Seminar at Komaba, University of Tokyo; *October 7th, 2004*

Introduction to

Gauged Linear Sigma Model

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References: E.Witten, "Phases of $\mathcal{N} = 2$ Theories in Two Dimensions," hep-th/9301042
K.Hori and C.Vafa, "Mirror Symmetry," hep-th/0002222
B.R.Greene, "String Theory on Calabi-Yau Manifolds," hep-th/9702155
and many papers

Purpose

quintic hypersurface $\mathbb{C}\mathrm{P}^4[5]$ (=CY 3-fold) を用いて

GLSM と Hori-Vafa theory に慣れ親しむ

そのため計算ツールとしての紹介をする

Gauged linear sigma model   E. Witten (1993), K. Hori and C. Vafa (2000)

$\mathcal{N}=(2,2)$ SUSY gauge theory with matters $(\mathrm{FI}:\ t\equiv r-i\theta)$

$$\mathscr{L} = \int \mathrm{d}^4\theta \Big\{ -\frac{1}{e^2}\overline{\Sigma}\Sigma + \sum_a \overline{\Phi}_a\, \mathrm{e}^{2Q_aV}\Phi_a \Big\} + \Big(\frac{1}{\sqrt{2}}\int \mathrm{d}^2\widetilde{\theta}\,(-\Sigma\, t) + c.c.\Big) + \Big(\int \mathrm{d}^2\theta\, W_{\mathrm{GLSM}}(\Phi_a) + c.c.\Big)$$

- Φ_a : charged chiral superfield, $\overline{D}_{\pm}\Phi_a = 0$
  Σ : twisted chiral superfield, $\overline{D}_{+}\Sigma = D_{-}\Sigma = 0$, $\Sigma = \frac{1}{\sqrt{2}}\overline{D}_{+}D_{-}V$
- 2つの phase:

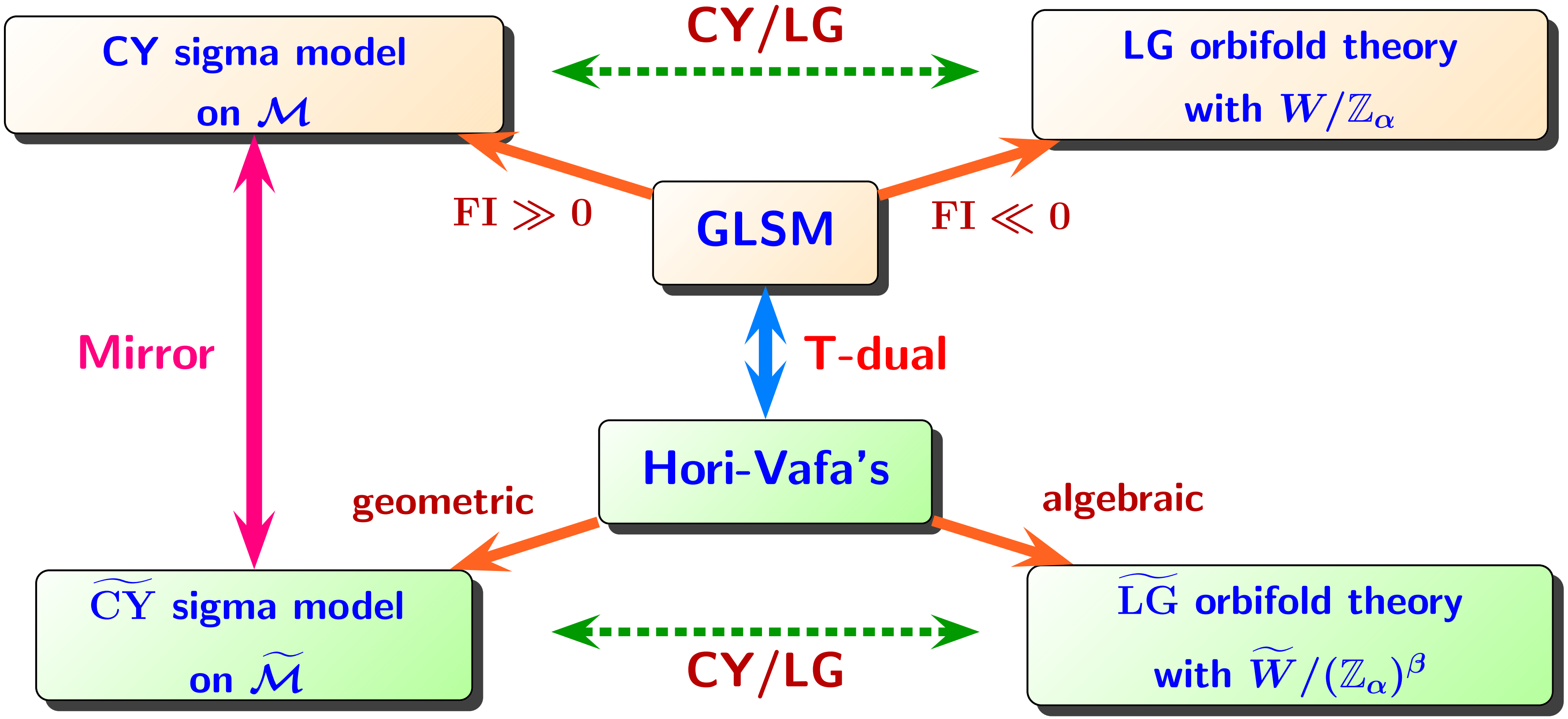
$\mathrm{FI} \gg 0$: differential-geometric phase $\longrightarrow$ SUSY NLSM

$\mathrm{FI} \ll 0$: algebro-gemetric phase $\longrightarrow$ LG, orbifold, SCFT

- LG model から CY 多様体の位相的情報を引き出せる (CY/LG対応)

harmonic forms $\longleftrightarrow$ NS-NS chiral primary states

- GLSM の T-dual model の IR limit には “Mirror” geometry が登場



CY sigma model on $\mathcal{M}$
CY/LG
LG orbifold theory with $W/\mathbb{Z}_\alpha$
FI $\gg 0$
GLSM
FI $\ll 0$
Mirror
T-dual
Hori-Vafa's
geometric
algebraic
$\widetilde{\mathrm{CY}}$ sigma model on $\widetilde{\mathcal{M}}$
CY/LG
$\widetilde{\mathrm{LG}}$ orbifold theory with $\widetilde{W}/(\mathbb{Z}_\alpha)^\beta$

Plan of this informal seminar

We will discuss 3-dim. compact CY hypersurfaces in $\mathbb{C}\mathrm{P}^4$:

- **GLSM**
  - **study $\mathrm{FI} \gg 0$ phase: CY sigma model, geometry**
  - **study $\mathrm{FI} \ll 0$ phase: LG theory (chiral ring)**
- **T-duality**
  - **study LG description: twisted LG theory**
  - **study geometric description: mirror CY geometry**
- **Comments**

