Spectrum of Eleven-dimensional Supergravity on a PP-wave Background

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  10-dim. type IIA (non-chiral), type IIB (chiral), etc...

  $AdS_4 \times S^7$ **(with cosmological constant)**

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- **only a classical theory**

Towards Quantum Theory

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- **Continuous spectrum, etc. $\longrightarrow$ UNSTABLE as a single object**
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- **Does this theory contain supergravity?**
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  that of supergravity on PP-wave background.)
- **We construct supergravity on PP-wave background**
  and check the spectrum of fields.

Field Contents

$e_M{}^{A}$: **vielbein**

Ψ_M : **gravitino (Majorana spinor)**

C_{MNP} : **three-form gauge field**

Lagrangian of Eleven-dimensional Supergravity

$$\begin{aligned}\mathcal{L} &= eR - \frac{1}{2} e\,\overline{\Psi}_M\,\widehat{\Gamma}^{MNP}\,D_N\Psi_P - \frac{1}{48} e\,F_{MNPQ}\,F^{MNPQ}\\ &+ \frac{1}{192} e\,\overline{\Psi}_M\,\widetilde{\Gamma}^{MNPQRS}\,\Psi_N\,F_{PQRS}\\ &+ \frac{1}{(144)^2}\varepsilon^{MNPQRSUVWXY}\,F_{MNPQ}\,F_{RSUV}\,C_{WXY}\end{aligned}$$

various conventions:

$$\begin{aligned}D_N\Psi_P &= \partial_N\Psi_P - \frac{1}{4}\omega_N{}^{AB}\widehat{\Gamma}_{AB}\Psi_P\\ \widetilde{\Gamma}^{NPQR}{}_M &= \widehat{\Gamma}^{NPQR}{}_M - 8\delta_M^{[N}\widehat{\Gamma}^{PQR]}\\ \widetilde{\Gamma}^{MNPQRS} &= \widehat{\Gamma}^{MNPQRS} + 12 g^{M[P}\widehat{\Gamma}^{QR}g^{S]N}\\ \varepsilon^{012\cdots 10} &= 1\end{aligned}$$

weight +1 invariant tensor density

Classical Field Equations

$$
\begin{aligned}
0 &= \frac{1}{2} g_{MN} R - R_{MN} - \frac{1}{96} g_{MN} F_{PQRS} F^{PQRS} + \frac{1}{12} F_{MPQR} F_N{}^{PQR} \\
0 &= \widehat{\Gamma}^{MNP} D_N \Psi_P - \frac{1}{96} \widetilde{\Gamma}^{MNPQRS} \Psi_N F_{PQRS} \\
0 &= \nabla^Q \{ e\, F_{QMNP} \} \\
&\quad + \frac{18}{(144)^2} g_{MZ}\, g_{NK}\, g_{PL}\, \varepsilon^{ZKLQRSUVWXY} F_{QRSU} F_{VWXY}
\end{aligned}
$$

Spectrum on $AdS_4 \times S^7$

$$\langle F_{\mu\nu\rho\sigma}\rangle \equiv 3m\,\epsilon_{\mu\nu\rho\sigma}$$

$$\Phi_{\mu\nu\cdots mn\cdots}(x,y) = \sum_I \phi^I_{\mu\nu\cdots}(x)\cdot Y^I_{mn\cdots}(y)$$

$d=11$	AdS_4	spin	$S^7\,(SO(8))$	number
$g_{MN}(x,y)$	$h_{\mu\nu}(x)$	2	1	1
	$V_\mu^{[IJ]}(x)$	1	$K_m^{[IJ]}(y)$	28
	$S^{[IJKL]}(x)$	0	$K^{[IJ}_{(m}K^{KL]}_{n)} - \frac{1}{9}g_{mn}K^{[IJ}_p K^{KL]p}$	35
$C_{MNP}(x,y)$	$P^{[IJKL]}(x)$	0	$K^{[IJKL]}_{[mnp]}(y)$	35
$\Psi_M(x,y)$	$\psi^I_\mu(x)$	$3/2$	$\eta^I(y)$	8
	$\chi^{[IJK]}(x)$	$1/2$	$\eta^{[IJK]}_m + \frac{1}{9}\widehat{\Gamma}_m \not{\eta}^{[IJK]}$	56

$K_m^{[IJ]}(y)$: **Killing vector** $\quad \eta^I(y)$: **Killing spinor** $(I=1,\cdots,8)$

