A理論
-U双対性を指導原理とするブレーン理論-

A-theory
-a brane world-volume theory with manifest U-duality-

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With Ondřej Hulík, William Linch, Warren Siegel, Di Wang & Yu-Ping Wang

References & collaborators

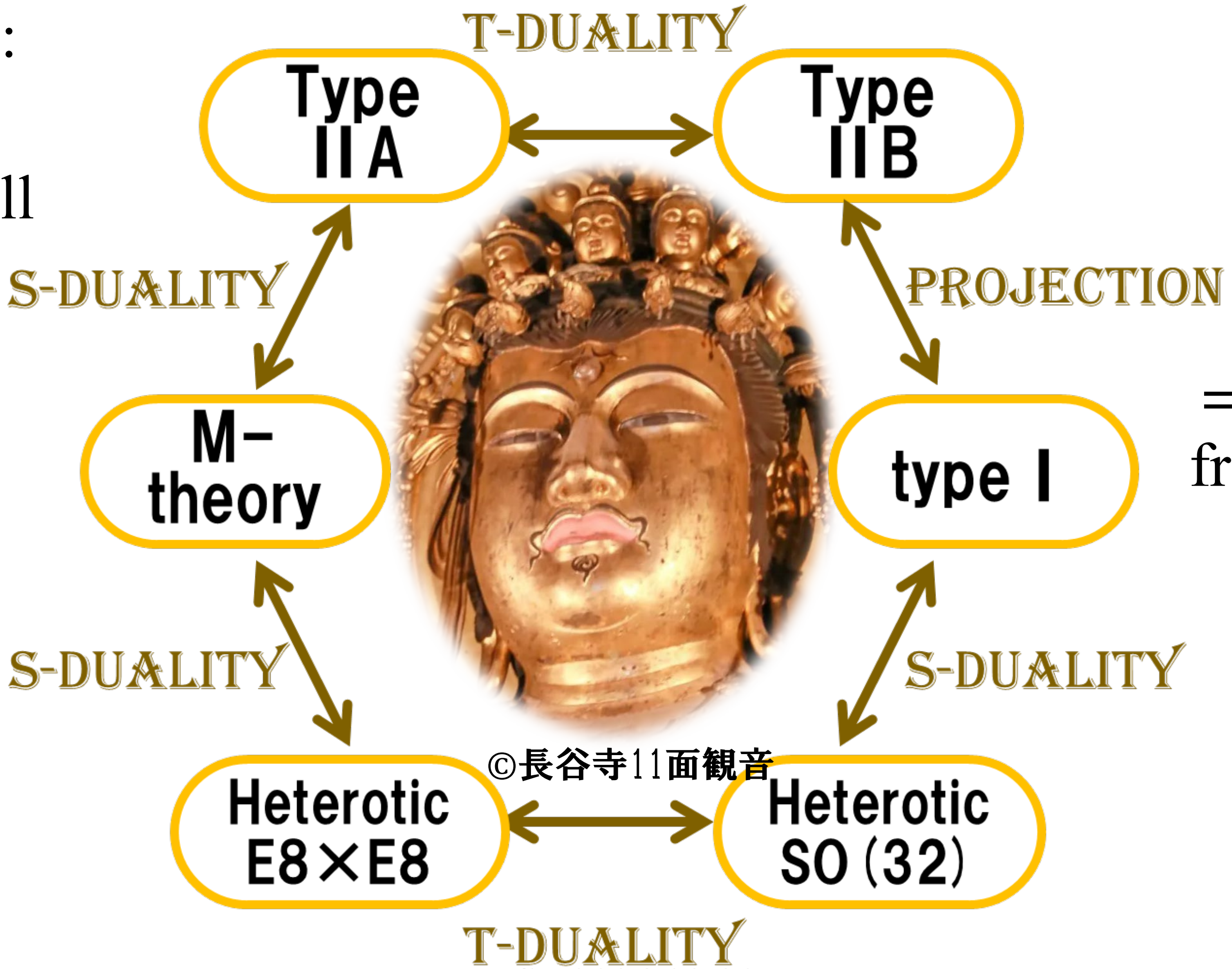
- T-duality off sell in 3D Thpe II superspace, Polacek, Siegel, arXive:1403.6904
- F-theory from Fundamental Five-branes, Linch, Siegel, arXiv:1502.00510
- F-theory with Worldvolume Sectioning, Linch, Siegel, arXiv:1503.00940
- Critical Super F-theories, Linch, Siegel, arXiv:1507.01669
- Enlarged exceptional symmetries of first-quantized F-theory, Siegel, Di Wang, arXiv:1806.02423
- F-theory superspace backgrounds, Siegel, D. Wang, arXiv:1910.01710
- M Theory from F Theory, Siegel, D. Wang, arXiv:2010.09564
- F-theory amplitudes, Siegel, Yu-Ping Wang, arXiv:2010.14590
- Perturbative F-theory 10-brane and M-theory 5-brane, M. H., Siegel, arXiv:2107.10568
- Open F-branes, M.H., Siegel, arXiv:2110.13010
- $\mathbb{A}$-theory -a brane worldvolume theory with manifest U-duality-, M.H., Hulík, Linch, Siegel, D. Wang and Y-P. Wang
- Strings and membranes from $\mathbb{A}$-theory five brane, M.H., Hulík, Linch, Siegel, Di Wang, Y-P Wang, arXiv:2410.11197

more…

String theories & dualities

Superstring theory:

candidate of the unified theory of all forces



T-DUALITY

Type IIA

Type IIB

S-DUALITY

PROJECTION

M-theory

type I

S-DUALITY

S-DUALITY

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Heterotic E8×E8

Heterotic SO(32)

T-DUALITY

Q: Why do dualities relate them?

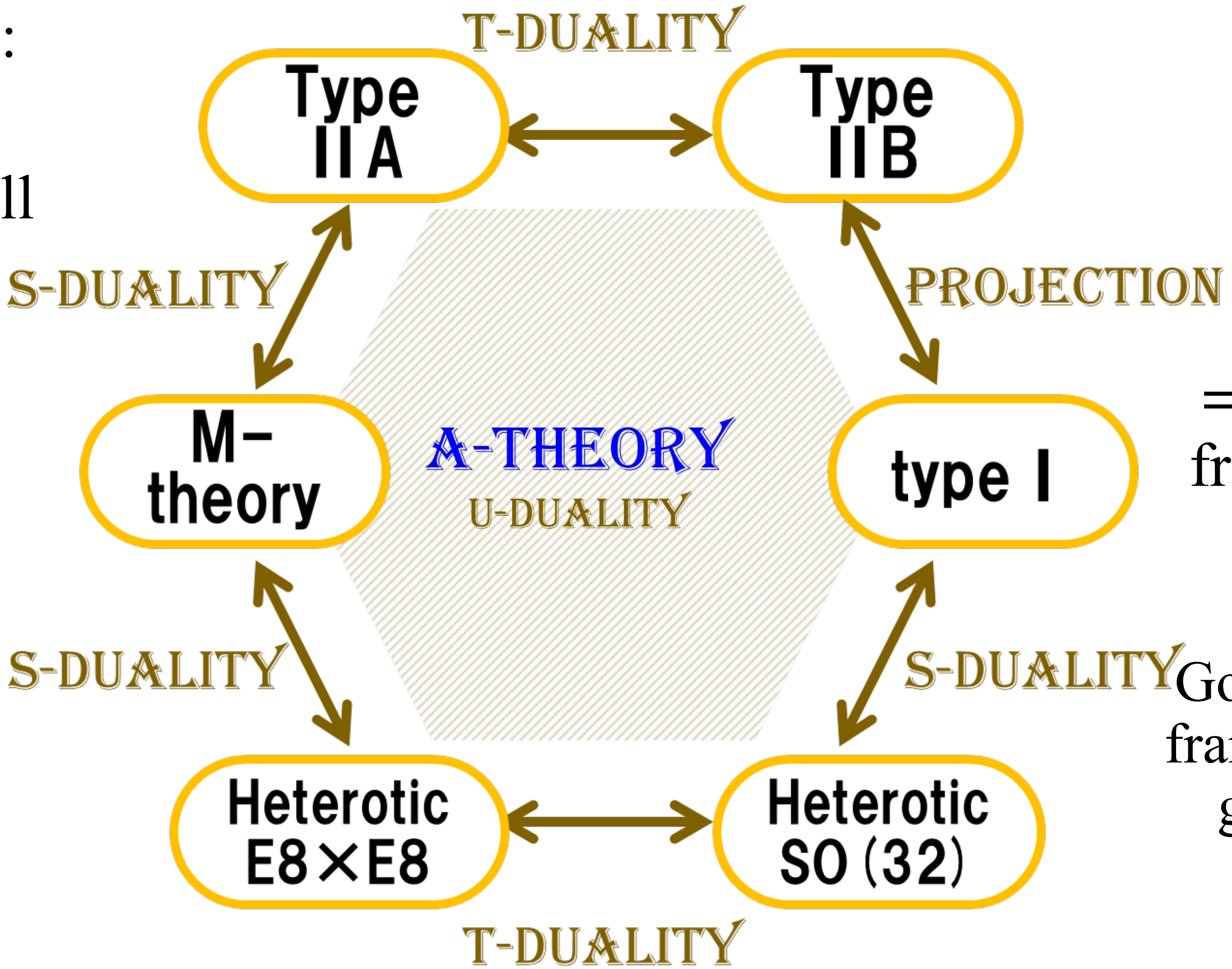
⇒Suppose $^{\exists}$ a unified framework based on a larger duality with many faces

Eleven-Faced Kannon at Hasedera Temple
Navihttps://enokama.jp/feature/2353/

String theories & dualities

Superstring theory:

candidate of the unified theory of all forces



T-DUALITY

Type IIA

Type IIB

S-DUALITY

PROJECTION

M-theory

A-THEORY

U-DUALITY

type I

S-DUALITY

S-DUALITY

Heterotic E8×E8

Heterotic SO(32)

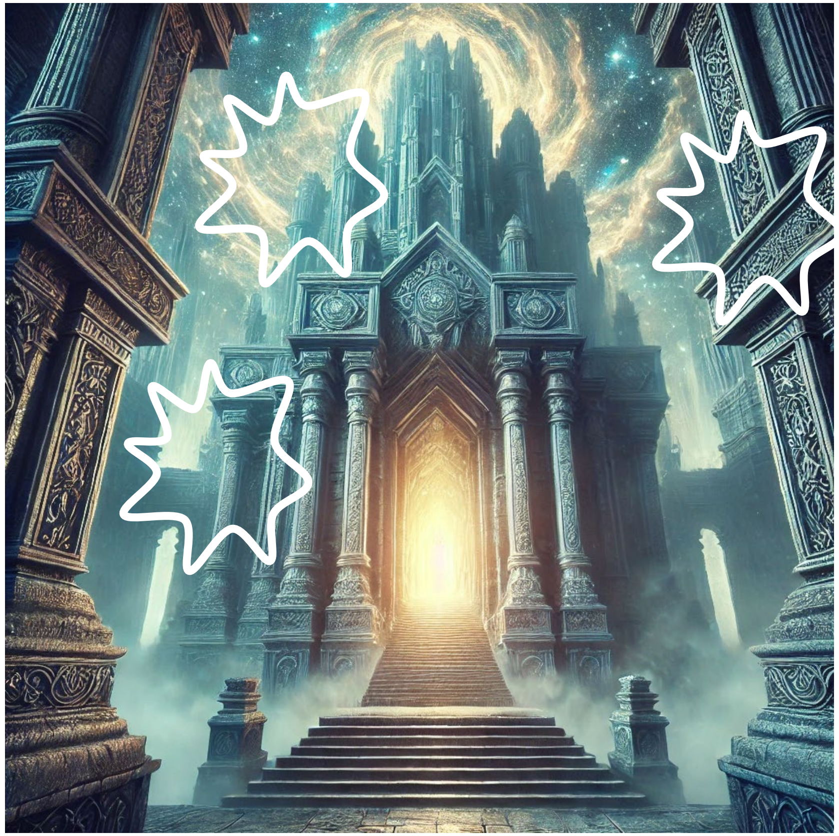
T-DUALITY

Q: Why related through dualities?

