# Femtoscopy and scattering of Λ-α with Λ potential from chiral EFT

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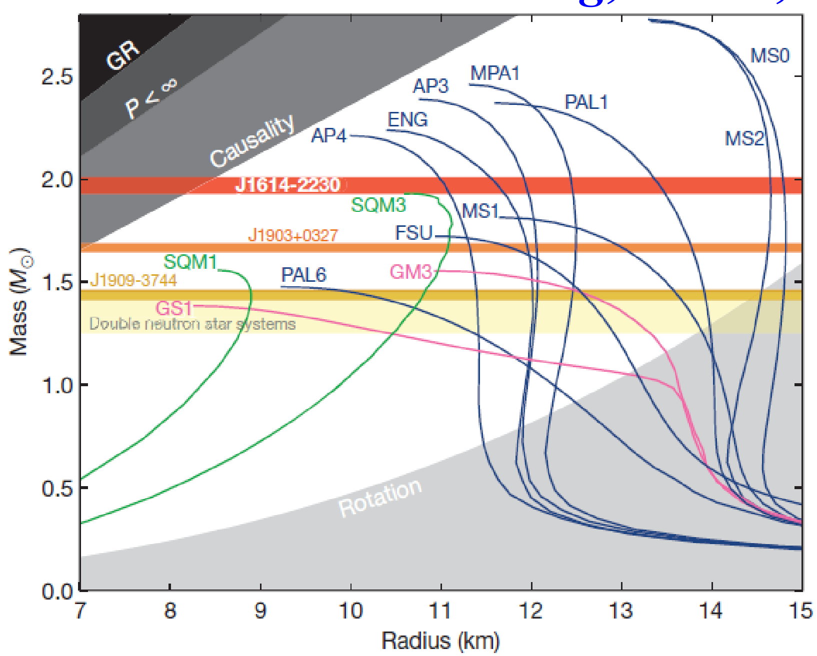
- Introduction – hyperon puzzle and Λ potential at high densities
- Λ-α potential from chiral effective field theory
- Λ-α correlation function
- Summary

**Work in progress**

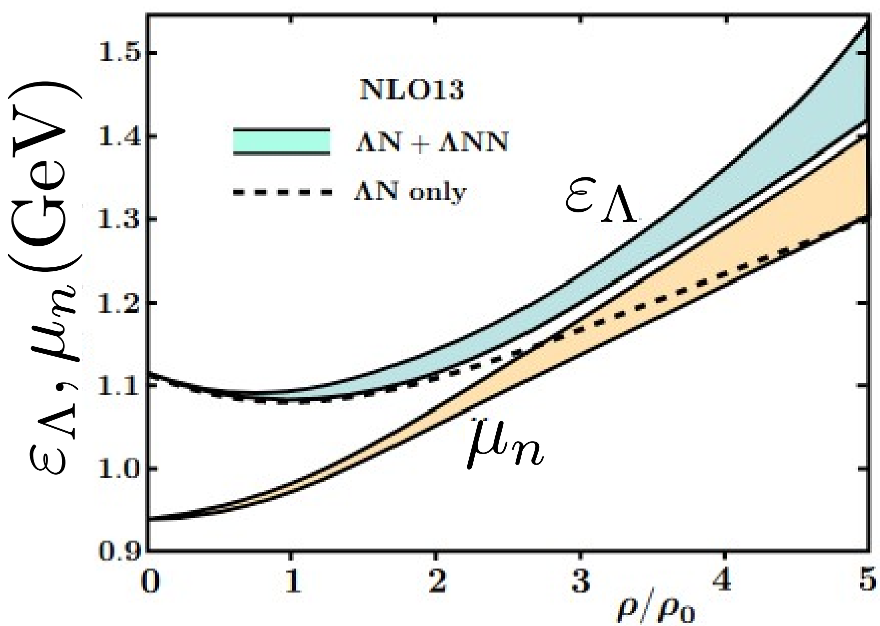
# Hyperon puzzle

- Hyperonic matter EOS with empirical Hyperon-Nucleon (YN) interactions cannot support 2 solar mass neutron stars
- Proposed solutions
  - YN interaction details ? YNN 3-body int. ? Many-body theories ? Gradual transition to quark matter ? Modified gravity ?
  - A plausible answer = repulsive Λ potential at high densities caused by YNN 3-body int.

*Chiral EFT: Gerstung, Kaiser, Weise (2001.10563), Kohno (1802.05388)*



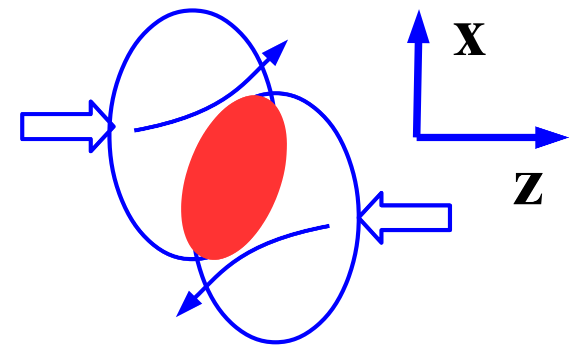

*Demorest et al., Nature 467 (2010) 1081 (Oct.28, 2010).*



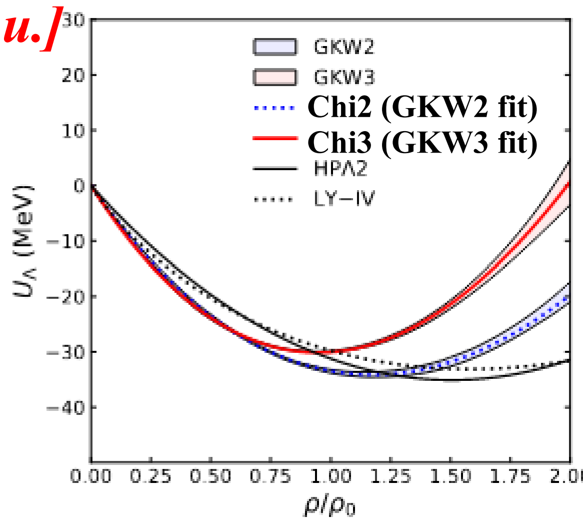

*Gerstung+('20)*

# Verification of repulsive $U_\Lambda$ at high densities

- **Directed flow of Λ in heavy-ion collisions**
  *Y. Nara, A. Jinno, K. Murase, AO (2208.01297)*
  - **$U_\Lambda$ from chiral EFT w/ 3-body force (Chi3) reasonably explains the directed flow slope.**
  - **Other softer potentials at high densities also explains the data (other effects are also important).**




