Challenge to the hyperon puzzle
– Density dependence of Λ potential in nuclear matter from heavy-ion collisions and hypernuclear spectroscopy –

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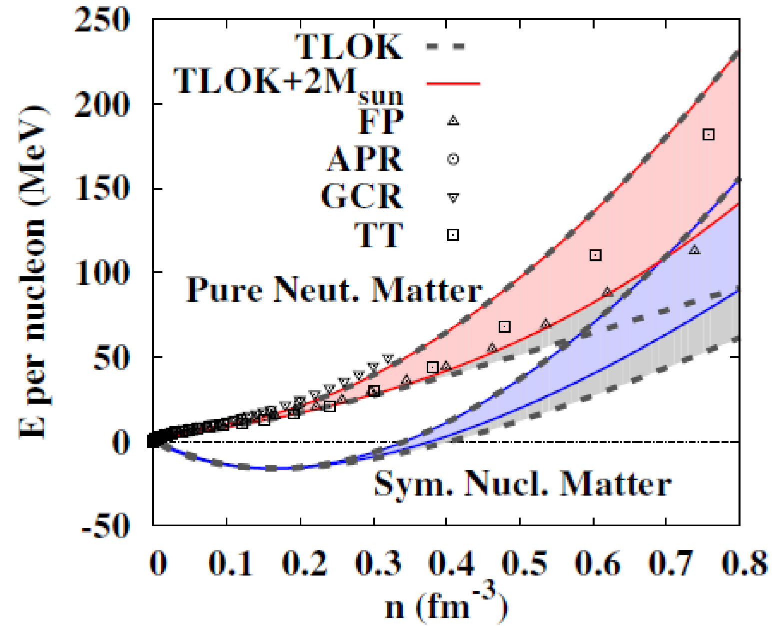
Fudan Nuclear Physics Forum #38, Jan. 4, Fudan U.

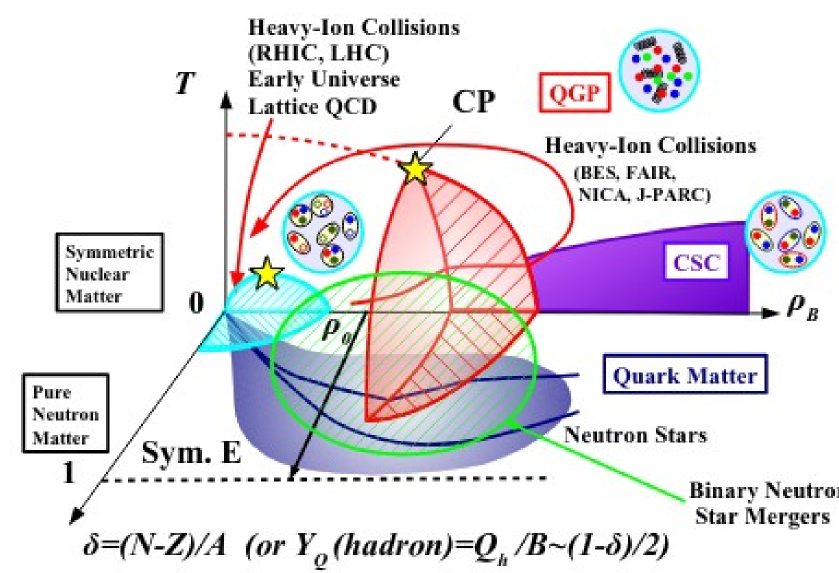
- **Introduction – Hyperon puzzle of neutron stars**
- **Λ potential from chiral effective field theory**
- **Directed flow of Λ**
- **Λ Binding Energy in Hypernuclei**
- **Summary**

Y.Nara, A. Jinno, K. Murase, AO, PRC106 ('22), 044902 [2208.01297];
A. Jinno, K. Murase, Y. Nara, AO, in prep.

A Grand Challenge in Nuclear Matter Physics

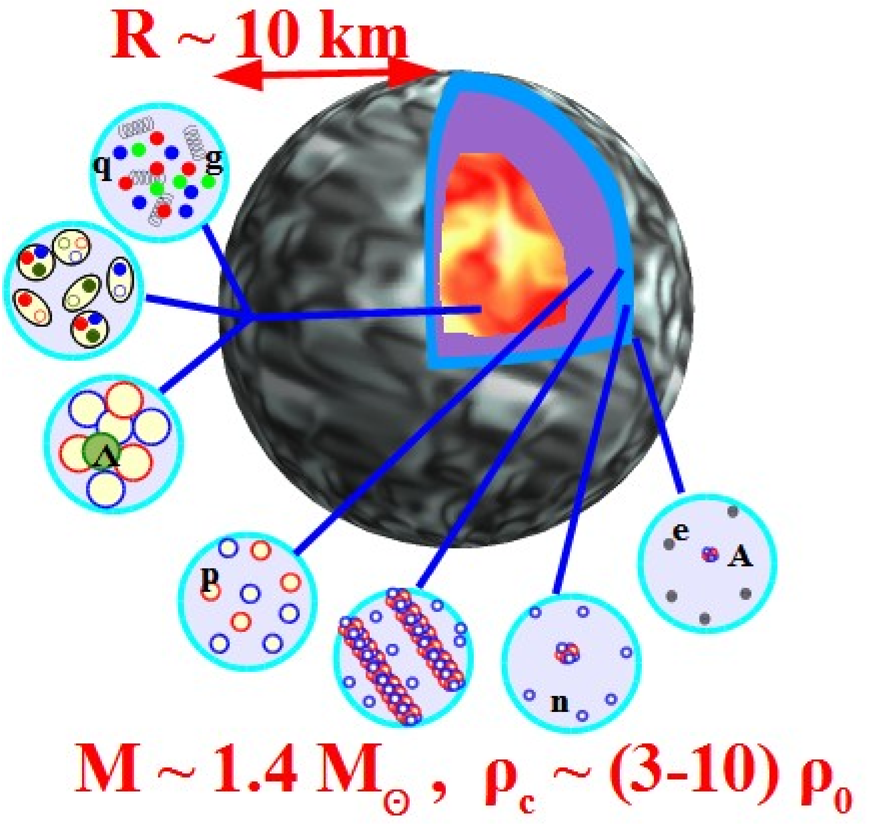
- Equation of State (EOS) and QCD phase diagram
- Current problems
  - **Symmetry energy**
    (Difference of E/A in
    pure neutron matter
    and symmetric nuclear matter)
  - **QCD phase transition**
    at high baryon densities
    (1st order or cross over ?)
  - **Hyperon Puzzle**
    (Empirical hyperon potentials
    cannot sustain $2M_\odot$ neutron stars)
- **We need data from many fascilities!**
  → BES(RHIC), FAIR, NICA, J-PARC,
  RIBF, FRIB, HIAF,
  LIGO/Virgo/KAGRA, ...





Neutron Star Composition

- Neutron star is a unique laboratory of dense matter physics
  - Envelope, Crust (A, n (drip), e) → Symmetry Energy
  - Outer Core (n, p, e, μ) → Symmetry Energy
  - Inner Core (hadrons? quarks?) → QCD phase transition Hyperon puzzle
- How can we distinguish ?
  - Mass-Radius (MR) curve → EOS (Tolman-Oppenheimer-Volkoff eq.)
  - Cooling rate
  - Neutron Star Oscillation

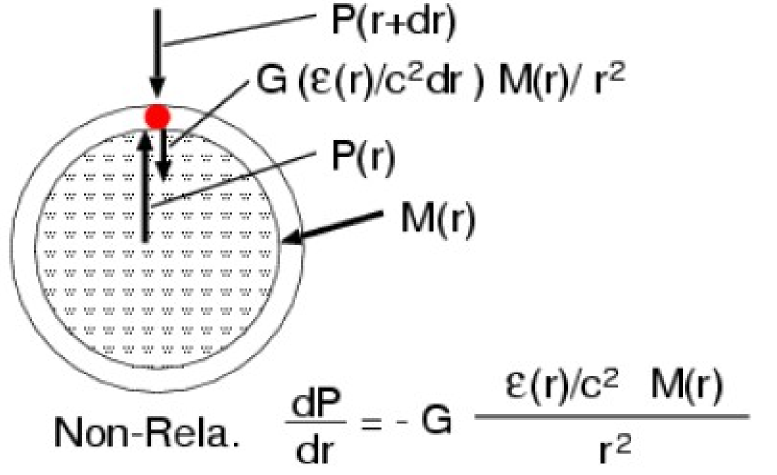


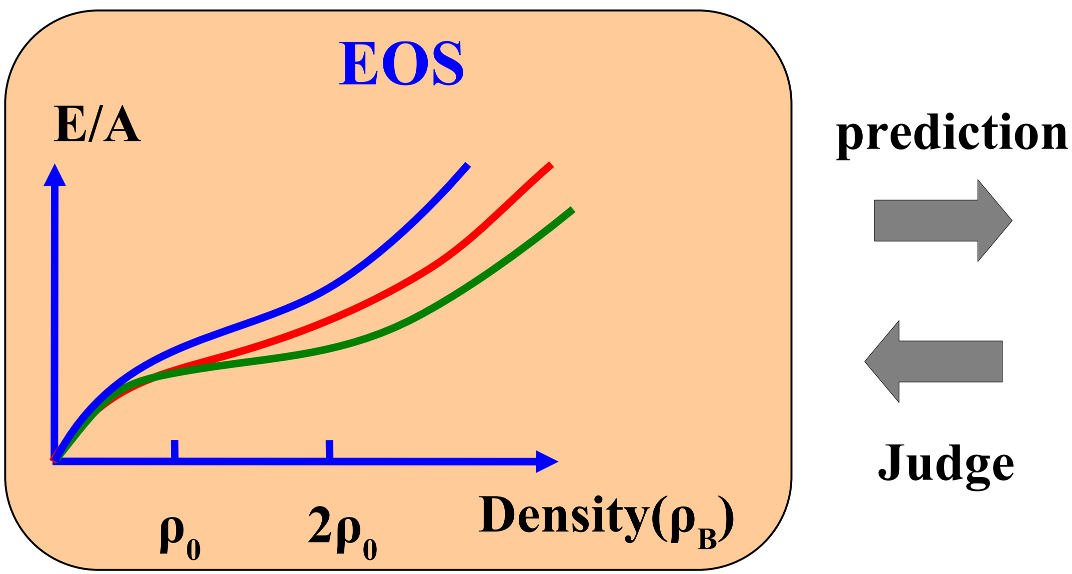
MR curve and EOS

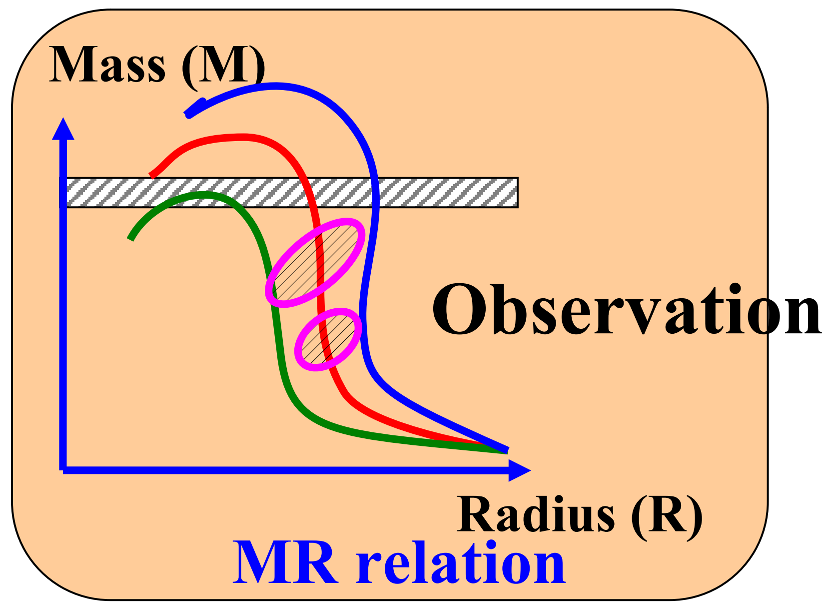
TOV equation

$$\frac{dP}{dr} = -G\frac{(\varepsilon/c^2 + P/c^2)(M + 4\pi r^3 P/c^2)}{r^2(1 - 2GM/rc^2)}$$

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon/c^2 , \quad P = P(\varepsilon) \quad (\text{EOS})$$



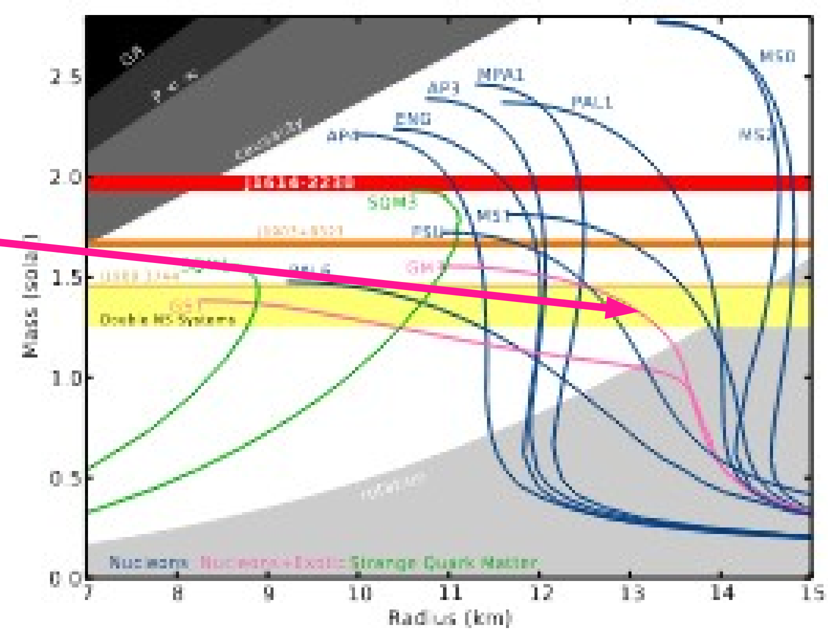




Hyperon Puzzle of Neutron Stars

- **Observation of massive neutron stars rules out hyperonic EOS ?**
  - **Attractive $U_\Lambda(\rho)$ causes hyperon mixing in NS at $(2\text{-}4)\rho_0$, softens the EOS, and reduces $M_{max} = (1.3\text{-}1.6)\ M_\odot$**
- **Proposed solutions**
  - **Three-body ΛNN repulsion → repulsive $U_\Lambda(\rho)$ at high density**
  - **Transition to quark matter before Λ appears**
  - **General relativity → Modified gravity**

Hyperons

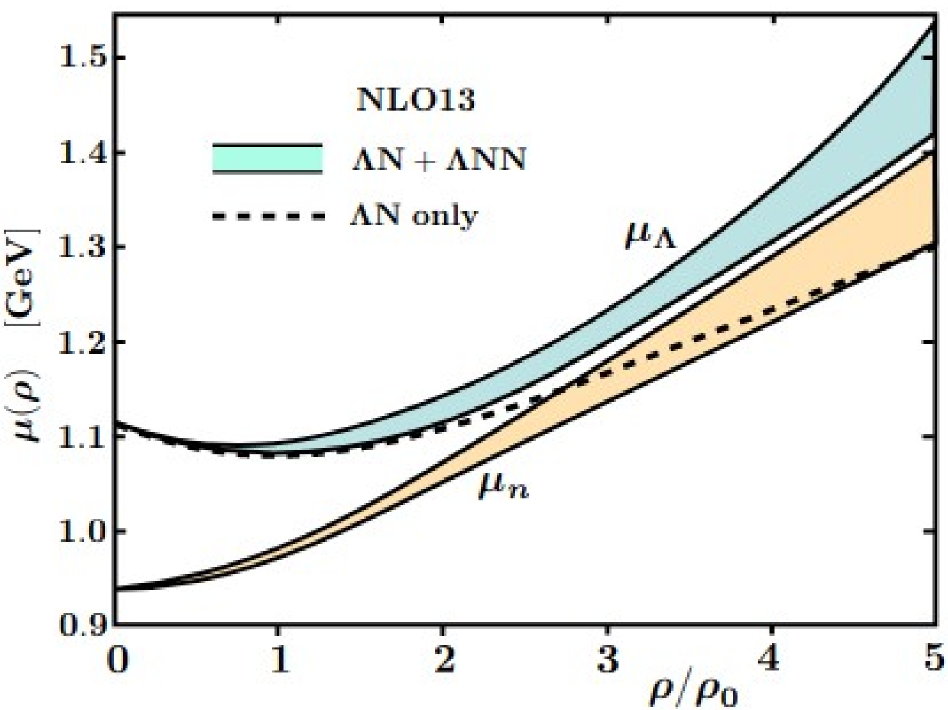


Demorest+(1010.5788)

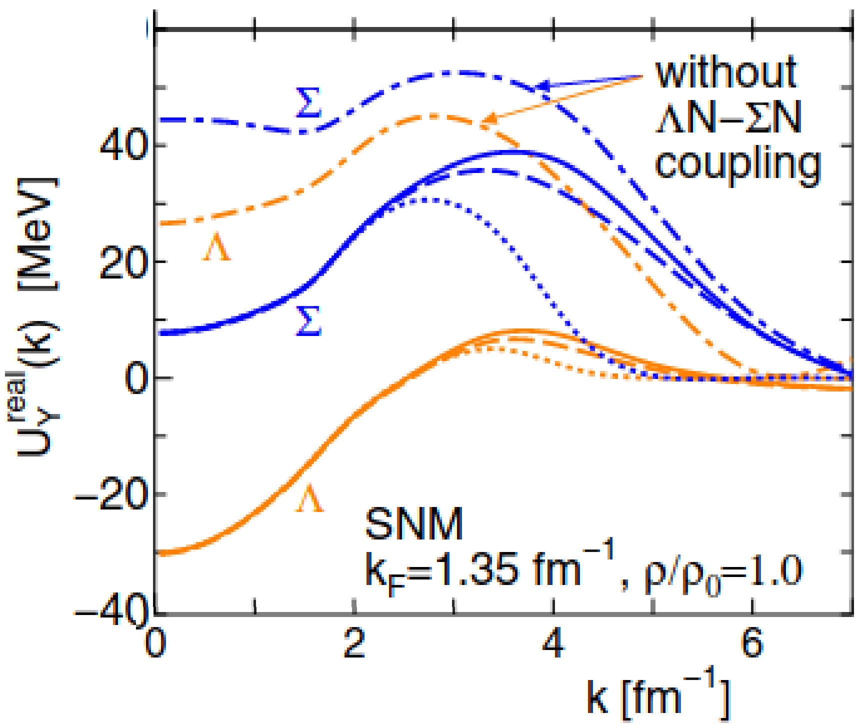
Challenging Subject in Mean Field Dynamics

Repulsive $U_\Lambda(\rho)$ at high density in chiral EFT

- **Chiral effective field theory (chiral EFT) may cause repulsive Λ potential at high densities**
  Gerstung, Kaiser, Weise (2001.10563), Kohno (1802.05388)
- **Yet unknown parameters are tuned to support 2 $M_\odot$ neutron stars.**
  → Repulsion at high densities needs to be verified !
  → E.g. Collective flows in heavy-ion collisions



Gerstung+('20)



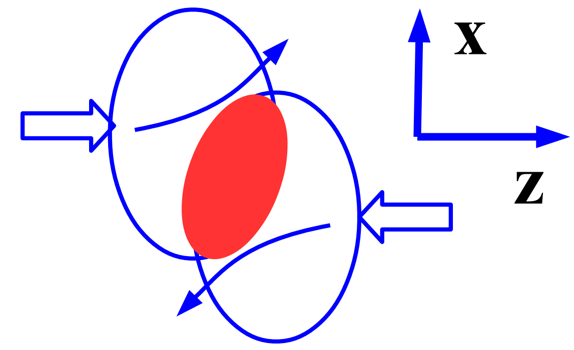
Kohno ('18)

Can we examine repulsive U_Λ at high densities ?

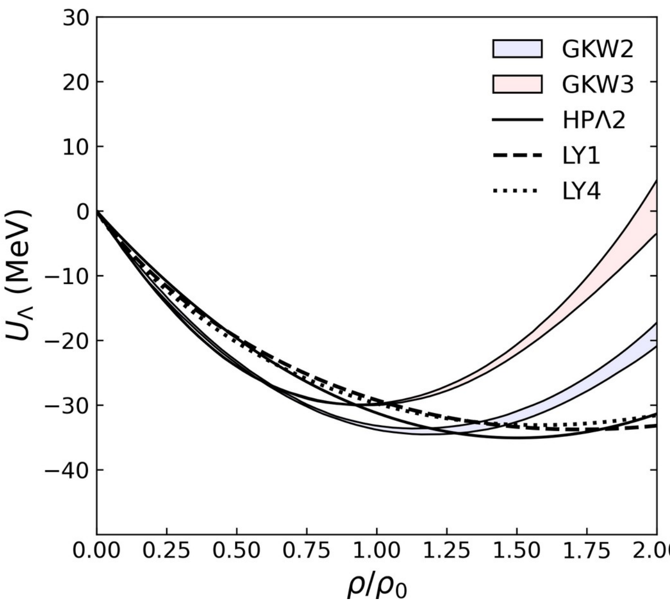
- **Candidate Observable 1: Directed flow from heavy-ion collisions**
  - **Directed flow has been utilized to study EOS**

$$v_1 = \langle \cos\phi \rangle \text{ (directed flow)}, \ \langle p_x \rangle \text{ (side flow)}$$

E.g. Sahu, Cassing, Mosel, AO (nucl-th/9907002); Snellings+(nucl-ex/9908001); Danielewicz, Lacey, Lynch (nucl-th/0208016);



  - **How about v_1 of Λ ?**
- **Candidate Observable 2: Hypernuclear Spectroscopy**
  - **Density dependence of U_Λ from chiral EFT is different from "Standard" potentials.**
    E.g. Lanskoy, Yamamoto ('97)
  - **Does U_Λ from chiral EFT explain the separation energy of Λ ?**



Jinno+ (in prep.)

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Λ potential from chiral EFT

- Directed flow of Λ
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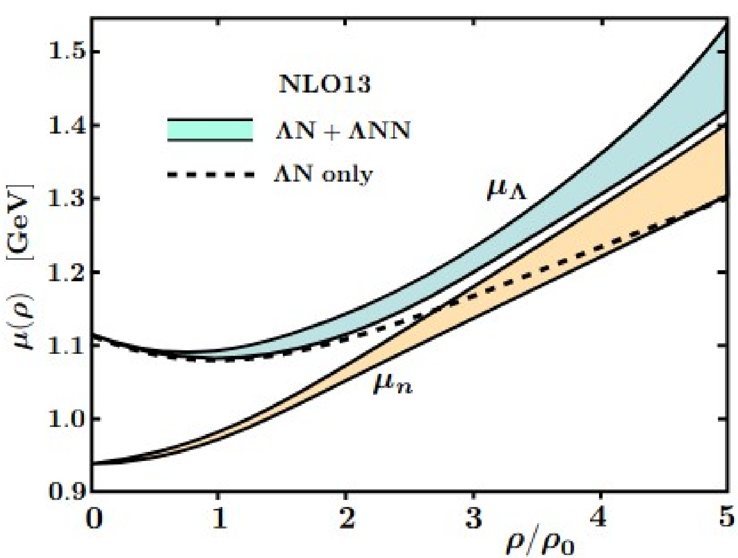
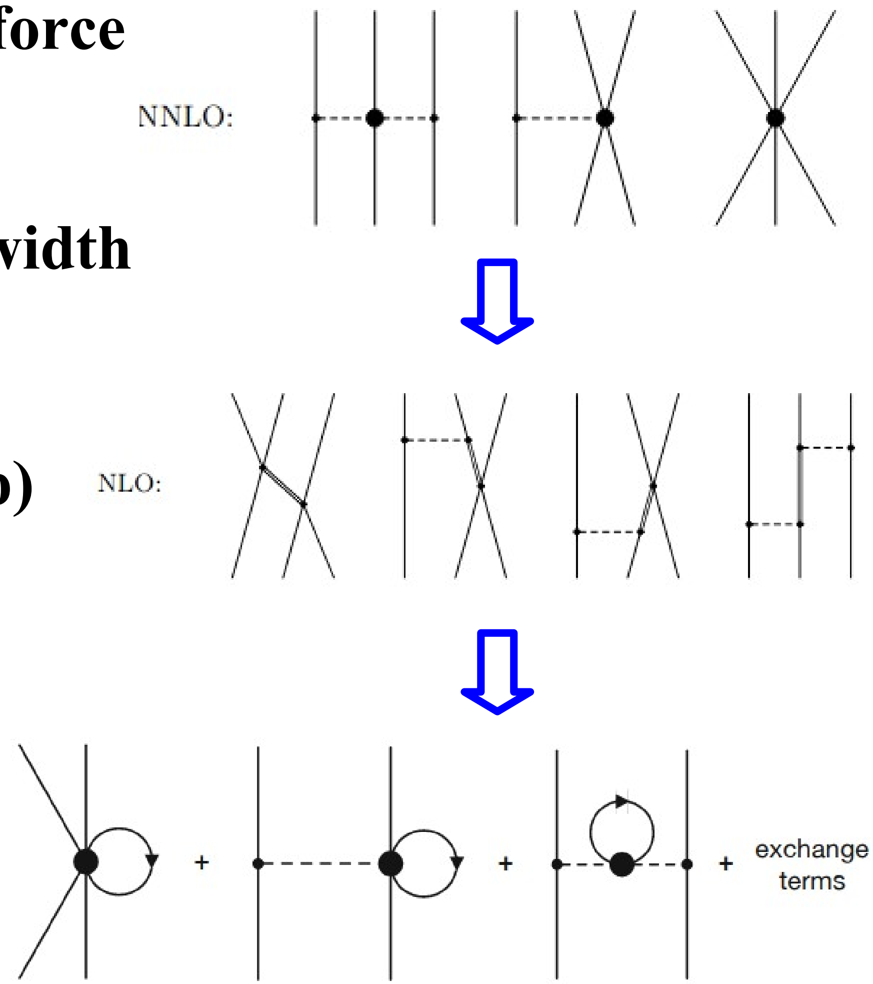
Chiral EFT