⇒Suppose $^{\exists}$ a unified framework based on a larger duality

Goal: construct a unified framework (A-THEORY) guided by U-DUALITY

T-duality



Large space, large R

Momentum modes are excited!

Winding modes are excited!

T-duality symmetry

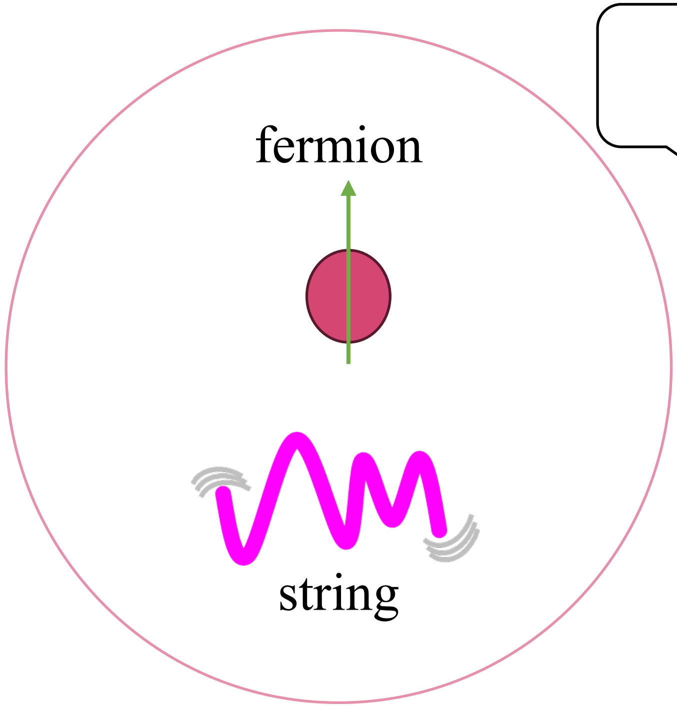
O(D,D)

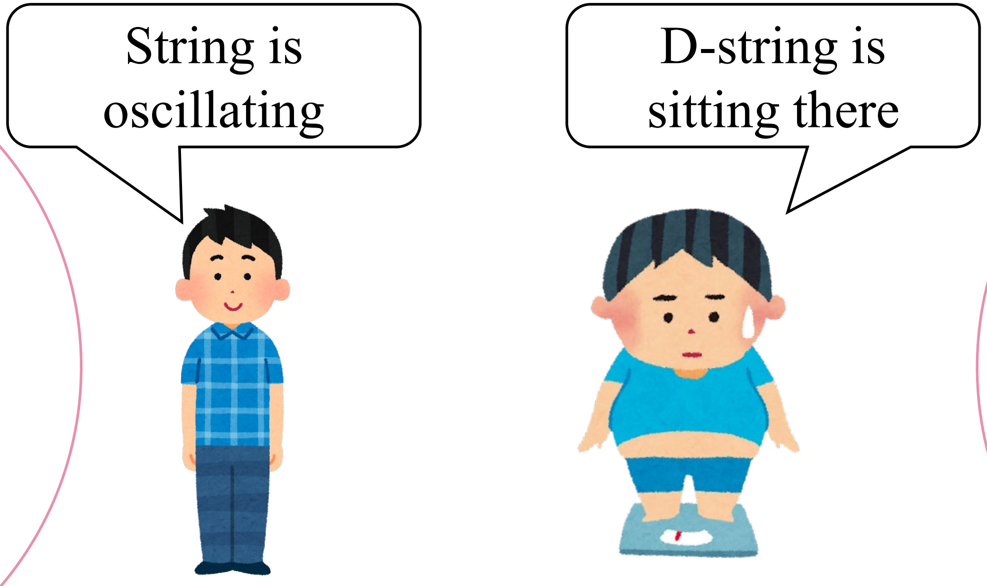
$$g_{mn}, B_{mn} \in \frac{O(D,D)}{SO(D-1,1)^2}$$

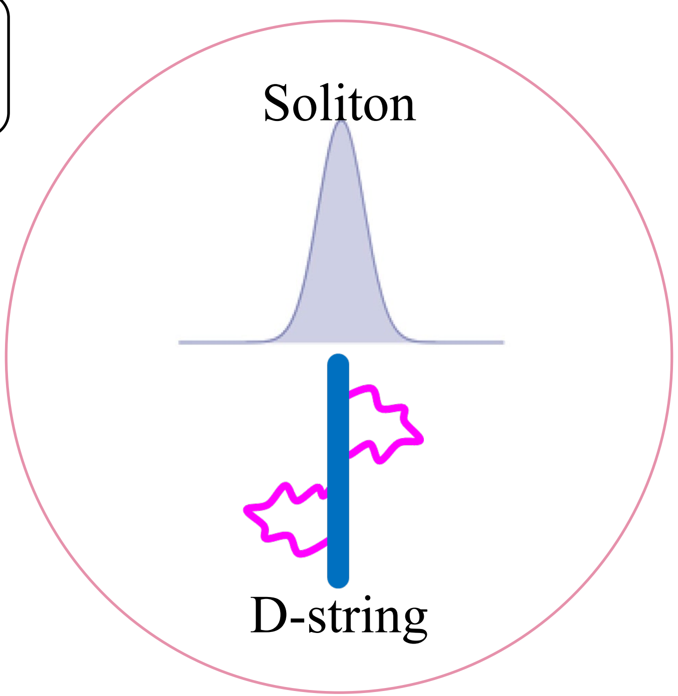
Small space, α’/R

Spaces: gpt4o
People: irasutoya

S-duality







Weak coupling, small g

S-duality symmetry

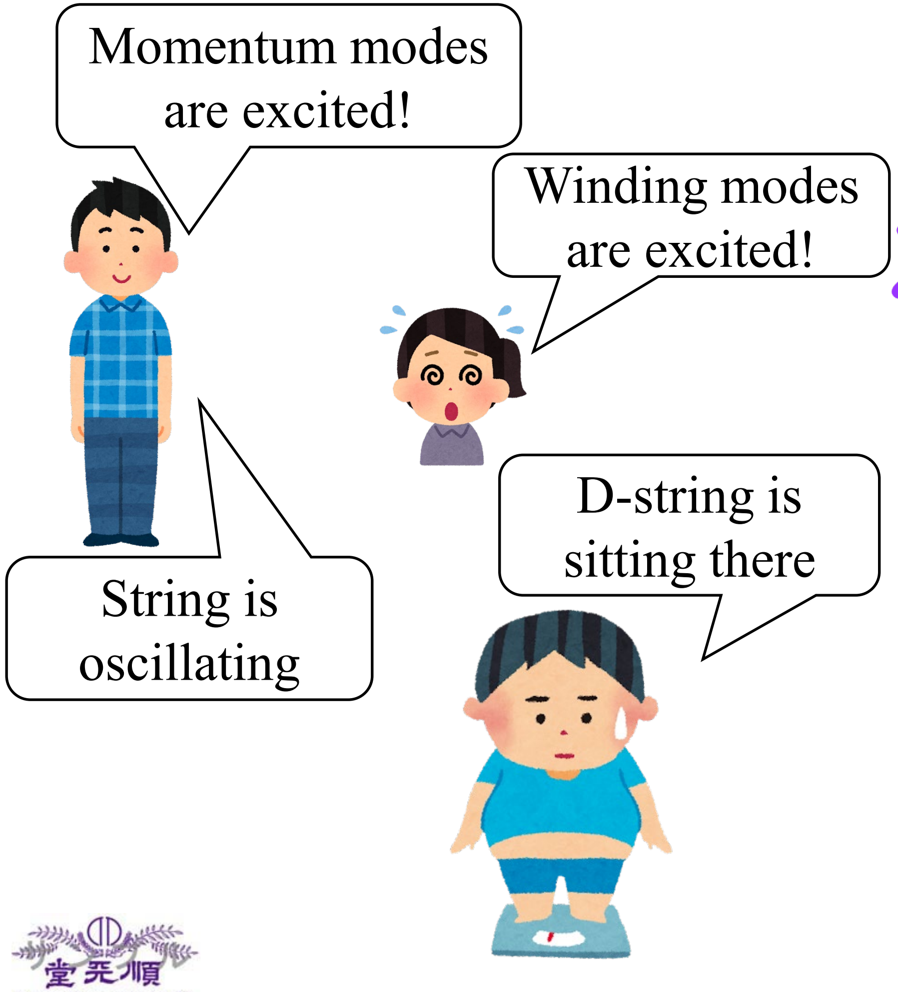
SL(2)

