Gauge Theoretical Construction of Non-compact Calabi-Yau Manifolds

木村 哲士

大阪大学大学院 理学研究科 素粒子論研究室

Phys. Lett. B518 (2001) 301, hep-th/0107100

hep-th/0108084 (to appear in Nucl.Phys.B)

hep-th/0110216 (to appear in Ann.of Phys.)

hep-th/0202XXX (to appear in arXiv)

in collaboration with K. Higashijima and M. Nitta

1. Introduction

1-loop 繰り込みで有限な SUSY Nonlinear Sigma Model の探求

Ricci-flat Kähler 多様体 (coset construction)

Non-compact Calabi-Yau 多様体を構成

SNLSM as Gauge Theories (compact Kähler) を応用

⇓

Complex Line Bundle の出現, 特に

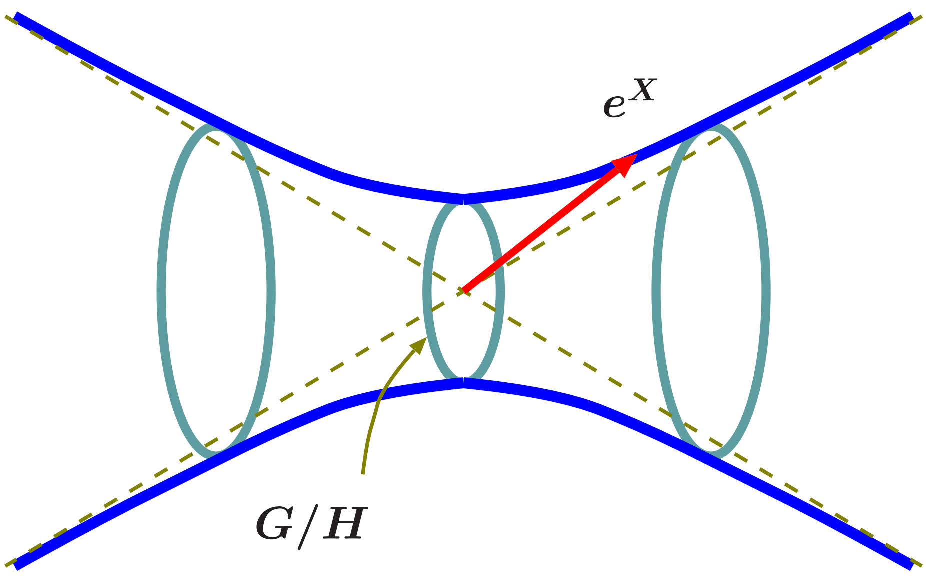
Hermitian Symmetric Spaces

$+$

complex line

total 空間は

特異点が回避されている多様体とみなされる



Contents

2. Compact Kähler manifolds

projective space: $\mathbb{C}P^{N-1} = SU(N)/[SU(N-1) \times U(1)]$

$\vec{\phi} \in \mathbb{C}^N$: $\vec{\phi} \sim \lambda\vec{\phi}$, $\lambda \in \mathbb{C}^1$ で同一視

$\vec{\phi}^T = \left(1, \varphi^i\right)$, $(i = 1, 2, \cdots, N-1)$ 座標系を選択

同一視の下で不変な Kähler potential K:

$$K = c \log \vec{\phi}^\dagger \vec{\phi} = c \log \left\{1 + |\varphi^i|^2\right\} , \quad c = \text{constant}$$

quadric surface: $Q^{N-2} = SO(N)/[SO(N-2) \times U(1)]$

$\mathbb{C}P^{N-1} + [\vec{\phi}^2 = 0]$:

$\vec{\phi}^T = \left(1, \varphi^i, -\frac{1}{2}(\varphi^i)^2\right)$, $(i = 1, 2, \cdots, N-2)$ 座標系を選択

$$K = c \log \vec{\phi}^\dagger \vec{\phi} = c \log \left\{1 + |\varphi^i|^2 + \frac{1}{4}(\varphi^i)^2(\varphi^{*j})^2\right\}$$

例外群: $E_6/[SO(10) \times U(1)]$, $E_7/[E_6 \times U(1)]$

$\mathbb{C}P^{26} + [\Gamma_{ijk}\phi^j\phi^k = 0]$, $\mathbb{C}P^{55} + [d_{\alpha\beta\gamma\delta}\phi^\beta\phi^\gamma\phi^\delta = 0]$:

Γ_{ijk}: rank-3 symmetric tensor invariant under E_6

$d_{\alpha\beta\gamma\delta}$: rank-4 symmetric tensor invariant under E_7

$[E_6]$: $\vec{\phi}^T = \left(1, \varphi_\alpha, -\frac{1}{2\sqrt{2}}\varphi C \sigma^\dagger_A \varphi\right)$, $(\alpha = 1, 2, \cdots, 16;\ A = 1, 2, \cdots, 10)$

$$K = c \log \left\{1 + |\varphi_\alpha|^2 + \frac{1}{8}|\varphi C \sigma^\dagger_A \varphi|^2\right\}$$

$[E_7]$: $\vec{\phi}^T = \left(1, \varphi^i, \frac{1}{2}\Gamma_{ijk}\varphi^j\varphi^k, \frac{1}{6}\Gamma_{ijk}\varphi^i\varphi^j\varphi^k\right)$, $(i = 1, 2, \cdots, 27)$

$$K = c \log \left\{1 + |\varphi^i|^2 + \frac{1}{4}|\Gamma_{ijk}\varphi^j\varphi^k|^2 + \frac{1}{36}|\Gamma_{ijk}\varphi^i\varphi^j\varphi^k|^2\right\}$$

Grassmannian: $G_{N,M} = U(N)/[U(N-M)\times U(M)]$

$\Phi : N \times M$ matrix, $\Phi \sim \Phi V$ $[V \in U(M)]$ で同一視

$\Phi = \begin{pmatrix} 1_M \\ \varphi_{Aa} \end{pmatrix}$, $(A = 1, 2, \cdots, N-M;\ a = 1, 2, \cdots, M)$ の座標系

同一視の下で不変な Kähler potential K:

$$K = c \log\det \Phi^\dagger \Phi = c \log\det \left\{1_M + \varphi^\dagger \varphi\right\}$$

$Sp(N)/U(N)$

$G_{2N,N} + [\varphi^T - \varphi = 0]$:

$\Phi = \begin{pmatrix} 1_N \\ \varphi_{ab} \end{pmatrix}$, $(1 \le a \le b \le N)$ の座標系

$$K = c \log\det \Phi^\dagger \Phi = c \log\det \left\{1_N + \varphi^\dagger \varphi\right\}$$

$SO(2N)/U(N)$

$G_{2N,N} + [\varphi^T + \varphi = 0]$:

$\Phi = \begin{pmatrix} 1_N \\ \varphi_{ab} \end{pmatrix}$, $(1 \le a < b \le N)$ の座標系

$$K = c \log\det \Phi^\dagger \Phi = c \log\det \left\{1_N + \varphi^\dagger \varphi\right\}$$

Hermitian symmetric spaces

type	G/H	$\dim_{\mathbb{C}}(G/H)$	Kähler potential K
AIII_1	$\mathbb{C}P^{N-1}$	$N-1$	$c\log\{1+\lvert\varphi^i\rvert^2\}$
AIII_2	$G_{N,M}$	$M(N-M)$	$c\log\det\{1_M+\varphi^\dagger\varphi\}$
BDI	Q^{N-2}	$N-2$	$c\log\{1+\lvert\varphi^i\rvert^2+\frac{1}{4}(\varphi^i)^2(\varphi^{*j})^2\}$
CI	$\dfrac{Sp(N)}{U(N)}$	$\dfrac{1}{2}N(N+1)$	$c\log\det\{1_N+\varphi^\dagger\varphi\}$
DIII	$\dfrac{SO(2N)}{U(N)}$	$\dfrac{1}{2}N(N-1)$	$c\log\det\{1_N+\varphi^\dagger\varphi\}$
EIII	$\dfrac{E_6}{SO(10)\times U(1)}$	16	$c\log\{1+\lvert\varphi_\alpha\rvert^2+\frac{1}{8}\lvert\varphi C\sigma_A^\dagger\varphi\rvert^2\}$
EVII	$\dfrac{E_7}{E_6\times U(1)}$	27	$c\log\{1+\lvert\varphi^i\rvert^2+\frac{1}{4}\lvert\Gamma_{ijk}\varphi^j\varphi^k\rvert^2+\frac{1}{36}\lvert\Gamma_{ijk}\varphi^i\varphi^j\varphi^k\rvert^2\}$

