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Spectrum of Eleven-dimensional Supergravity on a PP-wave Background

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Abstract

We calculate the spectrum of the linearized supergravity around the maximally supersymmetric pp-wave background in eleven dimensions. The resulting spectrum agrees with that of zero-mode Hamiltonian of a supermembrane theory on the pp-wave background. We also discuss the connection with the Kaluza-Klein zero modes of $AdS_4 \times S^7$ background.

Introduction

M-theory, or Supermembrane Theory in Eleven Dimensions

- strong coupling limit of type IIA string theory: $R_{10} \propto g_{\mathrm{IIA}}^{2/3}$
- eleven-dimensional supergravity as low energy theory
- three-form gauge field couples M2-brane (electrically) and M5-brane (magnetically)
- BFSS matrix theory ($0+1$-dim. $U(N)$ supersymmetric Yang-Mills)
- supermembrane theory

supermembrane on flat background

de Wit, Hoppe, Nicolai, Nucl. Phys. B305 (1988) 545

↓ but unstable (continuous spectrum)

de Wit, Lüscher, Nicolai, Nucl. Phys. B320 (1989) 135

supermembrane on pp-wave background

discretized energy spectrum

Nakayama, Sugiyama, Yoshida, Phys. Rev. D68 (2003) 026001

We would like to investigate fluctuation fields in eleven-dimensional supergravity and compare with the zero-modes of supermembrane on the pp-wave background.

Maximally Supersymmetric Spaces in Eleven-dimensional Supergravity

Kowalski-Glikman Solution (PP-wave Background)

$$\mathrm{d}s^2 = -2\mathrm{d}x^+\mathrm{d}x^- + G_{++}(\mathrm{d}x^+)^2 + \sum_{I=1}^{9}(\mathrm{d}x^I)^2 \qquad x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^{10})$$

$$G_{++} = -\left[\left(\frac{\mu}{3}\right)^2 \sum_{\tilde{I}=1}^{3}(x^{\tilde{I}})^2 + \left(\frac{\mu}{6}\right)^2 \sum_{I'=4}^{9}(x^{I'})^2\right] \qquad \mu = F_{+123} \neq 0$$

Affine connection Γ^P_{MN}, curvature tensor $R^R{}_{PMN}$ and spin connection $\omega_M{}^{AB}$:

$$\Gamma^{\tilde{I}}_{++} = \Gamma^-_{\tilde{I}+} = \omega_+{}^{\tilde{I}-} = \left(\frac{\mu}{3}\right)^2 x^{\tilde{I}} \qquad \Gamma^{I'}_{++} = \Gamma^-_{I'+} = \omega_+{}^{I'-} = \left(\frac{\mu}{6}\right)^2 x^{I'}$$

$$R^{\tilde{J}}{}_{+\tilde{I}+} = \delta_{\tilde{I}\tilde{J}}\left(\frac{\mu}{3}\right)^2 \qquad R^{J'}{}_{+I'+} = \delta_{I'J'}\left(\frac{\mu}{6}\right)^2$$

$$\mathcal{R}_{++} = \mathcal{R}^{--} = \frac{1}{2}\mu^2 \qquad \mathcal{R} = 0$$

Setup

Eleven-dimensional Supergravity

$e_M{}^A$: vielbein $\qquad E_A{}^M$: inverse vielbein

Ψ_M : gravitino (Majorana spinor) $\qquad \overline{\Psi} = i\Psi^\dagger\widehat{\Gamma}^0 = \Psi^T C$

C_{MNP} : three-form gauge field

Lagrangian of eleven-dimensional supergravity (without torsion):

$$\begin{aligned}\mathcal{L} \;=\; & e\mathcal{R} - \frac{1}{2}e\,\overline{\Psi}_M\,\widehat{\Gamma}^{MNP}\,D_N\Psi_P - \frac{1}{48}e\,F_{MNPQ}\,F^{MNPQ} \\ & + \frac{1}{192}e\,\overline{\Psi}_M\,\widetilde{\Gamma}^{MNPQRS}\,\Psi_N\,F_{PQRS} \\ & + \frac{1}{(144)^2}\varepsilon^{MNPQRSUVWXY}\,F_{MNPQ}\,F_{RSUV}\,C_{WXY}\end{aligned}$$

various conventions:

$$D_N\Psi_P \;=\; \partial_N\Psi_P - \frac{1}{4}\omega_N{}^{AB}\widehat{\Gamma}_{AB}\Psi_P$$

$$\widetilde{\Gamma}^{NPQR}{}_M \;=\; \widehat{\Gamma}^{NPQR}{}_M - 8\delta_M{}^{[N}\widehat{\Gamma}^{PQR]} \qquad \widetilde{\Gamma}^{MNPQRS} \;=\; \widehat{\Gamma}^{MNPQRS} + 12 g^{M[P}\widehat{\Gamma}^{QR}g^{S]N}$$

$$\varepsilon^{012\cdots 10} \;=\; 1 \qquad SO(10,1)\text{ Levi-Civita symbol}$$

Classical Field Equations

$$
\begin{aligned}
0 &= \frac{1}{2} g_{MN}\mathcal{R} - \mathcal{R}_{MN} - \frac{1}{96} g_{MN} F_{PQRS} F^{PQRS} + \frac{1}{12} F_{MPQR} F_N{}^{PQR} \\
0 &= \widehat{\Gamma}^{MNP} D_N \Psi_P - \frac{1}{96} \widetilde{\Gamma}^{MNPQRS} \Psi_N F_{PQRS} \\
0 &= \nabla^Q \{ e\, F_{QMNP} \} + \frac{18}{(144)^2}\, g_{MZ}\, g_{NK}\, g_{PL}\, \varepsilon^{ZKLQRSUVWXY}\, F_{QRSU}\, F_{VWXY}
\end{aligned}
$$

We neglected the terms derived from torsion, etc.

