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Emergence of Gravitational Spacetime from Quantum Information

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It from Qubit
Simons Collaboration

Grant-in-Aid for Transformative Research Areas A
Extreme Universe

① Introduction

In science, "microscopes" are basic important devices.

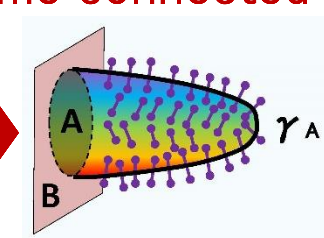
Biology, Chemistry
Materials Physics → (Optical, electronic, ..) Microscopes → Cells, DNAs, Atoms,…

High Energy Physics → Accelerators → Elementary particles

Quantum Gravity
(String Theory) → Holography (AdS/CFT) → Qubits !
Quantum Entanglement
~how spacetime connected

A thought experiment

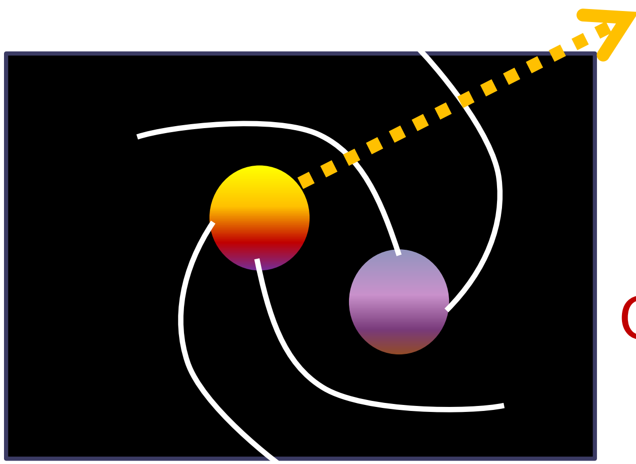
Holography magnifies
gravitational spacetimes.
The first target is black holes.

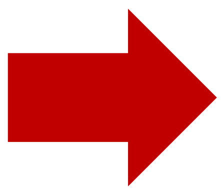


BH Entropy

After stars collapsed into a BH, outside observers cannot access the information inside the BH.

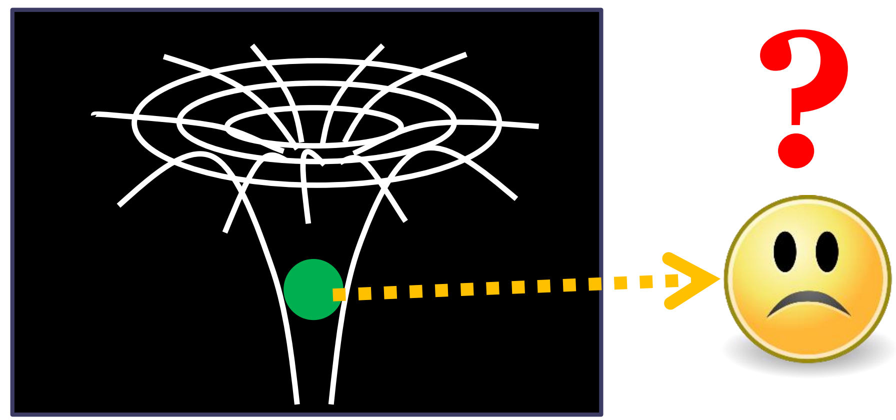
Stars





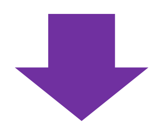
Gravitational collapse

BH



A lot of Information can be obtained !

Hidden Information !



BH entropy !

Bekentein–Hawking Formula of BH Entropy [1972–1976]

Calculations in general relativity show that a BH has the following entropy:

⇒Still mysterious !

$$S_{BH} = \frac{k_B c^3}{\hbar} \times \frac{A_{BH}}{4G_N}$$

➡ BH thermodynamics !

A_{BH}= Surface Area of Black hole ⇒ **Geometry**

G_N=Newton constant ⇒ **Gravity**

$\hbar$=Planck constant ⇒ **Quantum Mechanics**

k_B=Boltzmann const.⇒ **Stat. Mech. , Quantum Info.**

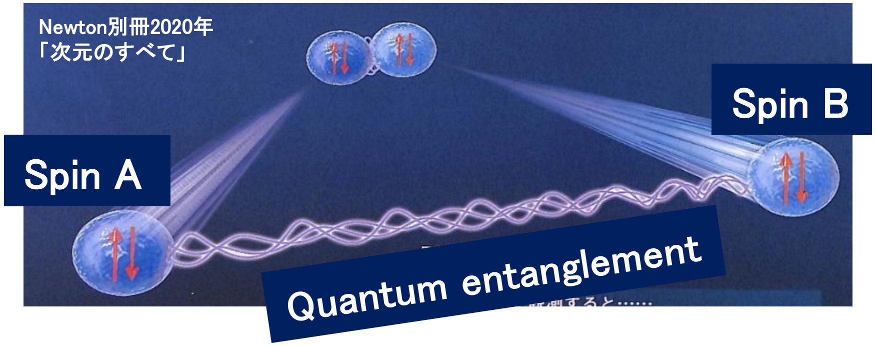
} **Quantum Gravity!**

[1] BH Entropy is proportional to the **area**, not to the volume !

[2] BH has the entropy even in the **classical theory** of Gravity !

What is Quantum Entanglement ?

Quantum Entanglement (QE) = Quantum correlation between two subsystems A and B



For pure states: Non-zero QE ⇔ $\left|\Psi\right\rangle_{AB} \neq \left|\Psi_1\right\rangle_A \otimes \left|\Psi_2\right\rangle_B$.

Not a Direct Product

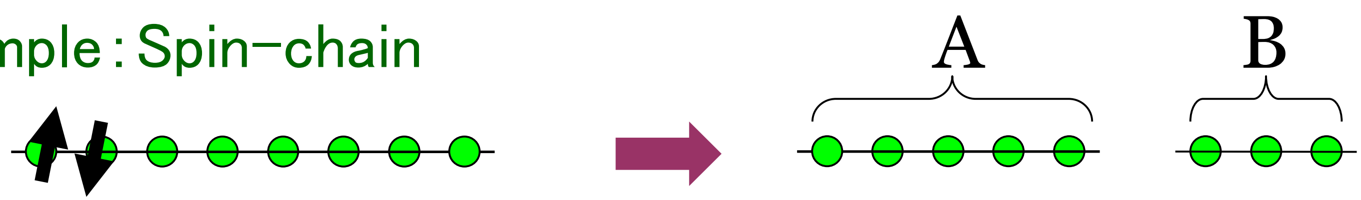
e.g. Bell pair : $\left|\Psi_{Bell}\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\uparrow\right\rangle_A \otimes \left|\downarrow\right\rangle_B + \left|\downarrow\right\rangle_A \otimes \left|\uparrow\right\rangle_B\right]$ ➡ Minimal Unit of Entanglement ↔ Planck length

Entanglement Entropy (EE)

An amount of QE is measured by **Entanglement Entropy (EE)**.

First we decompose the Hilbert space: $H_{tot} = H_A \otimes H_B$.

Example : Spin-chain



We introduce the reduced density matrix ρ_A

by tracing out B $\rho_A = \mathrm{Tr}_B\left[|\Psi_{tot}\rangle\langle\Psi_{tot}|\right]$.