- **Skyrme Hartree-Fock calculation using Chiral EFT $U_\Lambda$ w/ 3BF**
  *A. Jinno, K. Murase, Y. Nara, AO, in prep. [Talk by Jinno, Thu.]*
  - **Explains the Λ binding energies of hypernuclei comparably well to empirical SHF potentials.**
    *D. E. Lanskoy, Y. Yamamoto, PRC55, 2330 (1997);*
    *N. Guleria, S. K. Dhiman, R. Shyam, NPA886, 71 (2012).*

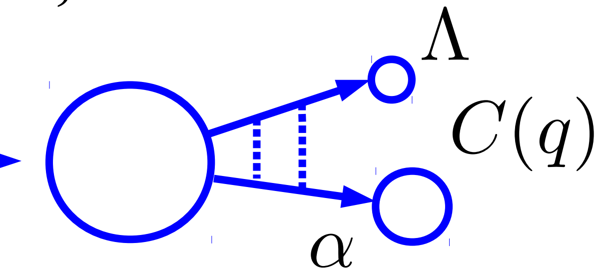



*Can we distinguish the density dep. of $U_\Lambda$?*
*→ How about Λ-nucleus femtoscopy or scattering ?*

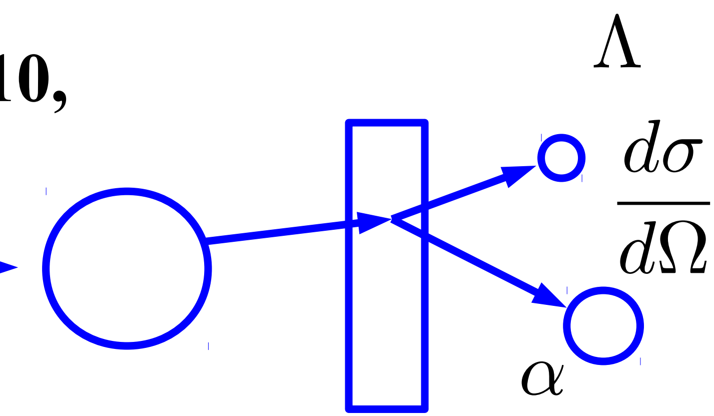
# Λ-α femtoscopy or scattering in HEFex

**High-p, π20, K10, or J-PARC-HI**

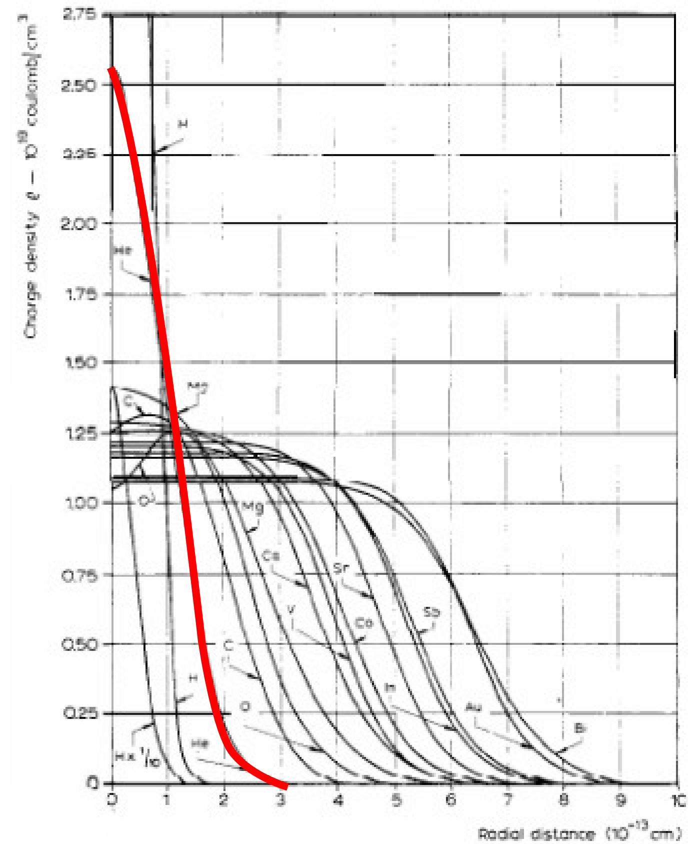
**HypTPC Catch**




**High-p, π20, K10, or J-PARC-HI**




**He target**




**Central density of α ~ $2\rho_0$**

*Hofstadter, http://www.nobelprize.org*

# Λ-α potential

# Λ potential ($U_\Lambda$)

- **Skyrme Hartree-Fock equation**

$$\left[-\boldsymbol{\nabla}\cdot\left(\frac{\hbar^2}{2m_B^*(\boldsymbol{r})}\right)\boldsymbol{\nabla}+U_B^{\rm local}(\boldsymbol{r})-i\boldsymbol{W}_B(\boldsymbol{r})\cdot(\boldsymbol{\nabla}\times\boldsymbol{\sigma})\right]\phi_i^B(\boldsymbol{r})=\varepsilon_i\ \phi_i^B(\boldsymbol{r})\,,$$

$$U_\Lambda^{\rm local}=a_1^\Lambda\rho_N+a_2^\Lambda\tau_N-a_3^\Lambda\triangle\rho_N+a_4^\Lambda\rho_N^{4/3}+a_5^\Lambda\rho_N^{5/3}\,,$$

$$\frac{\hbar^2}{2m_\Lambda^*}=\frac{\hbar^2}{2m_\Lambda}+a_2^\Lambda\rho_N\,,\ \rho_N=\sum_{i\in N}|\phi_i|^2\,,\ \tau_N=\sum_{i\in N}|\boldsymbol{\nabla}\phi_i|^2$$

- **Λ-α potential**

$$\left[-\frac{\hbar^2\boldsymbol{\nabla}^2}{2\mu}+U(\boldsymbol{r})\right]\phi(\boldsymbol{r})=\varepsilon_\Lambda\ \phi(\boldsymbol{r})\,,$$

$$U=a_1^\Lambda\rho_N+a_2^\Lambda\tau_N-a_3^\Lambda\triangle\rho_N+a_4^\Lambda\rho_N^{4/3}+a_5^\Lambda\rho_N^{5/3}-a_2^\Lambda\boldsymbol{\nabla}\cdot\rho_N\boldsymbol{\nabla}$$

  - **Nucleon densities are measured from the center of mass of α**
  - **μ = reduced mass of Λ-α**