They contribute to higher-order fluctuations.

Hamiltonian

We will encounter Klein-Gordon type equations of motion and have to evaluate its energy spectrum. Klein-Gordon type equation of motion for a field $\phi(x)$:

$$(\Box - \alpha\,\mu\,i\partial_-)\phi(x^+,x^-,x^I) \;=\; 0$$

α : arbitrary constant $\qquad$ x^+ : evolution parameter

d'Alembertian $\Box$ on the pp-wave background:

$$\Box \;=\; -\frac{1}{\sqrt{-g}}\partial_M\big(\sqrt{-g}g^{MN}\partial_N\big) \;=\; 2\partial_+\partial_- + G_{++}(\partial_-)^2 - (\partial_K)^2$$

Fourier transformed expression of $\phi(x)$:

$$\phi(x^+,x^-,x^I) \;=\; \int\frac{\mathrm{d}p_-\mathrm{d}^9p_I}{\sqrt{(2\pi)^{10}}}\,\mathrm{e}^{i(p_-x^-+p_Ix^I)}\,\widetilde{\phi}(x^+,p_-,p_I)$$

Then the operator $(\Box - \alpha\,\mu\,i\partial_-)$ is represented as:

$$0 \;=\; 2p_-\,i\partial_+ - \Big\{\sum_{\widetilde{I}=1}^{3}\Big(\frac{\mu}{3}\Big)^2(\partial_{p_{\widetilde{I}}})^2 + \sum_{I'=4}^{9}\Big(\frac{\mu}{6}\Big)^2(\partial_{p_{I'}})^2\Big\}(p_-)^2 + (p_I)^2 + \alpha\,\mu\,p_-$$

Introduce the standard technique of harmonic oscillators:

$$a^{\widetilde{I}} \equiv \frac{1}{\sqrt{2\widetilde{m}}}\{p_{\widetilde{I}} + \widetilde{m}\partial_{p_{\widetilde{I}}}\} \qquad \overline{a}^{\widetilde{I}} \equiv \frac{1}{\sqrt{2\widetilde{m}}}\{p_{\widetilde{I}} - \widetilde{m}\partial_{p_{\widetilde{I}}}\} \qquad \widetilde{m} \equiv -\frac{1}{3}\mu\, p_{-}$$
$$a^{I'} \equiv \frac{1}{\sqrt{2m'}}\{p_{I'} + m'\partial_{p_{I'}}\} \qquad \overline{a}^{I'} \equiv \frac{1}{\sqrt{2m'}}\{p_{I'} - m'\partial_{p_{I'}}\} \qquad m' \equiv -\frac{1}{6}\mu\, p_{-}$$

whose commutation relations are represented by

$$[a^{\widetilde{I}}, \overline{a}^{\widetilde{J}}] = \delta^{\widetilde{I}\widetilde{J}} \qquad [a^{I'}, \overline{a}^{J'}] = \delta^{I'J'} \qquad [a^{\widetilde{I}}, \overline{a}^{J'}] = [a^{I'}, \overline{a}^{\widetilde{J}}] = 0$$

Thus we express the Hamiltonian $H = i\partial_{+}$:

$$H = \frac{1}{3}\mu \sum_{\widetilde{I}} \overline{a}^{\widetilde{I}} a^{\widetilde{I}} + \frac{1}{6}\mu \sum_{I'} \overline{a}^{I'} a^{I'} + \frac{1}{2}\mu\,(2-\alpha)$$

Last term = zero-point energy E_0 of the system (eigenvalue of H):

$$E_0 = \frac{1}{2}\mu\, \mathcal{E}_0(\phi) \qquad \mathcal{E}_0(\phi) = 2-\alpha$$

Bosonic/Fermionic Spectrum

Fluctuation fields are expanded around classical fields as follows:

$$
\begin{aligned}
g_{MN} &\rightarrow g_{MN} + h_{MN} \\
\Psi_M &\rightarrow 0 + \psi_M \\
C_{MNP} &\rightarrow C_{MNP} + \mathcal{C}_{MNP}
\end{aligned}
$$

g_{MN} : pp-wave background

$$4\partial_{[+}C_{123]} = F_{+123} = \mu$$

Constraints:

$$
\left.
\begin{aligned}
h_{-M} = h^{+N} &= 0 \\
\psi_- &= 0 \\
\mathcal{C}_{-NP} &= 0
\end{aligned}
\right\}
$$

light-cone gauge-fixing

Bosonic Fields

$(--)$ component of (*A.1*):

$$0 = h_{II} \qquad \Longrightarrow \qquad \text{traceless condition}$$

Substitute this into $(-I)$ component of (*A.1*):

$$h_{I+} = \frac{1}{\partial_-}\partial_J h_{IJ} \qquad \Longrightarrow \qquad \text{non-dynamical}$$