- **Decuplet saturation model**
  S. Petschauer, Haidenbauer, Kaiser, Meißner, Weise (1607.04307)
  D. Gerstung, Kaiser, Weise (GKW)(2001.10563)
  - **NNLO Diagrams generating 3-body force are assumed to be saturated by decuplet baryon diagrams.**
  - **$U_\Lambda(\rho_0) \sim$ - 30 MeV + Decuplet decay width**
    → one remaining parameter gives uncertainty
  - **Bruckner-Hartree-Fock calc. → $U_\Lambda(\rho)$**





U_Λ from Chiral EFT

- **Chiral EFT with 3BF and hyperons**
  Gerstung+(2001.10563)(GKW, decuplet saturation model), Kohno (1802.05388)
  - **ρ-dep. potential using Fermi mom. expansion** *Tews+(1611.07133)*
    + momentum dep. fitted to *Kohno('18)*

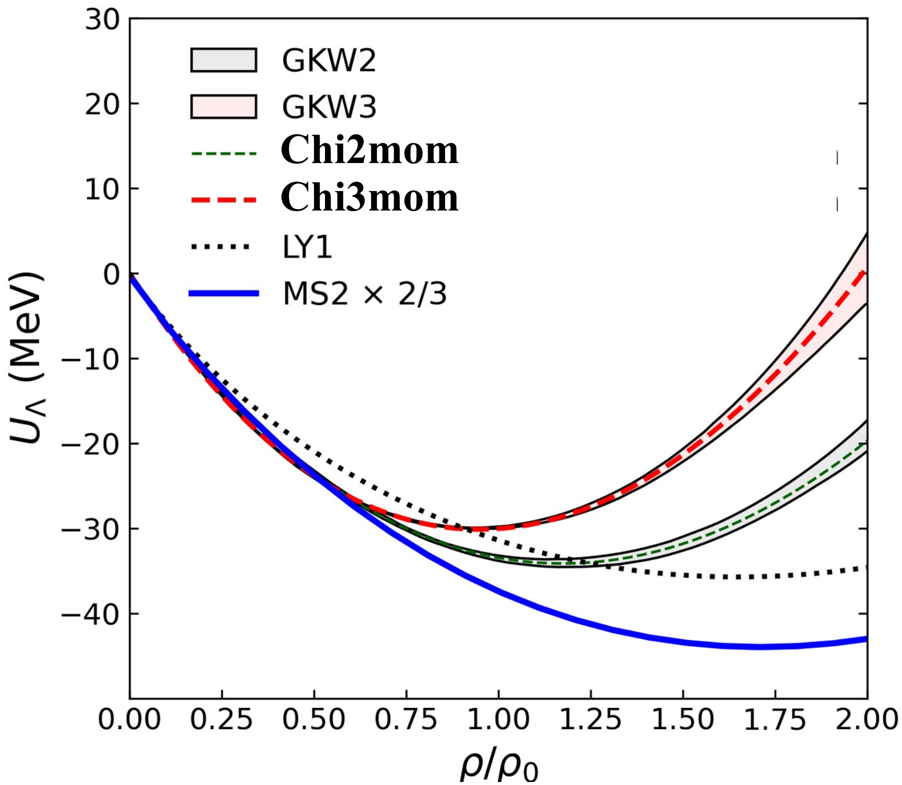
$$U_\Lambda(\rho,k) = a\frac{\rho}{\rho_0} + b\left(\frac{\rho}{\rho_0}\right)^{4/3} + c\left(\frac{\rho}{\rho_0}\right)^{5/3} + U_\Lambda^{(k)}(\rho,k)$$

$$U_\Lambda^{(k)}(\rho,k) = a_2^\Lambda \rho k^2 \ (\text{hypernuclei})\,,\ \sum_n \frac{C_n}{\rho_0}\int \frac{d\boldsymbol{k}'}{(2\pi)^3}\frac{f(\boldsymbol{r},\boldsymbol{k}')}{1+(\boldsymbol{k}-\boldsymbol{k}')^2/\mu_n^2}\ (\text{HIC})$$

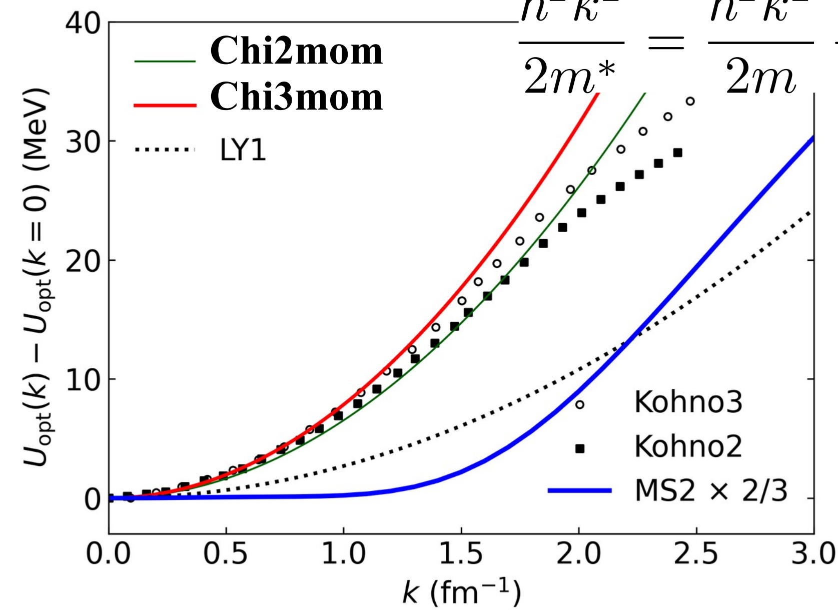
- **Parametrization** *Nara, Jinno, Murase, AO (2208.01297); Jinno+(in prep.)*
  - **Chi3 (GKW w/ 3BF, w/o momentum dep.)**
  - **Chi3mom (GKW w/ 3BF, w/ momentum dep.)**
  - **Chi2 (GKW w/o 3BF, w/o momentum dep.)**
  - **Chi2mom (GKW w/o 3BF, w/ momentum dep.)**

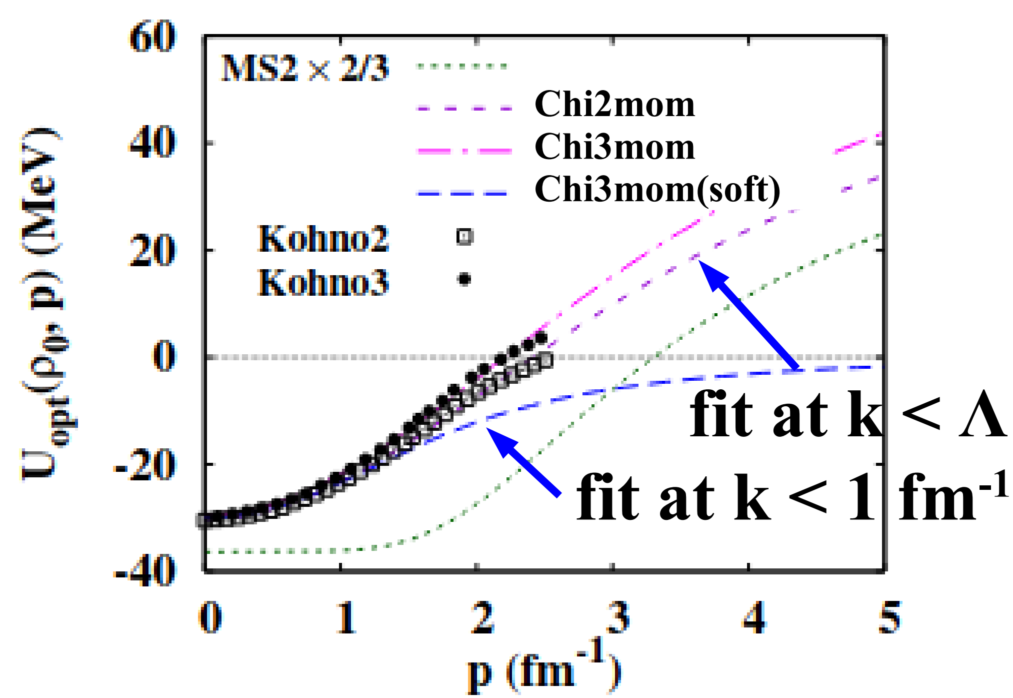
  - **For nulceons, SLy4 (hypernuclei), MS2 (mom. dep. soft) are used.**
  - **For comparison, Lanskoy, Yamamoto (LY('97)) potential is used.**

U_Λ from Chiral EFT



$$\frac{\hbar^2 k^2}{2m^*} = \frac{\hbar^2 k^2}{2m} + a_2^\Lambda \rho k^2$$





Nara, Jinno, Murase, AO (2208.01297);
Jinno, Murase, Nara, AO (in prep.)

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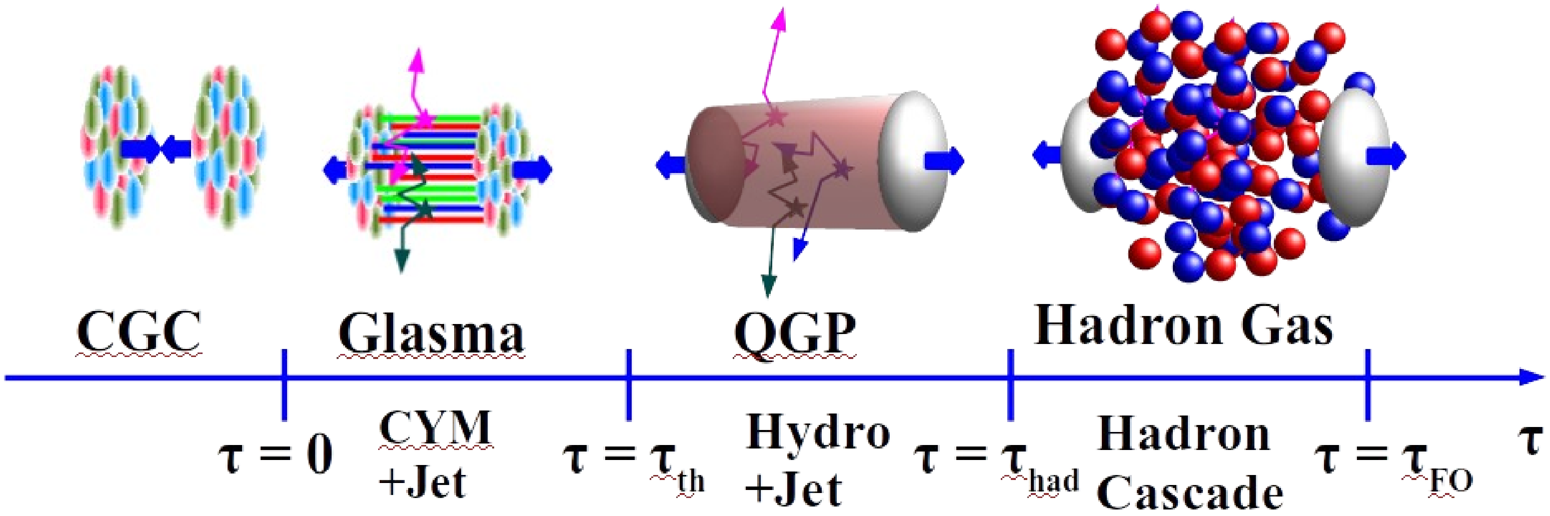
"High"-Energy Heavy-Ion Collisions

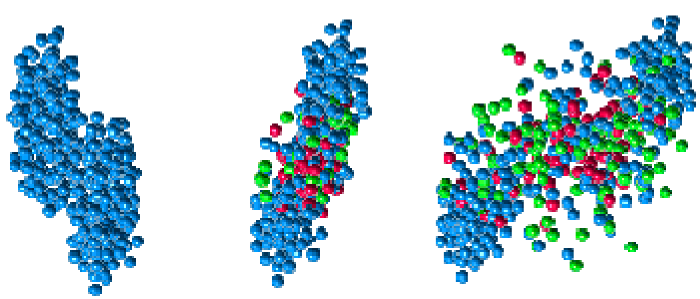
- **LHC and top energy at RHIC → QGP formation**



- **Heavy-Ion Collisions at**

$$\sqrt{s_{NN}} \leq \sqrt{s_{\rm tr}} \quad (\sqrt{s_{\rm tr}} \simeq 10\ \mathrm{GeV}(?)\,,\ E_{\rm inc} \lesssim 50\ \mathrm{GeV})$$

→ Dense Hadronic matter (partially quark-gluon matter?)



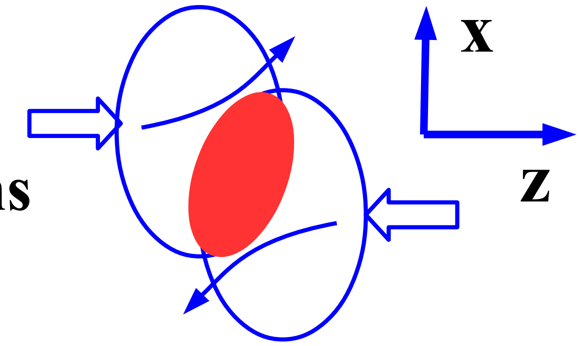
Collective Flow from Heavy-Ion Collisions

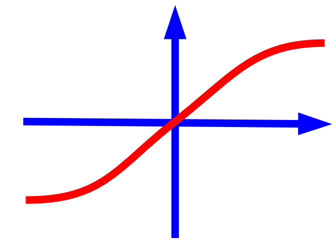
- **Directed flow (or side flow)**
  - **High pressure in hot and dense matter in the compressing stage kicks incoming nucleons**

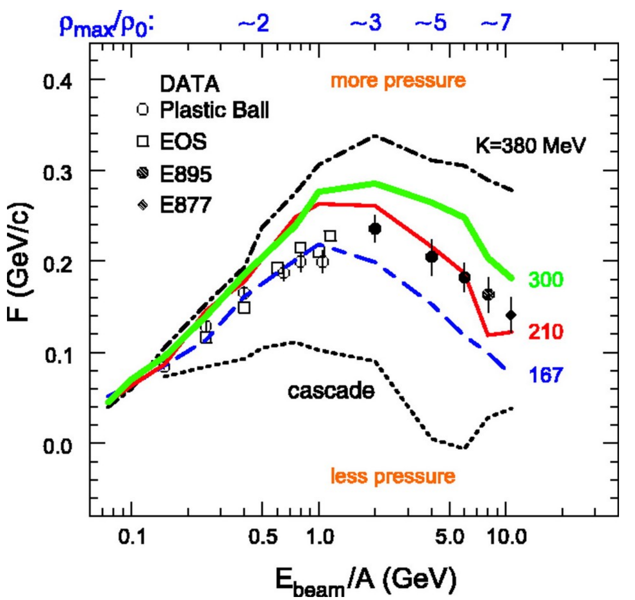
$$v_1(y) = \langle p_x/p_T \rangle_y = \langle \cos\phi \rangle_y$$

Side flow: $\langle p_x \rangle_y$

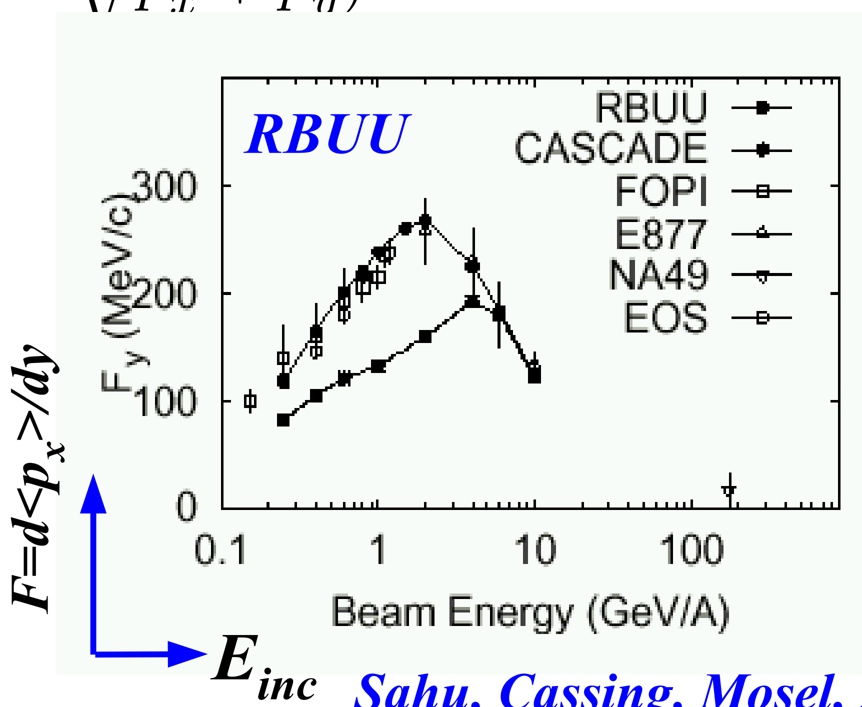
$$(y = \text{rapidity}, p_T = \sqrt{p_x^2 + p_y^2})$$







Danielewicz, Lacey, Lynch, Science 298('02)1592



Sahu, Cassing, Mosel, AO, NPA672('00)376

Nuclear Transport Theories

- **Transport equation = Mean field dynamics + Boltzmann Eq.**
  Boltzmann-Uehling-Uhlenbeck (BUU) equation
  G.F.Bertsch, S. Das Gupta, Phys. Rept. 160(1988), 190

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} f - \boldsymbol{\nabla} U \cdot \boldsymbol{\nabla}_p f = I_{\mathrm{coll}}[f]$$

$$I_{\mathrm{coll}}[f] = -\frac{1}{2} \int \frac{d\boldsymbol{p}_2}{(2\pi\hbar)^3} v_{12} \frac{d\sigma}{d\Omega} [f f_2 (1 - f_3)(1 - f_4) - f_3 f_4 (1 - f)(1 - f_2)]$$

  - **U=0 → Boltzmann equation (CASCADE, two-body collisions)**
  - **LHS=0 → Vlasov equation (~ Wigner transf. of TDHF)**
  - **Vlasov eq. can be simulated by solving classical equation for test particles, whose distribution gives the phase space dist. fn. (f).**
- **(Quantum) Molecular Dynamics**
  J. Aichelin, Phys.Rept. 202 (1991), 233
  - **Number of test particles = 1 per nucleon**
  - **Approximate way to solve BUU equation, but fluctuation strength is physical.**

Relativistic QMD (RQMD)

- **RQMD is developed based on constraint Hamiltonian dynamics**
  H. Sorge, H. Stoecker, W. Greiner, Ann. Phys. 192 (1989), 266.
  - **8N dof → 2N constraints → 6N (phase space)**
  - **Constraints = on-mass-shell constraints + time fixation**
- **RQMD/S uses simplified time-fixation**
  Tomoyuki Maruyama, et al. Prog. Theor. Phys. 96(1996),263.
  - **Single particle energy (on-mass-shell constraint) and EOM**

$$p_i^0 = \sqrt{\boldsymbol{p}_i^2 + m_i^2 + 2m_i V_i} \to \frac{d\boldsymbol{r}_i}{dt} = \frac{\boldsymbol{p}_i}{p_i^0} + \sum_j \frac{m_j}{p_j^0}\frac{\partial V_j}{\partial \boldsymbol{p}_i}, \quad \frac{d\boldsymbol{p}_i}{dt} = -\sum_j \frac{m_j}{p_j^0}\frac{\partial V_j}{\partial \boldsymbol{r}_i},$$

  - **Potential V_i is Lorentz scalar and becomes weaker at high E.**
- **Stronger potential effect is necessary → Vector-type potential**
  Y.Nara, T.Maruyama, H.Stoecker, PRC102('20)024913; Y.Nara,AO,arXiv:2109.07594

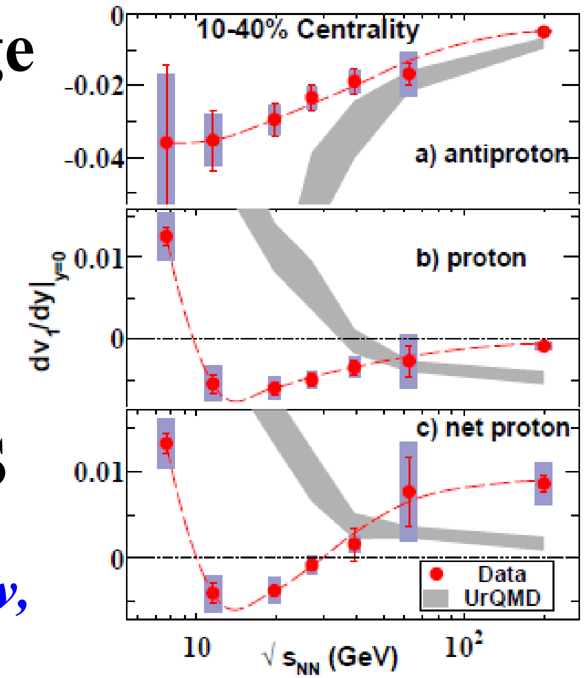
$$p_i^0 = \sqrt{\boldsymbol{p}_i^{*2} + m_i^2} + V_i^0 \to \frac{d\boldsymbol{r}_i}{dt} = \frac{\boldsymbol{p}_i^*}{p_i^{*0}} + \sum_j v_j^{*\mu}\frac{\partial V_{j\mu}}{\partial \boldsymbol{p}_i}, \quad \frac{d\boldsymbol{p}_i}{dt} = -\sum_j v_j^{*\mu}\frac{\partial V_{j\mu}}{\partial \boldsymbol{r}_i}$$

$$(p_j^{*\mu} = p_j^\mu - V_j^\mu, \; v_j^{*\mu} = p_j^{*\mu}/p_j^{*0})$$

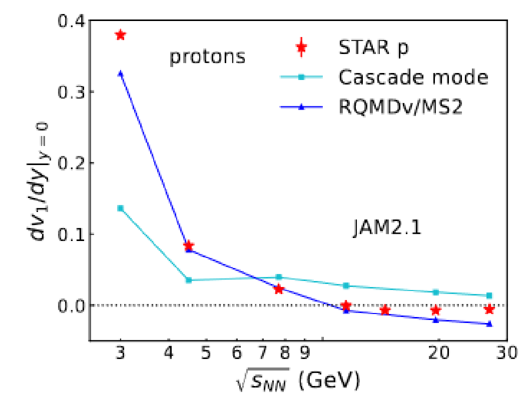
  - **Potential effect remains at high energy.**

Directed flow of protons

- **Directed flow is created in the overlapping stage of two nuclei → Sensitive to dense matter EOS.**
- **Non-monotonic beam E. dep. of v_1 slope**
  STAR, PRL 112('14)162301.
- **None of fluid and hybrid models explain the beam energy dependence with a single EOS**
  3FD: Y.B.Ivanov, A.A.Soldatov, PRC91('15)024915
  PHSD: V.P.Konchakovski, W.Cassing, Y.B.Ivanov, V.D.Toneev, PRC90('14)014903
  JAM: Y.Nara, H.Niemi, AO, H.Stoecker, PRC94 ('16)034906
- **A solution is found**
  Nara, AO, PRC105('22),014911[2109.07594]
  - **Strongly Coupled Hadronic Matter**
    (JAM-RQMDv, hadronic transport w/ vec. pot.)