Maximally Supersymmetric Spaces

Penrose Limit of $AdS_4 \times S^7$

$AdS_4 \times S^7$ coordinates:

$$\mathrm{d}s^2 = R_A^2 \Big\{ -\cosh^2\rho \cdot \mathrm{d}t^2 + \mathrm{d}\rho^2 + \sinh^2\rho \cdot \mathrm{d}\Omega_2^2 \Big\}$$
$$+ R_S^2 \Big\{ \cos^2\theta \cdot \mathrm{d}\varphi^2 + \mathrm{d}\theta^2 + \sin^2\theta \cdot \mathrm{d}{\Omega'_5}^2 \Big\}$$

We choose a null geodesics of $AdS_4 \times S^7$ as

$$R_S = 2R_A \qquad t = 2\varphi \qquad \rho = \theta = 0$$

Along this null direction we take $R_A \to \infty$:

$$\mathrm{d}s^2 = -2\mathrm{d}x^+\mathrm{d}x^- + G_{++}(\mathrm{d}x^+)^2 + \sum_{I=1}^{9}(\mathrm{d}x^I)^2$$

$$G_{++} = -\Big[\Big(\frac{\mu}{3}\Big)^2\sum_{\tilde{I}=1}^{3}(x^{\tilde{I}})^2 + \Big(\frac{\mu}{6}\Big)^2\sum_{I'=4}^{9}(x^{I'})^2\Big]$$

$$\mu = F_{+123} \neq 0$$

where

$$x = R_A\rho \qquad y = 2R_A\theta$$

$$x^+ = \frac{1}{2}(t+2\varphi)\cdot\frac{3}{\mu} \qquad x^- = R_A^2(t-2\varphi)\cdot\frac{\mu}{3}$$

Fluctuation Fields

$$\begin{aligned} g_{MN} &\longrightarrow g_{MN} + h_{MN} \\ \Psi_M &\longrightarrow 0 + \psi_M \\ C_{MNP} &\longrightarrow C_{MNP} + c_{MNP} \end{aligned}$$

g_{MN} : **pp-wave background**

$$4\partial_{[+}C_{123]} = F_{+123} = \mu$$

Field Equations for Fluctuations

From classical equation for g_{MN}:

$$
\begin{aligned}
0 \;=\; & -\frac{1}{2} g_{MN}\Big\{h^{PQ}\, R_{PQ} - \nabla^P \nabla^Q\, h_{PQ} + \nabla^P \nabla_P\, h_Q{}^Q\Big\} \\
& - \frac{1}{2}\Big\{\nabla^P \nabla_M\, h_{NP} + \nabla^P \nabla_N\, h_{MP} - \nabla_M \nabla_N\, h_P{}^P - \nabla^P \nabla_P\, h_{MN}\Big\} \\
& - \frac{1}{24} g_{MN}\Big\{2F^{PQRS}\, \partial_P \mathcal{C}_{QRS} - F_{PQRS}\, F_U{}^{QRS}\, h^{PU}\Big\} \\
& + \frac{1}{3} F_M{}^{PQR}\, \partial_{[N} \mathcal{C}_{PQR]} + \frac{1}{3} F_N{}^{PQR}\, \partial_{[M} \mathcal{C}_{PQR]} \\
& - \frac{1}{4} F_{MPQR}\, F_{NU}{}^{QR}\, h^{PU}
\end{aligned}
\qquad \text{(A-1)}
$$

Field Equations for Fluctuations

From classical equation for Ψ_M:

$$
\begin{aligned}
0 \;=\; & \widehat{\Gamma}^{MNP} D_N \psi_P - \frac{1}{4}\mu\, \widehat{\Gamma}^{MN+123}\psi_N \\
& - \frac{1}{4}\mu \Big\{ g^{M+}(\widehat{\Gamma}^{12} g^{3N} + \widehat{\Gamma}^{23} g^{1N} + \widehat{\Gamma}^{31} g^{2N}) \\
& \qquad - g^{M1}(\widehat{\Gamma}^{23} g^{+N} + \widehat{\Gamma}^{3+} g^{2N} + \widehat{\Gamma}^{+2} g^{3N}) \\
& \qquad + g^{M2}(\widehat{\Gamma}^{3+} g^{1N} + \widehat{\Gamma}^{+1} g^{3N} + \widehat{\Gamma}^{13} g^{+N}) \\
& \qquad - g^{M3}(\widehat{\Gamma}^{+1} g^{2N} + \widehat{\Gamma}^{12} g^{+N} + \widehat{\Gamma}^{2+} g^{1N}) \Big\} \psi_N
\end{aligned}
\tag{A-2}
$$

