The Physical Society of Japan 2003 Autumn Meeting at Miyazaki
September 9, 2003

# *Spectrum of Eleven-dimensional Supergravity on a PP-wave Background*

木村 哲士

KEK / 大阪大学

hep-th/0307193

*in collaboration with Kentaroh Yoshida (KEK)*

# Introduction: Eleven-dimensional Supergravity

## Lagrangian of eleven-dimensional supergravity (without torsion):

$$\mathcal{L} \;=\; e\mathcal{R} - \frac{1}{2}e\,\overline{\Psi}_M\,\widehat{\Gamma}^{MNP}\,D_N\Psi_P - \frac{1}{48}e\,F_{MNPQ}\,F^{MNPQ} + \frac{1}{192}e\,\overline{\Psi}_M\,\widetilde{\Gamma}^{MNPQRS}\,\Psi_N\,F_{PQRS} + \frac{1}{(144)^2}\varepsilon^{MNPQRSUVWXY}\,F_{MNPQ}\,F_{RSUV}\,C_{WXY}$$

## Classical Field Equations

$$0 \;=\; \frac{1}{2}g_{MN}\mathcal{R} - \mathcal{R}_{MN} - \frac{1}{96}g_{MN}F_{PQRS}F^{PQRS} + \frac{1}{12}F_{MPQR}{F_N}^{PQR}$$

$$0 \;=\; \widehat{\Gamma}^{MNP}D_N\Psi_P - \frac{1}{96}\widetilde{\Gamma}^{MNPQRS}\Psi_N F_{PQRS}$$

$$0 \;=\; \nabla^Q\{e\,F_{QMNP}\} + \frac{18}{(144)^2}\,g_{MZ}\,g_{NK}\,g_{PL}\,\varepsilon^{ZKLQRSUVWXY}\,F_{QRSU}\,F_{VWXY}$$

## Spectrum on $AdS_4 \times S^7$

$$\Phi_{\mu\nu\cdots mn\cdots}(x,y) = \sum_I \phi^I_{\mu\nu\cdots}(x) \cdot Y^I_{mn\cdots}(y)$$

| $d=11$ | $d=4$ $(AdS_4)$ | spin | $S^7$ $(SO(8))$ | number |
|---|---|---|---|---|
| $g_{MN}(x,y)$ | $h_{\mu\nu}(x)$ | 2 | 1 | 1 |
| | $V^{[ij]}_\mu(x)$ | 1 | $K^{[ij]}_m(y)$ | 28 |
| | $S^{[ijkl]}(x)$ | 0 | $K^{[ij}_{(m}K^{kl]}_{n)} - \frac{1}{9}g_{mn}K^{[ij}_p K^{kl]p}$ | 35 |
| $C_{MNP}(x,y)$ | $P^{[ijkl]}(x)$ | 0 | $\nabla_{[m}K^{[ij}_n \nabla_p K^{kl]}_{q]}$ | 35 |
| $\Psi_M(x,y)$ | $\psi^I_\mu(x)$ | $3/2$ | $\eta^I(y)$ | 8 |
| | $\chi^{[IJK]}(x)$ | $1/2$ | $\eta^{[IJK]}_m + \frac{1}{9}\widehat{\Gamma}_m \not\eta^{[IJK]}$ | 56 |

$K^{[ij]}_m(y)$ : Killing vector $(i = 1, \cdots, 8)$

$\eta^I(y)$ : Killing spinor $(I = 1, \cdots, 8)$

## *Maximally Supersymmetric Spaces*


## *Kowalski-Glikman Solution (PP-wave Background)*

$$ds^2 = -2dx^+dx^- + G_{++}(dx^+)^2 + \sum_{I=1}^{9}(dx^I)^2 \qquad x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^{10})$$

$$G_{++} = -\left[\left(\frac{\mu}{3}\right)^2 \sum_{\widetilde{I}=1}^{3}(x^{\widetilde{I}})^2 + \left(\frac{\mu}{6}\right)^2 \sum_{I'=4}^{9}(x^{I'})^2\right] \qquad \mu = F_{+123} \neq 0$$

## *Hamiltonian*

We will encounter Klein-Gordon type equations of motion and have to evaluate its energy spectrum. Klein-Gordon type equation of motion for a field $\phi(x)$:

$$\big(\Box - \alpha\,\mu\, i\partial_-\big)\phi(x^+, x^-, x^I) \;=\; 0$$

$\alpha$ : arbitrary constant $\qquad$ $x^+$ : evolution parameter

We express the Hamiltonian $H = i\partial_+$:

$$H \;=\; \frac{1}{3}\mu \sum_{\widetilde{I}} \overline{a}^{\widetilde{I}} a^{\widetilde{I}} + \frac{1}{6}\mu \sum_{I'} \overline{a}^{I'} a^{I'} + \frac{1}{2}\mu\,(2-\alpha)$$