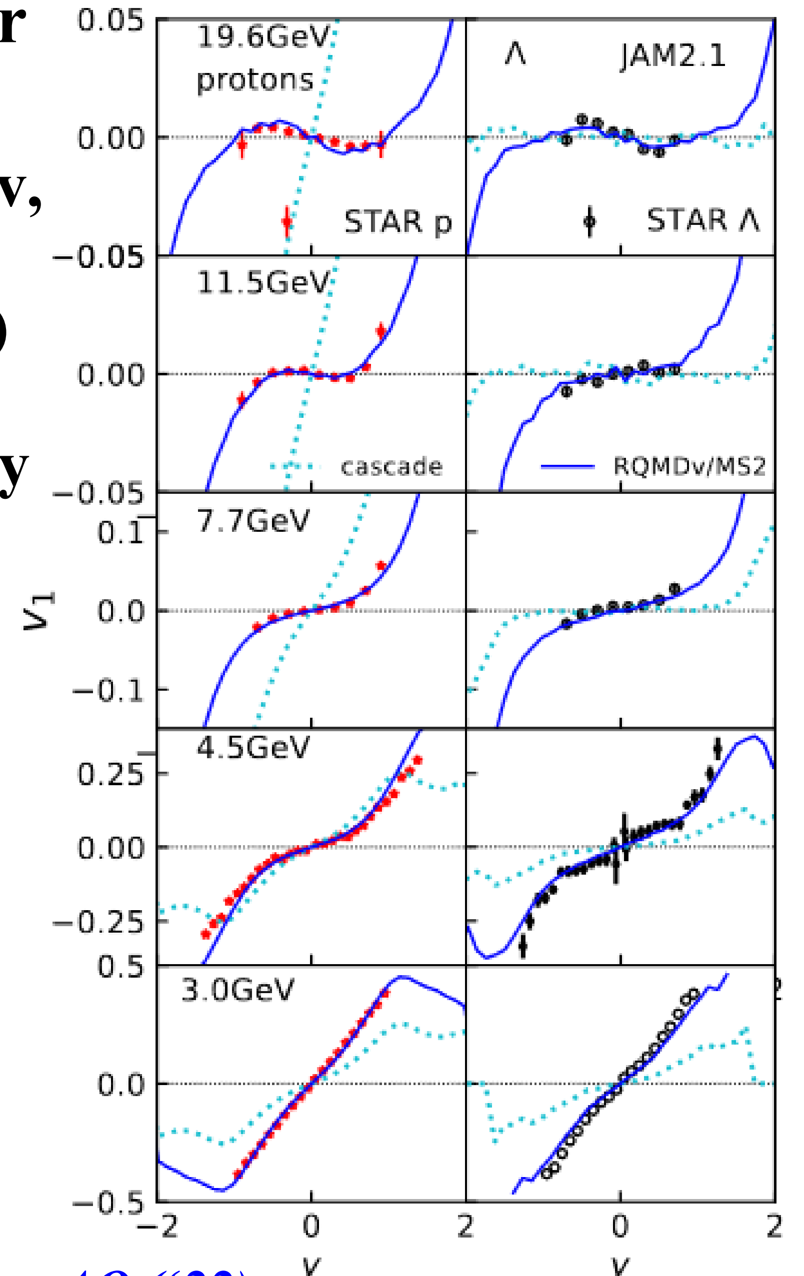
STAR ('14)



Nara, AO ('22); Nara, Jinno, Murase, AO ('22)

Directed flow of Λ

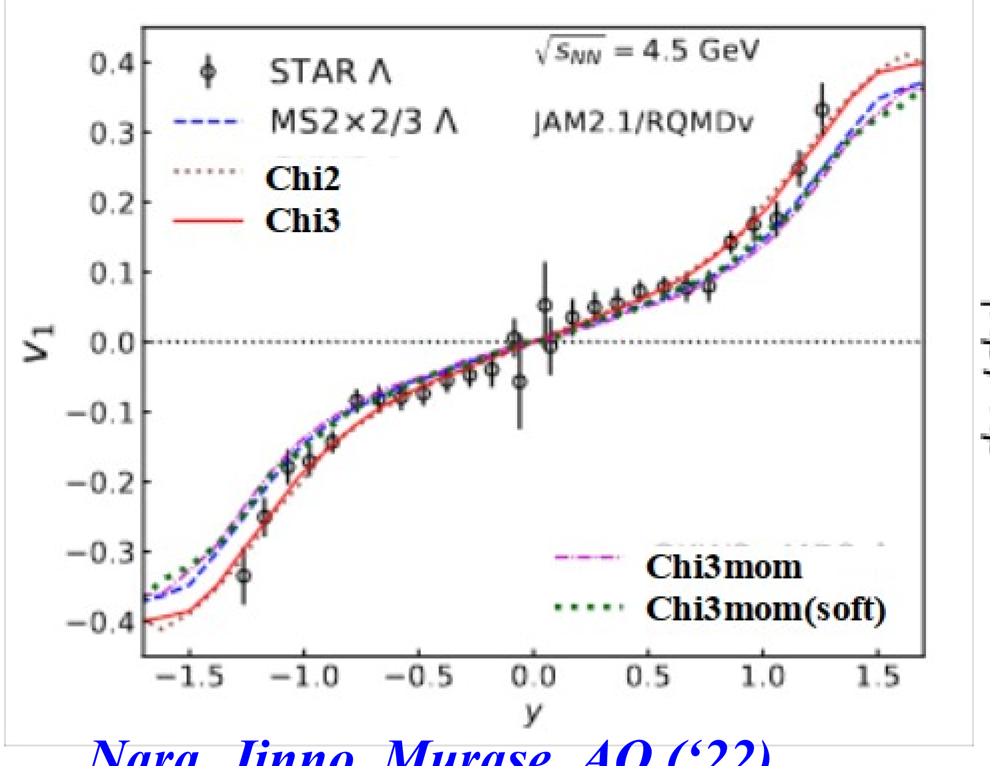
- **Calculation using JAM2 event generator**
  https://gitlab.com/transportmodel/jam2
  - **Potential effects are included in RQMDv, which solves the proton v1 puzzle. (Change of the v1 slope around 10 GeV)**
    Nara, AO, PRC105('22),014911[2109.07594]
  - **Directed flows of p and Λ are reasonably explained by using MS2 (momentum dep. soft potential) for non-strange baryons and MS2 x 2/3 for hyperons.**



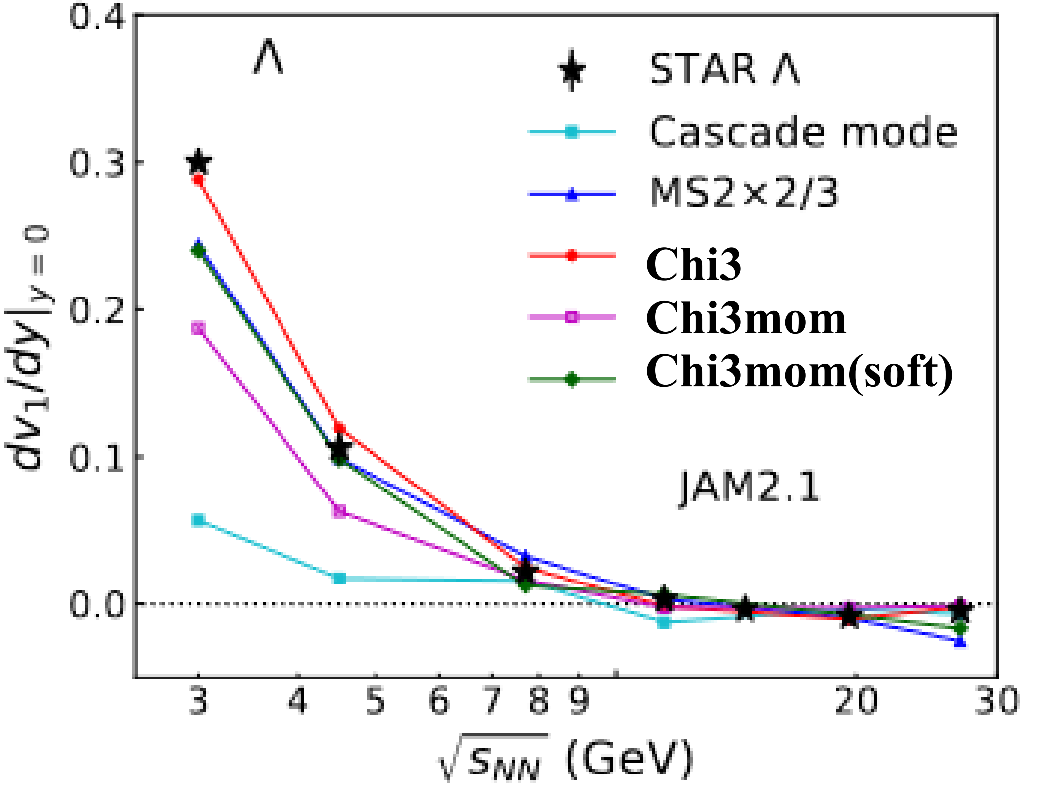
Nara, Jinno, Murase, AO ('22)

Directed flow of Λ with chiral EFT U_Λ

- Λ potential from chiral EFT is adopted.
  - Chi2/3: Fit to GKW $U_\Lambda(\rho)$ without/with 3-body force.
  - Chi2/3mom: Fit to Kohno('18) in the range k<500 and 200 MeV/c.
- GKW3 explains the data well.
  - Momentum dep. reduces v1 values.
  - Chi2 also explains the data.



Nara, Jinno, Murase, AO ('22)



Repulsive Λ potential (Chi3) enhances dv1/dy and gives results close to data, when momentum dependence is ignored.

Momentum dep. of U_Λ reduces v1 (and dv1/dy), while the density dep. affects less.
(Why? We haven't understood the mechanism yet.)

Provided that other effects(*) are included and dv1/dy(Λ) is enhanced, Chi3mom(soft) would be a reasonable solution.
(*: more repulsive Σ potentials, stiffer nuclear EOS with K~240 MeV, ...)

Any other way to constrain the density and momentum dependence of U_Λ?
→ Hypernuclear spectroscopy

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Λ Binding Energy in Hypernuclei

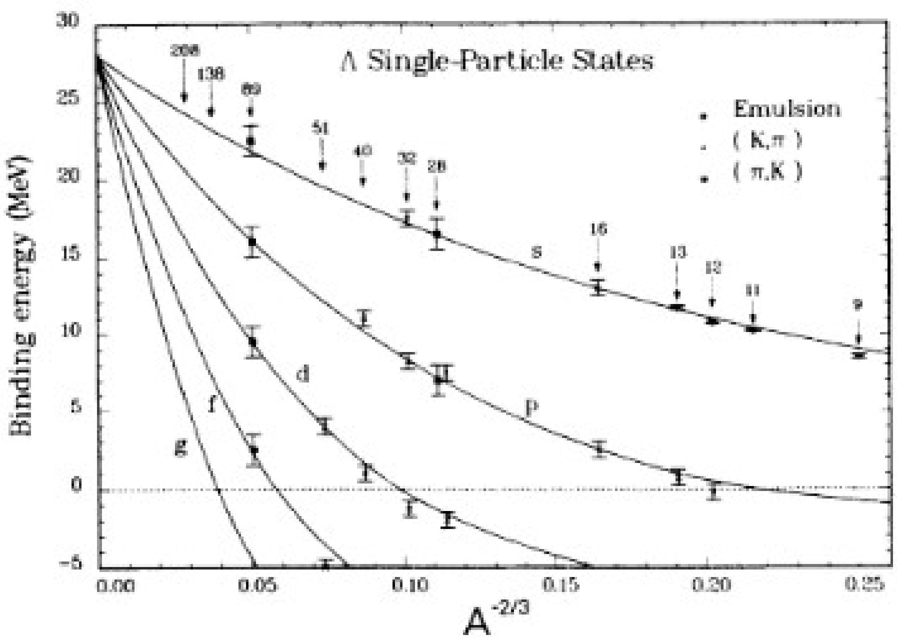
- Summary

Λ hypernuclei and Λ potential in nuclear matter

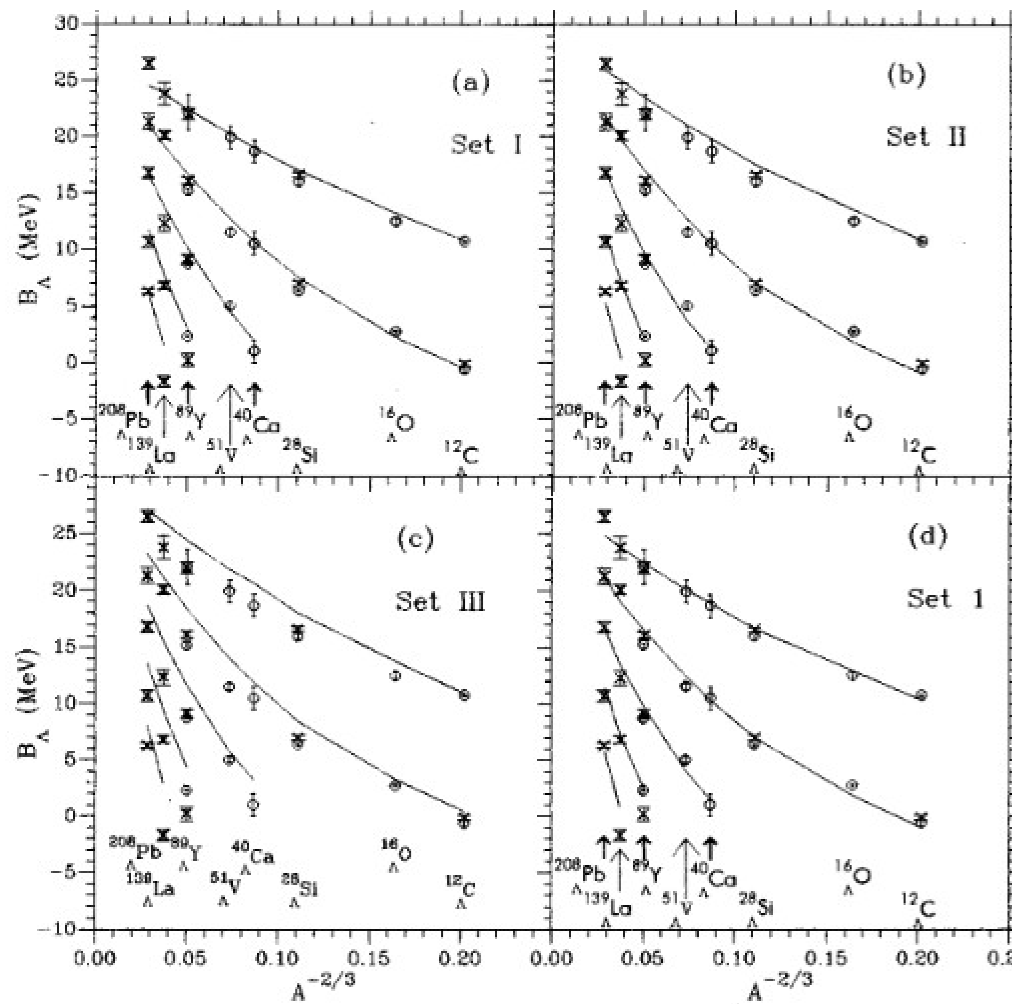
- **From binding energies of Λ from many Λ hypernuclei, people believed the density dependence of Λ is understood well.**
  D.J.Millener, C.B.Dover, A. Gal, PRC 38 (1988) 2700;
  D.E. Lanskoy, Y. Yamamoto, PRC 55 (1997) 2330.

$$A^{-2/3} \propto \text{K.E. of } \Lambda$$

→ **Hyperon Puzzle**



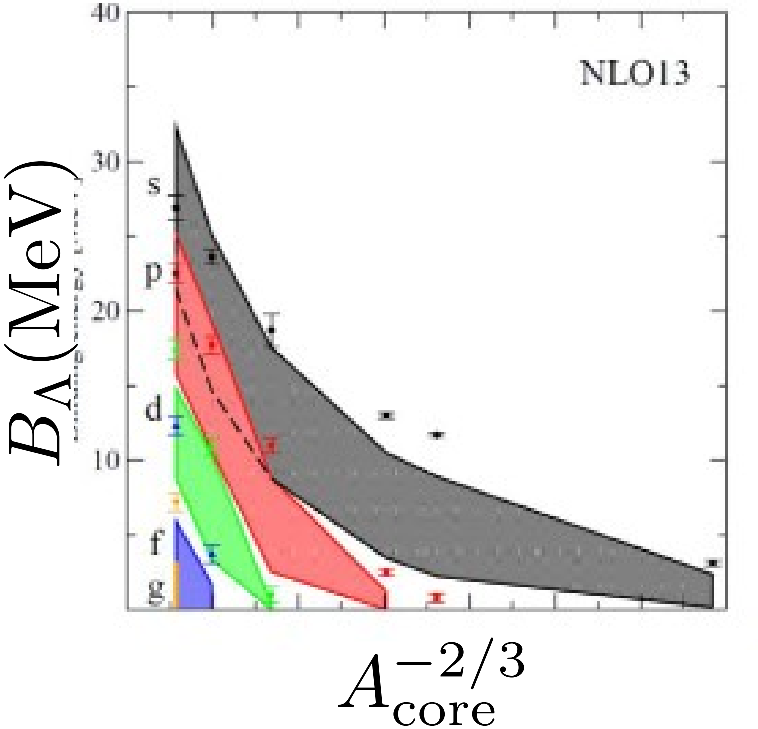
Millener, Dover, Gal ('88)



Lanskoy, Yamamoto ('97)

Λ hypernuclei and Λ potential in nuclear matter

- **Λ potential from chiral EFT w/ 3BF has not been examined in hypernuclear spectroscopy**
  - **It needs to be verified (and tuned) including the density and momentum dependence !**
    (before applying it to heavy-ion collisions (?))
  - **NLO result has been tested but does not explain the data well.**
    Haidenbauer, Vidaña ('21)
  - **How about NNLO with decouplet model ?**



Haidenbauer, Vidaña ('21)

Skyrme Hartree-Fock for Λ hypernuclei

- **Previous breakthrough works (spherical SHF)**
  - **Rayet('76,'81): Two-body SHF (w/o ρ dep.)**
  - **Lanskoy, Yamamoto (LY, '97): SHF w/ one ρ dep. term (as in standard HF for nucleons)**
  - **Choi, Hiyama et al. ('22): SHF w/ two or more ρ dep. terms (significant improvement w/ two ρ dep. terms.)**
- **SHF for Λ hypernuclei**
  - **HF equation**

$$\left[-\boldsymbol{\nabla}\cdot\left(\frac{\hbar^2}{2m_B^*(\boldsymbol{r})}\right)\boldsymbol{\nabla}+U_B(\boldsymbol{r})-i\boldsymbol{W}_B(\boldsymbol{r})\cdot(\boldsymbol{\nabla}\times\boldsymbol{\sigma})\right]\psi_{iB}(\boldsymbol{r})=\varepsilon_i\psi_{iB}(\boldsymbol{r})$$

  - **HF potential**

$$U_\Lambda(\boldsymbol{r})=a_1^\Lambda\rho_N+a_2^\Lambda\tau_N+a_3^\Lambda\triangle\rho_N+a_4^\Lambda\rho_N^{4/3}+a_5^\Lambda\rho_N^{5/3}$$

$$\frac{\hbar^2}{2m_\Lambda^*}=\frac{\hbar^2}{2m_\Lambda}+a_2^\Lambda\rho_N\,,\quad \tau_B=\sum_i\boldsymbol{\nabla}\psi_{iB}^*\cdot\boldsymbol{\nabla}\psi_{iB}$$

Parameters from Chiral EFT

- **Density dependence at zero momentum**

$$U_\Lambda(\boldsymbol{r}) = a_1^\Lambda \rho + a_2^\Lambda \tau_N + a_3^\Lambda \triangle\rho + a_4^\Lambda \rho^{4/3} + a_5^\Lambda \rho^{5/3} \quad (\rho = \rho_N)$$

$$\to U_\Lambda(\rho, \boldsymbol{k}=0) = a_1^\Lambda \rho + a_4^\Lambda \rho^{4/3} + \widetilde{a}_5^\Lambda \rho^{5/3} \text{ (uniform matter)}$$

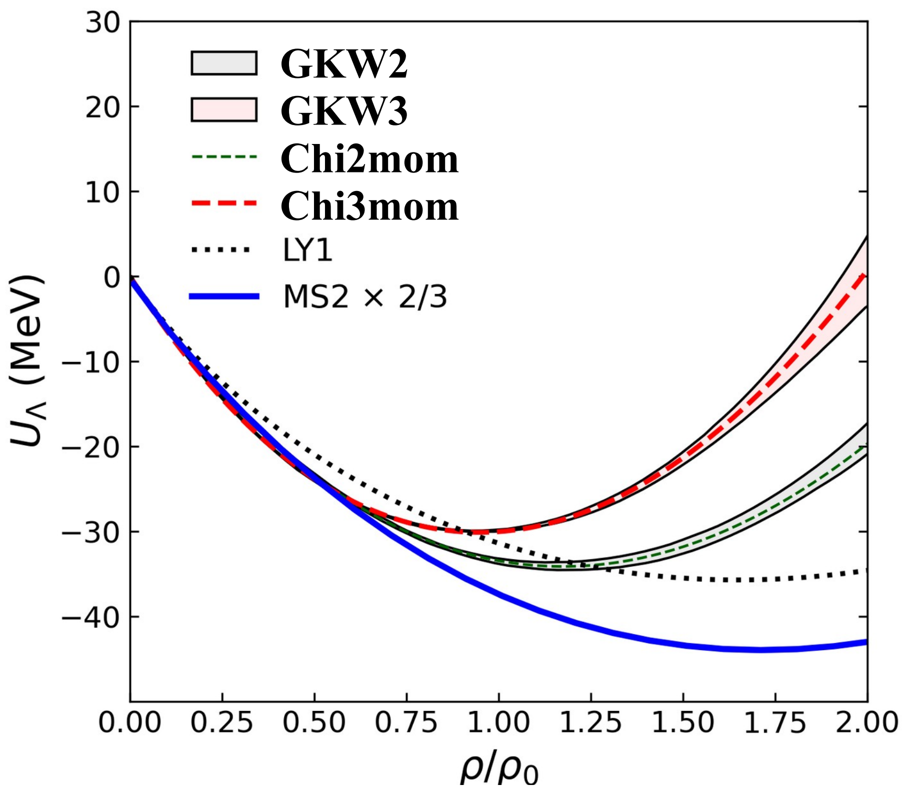
$$\widetilde{a}_5^\Lambda = a_5^\Lambda + \alpha a_2^\Lambda , \ \alpha = \text{const.}$$

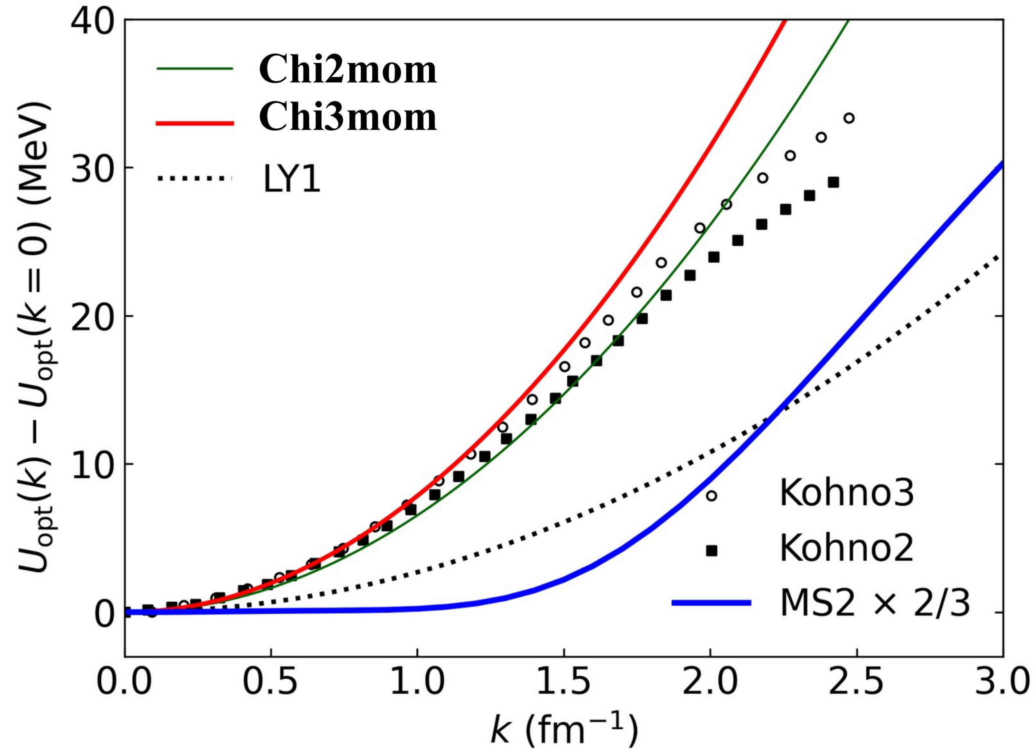
  - **Three parameters are tuned to reproduce Chiral EFT results.**
- **Momentum dependence**

$$U_\Lambda(\rho, \boldsymbol{k}) = U_\Lambda(\rho) + a_2^\Lambda \boldsymbol{k}^2 \rho$$

  - **a_2^Λ is tuned to reproduce Kohno's results at low momentum.**
- **Finite range effects**
  - **a_3^Λ is tuned to reproduce the separation energy of Λ in $^{13}_{\Lambda}C$ (even-even core nucleus)**

Parameters from Chiral EFT



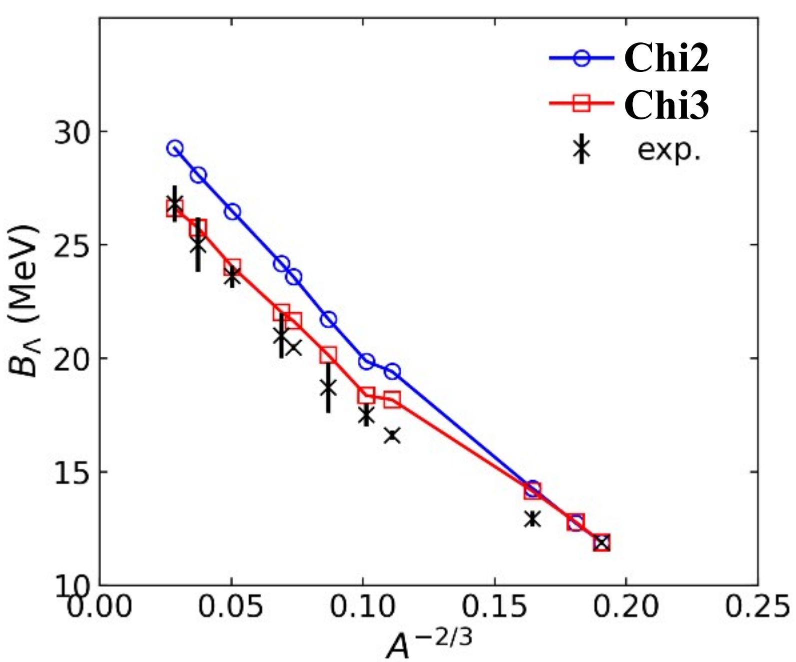


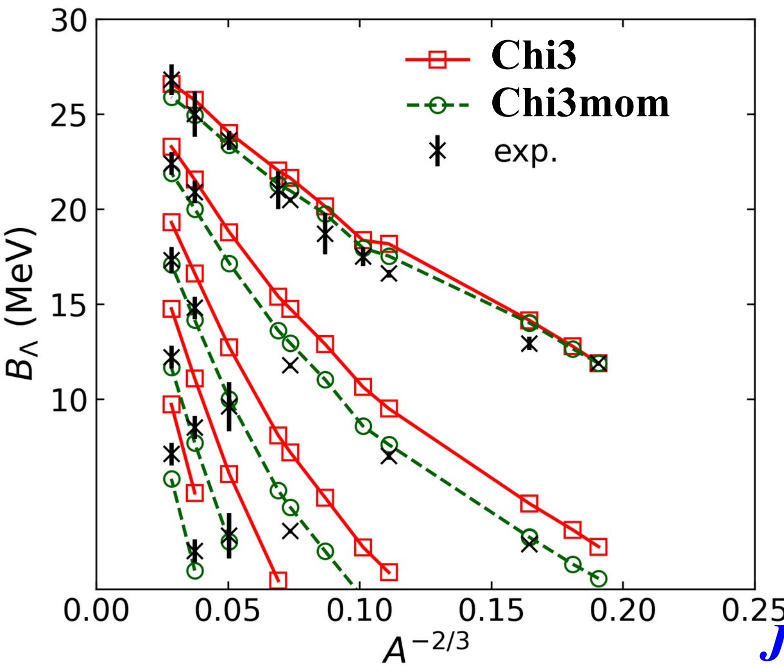
3 density-dep. terms + k^2 momentum dep. term are enough at $\rho/\rho_0 < 2$ and $k < 1.3$ fm^{-1}

Jinno+ (in prep.)

