

Smart and Human

常翔学園

摂南大学



# Impact of SUSY on the dynamical emergence of the spacetime in the type IIB matrix model with the Lorentz symmetry “gauge fixed”

Takehiro Azuma (Setsunan Univ.)

LATTICE 2025, Nov. 3<sup>rd</sup> 2025, 16:40~17:00

with Konstantinos N. Anagnostopoulos (NTUA),

Mitsuaki Hirasawa (Milano-Bicocca, INFN), Jun Nishimura (KEK, SOKENDAI),

Asato Tsuchiya (Shizuoka Univ.) and Naoyuki Yamamori (SOKENDAI)

# 1. Introduction

2



Type IIB matrix model (a.k.a. **IKKT model**)

⇒ Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$S = \underbrace{\frac{-N}{4} \text{tr}[A_\mu, A_\nu]^2}_{=S_b} + \underbrace{\frac{-N}{2} \text{tr}\bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta]}_{=S_f}$$

- Dimensional reduction of the **D=10** super-Yang-Mills theory to **0 dimension**
- $A_\mu$  ( $\mu=0, 1, \dots, 9$ ),  $\Psi_\alpha$  ( $\alpha=1, 2, \dots, 16$  after Weyl projection)  
⇒  $N \times N$  Hermitian traceless matrices.
- $N=2$  supersymmetry ⇒ **eigenvalues of  $A_\mu$**  are interpreted as the **spacetime coordinates**.

**How does our (3+1)-dim spacetime emerge dynamically?**

# 2. Lorentzian type IIB matrix model

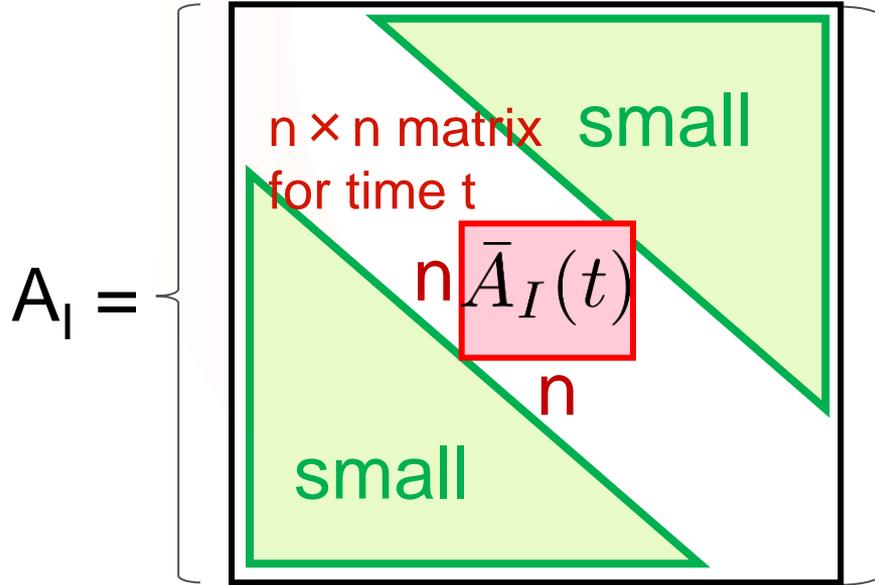
Lorentzian version : no Wick rotation  
 contracted by the **Lorentzian metric**  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, \dots, 1)$

Time development of the Lorentzian version:  
 $\Rightarrow$  gauge fixing to diagonalize  $A_0$  by  $SU(N)$ .

[S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

$A_0 = \text{diag}(\alpha_1, \alpha_2, \dots, \alpha_N)$ , where  $\alpha_1 < \alpha_2 < \dots < \alpha_N$ .

Band-diagonal structure of  $A_1$   $t_\nu = \sum_{k=1}^{\nu} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$ ,  $\bar{\alpha}_k = \frac{1}{n} \sum_{i=1}^n \alpha_{k+i}$



SSB of spatial  $SO(9)$  symmetry

Eigenvalues of  $T_{ij}(t) = \frac{1}{n} \text{tr}(\bar{A}_i(t) \bar{A}_j(t))$   
 $\lambda_{T,k}(t)$  ( $k = 1, 2, \dots, 9$ )

$$R^2(t) = \left\langle \frac{1}{N} \text{tr} \sum_{i=1}^9 (\bar{A}_i(t))^2 \right\rangle = |R^2(t)| \exp(2i\theta_s(t))$$

$\theta_s(t) \ll \frac{\pi}{8} \Rightarrow$  real space

# 2. Lorentzian type IIB matrix model

4



## "Gauge fixing" of the Lorentz symmetry

[Y. Asano, J. Nishimura, W. Piensuk and N. Yamamori, arXiv:2404.14045]

Diagonalizing  $A_0$  by  $SU(N)$  does not fully fix Lorentz sym.

$SO(9,1)$  Lorentz invariant quantity  $Q = \underbrace{\text{tr}(A_0^2)}_{=\mathcal{T}} - \underbrace{\sum_{i=1}^9 \text{tr}(A_i^2)}_{=\mathcal{U}}$

Minimize  $\mathcal{T}$  w.r.t the Lorentz transformation

$$\begin{pmatrix} A'_0 \\ A'_i \end{pmatrix} = \begin{pmatrix} \cosh \sigma & \sinh \sigma \\ \sinh \sigma & \cosh \sigma \end{pmatrix} \begin{pmatrix} A_0 \\ A_i \end{pmatrix}$$

$\text{tr}(A_0 A_j) = 0$  for all  $j = 1, 2, \dots, 9$

Gauge-fix the Lorentz symmetry by the Faddeev-Popov procedure:

$$Z = \int dA d\psi e^{iS} \underbrace{\Delta_{\text{FP}}(A)}_{=\det \Omega_{ij}} \prod_{j=1}^9 \delta(\text{tr}(A_0 A_j))$$

$$\Omega_{ij} = \text{tr}(A_0^2) \delta_{ij} + \text{tr}(A_i A_j) \quad (i, j = 1, 2, \dots, 9)$$

# 3. Complex Langevin Method

5



Sign problem of the type IIB matrix model

$$Z = \int dA \left( \underbrace{e^{iS_b}}_{\text{complex}} \underbrace{\int d\psi e^{iS_f}}_{=\text{Pf } \mathcal{M}} \right) \text{real}$$

[J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

$\mathcal{M}$  is a  $16(N^2-1) \times 16(N^2-1)$  sparse matrix

## Complex Langevin Method (CLM)

⇒ Promising method to solve complex-action systems.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