# Wave function of α particle

- **A simple gaussian wave function of α**

$$\psi = \prod_{i=1}^{A} \phi(\boldsymbol{r}_i)\,,\ \ \phi(\boldsymbol{r}) = \left(\frac{2\nu}{\pi}\right)^{3/4} \exp[-\nu \boldsymbol{r}^2]\,.$$

  - **Naive estimate of densities**

$$\rho(\boldsymbol{r}) = A\left(\frac{2\nu}{\pi}\right)^{3/2} \exp[-2\nu \boldsymbol{r}^2]\ (A=4)\,,$$

$$\tau(\boldsymbol{r}) = \sum_i |\boldsymbol{\nabla}\phi_i|^2 = 4\nu^2\boldsymbol{r}^2\rho(\boldsymbol{r})\,,\ \triangle\rho(\boldsymbol{r}) = \frac{1}{r}\frac{\partial^2}{\partial^2 r} r\rho(r) = -4\nu(3-4\nu r^2)\rho(\boldsymbol{r})$$

- **Densities measured from the center-of-mass**

$$\rho(\boldsymbol{r}) = \int d\boldsymbol{r}_1 \cdots d\boldsymbol{r}_A |\psi(\boldsymbol{r}_1,\cdots,\boldsymbol{r}_A)|^2 \sum_i \delta(\boldsymbol{r}_i - \boldsymbol{r}_G - \boldsymbol{r})$$

$$= A\left(\frac{2\nu_c}{\pi}\right)^{3/2} \exp[-2\nu_c \boldsymbol{r}^2]\ [\nu_c = \nu/(1-1/A)]\,,$$

$$\triangle\rho(\boldsymbol{r}) = -4\nu_c(3-4\nu_c r^2)\rho(\boldsymbol{r})\,,\ \tau(\boldsymbol{r}) = \rho(\boldsymbol{r})\left(4\nu^2\boldsymbol{r}^2 + \frac{3\nu}{A}\right)$$

- **Width parameter from rmsr=1.67824(83) fm**

*J.J. Krauth et al., Nature 589, 527 (2021);*

$$\nu = \frac{9}{16\langle \boldsymbol{r}^2\rangle_A} = 0.20\ \mathrm{fm}^{-2}$$

***Densities should include nucleon spread.***

# Λ potential parameters

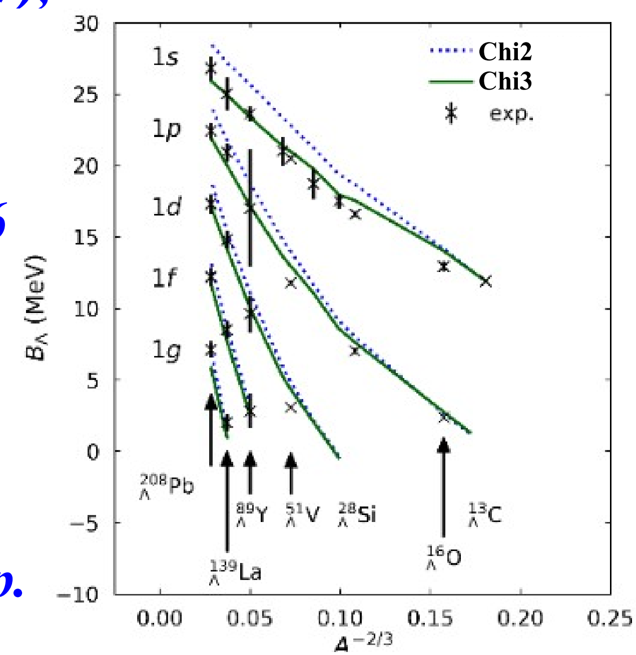
- **Parameters in Empirical SHF for Λ and in Chiral EFT**

| | a1 | a2 | a3 | a4 | a5 | BE(5HeΛ) | BE(pot.) |
|---|---|---|---|---|---|---|---|
| LY4 | -500.89 | 16.00 | 20.000 | 548.411 | 0 | 3.63184 | 2.70 |
| HPL2 | -302.76 | 23.72 | 29.853 | 514.263 | 0 | 2.48189 | – |
| Chi2 | -352.21 | 39.35 | 52.158 | -356.95 | 1000.81 | 3.61234 | 2.46 |
| Chi3 | -388.28 | 47.28 | 36.558 | -405.67 | 1256.75 | 4.78802 | 2.83 |

*$^{5}_{\Lambda}$He binding energy 3.12 ± 0.02 MeV, Juric et al., NPB52 (1973) 1;*
*LY4 from D. E. Lanskoy, Y. Yamamoto, PRC55, 2330 (1997);*
*HPL2 from N. Guleria, S. K. Dhiman, R. Shyam, NPA886, 71 (2012).*
*Chi2/Chi3 fit results of Chiral EFT w/o and w/ 3BF Gerstung, Kaiser, Weise EPJA56('20),175(2001.1056 Kohno, PRC97('18), 035206 (1802.05388).*

- **We ignore LS potential for Λ**
- **a3 parameter in Chi2 and Chi3 is tuned to fit $^{13}_{\Lambda}$C binding energy.**

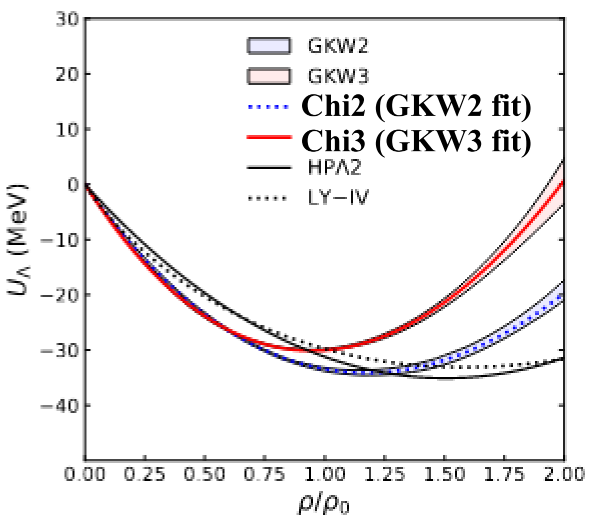
*Jinno, Murase, Nara, AO, in prep.*
*Jinno's talk on Thu.*