Gauged linear sigma model for $\mathbb{CP}^4[5]$

$$\mathscr{L} = \int d^4\theta \Big\{ -\frac{1}{e^2}\overline{\Sigma}\Sigma + \sum_a \overline{\Phi}_a \, e^{2Q_aV}\Phi_a \Big\} + \Big(\frac{1}{\sqrt{2}} \int d^2\widetilde{\theta}\,(-\Sigma t) + c.c. \Big) + \Big(\int d^2\theta \, W_{\rm GLSM} + c.c. \Big)$$

$$\left[\begin{array}{c} W_{\rm GLSM} = P \cdot G_5(S_i) \\ G_5(s) = \partial_1 G_5(s) = \cdots = \partial_5 G_5(s) = 0 \ \longrightarrow \ {}^\forall s_i = 0 \end{array} \right] \qquad t = r - i\theta$$

chiral superfield Φ_a	S_1	S_2	$\cdots$	S_5	P
$U(1)$ charge Q_a	1	1	$\cdots$	1	-5

Potential energy density:

$$\mathcal{U} = \frac{e^2}{2}\Big\{ r - \sum_{i=1}^5 |s_i|^2 + 5|p|^2 \Big\}^2 + \big|G_5(s)\big|^2 + |p|^2 \cdot \sum_{i=1}^5 \big|\partial_i G_5(s)\big|^2 + 2|\sigma|^2 \Big\{ \sum_a Q_a^2 |\phi_a|^2 \Big\}$$

"CY" condition: $\displaystyle\sum_a Q_a = 0 \ \longleftarrow \ r_0 = r(\mu) + \sum_a Q_a \log\Big(\frac{\Lambda_{\rm UV}}{\mu}\Big)$

FI parameter r 毎に SUSY vacuum manifold $\mathcal{U} = 0$ とその上の effective theory を考えよう

▼ $r \gg 0$ phase: SUSY nonlinear sigma model

SUSY vacuum manifold $\mathcal{M}_{\rm CY} = \{\mathcal{U} = 0\}$ を考える

$$r = \sum_{i=1}^{5} |s_i|^2 - 5|p|^2 > 0 \quad \longrightarrow \quad {}^{\exists}s_i \neq 0$$

$$G_5(s) = p\, \partial_i G_5(s) = 0 \quad \longrightarrow \quad p = 0$$

$$\therefore \quad \mathcal{M}_{\rm CY} \ = \ \Big\{ (s_1, s_2, \cdots, s_5) \in \mathbb{C}^5 \,\Big|\, r = \sum_{i=1}^{5} |s_i|^2 > 0 \ , \ G_5(s) = 0 \Big\} \Big/ U(1)$$

$$= \ \mathbb{C}\mathrm{P}^4[5]$$

fluctuations around a point ${}^{\exists}(\langle s_1 \rangle, \langle s_2 \rangle, \cdots, \langle s_5 \rangle) \in \mathcal{M}_{\rm CY}$: $U(1) \longrightarrow 1$

modes tangent to $\mathcal{M}_{\rm CY}$: massless

modes non-tangent to $\mathcal{M}_{\rm CY}$: massive $m^2 \propto e^2 r$ via Higgs mechanism

IR limit $e \longrightarrow \infty$ $\Rightarrow$ massive modes are decoupled

➡ massless effective theory = SUSY NLSM on $\mathbb{C}\mathrm{P}^4[5]$

▼ $r \ll 0$ phase: orbifolded LG theory

SUSY vacuum manifold $\mathcal{M}_{\text{orbifold}} = \{\mathcal{U} = 0\}$ を考える

$$r = \sum_{i=1}^{5} |s_i|^2 - 5|p|^2 < 0 \quad \rightarrow \quad p \neq 0$$

$$G_5(s) = p\,\partial_i G_5(s) = 0 \quad \rightarrow \quad {}^\forall s_i = 0$$

$$\therefore \quad \mathcal{M}_{\text{orbifold}} \; = \; \Big\{ (p) \in \mathbb{C}^* \,\Big|\, r = -5|p|^2 < 0 \Big\} \Big/ U(1)$$

fluctuations around a point $\{\langle p \rangle \neq 0\} \in \mathcal{M}_{\text{orbifold}}$: $U(1) \longrightarrow \mathbb{Z}_5$

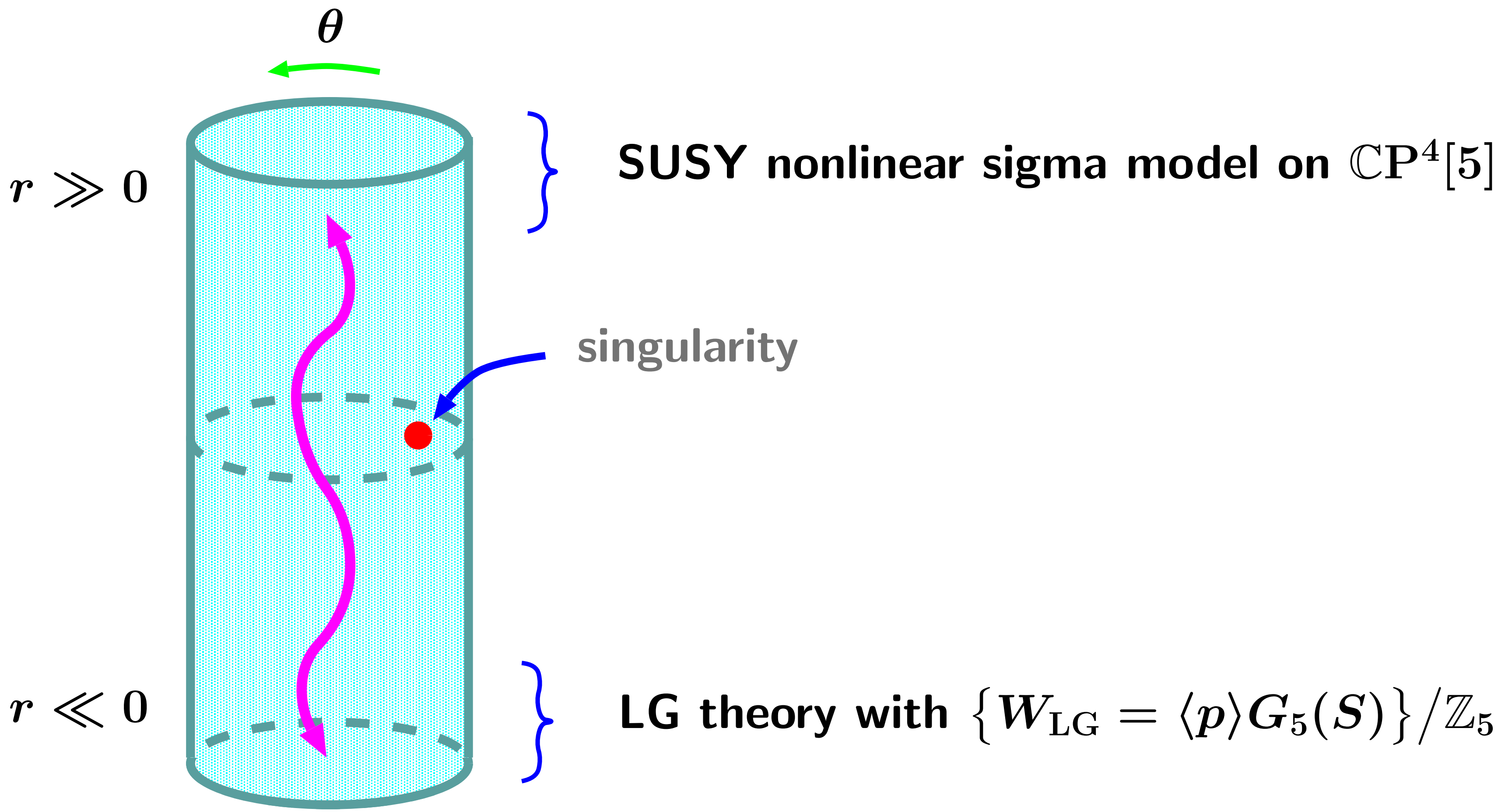
all S_i : massless with potential $\langle p \rangle G_5(S)$