GKW2 and GKW3

- NNLO chiral EFT with the decouplet saturation model without/with 3-body terms (Chi2 and Chi3)
- Chi2 overestimates B_Λ at large A.
  - Deeper potential at ρ_0. (~ -35 MeV)
  - Steeper A dep. is consistent with *Haidenbauer, Vidaña ('21)*
  - Mom. dep. and finite range terms does not help.
- Chi3mom (w/ mom. dep.) explains the data well.



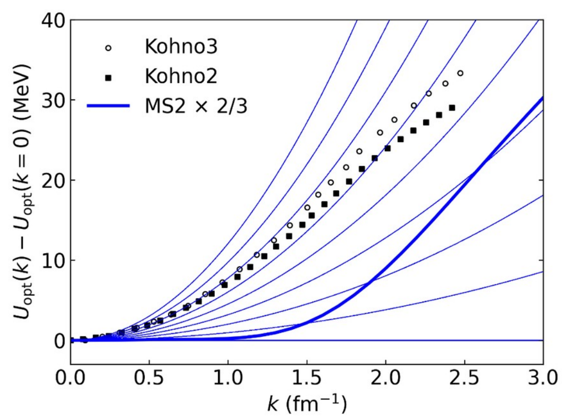


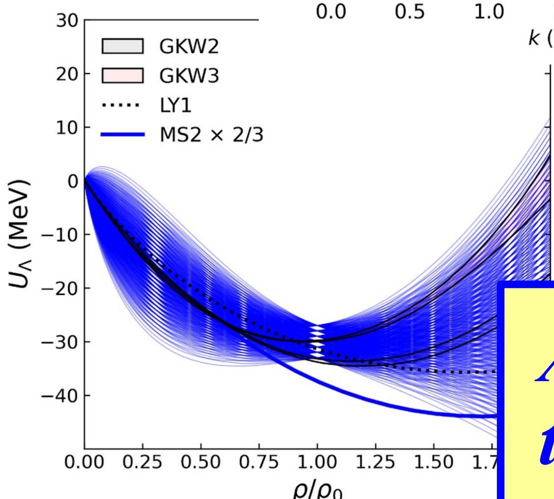
Jinno+ (in prog.)

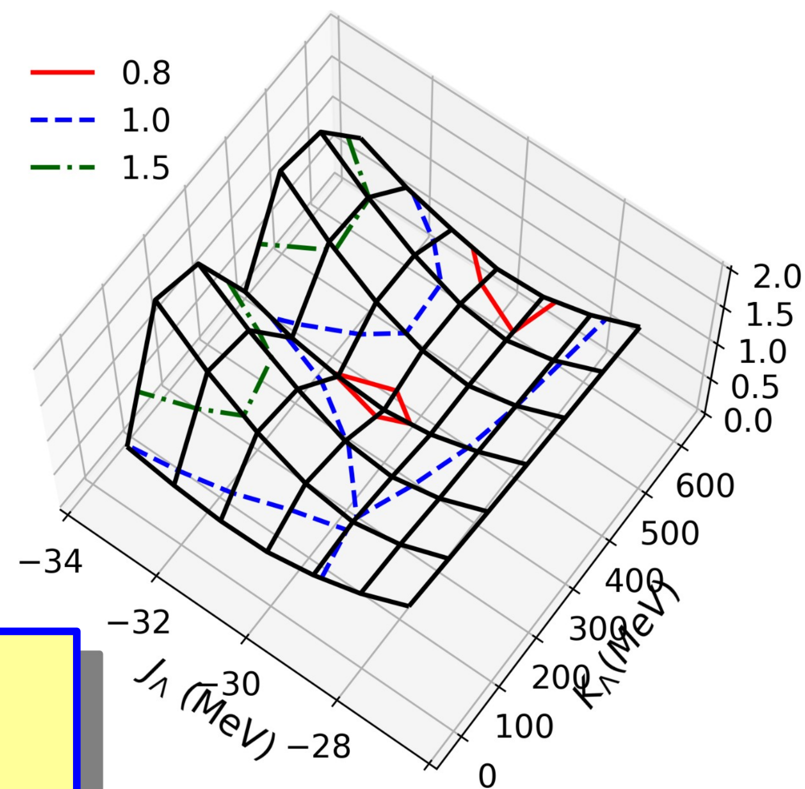
Question!

- **Chiral EFT and meson-exchange-based potentials have different density dependence. Why can both of them explain B_Λ?**
  → Global analysis using Taylor coeff. around ρ_0 !

$$U_\Lambda(\rho,k) = J_\Lambda + \frac{L_\Lambda}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_\Lambda}{9}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + a_2^\Lambda k^2\rho + \mathcal{O}\left[\left(\frac{\rho-\rho_0}{\rho_0}\right)^3, k^4\right]$$







Answer = There are two local minima.

Jinno+ (in prog.)

Summary

- Λ potential in nuclear matter from the chiral effective field theory with the 3-body force effect is examined via
  the directed flow of Λ from heavy-ion collisions
  and the binding energies of Λ in hypernuclei.
  - Chi2/3(mom): $U_\Lambda(\rho,k)$ (without/with 3-body force (mom. dep.))
    Gerstung, Kaiser, Weise ('20); Kohno ('18)
  - With Chi3(mom), repulsive U_Λ forbids Λ to appear in neutron stars and the hyperon puzzle is solved.
- Directed flow of Λ is well explained by Chi2/3.
  - With strong mom. dep., v1(Λ) is underestimted.
- Chi3mom can explain the Λ binding energies.
  - We need to tune the finite range term.
  - Chi3 (w/o mom. dep.) underestimate the binding energy of Λ for finite L states.
  - Chi2 is too attractive and cannot explain B_Λ.
- Hyperon puzzle would be accessible via laboratory experiments.

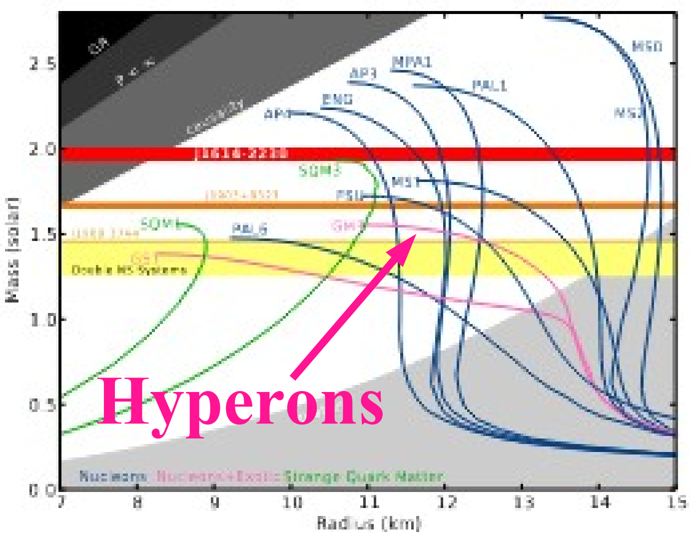
Conclusion, Conjecture, and To do

- Conclusion: Stiff U_Λ (K_Λ ~ (500-600) MeV) having momentum dependence (M_Λ^*~(0.7-0.9)M_Λ)
  explains both v1(Λ) in HIC and B_Λ in hypernuclei,
  provided that U_Λ is not very repulsive at high momentum.
- Conjecture: There may be two types of U_Λ which explains B_Λ.
  - Lanskoy-Yamamoto type: K_Λ~ 300 MeV
  - Chiral EFT type: K_Λ~ 600 MeV
  - Λ appears in neutron stars in the former,
    but Λ does not appear in neutron stars in the latter.
- To do
  - Comparison of the directed flows using U_Λ at two local minima.
  - Mechanism for the insensitivity of v1(Λ) to the density dependence of U_Λ via time-dependent analysis.
  - More serious estimate of B_Λ using chiral EFT. (E.g. HypAMD)
  - Other SHF parameters are close to local minima ?
    N.Guleria, S.K.Dhiman, R.Shyam(1108.0787)
  - Why does chiral EFT show strong k-dep. in N(N)LO?

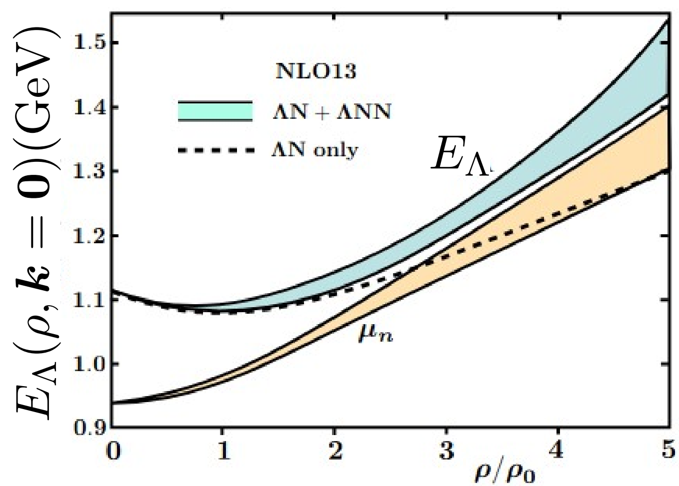
Thank you for your attention !

Hyperon Puzzle of Neutron Stars

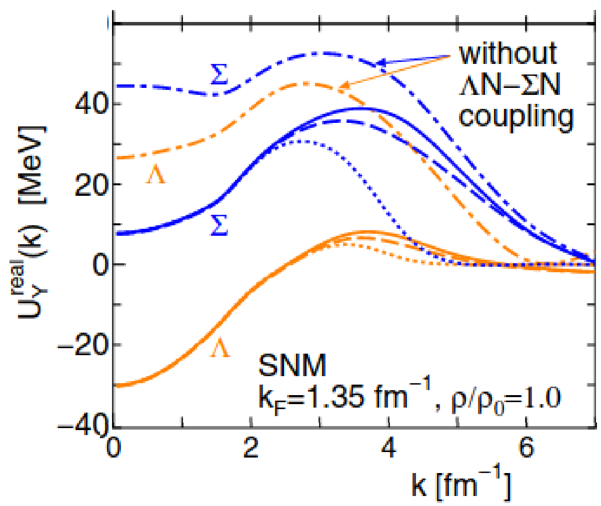
- **Hyperonic matter EOS cannot sustain $2M_\odot$ neutron stars.**
- **Proposed solutions**
  - **More repulsive hyperon potential ($U_\Lambda(\rho)$) at high density**
  - **Transition to quark matter before Λ appears**
  - **General relativity → Modified gravity**
- **Λ potential from chiral EFT**
  - **Three-body force may cause repulsive potential of Λ.**



Demorest+(1010.5788)



Gerstung, Kaiser, Weise ('20)



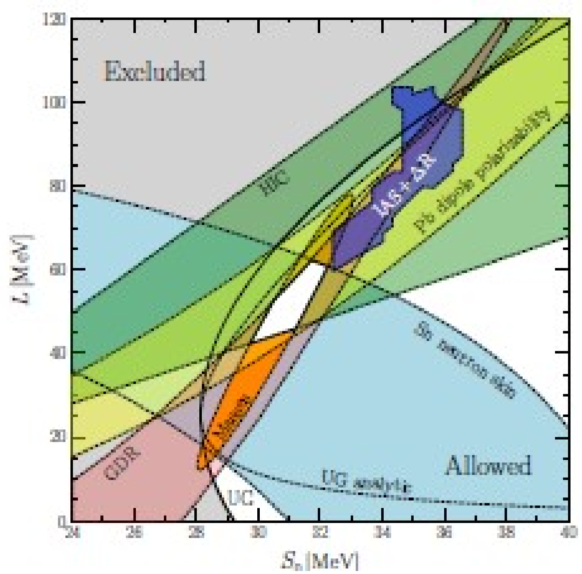
Kohno ('18)

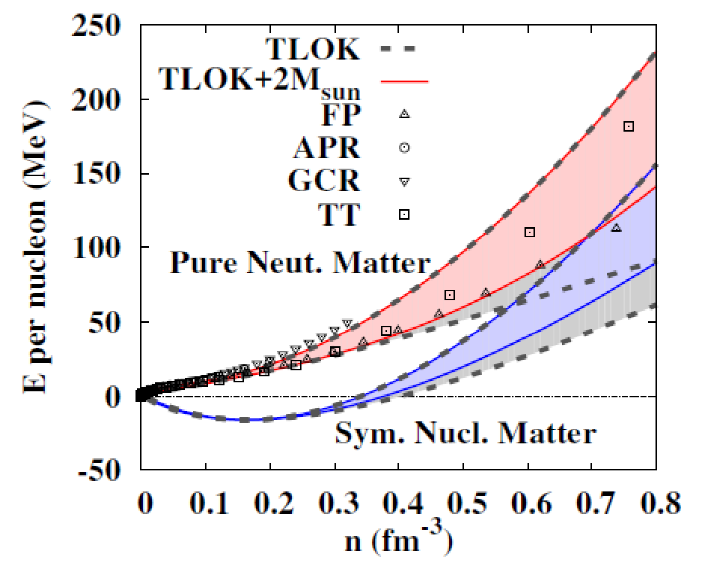
Fermi momentum expansion

- **Energy per nucleon would be expressed as the power series of k_F**

$$E = Tu^{2/3} + au + bu^{4/3} + cu^{5/3} + du^2$$
$$= J + \frac{L}{3}(u-1) + \frac{K}{18}(u-1)^2 + \frac{Q}{162}(u-1)^3 + \mathcal{O}((u-1)^4)$$
$$(u = \rho/\rho_0)$$

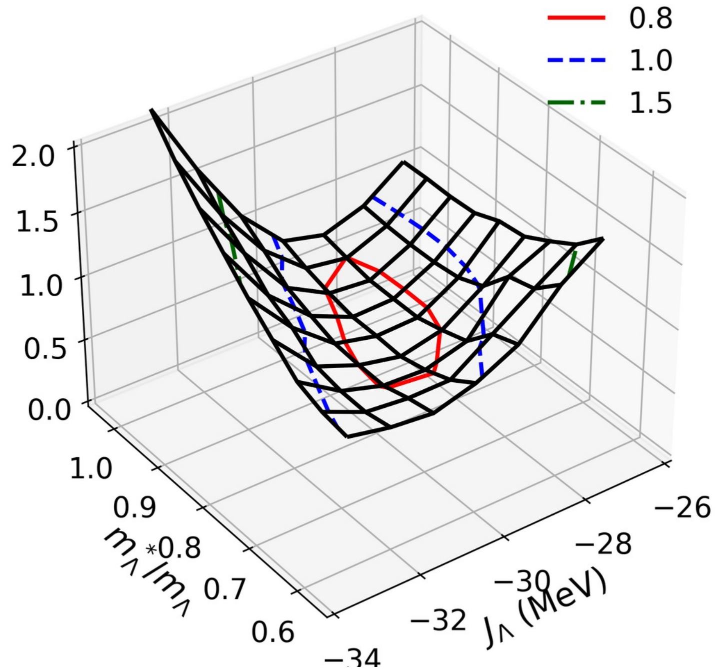
$$a = -4T + 20J - \frac{19}{3}L + K - \frac{1}{6}Q$$
$$b = 6T - 45J + 15L - \frac{5}{2}K + \frac{1}{2}Q$$
$$c = -4T + 36J - 12L + 2K - \frac{1}{2}Q$$
$$d = T - 10J + \frac{10}{3}L - \frac{1}{2}K + \frac{1}{6}Q$$

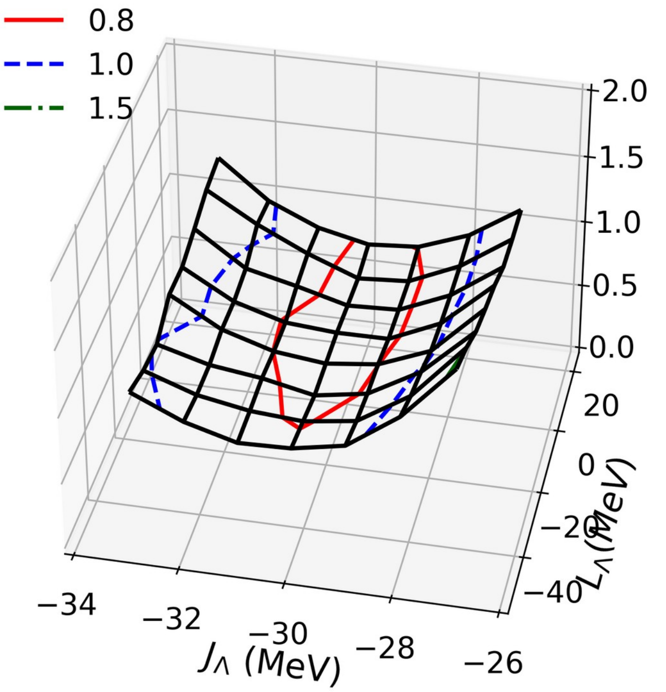




Tews, Lattimer, AO, Kolomeitsev ('17)

M*-J and J-L correlations





One local minimum

Parameter range

Root mean square deviations are calculated for 3528 parameter sets in total.

$$\left\langle \left(B_{\Lambda,\mathrm{exp}} - B_{\Lambda,\mathrm{cal}}\right)^2 \right\rangle^{1/2}$$

- $J_\Lambda = -33, -32, -31, \dots, -27$ MeV
- $L_\Lambda = -50, -40, -30, \dots, 20$ MeV
- $K_\Lambda = 0, 100, 200, \dots, 600$ MeV
- $m^*/m = 0.6, 0.65, 0.70, \dots, 1.0$

Semi-Classical Nuclear Transport Theories

- **Wigner(-Weyl) transform of TDHF = Vlasov equation**
  - **Wigner transform of density matrix=Wigner fn. (phase space dist.)**
  - **Wigner transform of commutator ~ i$\hbar\times$ Poisson bracket**

$$i\hbar\frac{d\rho}{dt}=[h,\rho]\to\frac{\partial f}{\partial t}+\boldsymbol{v}\cdot\boldsymbol{\nabla}f-\boldsymbol{\nabla}U\cdot\boldsymbol{\nabla}_p f=0$$

$$\left[f=\rho_W,[A,B]_W=i\hbar\{A_W,B_W\}_{PB}+\mathcal{O}(\hbar^2)\right]$$

  - **Test particle solution of the Vlasov equation → Classical EOM**

$$f(\boldsymbol{r},\boldsymbol{p})=\frac{(2\pi)^3}{N}\sum_{i=1,NA}\delta(\boldsymbol{r}-\boldsymbol{r}_i)\delta(\boldsymbol{p}-\boldsymbol{p}_i)$$

$$\to\frac{d\boldsymbol{r}_i}{dt}=\left.\frac{\partial h}{\partial\boldsymbol{p}}\right|_{\boldsymbol{p}=\boldsymbol{p}_i}=\frac{\boldsymbol{p}}{m}+\left.\frac{\partial U}{\partial\boldsymbol{p}}\right|_{\boldsymbol{p}=\boldsymbol{p}_i},\quad\frac{d\boldsymbol{p}_i}{dt}=-\left.\frac{\partial U}{\partial\boldsymbol{r}}\right|_{\boldsymbol{r}=\boldsymbol{r}_i}$$

- **Relativistic Quantum Molecular Dynamics**
  - **Transport model applicable to high energies**
    Sorge, Stoecker, Greiner ('89); Maruyama et al. ('96)
  - **Stronger potential effects are necessary → Vector potential**
    Nara et al. ('20), Nara, AO ('21)
  - **Stochastic collisions are also included**

U_Λ from Chiral EFT

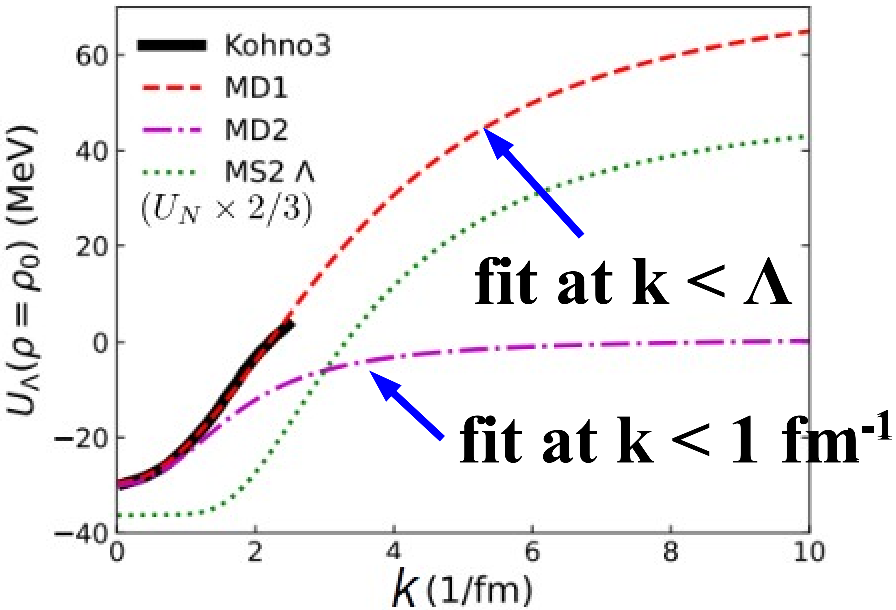
- **Chiral EFT with 3BF and hyperons**
  Gerstung+(2001.10563)(GKW, decuplet saturation model), Kohno (1802.05388)
  - **ρ-dep. potential using Fermi mom. expansion** *Tews+(1611.07133)* **+ momentum dep. fitted to Kohno ('18).**

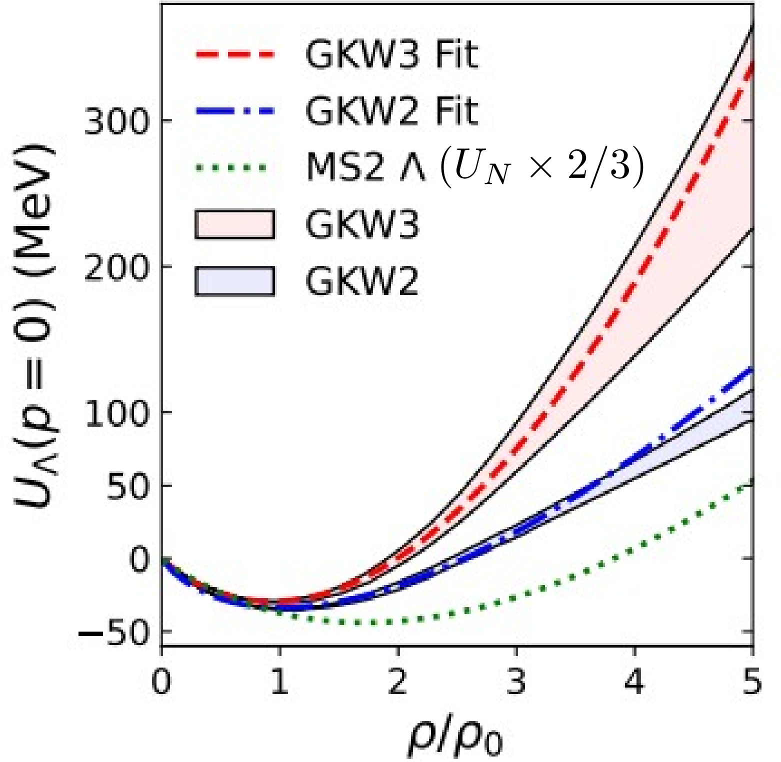
$$U_\Lambda(\rho,k) = a\frac{\rho}{\rho_0} + b\left(\frac{\rho}{\rho_0}\right)^{4/3} + c\left(\frac{\rho}{\rho_0}\right)^{5/3} + \sum_n \frac{C_n}{\rho_0}\int \frac{d\boldsymbol{k}'}{(2\pi)^3}\frac{f(\boldsymbol{r},\boldsymbol{k}')}{1+(\boldsymbol{k}-\boldsymbol{k}')^2/\mu_n^2}$$

  - **Range of fit**

$\rho \le 3.5\rho_0$ (unstable above $3.5\rho_0$)

$k \le \Lambda$ (MD1) or $k \le 1\ \mathrm{fm}^{-1}$ (MD2)

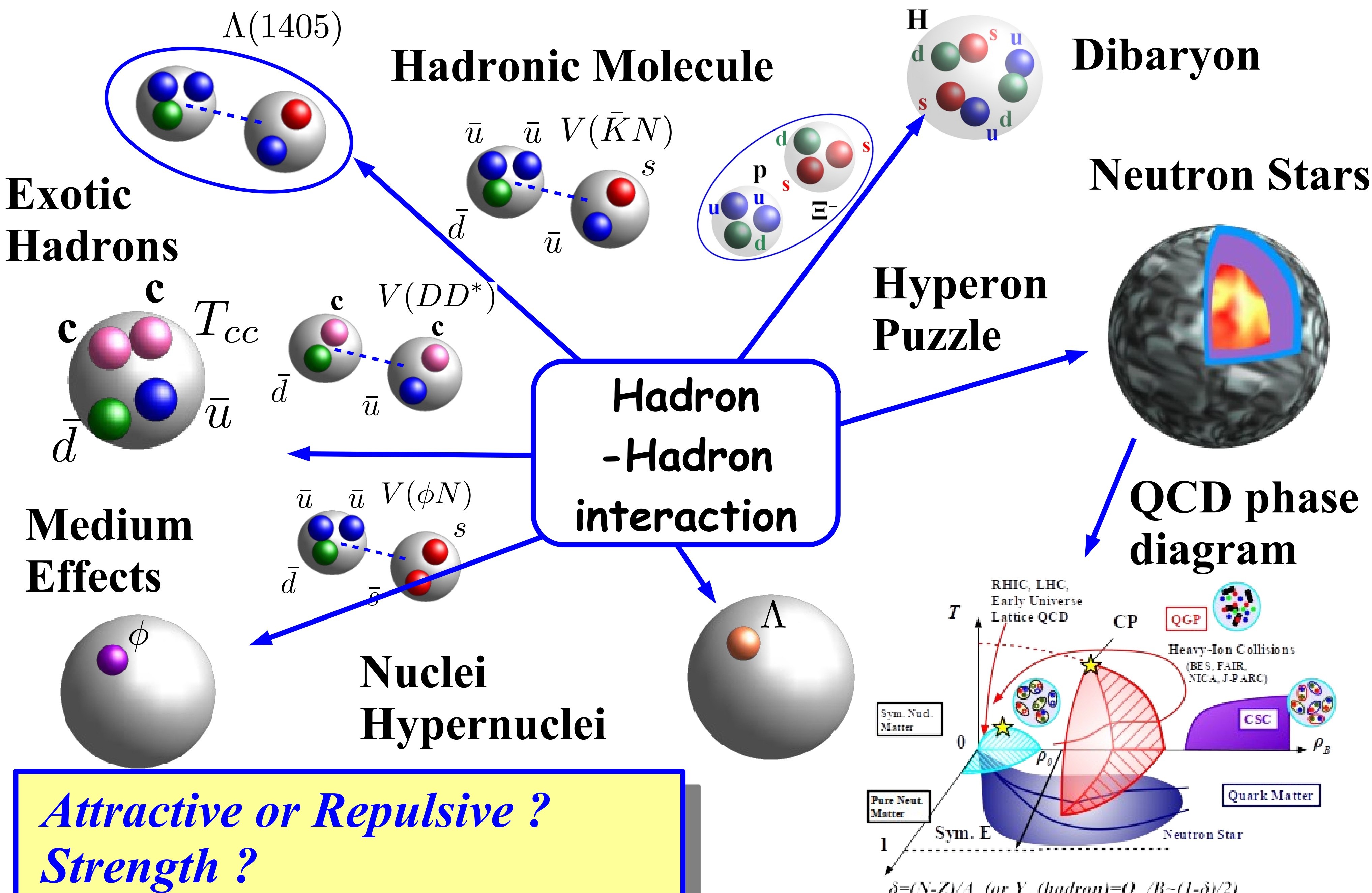




Nara, Jinno, Murase, AO ('22)

Somewhat different subjects...