$\tau = 4\pi i/g^2 + \theta/2\pi$

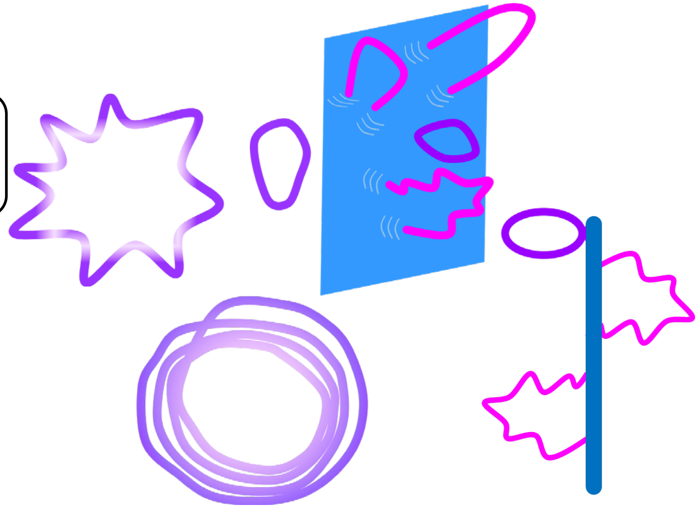
Strong coupling 1/g

$$\phi\,,\chi \in \frac{SL(2)}{SO(2)}$$

U-duality



String, D-string, D-brane



U-duality symmetry E_{D+1}

Exceptional group

$$g_{mn}, B_{mn}, C_{RR} \in \frac{\mathbf{E_{D+1}}}{\mathbf{H}}$$

D	Exceptional group U-duality symmetry $E_{D+1(D+1)}$
1	GL(2)
2	SL(3)SL(2)
3	SL(5)
4	SO(5,5)
5	$E_{6(6)}$
6	$E_{7(7)}$
7	$E_{8(8)}$

Hidden symmetry: ’78 Cremmer & Julia, de Wit &Nicolai,⋯

U-duality: '94 Hull& Townsend, ’98 Obers & Piolin⋯

Outline

I Introduction: string theories & string dualities

II 𝔸-theory: 𝔸5-brane SL(5) current algebra,

Virasoro constraints, Hamiltonian & Lagrangian

III Sectionings: 𝕋-string, M5-brane, SUGRA M2

IV Conclusions

I Introduction

String theories

String dualities

String dualities & duality covariant theories

- S & T dualities are unified into U-duality

  T ’84 Kikkawa & Yamasaki, ’86 Sakai & Senda, ‘87 Buscher, …
  U ‘78 Cremmer & Julia, de Wit &Nicolai,…'94 Hull& Townsend, ’98 Obers & Pioline …

- ‘95 Witten: What is the strong coupling limit of IIA?

  ⇒ M-theory← S-duality→IIA , I ← S-duality→ HO, M ← S-duality→ HE

- ‘96 Vafa: What is the geometric meaning of SL(2) S-duality of IIB?

  ⇒ F-theory ← U-duality→ IIB, F↔M

- ‘93 Siegel: What is the theory with manifest T-duality? ‘14 : What is the one for U-duality?

  ⇒ 𝕋-theory & 𝔸(F)-theory

String dualities & duality covariant theories

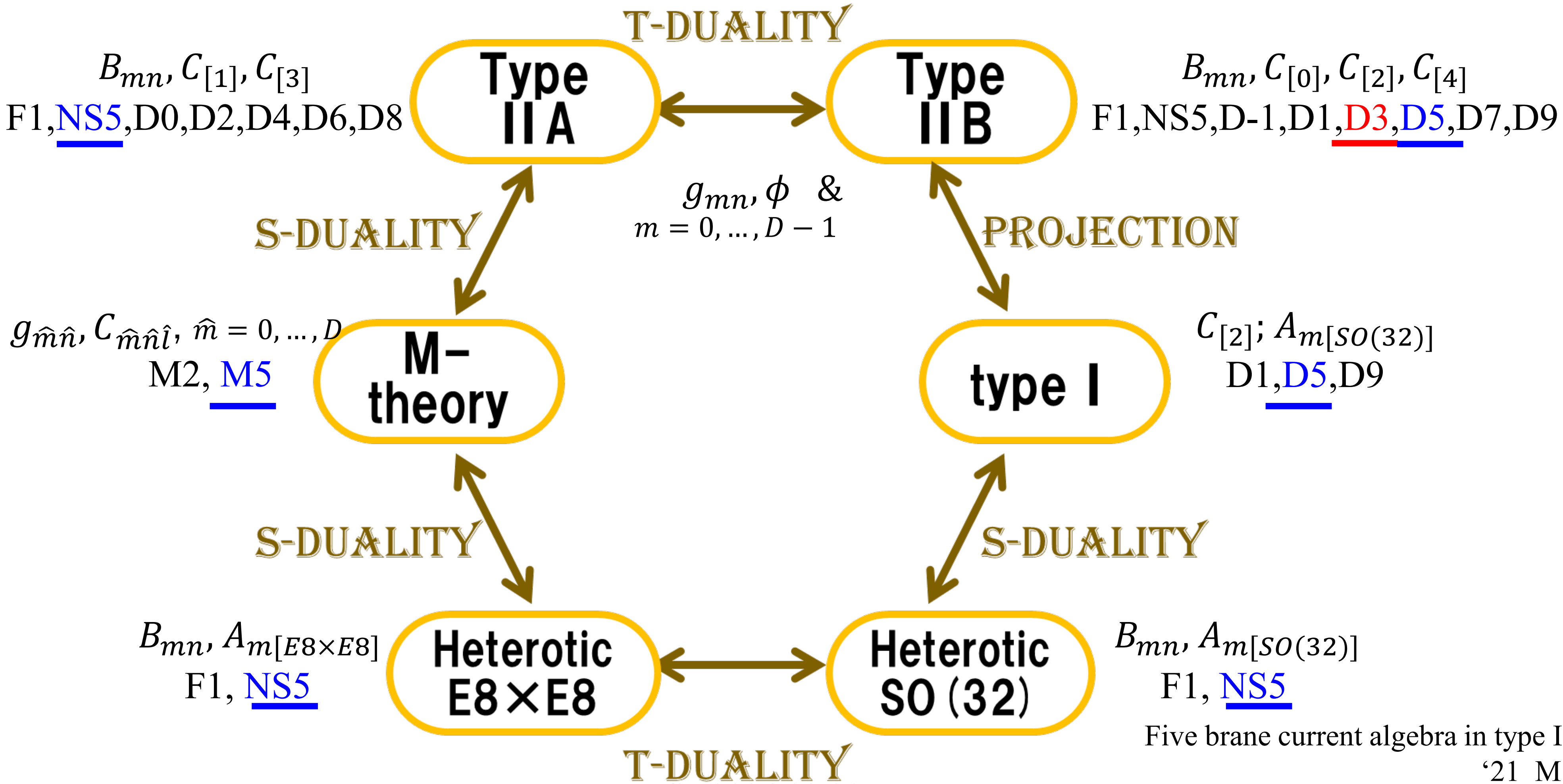
- Double Field theory (DFT) has been actively studied. DFT is a low energy effective theory of 0-modes of string with manifest T-duality, and DFT σ model has been developed.

’93 Siegel, ’09〜 Hull & Zwiebach, ’10〜 Hohm, Kwaw, Jeon, Lee, Park, Thompson, Berman, Aldazabal,Marques, Nuñez, Lust, Klimcik, Hassler,Ševera, Sfetsos, Demulder, Sakatani, Watamura, Sasaki, Yata, Mori, Polaceck, M.H. ,...

- Exceptional Field theory (ExFT) has been actively studied. ExFT is a low energy effective theory of 0-modes of superstring with manifest U-duality, and ExFT σ model has been developed.

’11〜 Coimbra, Strickland-Constable, Waldram; ’12〜 Berman, Cederwall, Kleinschmidt, Thompson; Godazgar, Godazgar, Hohm, Nicolai, Samtleben; ’14〜 Musaev, Samtleben, Henning , Blair, Malek, Baguet, M. Magro, Wang, Bossard, Ciceri, Inverso, Kleinschmidt, Abzalov, Bakhmatov, Sakatani, Siegel, Wang,… , Review ’20 Blair,…

String theories & gauge fields



T-DUALITY

$B_{mn}, C_{[1]}, C_{[3]}$
F1,NS5,D0,D2,D4,D6,D8

Type IIA

Type IIB

$B_{mn}, C_{[0]}, C_{[2]}, C_{[4]}$
F1,NS5,D-1,D1,D3,D5,D7,D9

g_{mn}, ϕ &
$m = 0, \dots, D-1$

S-DUALITY

PROJECTION

$g_{\widehat{m}\widehat{n}}, C_{\widehat{m}\widehat{n}\widehat{l}}, \widehat{m} = 0, \dots, D$
M2, M5

M-theory

type I

$C_{[2]}; A_{m[SO(32)]}$
D1,D5,D9

S-DUALITY

S-DUALITY

$B_{mn}, A_{m[E8\times E8]}$
F1, NS5

Heterotic E8×E8

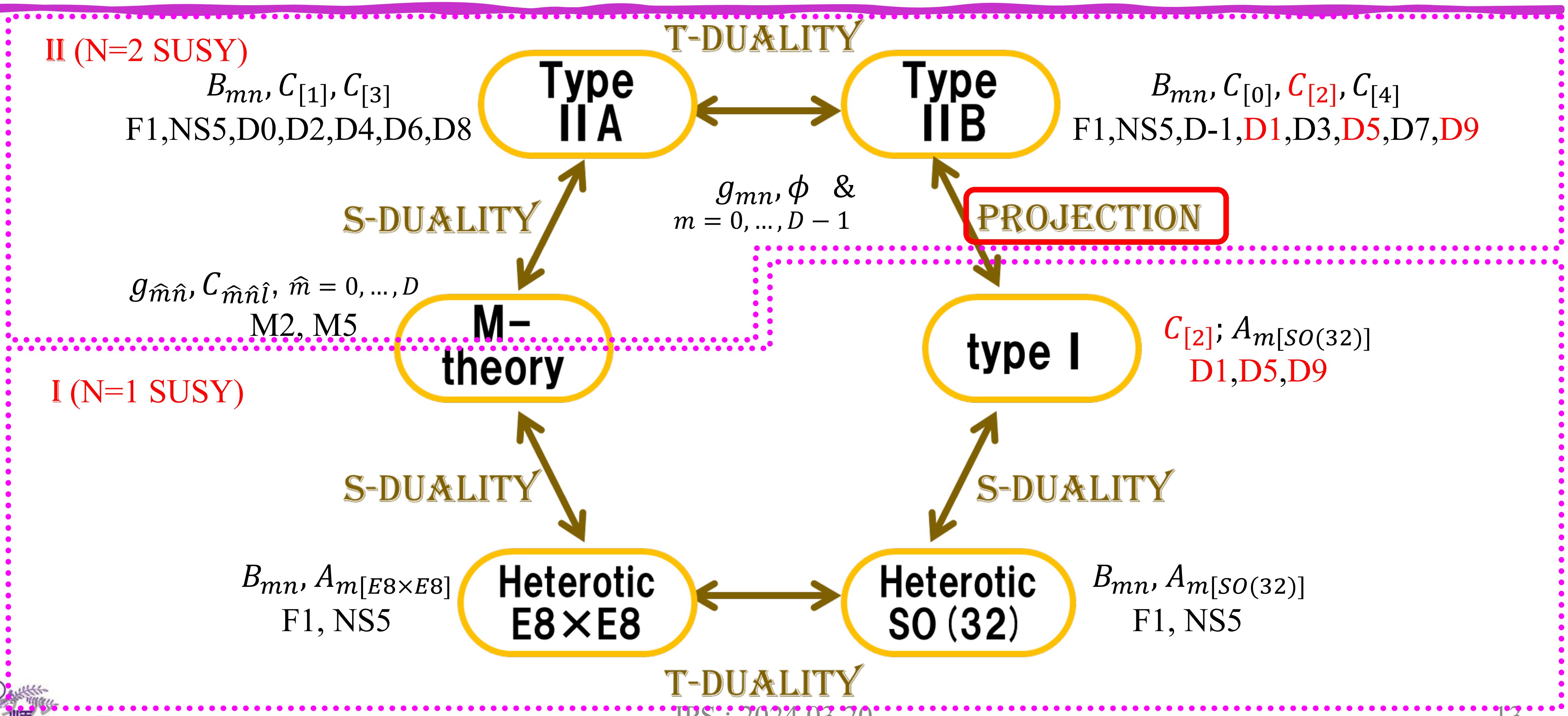
Heterotic SO(32)

