminary), Au+Au, APS2021.

Homeworks to hadron physicists from femtoscopy

Homework to Hadron Physics (1)

- **Present chiral models do not explain $D\pi$ and $D\bar{K}$ correlation.**
  - **Overestimate C($D^+\pi^-$) → Mystery ? Extrapolation to phys. mass ?**
    Leading order = Weinberg-Tomozawa (vector exch., repulsive)
    Further repulsive interaction ?
  - **Overestimate C(D^+K^-) → Further repulsion or bound state ?**





- L. Liu et al, Phys. Rev. D87 (2013) 014508
- X.-Y. Guo et al, Phys. Rev. D 98 (2018) 014510
- B.-L. Huang et al, Phys. Rev. D 105 (2022) 036016
- Z.-H. Guo et al Eur. Phys. J. C 79 (2019) 13

Fabrizio Grosa@QM2022

Homework to Hadron (Nuclear) Physics (2)

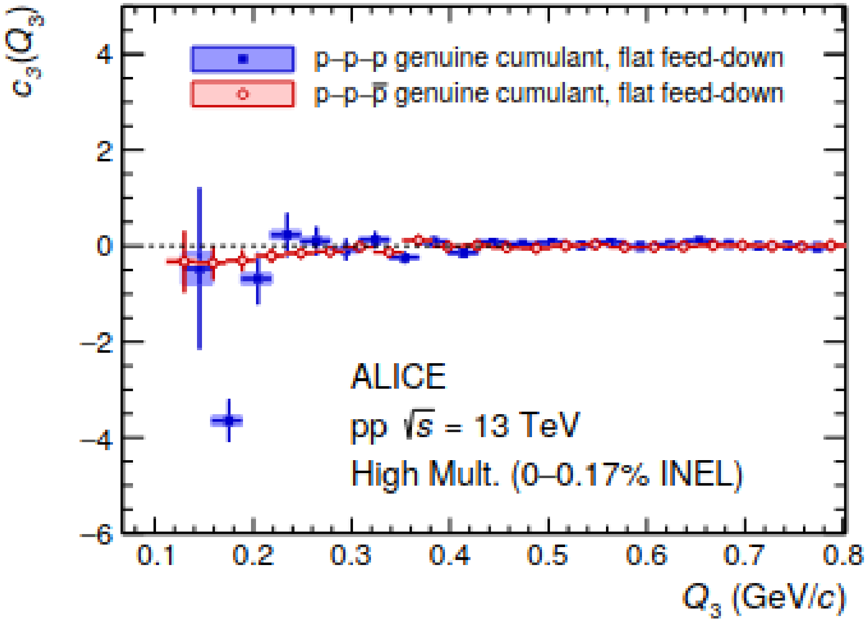
- **Proton-Deuteron Correlation function** *c.f. Bhawani Singh (Fri)*
  - **K^+d C(q) is well explained with $f_0 \sim -0.5$ fm** *Haidenbauer, Hyodo*
  - **Other hd corr. fn. are also described approximately by two-body potential** *S.Mrowczynski, P.Słon, Acta Phys.Pol.B51(‘20),1739; J.Haidenbauer, PRC102(‘20)034001; F.Etminan, M.M.Firoozabadi, (1908.11484); K.Ogata, TFukui, Y.Kamiya, AO, PRC103(‘21),065205.*
  - **pd correlation function cannot be explained by using LL formula or potential model with known f_0. One needs to solve Faddeev Eq.**
    Michael Viviani’s talk (Wed)
  - **Rearrangement?**
    Anti-sym. of
    two protons?

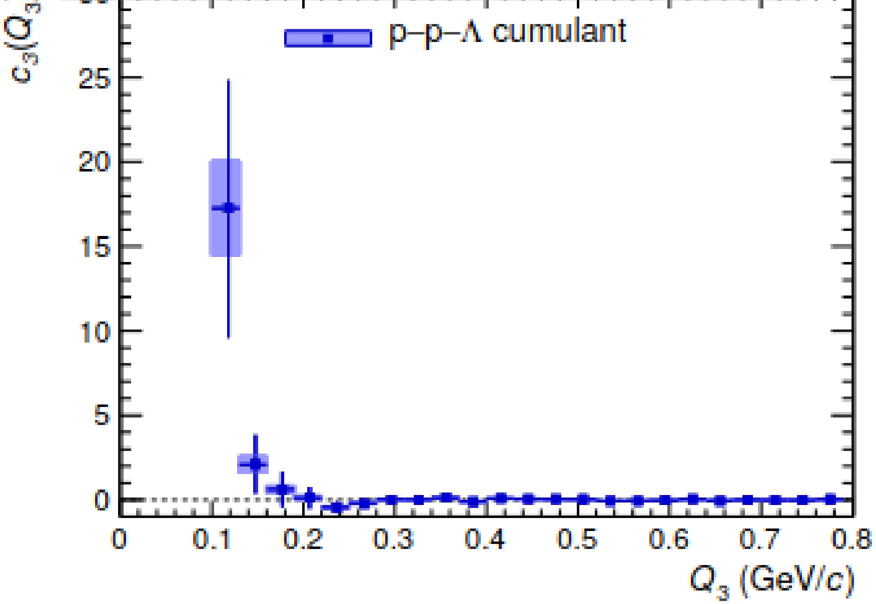
O. Vázquez@ FemTUM22

Homework to Hadron (Nuclear) Physics (3)

- **Three-body correlation function (c.f. Bhawani Singh (Fri))**
  - **Cumulant** $c_3 = C_{123} - C_{12} - C_{23} - C_{31} + 2$
  - **Can we extract three-baryon repulsion ?**
    (important to solve the hyperon puzzle)
    → One needs to solve continuum three-body w.f.
    with Coulmb potential.

Theoretical challenge





ALICE [2206.03344] (Raffaele Del Grande @QM2022)

Homework to Hadron Physics (4)

- **Correlation function including vector mesons**
  - **Femtoscopy** *ALICE (PRL, 2105.05578)*

$$a_0(\phi p) = 0.85 + i0.16 \text{ fm}$$

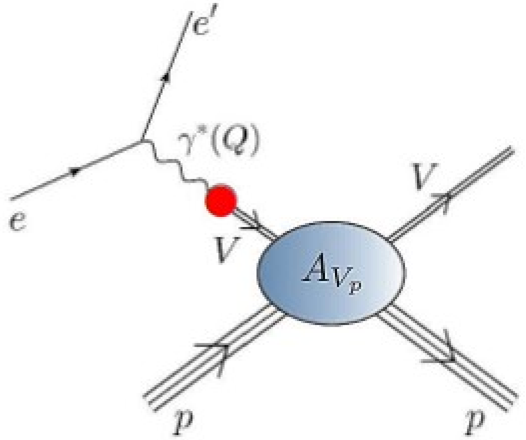
  - **Contradiction with the photo production ? scattering length is O(0.1 fm)**
    E.g. Strakovsky, Pentchev, Titov (2001.08851)

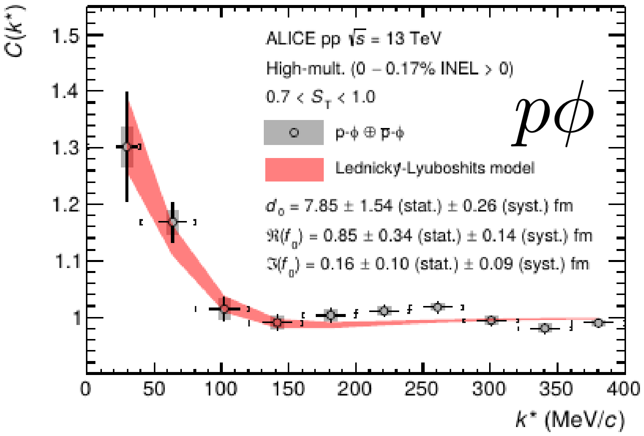
$$|a_0(\phi p)| = (0.063 \pm 0.010) \text{ fm}$$

  - **Smaller than lattice QCD result (J=3/2) ?**
    Lyu, Doi, Hatsuda, Ikeda (2205.10544)

$$a_0(\phi p, J = 3/2) = 1.43 \text{ fm}$$

  - **Bound state in J=1/2 channel**
    E. Chizzali, Y. Kamiya, R. Del Grande, T. Doi, L. Fabbietti, T. Hatsuda, Y.Lyu (2212.12690).