Last term = zero-point energy $E_0$ of the system (eigenvalue of $H$):

$$E_0 \;=\; \frac{1}{2}\mu\,\mathcal{E}_0(\phi) \qquad\qquad \mathcal{E}_0(\phi) \;=\; 2-\alpha$$

# *Bosonic/Fermionic Spectrum*

Fluctuation fields are expanded around classical fields as follows:

$$
\begin{aligned}
g_{MN} &\rightarrow g_{MN} + h_{MN} \qquad\qquad g_{MN}: \text{ pp-wave background} \\
\Psi_M &\rightarrow 0 + \psi_M \\
C_{MNP} &\rightarrow C_{MNP} + \mathcal{C}_{MNP} \qquad\qquad 4\partial_{[+}C_{123]} = F_{+123} = \mu
\end{aligned}
$$

Constraints:

$$
\left.
\begin{aligned}
h_{-M} = h^{+N} &= 0 \\
\psi_- &= 0 \\
\mathcal{C}_{-NP} &= 0
\end{aligned}
\right\} \qquad \text{light-cone gauge-fixing}
$$

# Bosonic Fields

$$\widetilde{I}\ :\ SO(3) \qquad I'\ :\ SO(6)$$

$$\begin{aligned}
0 &= \Box\, \mathcal{C}_{\widetilde{I}J'K'} \\
0 &= \left(\Box - \mu\, i\partial_-\right) H_{\widetilde{I}J'} \\
0 &= \Box\, h^{\perp}_{\widetilde{I}\widetilde{J}} \\
0 &= \left(\Box - 2\mu\, i\partial_-\right) h \\
0 &= \left(\Box - \mu\, i\partial_-\right) \mathcal{C}^{\ominus}_{I'J'K'}
\end{aligned}$$

$$\begin{aligned}
H_{\widetilde{I}J'} &= h_{\widetilde{I}J'} + i\,\mathcal{C}_{\widetilde{I}J'} \qquad \mathcal{C}_{\widetilde{I}J'} = \frac{1}{2}\varepsilon_{\widetilde{I}\widetilde{K}\widetilde{L}}\,\mathcal{C}_{\widetilde{K}\widetilde{L}J'} \\
h^{\perp}_{\widetilde{I}\widetilde{J}} &= h_{\widetilde{I}\widetilde{J}} - \frac{1}{3}\delta_{\widetilde{I}\widetilde{J}}\, h_{\widetilde{K}\widetilde{K}} \\
h &= h_{\widetilde{I}\widetilde{I}} + i\,\mathcal{C} \qquad \mathcal{C} = 2\,\mathcal{C}_{123} \\
\mathcal{C}^{\ominus}_{I'J'K'} &= -\frac{i}{3!}\varepsilon^{I'J'K'L'M'N'}\,\mathcal{C}^{\ominus}_{L'M'N'}
\end{aligned}$$

Now we have derived the spectrum:

| energy | bosonic fields | | | degrees of freedom |
|---|---|---|---|---|
| 4 | $\overline{h}$ | | | 1 |
| 3 | $\overline{H}_{\widetilde{I}J'}$ | $\mathcal{C}^{\oplus}_{I'J'K'}$ | | $18 + 10$ |
| 2 | $\mathcal{C}_{\widetilde{I}J'K'}$ | $h^{\perp}_{\widetilde{I}\widetilde{J}}$ | $h^{\perp}_{I'J'}$ | $45 + 5 + 20$ |
| 1 | $H_{\widetilde{I}J'}$ | $\mathcal{C}^{\ominus}_{I'J'K'}$ | | $18 + 10$ |
| 0 | $h$ | | | 1 |