The entanglement entropy (EE) S_A is defined by

$$S_A = -\mathrm{Tr}[\rho_A \log \rho_A]$$

$\propto$ # of Bell Pairs between A and B

Measurement of EE in Cond-mat Experiments

Example1: Ultracold bosonic atoms in optical lattices

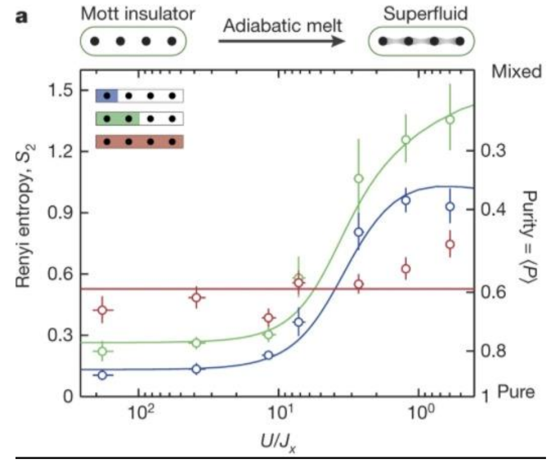
Published: 02 December 2015

Measuring entanglement entropy in a quantum many-body system

Rajibul Islam, Ruichao Ma, Philipp M. Preiss, M. Eric Tai, Alexander Lukin, Matthew Rispoli & Markus Greiner

Nature **528**, 77–83 (2015) | Cite this article

$$H = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i (n_i - 1) \qquad (4)$$



Example2: Trapped-ion quantum simulator

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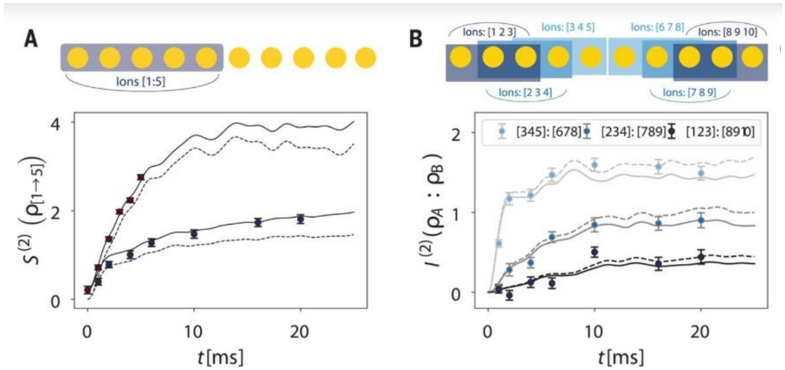
REPORT

Probing Rényi entanglement entropy via randomized measurements

TIFF BRYDGES, ANDREAS ELBEN, PETAR JURCEVIC, BENOÎT VERMERSCH, CHRISTINE MAIER, BEN P. LANYON, PETER ZOLLER, RAINER BLATT, AND CHRISTIAN F. ROOS Authors Info & Affiliations

SCIENCE • 19 Apr 2019 • Vol 364, Issue 6437 • pp. 260-263 • DOI: 10.1126/science.aau4963

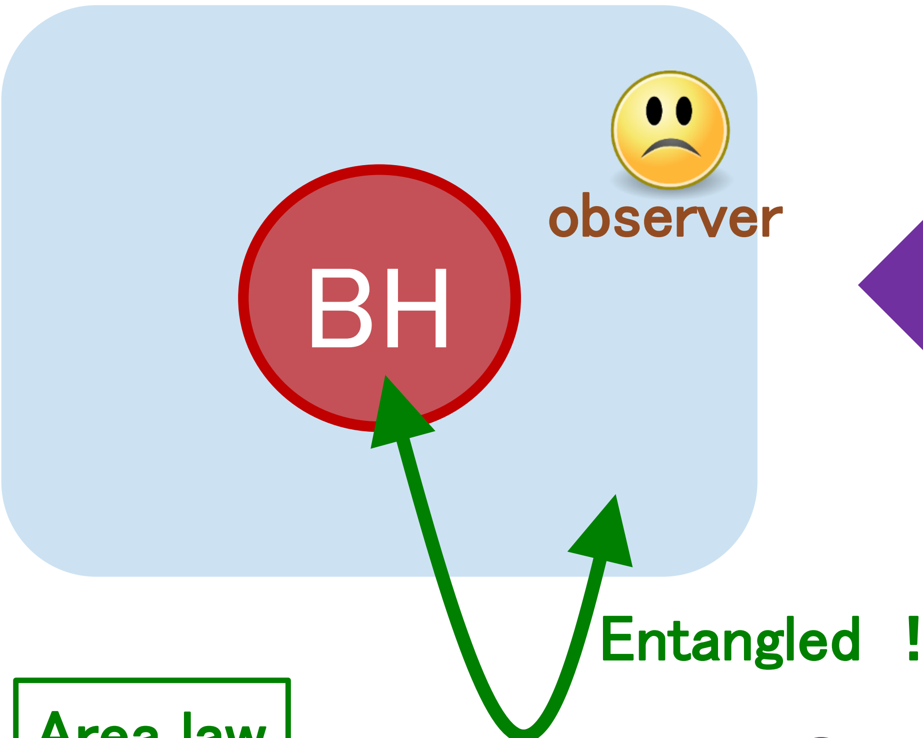
$$H_{XY} = \hbar \sum_{i<j} J_{ij} \left(\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+ \right) + \hbar B \sum_j \sigma_j^z$$



Analogy between BH and Qubits

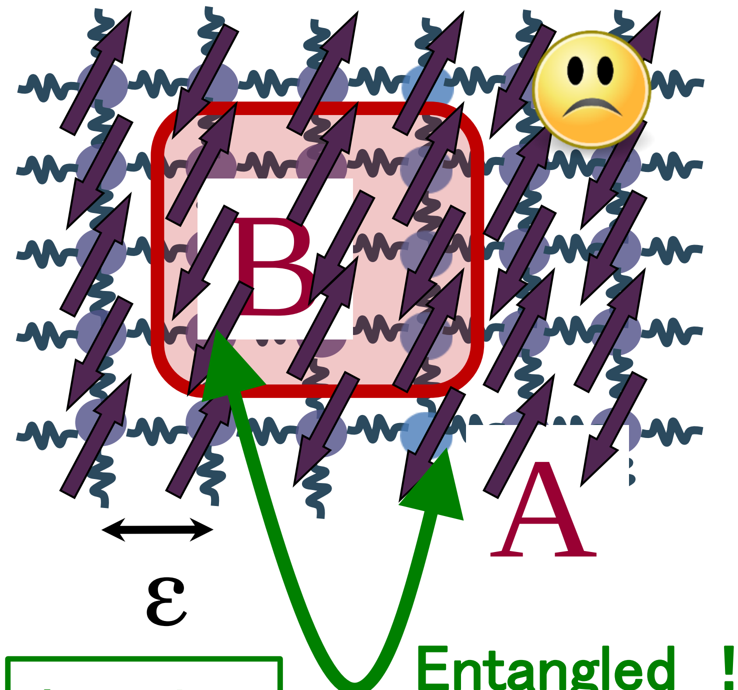
[Original motivation of studying EE in QFTs, Bombelli et.al. 1986, Srednicki 1993]