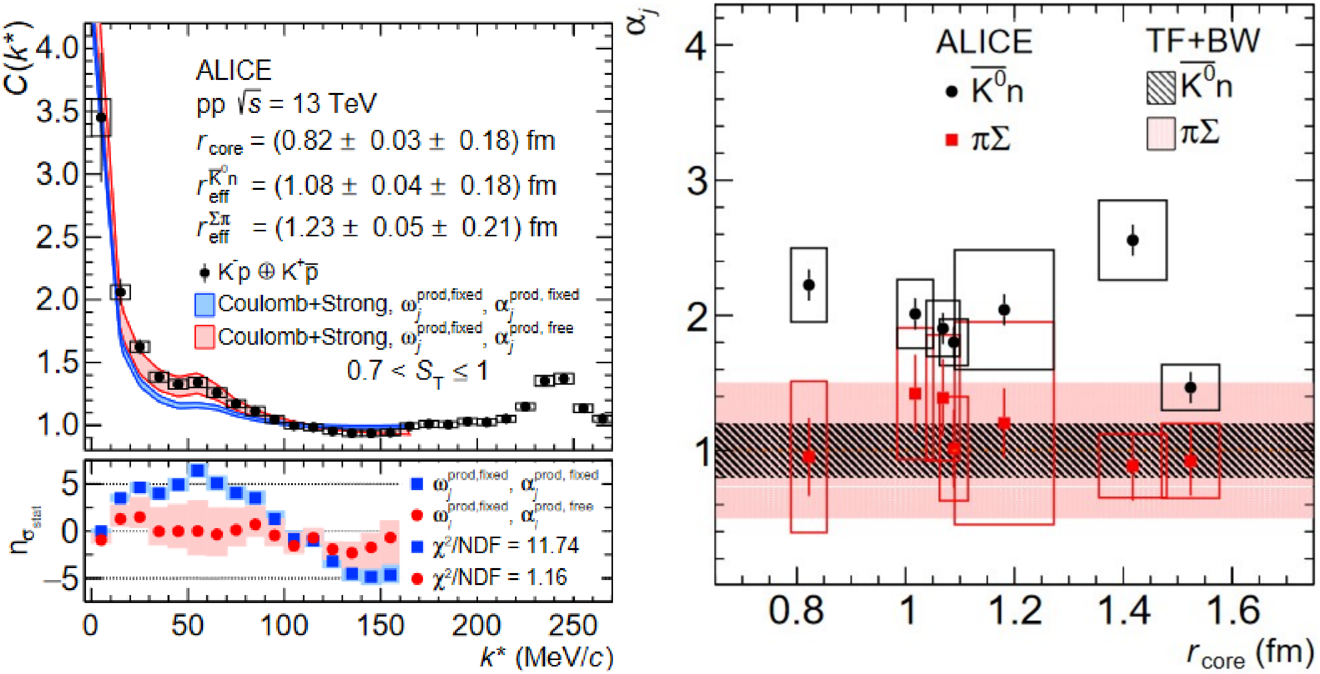


ALICE, 2105.05578

Homework to Hadron Physics (5)

- K^- p Correlation function
  - AA collisions → consistent with atomic data and the potential from chiral SU(3) dynamics ALICE('21)
  - pp collisions → C(q) can be explained with the potential from chiral SU(3) dynamics by tuning the source weight of $\pi\Sigma$ ALICE('20), Kamiya+('20)
  - pA collisions → Source weight of $\bar{K}^0$n needs to be doubled! ALICE (2205.15176)

$\bar{K}N$ potential needs update. (Transition pot. should be 1.4 times larger.)

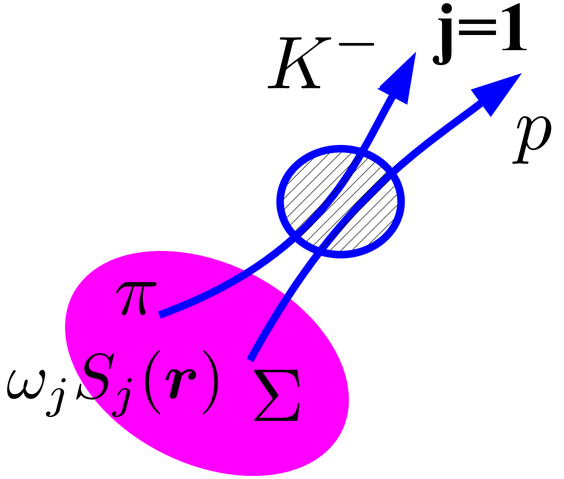


Backup (Details)

Correlation function with coupled-channel effects

- **KPLLL formula = CC Schrodinger eq. under Ψ⁽⁻⁾ boundary cond. + channel source**

  Koonin('77), Pratt+('86), Lednicky-Lyuboshits-Lyuboshits ('98), Heidenbauer ('19), Kamiya, Hyodo, Morita, AO, Weise ('20).



$$\Psi^{(-)}(\boldsymbol{q};\boldsymbol{r}) = [\phi(\boldsymbol{q};\boldsymbol{r}) - \phi_0(q;r)]\,\delta_{1j} + \psi^{(-)}(q;r)$$

$$\psi_j^{(-)}(q;r) \to \frac{1}{2iq_j}\left[\frac{u_j^{(+)}(q_j r)}{r}\delta_{1j} - A_j(q)\frac{u_j^{(-)}(q_j r)}{r}\right]$$

$$\boxed{C(q) = \int d\boldsymbol{r} S_1(r)\left[|\phi(\boldsymbol{q};\boldsymbol{r})|^2 - |\phi_0(q;r)|^2\right] + \sum_j \int d\boldsymbol{r}\omega_j S_j(r)|\psi_j^{(-)}(q;r)|^2}$$

- **No Coulomb** $\phi(\boldsymbol{q};\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}}, \phi_0(q;r) = j_0(qr), u_j^{(\pm)}(qr) = e^{\pm iqr},$

$$A_j(q) = \sqrt{(\mu_j q_j)/(\mu_1 q_1)} S_{1j}^\dagger(q_1)\ (S_{ji} = i \to j \text{ S-matrix})$$

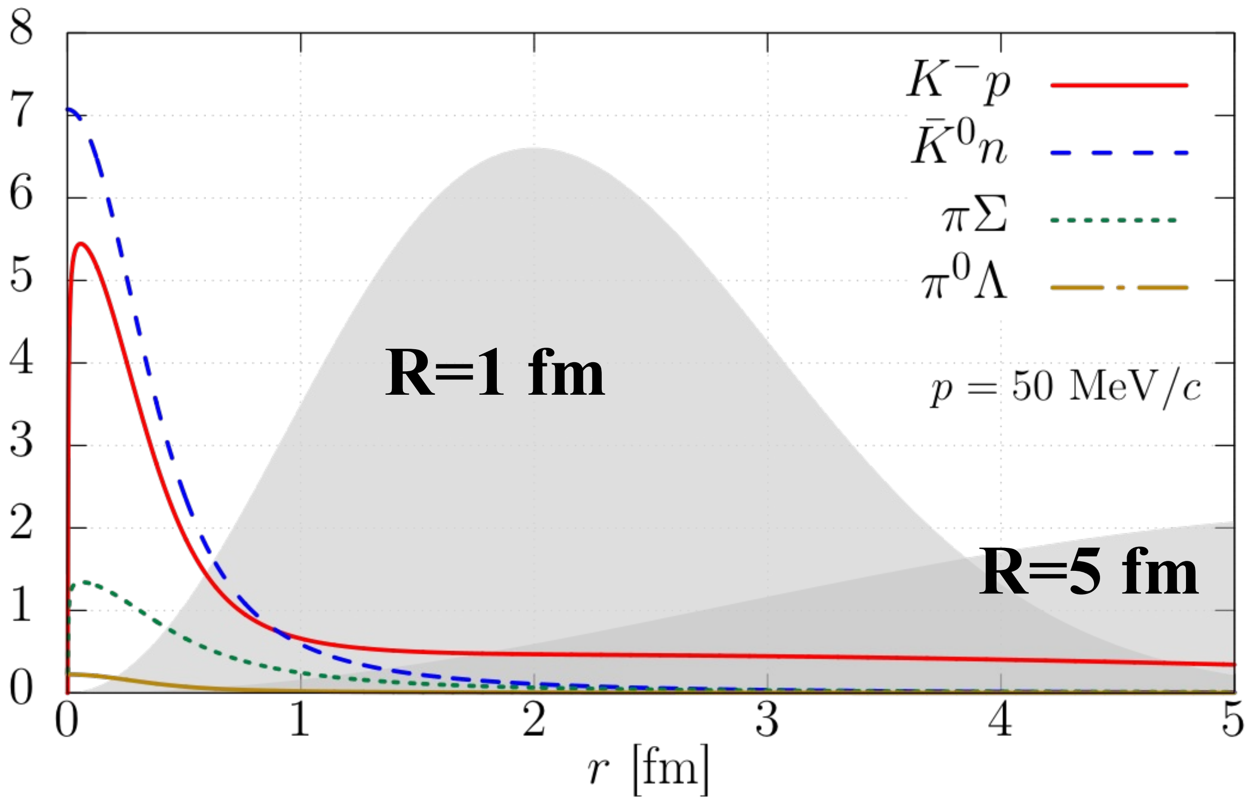
- **With Coulomb**

$$\phi(\boldsymbol{q};\boldsymbol{r}) = \text{Full Coulomb w.f.}, \phi_0(q;r) = \text{s-wave Coulomb w.f.},$$

$$u_j^{(\pm)}(qr) = \pm e^{\mp i\sigma_j}\,[iF(qr) \pm G(qr)]\,(F, G = \text{regular (irregular) Coulomb fn.})$$