P : massive $m^2 \propto e^2 r$ via Higgs mechanism

IR limit $e \longrightarrow \infty$:

➡ **massless effective theory =** $\left(\begin{array}{l} \text{SUSY orbifolded LG theory} \\ \qquad \text{with } \{W_{\text{LG}} = \langle p \rangle G_5(S)\}/\mathbb{Z}_5 \end{array} \right)$

CY/LG correspondence



昔はやっかいだったが

今や CY sigma model と LG theory は GLSM の異なる側面を表すに過ぎない

CY data and LG data

	CY 3-fold $\mathcal{M}_{\rm CY}$	$\mathcal{N}=(2,2)$ LG theory ($c=\overline{c}=9$)
$b_{3-p,q}$	$\dim H^{3-p,q}(\mathcal{M}_{\rm CY})$	# of (p,q) charged (c,c) primary states
$P(t,\overline{t})$	Poincaré polynomial	"Poincaré" polynomial ((c,c) primary)
$b_{p,q}=b_{3-p,3-q}$	Poincaré duality	charge conjugation invariance on R ground states
$b_{0,0}=1$	simply connected	uniqueness of vacuum: $(h_L,h_R)=(0,0)$
$b_{3,3}=1$	Poincaré duality	uniqueness of highest charge state: $(h_L,h_R)=(c/6,\overline{c}/6)$
$b_{3,0}=1$	holomorphic $(3,0)$-form	spectral flow from vacuum: $(\theta_L,\theta_R)=(-1/2,0)$

この関係を用いてLG theory から CY 3-fold の topological data を再現する

Information from LG superpotential

$$\{W_{\mathrm{LG}} = S_1^5 + S_2^5 + S_3^5 + S_4^5 + S_5^5\}/\mathbb{Z}_5$$

$$A_4 \otimes A_4 \otimes A_4 \otimes A_4 \otimes A_4 \text{ type}$$

A_4 type LG minimal model:

$$\text{chiral ring } \mathscr{R}_i = \mathbb{C}/[\mathrm{d}W] = \{1, S_i, S_i^2, S_i^3\}$$

$$\text{central charge } c = 3 - \frac{6}{4+1}$$

NS-NS (c, c) primary states $= \{\otimes_{i=1}^{5} S_i^{l_i}\}$ (但し $l_i = 0, 1, 2, 3$)

spectral flow を施して $\mathbb{Z}_5$ invariant Ramond ground states を構成する

(# of R gound states with $(p - 3/2, q - 3/2)$ charge)

$=$ (# of harmonic $(3 - p, q)$-forms on CY_3) $\equiv$ $b_{3-p,q}$

Hodge diamond:

$$
\begin{array}{ccccccc}
 & & & b_{0,0} & & & \\
 & & b_{1,0} & & b_{0,1} & & \\
 & b_{2,0} & & b_{1,1} & & b_{0,2} & \\
b_{3,0} & & b_{2,1} & & b_{1,2} & & b_{0,3} \\
 & b_{3,1} & & b_{2,2} & & b_{1,3} & \\
 & & b_{3,2} & & b_{2,3} & & \\
 & & & b_{3,3} & & &
\end{array}
=
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 1 & & 0 & \\
1 & & 101 & & 101 & & 1 \\
 & 0 & & 1 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

# of complex moduli $b_{2,1} = 101$:

"degree of $W_{\rm LG} = G_5(S)$ (126)" $-$ "deformation degree under $GL(5,\mathbb{C})$ (25)"

# of Kähler moduli $b_{1,1} = 1$:

"deformation degree of the FI parameter"

Euler number:

$$\chi(\mathcal{M}_{\rm CY}) = \sum_{p,q=0}^{3} (-1)^{p+q} b_{p,q} = 2(b_{1,1} - b_{2,1}) = -200$$

T-duality

$$\mathscr{L}' = \int d^4\theta \Big(\frac{R^2}{4} B^2 - \frac{1}{2}(Y + \overline{Y}) B \Big) , \qquad \begin{aligned} \overline{B} &= B \\ \overline{D}_+ Y &= D_- Y = 0 \end{aligned}$$

1. Theory on S^1 of radius R:

$$\left.\begin{aligned} d^4\theta &= \frac{1}{2} d^2\widetilde{\theta}\, \overline{D}_+ D_- \\ &\text{integrate out } Y, \overline{Y} \end{aligned}\right\} \Longrightarrow \overline{D}_+ D_- B = D_+ \overline{D}_- B = 0 \Longrightarrow B = \Phi + \overline{\Phi}$$

$$\therefore \quad \mathscr{L}'\Big|_{B=\Phi+\overline{\Phi}} = \int d^4\theta \, \frac{R^2}{2} \overline{\Phi}\Phi \Longrightarrow d^2 s = R^2 |d\phi|^2 = R^2 (d\rho^2 + d\varphi^2)$$

2. Theory on S^1 of radius $1/R$:

$$\text{integrate out } B \Longrightarrow B = \frac{1}{R^2}(Y + \overline{Y})$$

$$\therefore \quad \mathscr{L}'\Big|_{B=\frac{1}{R^2}(Y+\overline{Y})} = \int d^4\theta \Big(-\frac{1}{2R^2} \overline{Y} Y \Big) \Longrightarrow d^2 s = \frac{1}{R^2} |dy|^2 = \frac{1}{R^2} (R^4 d\rho^2 + d\vartheta^2)$$