$(-IJ)$ component of (*A.3*):

$$\mathcal{C}_{+IJ} = \frac{1}{\partial_-}\partial_K \mathcal{C}_{IJK} \qquad \Longrightarrow \qquad \text{non-dynamical}$$

Trace of (*A.1*) under the above conditions:

$$h_{++} = \frac{1}{(\partial_-)^2}\partial_I\partial_J h_{IJ} - \frac{1}{3\partial_-}\mu\,\mathcal{C}_{123} \qquad \Longrightarrow \qquad \text{non-dynamical}$$

Let us substitute the previous conditions into the other components of (𝒜.1) and (𝒜.3).

Non-trivial equations:

$$(\widetilde{I}\widetilde{J}) \text{ of } (\mathcal{A}.1): \qquad 0 = \Box h_{\tilde{I}\tilde{J}} + \frac{2}{3}\mu\,\delta_{\tilde{I}\tilde{J}}\,\partial_-\mathcal{C} \tag{1a}$$

$$(\widetilde{I}J') \text{ of } (\mathcal{A}.1): \qquad 0 = \Box h_{\tilde{I}J'} + \mu\,\partial_-\mathcal{C}_{\tilde{I}J'} \tag{1b}$$

$$(I'J') \text{ of } (\mathcal{A}.1): \qquad 0 = \Box h_{I'J'} - \frac{1}{3}\mu\,\delta_{I'J'}\partial_-\mathcal{C} \tag{1c}$$

$$(\widetilde{I}\widetilde{J}\widetilde{K}) \text{ of } (\mathcal{A}.3): \qquad 0 = \Box\mathcal{C} - 2\mu\,\partial_- h_{\tilde{I}\tilde{I}} \tag{1d}$$

$$(\widetilde{I}\widetilde{J}K') \text{ of } (\mathcal{A}.3): \qquad 0 = \Box\mathcal{C}_{\tilde{I}J'} - \mu\,\partial_- h_{\tilde{I}J'} \tag{1e}$$

$$(\widetilde{I}J'K') \text{ of } (\mathcal{A}.3): \qquad 0 = \Box\mathcal{C}_{\tilde{I}J'K'} \tag{1f}$$

$$(I'J'K') \text{ of } (\mathcal{A}.3): \qquad 0 = \Box\mathcal{C}_{I'J'K'} - \frac{1}{6}\mu\,\varepsilon^{I'J'K'W'X'Y'}\partial_-\mathcal{C}_{W'X'Y'} \tag{1g}$$

$\varepsilon^{I'J'K'W'X'Y'}$: $SO(6)$ Levi-Civita symbol $\quad (\varepsilon_{456789} = \varepsilon^{456789} = 1)$

$\varepsilon_{\tilde{I}\tilde{J}\tilde{K}}$: $SO(3)$ Levi-Civita symbol $\quad (\varepsilon_{123} = \varepsilon^{123} = 1)$

$$\mathcal{C}_{\tilde{I}J'} \equiv \frac{1}{2}\varepsilon_{\tilde{I}\tilde{K}\tilde{L}}\mathcal{C}_{\tilde{K}\tilde{L}J'} \qquad \mathcal{C} \equiv 2\mathcal{C}_{123}$$

(1f) leads to the zero-mode energy $\mathcal{E}_0(\mathcal{C}_{\tilde{I}J'K'})$ and degrees of freedom $\mathcal{D}(\mathcal{C}_{\tilde{I}J'K'})$:

$$\mathcal{E}_0(\mathcal{C}_{\tilde{I}J'K'}) \;=\; 2 \qquad\qquad \mathcal{D}(\mathcal{C}_{\tilde{I}J'K'}) \;=\; 45$$

Diagonalize $h_{\tilde{I}J'}$ and $\mathcal{C}_{\tilde{I}J'}$:

$$H_{\tilde{I}J'} \;=\; h_{\tilde{I}J'} + i\mathcal{C}_{\tilde{I}J'} \qquad\qquad \overline{H}_{\tilde{I}J'} \;=\; h_{\tilde{I}J'} - i\mathcal{C}_{\tilde{I}J'}$$

Thus modified (1b) and (1e) are

$$0 \;=\; \big(\Box - \mu\, i\partial_-\big) H_{\tilde{I}J'} \qquad\qquad 0 \;=\; \big(\Box + \mu\, i\partial_-\big) \overline{H}_{\tilde{I}J'}$$

$$\Longrightarrow \quad \mathcal{E}_0(H_{\tilde{I}J'}) \;=\; 1 \qquad \mathcal{E}_0(\overline{H}_{\tilde{I}J'}) \;=\; 3 \qquad \mathcal{D}(H_{\tilde{I}J'}) \;=\; \mathcal{D}(\overline{H}_{\tilde{I}J'}) \;=\; 18$$