Blackhole Spacetime



Area law

Quantum Spin System



Area law

? ⟺

BH entropy S_{BH}

Spacetime

? ⟺

Entanglement entropy S_A

Matter

Contents

② Gravitational Holography

BH Entropy Formula

$$S_{BH} = \frac{A_{BH}}{4G_N}$$

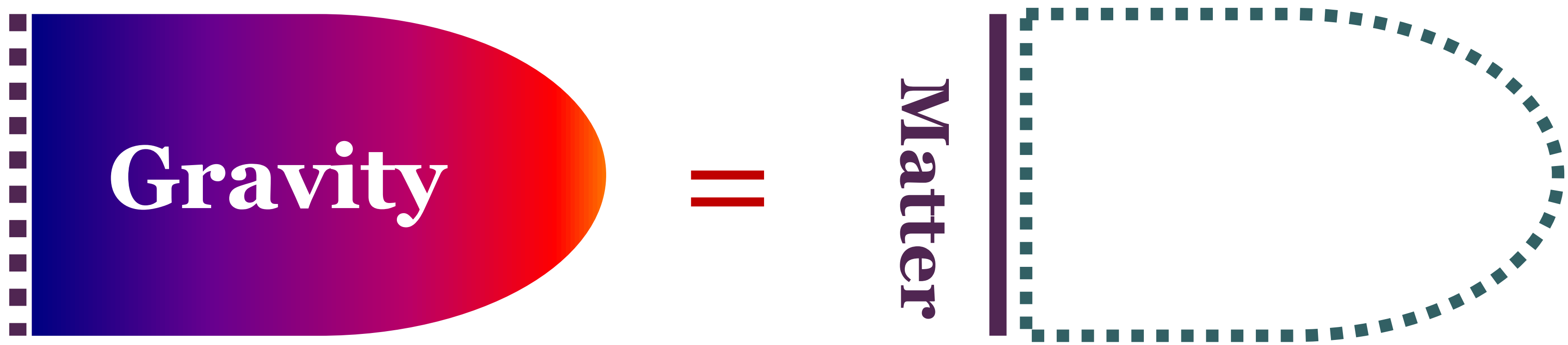
⇒ **Degrees of freedom in Gravity ∝ Area**

['t Hooft 1993, Susskind 1994]

Holography

Boundary of M

Gravity on M = Quantum Matter on ∂M



BH entropy(∝Area)= Thermal Entropy of Matter (∝Volume)

AdS/CFT Correspondence

[Maldacena 97]

AdS/CFT

Gravity (String theory) on D+1 dim. AdS (anti de-Sitter space)

=

Conformal Field Theory (CFT) on D dim. Minkowski spacetime

Massless (gapless) theory

Classical limit ↓

General relativity with $\Lambda < 0$

Large N + Strong coupling ↓

Strongly interacting Quantum Field Theories

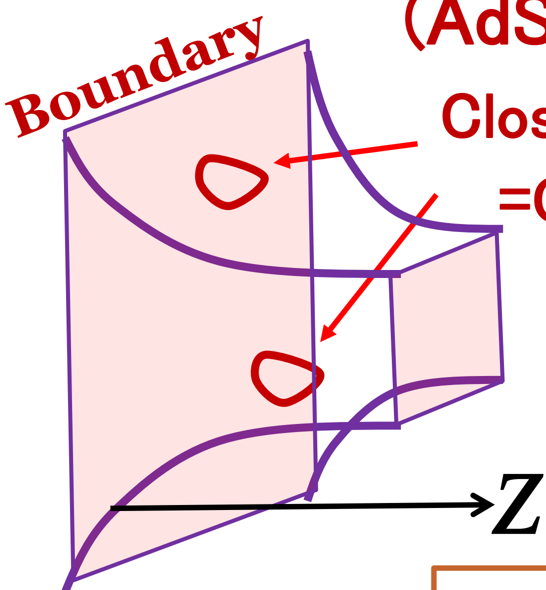
Basic Principle

(Bulk-Boundary relation) :

$$Z_{Gravity} = Z_{CFT}$$

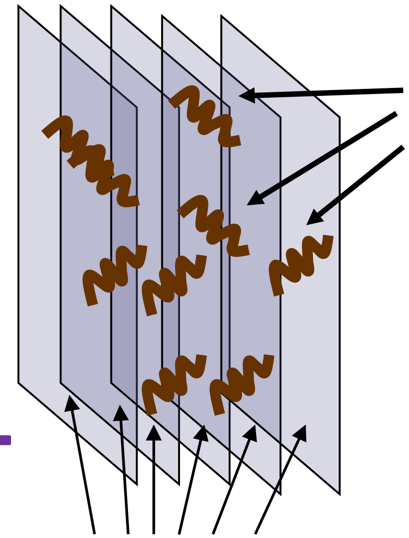
Partition Function

Gravity in Anti de-Sitter space (AdS)



Closed Strings
=Gravitons

Conformal Field Theories (CFTs)



Open Strings
=Gluons in U(N)
gauge theories

N D-branes

Equivalent

AdS/CFT

AdS metric

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

Thermodynamics of Black holes (branes)

Equivalent

Thermodynamics of various materials

dS/CFT: Holography for de Sitter Space $\Lambda > 0$

More realistic than AdS in Cosmology !

AdS/CFT: Gravity in 3D AdS = 2D (unitary) CFT

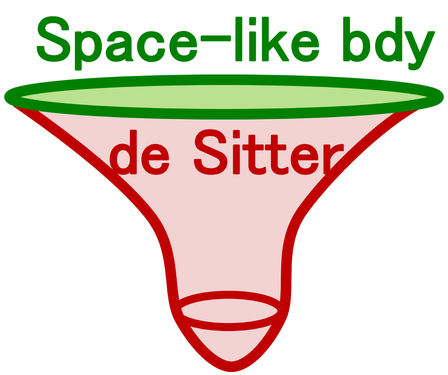
??

dS/CFT: Gravity in 3D dS = 2D (non-unitary) CFT [Strominger 2001]

First Explicit Example [Hikida-Nishioka-Taki-TT, 2021]

Large c limit of SU(2) WZW model (a 2dim. CFT)

= **Einstein Gravity** on 3D de Sitter (radius L_{ds})



Level

$$k \approx -2 + \frac{4iG_N}{L_{ds}}$$

Conformal dim.

$$\Delta \approx iL_{ds} \cdot E_{ds}$$

Energy in dS

Central charge

$$c = \frac{3k}{k+2} \approx i\frac{3L_{ds}}{2G_N}$$

$$Z[S^3, R_j] = |S_j^0|^2 \approx e^{\frac{\pi L_{ds}}{2G_N}\sqrt{1-8G_N E}}$$

CFT partition function

De Sitter Entropy

③ Holographic Entanglement Entropy (HEE)

HEE Formula (RT/HRT Formula)

[Ryu-TT 06, Hubeny-Rangamani-TT 07]

SA can be computed from the minimal area surface ΓA:

$$S_A = \min_{\Gamma_A}\left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N}\right]$$

Quantum Correction to HEE

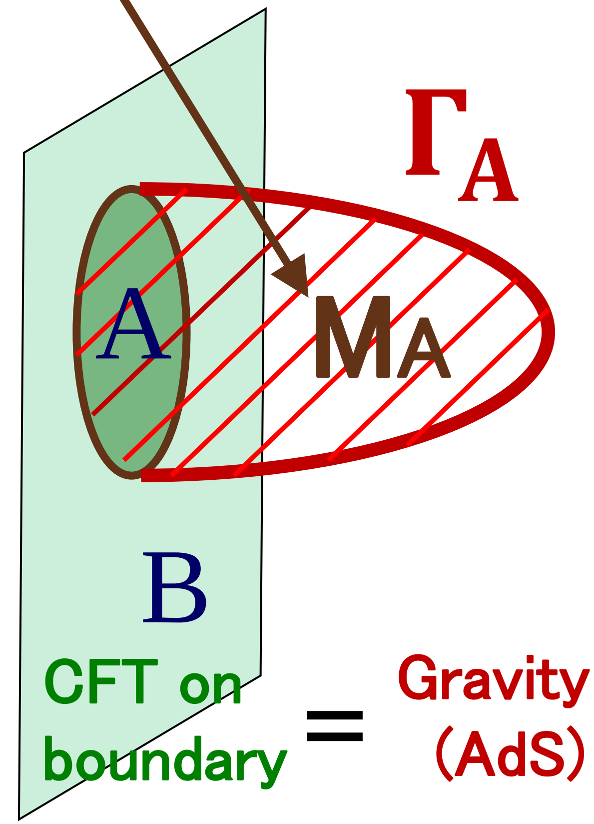
[Faulkner-Lewkowycz-Maldacena 13]

$$S_A = \min_{\Gamma_A}\left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N} + S_{bulk}(M_A)\right]$$

