$\mathcal{N}=2$ Supersymmetric $O(N)$ Model in Two Dimensions beyond the Leading Order of $1/N$ Expansion

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Contents

1. Introduction

$D = 4$, $\mathcal{N} = 1$ sigma model $\longrightarrow$ $D = 2$, $\mathcal{N} = 2$ sigma model

target space $G^{\mathbb{C}}/\hat{H}$: non-compact Kähler manifold (NCKM)

QNG bosons : non-compact directions

compact, homogeneous target space にするため

auxiliary gauge fields を導入 : D-term constraint

(ex.) $\mathbb{C}P^N$, $G_{N,M}(\mathbb{C})$, Q^N model[1], etc.

今回は gauge fields を導入せず、

non-compact, non-homogeneous のままのモデルの解析

$D = 2$, $\mathcal{N} = 2$, $O(N)$ model (F-term constraint only)

investigation

NCKM 上の場のダイナミクス

安定真空の存在

漸近的自由性, 非自由性

[1]K. Higashijima, T.K., M. Nitta and M. Tsuzuki, Prog. Theor. Phys. **105** (2001) 261.

2. Lagrangian

$$\mathcal{L} = \int d^4\theta \Phi_i^* \Phi_i + \frac{1}{2}\left\{ \int d^2\theta \Phi_0 \left(\Phi_i^2 - a^2\right) + (\mathrm{c.c.}) \right\} .$$

Φ_i : Dynamical Chiral Superfield, $O(N)^{\mathbb{C}}$ ベクトル表現

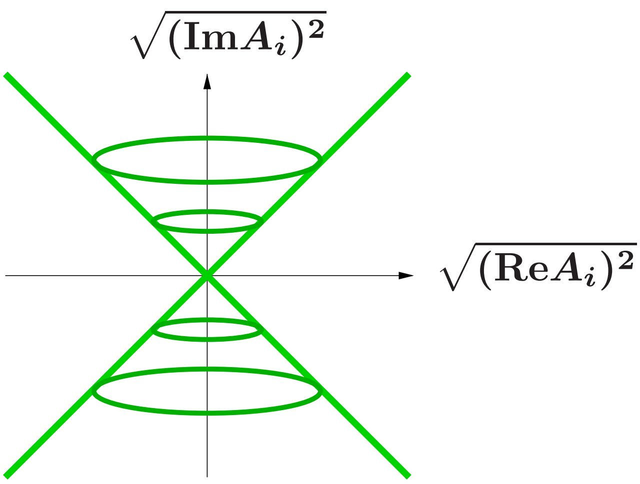
Φ_0 : Auxiliary Chiral Superfield, $O(N)^{\mathbb{C}}$ 一重項

a^2 : $= N/g^2$, 実定数, ***F*-term** 拘束を与える.

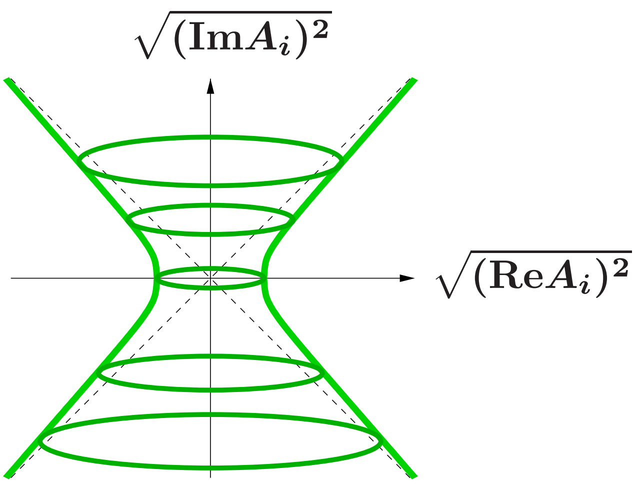
$$\textit{F}\text{-term 拘束条件 : } \Phi_i^2 = a^2 \ , \quad \Phi_i^{*2} = a^2$$

$$\Downarrow$$

$$(\mathrm{Re}A_i)^2 - (\mathrm{Im}A_j)^2 = a^2 \ , \quad (\mathrm{Re}A_i)(\mathrm{Im}A_i) = 0$$



