Toward Bound-state Approach to Strangeness in Holographic QCD

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arXiv:1009.0986, in press in PLB

RIKEN workshop, 17 Dec. 2010

#1: Skyrme from Sakai-Sugimoto

Sakai-Sugimoto model action

$$S = -\kappa \int d^4x dz \,\mathrm{Tr}\left[\frac{1}{2}h(z)F_{\mu\nu}^2 + k(z)F_{\mu z}^2\right] + \frac{N_c}{24\pi^2}\int_{\mathrm{M}_5} \omega_5$$

⇩ A_z=0 gauge + reduction to 4 dim.

Skyrme model action

$$S = \int d^4x \left[\frac{f_\pi^2}{4}\mathrm{Tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32e^2}\mathrm{Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2\right] + S_{\mathrm{WZW}}$$

	Skyrme	Sakai-Sugimoto
Baryon	Skyrmion	Instanton

#2: Hyperon = Skyrmion + Kaon

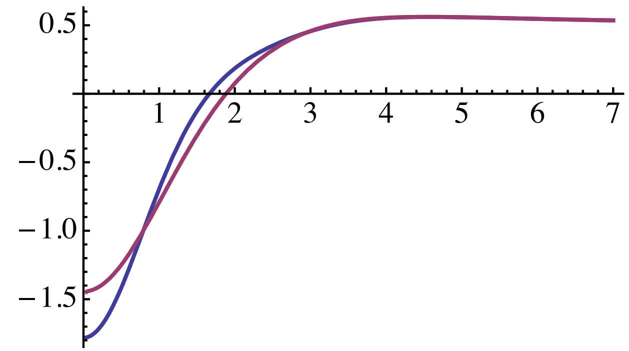
$$S = \int d^4x \left[\frac{f_\pi^2}{4} \mathrm{Tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32e^2} \mathrm{Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2 \right] + S_{\mathrm{WZW}}$$

Bound-state approach to strangeness [Callan-Klebanov]

ansatz: $U = \sqrt{U_\pi}\, U_K \sqrt{U_\pi}$

$$U_\pi = \begin{pmatrix} e^{iF(r)\hat{x}\cdot\tau} & 0_{1\times 2} \\ 0_{2\times 1} & 1 \end{pmatrix}, \quad U_K \sim \exp\left[\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix} \right]$$

A kaon in a binding potential provided by Skyrmion



The approach works well.

Is this a good approach?

**#1: Skyrme model action
from Sakai-Sugimoto model action**

- Baryon: Skyrmion ~ Instanton

**#2: The bound-state approach
works well in Skyrme model**

- There is a good ansatz.

**Any natural realization/explanation
from holographic QCD?**

Result: not straightforward

Kaon in Sakai-Sugimoto Model

$$S = -\kappa \int d^4x dz \, \mathrm{Tr}\left[\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2\right] + \frac{N_c}{24\pi^2} \int_{\mathrm{M}_5} \omega_5$$

Kaon as fluctuation around a baryon

2-flavor baryon: A^{inst}, Kaon: a_z $\quad a_z \sim K(x^\mu)\phi^{(0)}(z)$

$$A_\mu = \begin{pmatrix} A_\mu^{\mathrm{inst}} & 0 \\ 0 & 0 \end{pmatrix}, \quad A_z = \begin{pmatrix} A_z^{\mathrm{inst}} & a_z \\ a_z^\dagger & 0 \end{pmatrix}$$

- Quadratic in K: kaon in a potential

$$S = S_{\mathrm{inst}} + S_{\mathrm{kaon\ quad}}$$

- Kaon mass added (by hand): $\int d^4x \, m_K^2 K^\dagger K$

Different from Skyrme case

Kaon equation of motion:
Similar but **different** from Skyrme model case

$$\left[-\frac{1}{r^2}\partial_r\left(r^2\partial_r\right)+V(r)+m_K^2\right]k_n(r)=\left[E_n^2+2\Psi_0\,E_n\right]k_n(r)$$