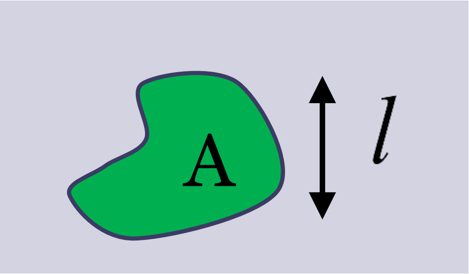
Bulk entanglement

Bulk Reconstruction

The information ρ_A in region A is encoded in the **entanglement wedge MA.**



General Behavior of HEE(=d+1 dim. CFT EE) [Ryu−TT 06,⋯]



$$S_A = \frac{\pi^{d/2} R^d}{2G_N^{(d+2)}\Gamma(d/2)}\left[p_1 \left(\frac{l}{\varepsilon}\right)^{d-1} + p_3 \left(\frac{l}{\varepsilon}\right)^{d-3} + \cdots \right.$$

$$\left. \cdots + \begin{cases} p_{d-1}\left(\frac{l}{\varepsilon}\right) + p_d & (\text{if } d+1=\text{odd}) \\ p_{d-2}\left(\frac{l}{\varepsilon}\right)^2 + q \log\left(\frac{l}{\varepsilon}\right) & (\text{if } d+1=\text{even}) \end{cases} \right],$$

where $p_1 = (d-1)^{-1},\ p_3 = -(d-2)/[2(d-3)],\ldots.$

$\ldots.. \quad q = (-1)^{(d-1)/2}(d-2)!!/(d-1)!! \quad .$

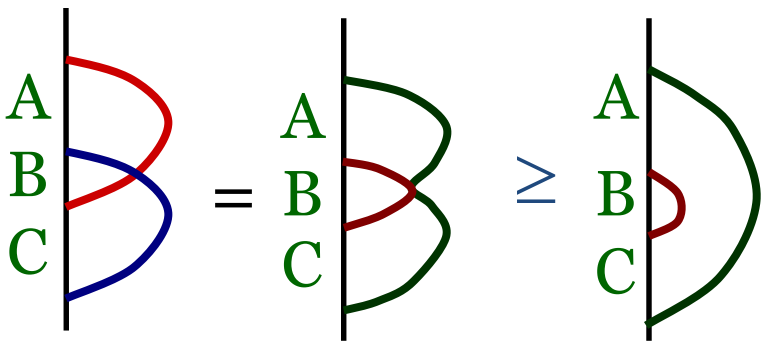
Area law divergence

A universal quantity (**F**) which characterizes **odd dim. CFT**.

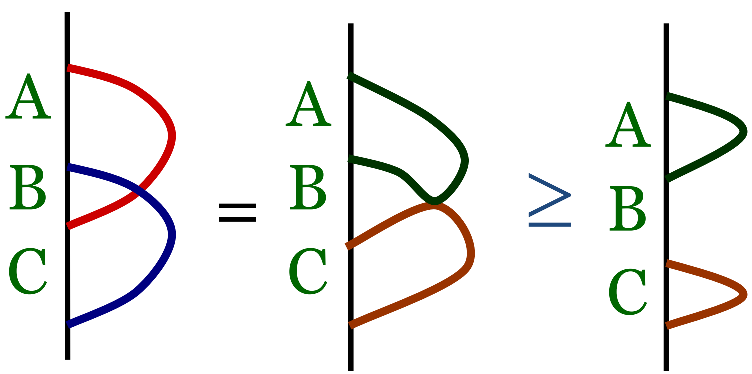
Agrees with **conformal anomaly** (central charge) in **even dim. CFT**

Algebraic properties in Quantum Information
⇔ **Geometric properties** in Gravity

Holographic Proof of Strong Subadditivity [Headrick-TT 07]



$$\Rightarrow \quad S_{AB} + S_{BC} \geq S_{ABC} + S_{B}$$



$$\Rightarrow \quad S_{AB} + S_{BC} \geq S_{A} + S_{C}$$

$$(\text{Note}: \quad AB \equiv A \cup B)$$

Einstein Equation from Quantum Entanglement

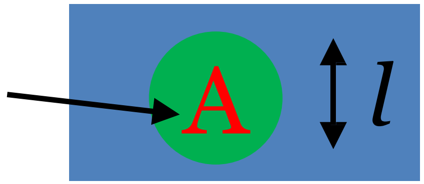
1st Law of EE

[Casini-Huerta-Myers 13, Bhattachrrya-Nozaki-Ugajin-TT 13]

$$\Delta S_A \simeq \Delta H_A$$

[$H_A = -\log \rho_A$: Modular Hamiltonian]

(t,x)



$$\left(\partial_l^2 - \partial_l - \partial_x^2 - \frac{3}{l^2}\right)\Delta S_A(t,\vec{x},l) = \langle O \rangle \langle O \rangle$$

[Nozaki-Numasawa-Prudenziati-TT 13]

[de Sitter space: de Boer-Haehl-Heller-Myers 16]