• Time order  $\alpha_1 < \alpha_2 < \dots < \alpha_N$  for **complexified**  $\alpha_i$  ( $k=2, \dots, N$ )

$$\alpha_1 = 0, \quad \alpha_k = \sum_{i=1}^{k-1} e^{\tau_i} \quad A_0 = \text{diag}(\alpha_1, \dots, \alpha_N)$$

[J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

$$S_{A\text{diag}} = -\log \prod_{1 \leq a < b \leq N} (\alpha_a - \alpha_b)^2 - \sum_{a=1}^{N-1} \tau_a.$$

# 3. Complex Langevin Method

6



• **Mass term** to avoid the near-zero modes of the

Dirac operator:  $\text{Pf } \mathcal{M} = \int d\psi e^{iS_{m_f}}$  ( $m_f \rightarrow \infty$ : bosonic model)

$$S_{m_f} = \frac{-N}{2} \text{tr} \{ \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta] + im_f \bar{\psi}_\alpha (\Gamma^7 \Gamma^8 \Gamma^9)_{\alpha\beta} \psi_\beta \}$$

• Deformation of the Lorentz-invariant mass term ( $\tilde{d} = 4, 5, 6$ )

[K.N. Anagnostopoulos, T. Azuma, M. Hirasawa, J. Nishimura and S. Papadoudis and A. Tsuchiya, arXiv:2509.XXXXX]

Control bosonic quantum fluctuations to **mimic SUSY cancellation**

$$S_\xi = \frac{N\gamma}{2} \left\{ \text{tr}(A_0)^2 - \sum_{I=1}^{\tilde{d}} \text{tr}(A_I)^2 - \xi \sum_{I=\tilde{d}+1}^9 \text{tr}(A_I)^2 \right\}$$

•  $\xi=1$ : reduces to  $S_\gamma = \frac{-N\gamma}{2} \text{tr}(A^\mu A_\mu)$

•  $\xi=3$ ,  $\tilde{d} = 6$  with Myers term  
(set  $\gamma, m_f$  appropriately)

⇒ supersymmetric deformation of  
the Lorentzian type IIB matrix model

[G. Bonelli, hep-th/0205213]

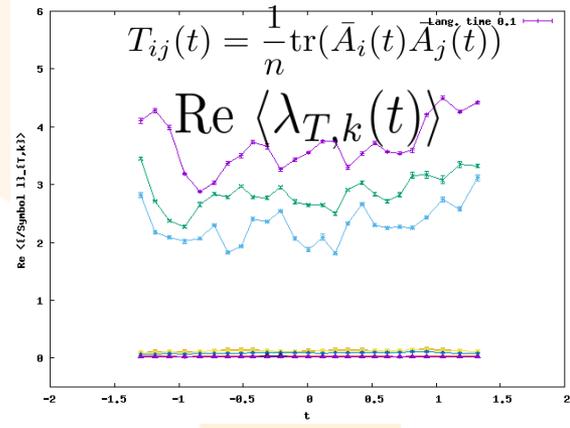
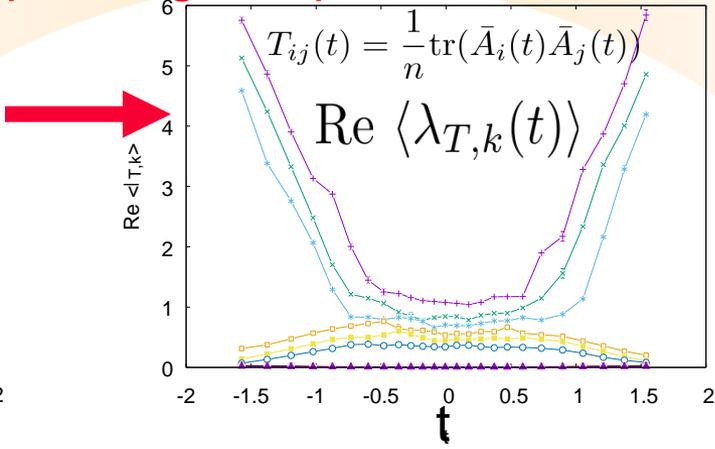
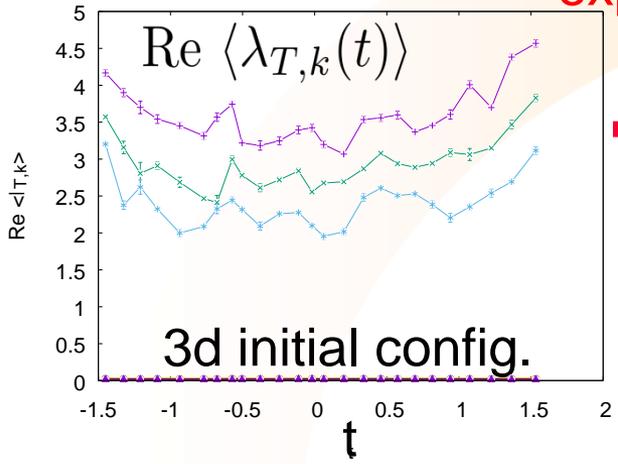
Break the spatial symmetry  $SO(9) \rightarrow SO(\tilde{d})$

⇒ explore the remaining symmetry

# 4. Result

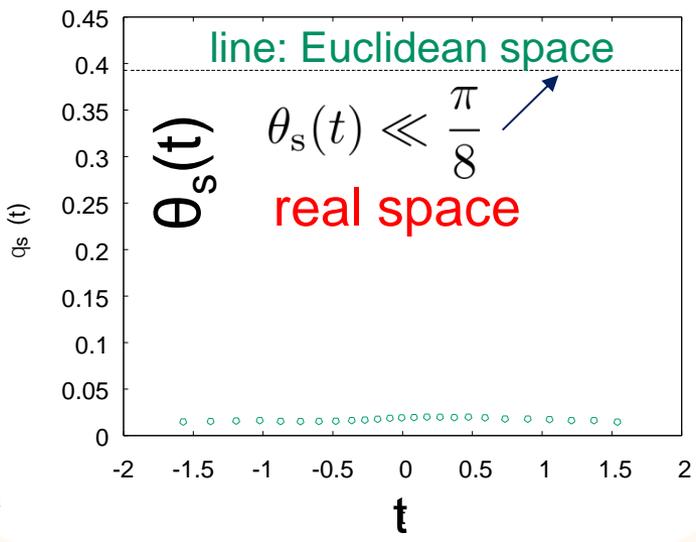
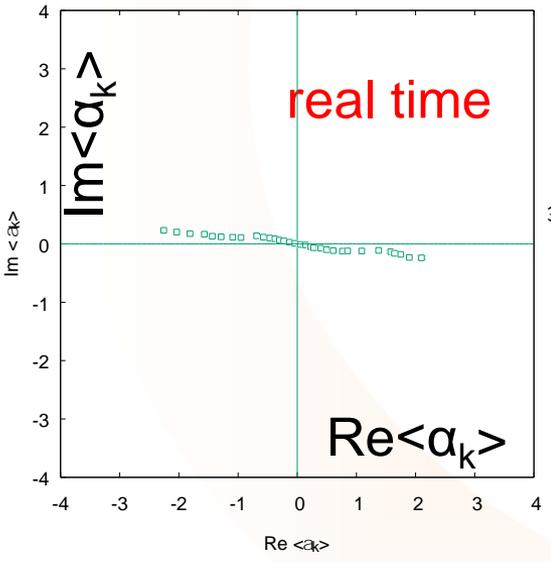
$N=32, n=8, m_f=2, \gamma=6, \xi=10, \tilde{d} = 6$

