Holographic chiral dynamics in confinement at finite baryon density

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Abstract

We study chiral condensate in a cold nuclear matter based on a holographic theory, which would be dual to a baryon system in quantum chromodynamics (QCD) in the confinement phase. We found that the magnitude of the chiral condensate C obtained in our model decreases with increasing baryon density. The level of decreasing depends on the quark mass m_q. When $m_q = 0$, C decreases to zero and the chiral symmetry is restored. In case of $m_q \neq 0$, C decreases but does not tend to 0 and the chiral breaking is reserved. We also calculate the renormalized action S_{ren} and obtain the equation of state (EoS) for the cold nuclear matter.

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1 Introduction

It is important to study chiral condensates in quantum chromodynamics (QCD) in order to understand the dynamical properties dominated by the Yang-Mills theory in the strong-coupling regime. The AdS/CFT duality is very useful to approach this kind of dynamics. It is also useful when we evaluate the value of such condensate considered in QCD in a hadronic matter system based on QCD.

We propose a holographic model, which is supposed to be dual to a low temperature baryonic matter. By using this model, we can execute the calculation of physical quantities in baryonic matter. Here they are performed in the confinement phase at low temperature and small baryon number density. Our model is based on the type IIB theory, and $N_f(=2)$ flavor probe D7 branes are introduced. According to the model mentioned above, various physical quantities are obtained by solving the equations of motion (EOMs) of bulk fields set appropriately through our holographic effective action for probes. Then we can obtain the chiral condensate $C = \langle \bar{q}q \rangle$ at some definite baryon number density which is evaluated by the chemical potential μ.

We also calculate the renormalized action S_{ren} for chiral breaking solution with $C \neq 0$. The action can be compared by that of chiral symmetric solution with $C = 0$. We conclude that C decreases according to the increased baryon density. When the quark mass $m_q = 0$, C decreases to zero and the chiral symmetry is restored. In case of $m_q \neq 0$, C decreases but does not reach zero and the chiral breaking is reserved. We calculate μ dependence of S_{ren} and estimate the equation of state (EoS) for the cold nuclear matter. Numerical analysis of C and S_{ren} is given in §4.

2 D3/D7 model for confining YM theory

We start from 10d IIB model retaining the dilaton Φ, axion χ and self-dual five form field strength $F_{(5)}$. Under the Freund-Rubin ansatz for $F_{(5)}$, $F_{\mu_1\cdots\mu_5} = -\sqrt{\Lambda}/2\ \epsilon_{\mu_1\cdots\mu_5}$ [1, 2], and for the 10d metric as $M_5 \times S^5$ or $ds^2 = g_{MN}dx^M dx^N + g_{ij}dx^i dx^j$, we find the solution.

The five dimensional M_5 part of the solution is obtained by solving the reduced 5d action,

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R + 3\Lambda - \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}e^{2\Phi}(\partial\chi)^2\right), \tag{2.1}$$

which is written in the string frame and taking $\alpha' = g_s = 1$.

The solution is obtained under the ansatz,

$$\chi = -e^{-\Phi} + \chi_0\ , \tag{2.2}$$

which is necessary to obtain supersymmetric solution. The solution is expressed as

$$ds^2_{10} = G_{MN}dX^M dX^N$$

$$= e^{\Phi/2}\left\{\frac{r^2}{R^2}A^2(r)\left(-dt^2+(dx^i)^2\right)+\frac{R^2}{r^2}dr^2+R^2d\Omega_5^2\right\} . \tag{2.3}$$

Then, in the non-supersymmetric case, the solution is given by (2.3) and

$$A(r)=\left((1-(\frac{r_0}{r})^8)\right)^{1/4}, \qquad e^{\Phi}=\left(\frac{(r/r_0)^4+1}{(r/r_0)^4-1}\right)^{\sqrt{3/2}}, \qquad \chi=0\,. \tag{2.4}$$

This configuration has a singularity at the horizon $r=r_0$. So we cannot extend our analysis to near this horizon where higher curvature contributions are important. This theory provides confinement and chiral symmetry breaking. The latter means that we find non-zero chiral condensate for the massless quark. In other words, a dynamical quark mass would be generated for a massless quark in this theory. This point is different from the above supersymmetric background solution. The confinement is sustained by the gauge condensate, which is proportional to r_0^4 in the present case [1], as in the supersymmetric case[1, 2, 3].