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$$

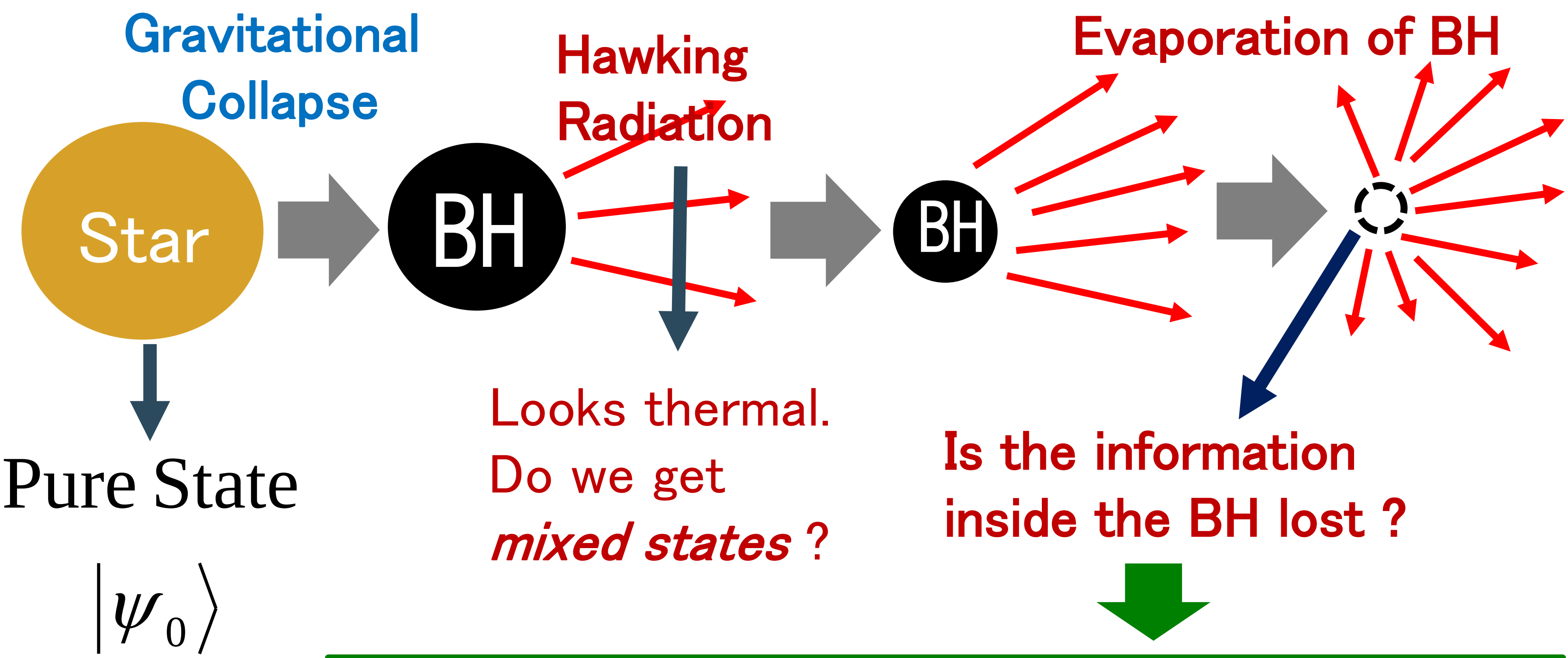
Kinetic term **C.C.** **Matter**

➡ The 1st law of EE explains the perturbative Einstein eq.

[Linear order: Raamsdonk et.al. 13, Non-linear order: Faulkner et.al 17, Sarosi-Ugajin 17]

④ Application to Black hole Information Problem

The black hole information problem is a touchstone in quantum gravity.



The information should not be lost if the evolution is unitary and the final state remains pure.

But how ?

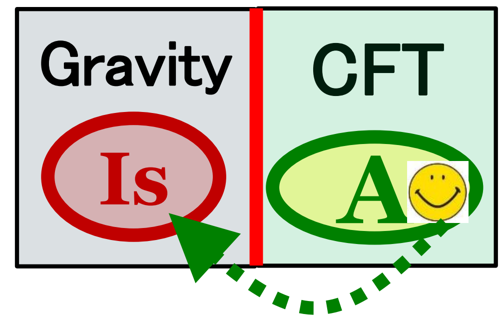
Black hole information problem and Island Formula

Recently, HEE was extended to CFTs which is coupled to gravity !

Island Formula: [Penington 2019, Almheiri et.al. 2019]

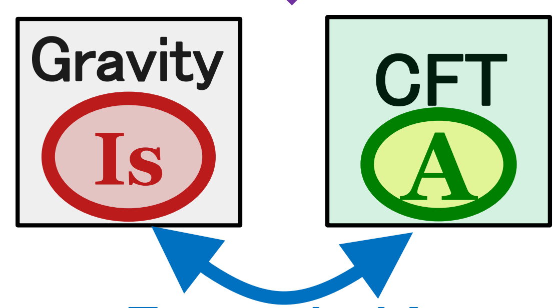
$$S_A = \underset{Is}{\mathrm{Min}}\left[\frac{\mathrm{Area}(\partial Is)}{4G_N} + S_{A\cup Is}\right]$$