Hadron-Hadron Interaction

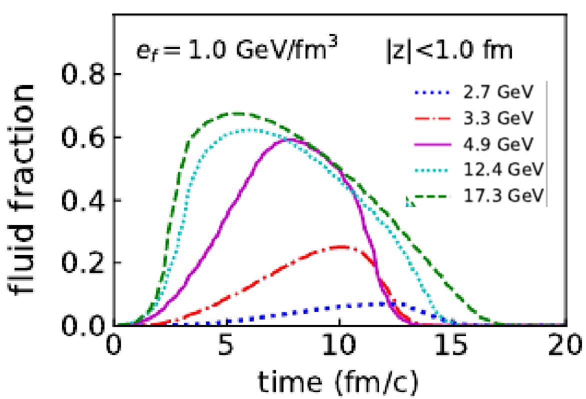


Attractive or Repulsive ?
Strength ?
Bound or not ?

Dense Baryonic Matter EOS from Collective Flow in HIC & Onset energy of QGP

Onset Beam Energy of (bulk) QGP formation

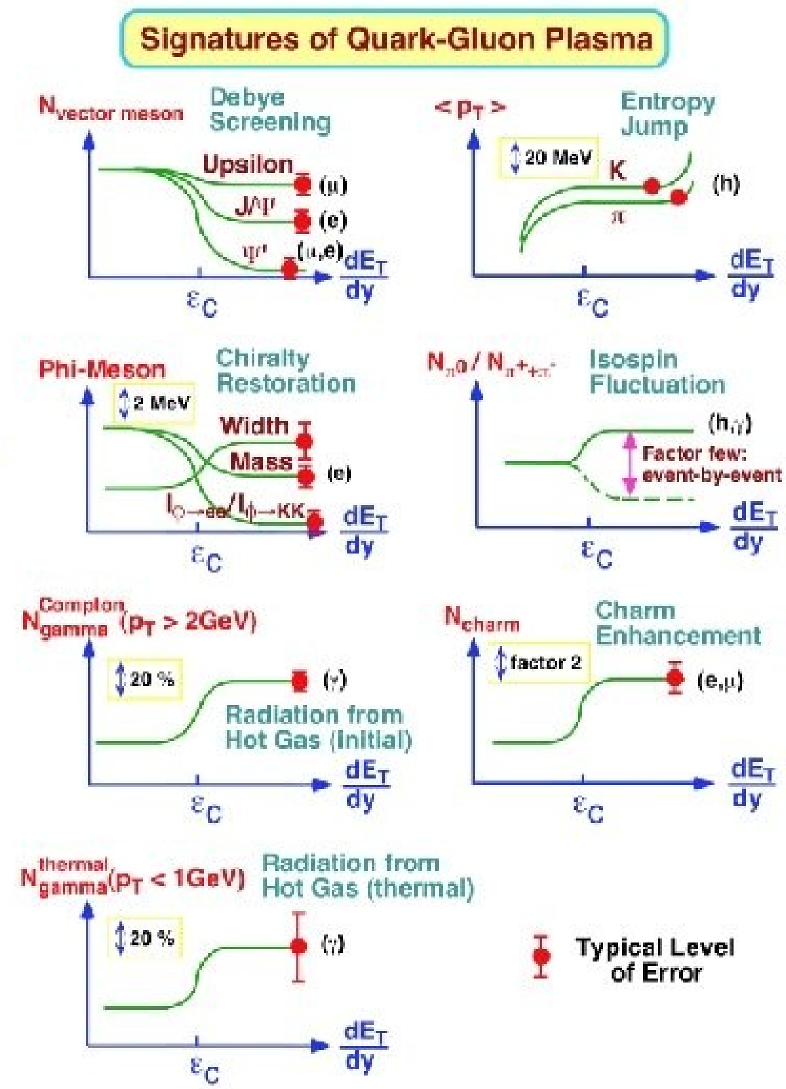
- **Nagamiya Plot**
  - **Simultaneous change of many signals**
  - **Finite volume smears sharp signal of 1st ord. p.t. even if it exists.**
    → Gradual increase of QGP fraction ?



Hadronic fluid or QGP ?

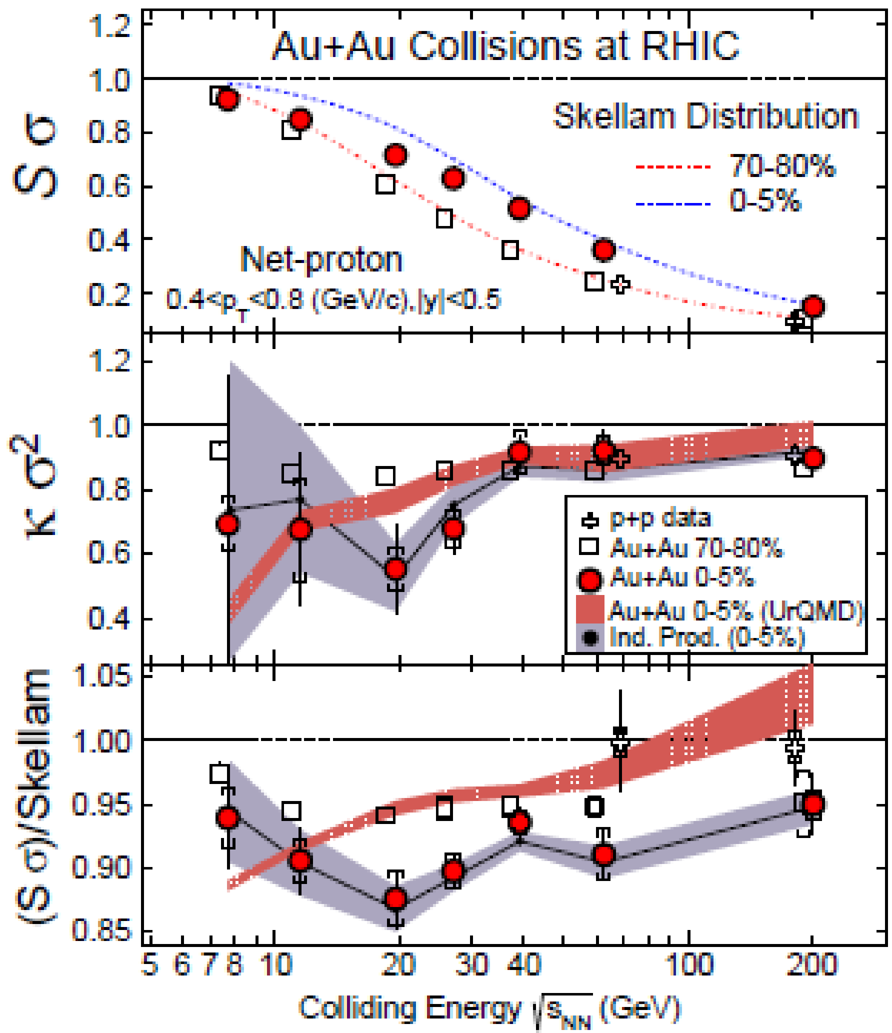
Y.Akamatsu, M.Asakawa, T.Hirano, M.Kitazawa, K.Morita, K.Murase, Y.Nara, C.Nonaka, AO, PRC98('18)024909.

- **Non-monotonic dep. on beam E.**
  - **Net-Proton Number Cumulants**
    STAR Collab. PRL 112('14)032302; PRC104 ('21) 024902.
  - **Directed Flow**
    STAR Collab., PRL 112('14)162301.

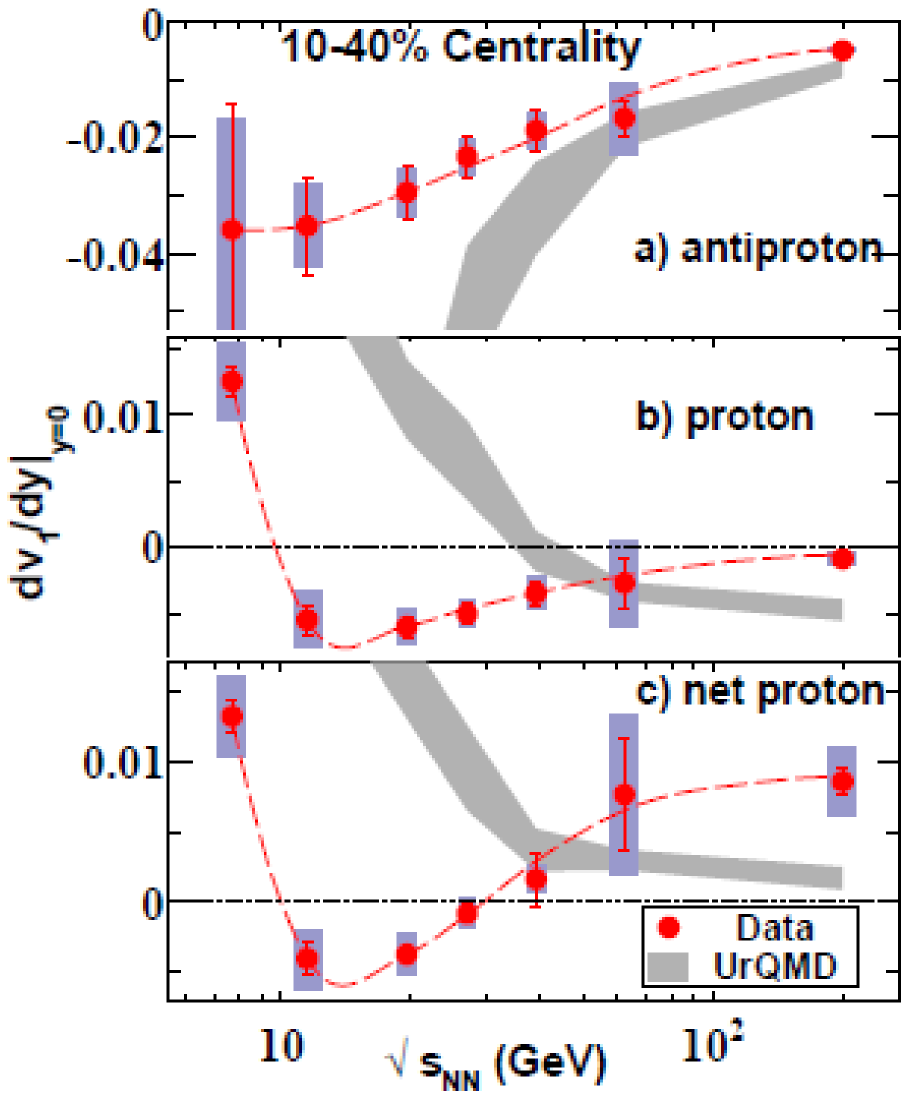


Taken from S. Nagamiya's Talk @ RIKEN, 2013/08/02

Net-Proton Number Cumulants & Directed Flow



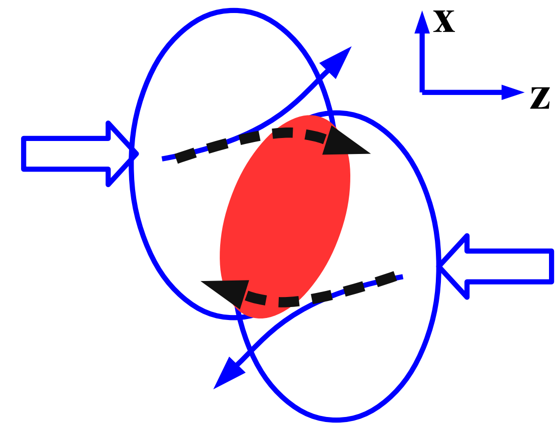
STAR, PRL 112('14)032302



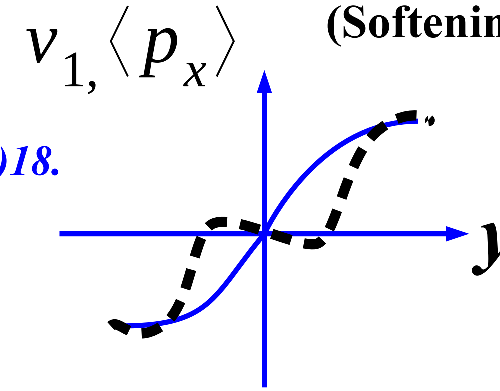
STAR, PRL 112('14)162301.

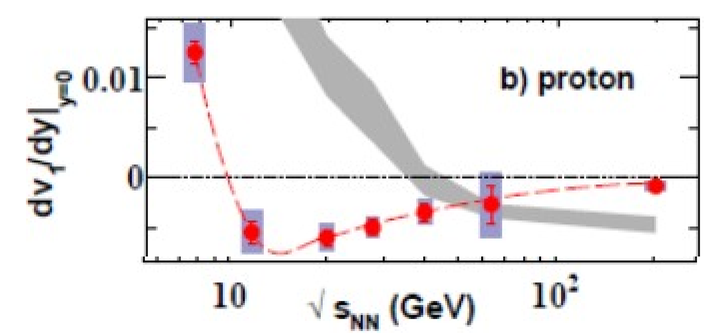
Non-Monotonic Beam E. dep. of v_1 slope

- **Directed flow (v_1 or $\langle p_x \rangle$) is created in the overlapping stage of two nuclei → Sensitive to the EOS of dense matter.**
- **BES (Beam Energy Scan) result → Non-monotonic beam E. dep. of v_1 slope**
  - **EOS softening ?**
    Y.Nara, H.Niemi, AO, H.Stoecker, PRC94 ('16)034906;
    Y.Nara, H.Niemi, J.Steinheimer, H.Stoecker, PLB769 ('17) 543;
    Y.Nara, H.Niemi, AO, J.Steinheimer, X.F.Luo, H.Stoecker, EPJA54 ('18)18.
  - **None of fluid and hybrid models explain the beam energy dependence with a single EOS**



Attraction (Softening)

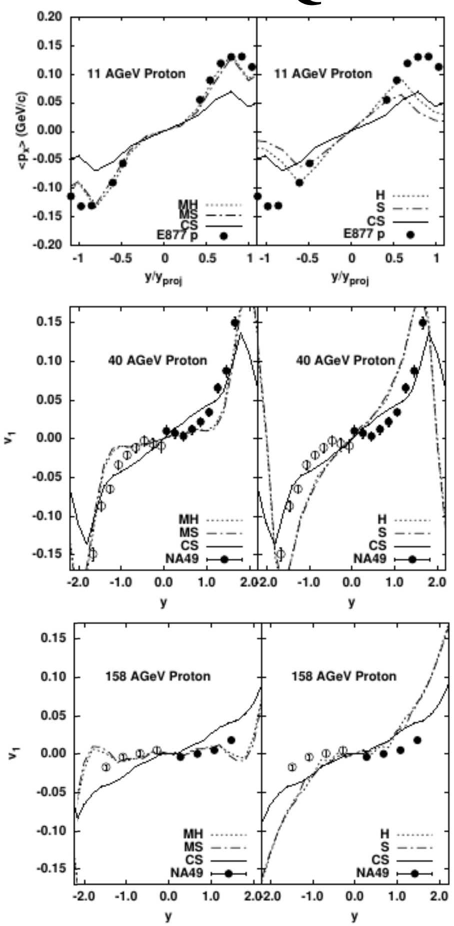




STAR, PRL112('14)162301

Past tries

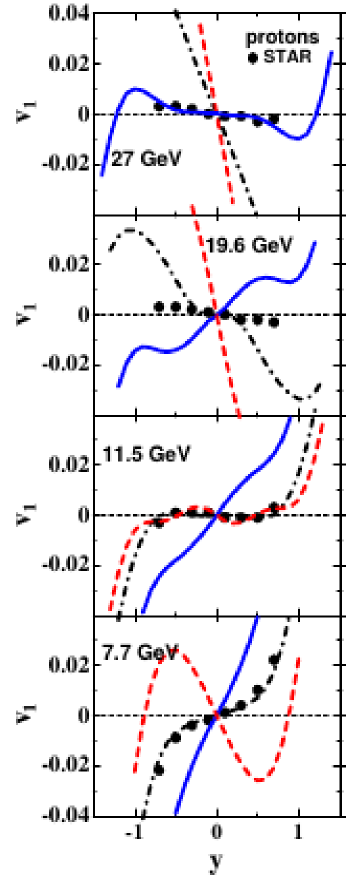
JAM-RQMD



p-dep. p-indep.

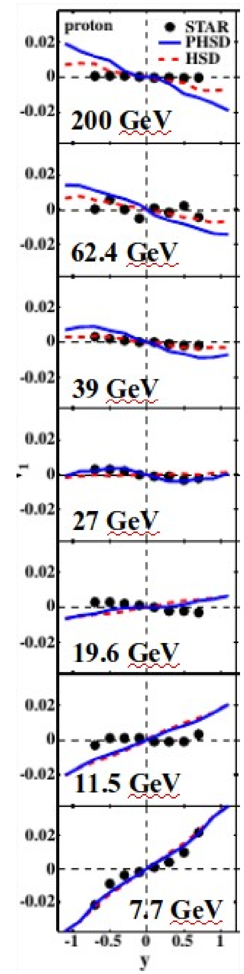
M.Isse, AO, N.Otuka, P.K.Sahu, Y.Nara, PRC72('05)064908 (There was a mistake...)

3FD



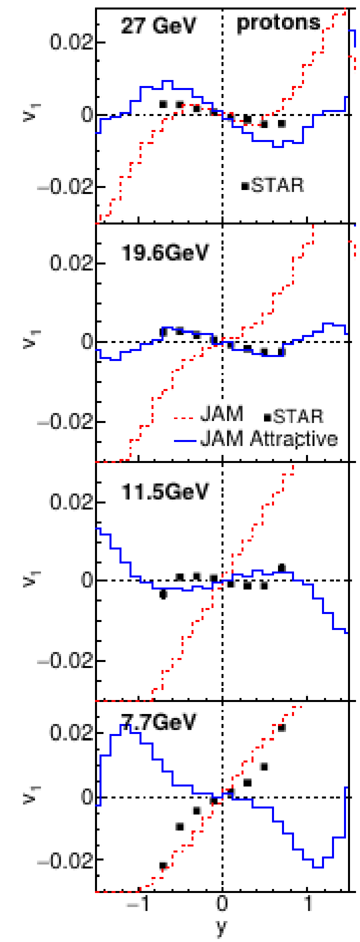
Y.B.Ivanov, A.A.Soldatov, PRC91('15) 024915

HSD/PHSD



V.P.Konchakovski, W.Cassing, Y.B.Ivanov, V. D. Toneev, PRC90('14)014903

JAM+Att.



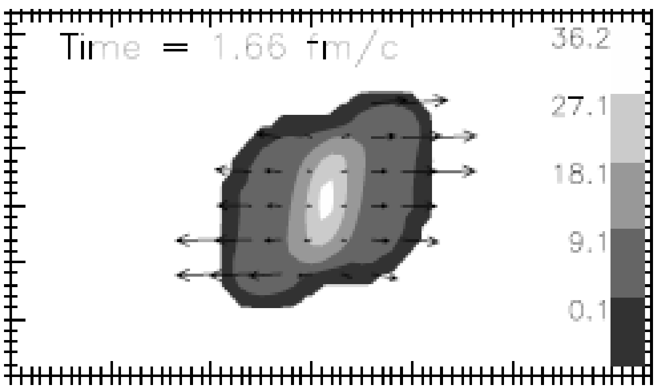
Y.Nara, H.Niemi, AO, H.Stoecker, PRC94 ('16)034906

What happens ?

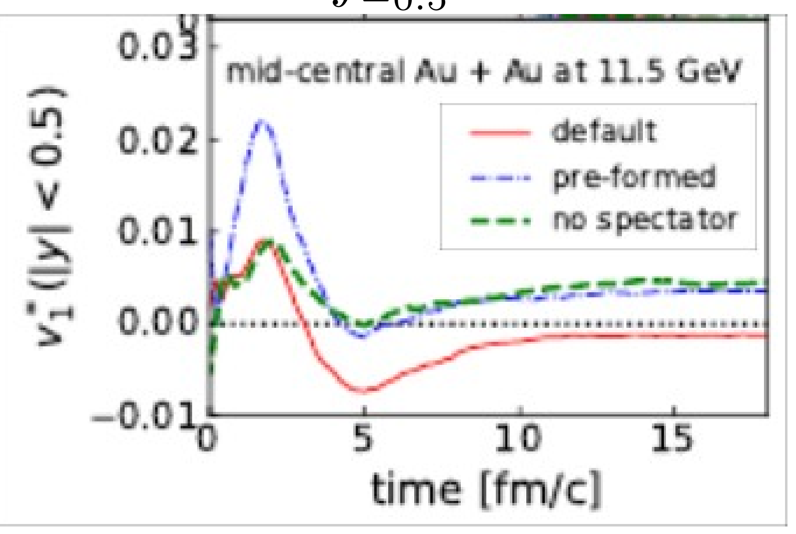
- **Origin of Positive & Negative Flow components**
  - **Repulsion during compression**
    → positive flow ($dv_1/dy > 0$)
  - **Emission from tilted matter during expansion**
    → negative flow ($dv_1/dy < 0$)
- **At the balance energy ($\sqrt{s_{NN}}$~ 10 GeV), either EOS softening or balance of compression/expansion contribution takes place.**

Nara+('16,'17,'18); Y. Nara, AO, arXIv:2109.07594

$$v_1^* = \int_{-0.5}^{0.5} dy v_1(y) \mathrm{sign}(y)$$



18 GeV, 3-fluid
Toneev et al. ('03)



Y. Nara, AO, arXiv:2109.07594

Relativistic QMD/Simplified (RQMD/S)

- **RQMD is developed based on constraint Hamiltonian dynamics**
  H. Sorge, H. Stoecker, W. Greiner, Ann. Phys. 192 (1989), 266.
  - **8N dof → 2N constraints → 6N (phase space)**
  - **Constraints = on-mass-shell constraints + time fixation**
- **RQMD/S uses simplified time-fixation**
  Tomoyuki Maruyama, et al. Prog. Theor. Phys. 96(1996),263.
  - **Single particle energy (on-mass-shell constraint) and EOM**

$$p_i^0 = \sqrt{\boldsymbol{p}_i^2 + m_i^2 + 2m_i V_i} \to \frac{d\boldsymbol{r}_i}{dt} = \frac{\boldsymbol{p}_i}{p_i^0} + \sum_j \frac{m_j}{p_j^0}\frac{\partial V_j}{\partial \boldsymbol{p}_i}\,,\quad \frac{d\boldsymbol{p}_i}{dt} = -\sum_j \frac{m_j}{p_j^0}\frac{\partial V_j}{\partial \boldsymbol{r}_i}\,,$$

  - **Potential V_i is Lorentz scalar and becomes weaker at high E.**
- **Stronger potential effect is necessary → Vector-type potential**
  Y.Nara, T.Maruyama, H.Stoecker, PRC102('20)024913; Y.Nara,AO,arXiv:2109.07594

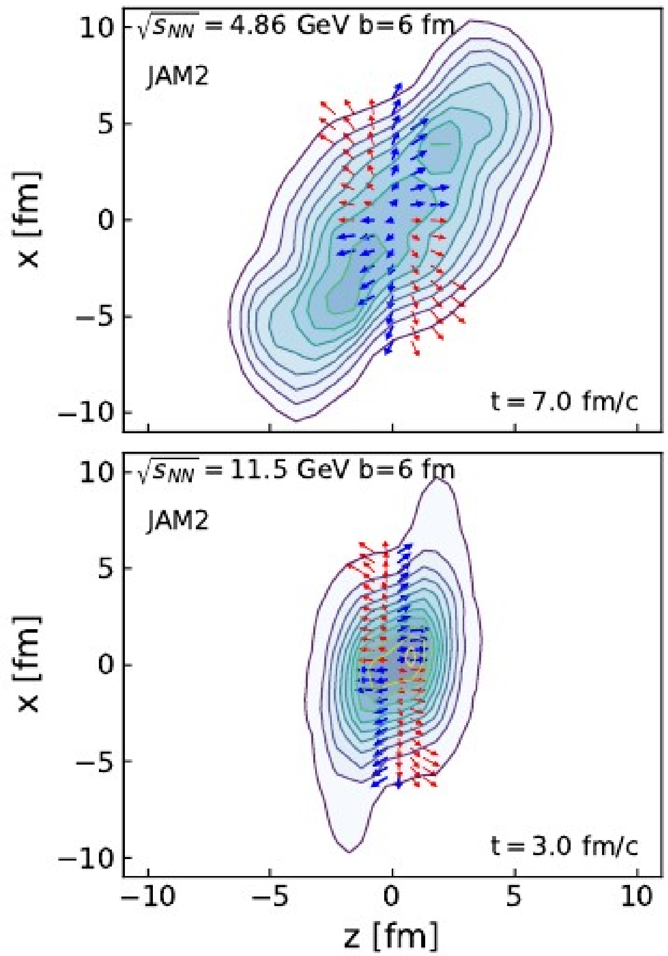
$$p_i^0 = \sqrt{\boldsymbol{p}_i^{*2} + m_i^2} + V_i^0 \to \frac{d\boldsymbol{r}_i}{dt} = \frac{\boldsymbol{p}_i^*}{p_i^{*0}} + \sum_j v_j^{*\mu}\frac{\partial V_{j\mu}}{\partial \boldsymbol{p}_i}\,,\quad \frac{d\boldsymbol{p}_i}{dt} = -\sum_j v_j^{*\mu}\frac{\partial V_{j\mu}}{\partial \boldsymbol{r}_i}$$