3. Complex line bundles

line bundle over $\mathbb{C}P^{N-1}$: Proto type

Kähler 構造を壊さずに複素 1 次元追加する最も簡単な方法:

$$\vec{\phi}^T \equiv \sigma(1, \varphi^i),\ \sigma \in \mathbb{C}^1$$

Assumption: Kähler potential $\mathcal{K}$ (non-compact manifold)

$$\mathcal{K} = \mathcal{K}(X)\ ,\quad X \equiv \log \vec{\phi}^\dagger \vec{\phi} = \log |\sigma|^2 + \Psi$$

X : non-compact direction σ × compact Ψ $(= K|_{c=1})$

$$\mathbb{C} \times \frac{SU(N)}{SU(N-1) \times U(1)}$$

多様体として複素 1 次元が non-compact 方向に伸びる

Ricci-flat condition

holomorphic coordinates: $\phi^\mu = \{\sigma, \varphi^i\}$

metric: $g_{\mu\nu^*} = \partial_\mu \partial_{\nu^*} \mathcal{K}(X)$

$$\text{Ricci tensor}:\quad (Ric)_{\mu\nu^*} = -\partial_\mu \partial_{\nu^*} \log \det g_{\kappa\lambda^*}$$

Ricci-flat condition:

$$(Ric)_{\mu\nu^*} = 0 \longrightarrow \begin{cases} \det g_{\mu\nu^*} = (\text{constant}) \times |F|^2 \\ F = \text{holomorphic function} \end{cases}$$

具体的には $\cdots$ ($\sigma \neq 0$ 領域で考える)

$$g_{\sigma\sigma^*} = \frac{d^2\mathcal{K}}{dX^2}\cdot\frac{\partial X}{\partial\sigma}\frac{\partial X}{\partial\sigma^*}, \quad g_{\sigma j^*} = \frac{d^2\mathcal{K}}{dX^2}\cdot\frac{\partial X}{\partial\sigma}\frac{\partial X}{\partial\varphi^{*j}},$$

$$g_{ij^*} = \frac{d^2\mathcal{K}}{dX^2}\cdot\frac{\partial X}{\partial\varphi^i}\frac{\partial X}{\partial\varphi^{*j}} + \frac{d\mathcal{K}}{dX}\cdot\frac{\partial^2 X}{\partial\varphi^i\partial\varphi^{*j}}$$

determinant:

$$\det g_{\mu\nu^*} = \frac{1}{|\sigma|^2}\frac{d^2\mathcal{K}}{dX^2}\cdot\det\left\{\frac{d\mathcal{K}}{dX}\cdot\frac{\partial^2 X}{\partial\varphi^i\partial\varphi^{*j}}\right\}$$

$$= \frac{1}{|\sigma|^2}\frac{d^2\mathcal{K}}{dX^2}\left(\frac{d\mathcal{K}}{dX}\right)^{N-1}\cdot\det\tilde{g}_{ij^*} \quad \left(\partial_i\partial_{j^*}X = \partial_i\partial_{j^*}\Psi \equiv \tilde{g}_{ij^*}\right)$$

Ricci-flat condition は偏微分方程式

一般には解析不能

しかし

$\mathbb{C}P^{N-1}$ は Einstein-Kähler

$$-\partial_i\partial_{j^*}\log\det\tilde{g}_{kl^*} = (\widetilde{Ric})_{ij^*} = \mathcal{C}\,\tilde{g}_{ij^*} = \mathcal{C}\,\partial_i\partial_{j^*}\Psi$$

$$\rightarrow \det\tilde{g}_{ij^*} \sim \exp\big(-\mathcal{C}\Psi\big) \sim \exp(-\mathcal{C}X)$$

Ricci-flat condition:

$$e^{-NX}\frac{d}{dX}\left(\frac{d\mathcal{K}}{dX}\right)^N \equiv (\text{constant})$$

$$\frac{d\mathcal{K}}{dX} = \left(\lambda e^{NX} + b\right)^{\frac{1}{N}}$$

λ : positive real parameter

b : integration constant, 非常に重要な parameter

特徴

$b \neq 0$ での metric: $\left(ds^2 = g_{\sigma\sigma^*} d\sigma d\sigma^* + \cdots \right)$

$$g_{\sigma\sigma^*} = \lambda\left(\lambda e^{NX} + b\right)^{\frac{1-N}{N}} e^{N\Psi} |\sigma|^{2N-2}$$

$\sigma = 0$ で metric が潰れる $\Longleftrightarrow$ $\sigma \to 0$ 極限で curvature は有限