EE of Radiations — Gravitational EE — QFT EE



Accesible to BH interior

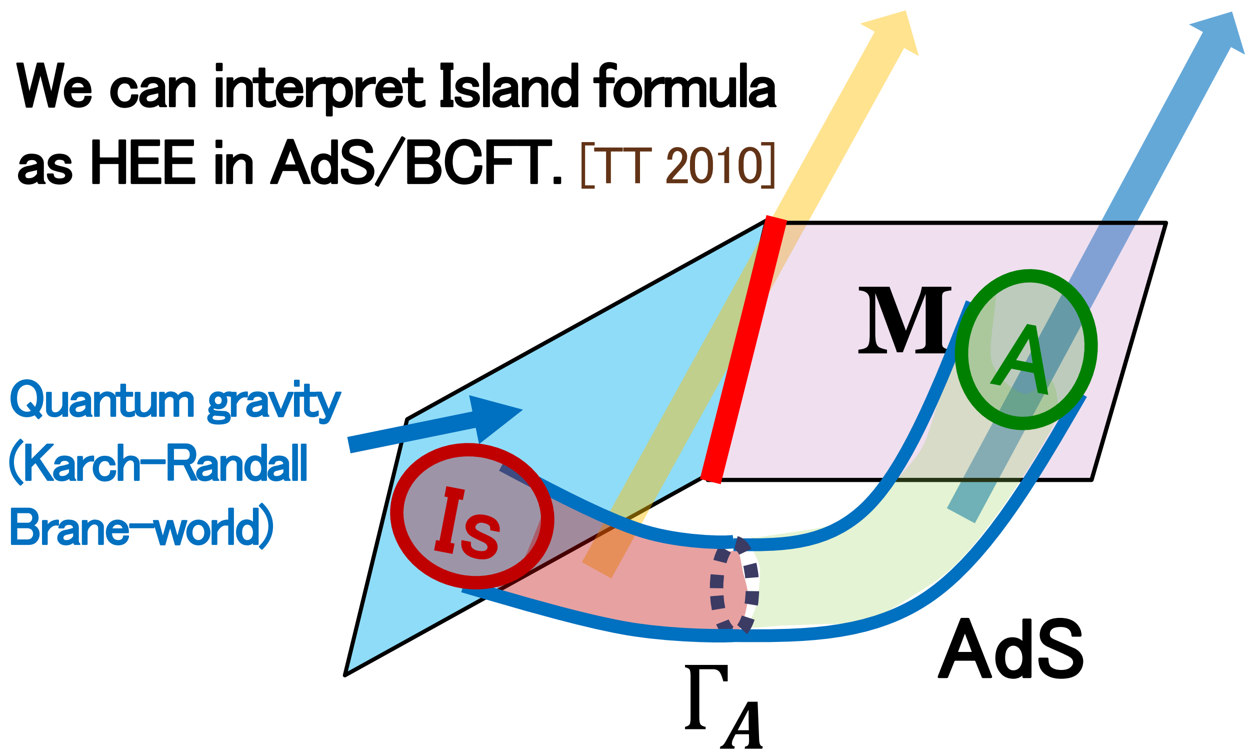
Generalization



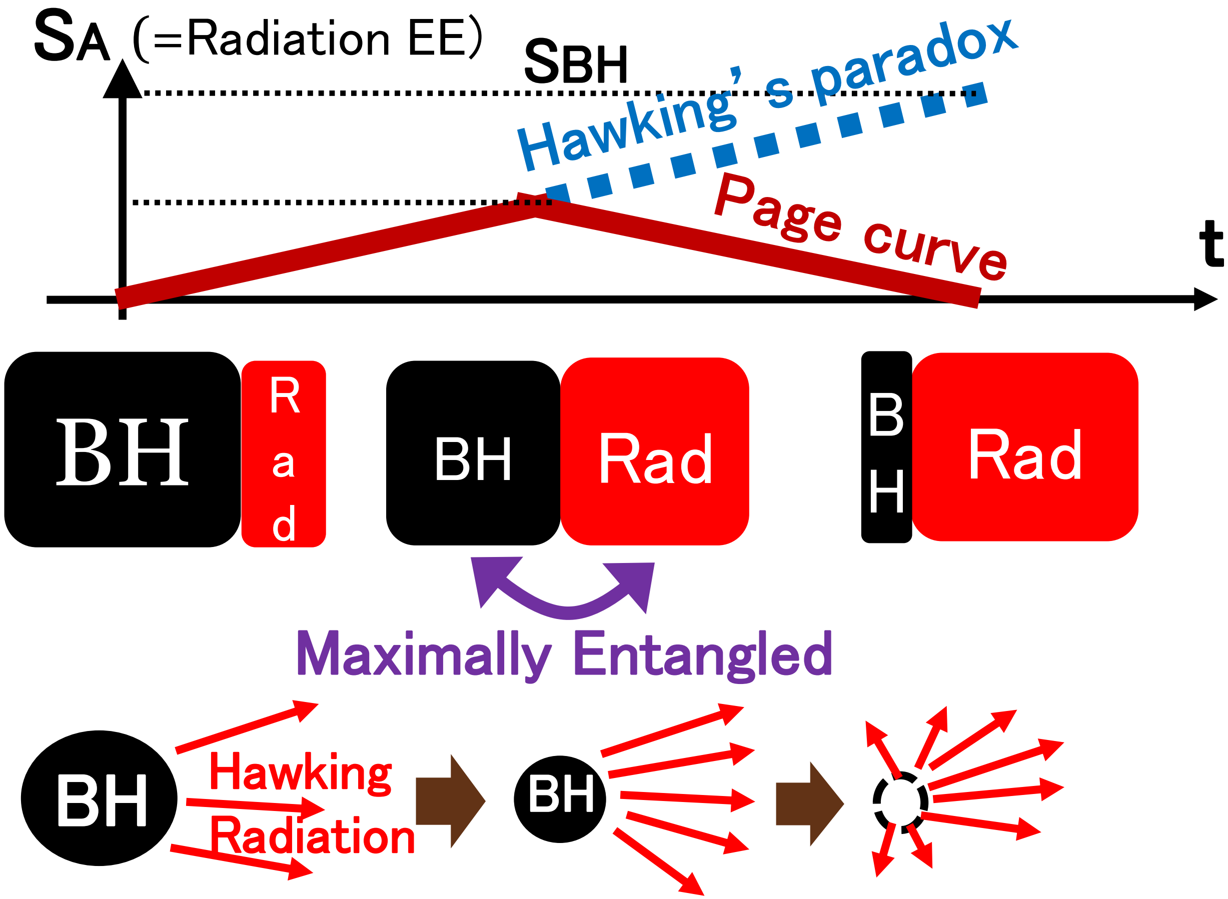
Entangled !

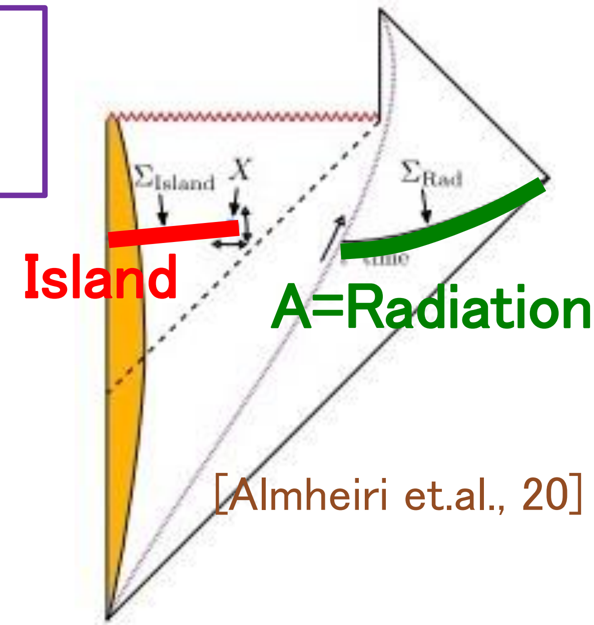
[Balasubramanian –Kar–Ugajin 2020–2021]

We can interpret Island formula as HEE in AdS/BCFT. [TT 2010]

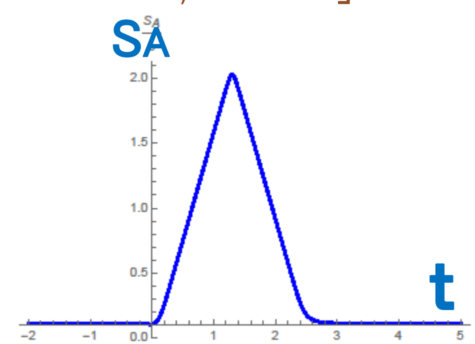


This Island formula explains the Page curve !
➔ Unitarity of BH evaporation !





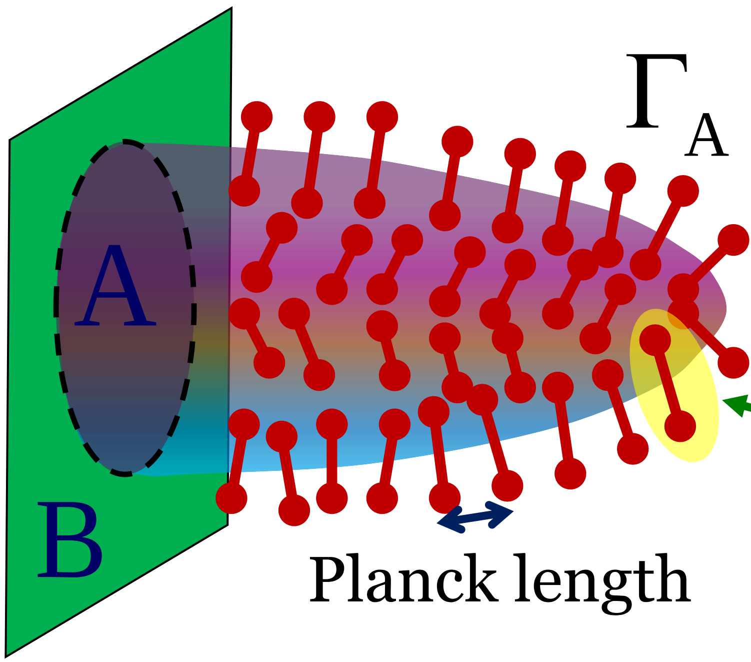
Page curve
from Moving Mirror
[Akal-Kusuki-Shiba
-Wei-TT, 20-21]



[See Kanato Goto's article this month in JPS Journal "Butsuri"]

⑤ Emergent Universe from Quantum Entanglement

The HEE suggests that there is one qubit of entanglement for each Planck length area !



$$S_A = \frac{\mathrm{Area}(\Gamma_A)}{4 l_{pl}^{D-1}}$$

~10^{65} qubits per 1cm² !

Spacetime may emerge from quantum entanglement !

➔ Tensor Network (TN) realizes this idea !

Tensor Network (TN)

[DMRG: White 92,.. CTM: Nishino-Okunishi 96, PEPS: Verstraete-Cirac 04, ⋯.]

Efficient variational ansatz for wave functions in quantum many-body systems, which respects quantum entanglement.