3 D7 brane embedding and chiral condensate

The D7 brane is embedded in the above background (2.3). First, the extra six dimensional part of the metric (2.3) is rewritten as

$$\frac{R^2}{r^2}dr^2+R^2d\Omega_5^2=\frac{R^2}{r^2}\left(d\rho^2+\rho^2d\Omega_3^2+(dX^8)^2+(dX^9)^2\right) , \tag{3.1}$$

where $r^2=\rho^2+(X^8)^2+(X^9)^2$. Then we obtain the induced metric for D7 brane,

$$\begin{aligned} ds_8^2 &= e^{\Phi/2}\left\{\frac{r^2}{R^2}A^2\left(-dt^2+(dx^i)^2\right)\right. \\ &\qquad \left.+\frac{R^2}{r^2}\left((1+(\partial_\rho w)^2)d\rho^2+\rho^2d\Omega_3^2\right)\right\} , \end{aligned} \tag{3.2}$$

where we set as $X^8=w(\rho)$ and $X^9=0$ without loss of generality due to the rotational invariance in X^8-X^9 plane.

The brane action for the D7-probe is given as

$$S_{\rm D7}=-\tau_7\int d^8\xi e^{-\Phi}\sqrt{-\det\left(\mathcal{G}_{ab}+2\pi\alpha' F_{ab}\right)}\,, \tag{3.3}$$

where $F_{ab}=\partial_aA_b-\partial_bA_a$. $\mathcal{G}_{ab}=\partial_{\xi^a}X^M\partial_{\xi^b}X^NG_{MN}$ $(a,\ b=0,\dots,7)$ and $\tau_7=[(2\pi)^7g_s\,\alpha'^4]^{-1}$ represent the induced metric and the tension of D7 brane, respectively.

[1] This point is easily assured by expanding e^{Φ} in (2.4) by the powers of r_0/r.

Then, by taking the canonical gauge, we arrive at the D7 brane action,

$$S_{\rm D7} = -2\pi^2\tau_7 \int d^4x d\rho \rho^3 A^3 e^{\Phi} \sqrt{A^2(1+(w')^2) - (\tilde{A}_0')^2 e^{-\Phi}}\,, \tag{3.4}$$

$$= -2\pi^2\tau_7 \int d^4x d\rho\ L_7\,, \tag{3.5}$$

where

$$L_7 = B\sqrt{A^2(1+(w')^2) - (\tilde{A}_0')^2 e^{-\Phi}}\,, \tag{3.6}$$

$$B = \rho^3 A^3 e^{\Phi} \tag{3.7}$$

and $\tilde{A}_0 = 2\pi\alpha' A_0$.

Equations of motion for w and $\tilde{A}_0$ are given as

$$-\partial_\rho \left(B \frac{A^2 w'}{\sqrt{A^2(1+(w')^2) - (\tilde{A}_0')^2 e^{-\Phi}}} \right) + \frac{w}{r}\partial_r \left(B\sqrt{A^2(1+(w')^2) - (\tilde{A}_0')^2 e^{-\Phi}} \right) = 0\,, \tag{3.8}$$

$$-\partial_\rho \left(B \frac{\tilde{A}_0' e^{-\Phi}}{\sqrt{A^2(1+(w')^2) - (\tilde{A}_0')^2 e^{-\Phi}}} \right) = 0\,. \tag{3.9}$$

The second equation (3.9) is integrated as

$$B \frac{\tilde{A}_0' e^{-\Phi}}{\sqrt{A^2(1+(w')^2) - (\tilde{A}_0')^2 e^{-\Phi}}} = d\,, \tag{3.10}$$

where d is a constant. Using d, $\tilde{A}_0'$ in the D_7 action (3.4) is eliminated.