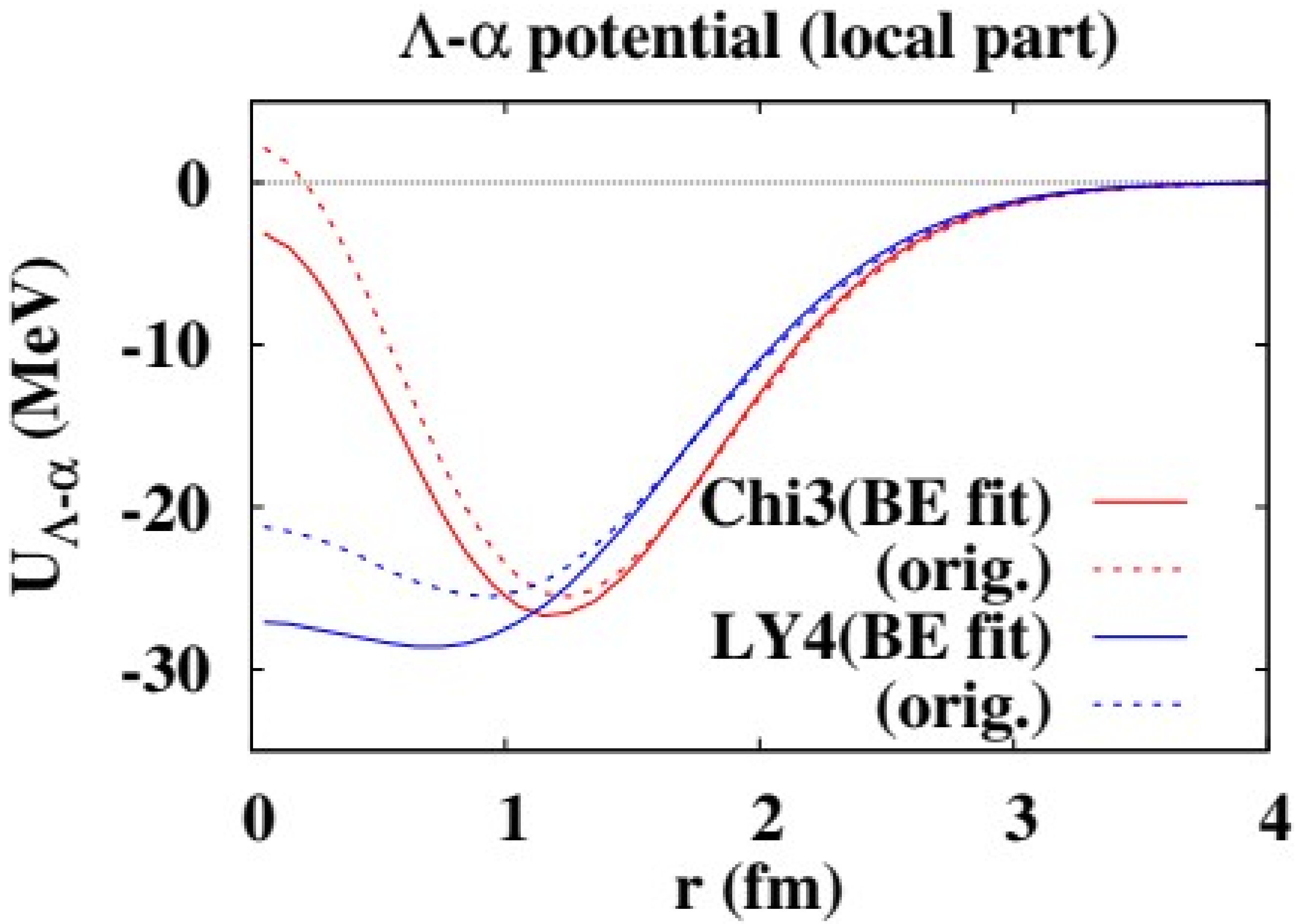


# Λ-α potential

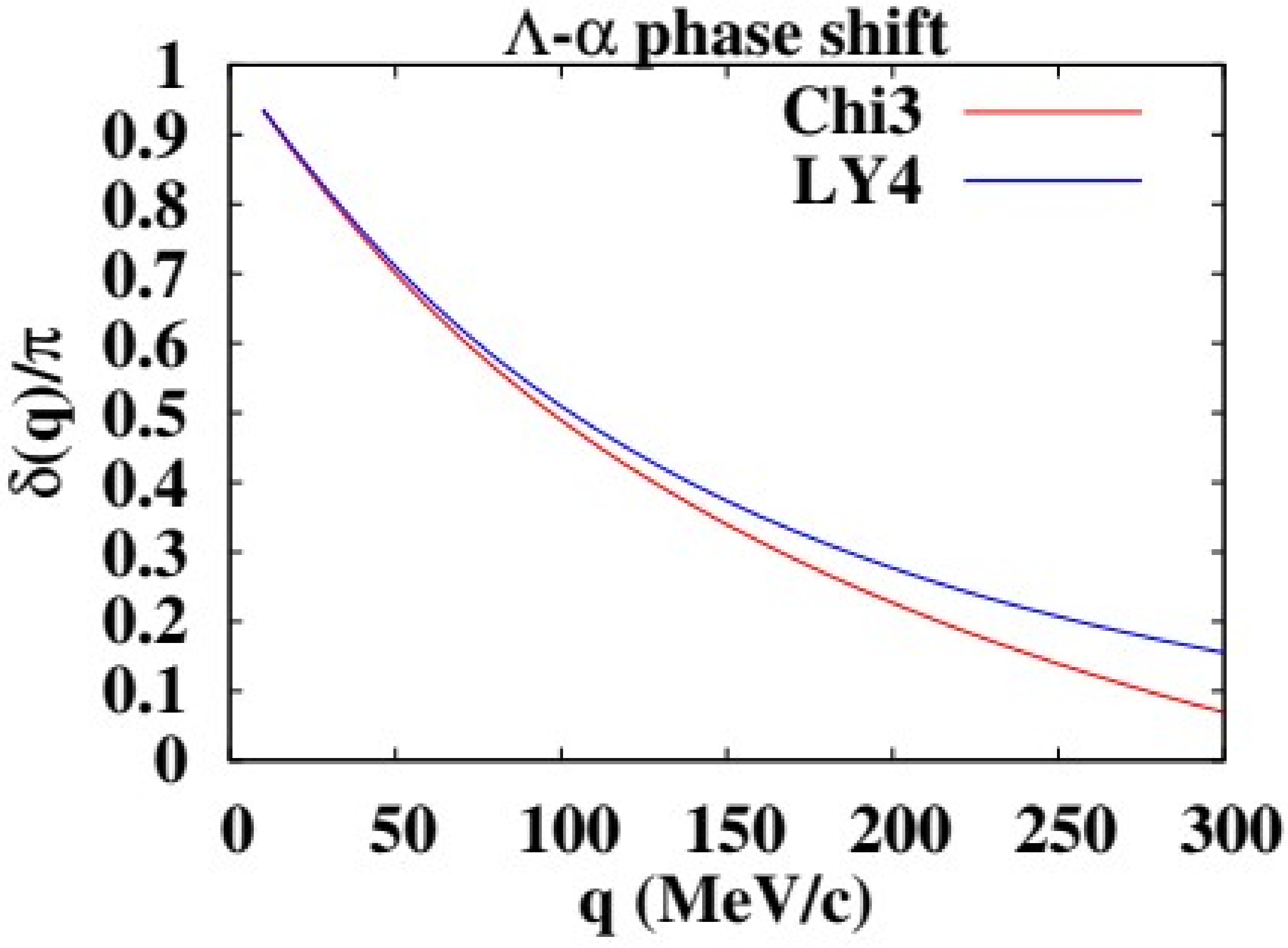
- **Significant difference in the potential shape (local part)**
  - **LY4 shows Woods-Saxon type Λ-α potential (volume type)**
  - **Chi3 shows a dip at r ~ 1.2 fm and central repulsion (surface type)**
  - **Potential shape reflects the density dependence of $U_\Lambda$**