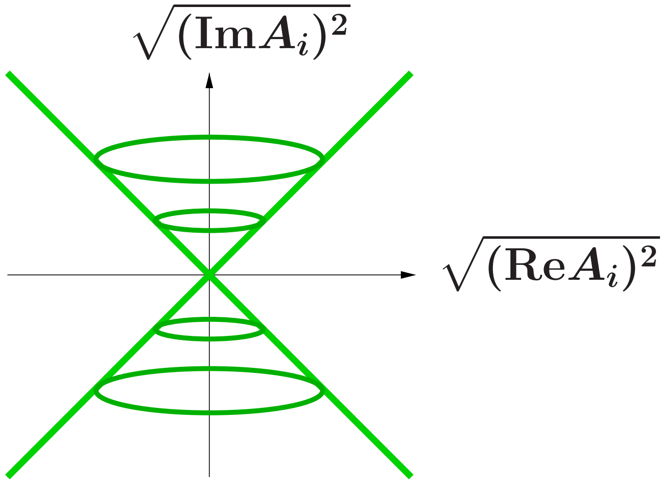
$a^2 = 0$



$a^2 \neq 0$

3. Various Vacua (leading order of $1/N$)

$a^2 = 0$ Lagrangian

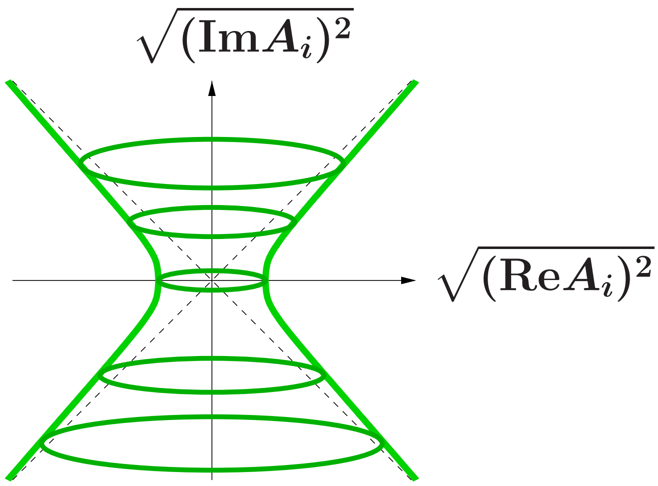


Ricci tensor $R_{i\bar{j}} = 0$

(1) $\langle A_0 \rangle \neq 0 \rightarrow \begin{cases} \Phi_i & : \text{Massive states} \\ \Phi_0 & : \text{Dynamical states (massless)} \end{cases}$

(2) $\langle A_i \rangle \neq 0 \rightarrow \begin{cases} \Phi_i & : \text{Massless states} \\ \Phi_0 & : \text{Non-dynamical states} \end{cases}$

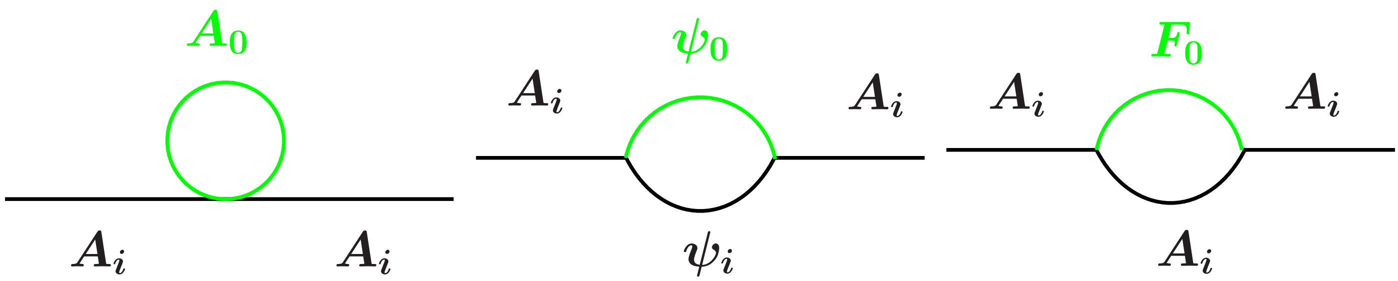
$a^2 \neq 0$ Lagrangian



Ricci tensor $R_{i\bar{j}} < 0$

(3) $\langle A_i \rangle \neq 0 \rightarrow \begin{cases} \Phi_i & : \text{Massless states} \\ \Phi_0 & : \text{Non-dynamical states} \end{cases}$