expanding 3d space at late time



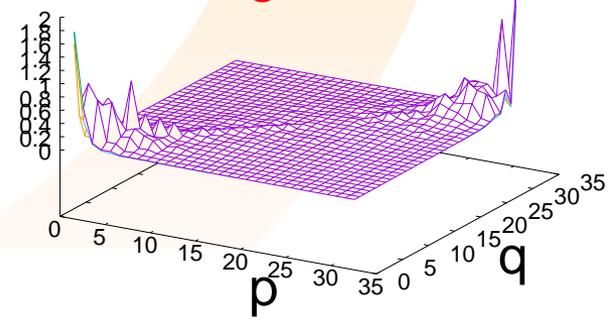
CLM cannot sample all the relevant saddle points in one simulation.

⇒ Try changing initial configs.



$$A_{pq} = \frac{1}{9} \text{Re} \left\langle \sum_{i=1}^9 (A_i)_{pq} (A_i)_{qp} \right\rangle$$

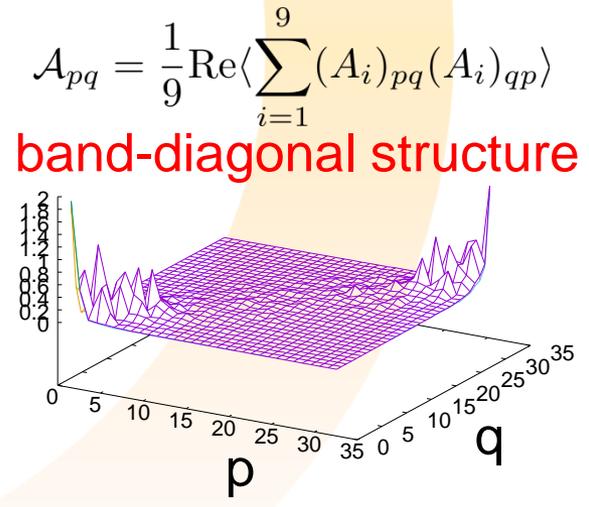
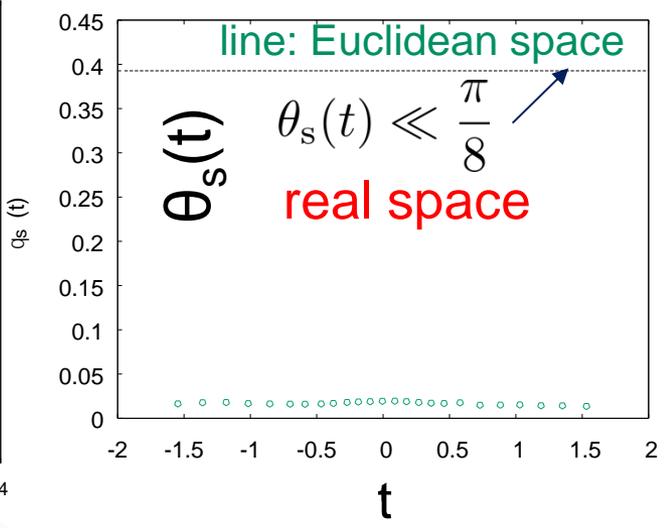
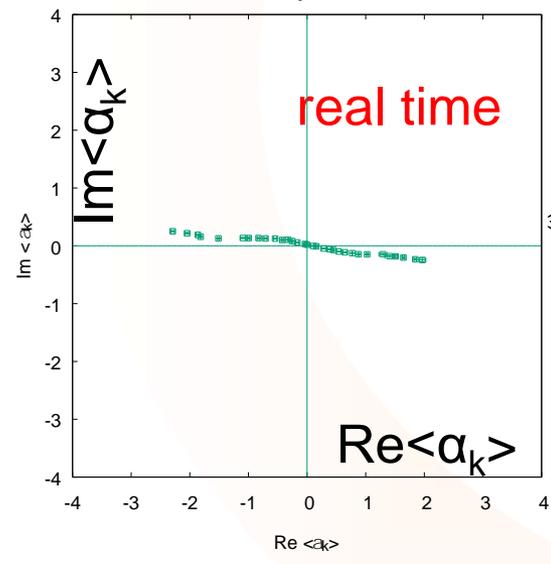
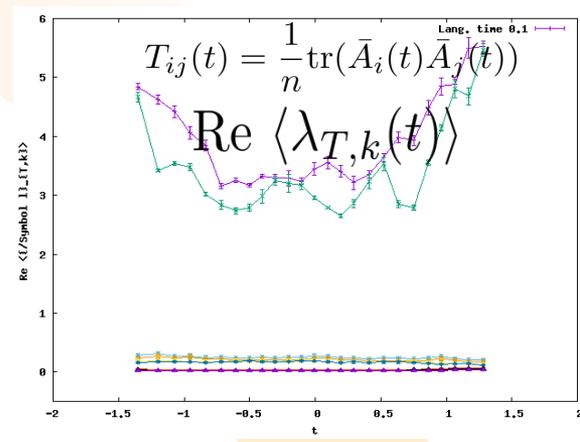
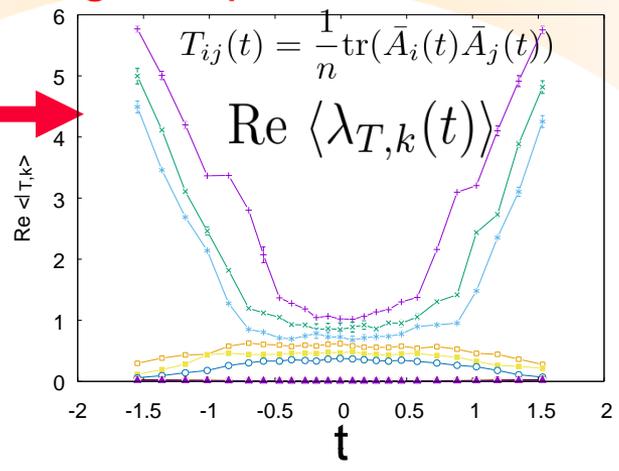
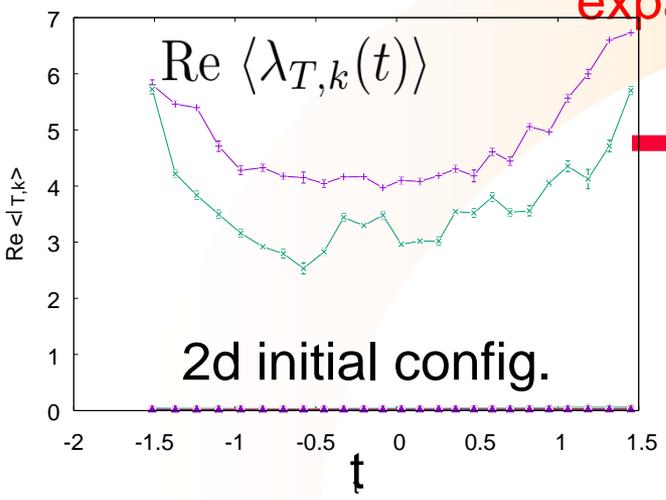
band-diagonal structure