Field Equations for Fluctuations

From classical equation for C_{MNP}:

$$
\begin{aligned}
0 \;=\; & 4g^{QR}\Big\{\partial_R\partial_{[Q}\mathcal{C}_{MNP]} - \Gamma^S_{RQ}\,\partial_{[S}\mathcal{C}_{MNP]} - \Gamma^S_{RM}\,\partial_{[Q}\mathcal{C}_{SNP]} \\
& \qquad - \Gamma^S_{RN}\,\partial_{[Q}\mathcal{C}_{MSP]} - \Gamma^S_{RP}\,\partial_{[Q}\mathcal{C}_{MNS]}\Big\} \\
& - \frac{1}{2}g^{QR}\Big\{F_{SMNP}(\nabla_R\,h_Q{}^S + \nabla_Q\,h_R{}^S - \nabla^S\,h_{RQ}) \\
& \qquad + F_{QSNP}(\nabla_R\,h_M{}^S + \nabla_M\,h_R{}^S - \nabla^S\,h_{RM}) \\
& \qquad + F_{QMSP}(\nabla_R\,h_N{}^S + \nabla_N\,h_R{}^S - \nabla^S\,h_{RN}) \\
& \qquad + F_{QMNS}(\nabla_R\,h_P{}^S + \nabla_P\,h_R{}^S - \nabla^S\,h_{RP})\Big\} \\
& + \frac{1}{144}g_{MZ}\,g_{NK}\,g_{PL}\,\varepsilon^{ZKLQRSUVWXY}\,F_{QRSU}\,\partial_V\mathcal{C}_{WXY}
\end{aligned}
\qquad \text{(A-3)}
$$

Gauge-fixing Conditions

$$\left.\begin{aligned} h_{-M} = h^{+N} &= 0 \\ \psi_{-} &= 0 \\ \mathcal{C}_{-NP} &= 0 \end{aligned}\right\}$$

light-cone gauge-fixing

Hamiltonian

We will encounter Klein-Gordon type equations of motion and have to evaluate its energy spectrum:

$$(\Box - \alpha\,\mu\,i\partial_-)\phi(x^+, x^-, x^I) \;=\; 0$$

α : **arbitrary constant** $\qquad$ x^+ : **evolution parameter**

We express the Hamiltonian $H = i\partial_+$**:**

$$H \;=\; \frac{1}{3}\mu\sum_{\widetilde{I}}\overline{a}^{\widetilde{I}}a^{\widetilde{I}} + \frac{1}{6}\mu\sum_{I'}\overline{a}^{I'}a^{I'} + \frac{1}{2}\mu\,(2-\alpha)$$

Last term = zero-point energy E_0 **of the system (eigenvalue of** H**):**

$$E_0 \;=\; \frac{1}{2}\mu\,\mathcal{E}_0(\phi) \qquad\qquad \mathcal{E}_0(\phi) \;=\; 2-\alpha$$

Bosonic Fields

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$$0 \;=\; h_{II} \qquad \Longrightarrow \qquad \textbf{traceless condition}$$

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- **Trace of (A-1) under the above conditions:**

$$h_{++} = \frac{1}{(\partial_-)^2}\partial_I\partial_J h_{IJ} - \frac{1}{3\partial_-}\mu\,\mathcal{C}_{123} \quad \Longrightarrow \quad \text{non-dynamical}$$

Non-trivial Equations

$(\widetilde{I}\widetilde{J})$ **of (A-1) :** $\quad 0 = \Box h_{\widetilde{I}\widetilde{J}} + \frac{2}{3}\mu\,\delta_{\widetilde{I}\widetilde{J}}\,\partial_-\mathcal{C}$ (1-a)

$(\widetilde{I}J')$ **of (A-1) :** $\quad 0 = \Box h_{\widetilde{I}J'} + \mu\,\partial_-\mathcal{C}_{\widetilde{I}J'}$ (1-b)

$(I'J')$ **of (A-1) :** $\quad 0 = \Box h_{I'J'} - \frac{1}{3}\mu\,\delta_{I'J'}\partial_-\mathcal{C}$ (1-c)

$(\widetilde{I}\widetilde{J}\widetilde{K})$ **of (A-3) :** $\quad 0 = \Box\mathcal{C} - 2\mu\,\partial_- h_{\widetilde{I}\widetilde{I}}$ (1-d)

$(\widetilde{I}\widetilde{J}K')$ **of (A-3) :** $\quad 0 = \Box\mathcal{C}_{\widetilde{I}J'} - \mu\,\partial_- h_{\widetilde{I}J'}$ (1-e)