Discriminating Coupled-Channel Effects

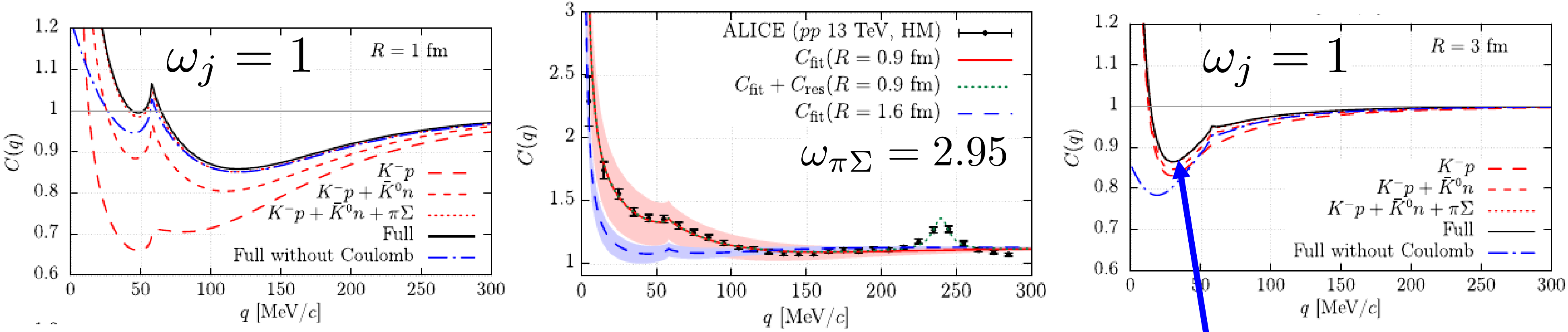
- **Source size dependence again !**
  - **Unmeasured coupled-channel wave functions disappear soon.**
    → CFs with large source is dominated
    by the measured channel wave function !
  - **Scattering parameters from CFs with large source**
    Coupled-channel effects from CFs with small source.



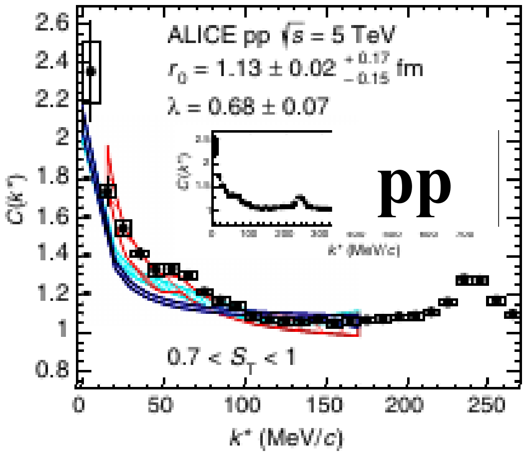
w.f. Kamiya+,arXiv:1911.01041v1

Source Size Dependence of $C(pK^-)$

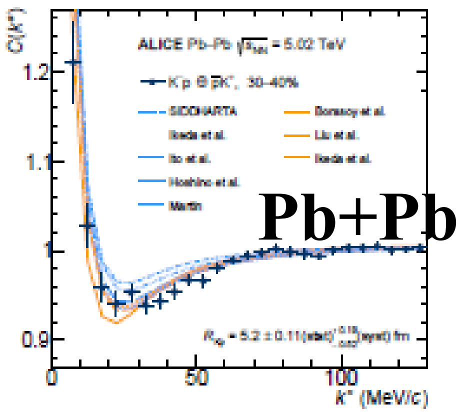
- **Coupled-channel effects are suppressed when R is large, and “pure” pK^- wave function may be observed in HIC.**



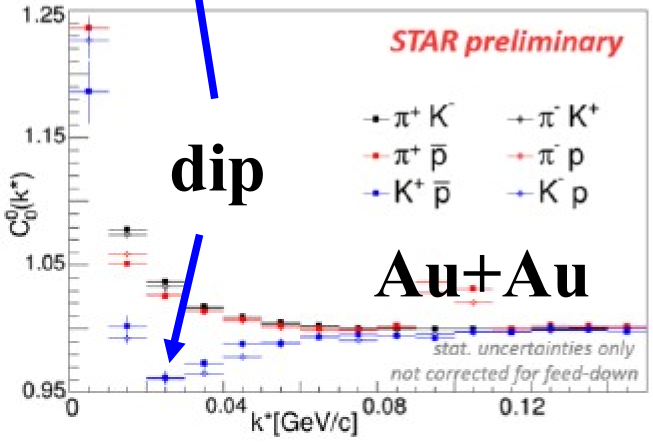
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124(‘20)132501.



S. Acharya+[ALICE], PRL124(‘20)092301



S. Acharya+[ALICE], 2105.05683



Siejka+[STAR, preliminary], NPA982 (‘19)359.

STAR(prel.) & new ALICE data show a dip at small q.

Scattering length from K⁻p correlation function

- **LL model fit (w/ Coulomb) to the correlation function data**

  S. Acharya+[ALICE], PLB 822 ('21) 136708 [2105.05683] (δ ~ +a_0q, HEP convention)

  $$a_0 = -0.91 \pm 0.03(\mathrm{stat})^{+0.17}_{-0.03}(\mathrm{syst}) + i[0.92 \pm 0.05(\mathrm{stat})^{+0.12}_{-0.33}(\mathrm{syst})]\ \mathrm{fm}$$

- **Consistent with SIDDHARTA (kaonic atom) data, and errors are comparable to previous dedicated experiments.**

  M. Bassi et al. [SIDDHARTA], NPA 881 ('12) 88 [1201.4635]

  $$a_0 = -0.65 \pm 0.10 + i[0.81 \pm 0.15]\ \mathrm{fm}$$

- **Femtoscopy reconfirmed $\bar{K}N$ bound state nature of Λ(1405)**

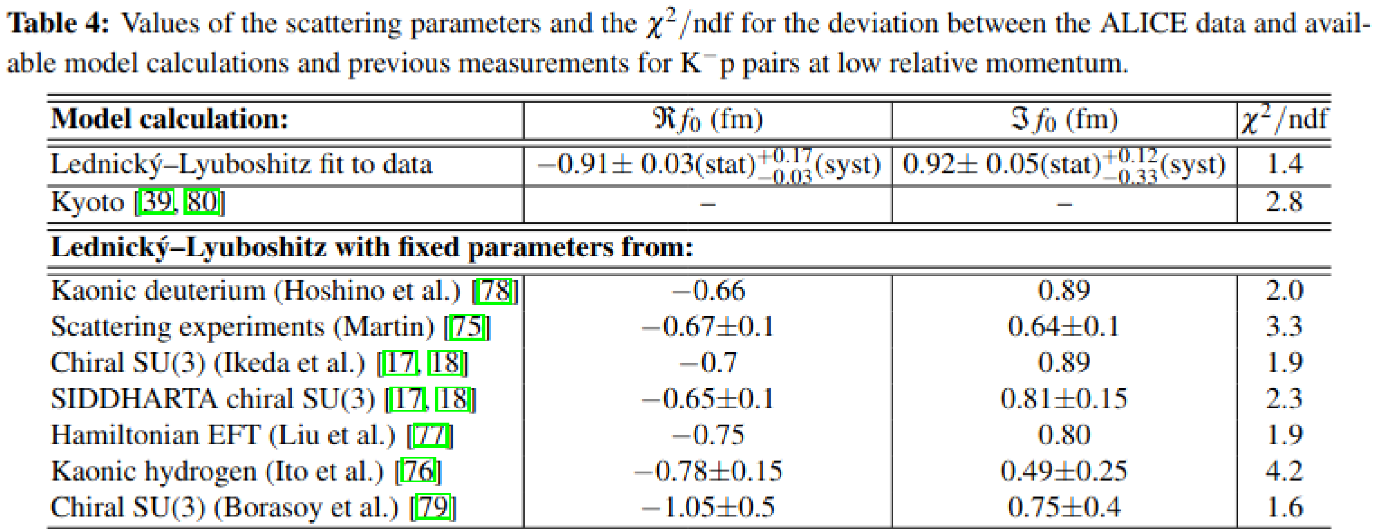
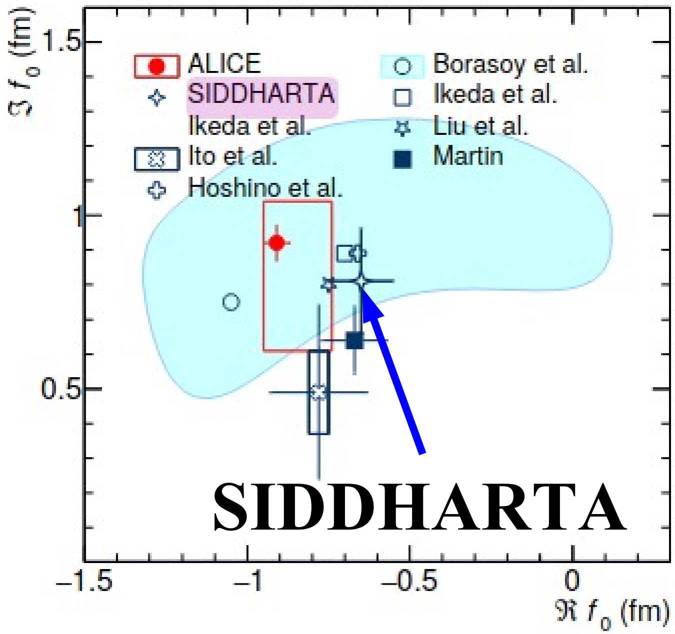