$B_{mn}, A_{m[SO(32)]}$
F1, NS5

T-DUALITY

Five brane current algebra in type II string theories
‘21 M.H. Sasaki, Yata

String theories & gauge fields



II (N=2 SUSY)

$B_{mn}, C_{[1]}, C_{[3]}$
F1,NS5,D0,D2,D4,D6,D8

Type IIA

T-DUALITY

Type IIB

$B_{mn}, C_{[0]}, C_{[2]}, C_{[4]}$
F1,NS5,D-1,D1,D3,D5,D7,D9

S-DUALITY

g_{mn}, ϕ &
$m = 0, \ldots, D-1$

PROJECTION

$g_{\hat{m}\hat{n}}, C_{\hat{m}\hat{n}\hat{l}}, \hat{m} = 0, \ldots, D$
M2, M5

M-theory

type I

$C_{[2]}; A_{m[SO(32)]}$
D1,D5,D9

I (N=1 SUSY)

S-DUALITY

S-DUALITY

$B_{mn}, A_{m[E8\times E8]}$
F1, NS5

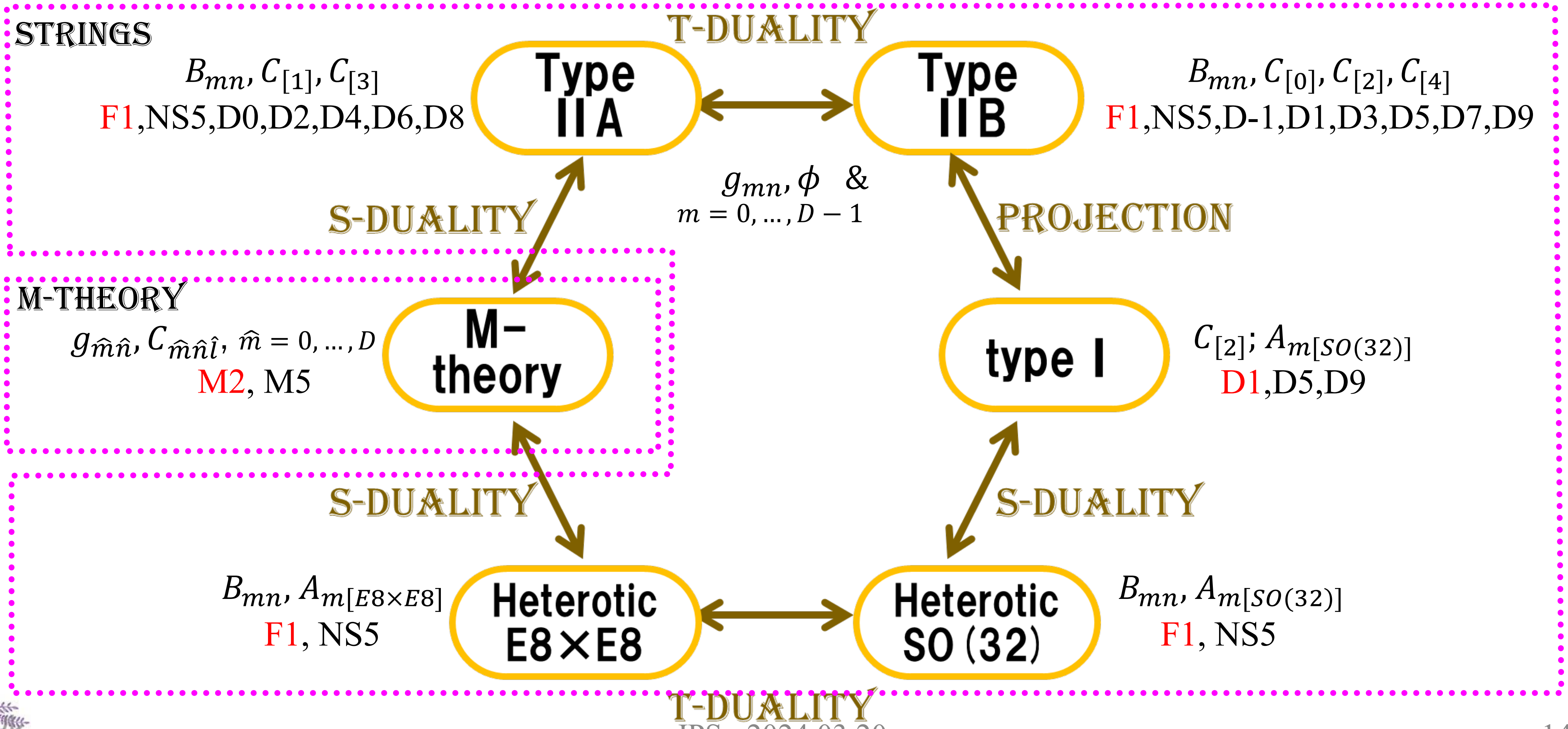
Heterotic E8×E8

Heterotic SO (32)

$B_{mn}, A_{m[SO(32)]}$
F1, NS5

T-DUALITY

String theories & gauge fields



STRINGS

$B_{mn}, C_{[1]}, C_{[3]}$
F1,NS5,D0,D2,D4,D6,D8

Type IIA

T-DUALITY

Type IIB

$B_{mn}, C_{[0]}, C_{[2]}, C_{[4]}$
F1,NS5,D-1,D1,D3,D5,D7,D9

g_{mn}, ϕ &
$m = 0, \dots, D-1$

S-DUALITY

PROJECTION

M-THEORY

$g_{\hat{m}\hat{n}}, C_{\hat{m}\hat{n}\hat{l}}, \hat{m} = 0, \dots, D$
M2, M5

M-theory

type I

$C_{[2]}; A_{m[SO(32)]}$
D1,D5,D9

S-DUALITY

S-DUALITY

$B_{mn}, A_{m[E8\times E8]}$
F1, NS5

Heterotic E8×E8

Heterotic SO(32)

$B_{mn}, A_{m[SO(32)]}$
F1, NS5

T-DUALITY

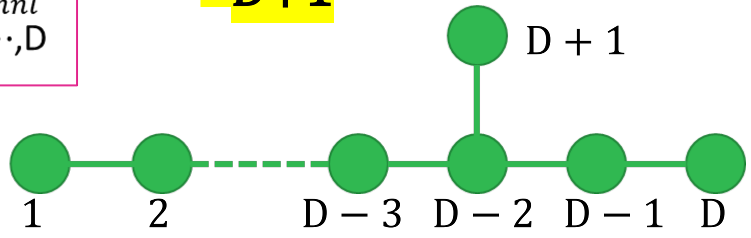
Dynkin of duality symmetry groups

$$\frac{E_{D+1}}{H_D} \ni g_{\hat{m}\hat{n}}, C_{\hat{m}\hat{n}\hat{l}} \quad \hat{m} = 0,\cdots,D$$

A-THEORY

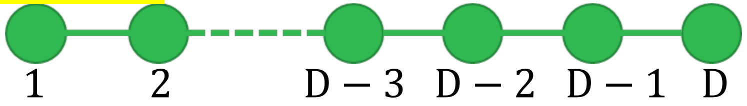
E_{D+1} U-duality symmetry

$$\frac{E_{D+1}}{H_D} \ni g_{mn}, B_{mn}, \phi, C_{RR} \quad m = 0,\cdots,D\text{-}1$$



M-THEORY

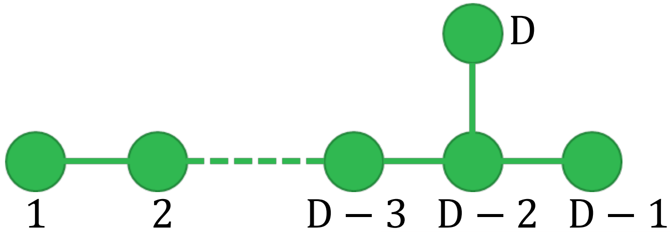
GL(D+1) (D+1)-dim. diffeo.