# 4. Result

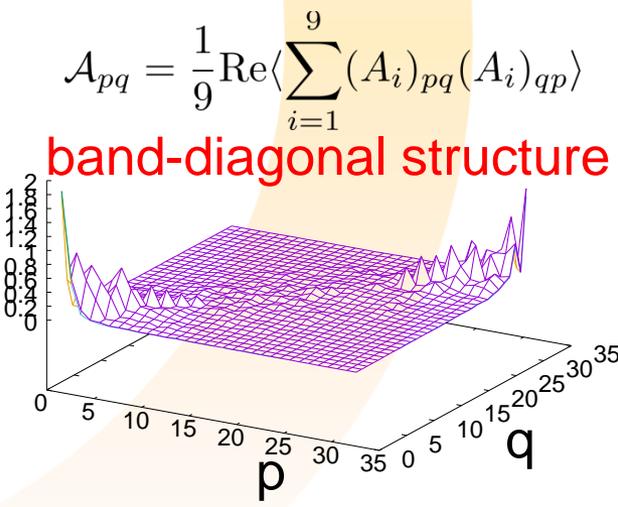
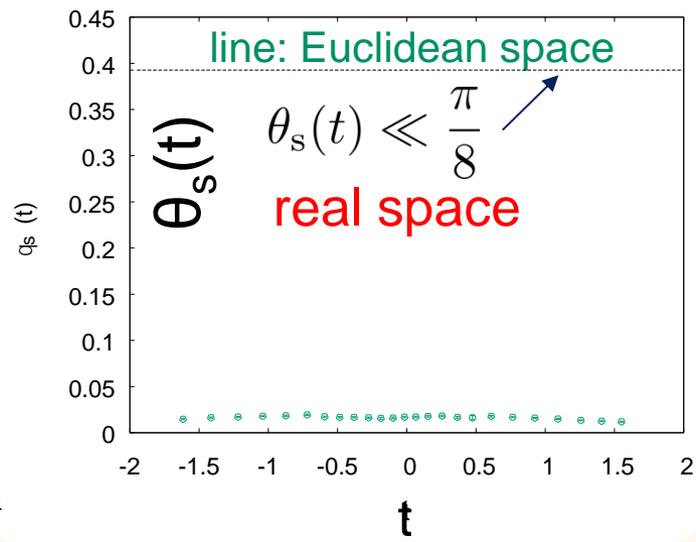
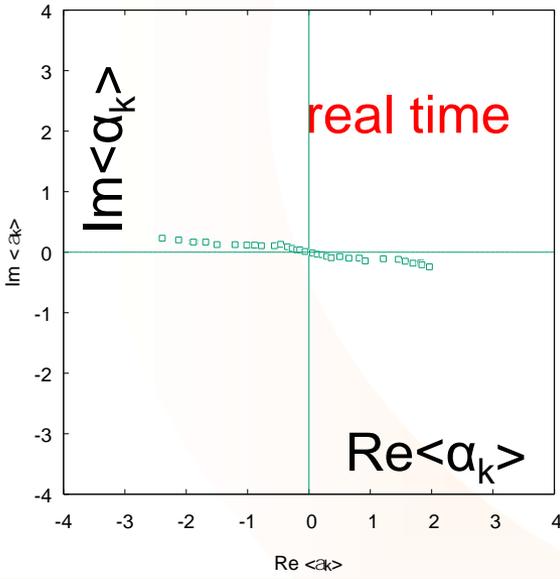
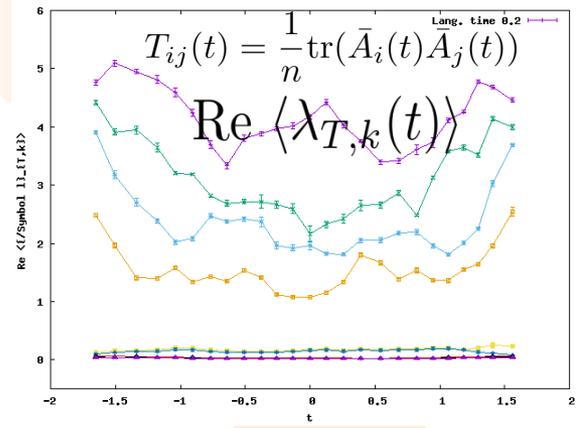
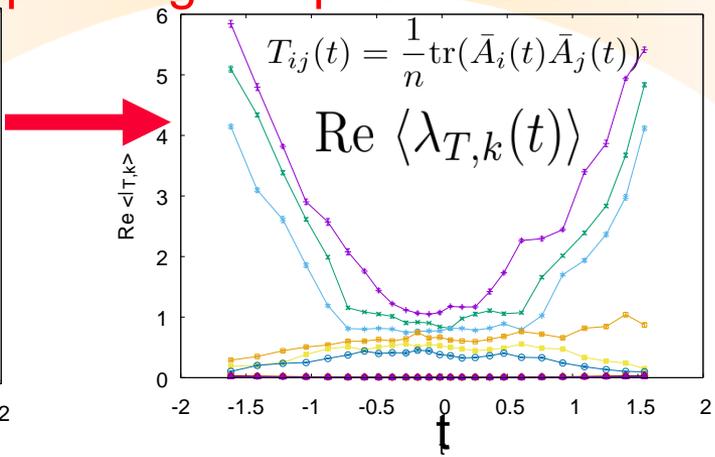
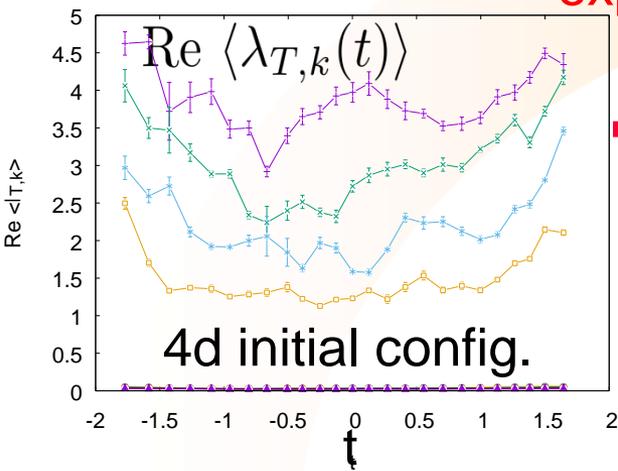
$N=32, n=8, m_f=2, \gamma=6, \xi=10, \tilde{d} = 6$