3.1 Chiral condensate and chemical potential

From the above EOM, we obtain w and A_0 numerically. By presumed asymptotic behaviour for w,

$$\lim_{\rho\to\infty} w = m_q + C/\rho^2 + \cdots, \tag{3.11}$$

we can read the value m_q and the chiral condensate C from the solutions.

Also, we obtain

$$\lim_{\rho\to\infty} A_0 = \mu - (d/2)/\rho^2 + \cdots, \tag{3.12}$$

where $(d/2)$ represents the baryon density and the chemical potential μ is given as

$$\mu = \int d\rho e^{-\Phi/2} d\sqrt{\frac{1+w'^2}{d^2 + B^2 e^{-\Phi}}}. \tag{3.13}$$

μ and d have one-to-one correspondence. The correspondence shifts slightly according to the solution. In Fig. 1, the correspondence is shown at the case of $C = 0$.

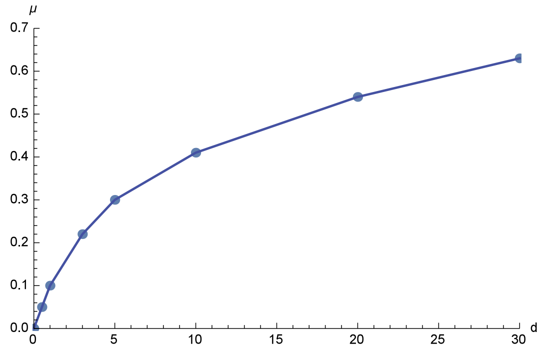

Fig. 1: μ vs. d in case of $C = 0$

3.2 Renormalized action

We obtain

$$S_{\rm D7} = -2\pi^2\tau_7 \int d^4x d\rho \; AB^2 e^{-\Phi/2} \sqrt{\frac{1+(w')^2}{B^2 e^{-\Phi} + d^2}} \,, \tag{3.14}$$

which is useful to estimate the on-shell action by substituting the solution w of Eq. (3.8). This integration diverges if we integrate to $\rho = \infty$. So we regulate with a cut-off at $\rho = \Lambda$. This divergence can be subtracted by the vacuum solution $w = 0$, $A_0 = 0$ [4].

$$S_0 = -2\pi^2\tau_7 \int_{r_0}^{\Lambda} \left(\frac{\rho^4 + r_0^4}{\rho^4 - r_0^4}\right)^{\sqrt{3/2}-1} \rho^3 d\rho \tag{3.15}$$

We can define the renormalized action by

$$S_{ren} = \lim_{\Lambda\to\infty} (S_{D7} - S_0) \,. \tag{3.16}$$

4 Numerical analysis of C and S_{ren}

For numerical solutions, we set $r_0 = 1.0$, $\Lambda = 200$, $-2\pi^2\tau_7 = 1$. As for the solution of w,we should solve by setting the integral constants C and m_q at $\rho = \infty$ but it is difficult. Practically, we put $w(r_0)$ and $w'(r_0)$ at $\rho = r_0$ and estimate C and m_q at $\rho = 200$. We show the example of solutions with $C = 0, 1.06$ at $d = 0$ in Fig. 2.

(i) Solutions $w(\rho)$ with $C = 0$ (chiral symmetric) and $C \neq 0$ (chiral breaking)

As mentioned above,we set initial conditions at $\rho = r_0$. We read C and m_q at $\rho = 200$. In case of $m_q = 0$,we set the initial condition in order to get $m_q < 10^{-5}$ at $\rho = 200$.