$$\frac{GL(D+1)}{SO(D,1)} \ni g_{\hat{m}\hat{n}}, \hat{m} = 0,\cdots,D$$

T-THEORY

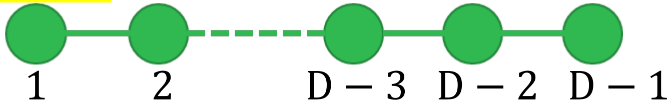
O(D,D) T-duality symmetry



$$\frac{O(D,D)}{SO(D-1,1)^2} \ni g_{mn}, B_{mn}$$

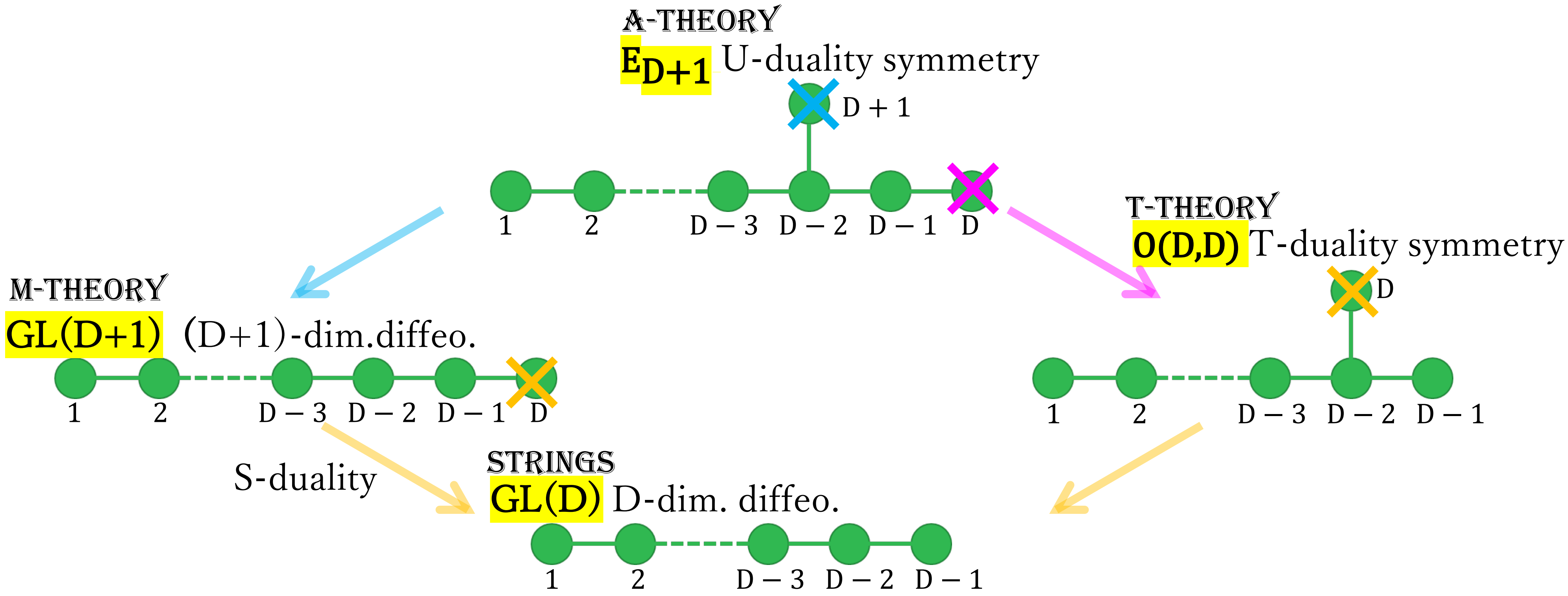
STRINGS

GL(D) D-dim. diffeo.



$$\frac{GL(D)}{SO(D-1,1)} \ni g_{mn}$$

Dynkin of duality symmetry groups



A-THEORY

E_{D+1} U-duality symmetry

T-THEORY

O(D,D) T-duality symmetry

M-THEORY

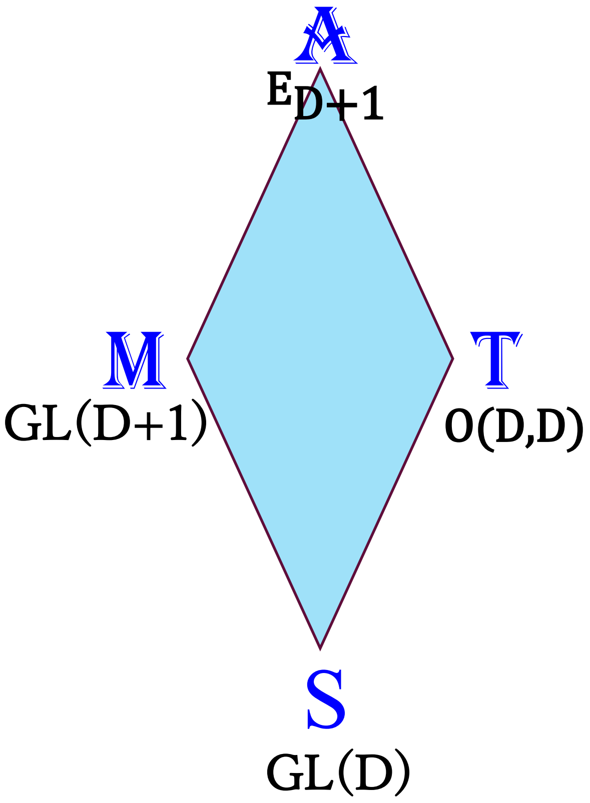
GL(D+1) (D+1)-dim.diffeo.

S-duality

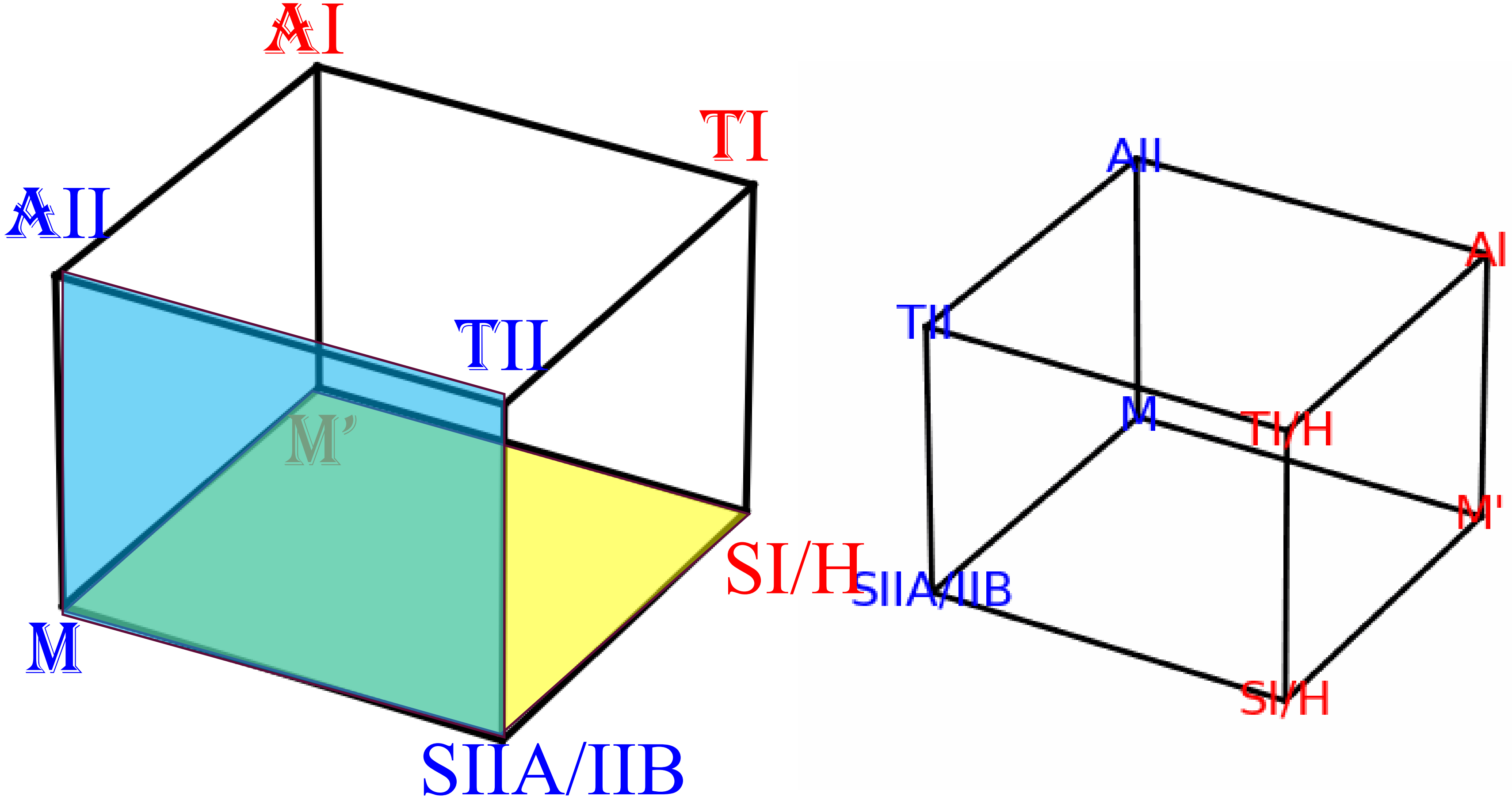
STRINGS

GL(D) D-dim. diffeo.

A-theory cube & diamond



“Diamond” in F-theory with worldvolime sectioning, ‘15 Linch & Siegel



Open F-branes ‘23 M.H. & Siegel

II A-theory

- Strategy
- A5-brane with SL(5) U-duality symmetry
- Current algebra
- Virasoro constraints
- Hamiltonian
- Lagrangian

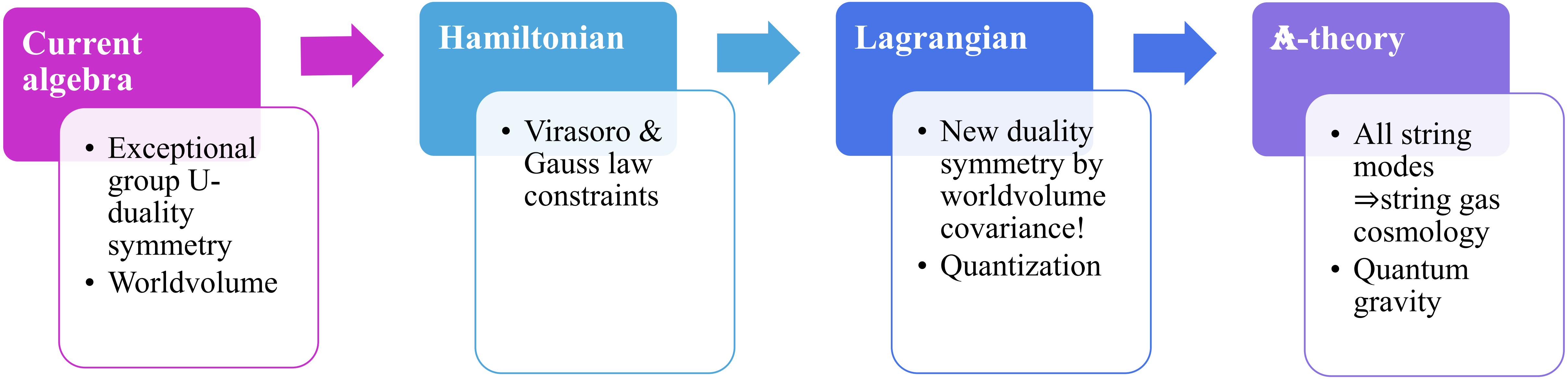
A-theory: strategy

‘93 Siegel: What is the string theory with manifest T-duality symmetry? ⇒ T-theory

’14 Siegel: What are the brane theories with manifest U-duality symmetry? ⇒ A-theory