$\phi^\mu = \{\sigma, \varphi^i\}$ は座標特異点 $(\sigma = 0)$ を持つ

$$\Downarrow$$

座標変換 : $\rho \equiv \dfrac{\sigma^N}{N}$

この変換後の metric:

$$g_{\rho\rho^*} = \lambda\left(\lambda e^{NX} + b\right)^{\frac{1-N}{N}} e^{N\Psi}$$

$$g_{\rho j^*} = \lambda N\left(\lambda e^{NX} + b\right)^{\frac{1-N}{N}} e^{N\Psi} \rho^* \partial_{j^*}\Psi$$

$$g_{ij^*} = \lambda N^2\left(\lambda e^{NX} + b\right)^{\frac{1-N}{N}} e^{N\Psi} |\rho|^2 \partial_i\Psi\partial_{j^*}\Psi + \left(\lambda e^{NX} + b\right)^{\frac{1}{N}} \partial_i\partial_{j^*}\Psi$$

$\rho = 0$ $(d\rho = 0)$ 部分多様体の metric:

$$g_{ij^*}\big|_{\rho=0} = b^{\frac{1}{N}} \partial_i\partial_{j^*}\Psi$$

$$\Uparrow$$

$\mathbb{C}P^{N-1}$ の Fubini-Study metric そのもの

$b = 0$ での Kähler potential: $d\mathcal{K}/dX = \lambda^{\frac{1}{N}} e^X$

$$\mathcal{K}\Big|_{b=0} = \lambda^{\frac{1}{N}} e^X = \lambda^{\frac{1}{N}} |\sigma|^2 (1 + |\varphi^i|^2)$$

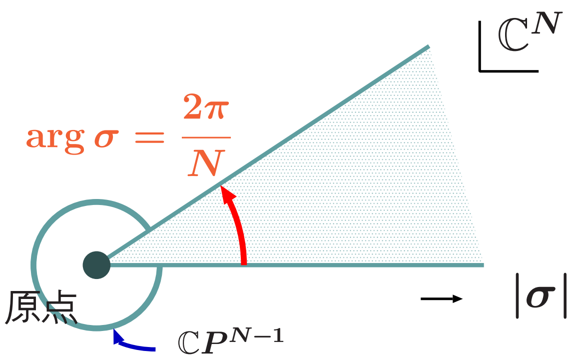
$\phi^1 = \sigma$, $\phi^i = \sigma\varphi^{i-1}$ と戻すと $\{\vec{\phi}^T = \sigma(1, \varphi^i)\}$

$$\mathcal{K} = \lambda^{\frac{1}{N}} \vec{\phi}^\dagger \vec{\phi} \quad \Leftarrow \quad \text{flat metric?}$$

$\rho = \sigma^N/N$ の座標変換

$\Downarrow$

Orbifold $\mathbb{C}^N/\mathbb{Z}_N$



したがってこの Kähler 多様体は

- $b \neq 0$ という特異点回避を示す parameter を持ち,
- 特異点は $\mathbb{C}P^{N-1}$ 多様体で回避され,
- $b = 0$ 極限で Orbifold $\mathbb{C}^N/\mathbb{Z}_N$ が出現する

$\Downarrow$

complex line bundle over $\mathbb{C}P^{N-1}$

line bundle over Q^{N-2}:

line bundle over $\mathbb{C}P^{N-1}$ と同様の設定:

$$\vec{\phi}^T = \sigma\left(\ 1,\ \varphi^i,\ -\frac{1}{2}(\varphi^i)^2\ \right)\ , \quad \sigma \in \mathbb{C}^1$$

line bundle over $\mathbb{C}P^{N-1}$ 同様, Kähler potential を次のように仮定:

$$\mathcal{K} = \mathcal{K}(X)\ , \quad X = \log \vec{\phi}^\dagger \vec{\phi} = \log |\sigma|^2 + \Psi$$

$$\Psi\text{: Kähler potential of } Q^{N-2}\ (c = 1)$$

Ricci-flat condition の解: $\quad \dfrac{d\mathcal{K}}{dX} = \left(\lambda e^{(N-2)X} + b\right)^{\frac{1}{N-1}}$