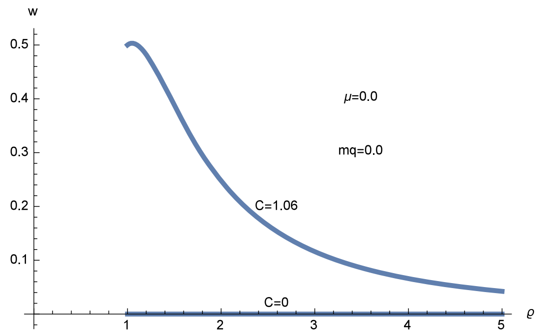

Fig. 2: $w(\rho)$ for $C = 0, 1.06$ with $m_q = 0$, $d = 0$

(ii) The action S_{ren} for solutions with various C

We obtain the action S_{ren} for various solutions with C in case of $m_q = 0$. They are shown in Fig. 3. We can see the value of C from the solution with the minimum S_{ren}. They are $C = 1.06,\ 0.98,\ 0.72$ at the baryon density $d/2 = 0,\ 1.5,\ 2.5$, respectively.

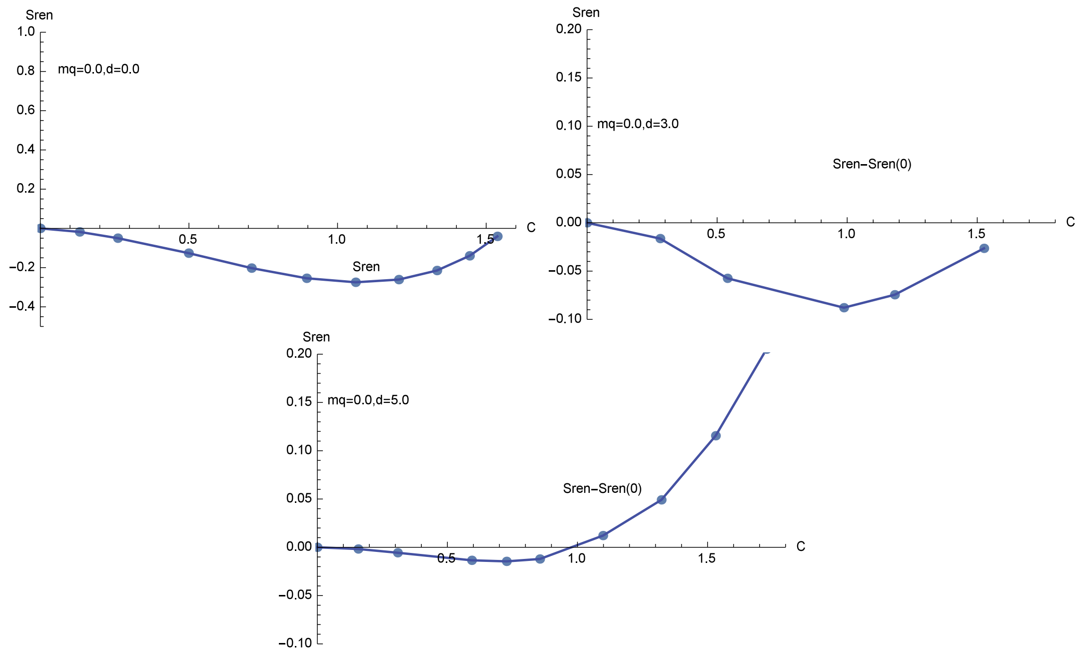

Fig. 3: S_{ren} vs. C at different baryon density. $S_{ren}(0)$ is the action of chiral symmetric solution.

(iii) The chiral condensate C in varying baryon density

We calculate μ dependence of C in case of $m_q = 0, 0.1, 0.5$. We can see that C decreases according to increasing baryon density. In case of $m_q = 0$ we can see that C tends to zero at $d \sim 7$ ($\mu \sim 0.36$). In case of $m_q \neq 0$, C decreases but does not reach zero. They are shown in Fig. 4.