これらの議論に $U(1)$ gauge symmetry を追加したもの = Hori-Vafa

T-dualized theory of GLSM for $\mathbb{CP}^4[5]$

Hori-Vafa's Lagrangian:

$$\mathscr{L} = \int d^4\theta \Big\{ -\frac{1}{e^2}\overline{\Sigma}\Sigma - \sum_a \Big(\frac{1}{2}(Y_a + \overline{Y}_a)\log(Y_a + \overline{Y}_a)\Big)\Big\} + \Big(\frac{1}{\sqrt{2}}\int d^2\widetilde{\theta}\,\widetilde{W} + c.c.\Big)$$

$$\widetilde{W} = \Sigma\Big(\sum_a Q_a Y_a - t\Big) + \sum_a e^{-Y_a}$$

chiral superfield	Φ_a	S_1	S_2	$\cdots$	S_5	P
$U(1)$ charge	Q_a	1	1	$\cdots$	1	-5
twisted chiral superfield	Y_a	Y_1	Y_2	$\cdots$	Y_5	Y_P

relation between chiral superfields $\{\Phi_a\}$ and twisted chiral superfields $\{Y_a\}$:

$$2\overline{\Phi}_a\, e^{2Q_a V}\Phi_a = Y_a + \overline{Y}_a$$

$U(1)$ phase rotation symmetry on Φ_a $\longrightarrow$ shift symmetry on Y_a:

$$Y_a \longrightarrow Y_a + 2\pi i$$

A powerful tool = "period integral":

$$\widehat{\Pi} \ \equiv \ \int \mathrm{d}\Sigma \prod_{i=1}^{5} \mathrm{d}Y_i \, \mathrm{d}Y_P \, (5\Sigma) \, \exp\big(- \widetilde{W} \big)$$

topological A-twisted sector のみを追う量

一言でいうと「partition function of topological A-theory with $W_{\mathrm{GLSM}} = P \cdot G_5(S_i)$」

Hori-Vafa の "proof of mirror symmetry" の不十分な点

しかし $G_5(S_i)$ に余分な条件がなければこの量で十分

$$5\Sigma \ \longrightarrow \ \begin{cases} 5\dfrac{\partial}{\partial t} : \text{ twisted LG theory を追う} \\ \dfrac{\partial}{\partial Y_P} : \text{ T-dualized geometry を追う} \end{cases}$$

twisted LG description

$$\widehat{\Pi} = 5\frac{\partial}{\partial t}\int\prod_{i=1}^{5}\mathrm{d}Y_i\,\mathrm{d}Y_P\,\delta\Big(\sum_{i=1}^{5}Y_i - 5Y_P - t\Big)\ \exp\Big\{-\sum_{i=1}^{5}\mathrm{e}^{-Y_i} - \mathrm{e}^{-Y_P}\Big\}$$

$$= \mathrm{e}^{t/5}\int\prod_{i=1}^{5}\big(\mathrm{e}^{-Y_i/5}\mathrm{d}Y_i\big)\ \exp\Big\{-\sum_{i=1}^{5}\mathrm{e}^{-Y_i} - \mathrm{e}^{t/5}\prod_{i=1}^{5}\mathrm{e}^{-Y_i/5}\Big\}$$

$$= \int\prod_{i=1}^{5}\mathrm{d}X_i\ \exp\Big\{-\big(X_1^5 + X_2^5 + \cdots + X_5^5 + \mathrm{e}^{t/5}X_1X_2\cdots X_5\big)\Big\} \equiv \int\prod_{i=1}^{5}\mathrm{d}X_i\ \exp\big(-\widetilde{W}_{\mathrm{LG}}\big)$$

但し $X_i \equiv \mathrm{e}^{-Y_i/5}$。

from shift symmetry $Y_a \to Y_a + 2\pi i$, this model has $(\mathbb{Z}_5)^4$ orbifold symmetry:

$$X_i \ \to\ \omega_i X_i\ , \qquad \omega_i^5 \ =\ \omega_1\omega_2\cdots\omega_5 \ =\ 1$$

⇒ massless effective theory = (SUSY orbifolded twisted chiral LG theory with $\{\widetilde{W}_{\mathrm{LG}}\}/(\mathbb{Z}_5)^4$)

Hodge diamond:

$$
\begin{array}{ccccccc}
 & & & \widetilde{b}_{0,0} & & & \\
 & & \widetilde{b}_{1,0} & & \widetilde{b}_{0,1} & & \\
 & \widetilde{b}_{2,0} & & \widetilde{b}_{1,1} & & \widetilde{b}_{0,2} & \\
\widetilde{b}_{3,0} & & \widetilde{b}_{2,1} & & \widetilde{b}_{1,2} & & \widetilde{b}_{0,3} \\
 & \widetilde{b}_{3,1} & & \widetilde{b}_{2,2} & & \widetilde{b}_{1,3} & \\
 & & \widetilde{b}_{3,2} & & \widetilde{b}_{2,3} & & \\
 & & & \widetilde{b}_{3,3} & & &
\end{array}
=
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 101 & & 0 & \\
1 & & 1 & & 1 & & 1 \\
 & 0 & & 101 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

\# of Kähler moduli $\widetilde{b}_{1,1} = 101$, \# of complex moduli $\widetilde{b}_{1,1} = 1$

Euler number: $$\widetilde{\chi} = \sum_{p,q=0}^{3} (-1)^{p+q} \widetilde{b}_{p,q} = 2(\widetilde{b}_{1,1} - \widetilde{b}_{2,1}) = 200$$

(X_i: twisted chiral superfield $\quad\leftrightarrow\quad$ S_i: chiral superfield)

T-dualized (mirror) geometric description

$$\widehat{\Pi} = \int d\Sigma \prod_{i=1}^{5} dY_i \, dY_P \frac{\partial}{\partial Y_P} \exp\left\{ -\Sigma\Big(\sum_i Y_i - 5Y_P - t\Big) - \sum_i e^{-Y_i} - e^{-Y_P} \right\}$$
$$= \int \prod_i dY_i \, dY_P \, e^{-Y_P} \, \delta\Big(\sum_i Y_i - 5Y_P - t\Big) \exp\left\{ -\sum_i e^{-Y_i} - e^{-Y_P} \right\}$$

field re-definition (preserving canonical measure of $\widehat{\Pi}$):

$$e^{-Y_P} = \widetilde{P} \, , \qquad e^{-Y_i} = \widetilde{P} U_i \, , \qquad U_i = e^{-t/5} \frac{Z_i^5}{Z_1 Z_2 \cdots Z_5}$$

period integral is re-written as

$$\widehat{\Pi} = \int \frac{1}{\mathrm{vol.}(\mathbb{C}^*)} \prod_{i=1}^{5} dZ_i \, \delta\Big(Z_1^5 + Z_2^5 + \cdots + Z_5^5 + e^{t/5} Z_1 \cdots Z_5\Big)$$