4. Beyond the Leading Order of $1/N$



$$A_i \,:\, D_{ij}(p) = \frac{\delta_{ij}}{-p^2}\,, \qquad \psi_i \,:\, S_{ij}(p) = \frac{\delta_{ij}}{\not{p}}\,.$$

$$A_0 \,:\, D_A(p) = -\big[\Pi(p)\big]^{-1}\,, \quad \psi_0 \,:\, S_\psi(p) = -\not{p}\cdot\big[\Pi(p)\big]^{-1}\,,$$

$$F_0 \,:\, D_F(p) = -p^2\cdot\big[\Pi(p)\big]^{-1}\,.$$

$$\Pi(p) = -|\langle A_i\rangle|^2 + \frac{N}{8\pi}\int_0^1 dx \frac{p^2}{\lambda^2 - x(1-x)p^2}\,,$$

$$Z^{-1} = 1 + \frac{1}{N}\log\left(\log\frac{\Lambda^2}{\lambda^2} + \frac{4\pi}{N}|\langle A_i\rangle|^2\right)\,.$$

$$\frac{N}{g_R^2} = Z^{-1}\frac{N}{g^2}\,, \quad g_R \to \text{finite}, \quad \Lambda \to \infty$$

$$\Downarrow$$

bare coupling constant $g \to \infty$

Asymptotically Non-Free

5. Summary and Future Problems

F-term 拘束のみの Lagrangian から出発

$a^2 = 0$ Lagrangian ($R_{i\bar{j}} = 0$)

$\langle A_0 \rangle \neq 0$: mass gap, auxiliary fields $\longrightarrow$ dynamical states

$\langle A_i \rangle \neq 0$: auxiliary fields $\longrightarrow$ non-dynamical states

$a^2 \neq 0$ Lagrangian ($R_{i\bar{j}} < 0$)

$\langle A_i \rangle \neq 0$: auxiliary fields $\longrightarrow$ non-dynamical states

Beyond the leading order of $1/N$

定性的に漸近的非自由であることを確認。

Future Problems

∘ 定量的な繰り込み群の考察

QNG bosons の自由度を復活させた Lagrangian を用いた

Wilson 的繰り込み群の展開

$$\mathcal{L} = \int d^4\theta \mathcal{K}(\Phi_i^* \Phi_i) + \int d^2\theta W(\Phi_i) + \int d^2\bar{\theta} W^*(\Phi_i^*) \, .$$

∘ NCKM の Ricci tensor と摂動論 (one-loop)

$$\beta_{i\bar{j}}(g) = -\frac{1}{2\pi} R_{i\bar{j}} \, .$$

∘ NCKM 上の補助場のダイナミクスの一般論

$$\text{non-compact}, R_{i\bar{j}} > 0 \xrightarrow{?} \beta > 0 \xrightarrow{\text{摂動論}} \text{漸近的非自由}$$

$$\xrightarrow{?} \text{mass gap なし} \xrightarrow{\text{非摂動論}} \text{non-dynamical}$$