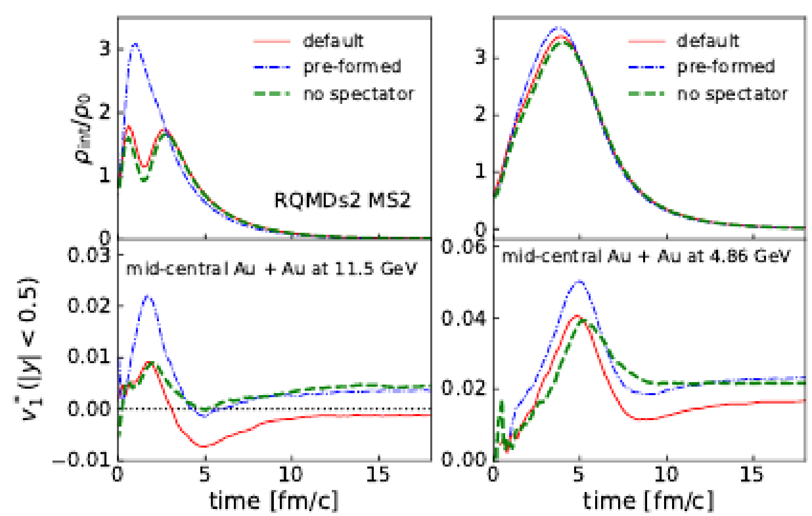
$$(p_j^{*\mu} = p_j^\mu - V_j^\mu\,,\ v_j^{*\mu} = p_j^{*\mu}/p_j^{*0})$$

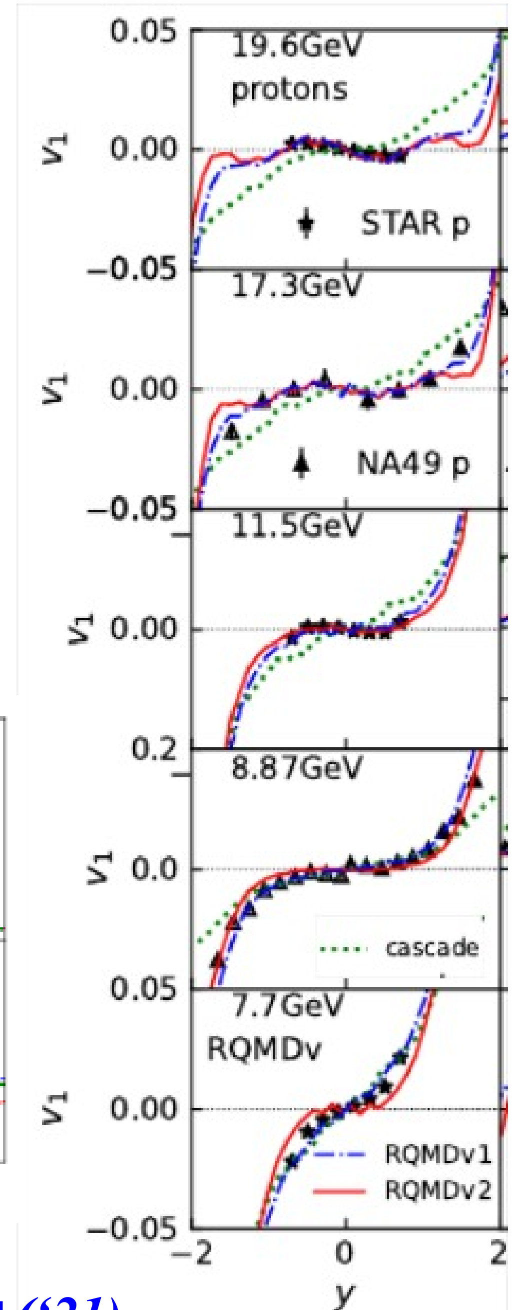
  - **Potential effect remains at high energy.**

JAM2 + RQMDv

- **Beam energy dependence of dv_1/dy can be explained in JAM2+RQMDv.**
- **The negative flow in the expansion stage becomes dominant at higher beam energies.**







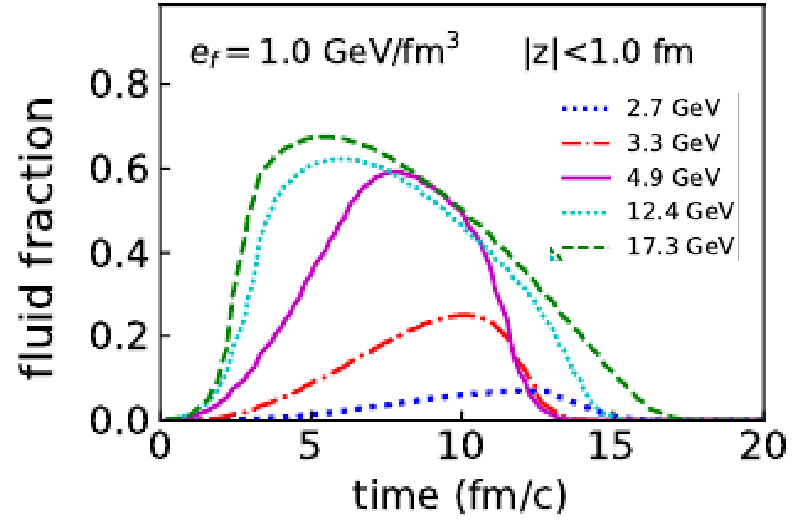
Nara+('21)

Onset energy, revisited

- **EOS softening at around $\sqrt{s_{NN}}$=10 GeV is not necessary.**
- **Even though, fluid-particle integrated model predicts formation of fluid component in 50% or more in volume at $\sqrt{s_{NN}}$=5-20 GeV.**
- **A part of fluid component (ε > 1 GeV/fm^3) should be QGP. (What is the signal of partial QGP formation?)**
- **Detailed analysis of dynamics using dynamical initialization and fluid-particle integrated evolution would be necessary.**

Kanakubo+(E.g. ATHIC); Akamatsu+.

Y.Akamatsu, M.Asakawa, T.Hirano, M.Kitazawa, K.Morita, K.Murase, Y.Nara, C.Nonaka, AO, PRC98('18)024909.

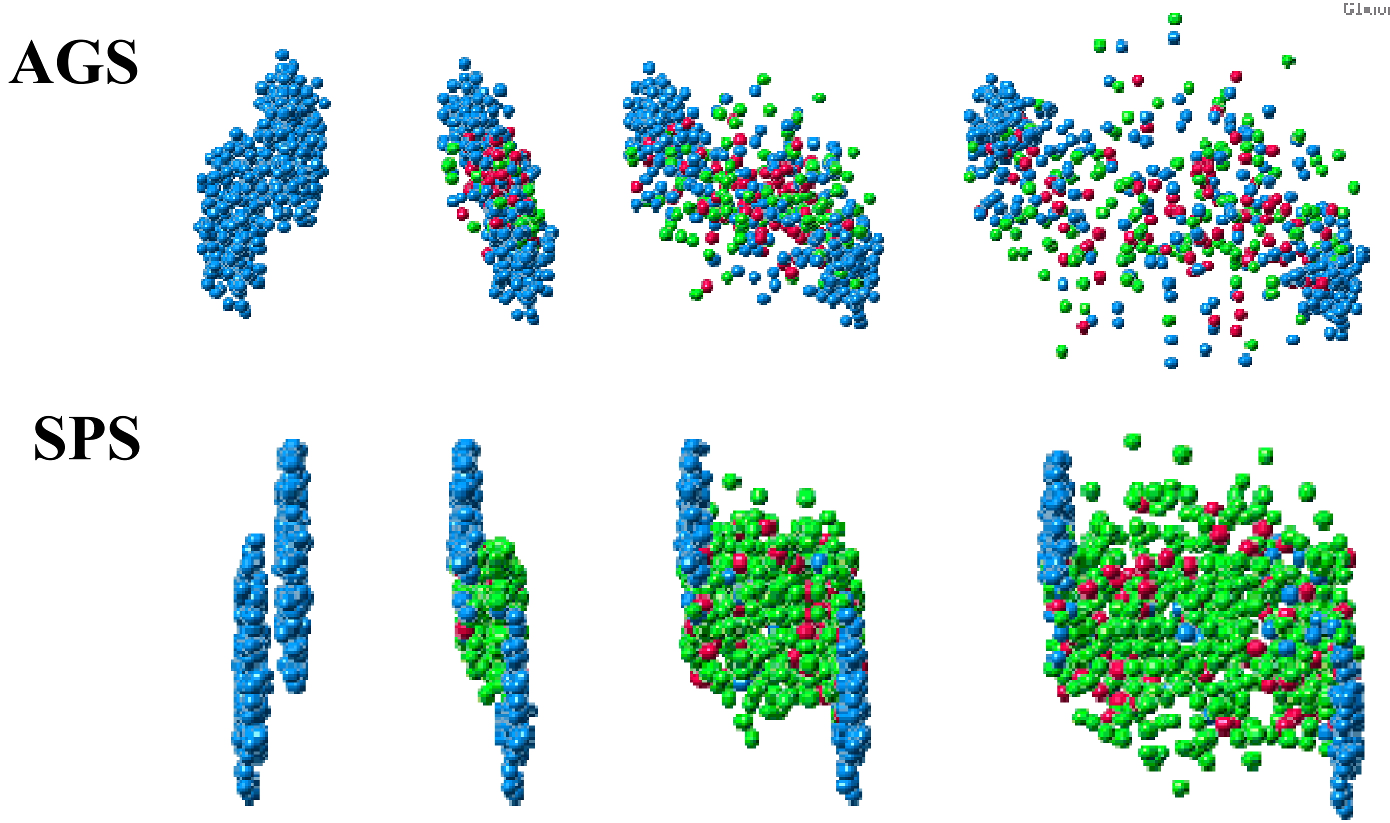


Nuclear Transport Models for Heavy-Ion Collisions and Collective Flows

Heavy-Ion Collisions at Einc ~ (1-100) A GeV

Study of Hot and Dense Hadronic Matter

→ Particle Yield, Collective Dynamics (Flow), EOS, ...



JAMming on the Web, linked from http://www.jcprg.org/

Nuclear Mean Field

MF has on both of ρ and p-deps.

ρ dep.: (ρ_0, E/A) = (0.15 fm^{-3}, -16.3 MeV) is known
Stiffness is not known well

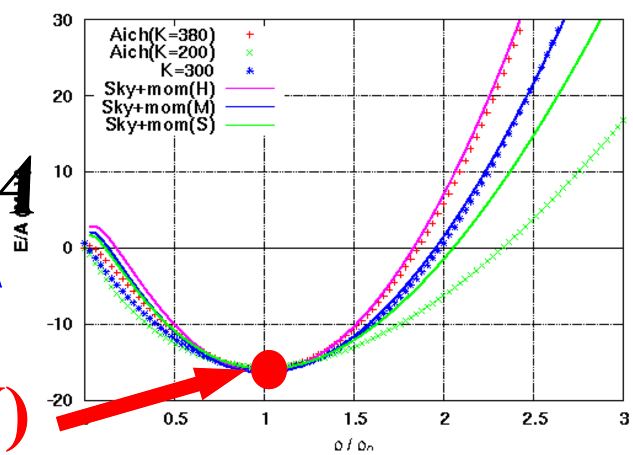
p dep.: Global potential up to E=1 GeV is known from pA scattering
$$U(\rho_0, E) = U(\rho_0, E=0) + 0.3\,E$$

Ab initio Approach; LQCD, GFMC, DBHF, G-matrix, ...
→ Not easy to handle, Not satisfactory for phen. purposes

Effective Interactions (or Energy Functionals):
Skyrme HF, RMF, ...

$U(E)=U(0)+0.3E$

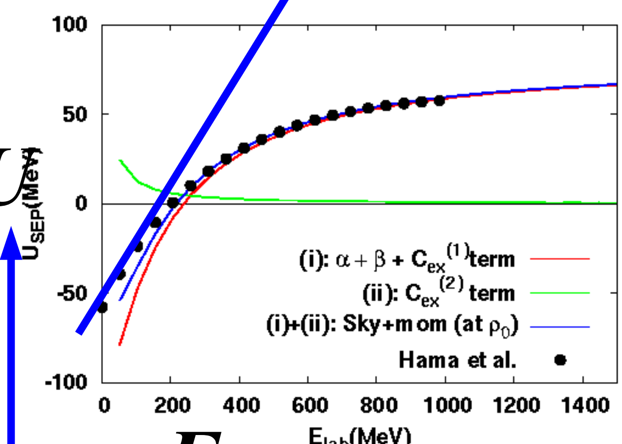
E/A



ρ

$(\rho_0 , E/A)$
$=(0.15\ \text{fm}^{-3} , -16.3\ \text{MeV})$

U



E_{inc}

HIC Transport Models: Major Four Origins

Nuclear Mean Field Dynamics

Basic Element of Low Energy Nuclear Physics,
and Critically Determines High Density EOS / Collective Flows

TDHF → Vlasov → BUU

NN two-body (residual) interaction

Main Source of Particle Production

Intranuclear Cascade Models

Partonic Interaction and String Decay

Main Source of high pT Particles at Collider Energies

JETSET + (previous) PYTHIA (Lund model) → (new) PYTHIA

Relativistic Hydrodynamics

Most Successful Picture at RHIC

TDHF and Vlasov Equation

Time-Dependent Mean Field Theory (e.g., TDHF) $i\hbar \frac{\partial \phi_i}{\partial t} = h\phi_i$

Density Matrix

$$\rho(r, r') = \sum_i^{Occ} \phi_i(r)\phi_i^*(r') \quad \rightarrow \quad \rho_W = f \text{ (phase space density)}$$

TDHF for Density Matrix

$$i\hbar \frac{\partial \rho}{\partial t} = [h, \rho] \quad \rightarrow \quad \frac{\partial f}{\partial t} = \{h_W, f\}_{P.B.} + O(\hbar^2)$$

Wigner Transformation and Wigner-Kirkwood Expansion
(Ref.: Ring-Schuck)

$$O_W(r, p) \equiv \int d^3 s \exp(-i p \cdot s/\hbar) < r + s/2 | O | r - s/2 >$$

$$(AB)_W = A_W \exp(i\hbar\Lambda) B_W \quad \Lambda \equiv \nabla'_r \cdot \nabla_p - \nabla'_p \cdot \nabla_r \quad (\nabla' \text{ acts on the left})$$

$$[A, B]_W = 2i A_W \sin(\hbar\Lambda/2) B_W = i\hbar\{A_W, B_W\}_{P.B.} + O(\hbar^3)$$

Test Particle Method

Vlasov Equation

$$\frac{\partial f}{\partial t} - \{h_W, f\}_{P.B.} = \frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = 0$$

Classical Hamiltonian

$$h_W(r,p) = \frac{p^2}{2m} + U(r,p)$$

Test Particle Method (C. Y. Wong, 1982)

$$f(r,p) = \frac{1}{N_0} \sum_i^{AN_0} \delta(r - r_i)\,\delta(p - p_i) \quad \rightarrow \quad \frac{dr_i}{dt} = \nabla_p h_w, \quad \frac{dp_i}{dt} = -\nabla_r h_w,$$

Mean Field Evolution can be simulated by Classical Test Particles
→ Opened a possibility to Simulate High Energy HIC including Two-Body Collisions in Cascade

BUU (Boltzmann-Uehling-Uhlenbeck) Equation

BUU Equation (Bertsch and Das Gupta, Phys. Rept. 160(88), 190)

$$\frac{\partial f}{\partial t} + v\cdot\nabla_r f - \nabla U\cdot\nabla_p f = I_{coll}[f]$$

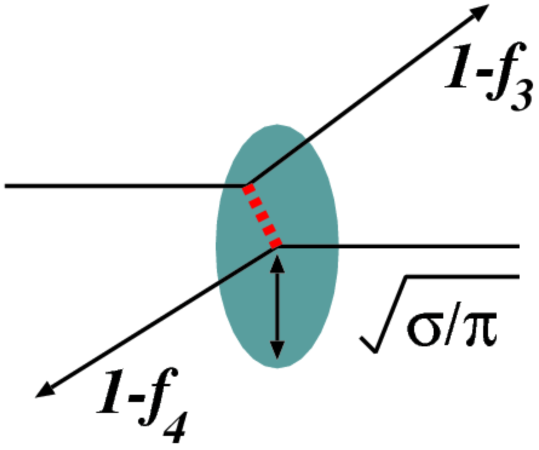
$$I_{coll}[f] = -\frac{1}{2}\int \frac{d^3 p_2\, d\Omega}{(2\pi\hbar)^3} v_{12} \frac{d\sigma}{d\Omega}$$
$$\times \quad [f f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f)(1-f_2)]$$

Incorporated Physics in BUU

Mean Field Evolution

(Incoherent) Two-Body Collisions

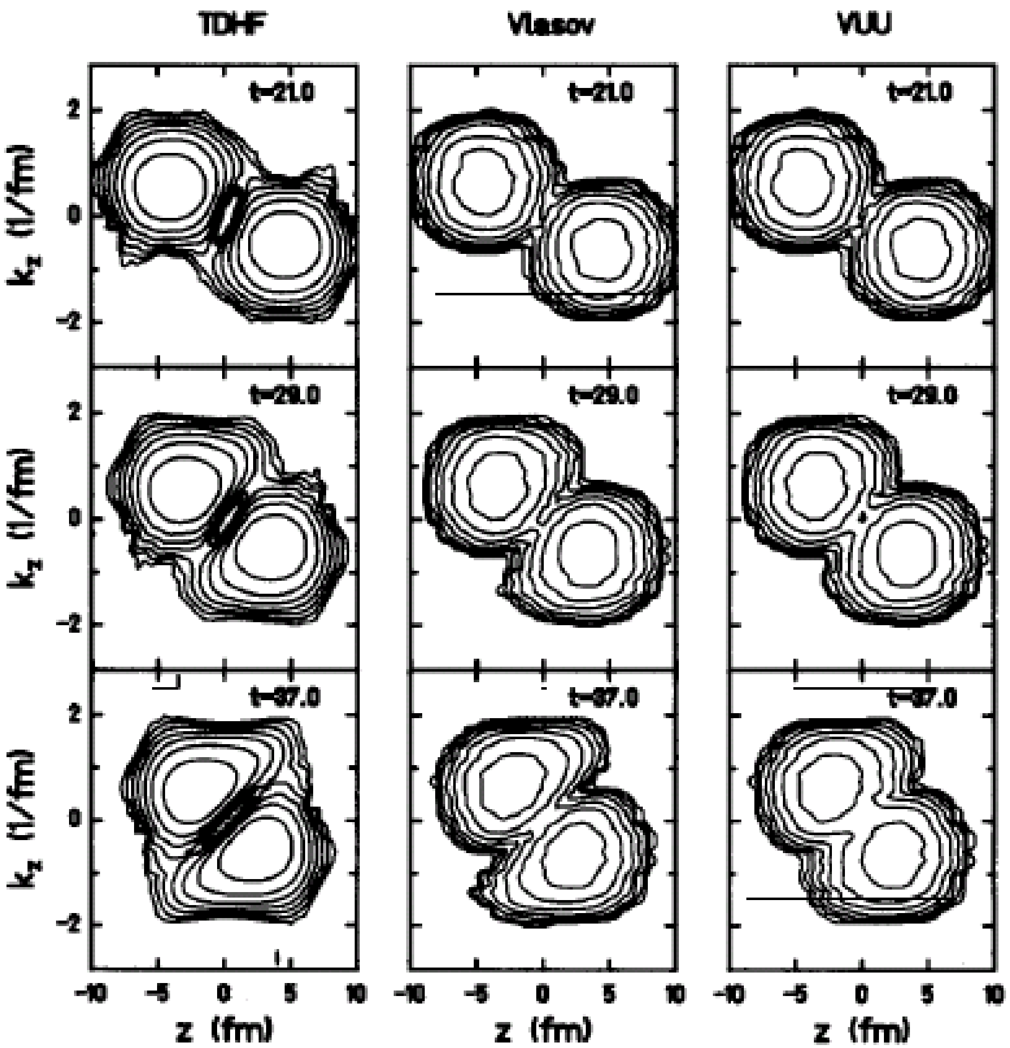
Pauli Blocking in Two-Body Collisions



O One-Body Observables (Particle Spectra, Collective Flow, ..)
X Event-by-Event Fluctuation (Fragment, Intermittency, ...)

Comarison of TDHF, Vlasov and BUU(VUU)

Ca+Ca, 40 A MeV
(Cassing-Metag-Mosel-Niita, Phys. Rep. 188 (1990) 363).



Relativistic Mean Field (II)

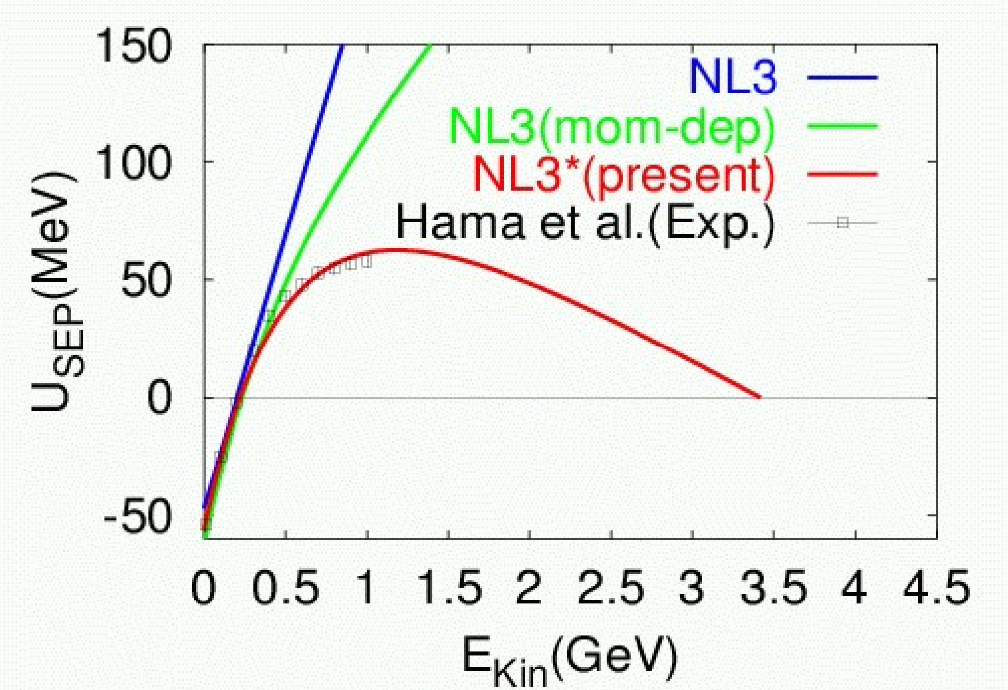
Dirac Equation $\left(i\gamma\partial-\gamma^0 U_v - M - U_s\right)\psi=0 \quad , \quad U_v = g_\omega \omega \quad , \quad U_s = -g_\sigma \sigma$

Schroedinger Equivalent Potential

$$\begin{pmatrix} E-U_v-M-U_s & i\sigma\cdot\nabla \\ -i\sigma\cdot\nabla & -E+U_v-M-U_s \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0$$

$$U_{sep} \sim U_S + \frac{E}{m} U_V = -g_\sigma \sigma + \frac{E}{m} g_\omega \omega$$

$$= -\frac{g_\sigma^2}{m_\sigma^2}\rho_s + \frac{E}{m}\frac{g_\omega^2}{m_\omega^2}\rho_B$$



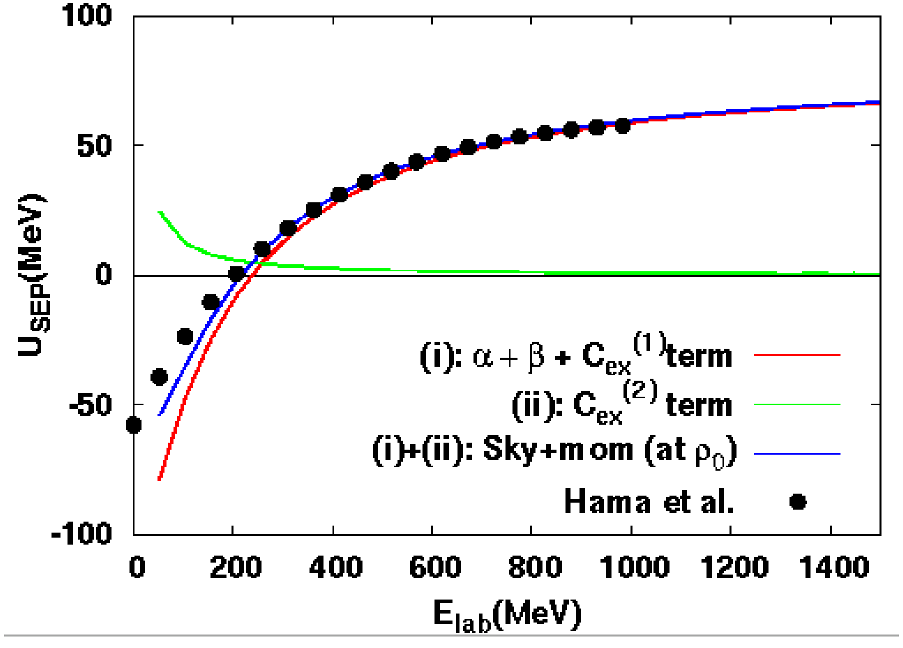
Saturation: -Scalar+Baryon Density
Linear Energy Dependence: Good at Low Energies,
Bad at High Energies (We need cut off !)