expanding 3d space at late time



# 4. Result

$N=32, n=8, m_f=2, \gamma=6, \xi=10, \tilde{d} = 6$   
 expanding 3d space at late time



$$A_{pq} = \frac{1}{9} \text{Re} \left\langle \sum_{i=1}^9 (A_i)_{pq} (A_i)_{qp} \right\rangle$$

# 5. Conclusion

Complex Langevin Method (CLM) of the Lorentzian type IIB matrix model.

Lorentz symmetry is "gauge-fixed" by Faddeev-Popov procedure.

Emergence of **expanding 3d space** from 2,3,4-dim initial configs.

Future prospects

- $N \rightarrow \infty \Rightarrow \gamma \rightarrow 0, m_f \rightarrow 0$  limit: does the 3d space emerge?
- Simulation of the supersymmetric deformation of the Lorentzian type IIB matrix model. [G. Bonelli, hep-th/0205213]

## Gauge fixing of the Lorentz symmetry

SO(d,1) Lorentz invariant quantity  $Q = \underbrace{\text{tr}(A_0^2)}_{=\mathcal{T}} - \underbrace{\sum_{i=1}^9 \text{tr}(A_i^2)}_{=\mathcal{U}}$

(1+1)d Lorentz transformation in the i-th direction

$$\begin{pmatrix} A'_0 \\ A'_i \end{pmatrix} = \begin{pmatrix} \cosh \sigma & \sinh \sigma \\ \sinh \sigma & \cosh \sigma \end{pmatrix} \begin{pmatrix} A_0 \\ A_i \end{pmatrix}$$

$$\mathcal{T} = \text{tr}(A_0^\dagger A_0) \Rightarrow \mathcal{T}' = \frac{1}{2}(a \cosh 2\sigma + b \sinh 2\sigma + (a - b))$$

$$a = \text{tr}(A_0^\dagger A_0) + \text{tr}(A_i^\dagger A_i), \quad b = 2\text{Re tr}(A_0^\dagger A_i)$$

Minimum of  $\mathcal{T}'$  :  $\sqrt{a^2 - b^2}$  at  $\tanh 2\sigma = \frac{-b}{a}$

## Gauge fixing of the Lorentz symmetry

Minimization of  $\mathcal{T} : (A^\dagger_0=A_0, A^\dagger_l=A_l)$

$$b = \text{tr}(A_0 A_i) = 0 \text{ for all } i = 1, 2, \dots, d$$

Gauge-fix the Lorentz symmetry by the Faddeev-Popov procedure:

$$A^\mathcal{O}_\mu = \mathcal{O}_{\mu\nu} A_\nu, \quad \mathcal{O} \in \text{SO}(d, 1)$$

$$\int d\mathcal{O} \underbrace{\Delta_{\text{FP}}(A^\mathcal{O})}_{=\det \Omega_{ij}} \prod_{j=1}^d \delta(\text{tr}(A^\mathcal{O}_0 A^\mathcal{O}_j)) = 1$$

$$\begin{pmatrix} A^\mathcal{O}_0(j, \sigma) \\ A^\mathcal{O}_j(j, \sigma) \end{pmatrix} = \begin{pmatrix} \cosh \sigma & \sinh \sigma \\ \sinh \sigma & \cosh \sigma \end{pmatrix} \begin{pmatrix} A^\mathcal{O}_0 \\ A^\mathcal{O}_j \end{pmatrix} \quad A^\mathcal{O}_k(j, \sigma) = A^\mathcal{O}_k \quad (k \neq j)$$

$$\begin{aligned} \Omega_{ij} &= \frac{d}{d\sigma} \text{tr}(A^\mathcal{O}_0(j, \sigma) A^\mathcal{O}_i(j, \sigma))|_{\sigma=0} \\ &= \begin{cases} \frac{d}{d\sigma} \left\{ \frac{1}{2} \text{tr}((A^\mathcal{O}_0)^\mathcal{O})^2 + (A^\mathcal{O}_i)^\mathcal{O})^2 \right\} \sinh 2\sigma + \text{tr}(A^\mathcal{O}_0 A^\mathcal{O}_i) \cosh 2\sigma \Big|_{\sigma=0} & (i = j) \\ \frac{d}{d\sigma} \left\{ \text{tr}(A^\mathcal{O}_0 A^\mathcal{O}_i) \cosh \sigma + \text{tr}(A^\mathcal{O}_i A^\mathcal{O}_j) \sinh \sigma \right\} \Big|_{\sigma=0} & (i \neq j) \end{cases} = \text{tr}(A^\mathcal{O}_0)^\mathcal{O})^2 \delta_{ij} + \text{tr}(A^\mathcal{O}_i A^\mathcal{O}_j) \end{aligned}$$