$(\widetilde{I}J'K')$ **of (A-3) :** $\quad 0 = \Box\mathcal{C}_{\widetilde{I}J'K'}$ (1-f)

$(I'J'K')$ **of (A-3) :** $\quad 0 = \Box\mathcal{C}_{I'J'K'} - \frac{1}{6}\mu\,\varepsilon^{I'J'K'W'X'Y'}\partial_-\mathcal{C}_{W'X'Y'}$ (1-g)

$$\mathcal{C}_{\widetilde{I}J'} \equiv \frac{1}{2}\varepsilon_{\widetilde{I}\widetilde{K}\widetilde{L}}\mathcal{C}_{\widetilde{K}\widetilde{L}J'} \qquad \mathcal{C} \equiv 2\mathcal{C}_{123}$$

(1-f): $0 = \Box \mathcal{C}_{\tilde{I}\tilde{J}K'}$

We find the zero-mode energy $\mathcal{E}_0(\mathcal{C}_{\tilde{I}J'K'})$ **and degrees of freedom** $\mathcal{D}(\mathcal{C}_{\tilde{I}J'K'})$**:**

$$\mathcal{E}_0(\mathcal{C}_{\tilde{I}J'K'}) = 2 \qquad \mathcal{D}(\mathcal{C}_{\tilde{I}J'K'}) = 45$$

(1-b), (1-e): $0 = \Box h_{\tilde{I}J'} + \mu\partial_- \mathcal{C}_{\tilde{I}J'} = \Box \mathcal{C}_{\tilde{I}J'} - \mu\partial_- h_{\tilde{I}J'}$

Diagonalize $h_{\tilde{I}J'}$ **and** $\mathcal{C}_{\tilde{I}J'}$**:**

$$H_{\tilde{I}J'} = h_{\tilde{I}J'} + i\mathcal{C}_{\tilde{I}J'} \qquad \overline{H}_{\tilde{I}J'} = h_{\tilde{I}J'} - i\mathcal{C}_{\tilde{I}J'}$$

Thus modified (1-b) and (1-e) are

$$0 = \left(\Box - \mu\, i\partial_-\right) H_{\tilde{I}J'} \qquad 0 = \left(\Box + \mu\, i\partial_-\right) \overline{H}_{\tilde{I}J'}$$

$$\Longrightarrow \mathcal{E}_0(H_{\tilde{I}J'}) = 1 \quad \mathcal{E}_0(\overline{H}_{\tilde{I}J'}) = 3$$

$$\mathcal{D}(H_{\tilde{I}J'}) = \mathcal{D}(\overline{H}_{\tilde{I}J'}) = 18$$

(1-a), (1-c), (1-d):

Apply similar consideration to (1-a), (1-c) and (1-d):

$$h^{\perp}_{\widetilde{I}\widetilde{J}} \equiv h_{\widetilde{I}\widetilde{J}} - \frac{1}{3}\delta_{\widetilde{I}\widetilde{J}} h_{\widetilde{K}\widetilde{K}} \quad h^{\perp}_{I'J'} \equiv h_{I'J'} - \frac{1}{6}\delta_{I'J'} h_{K'K'}$$

$$h \equiv h_{\widetilde{I}\widetilde{I}} + i\mathcal{C} \qquad \overline{h} \equiv h_{\widetilde{I}\widetilde{I}} - i\mathcal{C}$$

Then we find

$$\mathcal{E}_0(h^{\perp}_{\widetilde{I}\widetilde{J}}) = \mathcal{E}_0(h^{\perp}_{I'J'}) = 2 \quad \mathcal{D}(h^{\perp}_{\widetilde{I}\widetilde{J}}) = 5 \quad \mathcal{D}(h^{\perp}_{I'J'}) = 20$$

$$\mathcal{E}_0(h) = 0 \quad \mathcal{E}_0(\overline{h}) = 4 \quad \mathcal{D}(h) = \mathcal{D}(\overline{h}) = 1$$

(1-g):

Decomposing into self-dual part $\mathcal{C}^{\oplus}_{I'J'K'}$ and anti-self-dual part $\mathcal{C}^{\ominus}_{I'J'K'}$:

$$\mathcal{C}^{\oplus}_{I'J'K'} \equiv \frac{i}{3!}\varepsilon^{I'J'K'W'X'Y'}\mathcal{C}^{\oplus}_{W'X'Y'}$$

$$\mathcal{C}^{\ominus}_{I'J'K'} \equiv -\frac{i}{3!}\varepsilon^{I'J'K'W'X'Y'}\mathcal{C}^{\ominus}_{W'X'Y'}$$