T-theory’13～ Siegel, Polacek, M.H. , Kamimura, …

A-theory’14～ Siegel, Polacek, Linch, D. Wang, Y-T. Wang, Hulik, M.H. …



Current algebra

- Exceptional group U-duality symmetry
- Worldvolume

Hamiltonian

- Virasoro & Gauss law constraints

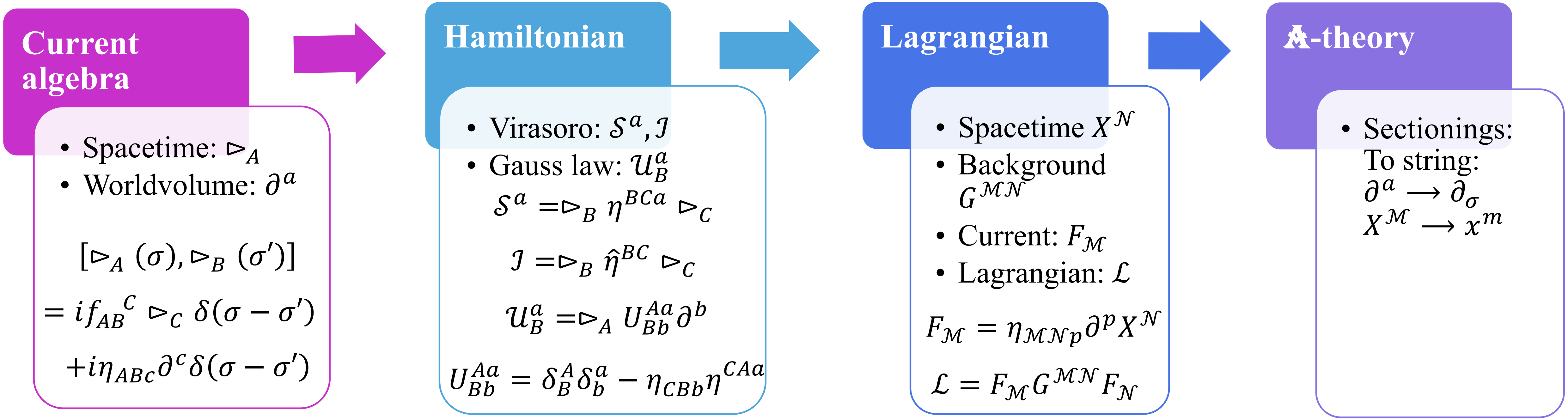
Lagrangian

- New duality symmetry by worldvolume covariance!
- Quantization

A-theory

- All string modes ⇒string gas cosmology
- Quantum gravity

A-theory: strategy



Current algebra

- Spacetime: $\rhd_A$
- Worldvolume: ∂^a

$$[\rhd_A(\sigma), \rhd_B(\sigma')]$$
$$= i f_{AB}{}^C \rhd_C \delta(\sigma-\sigma')$$
$$+ i\eta_{ABc}\partial^c\delta(\sigma-\sigma')$$

Hamiltonian

- Virasoro: $\mathcal{S}^a, \mathcal{J}$
- Gauss law: $\mathcal{U}_B^a$

$$\mathcal{S}^a = \rhd_B \, \eta^{BCa} \, \rhd_C$$
$$\mathcal{J} = \rhd_B \, \hat{\eta}^{BC} \, \rhd_C$$
$$\mathcal{U}_B^a = \rhd_A \, U_{Bb}^{Aa} \partial^b$$
$$U_{Bb}^{Aa} = \delta_B^A \delta_b^a - \eta_{CBb}\eta^{CAa}$$

Lagrangian

- Spacetime $X^{\mathcal{N}}$
- Background $G^{\mathcal{MN}}$
- Current: $F_{\mathcal{M}}$
- Lagrangian: $\mathcal{L}$

$$F_{\mathcal{M}} = \eta_{\mathcal{MN}p}\partial^p X^{\mathcal{N}}$$
$$\mathcal{L} = F_{\mathcal{M}} G^{\mathcal{MN}} F_{\mathcal{N}}$$

A-theory

- Sectionings: To string: $\partial^a \longrightarrow \partial_\sigma$ $X^{\mathcal{M}} \longrightarrow x^m$

◆ Translation $p_m = \frac{\partial}{i\partial x^m}$ → covariant derivative ∇_m→ stringy covariant derivative $\rhd_M$

𝔸5-brane with SL(5) U-duality symmetry

𝔸5-brane in 10-dim. spacetime

- Spacetime : 10 X^{mn} , $m = 1, \cdots 5$
- Worldvolume: 5+1 σ^m, τ

★ Both spacetime & worldvolume are representations of SL(5)

⇒ Current algebra is SL(5) covariant!

SL(5) 𝔸5 current algebra

$$[\rhd_{mn}(\sigma), \rhd_{lk}(\sigma')] = 2i\,\epsilon_{mnlkp}\,\partial^p \delta^{(5)}(\sigma - \sigma')$$

$$\rhd_{mn} = P_{mn} + \frac{1}{2}\epsilon_{mnlkp}\,\partial^l X^{kp}$$

SL(5) inv. tensor : $\epsilon_{m_1 \cdots m_5}$

D=3 A-theory ’14 Polacek & Siegel, ’15 Linch , Siegel, D.Wang, ’23 M.H., Hulek,Linch, Siegel, D.Wang, Y-P. Wang

A5 Virasoro & Gauss law constraints

A5-brane in 10-dim. spacetime

Virasoro : $\mathcal{S}^m = \frac{1}{16} \rhd_{nl} \, \underline{\epsilon^{mnlkp}} \, \rhd_{kp} = 0$ Worldvolume σ-diffeo.

$\mathcal{J} = \frac{1}{16} \rhd_{mn} \underline{\delta^{mn;lk}} \rhd_{lk} = 0$ Worldvolume τ-diffeo.

Gauss law: $\mathcal{U}_m = \partial^n \rhd_{mn} = 0$ Gauge transf. of X^{mn}

- SO(5) inv. tensor :$\delta^{mn;lk}$
- In curved background: $\frac{\mathrm{SL}(5)}{\mathrm{SO}(5)} \ni E_a{}^m$, $E_a{}^m E_b{}^n \, \delta^{ac} \delta^{bd} E_c{}^l E_d{}^k = G^{mn;lk}$, $\rhd_{ab} = E_a{}^m E_b{}^n \rhd_{mn}$

$\mathcal{S}^a$ inert ⇒ section condition , $\mathcal{J} = \frac{1}{16} \rhd_{mn} G^{mn;lk} \rhd_{lk}$ ⇒ kinetic term in curved b.g.

𝔄5 Hamiltonian

𝔄5-brane in 10-dim. spacetime

Hamiltonian

👇 auxiliary coordinate for the Gauss law

$$H = g\mathcal{J} + s_m\mathcal{S}^m + \tilde{g}\tilde{\mathcal{J}} + \tilde{s}_m\tilde{\mathcal{S}}^m + Y^m\mathcal{U}_m$$

$\mathcal{J}, \mathcal{S}^m$: Bilinears of selfdual currents $\rhd_{mn} = P_{mn} + \frac{1}{2}\epsilon_{mnlkp}\,\partial^l\,X^{kp}$ physical

$\tilde{\mathcal{J}}, \tilde{\mathcal{S}}^m$: Bilinears of anti-selfdual currents $\tilde{\rhd}_{mn} = P_{mn} - \frac{1}{2}\epsilon_{mnlkp}\,\partial^l\,X^{kp}$ unphysical

$\tilde{\rhd}_{mn} = 0$ Selfduality condition !

$$\text{with}\quad [\rhd_{mn}(\sigma), \tilde{\rhd}_{lk}(\sigma')] = 0$$

- Anti-selfdual parts are needed for the worldvolume covariance

A5 Lagrangian

A5-brane in 15-dim. spacetime

- Spacetime: 10+5 X^{mn} , Y^m, $m = 1, \cdots 5$
- Worldvolume: 5+1 σ^m, τ
- Field strength: 10+10' $F_\tau{}^{mn} = \dot{X}^{mn} - \partial^{[m}Y^{n]}$, $F_{\sigma\, mn} = \frac{1}{2}\epsilon_{mnlkp}\partial^l X^{kp}$; $F_{SD/\overline{SD}} = F_\tau \pm F_\sigma$
- Gauge parameter: 5+1 κ^m , κ

★ Using auxiliary anti-selfdual currents for conformal wv

'84 Siegel, '18∼ M.H. & Siegel, '22 M.H., Mori, Sasaki & Yata

Hamiltonian form action ⇒SL(5) A5 action

$$I = \int d\tau d^5\sigma \left[\underbrace{\varphi F_{SD} F_{\overline{SD}}}_{\text{👆 Free kinetic term}} + \underbrace{\phi F_{\overline{SD}}{}^2 + \phi_1^m \epsilon_{mnlkp} F_{\overline{SD}}{}^{nl} F_{\overline{SD}}{}^{kp} + \phi_{mn} F_{\overline{SD}}{}^{ml} F_{\overline{SD}}{}^{n}{}_{l}}_{\text{selfduality constraints}}\right]$$

in conformal gauge

$$I = \int d\tau d^5\sigma \left[\frac{1}{2}(\dot{X}^{mn} - \partial^{[m}Y^{n]})^2 - \frac{1}{12}(\partial^{[m}X^{nl]})^2\right]$$

A5 Lagrangian with new symmetry SL(6)

A5-brane in 15-dim. spacetime

- Spacetime: 15 $X^{\hat{m}\hat{n}}$, $\hat{m}=0,1,\cdots,5$
- Worldvolume: 6 $\sigma^{\hat{m}}$
- Field strength: 20 $F^{\hat{m}\hat{n}\hat{l}}=\partial^{[\hat{m}}X^{\hat{n}\hat{l}]}$
- Gauge parameter: 6 $\kappa^{\hat{m}}$

👉 New U-duality symmetry SL(6)

SL(6) A5 action

$$I=\int d^6\sigma\left[\underline{\varphi(F^{\hat{m}\hat{n}\hat{l}})^2}+\underline{\phi_{\hat{m}\hat{n}}F^{\hat{m}}{}_{\hat{l}\hat{k}}F^{\hat{n}\hat{l}\hat{k}}+\phi^{\hat{m}\hat{n}}\epsilon_{\hat{m}\hat{l}\hat{k}\hat{p}\hat{q}\hat{r}}F^{\hat{l}\hat{k}\hat{p}}F^{\hat{q}\hat{r}}{}_{\hat{n}}}\right]$$