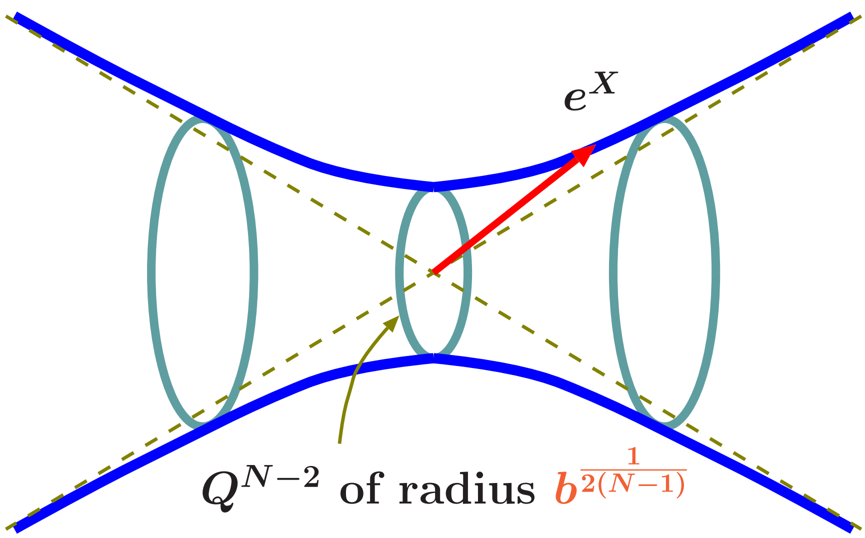
$b \neq 0$ での metric: $\left(ds^2 = g_{\sigma\sigma^*} d\sigma d\sigma^* + \cdots\right)$

$$g_{\sigma\sigma^*} = \frac{N-2}{N-1}\lambda\left(\lambda e^{(N-2)X} + b\right)^{-\frac{N-2}{N-1}} e^{(N-2)\Psi} |\sigma|^{2N-6}$$

座標特異点を除くための座標変換: $\rho \sim \sigma^{N-2}$

$\rho = 0$ $(d\rho = 0)$ 部分多様体:

$$g_{ij^*}\big|_{\rho=0} = b^{\frac{1}{N-1}} \partial_i \partial_{j^*} \Psi \quad \Leftarrow \quad Q^{N-2} \text{ metric そのもの}$$



$N = 3$ 解： Eguchi-Hanson gravitational instanton

$$\mathcal{K}(X) = 2\sqrt{\lambda e^X + b} + \sqrt{b}\log\left(\frac{\sqrt{\lambda e^X + b} - \sqrt{b}}{\sqrt{\lambda e^X + b} + \sqrt{b}}\right)$$

$\varrho^4 \equiv 4(\lambda e^X + b)$, $a^4 \equiv 4b$ と再定義

$$\mathcal{K} \;=\; \varrho^2 + \frac{a^2}{2}\log\left(\frac{\varrho^2 - a^2}{\varrho^2 + a^2}\right)$$

Eguchi-Hanson の Kähler potential そのもの

特異点 (原点) は $Q^1 = SO(3)/U(1) = S^2$ で回避

$N = 4$ 解： Q^2 Resolved Conifold

特異点回避は $Q^2 \simeq S^2 \times S^2$ (同半径) で行われる

deformation (S^3) でも small resolution (S^2) でもない

exceptional groups:

line bundle over $\mathbb{C}P^{26} + [\Gamma_{ijk}\phi^j\phi^k = 0]$

$$\Longrightarrow \text{line bundle over } E_6/[SO(10) \times U(1)]$$

line bundle over $\mathbb{C}P^{55} + [d_{\alpha\beta\gamma\delta}\phi^\beta\phi^\gamma\phi^\delta = 0]$

$$\Longrightarrow \text{line bundle over } E_7/[E_6 \times U(1)]$$

Ricci-flat condition の解と座標変換:

$$\frac{d\mathcal{K}}{dX} = \left(\lambda e^{\mathcal{C}X} + b\right)^{\frac{1}{D}}, \quad \rho = \sigma^n/n$$

line bundle	$\mathcal{C}$	D	n
$\mathbb{C} \times \dfrac{E_6}{SO(10) \times U(1)}$	12	17	12
$\mathbb{C} \times \dfrac{E_7}{E_6 \times U(1)}$	18	28	18