Apply similar consideration to (1a), (1c) and (1d):

$$h^{\perp}_{\tilde{I}\tilde{J}} \;\equiv\; h_{\tilde{I}\tilde{J}} - \frac{1}{3}\delta_{\tilde{I}\tilde{J}}\, h_{\tilde{K}\tilde{K}} \qquad\qquad h^{\perp}_{I'J'} \;\equiv\; h_{I'J'} - \frac{1}{6}\delta_{I'J'}\, h_{K'K'}$$

$$h \;\equiv\; h_{\tilde{I}\tilde{I}} + i\mathcal{C} \qquad\qquad \overline{h} \;\equiv\; h_{\tilde{I}\tilde{I}} - i\mathcal{C}$$

Then we find

$$\mathcal{E}_0(h^{\perp}_{\tilde{I}\tilde{J}}) \;=\; \mathcal{E}_0(h^{\perp}_{I'J'}) \;=\; 2 \qquad \mathcal{D}(h^{\perp}_{\tilde{I}\tilde{J}}) \;=\; 5 \qquad \mathcal{D}(h^{\perp}_{I'J'}) \;=\; 20$$

$$\mathcal{E}_0(h) \;=\; 0 \qquad \mathcal{E}_0(\overline{h}) \;=\; 4 \qquad \mathcal{D}(h) \;=\; \mathcal{D}(\overline{h}) \;=\; 1$$

Decomposing $\mathcal{C}_{I'J'K'}$ into self-dual part $\mathcal{C}^{\oplus}_{I'J'K'}$ and anti-self-dual part $\mathcal{C}^{\ominus}_{I'J'K'}$:

$$\mathcal{C}^{\oplus}_{I'J'K'} \equiv \frac{i}{3!}\varepsilon^{I'J'K'W'X'Y'}\mathcal{C}^{\oplus}_{W'X'Y'} \qquad \mathcal{C}^{\ominus}_{I'J'K'} \equiv -\frac{i}{3!}\varepsilon^{I'J'K'W'X'Y'}\mathcal{C}^{\ominus}_{W'X'Y'}$$

They satisfy the following equations:

$$(\Box + \mu\, i\partial_-)\mathcal{C}^{\oplus}_{I'J'K'} = 0 \qquad (\Box - \mu\, i\partial_-)\mathcal{C}^{\ominus}_{I'J'K'} = 0$$

$$\Longrightarrow \quad \mathcal{E}_0(\mathcal{C}^{\oplus}_{I'J'K'}) = 3 \quad \mathcal{E}_0(\mathcal{C}^{\ominus}_{I'J'K'}) = 1 \quad \mathcal{D}(\mathcal{C}^{\oplus}_{I'J'K'}) = \mathcal{D}(\mathcal{C}^{\ominus}_{I'J'K'}) = 10$$

Now we have fully solved the field equations for bosonic fluctuations and have derived the spectrum of graviton h_{MN} and three-form gauge field $\mathcal{C}_{MNP}$. The resulting spectrum is splitting with a certain energy difference in contrast to the flat case. The resulting spectrum is completely identical with that of zero-mode Hamiltonian in the supermembrane theory on the pp-wave background.

We summarize this result with fermion spectrum.

Fermionic Fields

$M = -$ component of ($\mathcal{A}.2$):

$$\widehat{\Gamma}^P \psi_P \;=\; 0 \qquad \Longrightarrow \qquad \text{Lorentz-type gauge-fixing condition}$$

$M = +$ component of ($\mathcal{A}.2$):

$$\partial^P \psi_P \;=\; 0 \quad \rightarrow \quad \psi_+ \;=\; \frac{1}{\partial_-}\partial_I \psi_I \qquad \Longrightarrow \qquad \text{non-dynamical}$$

$M = \widetilde{I}$ component of ($\mathcal{A}.2$):

$$\begin{aligned} \psi^{\ominus}_{\widetilde{I}} \;&=\; \frac{1}{2\partial_-}\widehat{\Gamma}^{+}\widehat{\Gamma}^{K}\partial_K \psi^{\oplus}_{\widetilde{I}} \qquad \Longrightarrow \qquad \text{non-dynamical} \\ 0 \;&=\; \Box\, \psi^{\oplus}_{\widetilde{I}} - \frac{1}{2}\mu \widehat{\Gamma}^{123}\left(\delta_{\widetilde{I}\widetilde{J}} - \widehat{\Gamma}_{\widetilde{I}}\widehat{\Gamma}_{\widetilde{J}}\right)\partial_- \psi^{\oplus}_{\widetilde{J}} \end{aligned} \tag{2}$$

where $\psi_{\widetilde{I}} \equiv \psi^{\oplus}_{\widetilde{I}} + \psi^{\ominus}_{\widetilde{I}}$: $\psi^{\oplus}_{\widetilde{I}} \equiv -\frac{1}{2}\widehat{\Gamma}^{-}\widehat{\Gamma}^{+}\psi_{\widetilde{I}}$ and $\psi^{\ominus}_{\widetilde{I}} \equiv -\frac{1}{2}\widehat{\Gamma}^{+}\widehat{\Gamma}^{-}\psi_{\widetilde{I}}$