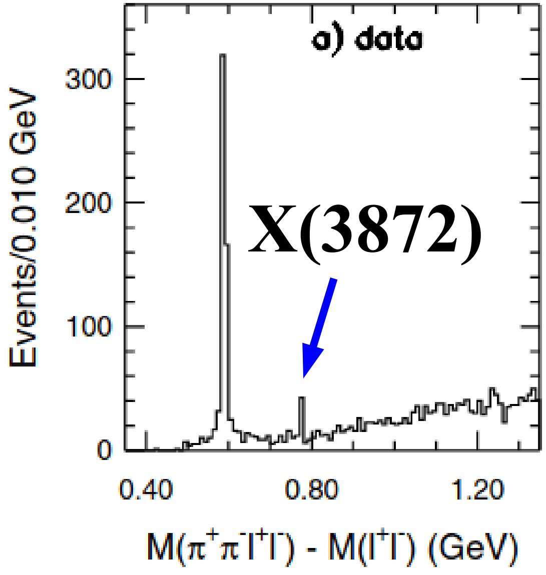
Table 4: Values of the scattering parameters and the χ^2/ndf for the deviation between the ALICE data and available model calculations and previous measurements for K⁻p pairs at low relative momentum.

Model calculation:	$\Re f_0$ (fm)	$\Im f_0$ (fm)	χ^2/ndf
Lednický–Lyuboshitz fit to data	$-0.91 \pm 0.03(\mathrm{stat})^{+0.17}_{-0.03}(\mathrm{syst})$	$0.92 \pm 0.05(\mathrm{stat})^{+0.12}_{-0.33}(\mathrm{syst})$	1.4
Kyoto [39, 80]	–	–	2.8
Lednický–Lyuboshitz with fixed parameters from:			
Kaonic deuterium (Hoshino et al.) [78]	−0.66	0.89	2.0
Scattering experiments (Martin) [75]	−0.67±0.1	0.64±0.1	3.3
Chiral SU(3) (Ikeda et al.) [17, 18]	−0.7	0.89	1.9
SIDDHARTA chiral SU(3) [17, 18]	−0.65±0.1	0.81±0.15	2.3
Hamiltonian EFT (Liu et al.) [77]	−0.75	0.80	1.9
Kaonic hydrogen (Ito et al.) [76]	−0.78±0.15	0.49±0.25	4.2
Chiral SU(3) (Borasoy et al.) [79]	−1.05±0.5	0.75±0.4	1.6

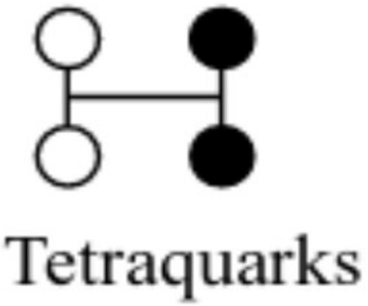


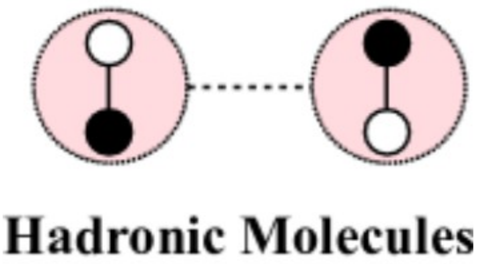
Exotic Hadrons including $c\bar{c}$ / cc / $\bar{c}\bar{c}$

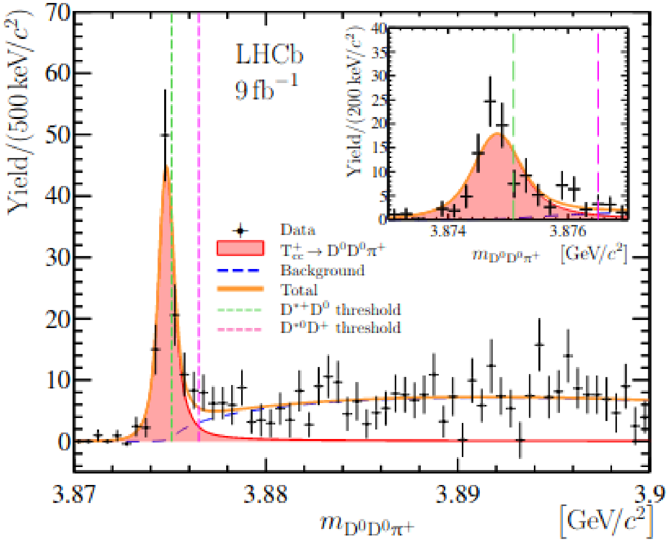
- **Main play ground of exotic hadron physics**
  - X(3872) *Belle ('03)* $c\bar{c}q\bar{q}$ **Beijing Spectrometer**
  - Many X,Y,Z states
    Belle, CDF, BaBar, LHCb, CMS, BESIII, ...
  - Charmed pentaquark Pc *LHCb ('15, '19)*
  - Doubly charmed tetraquark state Tcc
    LHCb ('21) $cc\bar{q}\bar{q}$
- **Structure of exotic hadrons**
  - Compact multiquark states
    → "good" [ud] diquark gains energy
  - Hadronic molecules
    → Many exotic states around thresholds
  - Their mixture...



S.K.Choi+[Belle], PRL91, 262001 ('03)


Tetraquarks


Hadronic Molecules

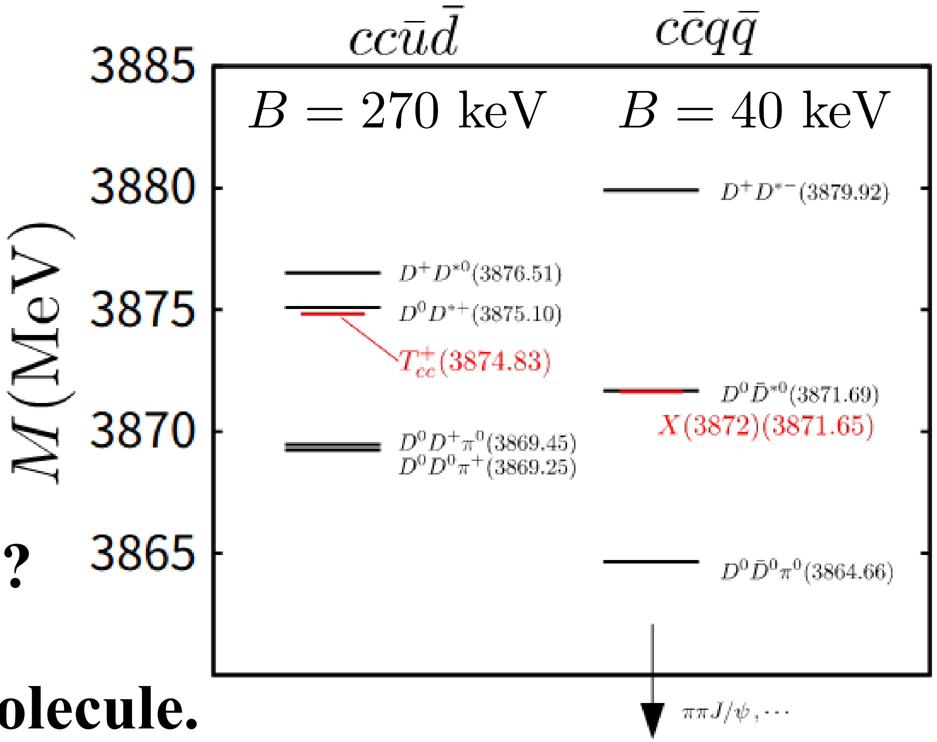


R. Aaji+ [LHCb], 2109.01038, 2109.01056

Compact Tetraquarks or Hadronic Molecules

- **Tcc = Compact Tetraquark ?**
  Good $[\bar{u}\bar{d}]$ diquark gains energy
  S. Zouzou+('86), ZPC30,457.