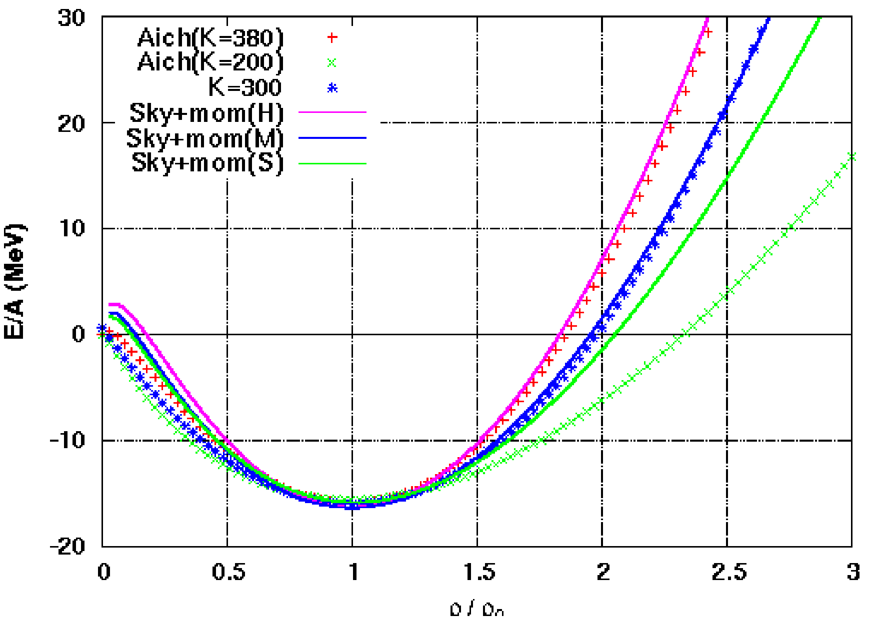
(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)

Skyrme type ρ-Dep. + Lorentzian p-Dep. Potential

$$V=\sum_i V_i=\int d^3r\left[\frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right)^2+\frac{\beta}{\gamma+1}\left(\frac{\rho}{\rho_0}\right)^{\gamma+1}\right]$$

$$+\sum_k\int d^3r\,d^3p\,d^3p'\,\frac{C_{ex}^{(k)}}{2\rho_0}\,\frac{f(r,p)\,f(r,p')}{1+(p-p')^2/\mu_k^2}$$

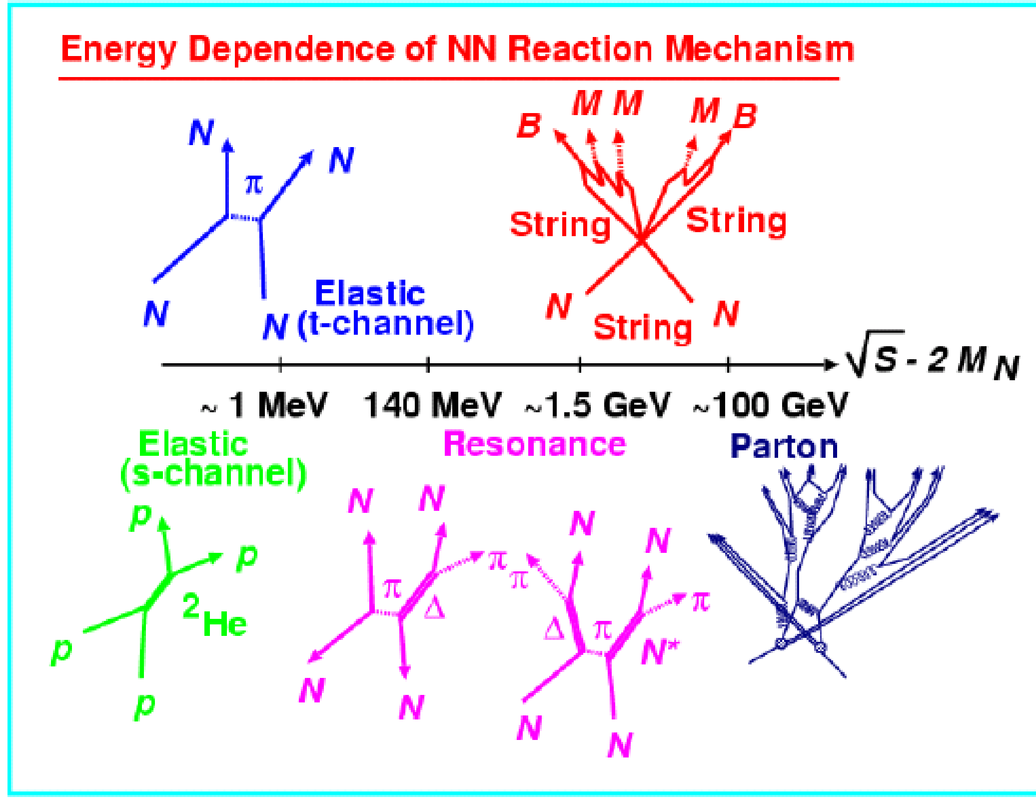




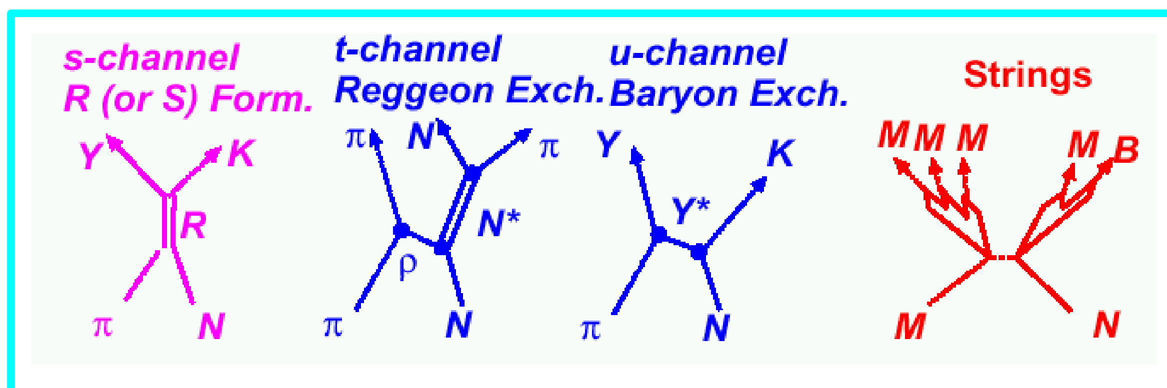
Isse, AO, Otuka, Sahu, Nara, Phys.Rev. C 72 (2005), 064908

Baryon-Baryon and Meson-Baryon Collisions

NN collision mechanism
Elastic
→ Resonance
→ String
→ Jet

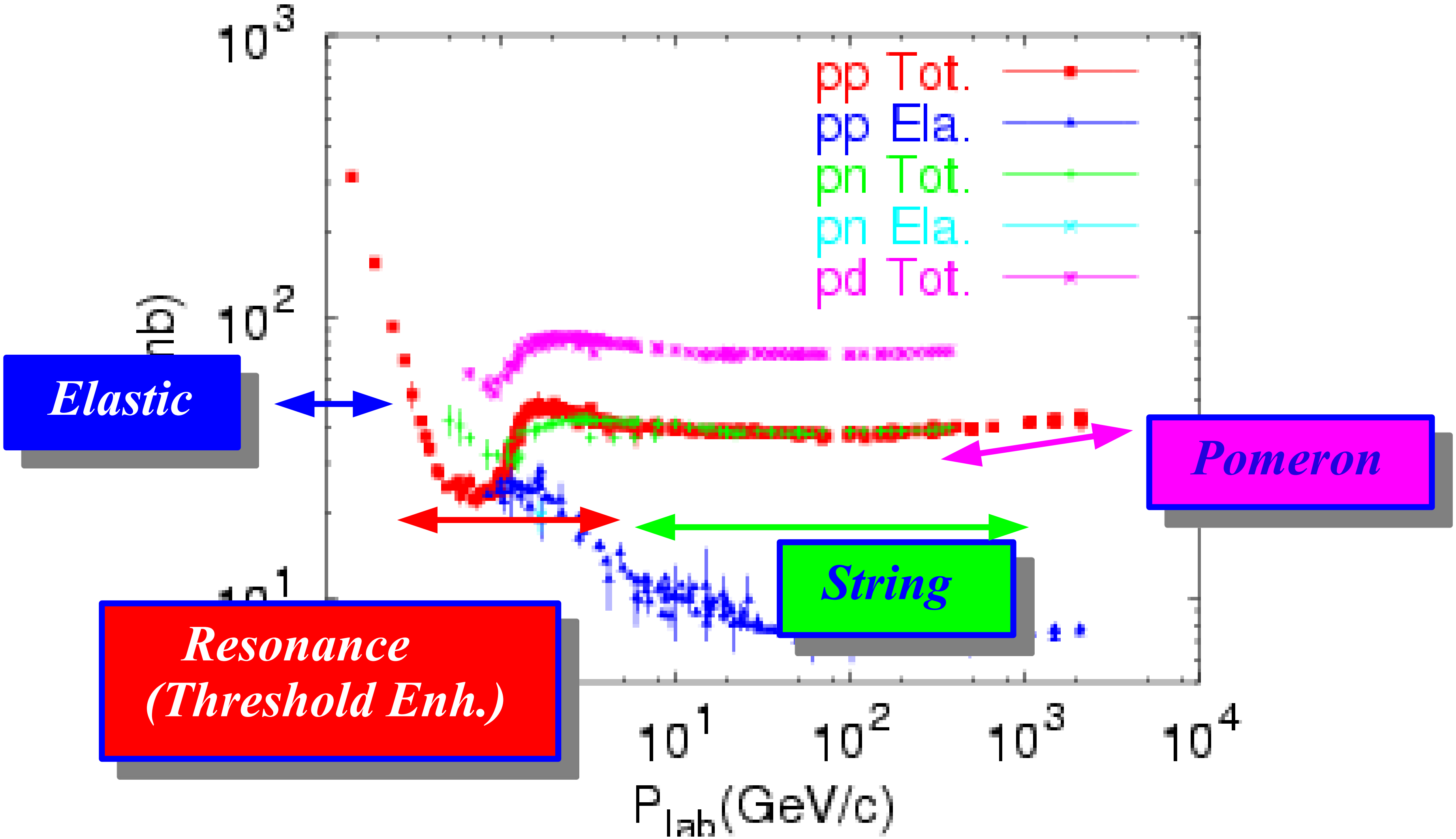


Meson-Nucleon Collision
→ s-channel Resonance
→ t-(u-) channel Res.
→ String formation

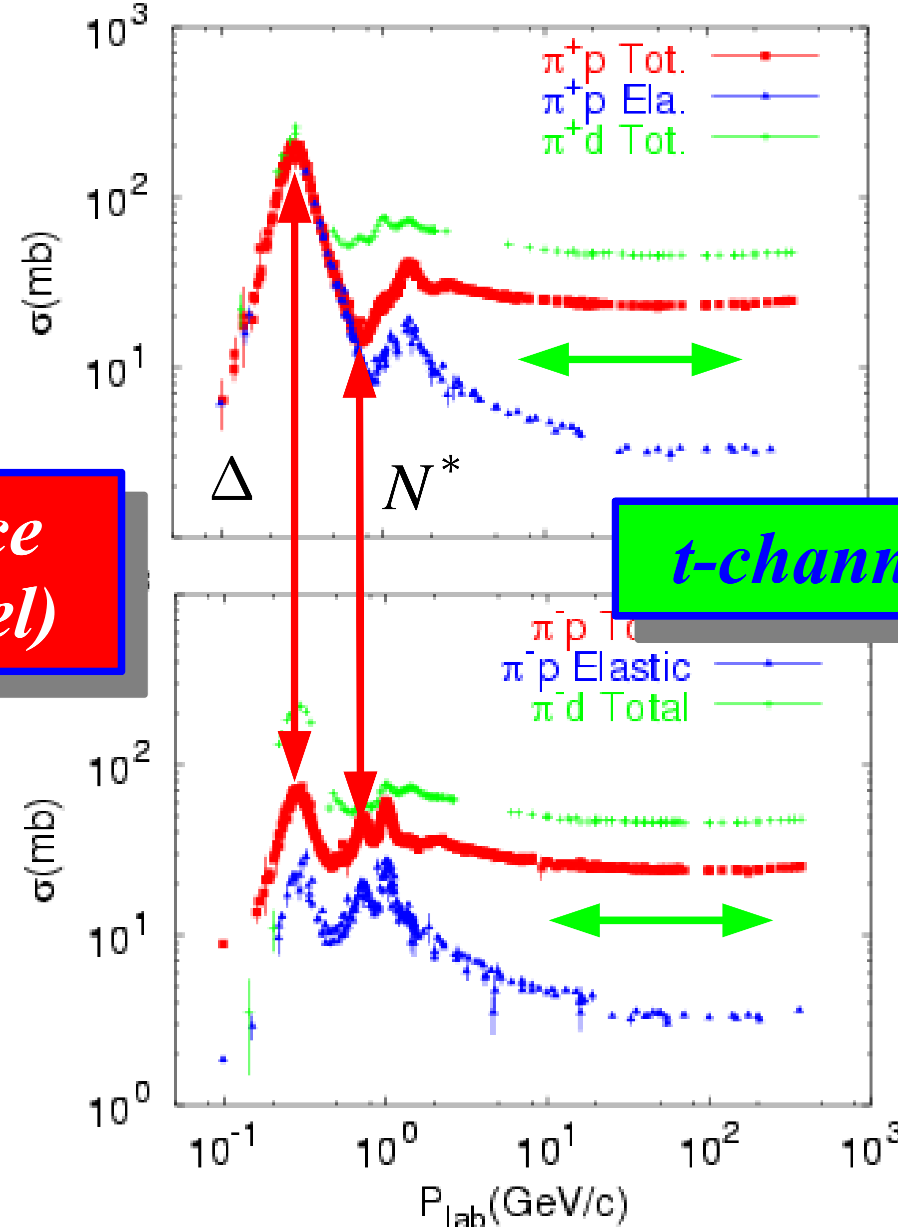


NN Cross Sections

From Particle Data Group



Meson-Baryon Cross Section



Resonance (s-channel)

t-channel and String

Reggeon Exchange

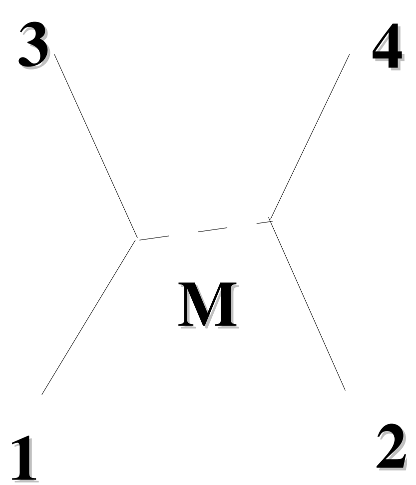
(Barger and Cline (Benjamin, 1969), H. Sorge, PRC (1995), RQMD2.1)

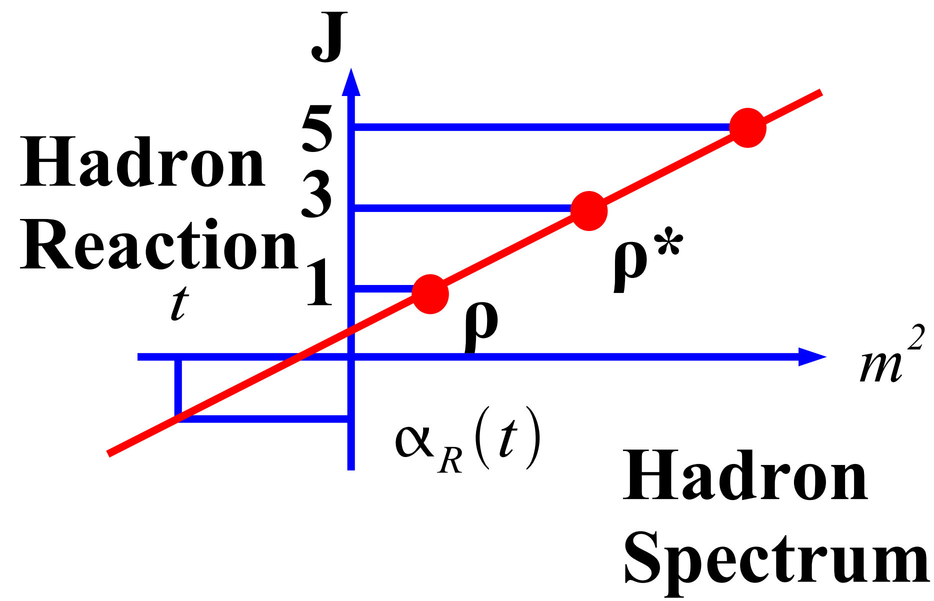
Regge Trajectory $J=\alpha_R(t)\sim\alpha_R(0)+\alpha'_R(0)\,t$

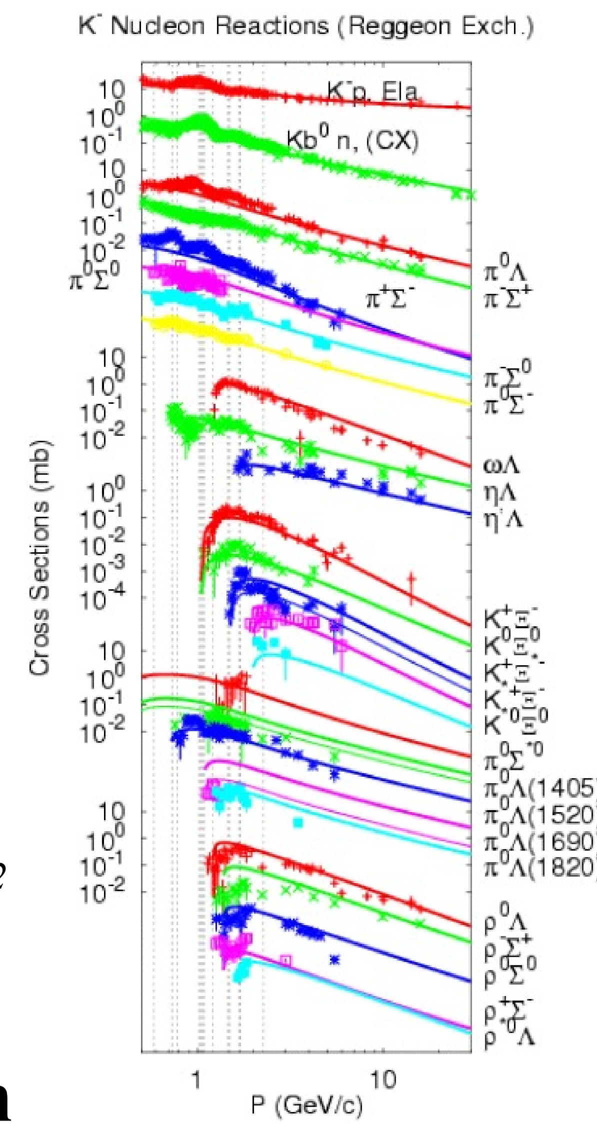
2 to 2 Cross Section

$$\frac{d\sigma}{d\Omega}=\frac{p_f}{64\pi s p_i}|M(s,t)|^2$$

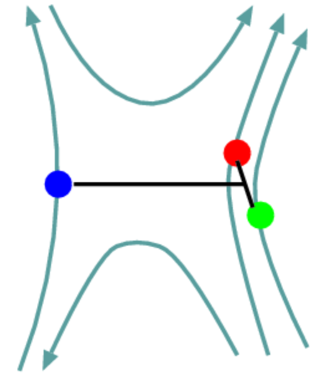
$$M(s,t)\sim\sum_R\frac{(p_i p_f)^J}{t-M_R}\sim F(t)\exp\left[\alpha_R(t)\log(s/s_0)\right]$$







String formation and decay

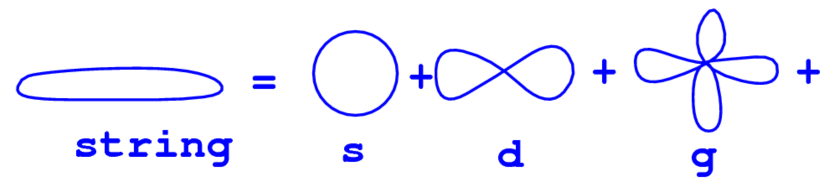


What does the regge trajectory suggest ?

→ Existence of (color- or hadron-)String !

$$M=2\int_0^R \frac{\kappa\, dr}{\sqrt{1-(r/R)^2}}=\pi\kappa R\ ,\quad J=2\int_0^R r\times\frac{\kappa\, dr}{\sqrt{1-(r/R)^2}}\frac{r}{R}=\frac{\pi\kappa R^2}{2}\pi$$

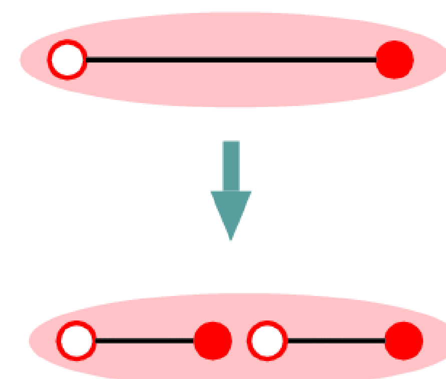
$$\rightarrow\ J=\frac{M^2}{2\pi\kappa}$$



String Tension

$$\frac{1}{2\pi\kappa}=\alpha'_R(0)\approx 0.9\, GeV^{-2}\ \rightarrow\ \kappa\approx 1\, GeV/fm$$

String decay



Extended String

→ Large E stored

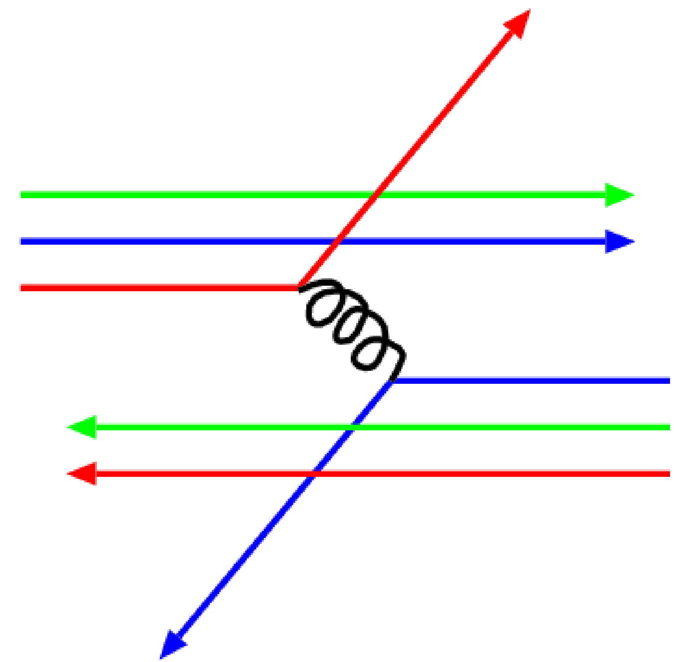
→ q qbar pair creation (Schwinger mech.)

String = Coherent superposition of hadron resonances with various J

Jet Production

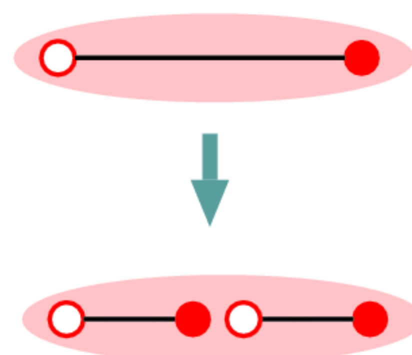
Elastic Scattering of Partons (mainly) with One Gluon Exch.

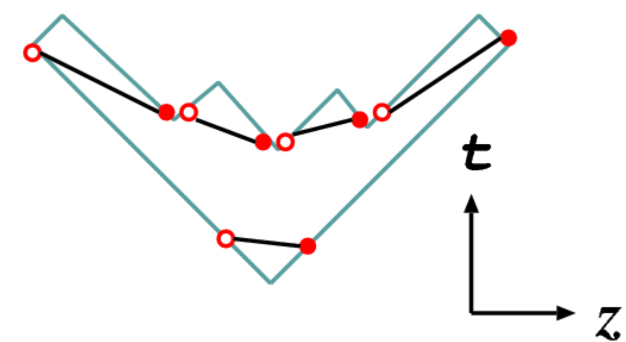
Color Exch. between Hadrons
→ Complex color flux starting from leading partons
→ many hadron production
→ Jet production



PYTHIA

Event Generator of High Energy Reactions
→ Jet production +String decay
for QCD processes





(T. Sjostrand et al., Comput. Phys. Commun. 135 (2001), 238.)

JAM (Jet AA Microscopic transport model)

Nara, Otuka, AO, Niita, Chiba, Phys. Rev. C61 (2000), 024901.