👆 Free kinetic term + selfduality constraints

in conformal gauge

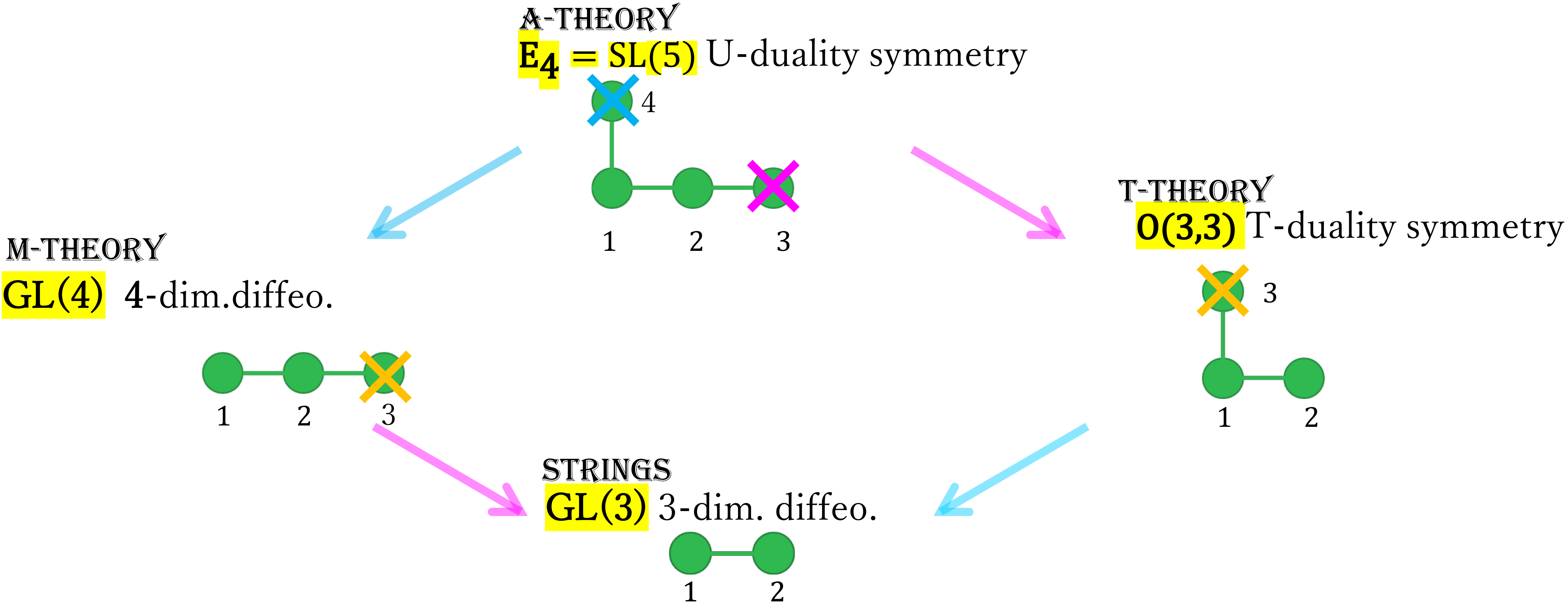
$$I=\int d^6\sigma\frac{1}{12}(\partial^{[\hat{m}}X^{\hat{n}\hat{l}]})^2$$

III Sectionings

- Consistent reduction
- T-string
- M5-brane
- SUGRA M2-brane

順天堂
Juntendo, Tokyo, established in 1838

Dynkin of D=3 theories



A-THEORY

$E_4 = SL(5)$ U-duality symmetry

M-THEORY

$GL(4)$ 4-dim.diffeo.

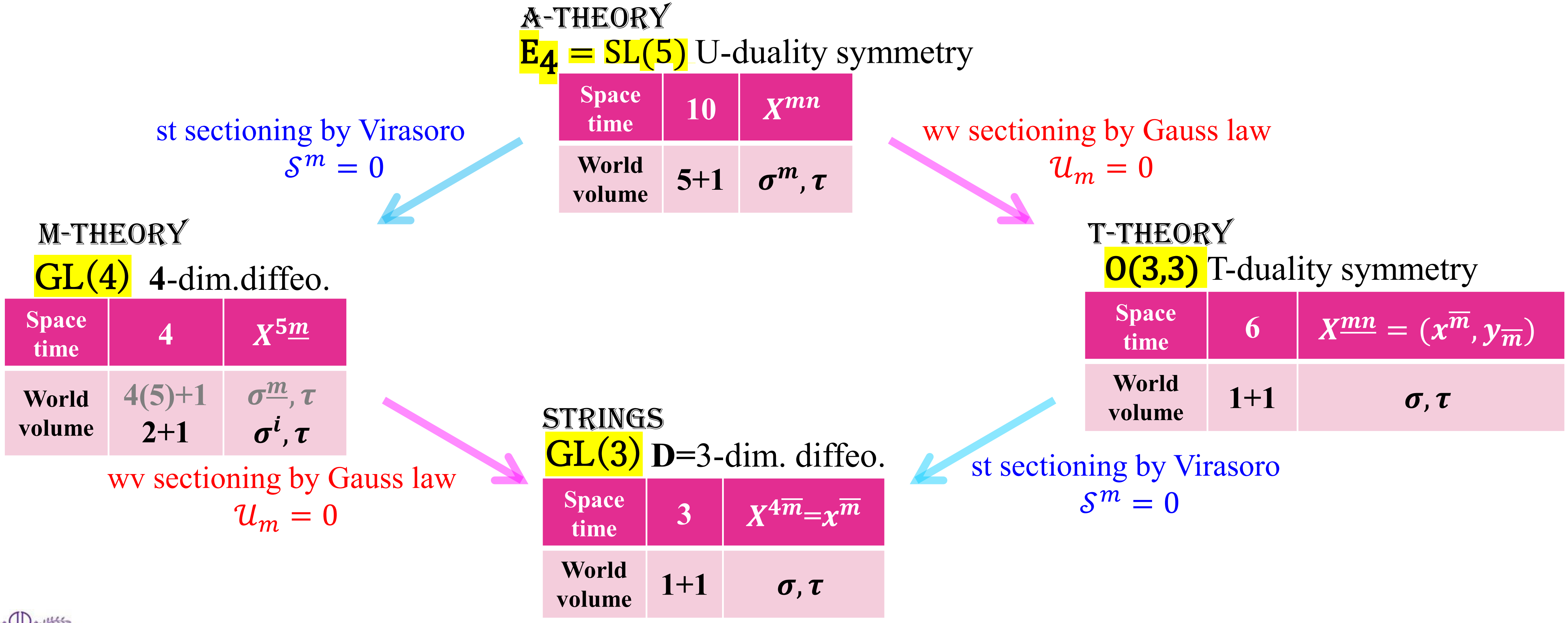
T-THEORY

$O(3,3)$ T-duality symmetry

STRINGS

$GL(3)$ 3-dim. diffeo.

Reducing spacetime & worldvolume



A-THEORY

$\mathrm{E}_4 = \mathrm{SL}(5)$ U-duality symmetry

Space time	**10**	X^{mn}
World volume	**5+1**	σ^m, τ

st sectioning by Virasoro
$\mathcal{S}^m = 0$

wv sectioning by Gauss law
$\mathcal{U}_m = 0$

M-THEORY

$\mathrm{GL}(4)$ 4-dim.diffeo.

Space time	**4**	$X^{5\underline{m}}$
World volume	**4(5)+1** **2+1**	$\sigma^{\underline{m}}, \tau$ σ^i, τ

T-THEORY

$\mathrm{O}(3{,}3)$ T-duality symmetry

Space time	**6**	$X^{\underline{mn}} = (x^{\overline{m}}, y_{\overline{m}})$
World volume	**1+1**	σ, τ

wv sectioning by Gauss law
$\mathcal{U}_m = 0$

STRINGS

$\mathrm{GL}(3)$ **D**=3-dim. diffeo.

Space time	**3**	$X^{4\overline{m}}{=}x^{\overline{m}}$
World volume	**1+1**	σ, τ

st sectioning by Virasoro
$\mathcal{S}^m = 0$

Reducing worldvolume by Gauss law

A-THEORY

$E_4 = SL(5)$ U-duality symmetry

Space time	**10**	X^{mn}
World volume	**5+1**	σ^m, τ

wv sectioning by Gauss law

$\mathcal{U}_m = 0$

T-THEORY

$O(3,3)$ T-duality symmetry

Space time	**6**	$X^{\underline{mn}} = (x^{\overline{m}}, y_{\overline{m}})$
World volume	**1+1**	σ, τ

Reduce WV by Gauss law: $\mathcal{U}_m|_{0-mode} = \partial^n P_{mn} = 0\,, m = 1\sim 5, \underline{m} = 1\sim 4$

$$\mathcal{U}_{\underline{m}}|_{0-mode} = \cancel{\partial^{\underline{n}}} P_{\underline{mn}} + \partial^5 P_{\cancel{\underline{m}5}} = 0\,, \mathcal{U}_5|_{0-mode} = \cancel{\partial^{\underline{m}}} P_{5\underline{m}} = 0$$

$\Rightarrow \quad \partial^{\underline{m}} = 0,\ \partial^5 \neq 0 = \partial_\sigma$ ➡ string

$P_{5\underline{m}} = 0,\ P_{\underline{mn}} \neq 0$ ➡ 6-dim. space: $X^{\underline{mn}} = (x^{\overline{m}}, y_{\overline{m}})\,, \overline{m} = 1,2,3$

Survived Virasoro:

$$\mathcal{S}^5 = \frac{1}{16}\ \epsilon^{\underline{mnlk}} (P_{\underline{mn}} + \frac{1}{2} \epsilon_{\underline{mnpq}} \partial^5 X^{\underline{pq}})\ (P_{\underline{lk}} + \frac{1}{2} \epsilon_{\underline{lkst}} \partial^5 x^{\underline{st}})$$ ➡ T-string

O(3,3) $\mathbb{T}$-string current algebra

$\mathbb{T}$-string in 6-dim. spacetime

- Spacetime: 3+3 $X^{4\overline{m}} = x^{\overline{m}},\ X^{\overline{nl}} = -\epsilon^{\overline{mnl}} y_{\overline{m}},\quad \overline{m} = 1,2,3$
- Worldvolume: 1+1 $\sigma^5 = \sigma, \tau$

O(3,3) current algebra

$$\left[\rhd_{\overline{m}}(\sigma), \rhd^{\overline{n}}(\sigma')\right] = 2i\,\delta_{\overline{m}}^{\overline{n}}\partial_\sigma\delta(\sigma-\sigma')$$

★ O(3,3) invariant metric: $\eta = \begin{pmatrix} 0 & \delta_{\overline{m}}^{\overline{n}} \\ \delta_{\overline{n}}^{\overline{m}} & 0 \end{pmatrix}$

$\rhd_{\overline{m}} \equiv \rhd_{4\overline{m}} = p_{\overline{m}} + \partial_\sigma y_{\overline{m}}$, $\rhd^{\overline{m}} \equiv -\frac{1}{2}\epsilon^{\overline{mnl}}\rhd_{\overline{nl}} = \tilde{p}^{\overline{m}} + \partial_\sigma x^{\overline{m}}$

O(3,3) 𝕋-string background

𝕋-string in 6-dim. spacetime

- Spacetime: 6 $X^{\underline{mn}}$, $\underline{m} = 1, \cdots, 4$
- Worldvolume: 2 $\sigma^5 = \sigma,\ \sigma^0 = \tau$

𝕋-string currents: $F^{0\underline{mn}} = \partial_\tau X^{\underline{mn}}$, $F^{5\underline{mn}} = \partial_\sigma X^{\underline{mn}}$, others=0

Curved background in SL(6) vielbein: $E_{\hat m}{}^{\hat a} E_{\hat n}{}^{\hat b} E_{\hat l}{}^{\hat c} F^{\hat m \hat n \hat l}$, $\hat m = 0{,}1 \sim 5$, $E_{\underline m}{}^{\underline a} \ni g_{\overline{mn}}\,, B_{\overline{mn}}$

SL(6) vielbein for 𝕋-string:

Worldsheet zweibein!

$$E_{\hat m}{}^{\hat a} = \begin{pmatrix} E_0{}^{\hat 0} & E_0{}^{\hat 5} & E_0{}^{\underline a} \\ E_5{}^{\hat 0} & E_5{}^{\hat 5} & E_5{}^{\underline a} \\ E_{\underline m}{}^{\hat 0} & E_{\underline m}{}^{\hat 5} & E_{\underline m}{}^{\underline a} \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} 1/g \\ -s/g \end{matrix}} & \begin{matrix} 0 \\ 1 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ 0 & 0 & g^{1/4}\boxed{E_{\underline m}{}^{\underline a}} \end{pmatrix}$$

★ Spacetime & worldsheet mixing!