- **X(3872)**
  - **$c\bar{c}$ component ? production cross section** *Bignamini+ (0906.0882)*
  - **Large yield in Pb+Pb → Molecule?**
    Sirunyan+ [CMS] (2102.13048)
    c.f. Δr/Δp is similar in HIC and molecule.
    ExHIC ('11,'11,'17)



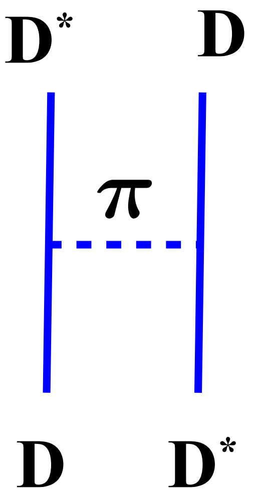
- **Hadronic Molecule Conditions**
  - **Appears around the threshold → OK**
  - **Have large size $R \simeq 1/\sqrt{2\mu B}$ → Yield**
  - **Described by the *hh* interaction**

How can we access hh int. with charm ?
→ Femtoscopy

Femtoscopic study of charmed hadron int.

- DD^* and $D\bar{D}^*$ correlation functions. *Kamiya, Hyodo, AO (2203.13814)*
  - Related with Tcc and X(3872)
  - *ALICE run3 can measure the correlation functions.*
- Model interaction
  - Range = one pion exchange *Yasui, Sudoh (0906.1452)*
  - Strength is fitted to the pole mass.
  - Isospin dep.
    - I=0: One range gaussian, strength fitted to the mass
    - I=1: ignored

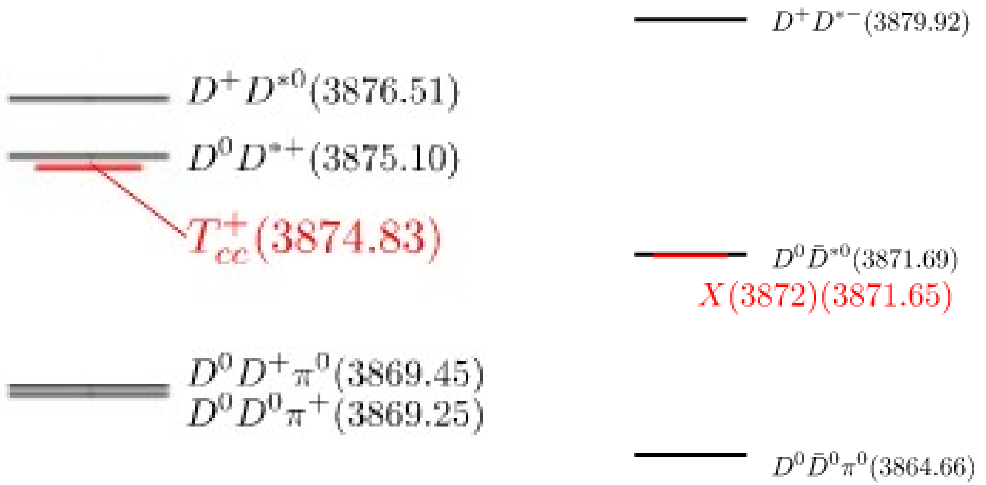
$$\{D^0\bar{D}^{*0}\} = (D^0\bar{D}^{*0} + \bar{D}^0 D^{*0})/\sqrt{2}\ (C = +1)$$
$$\{D^+D^{*-}\} = (D^+D^{*-} + D^-D^{*+})/\sqrt{2}\ (C = +1)$$

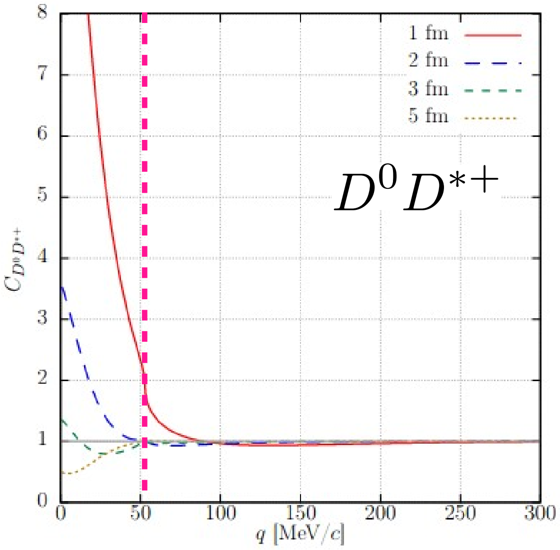


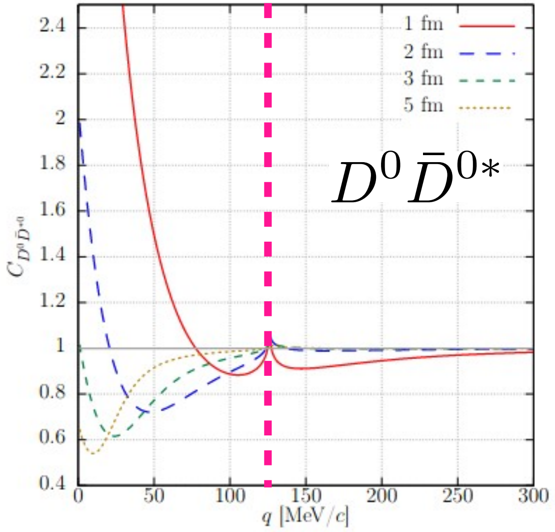
DD^*	V_0 [MeV]	$a_0^{D^0D^{*+}}$ [fm]	$a_0^{D^+D^{*0}}$ [fm]
	$-36.569 - i1.243$	$-7.16 + i1.85$	$-1.75 + i1.82$
$\{D\bar{D}^*\}$	V_0 [MeV]	$a_0^{\{D^0\bar{D}^{*0}\}}$ [fm]	$a_0^{\{D^+D^{*-}\}}$ [fm]
	$-43.265 - i6.091$	$-4.23 + i3.95$	$-0.41 + i1.47$

$D^0 D^{*+}$ and $D^+ \bar{D}^{*0}$ Correlation Functions

- **Features of C(q) with a bound state**
  - **Enhancement at small source, Dip at large source.**
  - **Modification of potential (Changing the range, V(I=1)=0 or ± V(I=0)/3) does not change C(q) significantly. (dominated by the pole)**
  - **Measurement in Run3 is awaited.**







Tcc and X(3872) structure

- **Hadronic molecule structure is assumed**
  → Eigenmomentum $k \simeq -i/a_0\,,\;\; a_0 \simeq R = 1/\sqrt{2\mu B}$
- **What happens when multiquark state mixes ?**
  → Deviation from weak binding relation (X=compositeness)

Weinberg, Phys. Rev. 137, B672 (1965), Hyodo, Jido, Hosaka (1108.5524), Kunigawa, Hyodo (2112.00249)

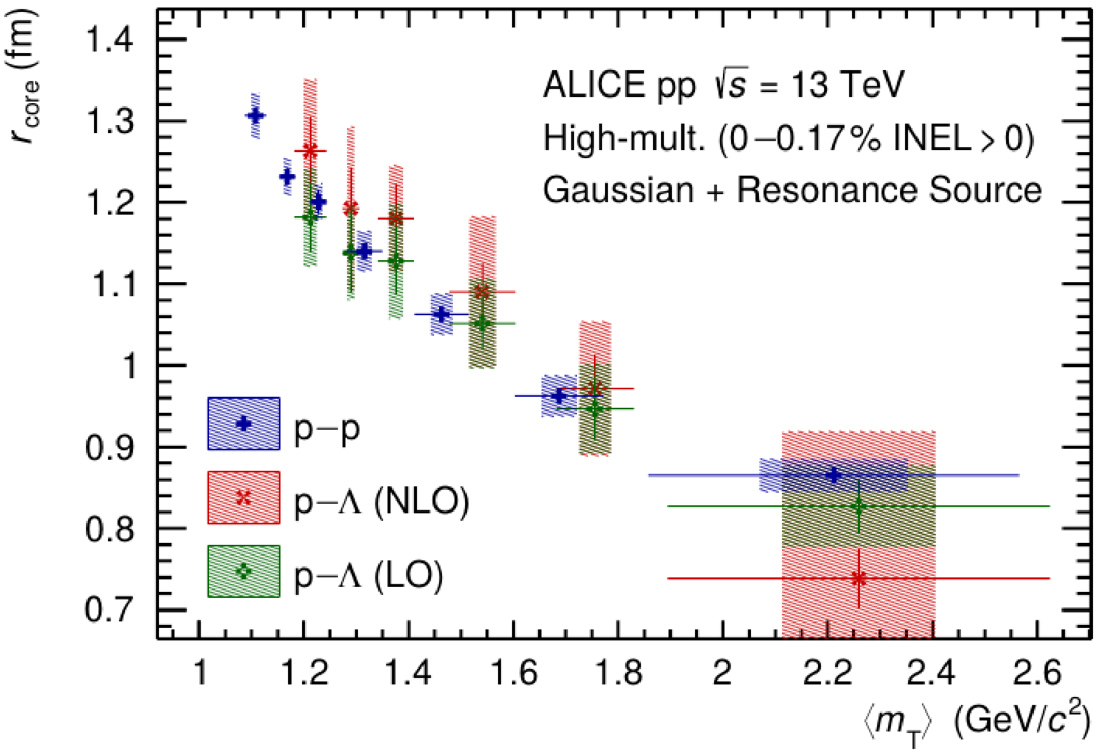
$$a_0 = R\left[\frac{2X}{1+X}\right] + \mathcal{O}(R_{\rm typ})$$