4. Gauge Theoretical Construction

line bundle over $\mathbb{C}P^{N-1}$ 同様の構成もあるが,
ここでは途中まで別の構成 (Gauge Theory) を用いる

$\Longrightarrow$ Supersymmetric Nonlinear Sigma Models を応用

Grassmannian $G_{N,M}$:

$$\Phi : N \times M \text{ matrix-valued chiral superfield}$$

$U(N) \times U(M)$ 群による変換:

$$\Phi \to \Phi' = g_{\rm L} \Phi g_{\rm R} \ , \quad (g_{\rm L}, g_{\rm R}) \in (U(N), U(M)) \ .$$

$U(M)$ gauge transformation:

$$\Phi \to \Phi' = \Phi e^{-i\Lambda} \ , \quad e^{V} \to e^{V'} = e^{i\Lambda} e^{V} e^{-i\Lambda^\dagger} \ .$$

$U(N) \times U(M)$ 不変な Lagrangian:

$$\mathcal{L} = \int d^4\theta \left\{ {\rm tr}(\Phi^\dagger \Phi e^V) - c \ {\rm tr} V \right\} .$$

V : 補助場, vector superfield ($U(M)^{\mathbb{C}}$ gauge group)

$\Longrightarrow$ 積分すると非線形模型, target 空間が Grassmannian $G_{N,M}$

gauge-fixing: $\Phi = \begin{pmatrix} 1_M \\ \varphi_{Aa} \end{pmatrix}$,

φ_{Aa} : $(N-M) \times M$ matrix-valued chiral superfield

$$\mathcal{K} = c \log \det \left(1_M + \varphi^\dagger \varphi \right)$$

$$G_{N,M} = \frac{U(N)}{U(N-M) \times U(M)}$$

$G_{2N,N}$ + F-term constraints:

$G_{2N,N}$ Lagrangian + Superpotential term: $W = \mathrm{tr}(\Phi_0 \Phi^T J' \Phi)$

Φ_0 : $N \times N$ matrix-valued auxiliary superfield

$$J' = \begin{pmatrix} 0 & \mathbf{1}_N \\ \epsilon \mathbf{1}_N & 0 \end{pmatrix}, \ \epsilon = \pm 1$$

$$\mathcal{L} = \int d^4\theta \, \mathcal{K}(\Phi, \Phi^\dagger, V) + \left(\int d^2\theta \, W(\Phi_0, \Phi) + \mathrm{c.c.} \right)$$

補助場 V, Φ_0 を積分して非線形模型

$$\mathcal{K} = c \log \det \left\{ \mathbf{1}_N + \varphi^\dagger \varphi \right\}$$

$$\frac{Sp(N)}{U(N)} \ \ (\epsilon = -1, \varphi^T - \varphi = 0) \ , \quad \frac{SO(2N)}{U(N)} \ \ (\epsilon = +1, \varphi^T + \varphi = 0)$$

line bundle over $G_{N,M}$:

$U(M)$ gauge symmetric Lagrangian:

$$\mathcal{K}_0(\Phi, \Phi^\dagger, V) = f(\mathrm{tr}(\Phi^\dagger \Phi e^V)) - c\,\mathrm{tr}V$$

f : $\mathrm{tr}(\Phi^\dagger \Phi e^V)$ の任意関数, $V = V^a T_a$, $T_a \in U(M)$

c : FI constant $\rightarrow$ vector superfield C に格上げ

equation of motion for V and C:

$$\partial \mathcal{L}/\partial V = f' \cdot \Phi^\dagger \Phi e^V - C \cdot 1_M = 0$$

$$\partial \mathcal{L}/\partial C = -\mathrm{tr}V = 0$$

$\mathrm{tr}V = 0$ のもと, 第1式の trace, determinant をとる:

$$f' \cdot \mathrm{tr}(\Phi^\dagger \Phi e^V) = M \cdot C\,, \quad (f')^M \cdot \det \Phi^\dagger \Phi = C^M$$

$$C \text{ を消去: } \mathrm{tr}(\Phi^\dagger \Phi e^V) = M \left[\det \Phi^\dagger \Phi \right]^{\frac{1}{M}}$$

$$\Downarrow$$

$\partial \mathcal{L}/\partial C = 0$ ($U(M) \rightarrow SU(M)$ gauge group) のもとで

$$\mathcal{K}_0 = f(M[\det \Phi^\dagger \Phi]^{\frac{1}{M}}) \equiv \mathcal{K}(\underbrace{\log \det \Phi^\dagger \Phi}_{\textstyle \equiv X})$$