Total effective action:

$$Z_{\text{eff}} = \int dA dk d\tau e^{-S_{\text{eff}}}, \quad \text{where } S_{\text{eff}} = -i(S_b + S_\xi) + S_{\text{gf,L}} + S_{\text{Adiag}} - \log \text{Pf} \mathcal{M}$$

$$S_b = \frac{-N}{4} \text{tr}[A_\mu, A_\nu]^2 \quad S_\xi = \frac{N\gamma}{2} \left\{ \text{tr}(A_0)^2 - \sum_{I=1}^{\tilde{d}} \text{tr}(A_I)^2 - \xi \sum_{I=\tilde{d}+1}^9 \text{tr}(A_I)^2 \right\}$$

$$S_{\text{gf,L}} = \frac{\alpha}{2} \{ \text{tr}(\tilde{A}_0 \tilde{A}_j) \}^2 + \left( -iN\kappa \sum_{j=1}^9 k_j \text{tr}(\tilde{A}_0 \tilde{A}_j) \right) - \log \det \Omega_{ij} \quad \Omega_{ij} = \text{tr}(A_0^2) \delta_{ij} + \text{tr}(A_i A_j) \quad (i, j = 1, 2, \dots, 9)$$

$$S_{\text{Adiag}} = -\log \prod_{1 \leq a < b \leq N} (\alpha_a - \alpha_b)^2 - \sum_{a=1}^{N-1} \tau_a \quad \text{Pf} \mathcal{M} = \int d\psi e^{iS_{m_f}} \quad S_{m_f} = \frac{-N}{2} \text{tr} \{ \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta] + im_f \bar{\psi}_\alpha (\Gamma^7 \Gamma^{8\dagger} \Gamma^9)_{\alpha\beta} \psi_\beta \}$$

$$\frac{d(A_i)_{ab}(\sigma)}{d\sigma} = - \frac{dS_{\text{eff}}}{d(A_i)_{ba}} + \eta_{i,ab}(\sigma), \quad \frac{d\tau_a(\sigma)}{d\sigma} = - \frac{dS_{\text{eff}}}{d\tau_a} + \eta_{a,\tau}(\sigma)$$

$$\frac{dk_j(\sigma)}{d\sigma} = - \frac{dS_{\text{eff}}}{dk_j} + \eta_{j,k}(\sigma)$$

fictitious Langevin time

- $A_i$ : Hermitian  $\rightarrow$  general complex traceless matrices
- $\tau_a, k_j$ : Real number  $\rightarrow$  complex number.

The condition to justify the CLM: [K. Nagata, J. Nishimura and S. Shimasaki, arXiv:1606.07627]

The probability distribution of the drift norms falls exponentially or faster.

$$u_A = \sqrt{\frac{1}{9N^3} \sum_{I=1}^9 \sum_{a,b=1}^N \left| \frac{dS_{\text{eff}}}{d(A_I)_{ba}} \right|^2}, \quad u_\alpha = \sqrt{\frac{1}{N} \sum_{a=1}^{N-1} \left| \frac{dS_{\text{eff}}}{d\tau_a} \right|^2}$$

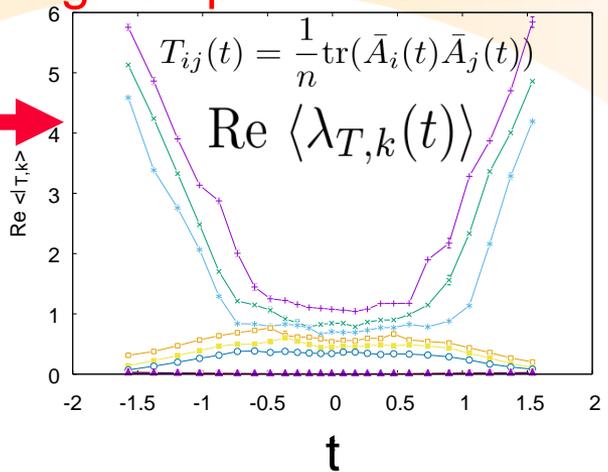
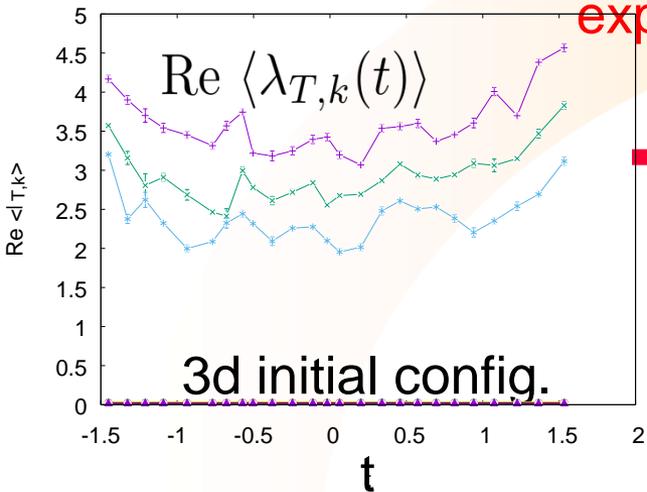
• Dynamical stabilization: [F. Attanasio and B. Jäger arXiv:1808.04400]

After each Langevin step,  $A_I \rightarrow \frac{A_I + \eta A_I^\dagger}{1 + \eta}$  Here,  $\eta = 5 \times 10^{-3}$ .  
( $\eta=0$ : do nothing,  $\eta=1$ : Hermitize completely)

# A. backup

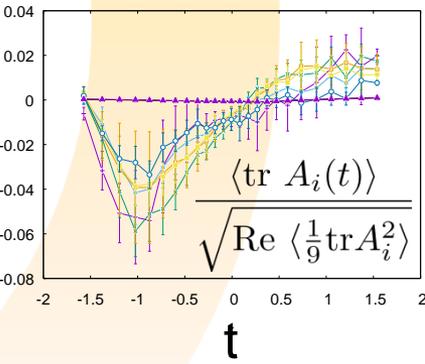
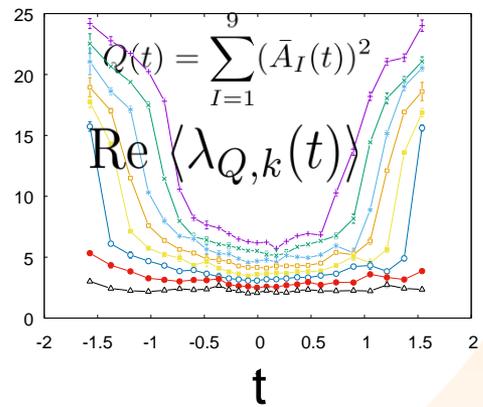
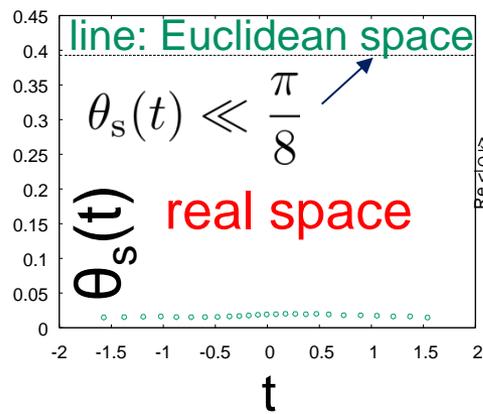
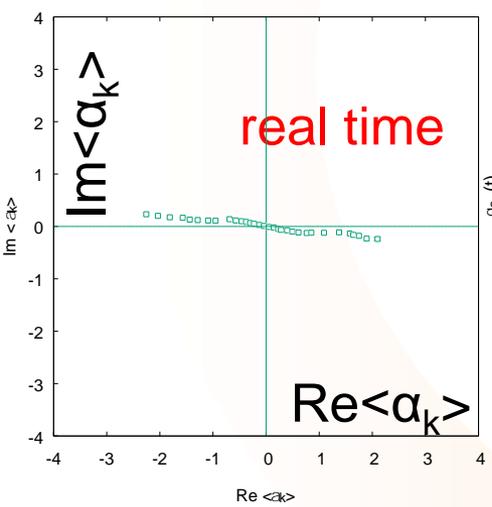
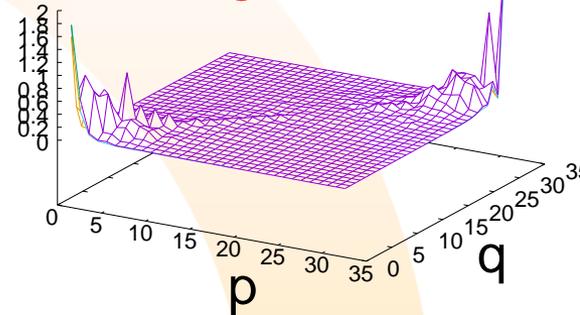
$N=32, n=8, m_f=2, \gamma=6, \xi=10, \tilde{d} = 6$