Hadron-String Cascade with Jet production

hh collision with Res. up to m < 2 GeV (3.5 GeV) for M (B)

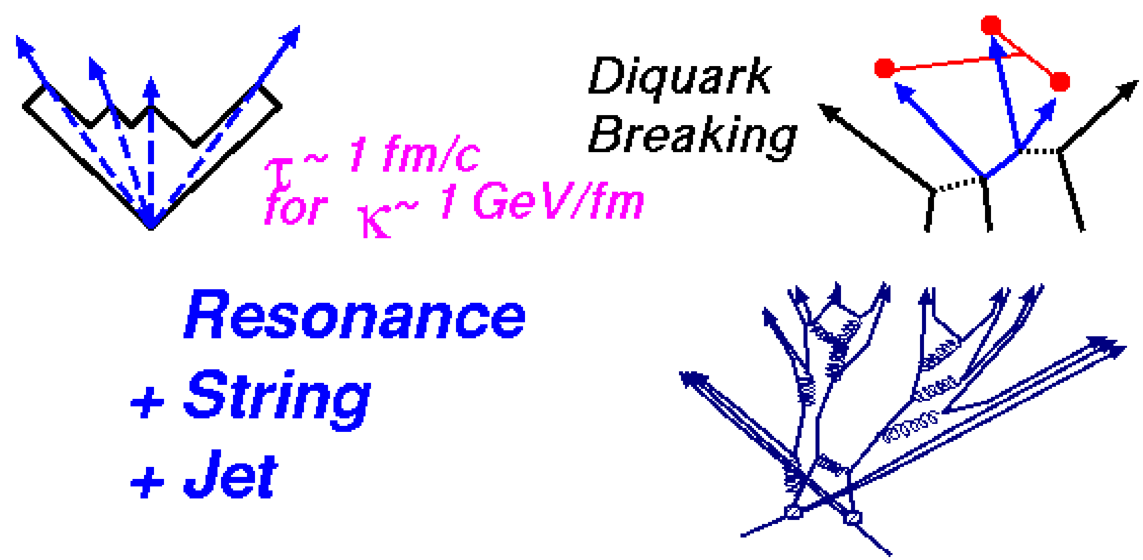
String excitation and decay

String-Hadron collisions are simulated by hh collisions in the formation time.

jet production is incl. using PYTHIA

Secondary partonic int.:
NOT incl.

Color transparency:
NOT taken care of



Collective Flow and EOS: Old Problem ?

1970's-1980's: First Suggestions and Measurement

Hydrodynamics suggested the Exsitence of Flow.

Strong Collective Flow suggests Hard EOS

1980's-1990's: Deeper Discussions in Wider E_{inc} Range

Momentum Dep. Pot. can generate Strong Flows.

Einc deps. implies the importance of Momentum Deps.

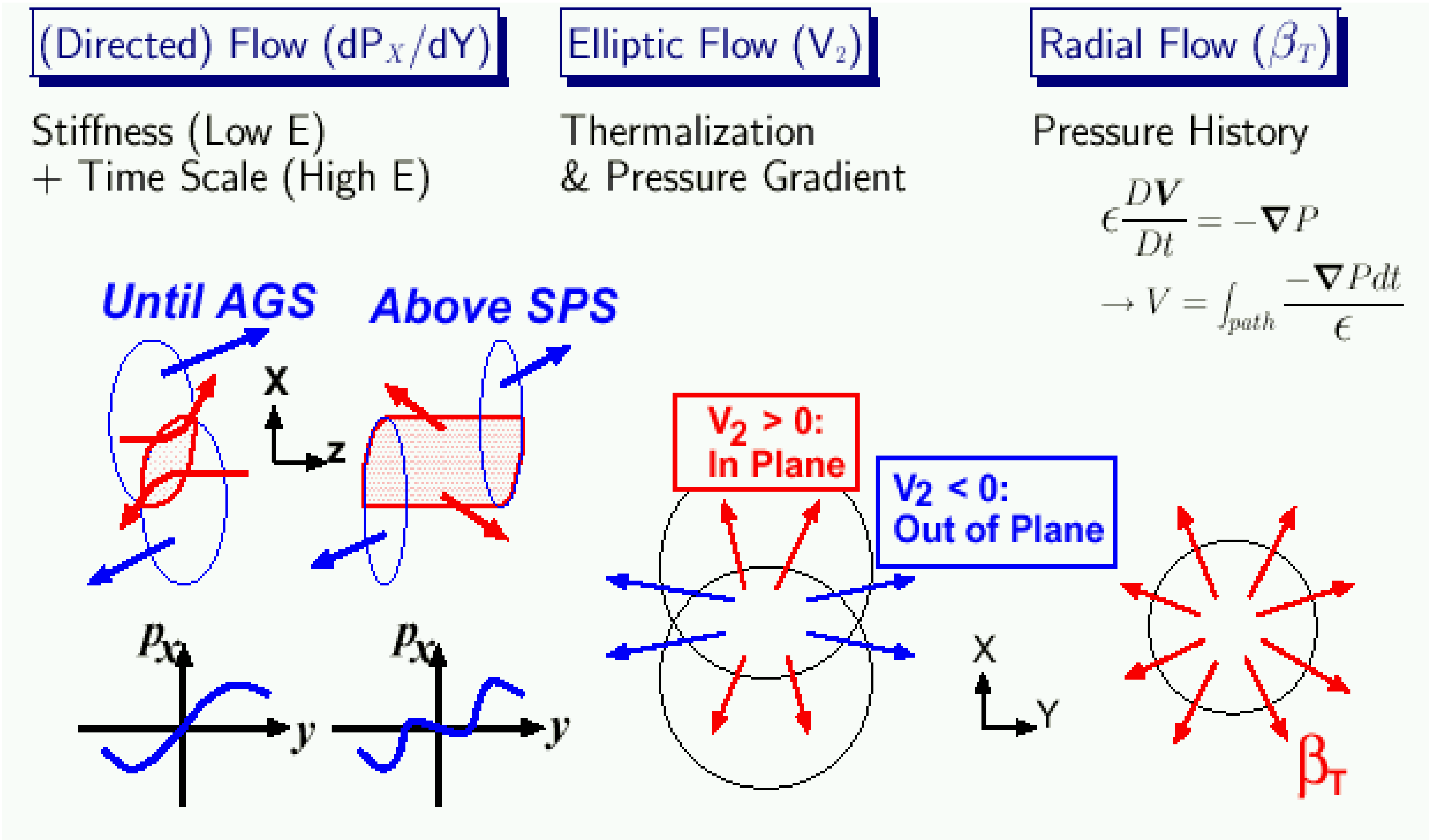
Flow Measurement up to AGS Energies.

2000's: Extention to SPS and RHIC Energies

EOS is determined with Mom. AND Density Dep. Pot. ?

Old but New (Continuing) Problem !

What is Collective Flow ?



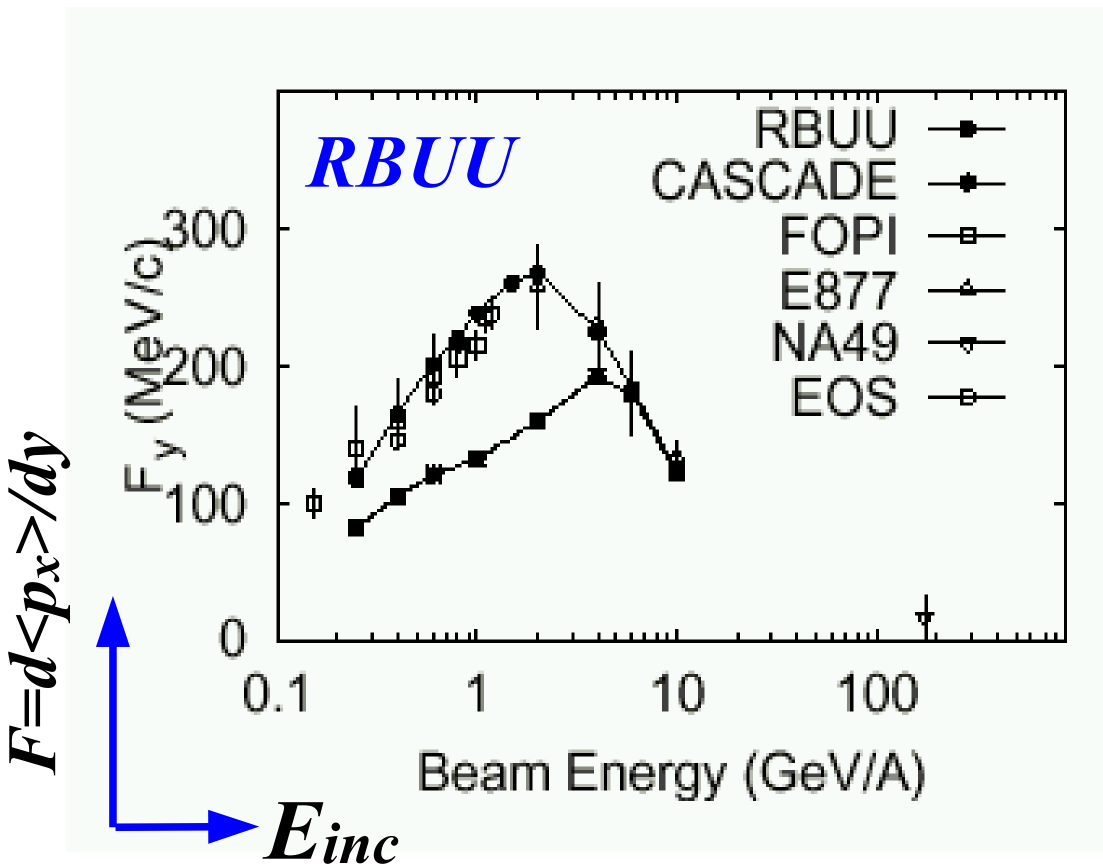
Side Flow at AGS Energies

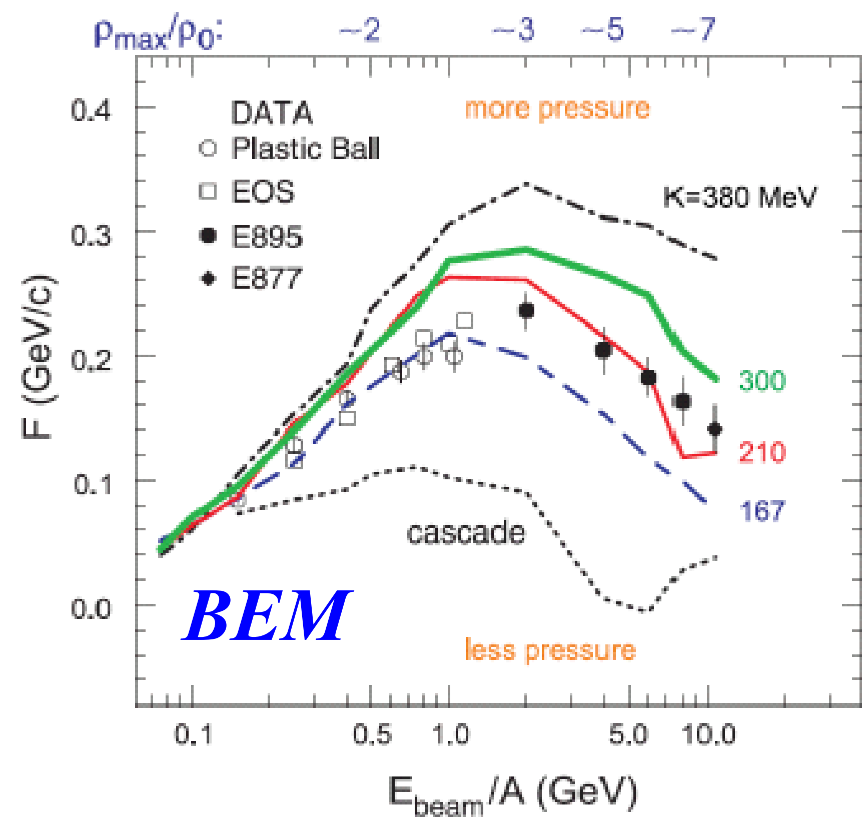
Relativistic BUU (RBUU) model: ***K ~ 300 MeV***

(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)

Boltzmann Equation Model (BEM): ***K=167~210 MeV***

(P. Danielewicz, R. Lacey, W.G. Lynch, Science 298(2002), 1592.)





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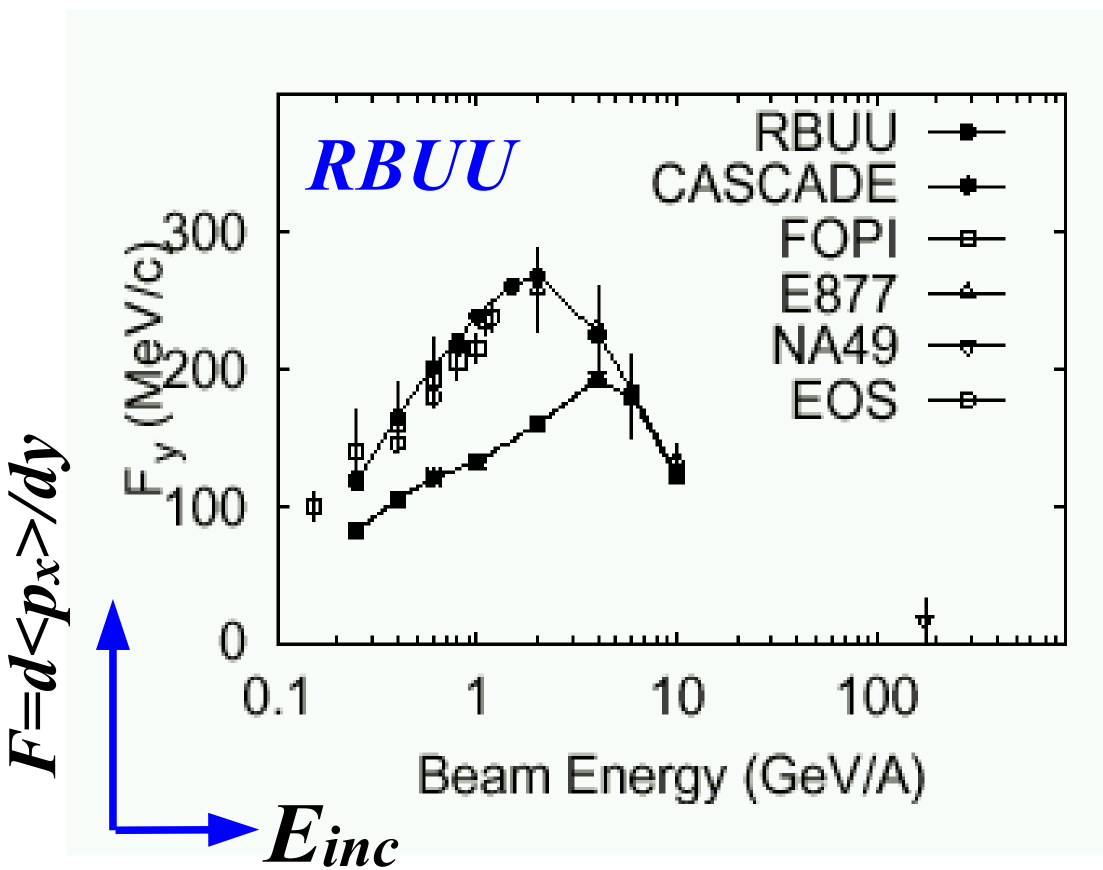
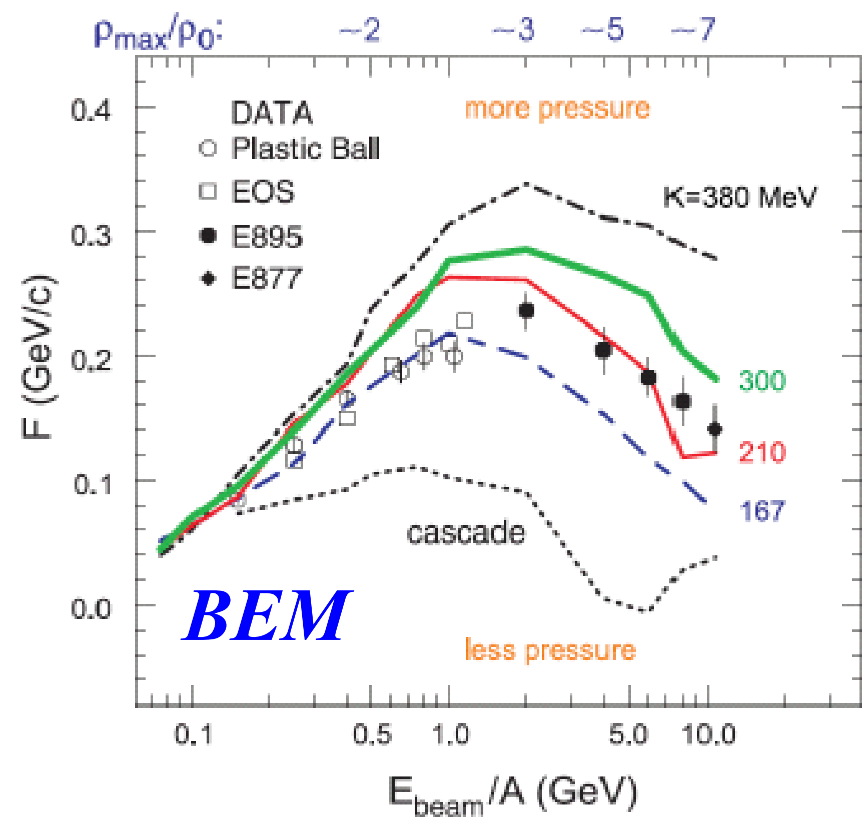




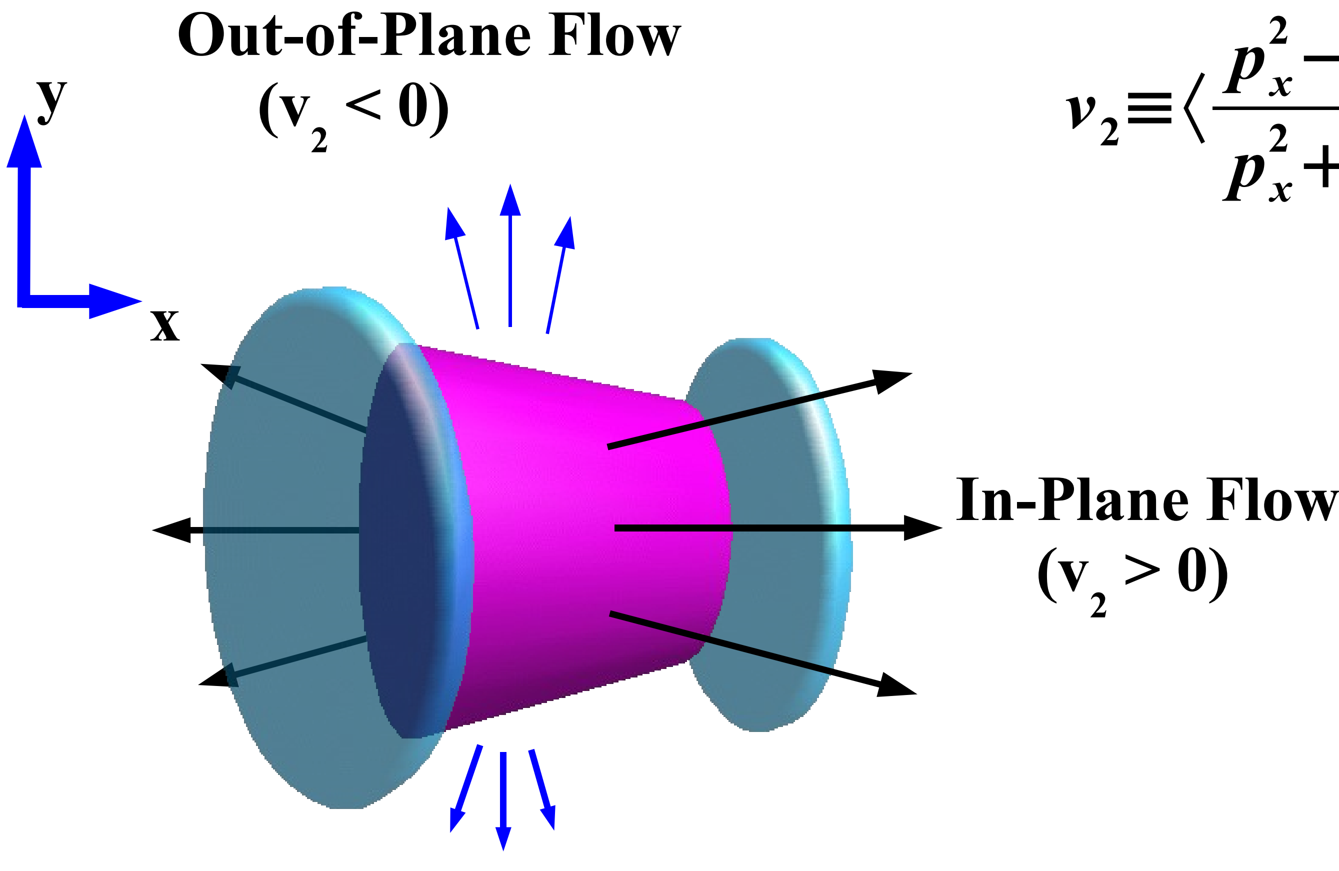
Elliptic Flow

What is Elliptic Flow ? → Anisotropy in P space

Hydrodynamical Picture

Sensitive to the Pressure Anisotropy in the Early Stage

Early Thermalization is Required for Large V2



$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos 2\phi \rangle$$

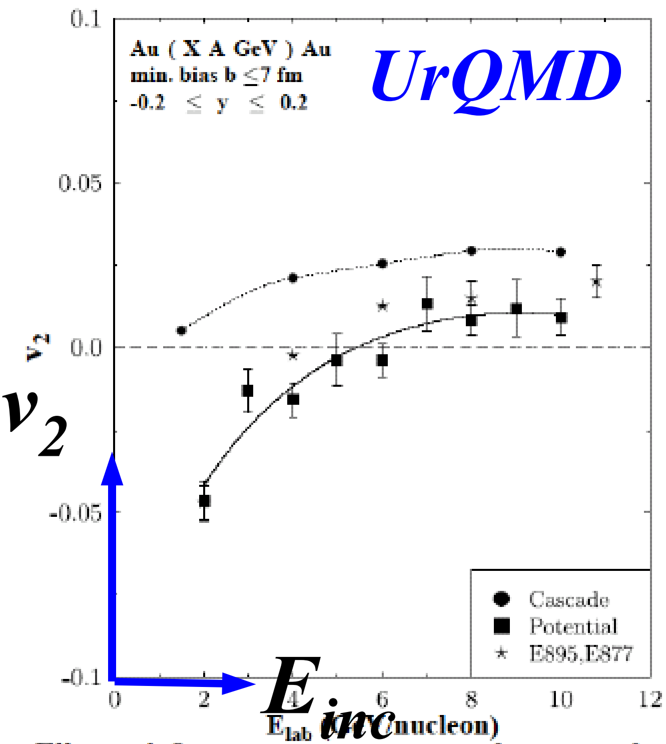
Elliptic Flow at AGS

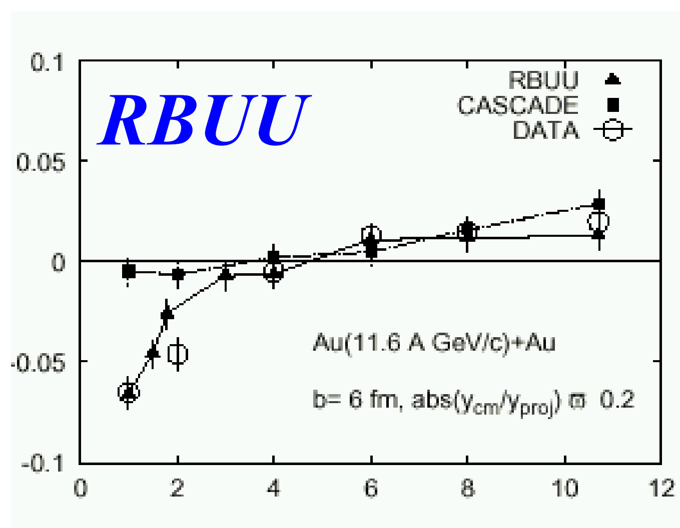
Strong Squeezing Effects at low E (2-4 A GeV)

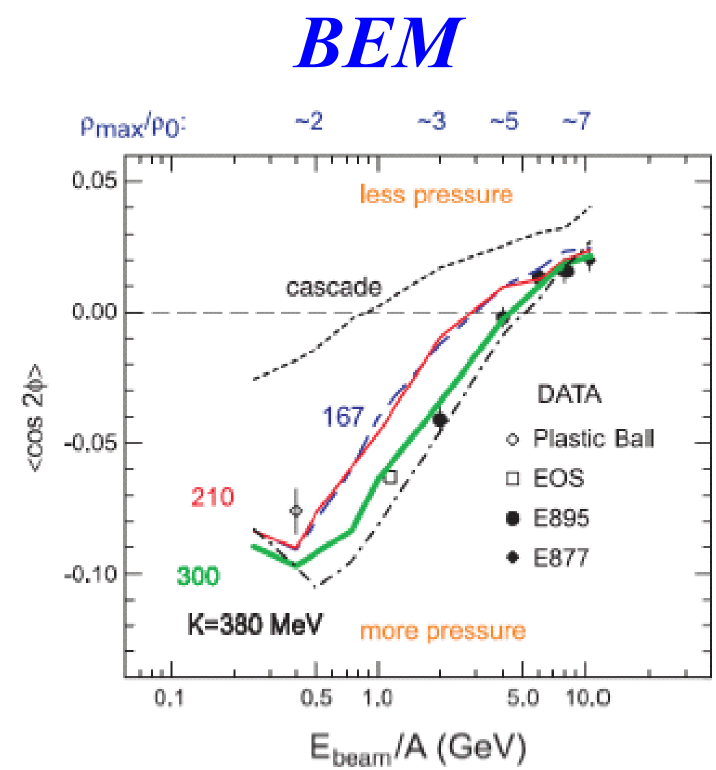
UrQMD: Hard EOS (S.Soff et al., nucl-th/9903061)

RBUU (Sahu-Cassing-Mosel-AO, 2000): K ~ 300 MeV

BEM(Danielewicz2002): K = 167 → 300 MeV







Elliptic Flow from AGS to SPS

JAM-MF with p dep. MF explains proton v2 at 1-158 A GeV

- **v2 is not very sensitive to K (incompressibility)**
- **Data lies between MS(B) and MS(N)**