***cf: I. Kumagai-Fuse, S. Okabe, Y. Akaishi, PLB 345 ('95) 386. (Isle potential)***



# Λ-α scattering phase shift

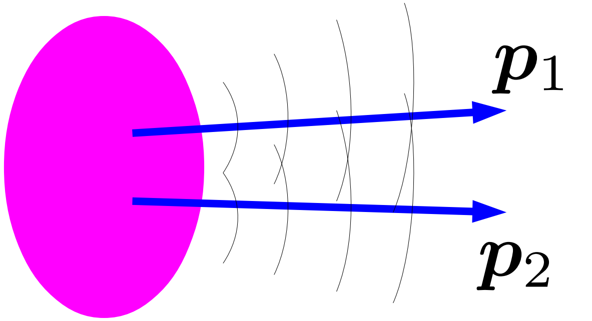



$$(a_0, r_{\rm eff}) = (4.13, 1.72){\rm fm}\ ({\rm Chi3}), (3.94, 1.76){\rm fm}\ ({\rm LY4})$$

# Λ-α correlation function

# Femtoscopic study of hadron-hadron interaction

- How can we study interactions between short-lived particles ? → Femtoscopy !




- Correlation function (CF)
  - Koonin-Pratt formula
    *Koonin('77), Pratt+('86), Lednicky+('82)*

source fn. relative w.f.

$$C(\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r}\, \underline{S_{12}(\boldsymbol{r})} \underline{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}$$

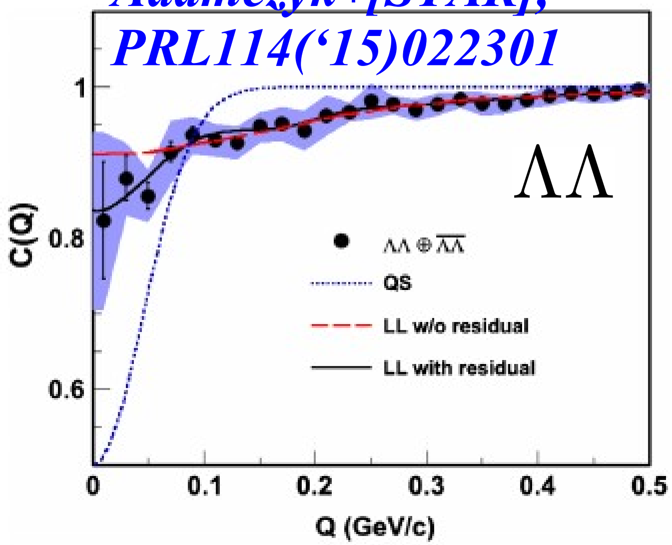
- Source size from quantum stat. + CF (Femtoscopy)
  *Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)*
- Hadron-hadron interaction from source size + CF
  - CF of non-identical pair from Gaussian source
    *R. Lednicky, V. L. Lyuboshits ('82); K. Morita, T. Furumoto, AO ('15)*

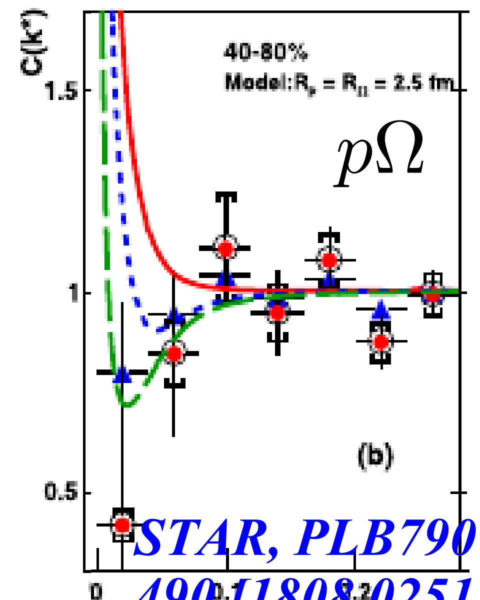
$$C(\boldsymbol{q}) = 1 + \int d\boldsymbol{r}\, S(r) \left\{ |\varphi_0(r)|^2 - |j_0(qr)|^2 \right\} \quad (\varphi_0 = \text{s-wave w.f.})$$