expanding 3d space at late time



$$A_{pq} = \frac{1}{9} \text{Re} \left\langle \sum_{i=1}^9 (A_i)_{pq} (A_i)_{qp} \right\rangle$$

band-diagonal structure



no Pauli-matrix structure  
 $\Rightarrow$  continuous space

Lorentz boost is mitigated.

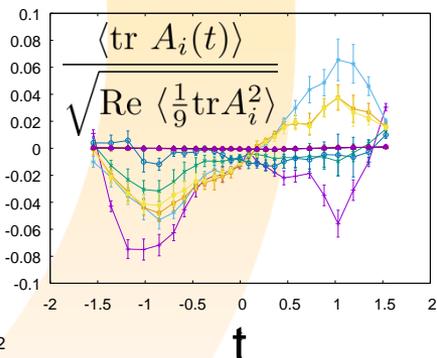
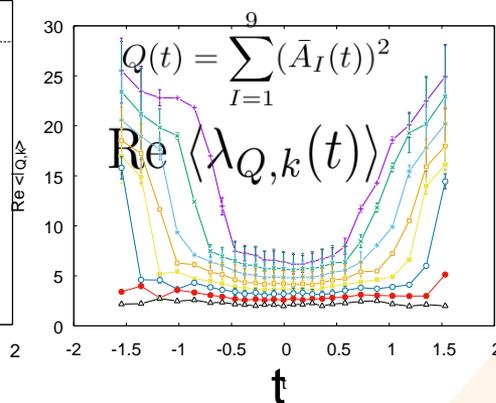
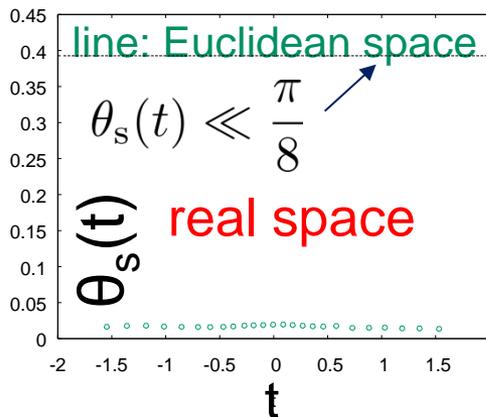
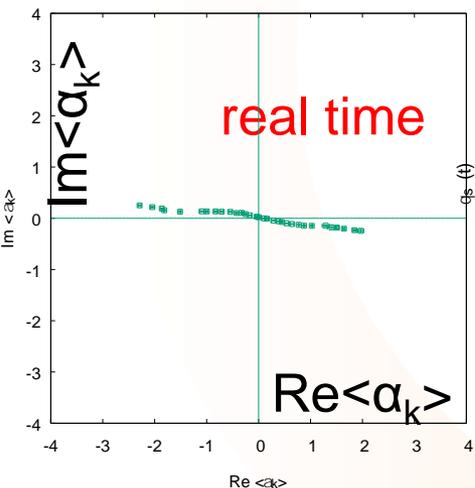
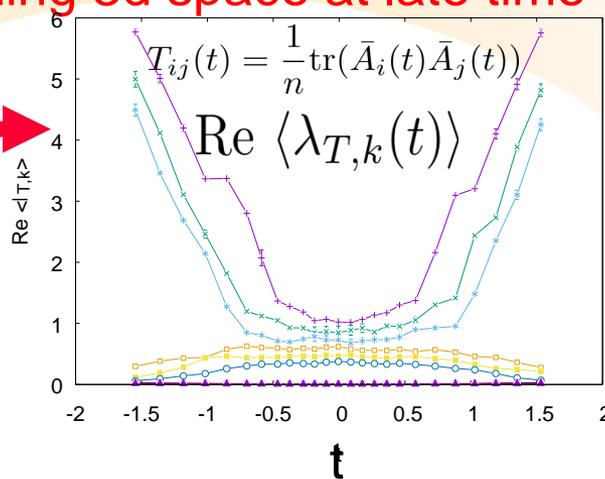
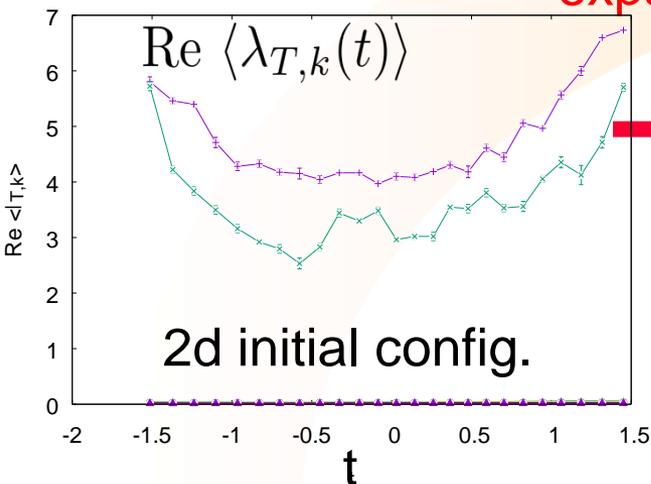
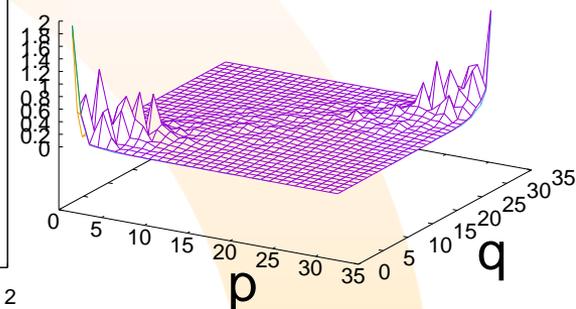
# A. backup