They satisfy the following equations:

$$(\Box + \mu\, i\partial_-)\mathcal{C}^{\oplus}_{I'J'K'} = 0 \qquad (\Box - \mu\, i\partial_-)\mathcal{C}^{\ominus}_{I'J'K'} = 0$$

$$\Longrightarrow \mathcal{E}_0(\mathcal{C}^{\oplus}_{I'J'K'}) = 3 \quad \mathcal{E}_0(\mathcal{C}^{\ominus}_{I'J'K'}) = 1$$

$$\mathcal{D}(\mathcal{C}^{\oplus}_{I'J'K'}) = \mathcal{D}(\mathcal{C}^{\ominus}_{I'J'K'}) = 10$$

Results

We have fully solved the field equations for bosonic fluctuations and have derived the spectrum of graviton h_{MN} and three-form gauge field $\mathcal{C}_{MNP}$. The resulting spectrum is splitting with a certain energy difference in contrast to the flat case.

energy	bosonic fields			degrees of freedom
4	$\overline{h}$			1
3	$\overline{H}_{\tilde{I}J'}$	$\mathcal{C}^{\oplus}_{I'J'K'}$		$18+10$
2	$\mathcal{C}_{\tilde{I}J'K'}$	$h^{\perp}_{\tilde{I}\tilde{J}}$	$h^{\perp}_{I'J'}$	$45+5+20$
1	$H_{\tilde{I}J'}$	$\mathcal{C}^{\ominus}_{I'J'K'}$		$18+10$
0	h			1

Fermionic Fields

- $M = -$ **component of (A-2):**

$$\widehat{\Gamma}^{P}\psi_{P} = 0 \qquad \Longrightarrow \qquad \text{Lorentz-type gauge-fixing condition}$$

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- $M = -$ **component of (A-2):**

$$\widehat{\Gamma}^P \psi_P = 0 \qquad \Longrightarrow$$ **Lorentz-type gauge-fixing condition**

- $M = +$ **component of (A-2):**

$$\partial^P \psi_P = 0 \quad \longrightarrow \quad \psi_+ = \frac{1}{\partial_-}\partial_I \psi_I \qquad \Longrightarrow$$ **non-dynamical**

$M = \widetilde{I}$ component of (A-2):

$$
\begin{aligned}
0 \;=\; & \left\{ \widehat{\Gamma}^{+} \left(\partial_{+} + \frac{1}{2} G_{++} \partial_{-} \right) + \widehat{\Gamma}^{-} \partial_{-} + \widehat{\Gamma}^{K} \partial_{K} \right\} \psi_{\widetilde{I}} \\
& - \frac{1}{4} \mu \widehat{\Gamma}^{+123} \left(\delta_{\widetilde{I}\widetilde{J}} - \widehat{\Gamma}_{\widetilde{I}} \widehat{\Gamma}_{\widetilde{J}} \right) \psi_{\widetilde{J}}
\end{aligned}
$$

We decompose

$$
\psi^{\oplus}_{\widetilde{I}} \;=\; -\frac{1}{2} \widehat{\Gamma}^{-} \widehat{\Gamma}^{+} \psi_{\widetilde{I}} \qquad \psi^{\ominus}_{\widetilde{I}} \;=\; -\frac{1}{2} \widehat{\Gamma}^{+} \widehat{\Gamma}^{-} \psi_{\widetilde{I}}
$$

Then we obtain

$$
\psi^{\ominus}_{\widetilde{I}} \;=\; \frac{1}{2\partial_{-}} \widehat{\Gamma}^{+} \widehat{\Gamma}^{K} \partial_{K} \psi^{\oplus}_{\widetilde{I}} \qquad \Longrightarrow \qquad \text{non-dynamical}
$$

$$
0 \;=\; \Box \, \psi^{\oplus}_{\widetilde{I}} - \frac{1}{2} \mu \widehat{\Gamma}^{123} \left(\delta_{\widetilde{I}\widetilde{J}} - \widehat{\Gamma}_{\widetilde{I}} \widehat{\Gamma}_{\widetilde{J}} \right) \partial_{-} \psi^{\oplus}_{\widetilde{J}}
$$

we shall introduce the following fi elds:

$$\psi^{\oplus\perp}_{\tilde{I}} \equiv \left(\delta_{\tilde{I}\tilde{J}} - \frac{1}{3}\widehat{\Gamma}_{\tilde{I}}\widehat{\Gamma}_{\tilde{J}}\right)\psi^{\oplus}_{\tilde{J}}$$ $\widehat{\Gamma}$-transverse mode

$$\psi^{\oplus\|}_{1} \equiv \widehat{\Gamma}^{\tilde{I}}\psi^{\oplus}_{\tilde{I}}$$ $\widehat{\Gamma}$-parallel mode