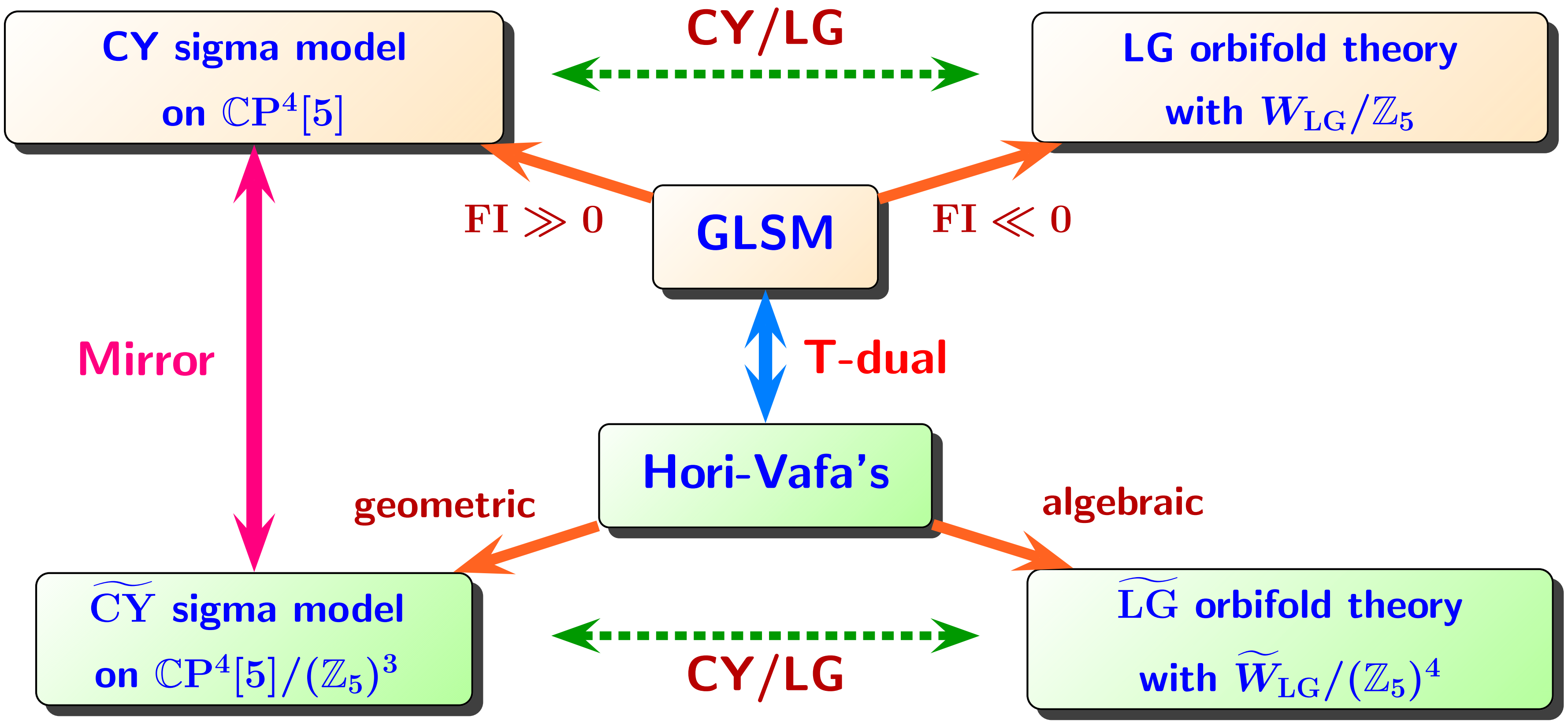
twisted chiral superfields Z_i has the following symmetry:

$$Z_i \to \lambda \omega_i Z_i \, , \qquad \omega_i^5 = \omega_1 \omega_2 \cdots \omega_5 = 1 \, , \qquad \lambda \in \mathbb{C}^*$$

This is nothing but the definition of $\mathbb{C}\mathrm{P}^4[5]/(\mathbb{Z}_5)^3$!

(the mirror geometry appears!)

Result



$$W_{\rm LG} \;=\; G_5(S)$$

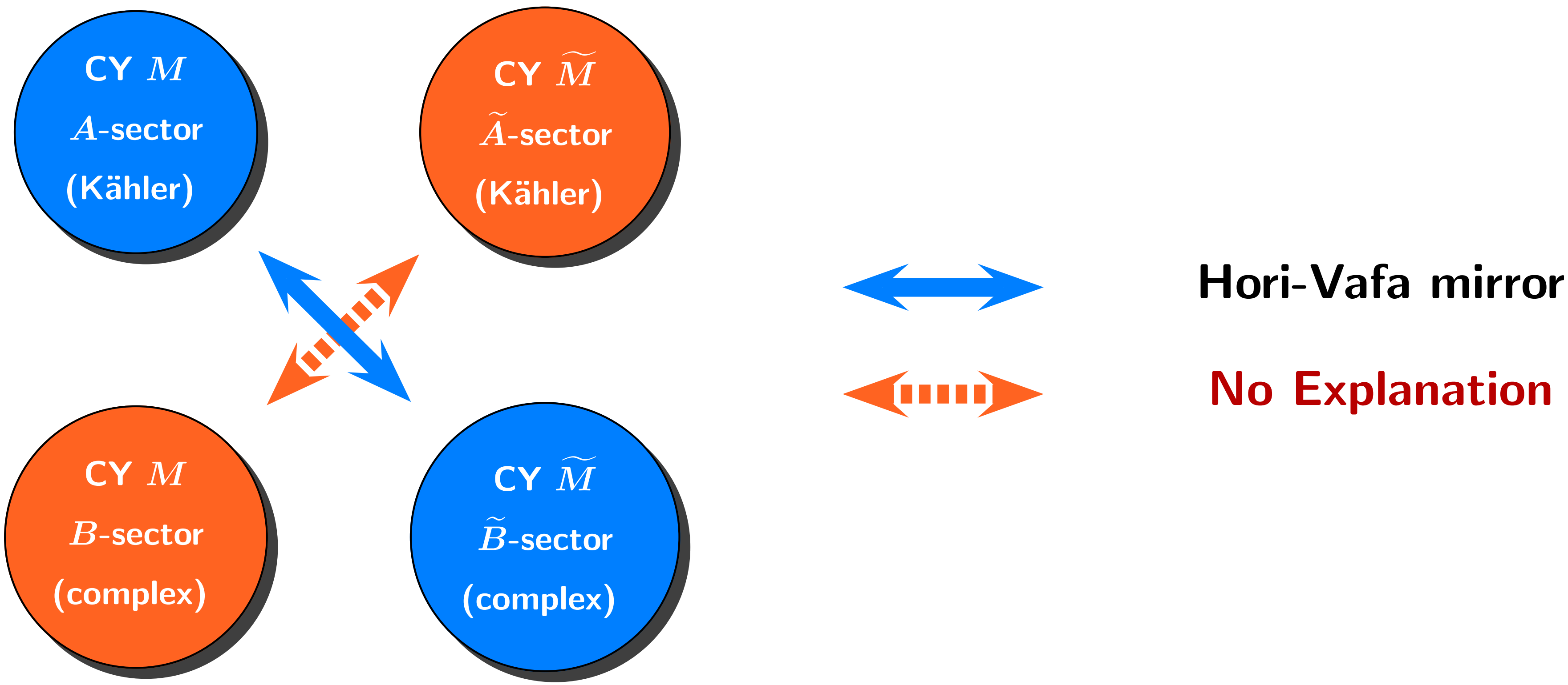
$$\widetilde{W}_{\rm LG} \;=\; X_1^5 + X_2^5 + X_3^5 + X_4^5 + X_5^5 + e^{t/5} X_1 X_2 X_3 X_4 X_5$$

$$b_{2,1}(\mathbb{C}P^4[5]) = \widetilde{b}_{1,1}(\mathbb{C}P^4[5]/(\mathbb{Z}_5)^3) = 101 \;, \qquad b_{1,1}(\mathbb{C}P^4[5]) = \widetilde{b}_{2,1}(\mathbb{C}P^4[5]/(\mathbb{Z}_5)^3) = 1$$