$N=32, n=8, m_f=2, \gamma=6, \xi=10, \tilde{d} = 6$

expanding 3d space at late time

$$A_{pq} = \frac{1}{9} \text{Re} \langle \sum_{i=1}^9 (A_i)_{pq} (A_i)_{qp} \rangle$$

band-diagonal structure



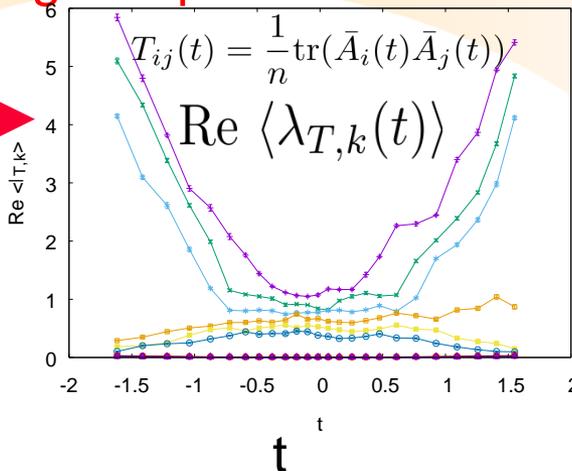
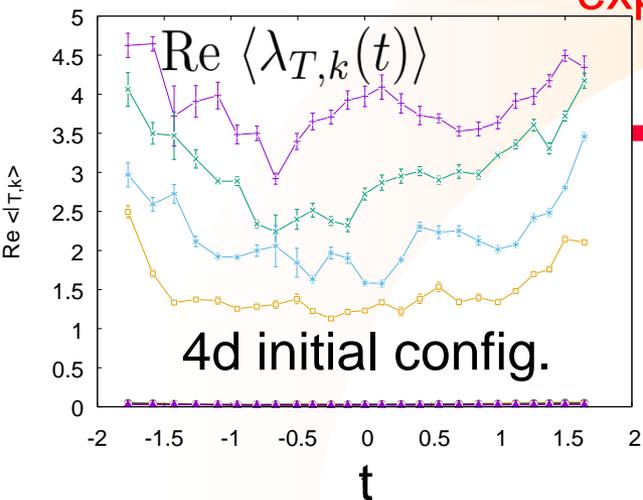
no Pauli-matrix structure  
=> continuous space

Lorentz boost is mitigated.

# A. backup

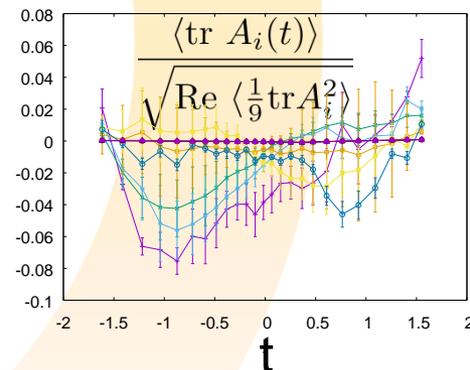
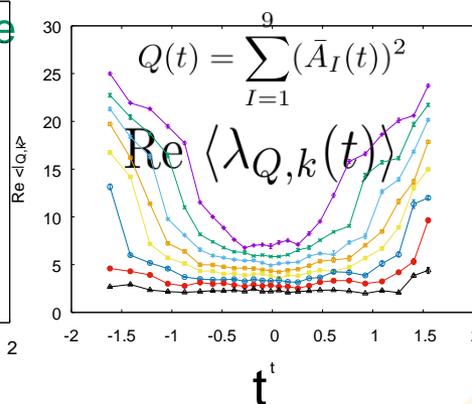
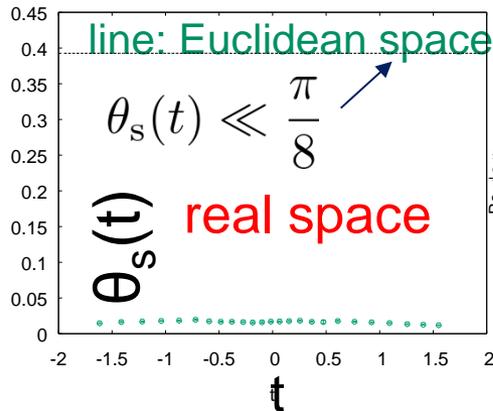
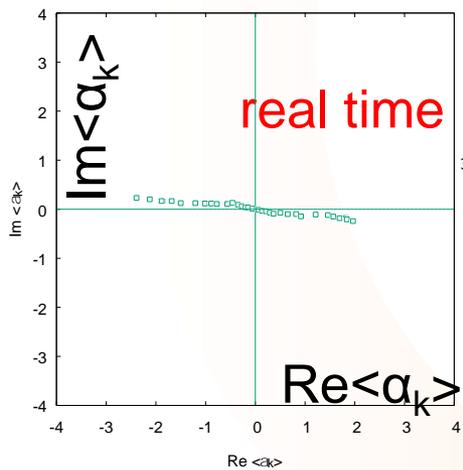
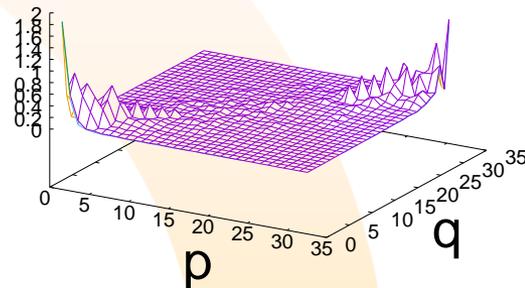
$N=32, n=8, m_f=2, \gamma=6, \xi=10, \tilde{d} = 6$

expanding 3d space at late time



$$A_{pq} = \frac{1}{9} \text{Re} \left\langle \sum_{i=1}^9 (A_i)_{pq} (A_i)_{qp} \right\rangle$$

band-diagonal structure



no Pauli-matrix structure  
 $\Rightarrow$  continuous space

Lorentz boost is mitigated.