$$\left[R_{\rm typ} = \max(m_\pi^{-1}, r_{\rm eff}), R = 1/\sqrt{2\mu B}\right]$$

- **Hadronic molecule assumption → X=1**
  Pure multiquark state → X=0
- **Smaller scattering length in DD* may signal the *genuine* tetraquark nature of Tcc.**

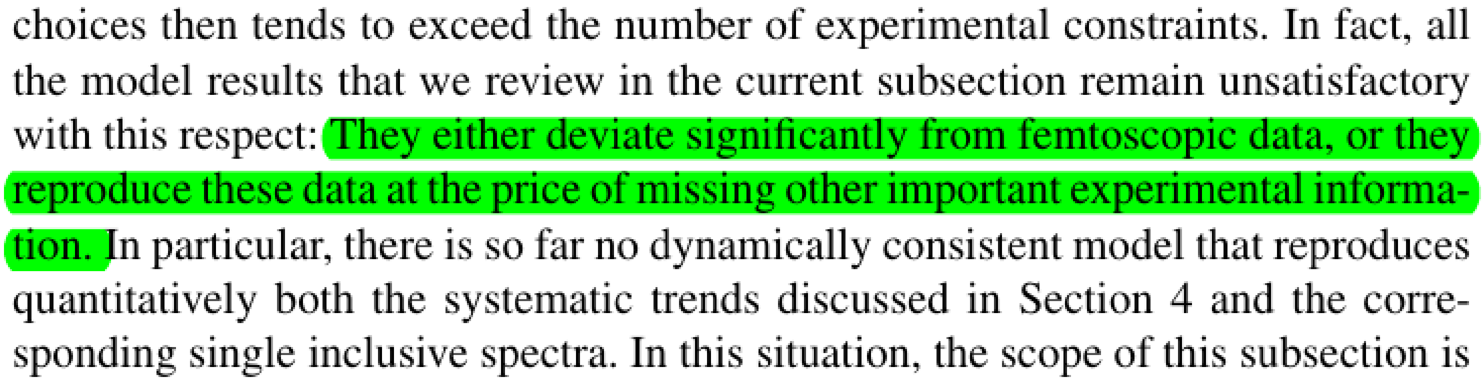
We are sorry, but we use a Gaussian Source !

- **Calculating HBT radius in dynamical models is not easy**
  M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014]; S. Pratt, PRL102('09)232301 [0811.3363].
- **and a Gaussian source seems to work at the current precision of hh interaction studies.**
  S. Acharya+[ALICE], PLB811('20)135849.
  - **primary (universal ?)+ decay of short-lived resonances ~ eff. Gaussian**
  - **Flow and source geometry effects are seen in CF, but the uncertainty of hh int. is the largest.**



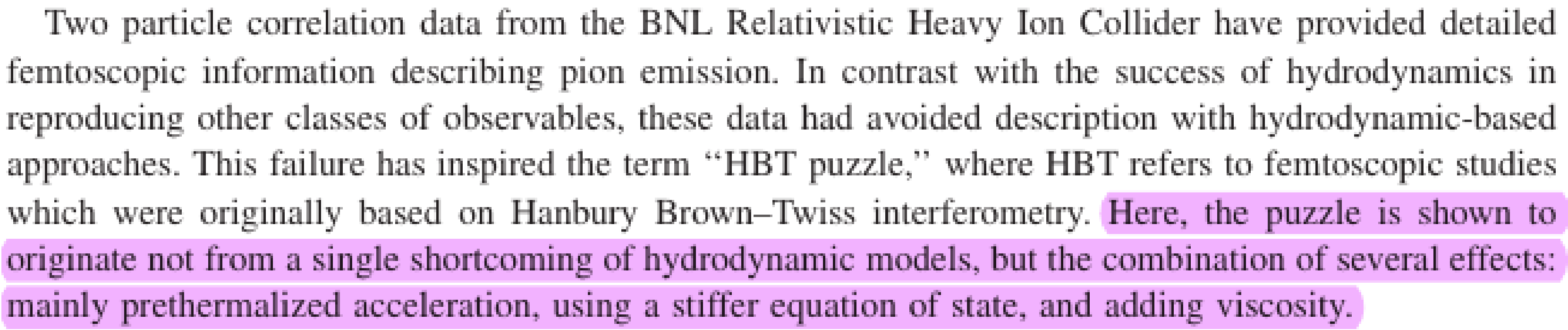
Toward dynamical source

- **Calculating HBT radius in dynamical models is not easy (HBT puzzle).**
  M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014];



choices then tends to exceed the number of experimental constraints. In fact, all the model results that we review in the current subsection remain unsatisfactory with this respect: They either deviate significantly from femtoscopic data, or they reproduce these data at the price of missing other important experimental information. In particular, there is so far no dynamically consistent model that reproduces quantitatively both the systematic trends discussed in Section 4 and the corresponding single inclusive spectra. In this situation, the scope of this subsection is

- **But carefully constructed hydrodynamic model may answer.**
  S. Pratt, PRL102('09)232301 [0811.3363].



Two particle correlation data from the BNL Relativistic Heavy Ion Collider have provided detailed femtoscopic information describing pion emission. In contrast with the success of hydrodynamics in reproducing other classes of observables, these data had avoided description with hydrodynamic-based approaches. This failure has inspired the term "HBT puzzle," where HBT refers to femtoscopic studies which were originally based on Hanbury Brown–Twiss interferometry. Here, the puzzle is shown to originate not from a single shortcoming of hydrodynamic models, but the combination of several effects: mainly prethermalized acceleration, using a stiffer equation of state, and adding viscosity.

- **How about afterburner effects ?**

Wave function around threshold (S-wave, attraction)