Effective Potential and Gap Equations
Strong Coupling Theory

$$
\begin{aligned}
V_{\text{eff}} = & \frac{N}{2}\int \frac{d^2k}{(2\pi)^2 i} \log\left[(-k^2+m^2)^2 - f^2\right] \\
& - N\int \frac{d^2k}{(2\pi)^2 i}\log\left[-k^2+m^2\right] \\
& - \frac{1}{2} f\left(\phi_i^2 + \phi_i^{*2}\right) + m^2\phi_i^*\phi_i \, .
\end{aligned}
$$

$$
\begin{aligned}
0 &= \phi_i\left(m^4 - f^2\right) , \\
0 &= \phi_i^*\left(m^4 - f^2\right) , \\
0 &= 2m\left\{\phi_i^*\phi_i + N\int\frac{d^2k}{(2\pi)^2 i}\frac{-k^2+m^2}{(-k^2+m^2)^2 - f^2}\right. \\
& \qquad \left. - N\int\frac{d^2k}{(2\pi)^2 i}\frac{1}{-k^2+m^2}\right\} , \\
0 &= -\frac{1}{2}(\phi_i^2+\phi_i^{*2}) - N\int\frac{d^2k}{(2\pi)^2 i}\frac{f}{(-k^2+m^2)^2 - f^2} \, .
\end{aligned}
$$

Effective Potential and Gap Equations

Weak Coupling Theory

$$
\begin{aligned}
V_{\mathrm{eff}} = {} & \frac{N}{2}\int \frac{d^2k}{(2\pi)^2 i}\log\left[(-k^2+m^2)^2 - f^2\right] \\
& - N\int \frac{d^2k}{(2\pi)^2 i}\log\left[-k^2+m^2\right] \\
& - \frac{1}{2} f e^{i\theta}(\phi_i^2 - a^2) - \frac{1}{2} f e^{-i\theta}(\phi_i^{*2} - a^2) + m^2 \phi_i^* \phi_i \,.
\end{aligned}
$$

$$
\begin{aligned}
0 &= \phi_i (m^4 - f^2) = \phi_i^* (m^4 - f^2) \,, \\
0 &= 2m \left\{ \phi_i^* \phi_i + N\int \frac{d^2k}{(2\pi)^2 i}\frac{-k^2+m^2}{(-k^2+m^2)^2 - f^2} \right. \\
& \qquad \left. - N\int \frac{d^2k}{(2\pi)^2 i}\frac{1}{-k^2+m^2} \right\} \,, \\
\frac{N}{g^2} &= \phi_i^2 + f e^{-i\theta} N \int \frac{d^2k}{(2\pi)^2 i}\frac{1}{(-k^2+m^2)^2 - f^2} \,, \\
\frac{N}{g^2} &= \phi_i^{*2} + f e^{i\theta} N \int \frac{d^2k}{(2\pi)^2 i}\frac{1}{(-k^2+m^2)^2 - f^2} \,.
\end{aligned}
$$

Lagrangian – component fields –

$$\mathcal{L} = \partial_\mu A_i^* \partial^\mu A_i + i\overline{\psi}_i \gamma^\mu \partial_\mu \psi_i$$
$$+\frac{1}{2} F_0 \left(A_i^2 - a^2\right) + \frac{1}{2} F_0^* \left(A_i^{*2} - a^2\right) - A_0^* A_0 A_i^* A_i$$
$$-A_i \overline{\psi_0^c} \psi_i - A_i^* \overline{\psi_i^c} \psi_0 - \frac{1}{2} A_0 \overline{\psi}_i \psi_i - \frac{1}{2} A_0^* \overline{\psi}_i \psi_i^c \quad .$$

charge	Φ_i	A_i	ψ_i	F_i	$\Phi_i^\dagger$	A_i^*	$\overline{\psi}_i$	F_i^*	Φ_0	A_0	ψ_0	F_0	$\Phi_0^\dagger$	A_0^*	$\overline{\psi}_0$	F_0^*	θ	$\overline{\theta}$
R	0	0	−1	−2	0	0	1	2	2	2	1	0	−2	−2	−1	0	1	−1
$U(1)_{\mathrm{A}}$	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	1	1
$U(1)_{\mathrm{V}}$	1	1	1	1	−1	−1	−1	−1	−2	−2	−2	−2	2	2	2	2	0	0
D	−	0	$\frac{1}{2}$	1	−	0	$\frac{1}{2}$	1	−	1	$\frac{3}{2}$	2	−	1	$\frac{3}{2}$	2	$\frac{1}{2}$	$\frac{1}{2}$

$\Phi = A + \sqrt{2}\theta\psi + \theta\theta F$: Chiral superfield in two-dimensional spacetime.