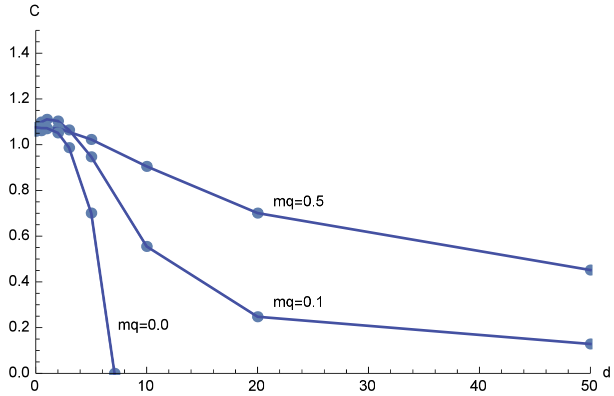

Fig. 4: C vs. d for $m_q = 0$, 0.1, 0.5.

(iv) μ dependence of the action S_{ren} for the solutions

In the following, we restrict the case of $m_q = 0$. We show the calculated S_{ren} of various phases in Fig. 5. We can evaluate them as

$$S(C = 0) \sim -28\mu^2 - 140\mu^4, \tag{4.1}$$

$$S(C \neq 0) \sim -0.2759 - 40\mu^2 - 30\mu^4 \tag{4.2}$$

We also refer S_{ren} of the deconfinement phase which is obtained analytically.

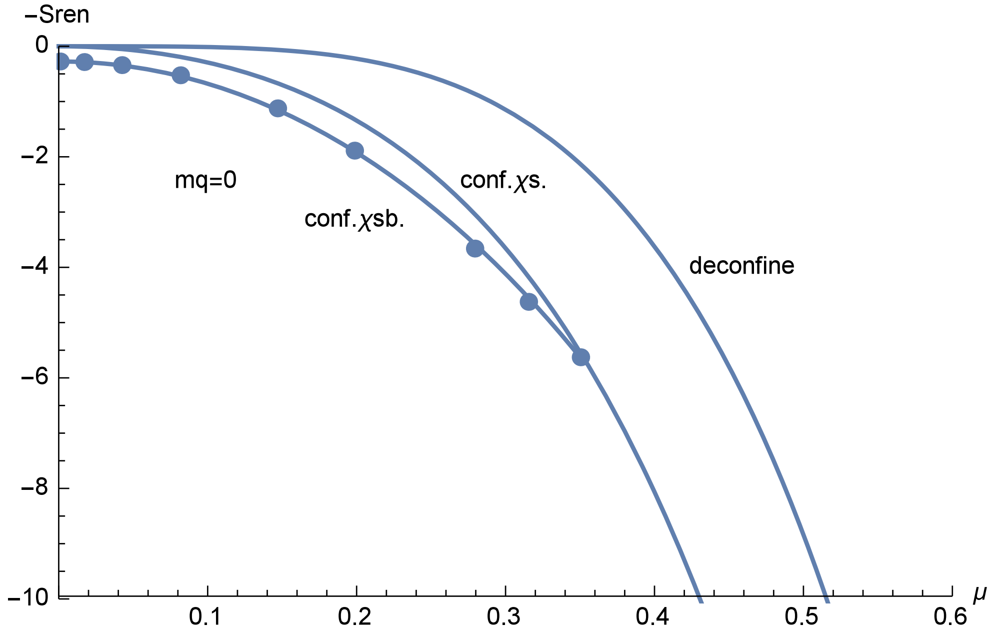

Fig. 5: μ dependence of S_{ren}. $\chi_s(\chi_{sb})$ means the chiral symmetric (broken) phase.

(v) EoS for cold nuclear matter

We can estimate the pressure $P(\mu)$ and the energy density $\epsilon(\mu)$ for cold nuclear matter from the relation $P(\mu) = -S_{ren}$ and $\epsilon(\mu) = \mu\partial P/\partial\mu - P$.