- **Low energy w.f. and phase shift**

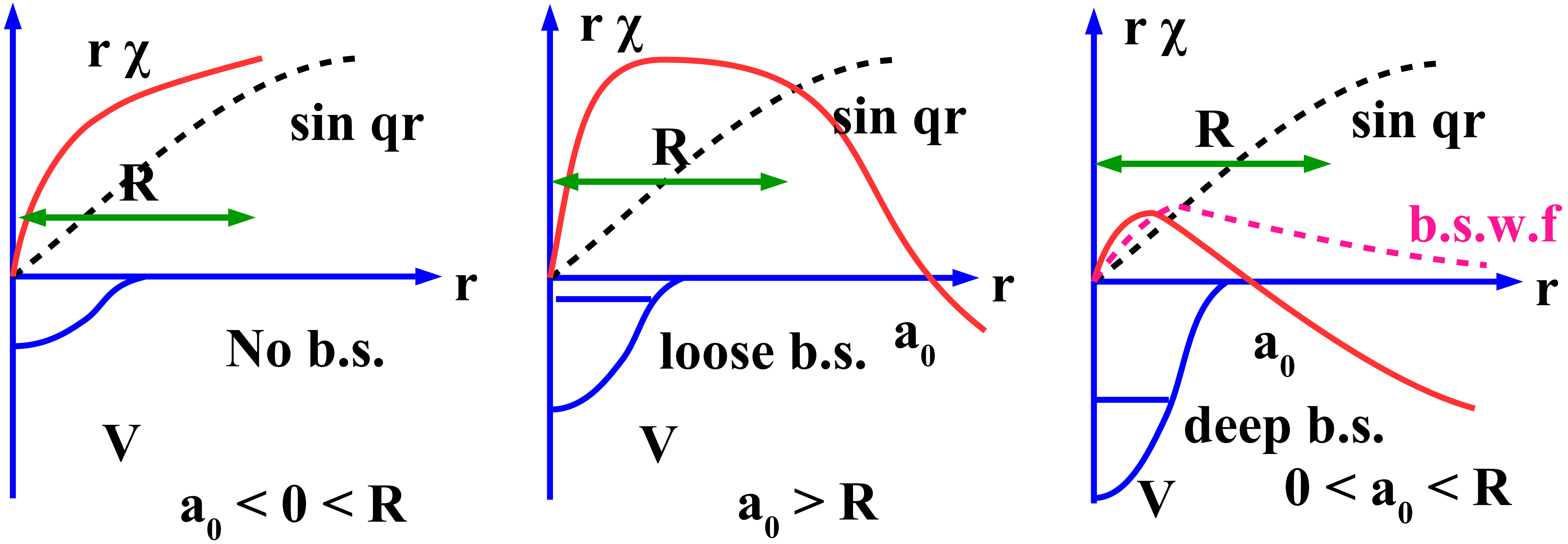
a_0 =scatt. length
r_{eff}=eff. range

$$u(r) = qr\chi_q(r) \to \sin(qr + \delta(q)) \sim \sin(q(r - a_0))$$

$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}q^2 + \mathcal{O}(q^4) \ (\delta \sim -a_0 q)$$

Nucl. and Atomic Phys. convention

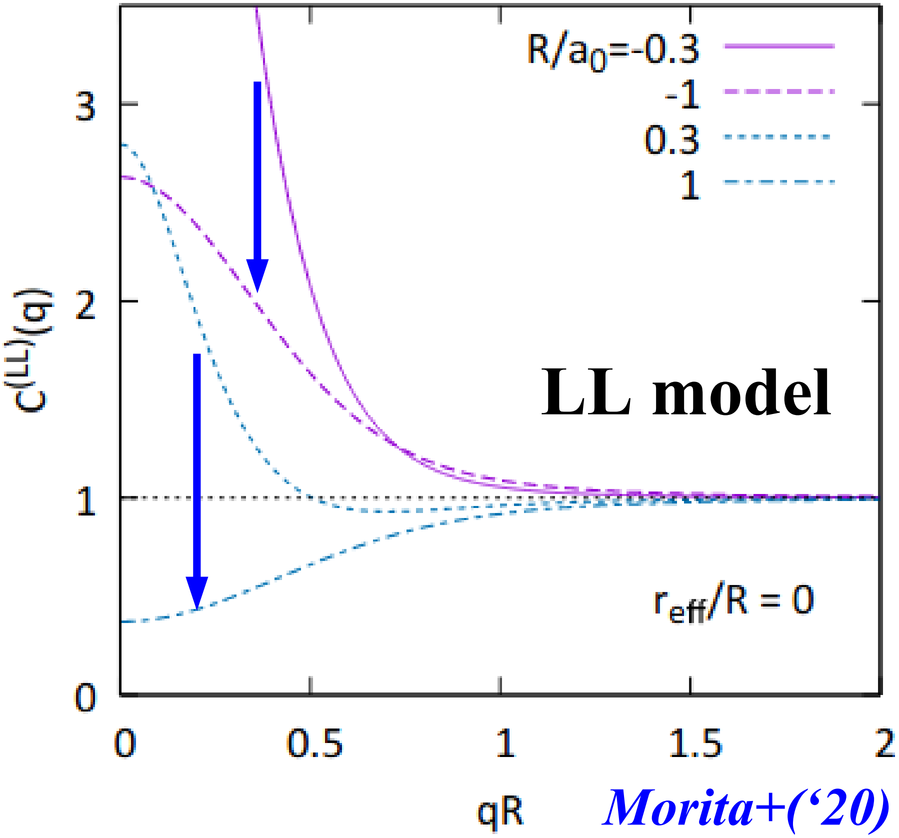
- **Wave function grows rapidly at small r with attraction.**
- **With a bound state (a_0>0), a node appears around r=a_0**
  → Suppressed |w.f.|^2 on average



Interaction Dependence of C(q)

- **Repulsive interaction → C(q) is suppressed.**
- **Attractive interaction**
  - **Wave function grows rapidly at small r with attraction.
→ C(q) is enhanced for small source.**
  - **Without a bound state ($a_0 < 0$)
→ C(q) > 1**
  - **With a bound state ($a_0 > 0$)
→ Region with C(q) < 1 appears**

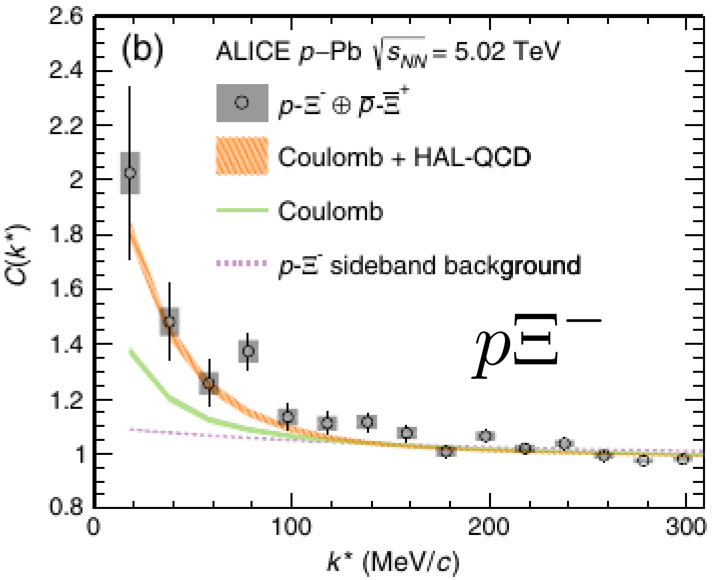
Why is C(q) suppressed when there is a bound state ? Do we really see enhanced C(q) for small R and suppressed C(q) for large R when there is a bound state ?



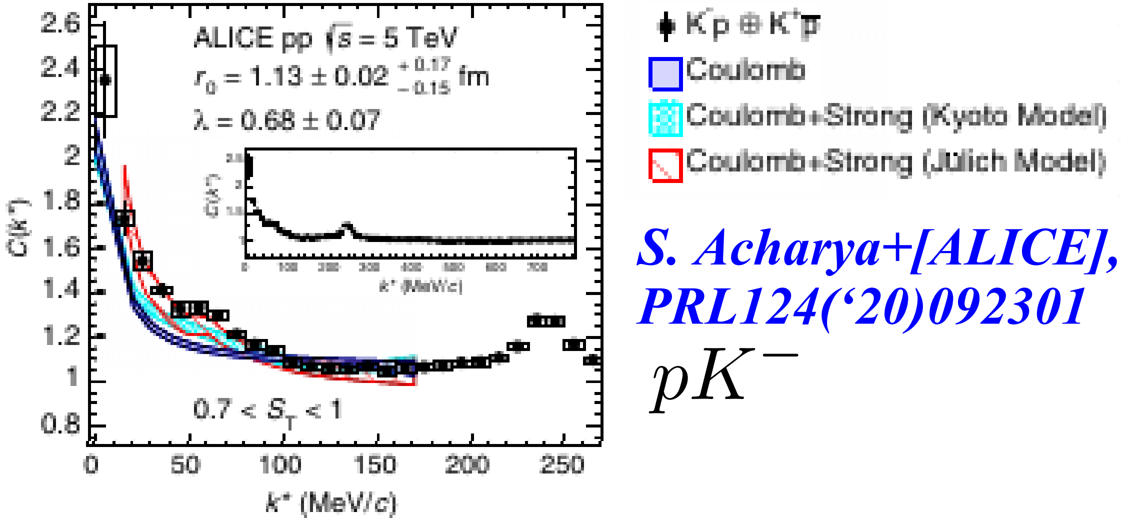
Morita+('20)

