Seminar at Komaba, University of Tokyo; *October 7th, 2004*

Towards Mirror Symmetry on Noncompact Calabi-Yau Manifolds

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Preprint: hep-th/0409003 (Proceeding of SUSY 2004)

References: Witten, hep-th/9301042

Hori and Vafa, hep-th/0002222

Higashijima, Nitta and TK, hep-th/0110216, 0202064

Introduction

String theory is a powerful framework

- Perturbative calculation around smooth spacetime backgrounds
- (Ultimately) unified theory

  gravitational/strong/weak/electromagnetic interactions
- Possibility to generate 4-dim. spacetime

  heterotic/IIA/IIB string on compact Calabi-Yau 3-fold, on D3-branes, etc.
- Mirror Symmetry between string theories on different CY's

  unification of various SCFTs and (algebraic) geometries

Gauged linear sigma model

E. Witten (1993), K. Hori and C. Vafa (2000)

$\mathcal{N} = (2,2)$ SUSY gauge theory with matters $(\mathrm{FI}: \ t \equiv r - i\theta)$

$$\mathscr{L} = \int \mathrm{d}^4\theta \Big\{ -\frac{1}{e^2}\overline{\Sigma}\Sigma + \sum_a \overline{\Phi}_a \, \mathrm{e}^{2Q_a V} \Phi_a \Big\} + \Big(\frac{1}{\sqrt{2}} \int \mathrm{d}^2\widetilde{\theta} \, (-\Sigma \, t) + c.c. \Big) + \Big(\int \mathrm{d}^2\theta \, W_{\mathrm{GLSM}}(\Phi_a) + c.c. \Big)$$

- Φ_a : charged chiral superfield, $\overline{D}_{\pm}\Phi_a = 0$
  Σ : twisted chiral superfield, $\overline{D}_{+}\Sigma = D_{-}\Sigma = 0$, $\Sigma = \frac{1}{\sqrt{2}}\overline{D}_{+}D_{-}V$
- 2つの phase:

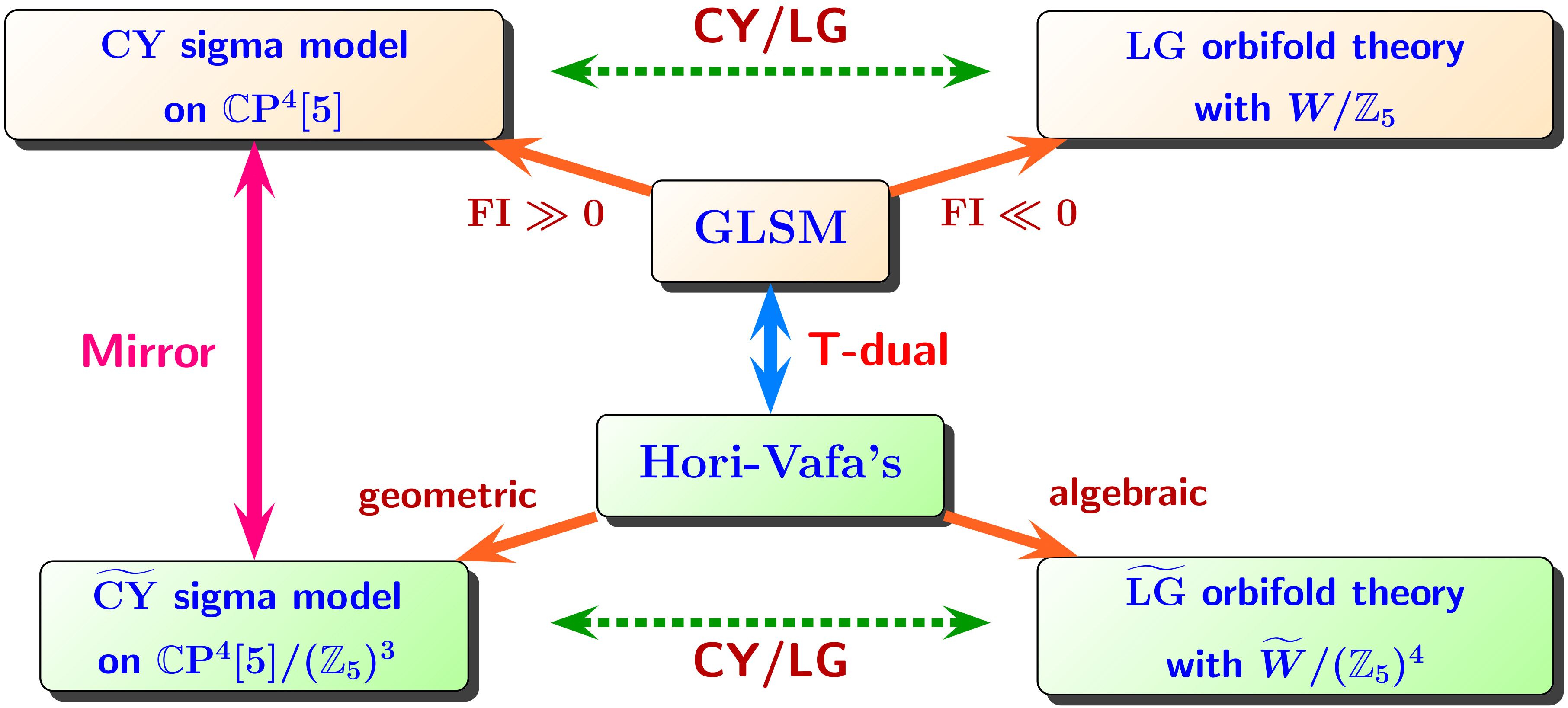
  $\mathrm{FI} \gg 0$: differential-geometric phase $\longrightarrow$ SUSY NLSM