In the case of $m_q = 0$, for $\mu \leq 0.36$,

$$P \sim 0.2759 + 40\mu^2 + 30\mu^4, \tag{4.3}$$
$$\epsilon \sim -0.2759 + 40\mu^2 + 3 \times 30\mu^4. \tag{4.4}$$

For $\mu > 0.36$,

$$P \sim 28\mu^2 + 140\mu^4, \tag{4.5}$$
$$\epsilon \sim 28\mu^2 + 3 \times 140\mu^4. \tag{4.6}$$

EoS is shown in Fig. 6.

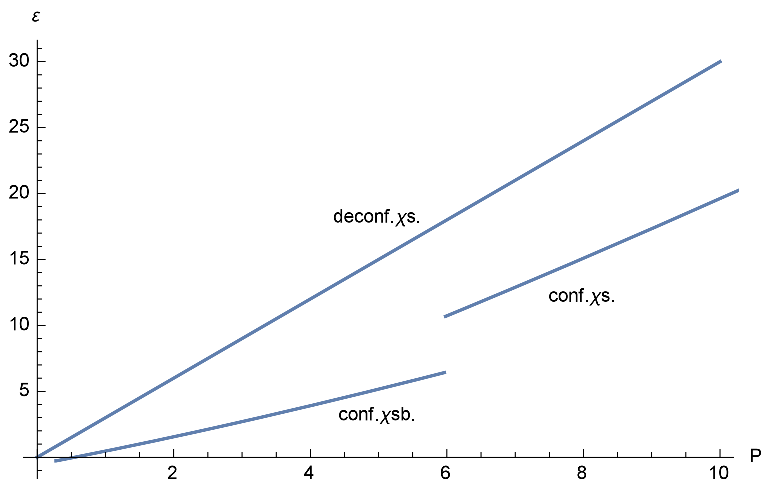

Fig. 6: The relation for ϵ and P. The line of $\epsilon = 3P$ shows the deconfined phase.

Also we get the sound velocity Cs as

$$Cs^2 = (\partial P(\mu)/\partial\mu)/(\partial\epsilon(\mu)/\partial\mu). \tag{4.7}$$

They are shown in Fig. 7.

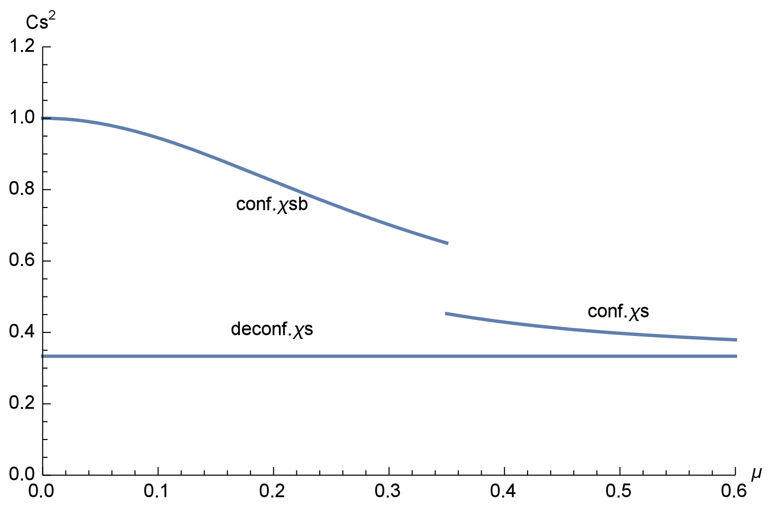

Fig. 7: μ dependence of Cs^2 in case of $m_q = 0$.

5 Summary and Discussion

We have calculated the renormalized DBI action in confinement phase at finite baryon number density. It is shown that the chiral condensate C decreases with increasing baryon density and chiral symmetry would be restored in case of $m_q = 0$. When $m_q \neq 0$, C decreases but does not reach zero and seems to approach the constant.

From μ dependence of the renormalized action we estimate the EoS for the cold nuclear matter in case of $m_q = 0$. In case of $m_q \neq 0$, we will discuss in the near future.

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