Wave function = Geometry of Network of Entanglement

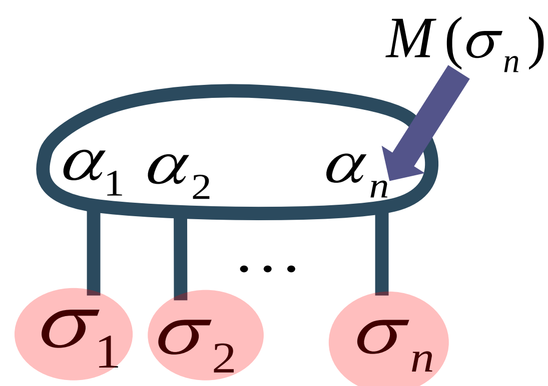
(1) The simplest example: EPR pair

$$|EPR\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

(2) Matrix product states

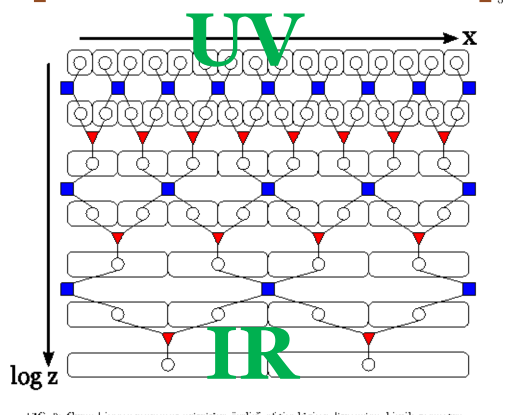
$$|\Psi\rangle = \sum_{\sigma_1,\sigma_2,\cdots,\sigma_n} \mathrm{Tr}[M(\sigma_1)M(\sigma_2)\cdots M(\sigma_n)]\,\underbrace{|\sigma_1,\sigma_2,\cdots,\sigma_n\rangle}_{n\ \mathrm{Spins}}$$

$$\alpha_i = 1,2,\ldots,\chi, \qquad \sigma_i = 0 \text{ or } 1 .$$



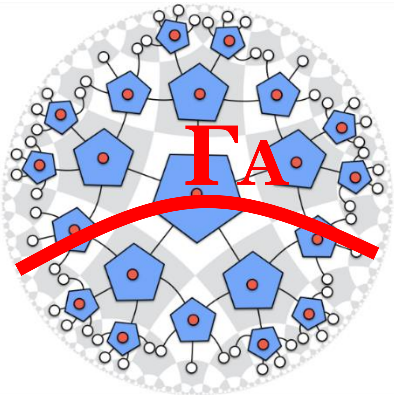
MERA TN

[TN for CFT: Vidal 05]



HaPPY TN

[Pastawski-Yoshida -Harlow-Preskill 15]



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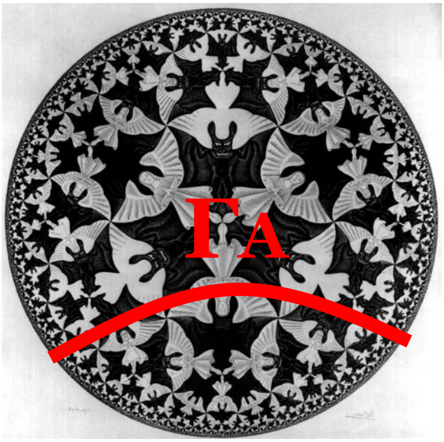
AdS



[Swingle 09,……]

Conjecture: "AdS geometry" = TN

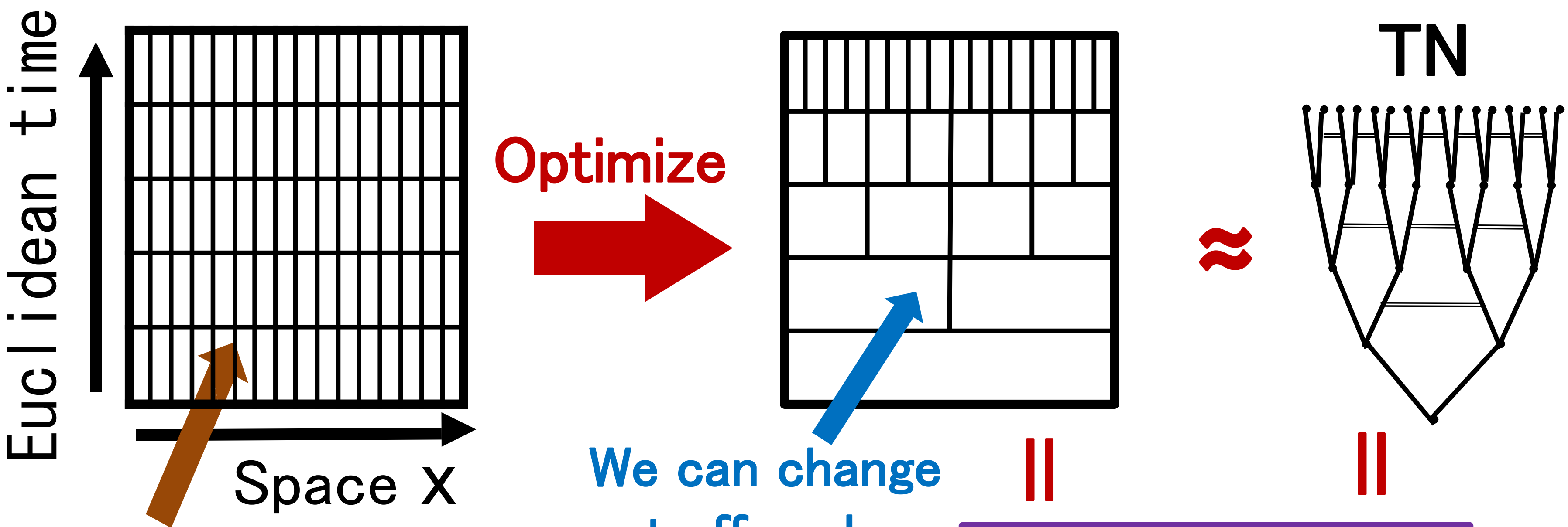
Time-like, Light-like or Space-like Slice ?? [∃Lattice artifacts]

Let us take the continuum limit to treat genuine CFTs !

Path-integral Optimization

[Caputa-Kundu-Miyaji-Watanabe-TT 2017,…..]

Principle: We minimize the computational cost of (discretized) path-integral to obtain a target state.



Initially, short wave length modes can be neglected.

We can change cut off scale locally.

Time slice of AdS

How to optimize path-integral (in 2 dim. CFT)

Idea: Local change of UV cut off scale = Metric change

$$ds^2 = e^{2\omega(x,z)}(dx^2 + dz^2).$$

Owing to conformal symmetry, the wave function behaves as

$$\Psi[\phi,\omega] = e^{C[\omega]} \cdot \Psi[\phi,\omega=0].$$

Optimization ⇒ Minimize the cost C[ω] !

[C[ω]≈Computational Complexity]