In order to solve this equation, we shall introduce the following fields:

$$\psi^{\oplus\perp}_{\widetilde{I}} \;\equiv\; \left(\delta_{\widetilde{I}\widetilde{J}} - \frac{1}{3}\widehat{\Gamma}_{\widetilde{I}}\widehat{\Gamma}_{\widetilde{J}}\right)\psi^{\oplus}_{\widetilde{J}} \qquad \widehat{\Gamma}\text{-transverse mode}$$

$$\psi^{\oplus\parallel}_{1} \;\equiv\; \widehat{\Gamma}^{\widetilde{I}}\psi^{\oplus}_{\widetilde{I}} \qquad \widehat{\Gamma}\text{-parallel mode}$$

Acting $\widehat{\Gamma}^{\widetilde{I}}$ or $(\delta_{\widetilde{K}\widetilde{I}} - \frac{1}{3}\widehat{\Gamma}_{\widetilde{K}}\widehat{\Gamma}_{\widetilde{I}})$ on (2):

$$0 = \Box\,\psi_1^{\oplus\parallel} - \mu\widehat{\Gamma}^{123}\partial_-\psi_1^{\oplus\parallel} \qquad 0 = \Box\,\psi_{\widetilde{K}}^{\oplus\perp} - \frac{1}{2}\mu\widehat{\Gamma}^{123}\partial_-\psi_{\widetilde{K}}^{\oplus\perp} \tag{3}$$

Decompose $\psi_{\widetilde{I}}^{\oplus\perp}$ and $\psi_1^{\oplus\parallel}$ according to the chirality in terms of $i\widehat{\Gamma}^{123}$:

$$\psi_{\widetilde{I}\mathrm{R}}^{\oplus\perp} \equiv \frac{1+i\widehat{\Gamma}^{123}}{2}\psi_{\widetilde{I}}^{\oplus\perp} \qquad \psi_{\widetilde{I}\mathrm{L}}^{\oplus\perp} \equiv \frac{1-i\widehat{\Gamma}^{123}}{2}\psi_{\widetilde{I}}^{\oplus\perp}$$

$$\psi_{1\mathrm{R}}^{\oplus\parallel} \equiv \frac{1+i\widehat{\Gamma}^{123}}{2}\psi_1^{\oplus\perp} \qquad \psi_{1\mathrm{L}}^{\oplus\parallel} \equiv \frac{1-i\widehat{\Gamma}^{123}}{2}\psi_1^{\oplus\perp}$$

Multiplying chiral projection operator $\frac{1}{2}(1 \pm i\widehat{\Gamma}^{123})$ to (3):

$$0 = \left(\Box + \mu\, i\partial_-\right)\psi_{1\mathrm{R}}^{\oplus\parallel} \qquad 0 = \left(\Box - \mu\, i\partial_-\right)\psi_{1\mathrm{L}}^{\oplus\parallel}$$

$$0 = \left(\Box + \frac{1}{2}\mu\, i\partial_-\right)\psi_{\widetilde{I}\mathrm{R}}^{\oplus\perp} \qquad 0 = \left(\Box - \frac{1}{2}\mu\, i\partial_-\right)\psi_{\widetilde{I}\mathrm{L}}^{\oplus\perp}$$

Zero-mode energies and degrees of freedom of $\psi_{\widetilde{I}\mathrm{R}}^{\oplus\perp}$ and $\psi_{\widetilde{I}\mathrm{L}}^{\oplus\perp}$:

$$\mathcal{E}_0(\psi_{\widetilde{I}\mathrm{R}}^{\oplus\perp}) = \frac{5}{2} \quad \mathcal{E}_0(\psi_{\widetilde{I}\mathrm{L}}^{\oplus\perp}) = \frac{3}{2} \quad \mathcal{D}(\psi_{\widetilde{I}\mathrm{R}}^{\oplus\perp}) = \mathcal{D}(\psi_{\widetilde{I}\mathrm{L}}^{\oplus\perp}) = 8\times(3-1) = 16$$

We will discuss $\mathcal{E}_0$ and $\mathcal{D}$ of $\widehat{\Gamma}$-parallel modes later on.

$M = I'$ component of ($\mathcal{A}.2$):

$$0 = \left\{\widehat{\Gamma}^{+}\left(\partial_{+} + \frac{1}{2}G_{++}\partial_{-}\right) + \widehat{\Gamma}^{-}\partial_{-} + \widehat{\Gamma}^{K}\partial_{K}\right\}\psi_{I'} + \frac{1}{4}\mu\widehat{\Gamma}^{+123}\left(\delta_{I'J'} - \widehat{\Gamma}_{I'}\widehat{\Gamma}_{J'}\right)\psi_{J'}$$