  $\mathrm{FI} \ll 0$: algebro-gemetric phase $\longrightarrow$ LG, orbifold, SCFT

- LG model から CY 多様体の位相的情報を引き出せる (CY/LG対応)

  harmonic forms $\longleftrightarrow$ NS-NS chiral primary states

- GLSM の T-dual model の IR limit には "Mirror" geometry が登場



$$W = X_1^5 + X_2^5 + X_3^5 + X_4^5 + X_5^5$$

$$\widetilde{W} = \widetilde{X}_1^5 + \widetilde{X}_2^5 + \widetilde{X}_3^5 + \widetilde{X}_4^5 + \widetilde{X}_5^5 + e^{t/5}\widetilde{X}_1\widetilde{X}_2\widetilde{X}_3\widetilde{X}_4\widetilde{X}_5$$

$$h_{21}(\mathbb{C}P^4[5]) = h_{11}(\mathbb{C}P^4[5]/(\mathbb{Z}_5)^3) = 101 \,, \qquad h_{11}(\mathbb{C}P^4[5]) = h_{21}(\mathbb{C}P^4[5]/(\mathbb{Z}_5)^3) = 1$$

Towards noncompact Calabi-Yau

inspired by Seiberg-Witten theory for 4-dim. $\mathcal{N}=2$ $SU(2)$ SYM

- **Powerful tool to understand nonperturbative effects in field theories**

  massless particles coming from singularity in CY

  lift up to M-theory on (non)singular manifolds with special holonomies

- **We can construct metrics on noncompact CY**

  research supersymmetric NLSMs as one-loop finite models

Metrics on Noncompact Calabi-Yau

(K.Higashijima, M.Nitta and TK, 2001, 2002)

line bundles	total dim. D	dual Coxeter C	"orbifolding" ℓ
$\mathbb{C} \ltimes \left(\mathbb{C}\mathrm{P}^{N-1} = \frac{SU(N)}{SU(N-1)\times U(1)}\right)$	$1+(N-1)$	N	N
$\mathbb{C} \ltimes \left(Q^{N-2} = \frac{SO(N)}{SO(N-2)\times U(1)}\right)$	$1+(N-2)$	$N-2$	$N-2$
$\mathbb{C} \ltimes E_6/[SO(10)\times U(1)]$	$1+16$	12	12
$\mathbb{C} \ltimes E_7/[E_6\times U(1)]$	$1+27$	18	18
$\mathbb{C} \ltimes \left(\mathrm{G}_{N,M} = \frac{U(N)}{U(N-M)\times U(M)}\right)$	$1+M(N-M)$	N	MN
$\mathbb{C} \ltimes SO(2N)/U(N)$	$1+\frac{1}{2}N(N-1)$	$N-1$	$N(N-1)$
$\mathbb{C} \ltimes Sp(N)/U(N)$	$1+\frac{1}{2}N(N+1)$	$N+1$	$N(N+1)$

$$\mathcal{K}'_{\mathrm{noncompact}}(\rho,\varphi) \;=\; \left(\mathrm{e}^{CX}+b\right)^{1/D}, \qquad X \;=\; \log|\rho^{1/\ell}|^2 + K_{\mathrm{compact}}(\varphi)$$

$$K_{\mathbb{C}\mathrm{P}^{N-1}}(\varphi) = r\log\left(1+\sum_{i=1}^{N-1}|\varphi_i|^2\right)$$

What are their mirror geometries?

Another motivation

- **Higashijima-Nitta-TK (2001∼2002):**

SUSY nonlinear sigma models = geometric description of noncompact CY

How about SCFT description?