Acting $\widehat{\Gamma}^{\tilde{I}}$ or $(\delta_{\tilde{K}\tilde{I}} - \frac{1}{3}\widehat{\Gamma}_{\tilde{K}}\widehat{\Gamma}_{\tilde{I}})$:

$$0 = \Box\,\psi^{\oplus\|}_{1} - \mu\widehat{\Gamma}^{123}\partial_{-}\psi^{\oplus\|}_{1} \qquad 0 = \Box\,\psi^{\oplus\perp}_{\tilde{K}} - \frac{1}{2}\mu\widehat{\Gamma}^{123}\partial_{-}\psi^{\oplus\perp}_{\tilde{K}}$$

Decompose $\psi^{\oplus\perp}_{\tilde{I}}$ and $\psi^{\oplus\|}_{1}$ according to the chirality:

$$\psi^{\oplus\perp}_{\tilde{I}\mathrm{R}} \equiv \frac{1 + i\widehat{\Gamma}^{123}}{2}\psi^{\oplus\perp}_{\tilde{I}} \qquad \psi^{\oplus\perp}_{\tilde{I}\mathrm{L}} \equiv \frac{1 - i\widehat{\Gamma}^{123}}{2}\psi^{\oplus\perp}_{\tilde{I}}$$

$$\psi^{\oplus\|}_{1\mathrm{R}} \equiv \frac{1 + i\widehat{\Gamma}^{123}}{2}\psi^{\oplus\|}_{1} \qquad \psi^{\oplus\|}_{1\mathrm{L}} \equiv \frac{1 - i\widehat{\Gamma}^{123}}{2}\psi^{\oplus\|}_{1}$$

Multiplying chiral projection operator $\frac{1}{2}(1 \pm i\widehat{\Gamma}^{123})$:

$$0 = \left(\Box + \mu\, i\partial_-\right)\psi^{\oplus\parallel}_{1\mathrm{R}} \qquad 0 = \left(\Box - \mu\, i\partial_-\right)\psi^{\oplus\parallel}_{1\mathrm{L}}$$

$$0 = \left(\Box + \frac{1}{2}\mu\, i\partial_-\right)\psi^{\oplus\perp}_{\widetilde{I}\mathrm{R}} \qquad 0 = \left(\Box - \frac{1}{2}\mu\, i\partial_-\right)\psi^{\oplus\perp}_{\widetilde{I}\mathrm{L}}$$

Zero-mode energies and degrees of freedom of $\psi^{\oplus\perp}_{\widetilde{I}\mathrm{R}}$ and $\psi^{\oplus\perp}_{\widetilde{I}\mathrm{L}}$:

$$\mathcal{E}_0(\psi^{\oplus\perp}_{\widetilde{I}\mathrm{R}}) = \frac{5}{2} \qquad \mathcal{E}_0(\psi^{\oplus\perp}_{\widetilde{I}\mathrm{L}}) = \frac{3}{2}$$

$$\mathcal{D}(\psi^{\oplus\perp}_{\widetilde{I}\mathrm{R}}) = \mathcal{D}(\psi^{\oplus\perp}_{\widetilde{I}\mathrm{L}}) = 8 \times (3-1) = 16$$

$M = I'$ **component of (A-2):**

$$0 = \Big\{\widehat{\Gamma}^{+}\Big(\partial_{+} + \frac{1}{2}G_{++}\partial_{-}\Big) + \widehat{\Gamma}^{-}\partial_{-} + \widehat{\Gamma}^{K}\partial_{K}\Big\}\psi_{I'}$$
$$+\frac{1}{4}\mu\widehat{\Gamma}^{+123}\Big(\delta_{I'J'} - \widehat{\Gamma}_{I'}\widehat{\Gamma}_{J'}\Big)\psi_{J'}$$