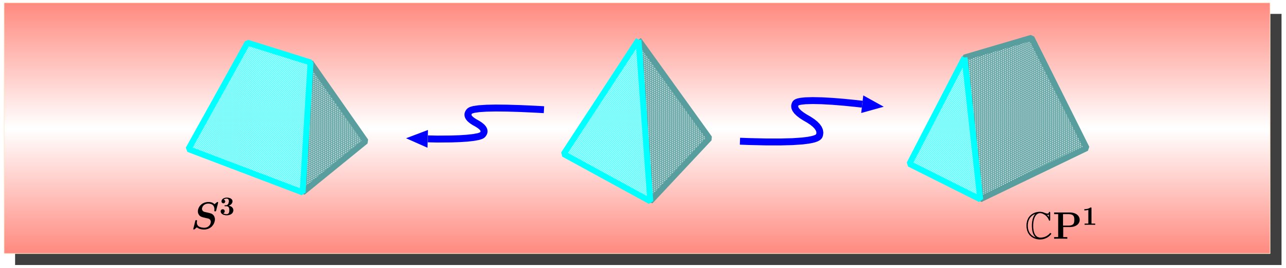
Comments

Hori-Vafa's explanation of Mirror symmetry is not complete!

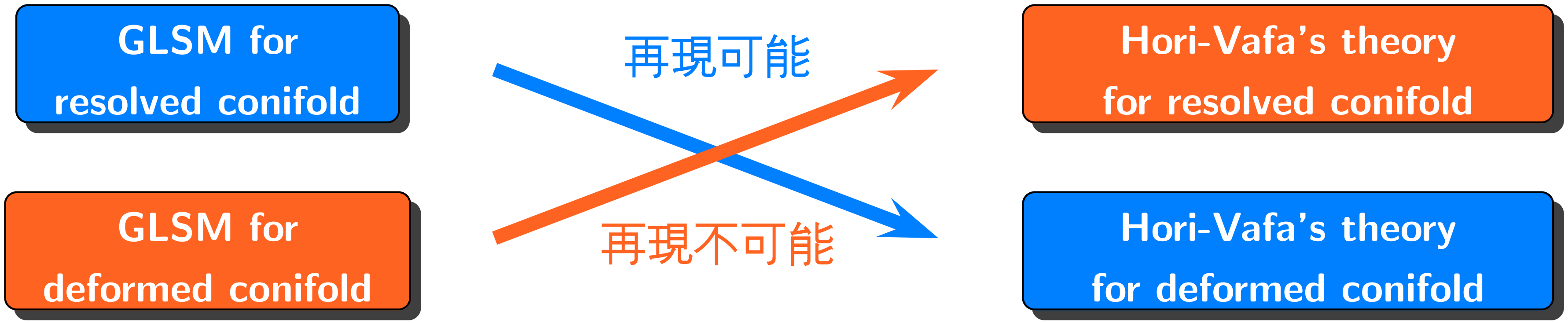
topological A-twisted sector only $\longrightarrow$ How about B-sector?



Hori-Vafa の未解決部分の例: resolved/deformed conifold

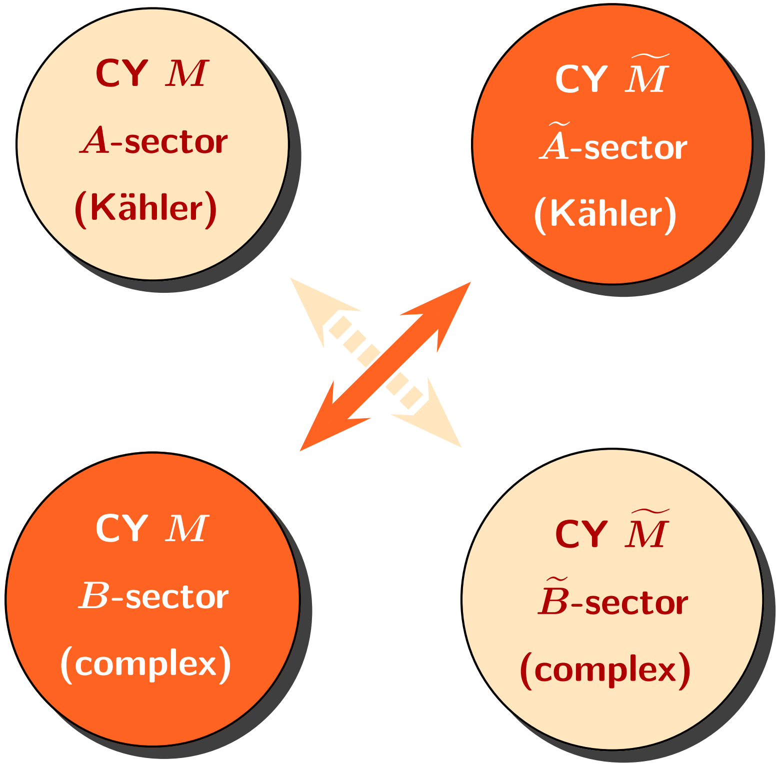


- deformed conifold: deformation of complex moduli $(b_{2,1})$
- resolved conifold: deformation of Kähler moduli $(b_{1,1})$