Spacetime vielbein

Reducing spacetime by Virasoro

A-THEORY

$\mathbf{E_4} = \mathrm{SL}(5)$ U-duality symmetry

Space time	**10**	X^{mn}
World volume	**5+1**	σ^m, τ

st sectioning by Virasoro

$\mathcal{S}^m = 0$

M-THEORY

$\mathrm{GL}(4)$ 4-dim.diffeo.

Space time	**4**	$X^{5\underline{m}}$
World volume	**4(5)+1** **2+1**	$\sigma^{\underline{m}}, \tau$ σ^i, τ

Reduce ST by Virasoro: $\mathcal{S}^m|_{0-mode} = \frac{1}{16} P_{nl}\epsilon^{mnlkp}P_{kp} = 0\,, \underline{m} = 1 \sim 4$

$$\mathcal{S}^{\underline{m}}|_{0-mode} = \frac{1}{4} P_{5\underline{n}}\epsilon^{\underline{m}5\underline{nlk}}\cancel{P_{\underline{lk}}} = 0,\ \mathcal{S}^5|_{0-mode} = \frac{1}{16} \cancel{P_{\underline{mn}}}\epsilon^{\underline{mnlk}}\cancel{P_{\underline{lk}}} = 0$$

$\Rightarrow P_{\underline{mn}} = 0, P_{5\underline{m}} \neq 0$ ➡ 4-dim. space: $X^{5\underline{m}} = x^{\underline{m}}$

Survived constraints

Virasoro: $\mathcal{S}^{\underline{m}} = \frac{1}{2} p_{\underline{n}}\partial^{[\underline{m}}x^{n]}, \mathcal{S}^5 = \frac{1}{4}\epsilon_{\underline{mnlk}}(\partial^{\underline{m}}x^{\underline{n}})(\partial^{\underline{l}}x^{\underline{k}})$ ➡ M5-brane

Gauss law: $\mathcal{U}_5 = \partial^{\underline{m}}\, p_{\underline{m}}\,,\ \mathcal{U}_{\underline{m}} = \partial^5 p_{\underline{m}}$

GL(4) 𝕄5-brane

𝕄5-brane in 4-dim. spacetime

- Spacetime: 4 $X^{5\underline{m}} = x^{\underline{m}}, \quad (X^{05} = Y), \underline{m} = 1, \cdots, 4$
- Worldvolume: 6 $\sigma^{\hat{m}}$
- Field strength: 10 $F^{5\check{m}\check{n}} = \partial^{[\check{m}} X^{\check{n}]5} \quad , \ \check{m} = 0,1, \dots ,4$
- Gauge parameter: 5 $\kappa^{\check{m}}$

𝕄5 Lagrangian in 4-dim. SUGRA b.g.

$$L_0 = \frac{\mathrm{e}}{2}\left[\left(\dot{x}^{\underline{m}} + \partial^{\underline{m}}Y + s_{\underline{l}}\partial^{[\underline{m}}x^{\underline{l}]}\right) g_{\underline{mn}} \left(\dot{x}^{\underline{n}} + \partial^{\underline{n}}Y + s_{\underline{k}}\partial^{[\underline{n}}x^{\underline{k}]}\right) - \frac{1}{2}\partial^{[\underline{m}}x^{\underline{l}]}g_{\underline{mn}}g_{\underline{kl}}\partial^{[\underline{n}}x^{\underline{k}]}\right]$$

$$L_{WZ} = (\dot{x}^{\underline{m}} + \partial^{\underline{m}}Y)C_{\underline{mnl}}\partial^{\underline{n}}x^{\underline{l}} + \frac{1}{6}\partial^{\underline{m}}x^{\underline{n}}s_{\underline{[m}}C_{\underline{nlk}]}\partial^{\underline{l}}x^{\underline{k}}$$

GL(4) 𝕄5-brane background

𝕄5-brane in 4-dim. spacetime

SL(6) vielbein: $m = 1, \cdots, 5$ Worldvolume vielbein

$$E_{\hat m}{}^{\hat a} = \begin{pmatrix} E_0{}^{\hat 0} & E_0{}^{a} \\ E_m{}^{\hat 0} & E_m{}^{a} \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} 1/g \\ -s_m/g \end{matrix}} & \begin{matrix} 0 \\ g^{1/5}\boxed{E_m{}^{a}} \end{matrix} \end{pmatrix}$$

Spacetime vielbein

👇 3-form gauge fields

$$\boxed{E_m{}^{a}} = \begin{pmatrix} E_5{}^{5} & E_5{}^{\underline{a}} \\ E_{\underline{m}}{}^{5} & E_{\underline{m}}{}^{\underline{a}} \end{pmatrix} = \begin{pmatrix} \mathbf{e}^{3/5} & \mathbf{e}^{-2/5}\tilde{C}^{\underline{m}} e_{\underline{n}}{}^{\underline{a}} \\ 0 & \mathbf{e}^{-2/5} e_{\underline{m}}{}^{\underline{a}} \end{pmatrix}$$

$$\tilde{C}^{\underline{m}} = \frac{1}{3!}\epsilon^{\underline{mnlk}} C_{\underline{nlk}}, \quad \mathbf{e} = \det e_{\underline{m}}{}^{\underline{a}}$$

SUGRA M2-brane

- Action for a M2 in SUGRA : $I = \int d^3\sigma\,(L_0 + L_{WZ})$, $\mu = 0,1,2,\ i = 1,2$

  $L_{NG} = -T\sqrt{-\mathit{det}\ \partial_\mu x^m\, \partial_\nu x^n\, g_{mn}}$, $L_{WZ} = \frac{T}{3!}\epsilon^{\mu\nu\rho}\ \partial_\mu x^m\, \partial_\nu x^n\, \partial_\rho x^l\, C_{mnl}$ Non-perturbative

  U-duality symmetry of 4+7=11 dim. is SL(5) ⇒ Focus 4-dim. part $x^{\underline{m}}$, $\underline{m} = 1,\cdots 4$

- Currents: ’12 M.H. & Kamimura

  $\rhd_{\underline{m}} = p_{\underline{m}}$ Momentum , $\rhd_{\underline{mn}} = \frac{1}{2}\epsilon_{\underline{mnlk}}\,\epsilon^{ij}\partial_i x^{\underline{l}}\,\partial_j x^{\underline{k}}$ M2-brane volume

- SUGRA M2-brane current algebra:

$$\begin{aligned}
&\left[\rhd_{\underline{m}}(\sigma), \rhd_{\underline{n}}(\sigma')\right] = 0\\
&\left[\rhd_{\underline{m}}(\sigma), \rhd_{\underline{nl}}(\sigma')\right] = 2i\epsilon_{\underline{mnlk}}\epsilon^{ij}\partial_i x^{\underline{l}}\ \partial_j\ \ \delta^2(\sigma - \sigma')\\
&\left[\rhd_{\underline{mn}}(\sigma), \rhd_{\underline{lk}}(\sigma')\right] = 0
\end{aligned}$$

SUGRA M2-brane from M5

- M5 Lagrangian:

$$L = (F^{05\underline{a}})^2 - (F^{5\underline{ab}})^2 + \epsilon_{\underline{mnlk}}\left[\partial_\tau\left(x^{\underline{m}}\,\partial^{\underline{n}} y^{\underline{lk}}\right) - \partial^{\underline{n}}(x^{\underline{m}}\,\partial_\tau y^{\underline{lk}})\right] + \cdots$$

Perturbative

- Spacetime: 4 $X^{5\underline{m}} = x^{\underline{m}}, X^{\underline{mn}} = y^{\underline{mn}}, X^{0m} = Y^m, \underline{m} = 1,\cdots,4$
- Worldvolume: 2+1 $\boxed{\partial^{\underline{m}} = \epsilon^{ij}\partial_i x^{\underline{m}}\partial_j}, \partial^0 = \tau, \partial^5 = 0\,,\ i = 1{,}2$

Non-perturbative projection

’24 M. H., Hulik, Linch, Siegel, D. Wang, Y-P. Wang

- Field strengths: 10 $F^{05\underline{m}} = \partial_\tau x^{\underline{m}}\,, F^{5\underline{mn}} = \partial^{[\underline{n}} x^{\underline{m}]}\,, F^{0\underline{mn}} = \partial_\tau y^{\underline{mn}}\,, F^{\underline{mnl}} = \frac{\partial^{[\underline{m}} y^{\underline{nl}]}}{2}$
- Curved background: $E_{\hat{m}}{}^{\hat{a}} E_{\hat{n}}{}^{\hat{b}} E_{\hat{l}}{}^{\hat{c}} F^{\hat{m}\hat{n}\hat{l}}$
- 6-bein gauge fixing

$$s_{\underline{m}}\epsilon^{ij}\partial_j x^{\underline{m}} = -\frac{h^{0i}}{h^{00}}, g^2 = \frac{-1}{h(h^{00})^2}, s_5 = 0, h_{ij} = \partial_i x^{\underline{m}}\, g_{\underline{mn}}\partial_j x^{\underline{n}}\,, \phi = -\frac{\sqrt{-h}h^{00}}{2}\,, h = \det h_{\mu\nu}$$

⇒ Non-perturbative SUGRA M2 Lagrangian !

IV Conclusions

Conclusion & future topics

- SL(5) U-duality symmetry is realized by the 𝔸5-brane current algebras and actions.

- The worldvolume Lorentz covariance requires SL(5) to SL(6) which is new duality symmetry!

- We showed how perturbative 𝔸5-brane with SL(5) reduces to 𝕋-string with O(3,3) , string with GL(3) & non-perturbative M2-brane in SUGRA.

<u>Future topics</u>

- Higher dimensional generalization, quantization, scattering amplitude, constructing gravity theory, application to plank scale gravity theory,…