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Kähler Potentials on Toric Varieties

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INTRODUCTION

Motivations: Algebraic geometry $\longrightarrow$ Differential geometry

Vafa's large-N duality (before "*Dijkgraaf-Vafa*" appears)

is investigated in terms of some algebraic geometries

CY, vector bundles on toric varieties, described by GLSM

large-N duality is based on topological field theory

only algebro-geometric properties

We want to know this duality beyond the topological aspects

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Field Theory and Toric Varieties

$D = 2$, $\mathcal{N} = 2$ GLSM Lagrangian:

$$\mathcal{L}_{\text{GLSM}} = \int \mathrm{d}^4\theta \left\{ \sum_a -\frac{1}{e_a^2}\overline{\Sigma}_a\Sigma_a + \sum_i \overline{\Phi}^i \, \mathrm{e}^{\sum_a 2Q_i^a V_a}\Phi^i \right\}$$

$$+ \left(\frac{1}{2\sqrt{2}} \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^- \sum_a \tau_a \Sigma_a + (h.c.)\right)$$

$\tau_a = r_a - i\dfrac{\theta_a}{2\pi}$: complexified FI parameter

Potential energy density (bosonic part):

$$U = \sum_a \left\{\frac{1}{2e_a^2} D_a^2\right\} + 2\sum_{a,b} \overline{\sigma}_a \sigma_b \left(\sum_i Q_i^a Q_i^b \, |\phi^i|^2\right)$$

$$\frac{1}{e_a^2} D_a = r_a - \sum_i Q_i^a |\phi^i|^2$$

Supersymmetric vacua: $\boldsymbol{U = 0}$ **and** $\boldsymbol{D_a = 0}$

consider Higgs branch

We rewrite GLSM to "frozen" Lagrangian (non-dynamical gauge fields):

$$\mathcal{L}_{\rm F} = \int {\rm d}^4\theta \left\{ \sum_i \overline{\Phi}^i \, {\rm e}^{\sum_a 2Q_i^a V_a} \Phi^i + \sum_a \big(- 2 r_a \, V_a \big) \right\} \equiv \int {\rm d}^4\theta \, K$$

Integrating out gauge fields ⟶ supersymmetric NLSM

Hirzebruch Surfaces $\mathbb{F}_k$

$\mathbb{C}\mathbf{P}^1$ *bundle on weighted projective space* $W\mathbb{C}\mathbf{P}^{1,1,k}$

	Φ^1	Φ^2	Φ^3	Φ^4
$U(1)_1$	1	1	k	0
$U(1)_2$	0	0	1	1

[$U(1) \times U(1)$ *charges*]

$$\begin{aligned} K &= \sum_{i=1}^{4} |\Phi^i|^2 \exp(2Q_i^1 V_1 + 2Q_i^2 V_2) + \sum_{a=1}^{2}(-2r_a V_a) \\ &= |\Phi^1|^2\, e^{2V_1} + |\Phi^2|^2\, e^{2V_1} + |\Phi^3|^2\, e^{2kV_1+2V_2} + |\Phi^4|^2\, e^{2V_2} \\ &\quad - 2r_1 V_1 - 2r_2 V_2 \end{aligned}$$

$\mathbb{F}_0 = \mathbb{C}\mathbf{P}^1 \times \mathbb{C}\mathbf{P}^1$: **no fibration, direct product**

$k = 1$ case: $\mathbb{F}_1$

$$K \;=\; (|\Phi^1|^2 + |\Phi^2|^2)\, e^{2V_1} + |\Phi^3|^2\, e^{2V_1+2V_2} + |\Phi^4|^2\, e^{2V_2} - 2r_1 V_1 - 2r_2 V_2$$

Equations of motion for gauge multiplets:

$$\frac{\partial \mathcal{L}_F}{\partial V_1} = 0 \quad : \quad r_1 \;=\; \big(|\Phi^1|^2 + |\Phi^2|^2\big)\, e^{2V_1} + |\Phi^3|^2\, e^{2V_1+2V_2}$$

$$\frac{\partial \mathcal{L}_F}{\partial V_2} = 0 \quad : \quad r_2 \;=\; |\Phi^3|^2\, e^{2V_1+2V_2} + |\Phi^4|^2\, e^{2V_2}$$

Local coordinates:

$$(X^1, X^2) \;=\; \Big(\frac{\Phi^1}{\Phi^2}, \frac{\Phi^3}{\Phi^2\Phi^4}\Big) \;\in\; U_{\sigma_1} \qquad \text{where} \;\; \Phi^2, \Phi^4 \;\neq\; 0$$

Substituting the above variables into Kähler potential:

$$K(X^a, \overline{X}^a) \;=\; r_1 \log \widetilde{D}_+ + r_2 \log \Big\{1 + \frac{\widetilde{D}_-}{2(1+|X^1|^2)}\Big\} - \frac{r_2\, \widetilde{D}_-}{2(1+|X^1|^2) + \widetilde{D}_-}$$

where

$$\widetilde{D}_\pm \;=\; \pm \widetilde{B} + \sqrt{\widetilde{B}^2 + 4(1+|X^1|^2)|X^2|^2}$$

$$\widetilde{B} \;=\; (1+|X^1|^2) - (r_1 - r_2)|X^2|^2$$

Summary and Discussions

GLSM on toric varieties ($\mathbb{F}_k$, $\mathbb{B}_k$)

$$\Downarrow$$

concrete expressions of Kähler potentials

Next Directions

- **check Euler numbers and other topological charges**
- **introduce antisymmetric tensor field**

  WZW models, CFT and string worldsheet theory
- **explicit investigation of large-N duality, gauge/gravity duality, etc.**

  D-branes and mirror symmetry **(Hori, Iqbal, Vafa)**

  a mysterious duality **(Iqbal, Neitzke, Vafa)**