## Results

| energy | bosons | fermions | D.O.F. |
|---|---|---|---|
| 4 | $\overline{h}$ | | 1 |
| 7/2 | | $\psi_{\mathrm{L}}^{\oplus\parallel}$ | 8 |
| 3 | $\overline{H}_{\tilde{I}J'}$ $\mathcal{C}^{\oplus}_{I'J'K'}$ | | 28 |
| 5/2 | | $\psi^{\oplus\perp}_{\tilde{I}\mathrm{R}}$ $\psi^{\oplus\perp}_{I'\mathrm{L}}$ | 56 |
| 2 | $\mathcal{C}_{\tilde{I}J'K'}$ $h^{\perp}_{\tilde{I}\tilde{J}}$ $h^{\perp}_{I'J'}$ | | 70 |
| 3/2 | | $\psi^{\oplus\perp}_{\tilde{I}\mathrm{L}}$ $\psi^{\oplus\perp}_{I'\mathrm{R}}$ | 56 |
| 1 | $H_{\tilde{I}J'}$ $\mathcal{C}^{\ominus}_{I'J'K'}$ | | 28 |
| 1/2 | | $\psi_{\mathrm{R}}^{\oplus\parallel}$ | 8 |
| 0 | $h$ | | 1 |

*Energy spectrum and degrees of freedom of physical modes in 11-dimensional supergravity on the pp-wave background.*

## *Discussions: Developments around 11-dimensional Theory*

**11-dimensional Supergravity was born (1978)**

**Freund-Rubin ansatz, spontaneous compactification ($AdS_{4(7)} \times S^{7(4)}$) ($\sim$ 1982)**

**but,** *Supergravity is Dead!* **(by A.Salam, 1987)**

**Supermembrane in flat 11-dimensions (1988)**

*unstable, continuous spectrum* **(1989)**

$\vdots$

*M-theory, BFSS conjecture* **(1996)**

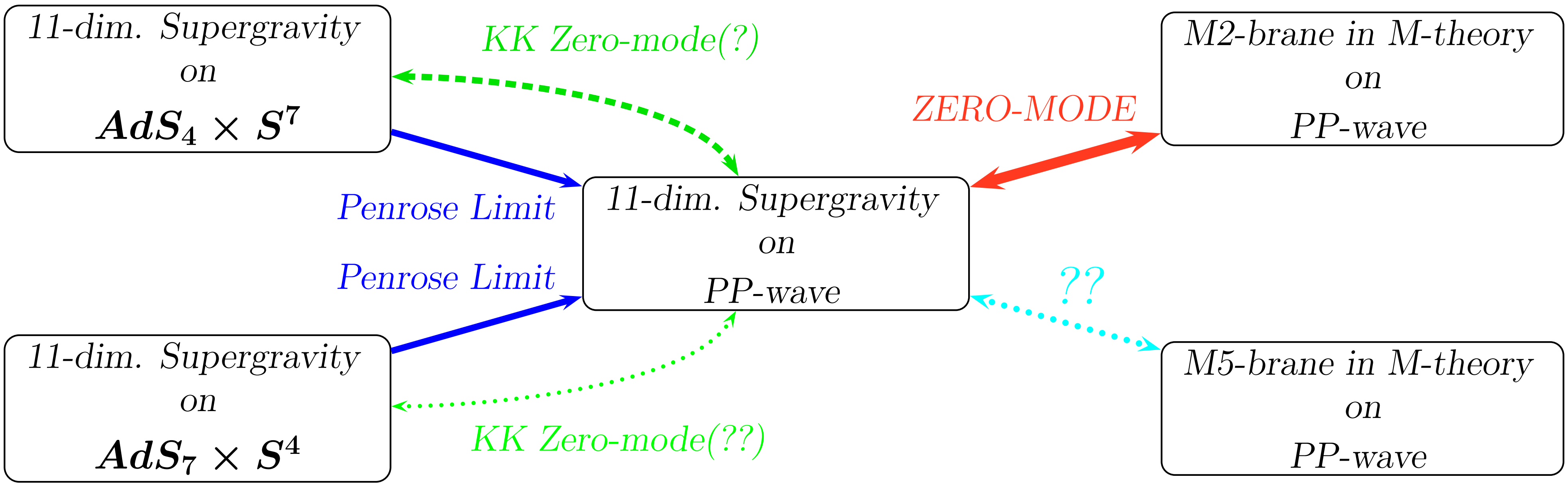
**matrix model (supermembrane) revisited**

**supermembrane (matrix model) on PP-wave background (2002)**

*Discretized Spectrum, Mass Gap, etc.* **(2002)**

$\Downarrow$

*Supergravity revisited!!*




## *Work in Progress*

- **Comparison with KK zero-modes of $AdS_p \times S^q$**
- **Propagators and energy-momentum tensors of $h_{MN}$, $\mathcal{C}_{MNP}$ and $\psi_M$**
- **Dimensional reduction to type IIA supergravity (only 24 supercharges)**