**CF shows how much $|\varphi|^2$ is enhanced → $V_{hh}$ effects !**

# Measured Correlation Functions (examples)

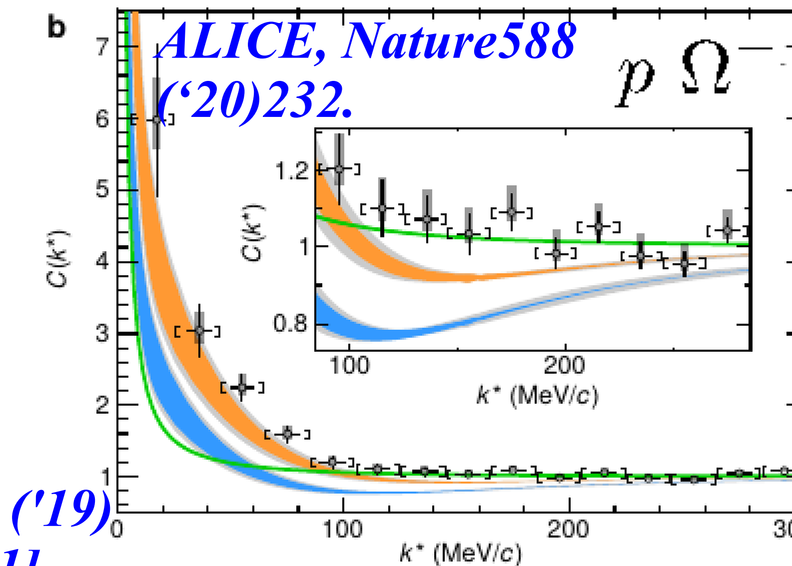
*Adamczyk+[STAR], PRL114('15)022301*

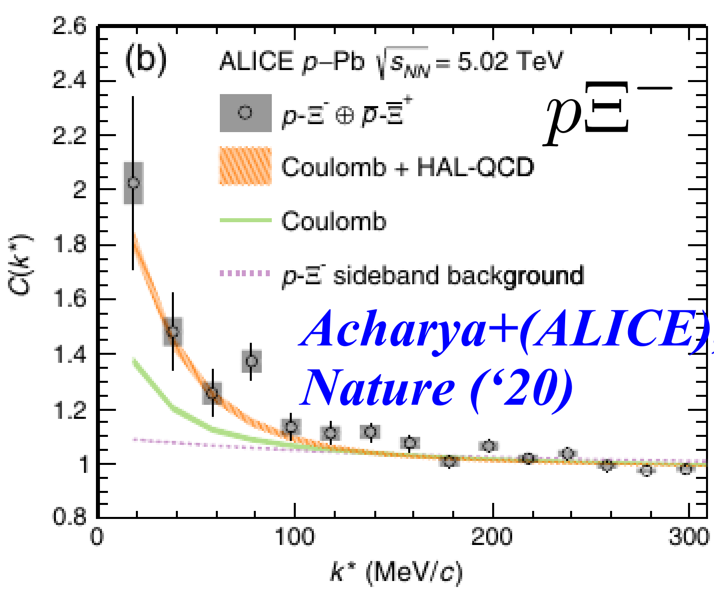


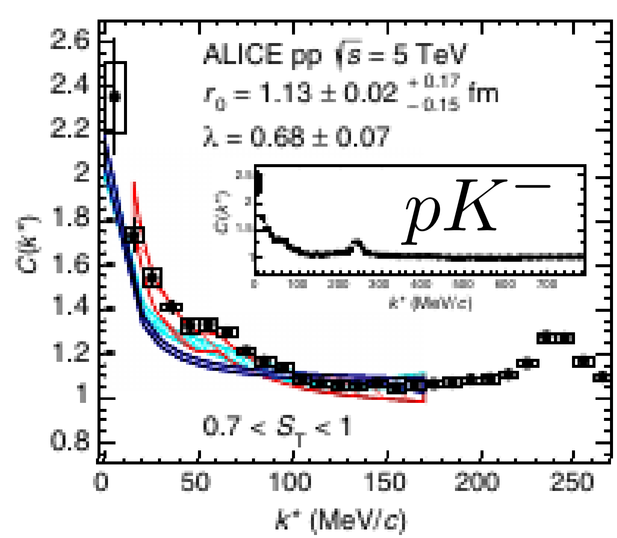

*STAR, PLB790 ('19) 490 [1808.02511].*

*ALICE, Nature588 ('20)232.*

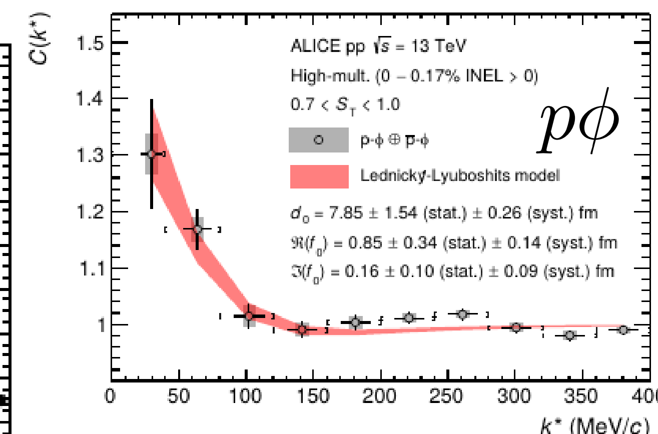



*Acharya+(ALICE), Nature ('20)*



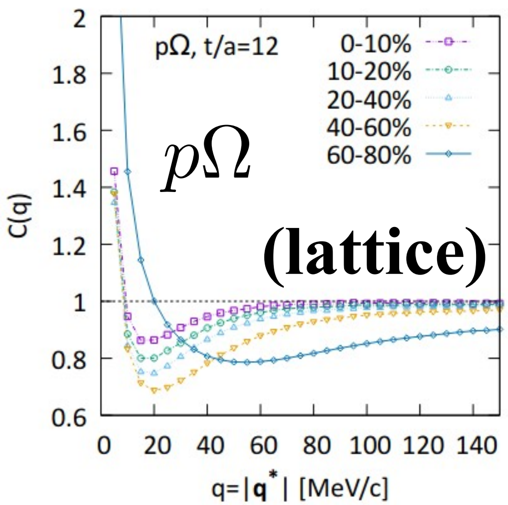

*S. Acharya+[ALICE], PRL124('20)092301*



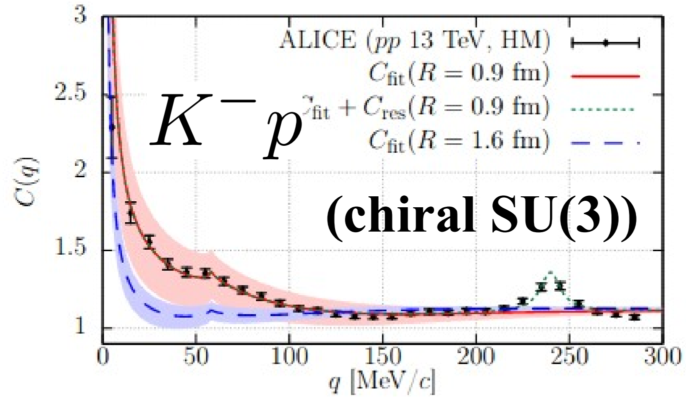

*ALICE, 2105.05578*

*c.f. talk by V. Mantovani Sarti,*

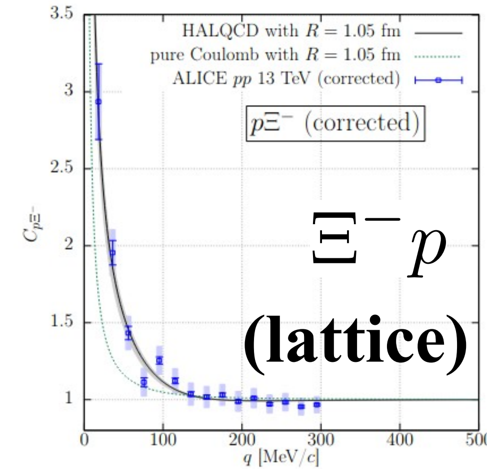
# Theoretical femtoscopic study of hh int. (examples)