My Present Study

▼ 1. investigation beyond the Hori-Vafa



$W_{\text{GLSM}} = P \cdot G_\ell(S)$ に additional な対称性

example: sigma model on quadric surface

and its line bundle

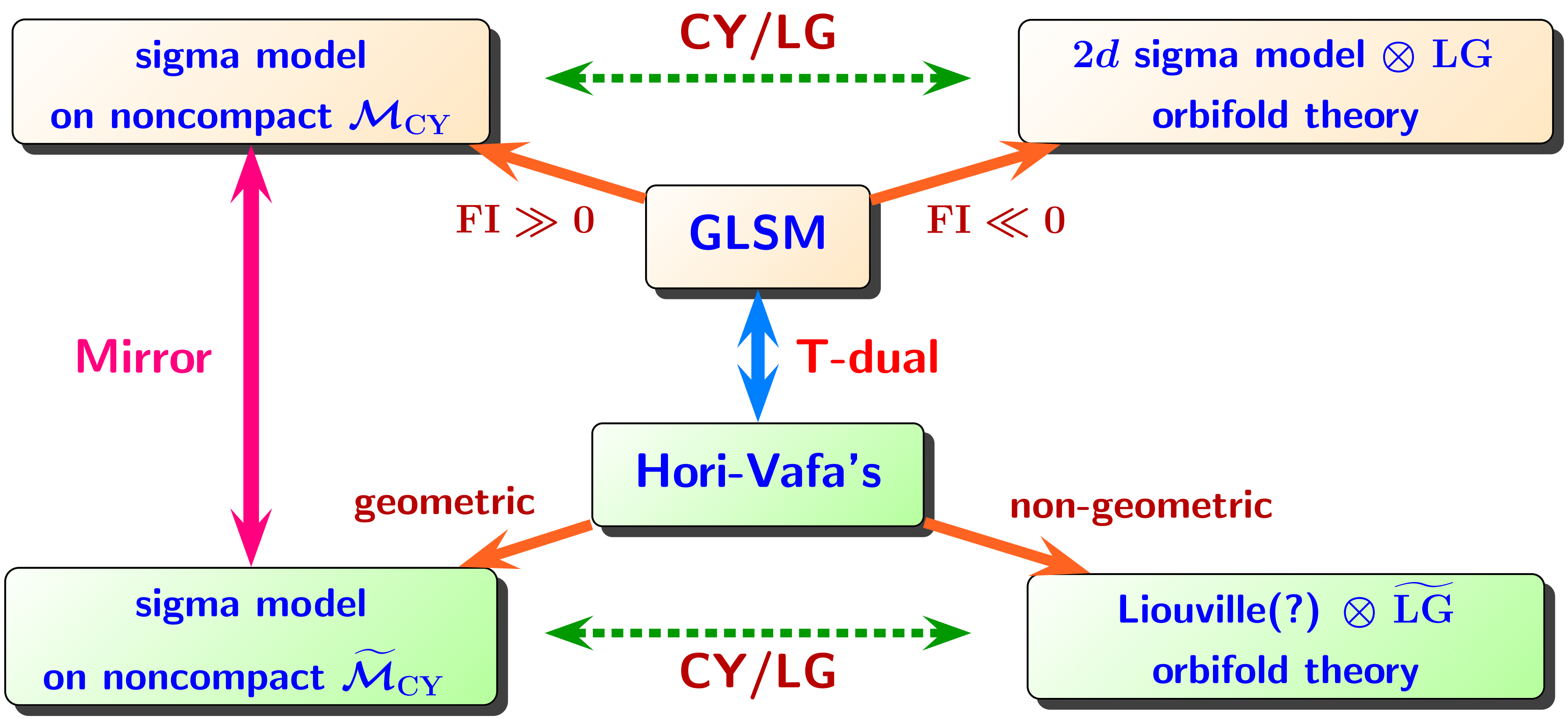
$$\left(\mathbb{C} \ltimes\right) \frac{SO(N)}{SO(N-2) \times U(1)}$$

$$\Downarrow$$

$SO(N)$ symmetry on $G_{\ell=2}(S)$

My Present Study

- 2. mirror symmetry on noncompact CY



Landau-Ginzburg theory

well-defined LG superpotential

$$W(\Phi)|_{\Phi=0} = \left.\frac{\partial W}{\partial \Phi^i}\right|_{\Phi=0} = 0 \iff \text{existence of the critical point at } \Phi^i = 0$$

$$\left.\det_{i,j}\left(\frac{\partial^2 W}{\partial \Phi^i \partial \Phi^j}\right)\right|_{\Phi=0} = 0$$

$$\rho \equiv \det_{i,j}\left(\frac{\partial^2 W}{\partial \Phi^i \partial \Phi^j}\right) \neq 0 \iff \text{existence of isolated singularity}$$

$$W(\lambda^{\omega_i}\Phi^i) = \lambda^{+1} W(\Phi^i) \quad \text{where} \quad h_i = \bar{h}_i = \frac{\omega_i}{2} > 0$$

A_k type singularity: (level $= k-1$)

$$W = x^{k+1}, \quad k \geq 1$$

$$\omega_x = \tfrac{1}{k+1}$$

$$p_x = 1, \quad N = k+1$$

$$\tfrac{\partial W}{\partial x} = (k+1)x^k$$

$$\mathscr{R} = \{1, x, x^2, \cdots, x^{k-1}\}$$

$$\mu = \left(\tfrac{1}{\omega_x} - 1\right) = k = \dim \mathscr{R}$$

$$\beta = \tfrac{1}{2} - \omega_x = \tfrac{1}{2} - \tfrac{1}{k+1}$$

$$c = 6\beta = 3 - \tfrac{6}{k+1}$$

field	dimension
1	$\frac{0}{k+1}$
x	$\frac{1}{k+1}$
$\vdots$	$\vdots$
x^{k-1}	$\frac{k-1}{k+1}$

NS chiral primary

$\mathcal{N}=(2,2)$ **superconformal algebra:**

$$
\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n} \\
[L_m, G_r^\pm] &= \Big(\frac{n}{2}-r\Big)G_{n+r}^\pm \\
[L_m, J_n] &= -nJ_{m+n} \\
[J_m, J_n] &= \frac{c}{3}m\delta_{m+n} \\
[J_n, G_r^\pm] &= \pm G_{n+r}^\pm \\
\{G_r^-, G_s^+\} &= 2L_{r+s} - (r-s)J_{r+s} + \frac{c}{3}\Big(r^2-\frac{1}{4}\Big)\delta_{r+s}
\end{aligned}
$$

conformal vacuum:

$$
L_m|\,0\,\rangle = 0 \quad \text{for} \quad m \geq -1\,, \qquad G_r^\pm|\,0\,\rangle = 0 \quad \text{for} \quad \begin{cases} r \geq -\frac{1}{2} & \text{(NS sector)} \\ r \geq -1 & \text{(R sector)} \end{cases}
$$

primary field:

$$\phi'(z',\overline{z}') = \Big(\frac{\partial z}{\partial z'}\Big)^{h}\Big(\frac{\partial \overline{z}}{\partial \overline{z}'}\Big)^{\overline{h}}\phi(z,\overline{z})$$

primary state:

$$|\,\phi\,\rangle = \lim_{z,\overline{z}\to 0}\phi(z,\overline{z})|\,0\,\rangle$$

$$L_0|\,\phi\,\rangle = h|\,\phi\,\rangle\,, \quad L_m|\,\phi\,\rangle = 0 \quad \text{for} \quad m > 0$$

$$J_0|\,\phi\,\rangle = q|\,\phi\,\rangle\,, \quad J_m|\,\phi\,\rangle = 0 \quad \text{for} \quad m > 0$$

NS chiral primary state:

$$G^{+}_{n-1/2}|\,\phi\,\rangle = G^{-}_{n+1/2}|\,\phi\,\rangle = 0 \quad \text{for} \quad n \geq 0\,, \qquad \text{left chiral}$$

$$G^{+}_{n+1/2}|\,\phi\,\rangle = G^{-}_{n-1/2}|\,\phi\,\rangle = 0 \quad \text{for} \quad n \geq 0\,, \qquad \text{left anti-chiral}$$

$$\overline{G}^{+}_{n-1/2}|\,\phi\,\rangle = \overline{G}^{-}_{n+1/2}|\,\phi\,\rangle = 0 \quad \text{for} \quad n \geq 0\,, \qquad \text{right chiral}$$

$$\overline{G}^{+}_{n+1/2}|\,\phi\,\rangle = \overline{G}^{-}_{n-1/2}|\,\phi\,\rangle = 0 \quad \text{for} \quad n \geq 0\,, \qquad \text{right anti-chiral}$$

$$0 = \{G^{-}_{1/2}, G^{+}_{-1/2}\}|\,\phi\,\rangle = (2L_0 - J_0)|\,\phi\,\rangle \quad \longrightarrow \quad h = \frac{1}{2}q$$