The $\widehat{\Gamma}$-transverse mode and $\widehat{\Gamma}$-parallel mode are defined as

$$\psi^{\oplus}_{I'} = -\frac{1}{2}\widehat{\Gamma}^{-}\widehat{\Gamma}^{+}\psi_{I'}$$

$$\psi^{\oplus\perp}_{I'\mathrm{R}} = \frac{1+i\widehat{\Gamma}^{123}}{2}\psi^{\oplus\perp}_{I'} \qquad \psi^{\oplus\perp}_{I'\mathrm{L}} = \frac{1-i\widehat{\Gamma}^{123}}{2}\psi^{\oplus\perp}_{I'}$$

$$\psi^{\oplus\parallel}_{2\mathrm{R}} = \frac{1+i\widehat{\Gamma}^{123}}{2}\psi^{\oplus\parallel}_{2} \qquad \psi^{\oplus\parallel}_{2\mathrm{L}} = \frac{1-i\widehat{\Gamma}^{123}}{2}\psi^{\oplus\parallel}_{2}$$

Equations for the $\widehat{\Gamma}$-parallel mode and $\widehat{\Gamma}$-transverse mode:

$$0 = \left(\Box - \frac{5}{2}\mu\, i\partial_-\right)\psi^{\oplus\|}_{2R} \qquad 0 = \left(\Box + \frac{5}{2}\mu\, i\partial_-\right)\psi^{\oplus\|}_{2L}$$

$$0 = \left(\Box - \frac{1}{2}\mu\, i\partial_-\right)\psi^{\oplus\perp}_{I'R} \qquad 0 = \left(\Box + \frac{1}{2}\mu\, i\partial_-\right)\psi^{\oplus\perp}_{I'L}$$

We find that the zero-mode energies and degrees of freedom:

$$\mathcal{E}_0(\psi^{\oplus\perp}_{I'R}) = \frac{3}{2} \qquad \mathcal{E}_0(\psi^{\oplus\perp}_{I'L}) = \frac{5}{2}$$

$$\mathcal{D}(\psi^{\oplus\perp}_{I'R}) = \mathcal{D}(\psi^{\oplus\perp}_{I'L}) = 8 \times (6-1) = 40$$

Linear combination of $\widehat{\Gamma}$-parallel modes:

$$\psi_{\mathbf{R}}^{\oplus\|} \equiv \frac{2}{5}\psi_{\mathbf{1R}}^{\oplus\|} - \psi_{\mathbf{2R}}^{\oplus\|} \qquad \psi_{\mathbf{L}}^{\oplus\|} \equiv \frac{2}{5}\psi_{\mathbf{1L}}^{\oplus\|} - \psi_{\mathbf{2L}}^{\oplus\|}$$

We can easily see that the re-defi ned fermions satisfy:

$$0 = \left(\Box - \frac{3}{2}\mu\, i\partial_-\right)\psi_{\mathbf{R}}^{\oplus\|} \qquad 0 = \left(\Box + \frac{3}{2}\mu\, i\partial_-\right)\psi_{\mathbf{L}}^{\oplus\|}$$

$$\Longrightarrow \qquad \mathcal{E}_0(\psi_{\mathbf{R}}^{\oplus\|}) = \frac{1}{2} \quad \mathcal{E}_0(\psi_{\mathbf{L}}^{\oplus\|}) = \frac{7}{2}$$

$$\mathcal{D}(\psi_{\mathbf{R}}^{\oplus\|}) = \mathcal{D}(\psi_{\mathbf{L}}^{\oplus\|}) = 8$$

Results

We have solved and derived the spectrum of gravitino ψ_M in the case of pp-wave background. As a result, we have found that the spectrum is splitting with a certain energy difference in the same manner with the spectrum of bosons.

energy	fermionic fields	degrees of freedom
$7/2$	$\psi_{\mathrm{L}}^{\oplus\parallel}$	8
$5/2$	$\psi_{\tilde{I}\mathrm{R}}^{\oplus\perp}$ $\psi_{I'\mathrm{L}}^{\oplus\perp}$	$16+40$
$3/2$	$\psi_{\tilde{I}\mathrm{L}}^{\oplus\perp}$ $\psi_{I'\mathrm{R}}^{\oplus\perp}$	$16+40$
$1/2$	$\psi_{\mathrm{R}}^{\oplus\parallel}$	8

Results

energy	bosons			fermions		D.O.F.
4	$\overline{h}$					1
$7/2$				$\psi_{\mathrm{L}}^{\oplus\parallel}$		8
3	$\overline{H}_{\tilde{I}J'}$	$\mathcal{C}^{\oplus}_{I'J'K'}$				28
$5/2$				$\psi^{\oplus\perp}_{\tilde{I}\mathrm{R}}$	$\psi^{\oplus\perp}_{I'\mathrm{L}}$	56
2	$\mathcal{C}_{\tilde{I}J'K'}$	$h^{\perp}_{\tilde{I}\tilde{J}}$	$h^{\perp}_{I'J'}$			70
$3/2$				$\psi^{\oplus\perp}_{\tilde{I}\mathrm{L}}$	$\psi^{\oplus\perp}_{I'\mathrm{R}}$	56
1	$H_{\tilde{I}J'}$	$\mathcal{C}^{\ominus}_{I'J'K'}$				28
$1/2$				$\psi_{\mathrm{R}}^{\oplus\parallel}$		8
0	h					1