Examples of Enhanced C(q) from small source

High-Multiplicity events from pp

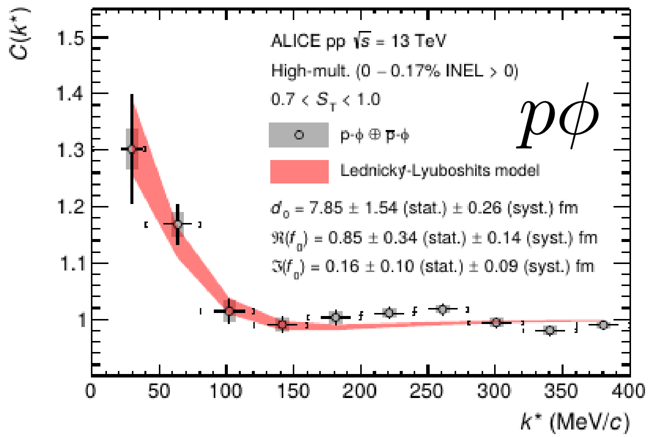


S. Acharya+[ALICE], PRL123('19)112002.

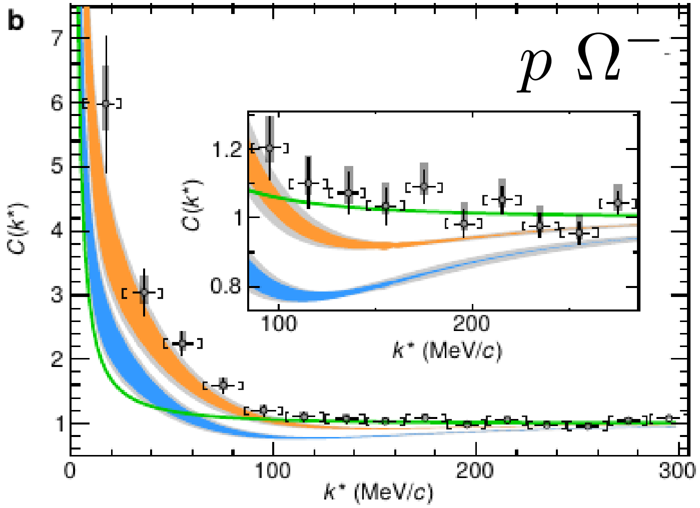


S. Acharya+[ALICE], PRL124('20)092301

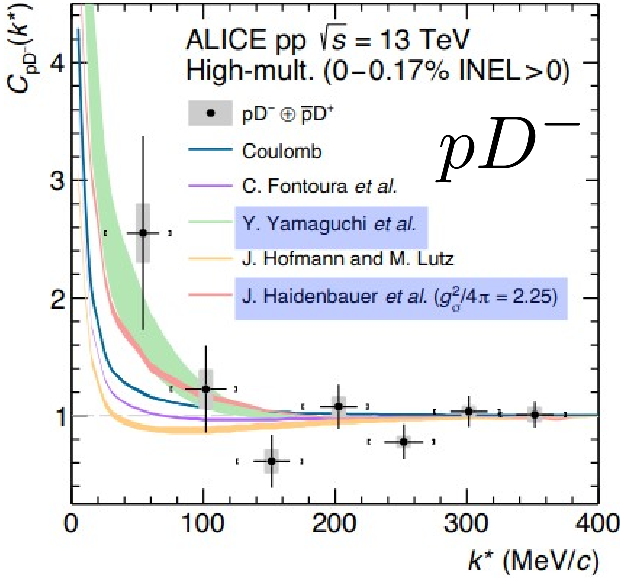
pK^-



ALICE, 2105.05578

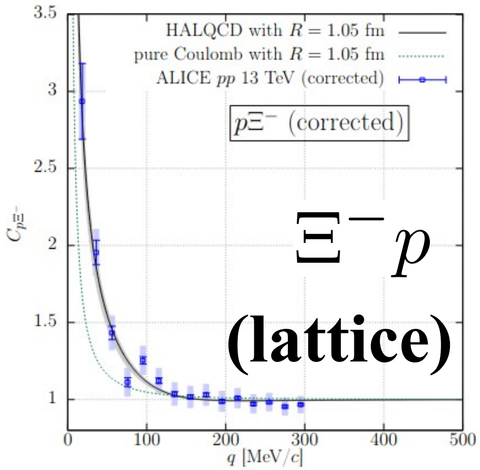


S. Acharya+[ALICE], 2005.11495 [nucl-ex] (pp 13 TeV)

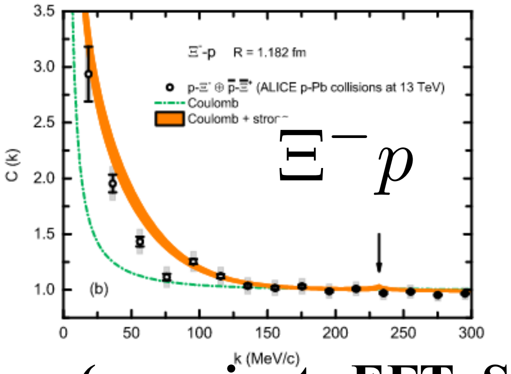


Acharya+[ALICE] (2201.05352)

Theoretical femtoscopic study of hh int. (examples)

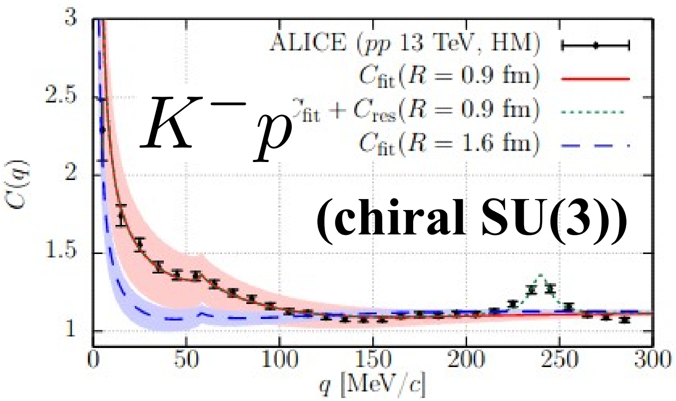


Y.Kamiya, K.Sasaki, et al., (2108.09644)

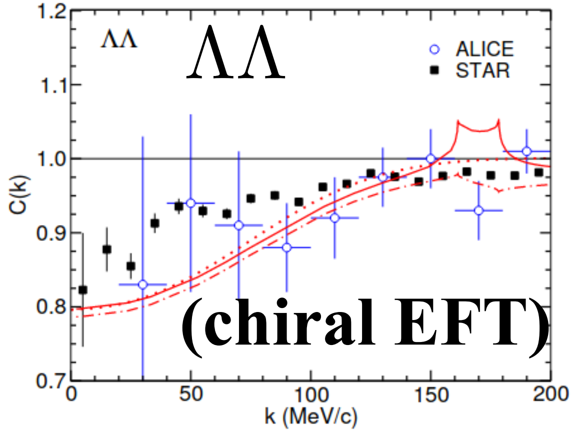


(covariant χEFT, S=-2)

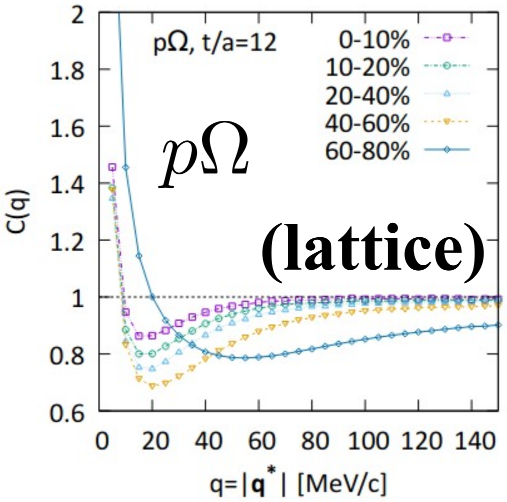
Z.-W. Liu, K.-W. Li, L.-S. Geng (2201.04997)



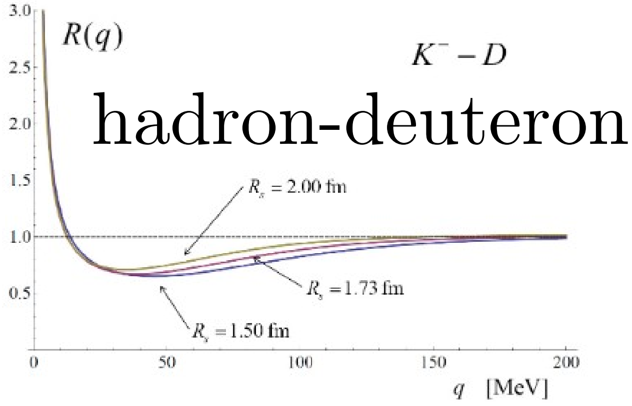
Kamiya+(1911.01041)



Haidenbauer(1808.05049), Morita+(1408.6682)



Morita, Gongyo et al., (1908.05414), Morita, AO, Etminan, Hatsuda (1605.06765)



Mrówczyński, Słón (1904.08320, K⁻d), Haidenbauer (2005.05012, Λd), Etminan, Firoozabadi (1908.11484, Ωd), K.Ogata+ (Ξ⁻ d,2103.00100)

High-Energy Heavy-Ion Collisions

- Main Goal of HIC physics = Discovery and Properties of QGP
- HIC as a playground / tool
  - Development of dynamical models
  - Physics of extreme conditions and/or strong field
  - Hadron physics

Hadron Physics using HIC as Hadron Factories

- *Simultaneous Prod. of many hadrons statistically*
- *Nearly 4π detectors & Vertex detectors*