C を導入 = gauge 群のうち $U(1)$ 部分を ungauged

line bundle を構成する際最も簡単な 1 次元追加方法:

$$\Phi = \sigma \begin{pmatrix} 1_M \\ \varphi_{Aa} \end{pmatrix}, \sigma \in \mathbb{C}^1 \quad \Longrightarrow \quad X = M^2 \log |\sigma|^2 + \Psi$$

以後の解析方法は line bundle over $\mathbb{C}P^{N-1}$ と同じ

Ricci-flat condition の解と座標変換:

$$\frac{d\mathcal{K}}{dX} = \left(\lambda e^{\mathcal{C}X} + b\right)^{\frac{1}{D}}, \quad \rho = \sigma^n/n$$

line bundle	$\mathcal{C}$	D	n
$\mathbb{C} \times G_{N,M}$	N	$1 + M(N-M)$	MN
$\mathbb{C} \times \dfrac{Sp(N)}{U(N)}$	$N+1$	$1 + \frac{1}{2}N(N+1)$	$N(N+1)$
$\mathbb{C} \times \dfrac{SO(2N)}{U(N)}$	$N-1$	$1 + \frac{1}{2}N(N-1)$	$N(N-1)$

すべて特異点はそれぞれの compact な多様体で回避

Isomorphism and duality:

base manifold にある同型性が line bundle にも存在する

Eguchi-Hanson space (complex two-dimensions)

$$\mathbb{C}P^1 \simeq \frac{SO(4)}{U(2)} \simeq \frac{Sp(1)}{U(1)} \simeq Q^1 = \frac{SO(3)}{U(1)}$$

Complex line bundle over $\mathbb{C}P^3$ (complex four-dimensions)

$$\mathbb{C}P^3 \simeq \frac{SO(6)}{U(3)}$$

Another four-dimensional manifolds

$$\frac{Sp(2)}{U(2)} \simeq Q^3 = \frac{SO(5)}{SO(3) \times U(1)}$$

Line bundle over the Klein quadric (complex five-dimensions)

$$G_{4,2} \simeq Q^4 = \frac{SO(6)}{SO(4) \times U(1)}$$

Grassmannian, line bundle 共に次の duality が存在する

Duality between Grassmannians

$$G_{N,M} \simeq G_{N,N-M}$$

Ricci-flat solution and coordinate transformation:

$$\frac{d\mathcal{K}}{dX} = \left(\lambda e^{\mathcal{C}X} + b\right)^{\frac{1}{D}} \,, \quad \rho = \sigma^n/n \,.$$

Hermitian symmetric spaces:

type	$\mathbb{C} \ltimes G/H$	$\mathcal{C}$	D	n
AIII_1	$\mathbb{C} \ltimes \mathbb{C}P^{N-1}$	N	$1 + (N-1)$	N
AIII_2	$\mathbb{C} \ltimes G_{N,M}$	N	$1 + M(N-M)$	MN
BDI	$\mathbb{C} \ltimes Q^{N-2}$	$N-2$	$1 + (N-2)$	$N-2$
CI	$\mathbb{C} \ltimes Sp(N)/U(N)$	$N+1$	$1 + \frac{1}{2}N(N+1)$	$N(N+1)$
DIII	$\mathbb{C} \ltimes SO(2N)/U(N)$	$N-1$	$1 + \frac{1}{2}N(N-1)$	$N(N-1)$
EIII	$\mathbb{C} \ltimes E_6/[SO(10) \times U(1)]$	12	$1 + 16$	12
EVII	$\mathbb{C} \ltimes E_7/[E_6 \times U(1)]$	18	$1 + 27$	18

$$D = \dim_{\mathbb{C}}(\mathbb{C} \ltimes G/H) \,, \quad \mathcal{C} = \frac{1}{2} C_2(G)$$

$$Q^1 \simeq \mathbb{C}P^1 \simeq SO(4)/U(2) \simeq Sp(1)/U(1) \qquad \mathbb{C}P^3 \simeq SO(6)/U(3)$$

$$Q^2 \simeq \mathbb{C}P^1 \times \mathbb{C}P^1 \qquad Q^4 \simeq G_{4,2}$$

$$Q^3 \simeq Sp(2)/U(2) \qquad G_{N,M} \simeq G_{N,N-M}$$

5. Generalizations

(to appear in hep-th/0202XXX)

Kähler coset G/H には Einstein 計量が入れられる

$\Downarrow$

Kähler G/H を base にした line bundle が作られる

Non-symmetric Spaces

$$(\text{ex.}) \qquad \mathbb{C} \ltimes \frac{SU(\ell+m+n)}{S[U(\ell)\times U(m)\times U(n)]}$$