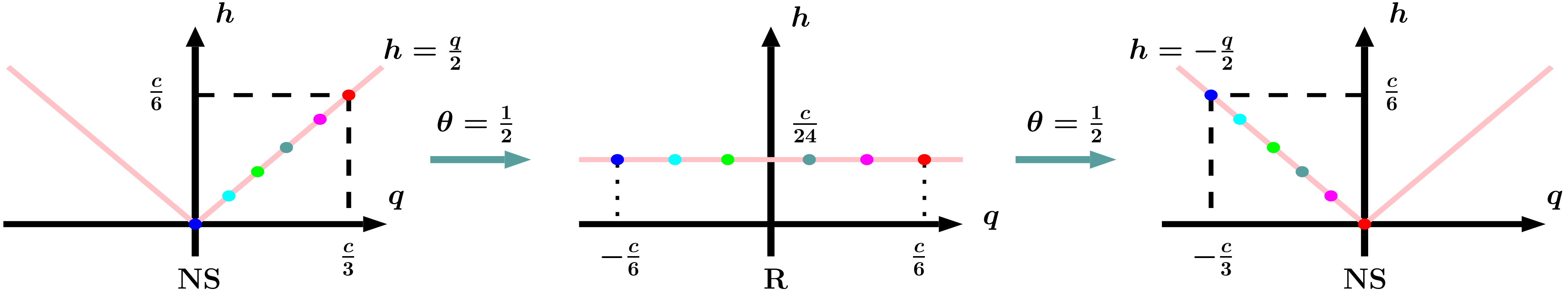
Spectral flow

W.Lerche, C.Vafa and N.P.Warner, Nucl. Phys. B324 (1989) 427.

$$L_n^\theta = \mathscr{U}_\theta L_n \mathscr{U}_\theta^{-1} = L_n + \theta J_n + \frac{c}{6}\theta^2 \delta_{n,0} , \qquad J_n^\theta = \mathscr{U}_\theta J_n \mathscr{U}_\theta^{-1} = J_n + \frac{c}{3}\theta\delta_{n,0}$$

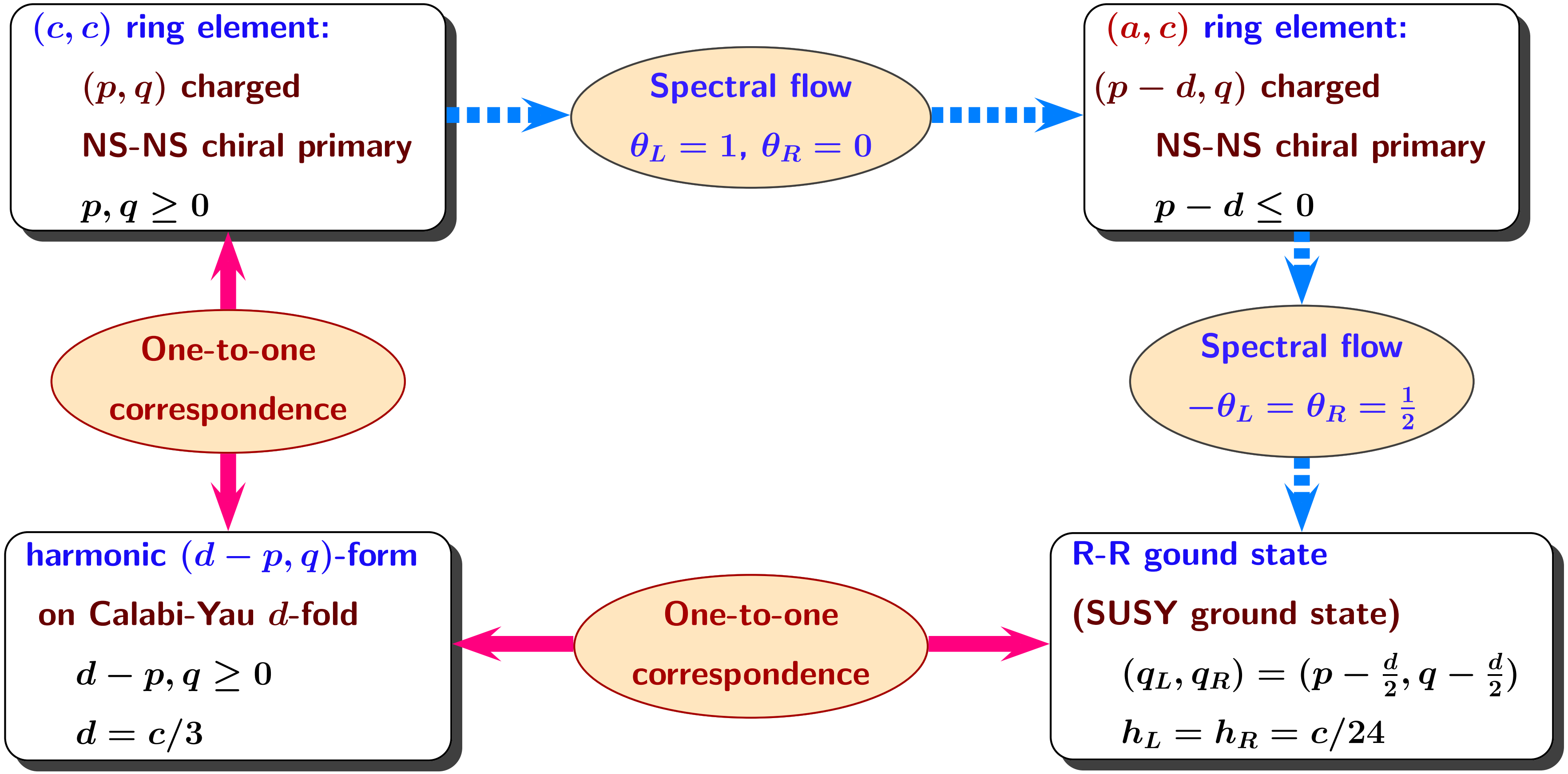
$$G_r^{\pm,\theta} = \mathscr{U}_\theta G_r{}^{\pm} \mathscr{U}_\theta^{-1} = G^{\pm}_{r\pm\theta}$$

$$h_\theta = h - q\theta + \frac{c}{6}\theta^2 , \qquad q_\theta = q - \frac{c}{3}\theta$$



Neveu-Schwarz	spectral flow	Ramond
$(h = \frac{q}{2}, q)$ chiral primary state	$\longrightarrow (\theta = \frac{1}{2}) \longrightarrow$	$(c/24, q - c/6)$ R ground state

$\mathcal{N} = (2,2)$ SCFT and Calabi-Yau geometry



NS-NS chiral primary states, R-R ground states と harmonic formsの相関