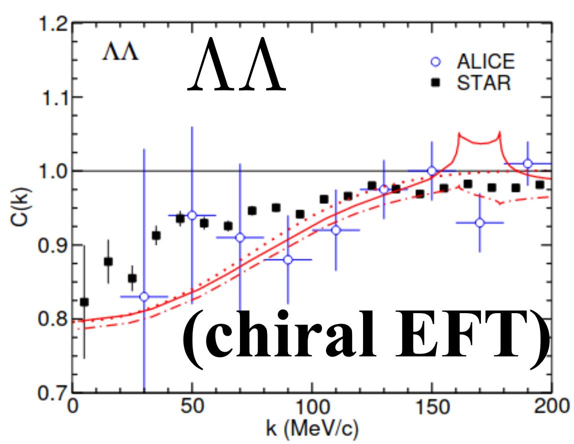

*Morita, Gongyo et al., (1908.05414),*
*Morita, AO, Etminan, Hatsuda (1605.06765)*



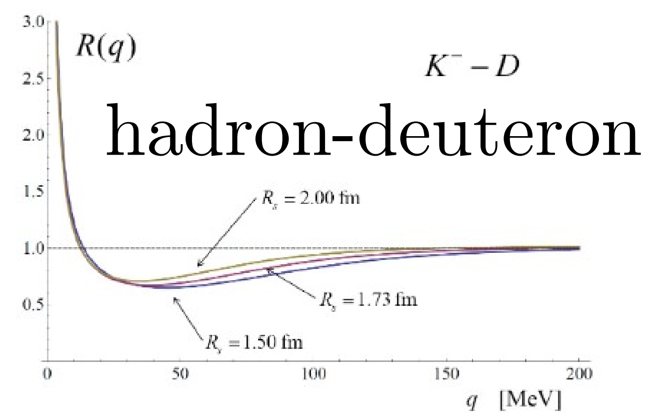

*Kamiya+(1911.01041)*



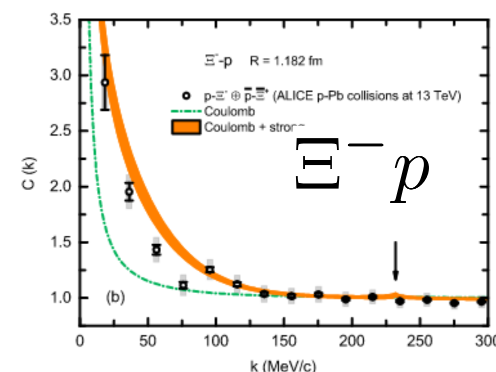

*Y.Kamiya, K.Sasaki, et al., (2108.09644)*




*Haidenbauer(1808.05049), Morita+(1408.6682)*




*Mrówczyński, Słón (1904.08320, K⁻d), Haidenbauer (2005.05012, Λd), Etminan, Firoozabadi (1908.11484, Ωd), K.Ogata+ (Ξ⁻ d,2103.00100)*