† 複素構造が 2 種類存在する

Direct Product

$$(\text{ex.}) \qquad \mathbb{C} \ltimes \{\mathbb{C}P^{N-1} \times \mathbb{C}P^{M-1}\}$$

† それぞれの半径は任意ではない $(N : M)$

6. Summary and Discussions

Gauge theory を用いた compact な Kähler 多様体

$\Downarrow$

$U(1)$ ungauged $\Rightarrow$ non-compact Kähler 多様体の導出

Einstein-Kähler $\Rightarrow$ 「Ricci-flat 条件 $=$ 常微分方程式」

座標変換 $\rho \sim \sigma^n$ $\Rightarrow$ 座標特異点消失

積分定数 $b \neq 0$ $\Rightarrow$ 特異点消失

$\rho = 0$ 部分多様体 $=$ 「compact Kähler 多様体」

$\Downarrow$

Complex line bundle over compact Kähler manifolds

$\Downarrow$

Non-compact Calabi-Yau manifolds

課題:

多様体の大域的な構造/無限遠の構造

超共形場理論の構成

Supergravity/Superstring, D-branes, M-branes への応用

付録

NLSM = Non-Linear Sigma Model

対称性の破れを記述 (ex. $D = 4$)

Riemann 多様体, coset space ($M = G/H$)

$$\mathcal{L} = g_{ab}(\varphi)\partial_\mu \varphi^a(x)\partial^\mu \varphi^b(x)$$

μ : 時空の添字

φ^a : Nambu-Goldstone 場, Riemann 多様体の座標

$g_{ab}(\varphi)$: Riemann 多様体の計量

SNLSM and Kähler Potential

SNLSM = Supersymmetric NLSM

Kähler 多様体 ($D = 4$, $\mathcal{N} = 1$)

$$\begin{aligned}\mathcal{L} &= g_{ab^*}\partial_\mu \varphi^a \partial^\mu \varphi^{*b} + i g_{ab^*}\bar{\psi}^b (\not{D}\psi)^a + \frac{1}{4} R_{ab^*cd^*}\psi^a \psi^c \bar{\psi}^b \bar{\psi}^d \\ &= \int d^4\theta \, \mathcal{K}(\Phi, \Phi^\dagger)\end{aligned}$$

$\Phi^a = \varphi^a + \sqrt{2}\theta\psi^a + \theta\theta F^a$: chiral superfield

$\mathcal{K}(\Phi, \Phi^\dagger)$: Kähler potential

計量, 曲率は Kähler potential で表現される

$$g_{ab^*} = \partial_a \partial_{b^*} \mathcal{K} \,, \quad (\partial_a \equiv \partial / \partial \varphi^a)$$

Kähler potential の一般解と積分因子:

$$\frac{d\mathcal{K}}{dX} = \left(\lambda e^{\mathcal{C}X} + b\right)^{\frac{1}{D}}$$

$$\mathcal{K}(X) = \frac{D}{\mathcal{C}}\left[\left(\lambda e^{\mathcal{C}X} + b\right)^{\frac{1}{D}} + b^{\frac{1}{D}} \cdot I\left(b^{-\frac{1}{D}}\left(\lambda e^{\mathcal{C}X} + b\right)^{\frac{1}{D}}; D\right)\right]$$

$$\begin{aligned} I(y; D) \equiv \int^{y} \frac{dt}{t^D - 1} = \frac{1}{D}\left[\log(y-1) - \frac{1+(-1)^D}{2}\log(y+1)\right] \\ + \frac{1}{D}\sum_{r=1}^{\left[\frac{D-1}{2}\right]} \cos\frac{2r\pi}{D} \log\left(y^2 - 2y\cos\frac{2r\pi}{D} + 1\right) \\ + \frac{2}{D}\sum_{r=1}^{\left[\frac{D-1}{2}\right]} \sin\frac{2r\pi}{D} \arctan\left[\frac{\cos(2r\pi/D) - y}{\sin(2r\pi/D)}\right] \end{aligned}$$

Deformation, Small resolution

Six-dimensional manifold:

$$\sum_{A=1}^{4} (w^A)^2 = 0 \ , \quad \sum_{A=1}^{4} |w^A|^2 = r^2$$

$$\Downarrow$$

$$\mathbb{R} \times S^2 \times S^3$$