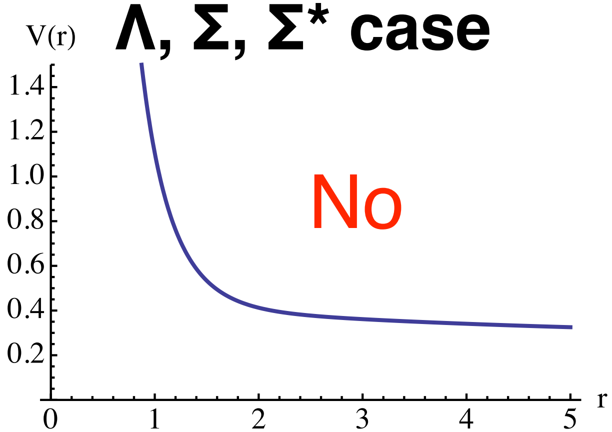
bad: canonical kinetic term

bad: repulsive

good: contrib. from A_0^{inst}

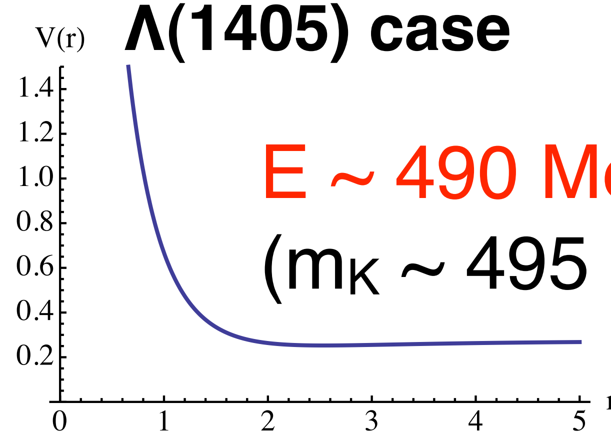
How about bound-state?

Λ, Σ, Σ* case



No

Λ(1405) case



E ~ 490 MeV
(m_K ~ 495 MeV)

Comment on Mass Term

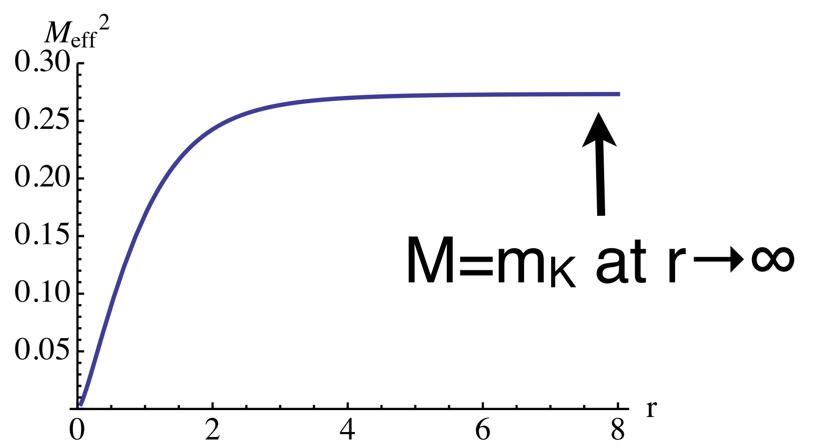
A quark-mass term in Sakai-Sugimoto model

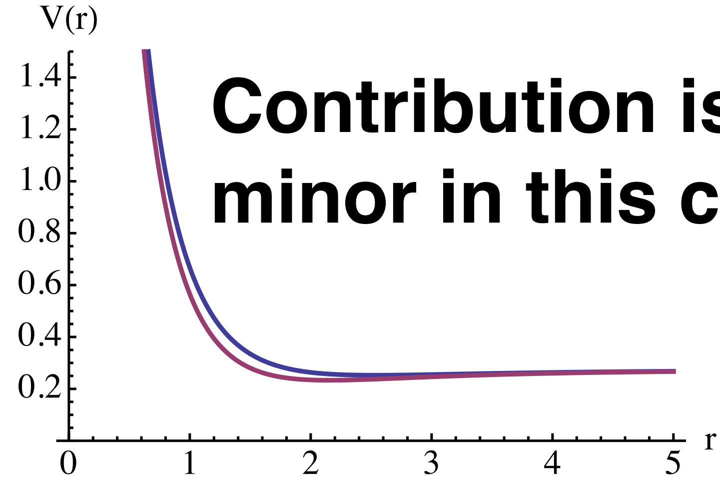
[Hashimoto-Hirayama-Lin-Yee, Aharony-Kutasov]

$$S_{\text{mass}} \sim \int d^4x \, \mathrm{PTr}\left[M_q e^{-i\int_{-\infty}^{\infty} A_z dz} \right]$$

$$A_z = A_z^{\text{inst}} + \delta A_z^{\text{kaon}}$$

$$S_{\text{kaon mass}} = -\int d^4x \, M_{\text{eff}}^2(r) K^\dagger(x^\mu) K(x^\mu)$$





Contribution is minor in this case.

Summary

Bound-state approach to strangeness
- not natural in Sakai-Sugimoto model
- A weakly bound Λ(1405)
  (Is Λ(1405) a N-$\bar{K}$ weak bound-state?)

A quark mass term in Sakai-Sugimoto model
- Radial dependence of effective kaon mass
  due to the path-ordering of the Wilson line

There remain challenging tasks
- Systems with both mesons and baryons
- A_z=0 gauge, where the mass term unclear