Decompose $\psi_{I'}$ into the $\widehat{\Gamma}$-parallel mode and $\widehat{\Gamma}$-transverse mode:

$$0 = \left(\Box - \frac{5}{2}\mu\, i\partial_{-}\right)\psi_{2\mathrm{R}}^{\oplus\parallel} \qquad 0 = \left(\Box + \frac{5}{2}\mu\, i\partial_{-}\right)\psi_{2\mathrm{L}}^{\oplus\parallel}$$

$$0 = \left(\Box - \frac{1}{2}\mu\, i\partial_{-}\right)\psi_{I'\mathrm{R}}^{\oplus\perp} \qquad 0 = \left(\Box + \frac{1}{2}\mu\, i\partial_{-}\right)\psi_{I'\mathrm{L}}^{\oplus\perp}$$

where the $\widehat{\Gamma}$-transverse mode and $\widehat{\Gamma}$-parallel mode are defined as

$$\psi_{I'}^{\oplus} = -\frac{1}{2}\widehat{\Gamma}^{-}\widehat{\Gamma}^{+}\psi_{I'} \qquad \psi_{I'\mathrm{R}}^{\oplus\perp} = \frac{1 + i\widehat{\Gamma}^{123}}{2}\psi_{I'}^{\oplus\perp} \qquad \psi_{I'\mathrm{L}}^{\oplus\perp} = \frac{1 - i\widehat{\Gamma}^{123}}{2}\psi_{I'}^{\oplus\perp}$$

$$\psi_{2\mathrm{R}}^{\oplus\parallel} = \frac{1 + i\widehat{\Gamma}^{123}}{2}\psi_{2}^{\oplus\parallel} \qquad \psi_{2\mathrm{L}}^{\oplus\parallel} = \frac{1 - i\widehat{\Gamma}^{123}}{2}\psi_{2}^{\oplus\parallel}$$

We find that the zero-mode energies and degrees of freedom:

$$\mathcal{E}_0(\psi_{I'\mathrm{R}}^{\oplus\perp}) = \frac{3}{2} \qquad \mathcal{E}_0(\psi_{I'\mathrm{L}}^{\oplus\perp}) = \frac{5}{2} \qquad \mathcal{D}(\psi_{I'\mathrm{R}}^{\oplus\perp}) = \mathcal{D}(\psi_{I'\mathrm{L}}^{\oplus\perp}) = 8 \times (6-1) = 40$$

Linear combination of $\widehat{\Gamma}$-parallel modes:

$$\psi_{\rm R}^{\oplus\|} \equiv \frac{2}{5}\psi_{\rm 1R}^{\oplus\|} - \psi_{\rm 2R}^{\oplus\|} \qquad \psi_{\rm L}^{\oplus\|} \equiv \frac{2}{5}\psi_{\rm 1L}^{\oplus\|} - \psi_{\rm 2L}^{\oplus\|}$$

We can easily see that the re-defined fermions satisfy:

$$0 = \Big(\Box - \frac{3}{2}\mu\, i\partial_-\Big)\psi_{\rm R}^{\oplus\|} \qquad 0 = \Big(\Box + \frac{3}{2}\mu\, i\partial_-\Big)\psi_{\rm L}^{\oplus\|}$$

$$\Longrightarrow \quad \mathcal{E}_0(\psi_{\rm R}^{\oplus\|}) = \frac{1}{2} \quad \mathcal{E}_0(\psi_{\rm L}^{\oplus\|}) = \frac{7}{2} \quad \mathcal{D}(\psi_{\rm R}^{\oplus\|}) = \mathcal{D}(\psi_{\rm L}^{\oplus\|}) = 8$$

We have solved and derived the spectrum of gravitino in the case of pp-wave background. As a result, we have found that the spectrum is splitting with a certain energy difference in the same manner with the spectrum of bosons.