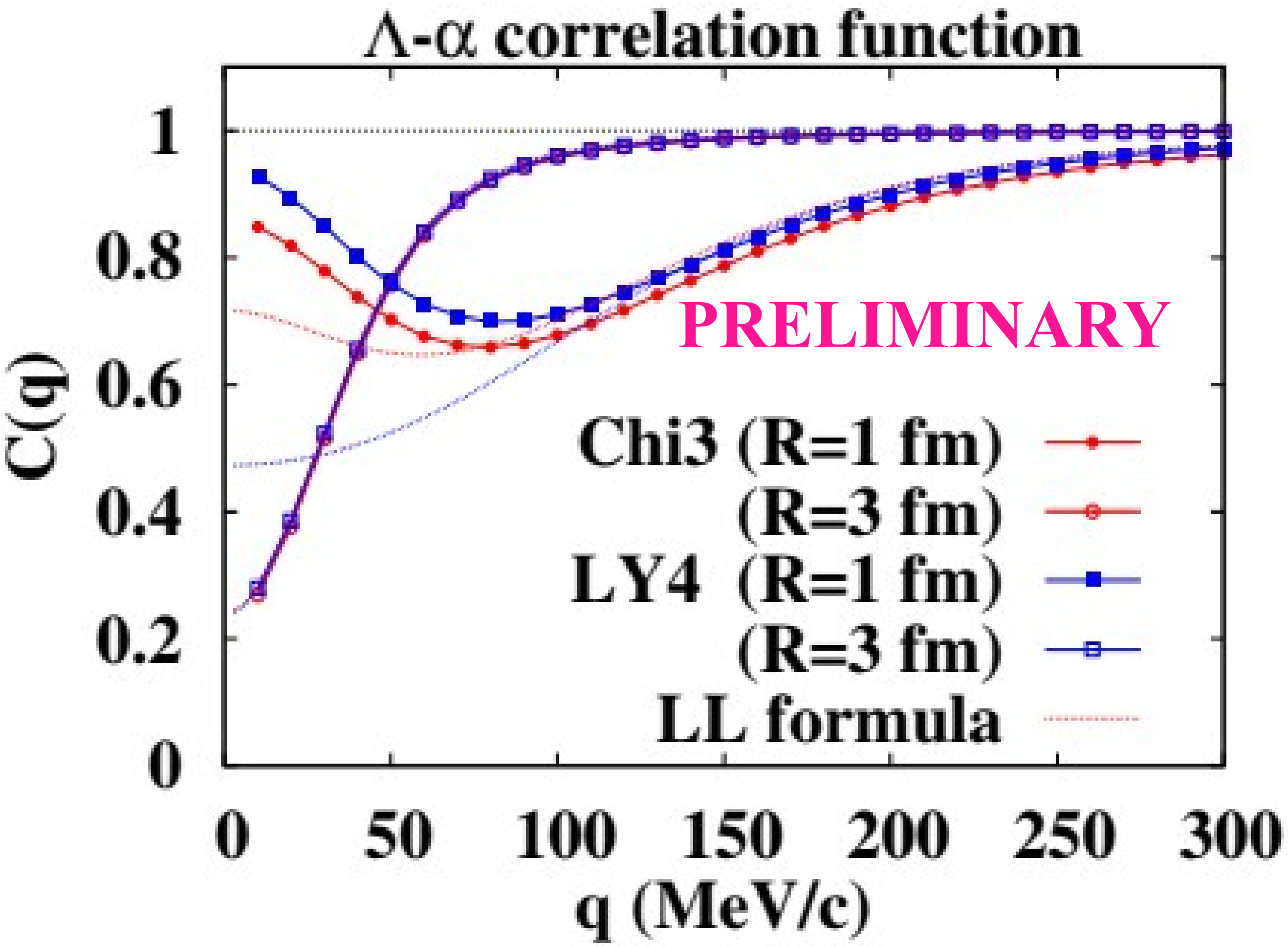



**(covariant χEFT, S=-2)**

*Z.-W. Liu, K.-W. Li, L.-S. Geng (2201.04997)*

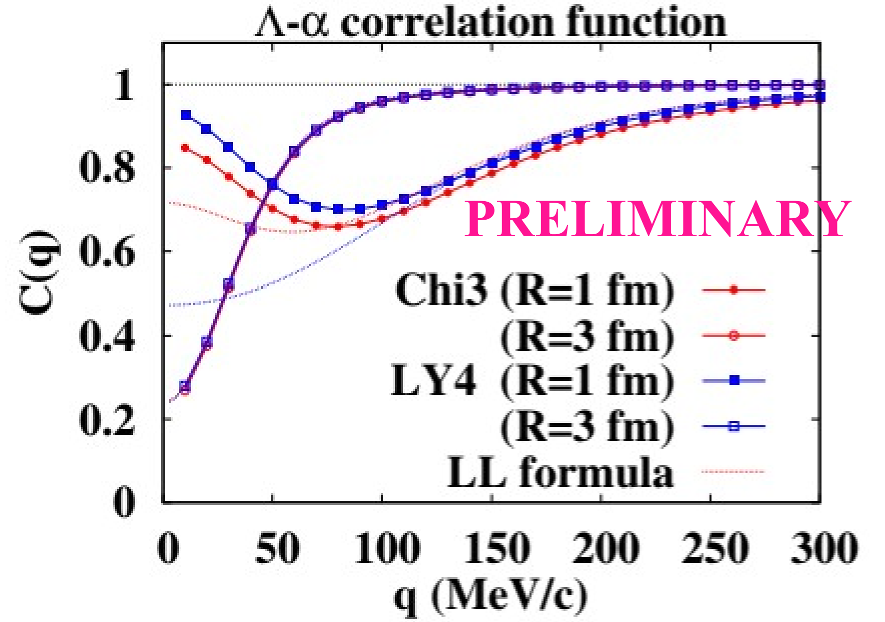
# Λ-α correlation function

- **Comparison of Λ-α correlation functions from empirical (LY4) and chiral (Chi3) potentials for gaussian source with R=1 and 3 fm.**



# Λ-α correlation function

- **For a large source (R=3 fm), LY4 and Chi3 give almost the same C(q).**
  - **Similar scattering parameters → Similar asymptotic wave function**
  - **Lednicky-Lyuboshitz formula based on asymptotic w.f. works well.**
- **For a small source (R=1 fm), Chi3 gives a slightly smaller C(q) at low momenta.**
  - **Central repulsion pushes out the w.f. to the outer region (r>1 fm).**
- **In all cases, the bound state ($^{5}_{\Lambda}$He) causes a dip in C(q).**
  - **The bound state eats the yield at low momenta.**

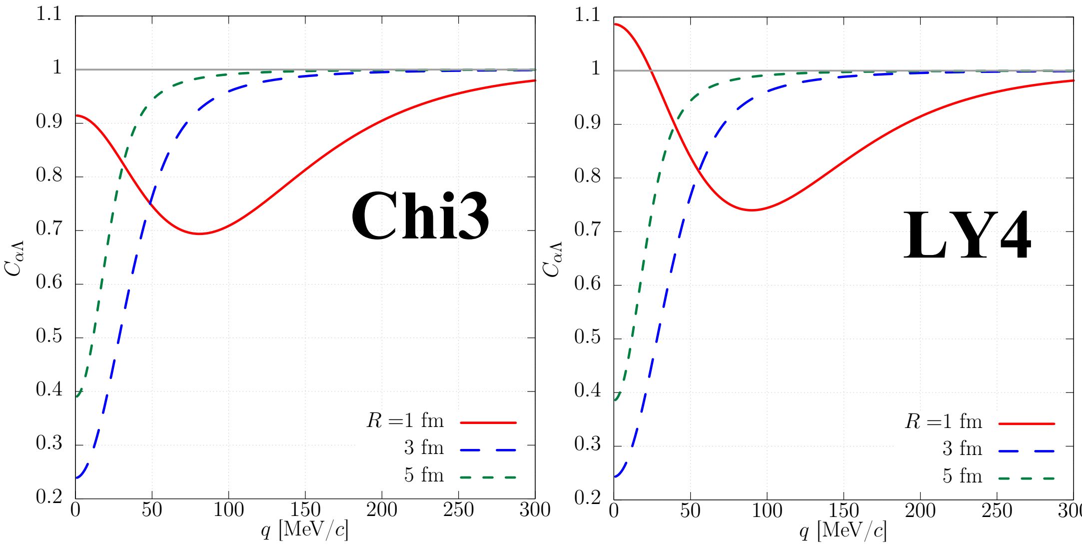


# Summary

- **The density dependence of Λ, $U_\Lambda(\rho)$, is important to understand and/or to solve the hyperon puzzle.**
- **Repulsive $U_\Lambda$ at high densities from chiral EFT with 3-body force effects seems to be consistent with**
  **the flow data of Λ from heavy-ion collisions**
  **and the hypernuclear binding energies.**
  *Gerstung, Kaiser, Weise EPJA56('20),175(2001.10563);*
  *Kohno, PRC97('18), 035206 (1802.05388).*
  *Nara, Jinno, Murase, AO (PRC106 ('22), 044902 (2208.01297);*
  *Jinno, Murase, Nara, AO, in prep.*
  **However, $U_\Lambda$ with different density dependence can also explain the data.**
- **Λ-α correlation function may be helpful to distinguish the density dependence, if C(q) is obtained at the precision of a few %.**
  - **Large source → scattering parameters (STAR BES II?)**
  - **Small source → difference in the potential shape (J-PARC?)**

# Summary (cont.)

- There are many problems…
  - Feasible in experiment ?
  - α production from a small source (R=1 fm → pp or pA)?
  - Does pA collision at $\sqrt{s_{NN}} < 5$ GeV lead to "chaotic" source ?
  - Is a few % precision achievable in experiment ?
  - Other effects may modify C(q)
    E.g. m*=m approximation leads to different shape of C(q).
  - Other "smoking gun" observable?




**Thank you for your attention!**