Towards Quantum Theory, revisited

- **Supermembrane as a matrix model:**
  Nakayama, Sugiyama and Yoshida (2002), etc.
  discrete spectrum, STABLE as a single object

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- **Supermembrane as a matrix model:**

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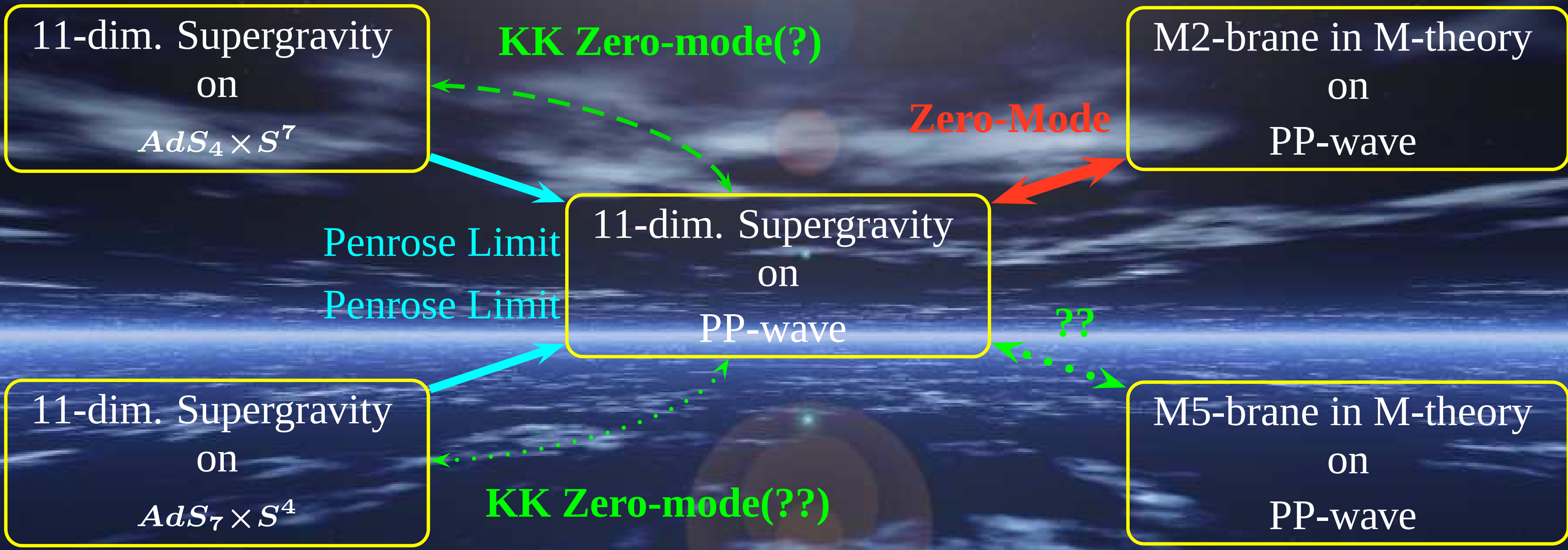
  discrete spectrum, STABLE as a single object

- **Zero-mode spectrum corresponds to that of Supergravity**

- **WE SHOWED HERE!**

Conclusion


11-dim. Supergravity on $AdS_4 \times S^7$
KK Zero-mode(?)
M2-brane in M-theory on PP-wave
Zero-Mode
Penrose Limit
11-dim. Supergravity on PP-wave
Penrose Limit
??
11-dim. Supergravity on $AdS_7 \times S^4$
KK Zero-mode(??)
M5-brane in M-theory on PP-wave

Discussions, Future Problems

- **Comparison with KK zero-modes and algebras of** $AdS_{4(7)} \times S^{7(4)}$

Discussions, Future Problems

- **Comparison with KK zero-modes and algebras of $AdS_{4(7)} \times S^{7(4)}$**
- **Propagators and energy-momentum tensors of h_{MN}, c_{MNP} and ψ_M**

Discussions, Future Problems

- **Comparison with KK zero-modes and algebras of $AdS_{4(7)} \times S^{7(4)}$**
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- **Dimensional reduction to type IIA supergravity (only 24 supercharges)**