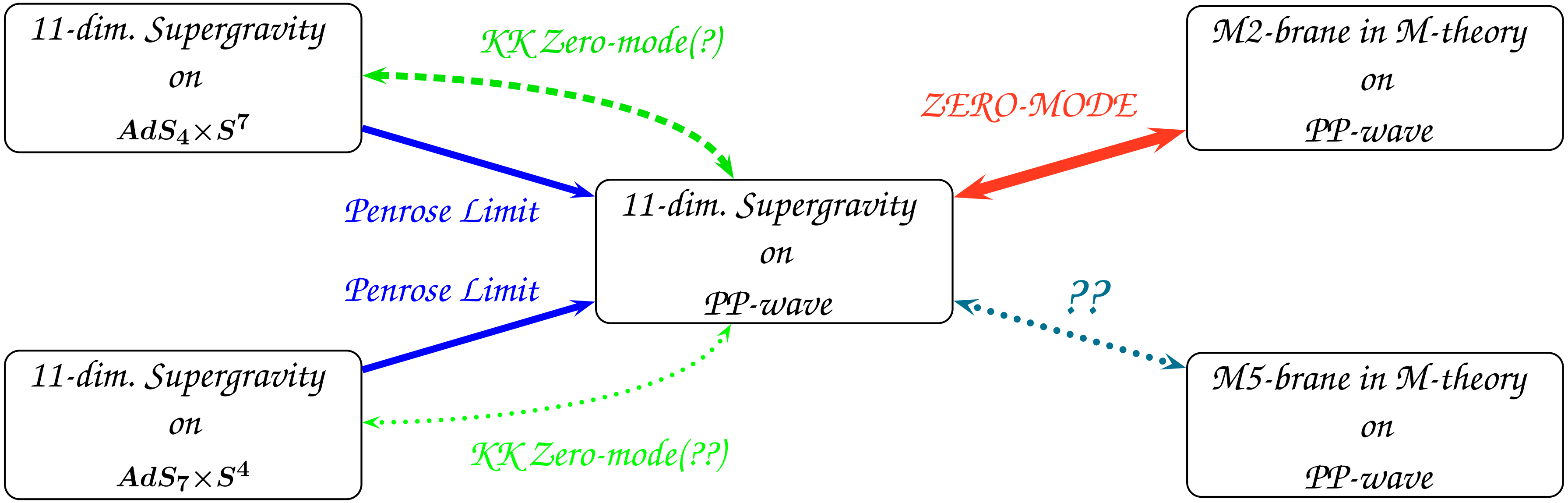
Results

energy $\mathcal{E}_0$	bosons $(\mathcal{D})$	fermions $(\mathcal{D})$	degrees
4	$\overline{h}(1)$		1
7/2		$\psi^{\oplus\parallel}_{\mathrm{L}}(8)$	8
3	$\overline{H}_{\tilde{I}J'}(18)$ $\mathcal{C}^{\oplus}_{I'J'K'}(10)$		28
5/2		$\psi^{\oplus\perp}_{\tilde{I}\mathrm{R}}(16)$ $\psi^{\oplus\perp}_{I'\mathrm{L}}(40)$	56
2	$\mathcal{C}_{\tilde{I}J'K'}(45)$ $h^{\perp}_{\tilde{I}\tilde{J}}(5)$ $h^{\perp}_{I'J'}(20)$		70
3/2		$\psi^{\oplus\perp}_{\tilde{I}\mathrm{L}}(16)$ $\psi^{\oplus\perp}_{I'\mathrm{R}}(40)$	56
1	$H_{\tilde{I}J'}(18)$ $\mathcal{C}^{\ominus}_{I'J'K'}(10)$		28
1/2		$\psi^{\oplus\parallel}_{\mathrm{R}}(8)$	8
0	$h(1)$		1

Energy spectrum and degrees of freedom of physical modes in 11-dimensional supergravity on the pp-wave background.

This result corresponds to that of the zero-mode Hamiltonian in the supermembrane theory on the pp-wave background.

Conclusion and Discussions



Work in Progress

- **Propagators and energy-momentum tensors of h_{MN}, $\mathcal{C}_{MNP}$ and ψ_M**
- **Dimensional reduction to type IIA supergravity (only 24 supercharges)**
- **Comparison with KK zero-mode and algebra of $AdS_7 \times S^4$**
- **Relationship to Zero-mode spectrum and superalgebra of M5-brane**

Appendix A *Penrose Limit of Anti-de Sitter Spaces*

- $AdS_4 \times S^7 \longrightarrow$ **Kowalski-Glikman**

$$\mathrm{d}s^2 = R^2\Big\{-(\cosh\rho)^2\mathrm{d}t^2 + \mathrm{d}\rho^2 + (\sinh\rho)^2\mathrm{d}\Omega_2^2\Big\} + (2R)^2\Big\{(\cos\theta)^2\mathrm{d}\varphi^2 + \mathrm{d}\theta^2 + (\sin\theta)^2\mathrm{d}\Omega_5^2\Big\}$$

$$x^+ = \frac{\mu}{6}(t + 2\varphi) \quad x^- = \frac{3R^2}{\mu}(t - 2\varphi) \quad x = R\rho \quad y = 2R\theta\,, \quad R \longrightarrow \infty : \quad \text{Penrose limit}$$

$$\Longrightarrow \quad \mathrm{d}s^2 = -2\mathrm{d}x^+\mathrm{d}x^- - \Big(\frac{\mu}{3}\Big)^2\Big\{x^2 + \frac{1}{4}y^2\Big\}(\mathrm{d}x^+)^2 + \big\{\mathrm{d}x^2 + x^2\mathrm{d}\Omega_2^2\big\} + \big\{\mathrm{d}y^2 + y^2\mathrm{d}\Omega_5^2\big\}$$

- $AdS_7 \times S^4 \longrightarrow$ **Kowalski-Glikman**

$$\mathrm{d}s^2 = (2R)^2\Big\{-(\cosh\rho)^2\mathrm{d}t^2 + \mathrm{d}\rho^2 + (\sinh\rho)^2\mathrm{d}\Omega_5^2\Big\} + R^2\Big\{(\cos\theta)^2\mathrm{d}\varphi^2 + \mathrm{d}\theta^2 + (\sin\theta)^2\mathrm{d}\Omega_2^2\Big\}$$

$$x^+ = \frac{\mu}{2}\Big(t + \frac{1}{2}\varphi\Big) \quad x^- = \frac{4R^2}{\mu}\Big(t - \frac{1}{2}\varphi\Big) \quad x = 2R\rho \quad y = R\theta\,, \quad R \longrightarrow \infty : \quad \text{Penrose limit}$$