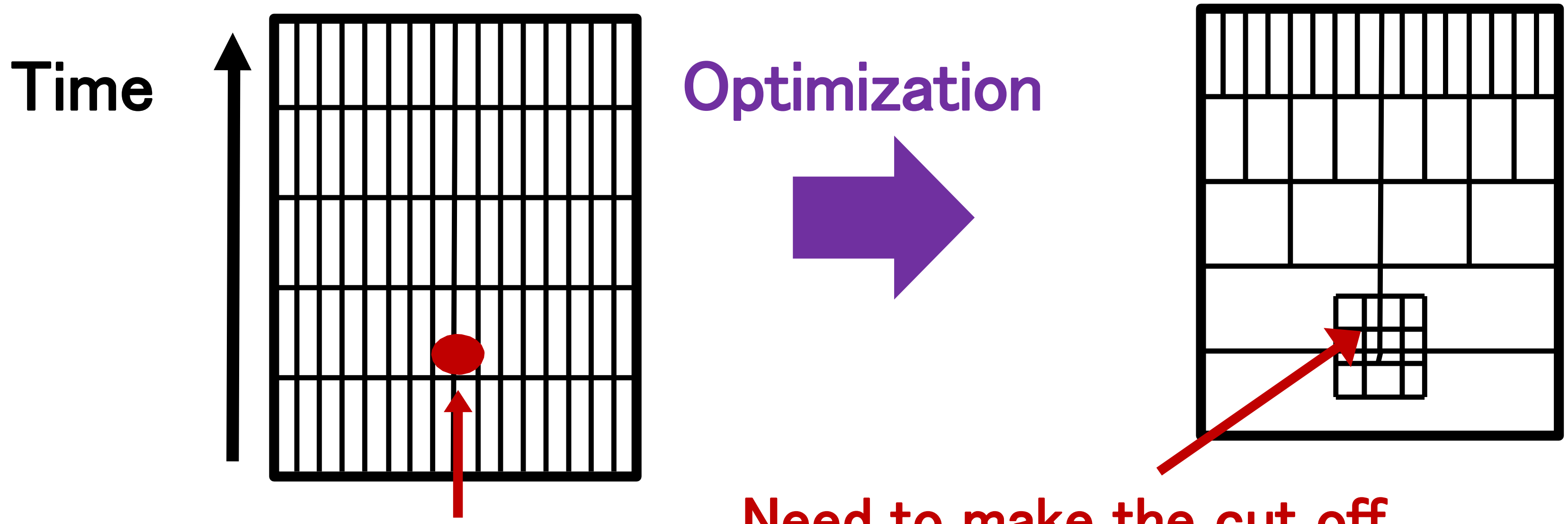
In two dim. CFT, C[ω] is given by Liouville action:

$$C_{2D}[\omega] = \frac{c}{24\pi}\int dxdz\left[(\partial_x\omega)^2 + (\partial_z\omega)^2 + e^{2\omega}\right].$$

Minimization leads to AdS metric !

$$e^{2\omega} = z^{-2}.$$

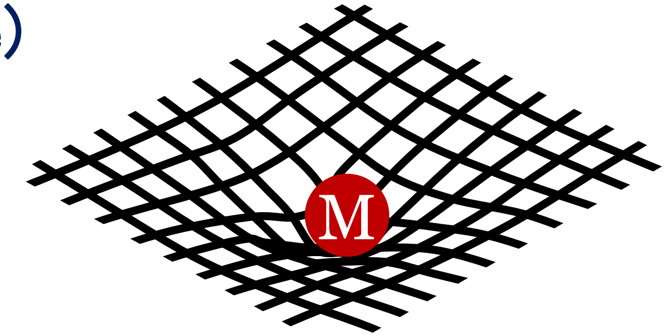
Path-integral Optimization for excited states



Local excitation
(Energy source)

Need to make the cut off
to be fine-grained
⇒ Metric is deformed !

Energetic source (= information source)
back-reacts the geometry !
⇒ Essence of GR !



Holographic Interpretation

[Boruch-Caputa-Ge-TT 2021]

Hartle-Hawking wave function on a time slice Q in AdS3 gravity ⇒ **Liouville action $C_{2D}[\phi]$!**

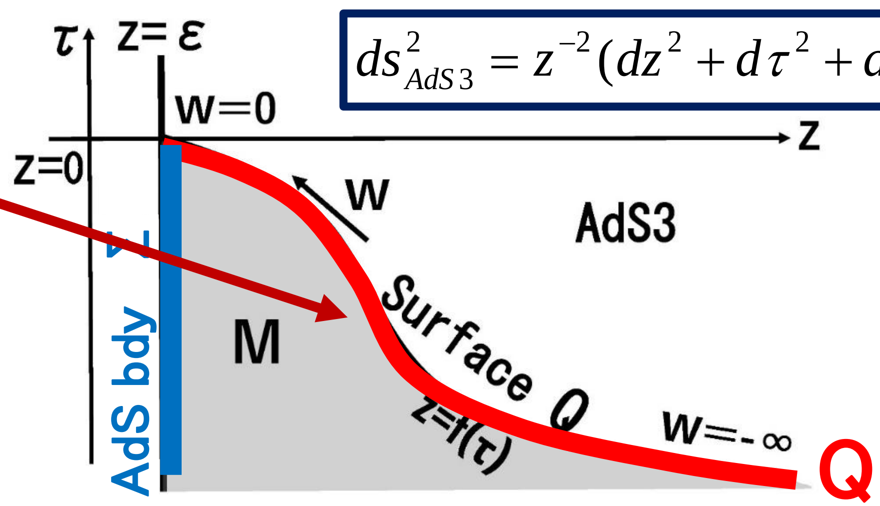
Wave Function of Universe

$$\Psi_{HH}^{(T)}[e^{2\phi}\delta_{ab}] \equiv \int Dg_{\mu\nu}\, e^{-I_G[g]}\, \delta\left(g|_Q - e^{2\phi}\delta_{ab}\right) \approx e^{-C_{2D}[\phi]},.$$

Metric on Q

$$ds^2|_Q = e^{2\phi}(dw^2 + dx^2)$$

$$ds^2_{AdS3} = z^{-2}(dz^2 + d\tau^2 + dx^2)$$

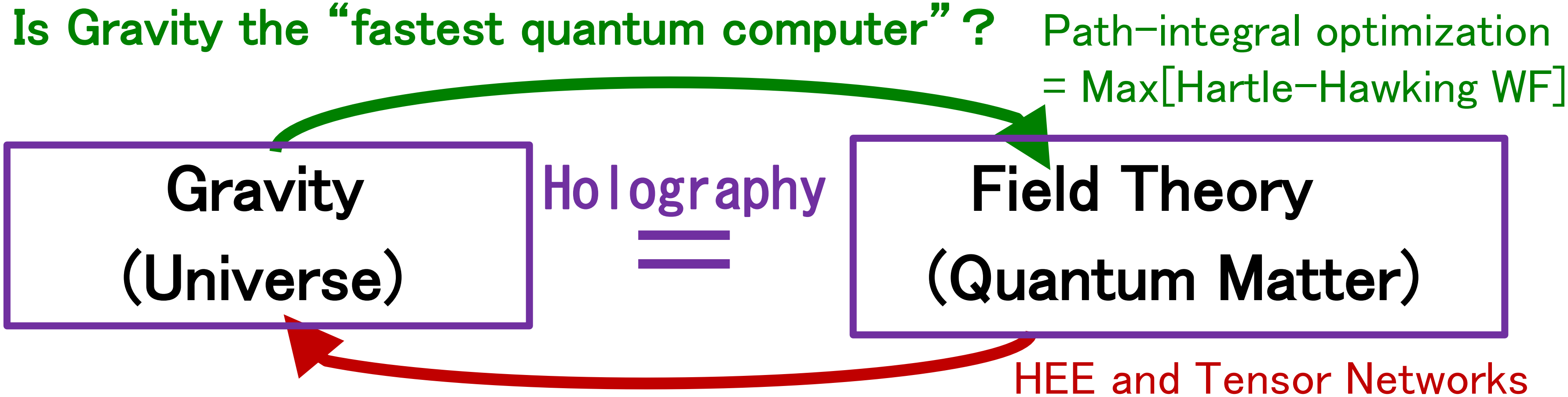


Path-int. optimization = Minimization $C_{2D}[\phi]$ i.e. Complexity
= Maximization of HH wave function

⑥ Conclusions

Starting from black holes entropy, we reviewed recent progresses on holography from the viewpoint of quantum information.

Is Gravity the "fastest quantum computer"?



Does the Universe emerge from Quantum Entanglement ?

Future problems

- Proof of AdS/CFT via quantum information
- Quantum gravity origin of black hole entropy (though we know it from CFT)
- Complete solution of black hole information problem
- Understanding of holography in dS and more generally in cosmology

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Thank you very much !