$$\Rightarrow \quad \mathrm{d}s^2 = -2\mathrm{d}x^+\mathrm{d}x^- - \mu^2\Big\{x^2 + 4y^2\Big\}(\mathrm{d}x^+)^2 + \big\{\mathrm{d}x^2 + x^2\mathrm{d}\Omega_5^2\big\} + \big\{\mathrm{d}y^2 + y^2\mathrm{d}\Omega_2^2\big\}$$

$$x^+ \longrightarrow \frac{1}{2}x^+ \quad x^- \longrightarrow 2x^- \quad \mu \longrightarrow \frac{1}{3}\mu \quad x \longleftrightarrow y$$

$$\Longrightarrow \quad \mathrm{d}s^2 = -2\mathrm{d}x^+\mathrm{d}x^- - \Big(\frac{\mu}{3}\Big)^2\Big\{x^2 + \frac{1}{4}y^2\Big\}(\mathrm{d}x^+)^2 + \big\{\mathrm{d}x^2 + x^2\mathrm{d}\Omega_2^2\big\} + \big\{\mathrm{d}y^2 + y^2\mathrm{d}\Omega_5^2\big\}$$

Appendix B *Linearized Field Equations for Fluctuations on the PP-wave Background*

$$
\begin{aligned}
0 \;=\; & -\frac{1}{2}g_{MN}\Big\{h^{PQ}\,\mathcal{R}_{PQ} - \nabla^P\nabla^Q\,h_{PQ} + \nabla^P\nabla_P\,h_Q{}^Q\Big\} \\
& - \frac{1}{2}\Big\{\nabla^P\nabla_M\,h_{NP} + \nabla^P\nabla_N\,h_{MP} - \nabla_M\nabla_N\,h_P{}^P - \nabla^P\nabla_P\,h_{MN}\Big\} \\
& - \frac{1}{24}g_{MN}\Big\{2F^{PQRS}\,\partial_P\mathcal{C}_{QRS} - F_{PQRS}\,F_U{}^{QRS}\,h^{PU}\Big\} \\
& + \frac{1}{3}F_M{}^{PQR}\,\partial_{[N}\mathcal{C}_{PQR]} + \frac{1}{3}F_N{}^{PQR}\,\partial_{[M}\mathcal{C}_{PQR]} - \frac{1}{4}F_{MPQR}\,F_{NU}{}^{QR}\,h^{PU} \qquad (\mathcal{A}.1)
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\; & \widehat{\Gamma}^{MNP}\,D_N\psi_P - \frac{1}{4}\mu\,\widehat{\Gamma}^{MN+123}\psi_N \\
& - \frac{1}{4}\mu\Big\{g^{M+}(\widehat{\Gamma}^{12}g^{3N} + \widehat{\Gamma}^{23}g^{1N} + \widehat{\Gamma}^{31}g^{2N}) - g^{M1}(\widehat{\Gamma}^{23}g^{+N} + \widehat{\Gamma}^{3+}g^{2N} + \widehat{\Gamma}^{+2}g^{3N}) \\
& \quad + g^{M2}(\widehat{\Gamma}^{3+}g^{1N} + \widehat{\Gamma}^{+1}g^{3N} + \widehat{\Gamma}^{13}g^{+N}) - g^{M3}(\widehat{\Gamma}^{+1}g^{2N} + \widehat{\Gamma}^{12}g^{+N} + \widehat{\Gamma}^{2+}g^{1N})\Big\}\psi_N \qquad (\mathcal{A}.2)
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\; & -\Big(\frac{1}{2}g^{QR}\,h_U{}^U - h^{QR}\Big)\Big\{\Gamma^S_{RQ}\,F_{SMNP} + \Gamma^S_{RM}\,F_{QSNP} + \Gamma^S_{RN}\,F_{QMSP} + \Gamma^S_{RP}\,F_{QMNS}\Big\} \\
& + 4g^{QR}\Big\{\partial_R\partial_{[Q}\mathcal{C}_{MNP]} - \Gamma^S_{RQ}\,\partial_{[S}\mathcal{C}_{MNP]} - \Gamma^S_{RM}\,\partial_{[Q}\mathcal{C}_{SNP]} - \Gamma^S_{RN}\,\partial_{[Q}\mathcal{C}_{MSP]} - \Gamma^S_{RP}\,\partial_{[Q}\mathcal{C}_{MNS]}\Big\} \\
& - \frac{1}{2}g^{QR}\Big\{F_{SMNP}(\nabla_R\,h_Q{}^S + \nabla_Q\,h_R{}^S - \nabla^S\,h_{RQ}) + F_{QSNP}(\nabla_R\,h_M{}^S + \nabla_M\,h_R{}^S - \nabla^S\,h_{RM}) \\
& \quad + F_{QMSP}(\nabla_R\,h_N{}^S + \nabla_N\,h_R{}^S - \nabla^S\,h_{RN}) + F_{QMNS}(\nabla_R\,h_P{}^S + \nabla_P\,h_R{}^S - \nabla^S\,h_{RP})\Big\} \\
& + \frac{1}{144}g_{MZ}\,g_{NK}\,g_{PL}\,\varepsilon^{ZKLQRSUVWXY}\,F_{QRSU}\,\partial_V\mathcal{C}_{WXY} \qquad (\mathcal{A}.3)
\end{aligned}
$$