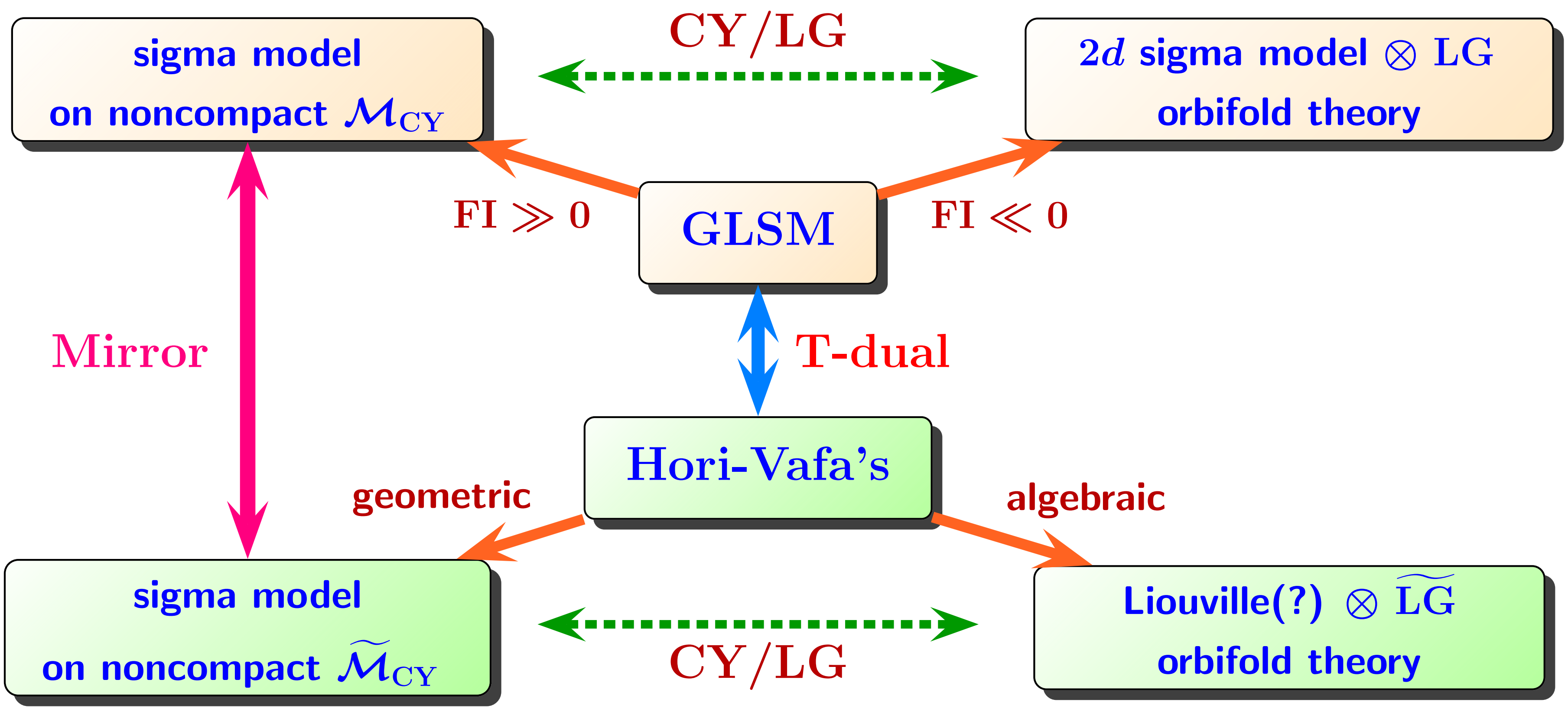
- **Giveon-Kutasov, Eguchi-Sugawara (1999∼), etc.:**

$\mathcal{N}=2$ **Liouville** $\otimes$ **Landau-Ginzburg = SCFT description of noncompact CY**

Why direct product? (holographic picture of $\mathbb{R}^{d-1,1}\times$ (non)singular CY)

↓

- **T. Kimura (2002∼): GLSM for $\mathcal{O}(-N+\ell)$ bundle on $\mathbb{CP}^{N-1}[\ell]$**



Some problems:

- There are no precise methods to calculate invariants (Gromov-Witten, etc.) for general CY
- There appear many vacua... ($\leftarrow$ now studying whether they disappear via nonperturbative effects)

Gauged Linear Sigma Model for $\mathcal{O}(-N+\ell)$ bundle on $\mathbb{C}\mathrm{P}^{N-1}[\ell]$

chiral superfield	S_1	$\cdots$	S_N	P_1	P_2
$U(1)$ charge	1	$\cdots$	1	$-\ell$	$-N+\ell$

$$W_{\mathrm{GLSM}} = P_1 \cdot G_\ell(S_i)$$

$G_\ell(S_i)$: homogeneous polynomial of degree ℓ

potential energy density:

$$\begin{aligned}\mathcal{U} &= \frac{e^2}{2}\Big[r - \sum_{i=1}^{N}|s_i|^2 + \ell|p_1|^2 + (N-\ell)|p_2|^2\Big]^2 \\ &\quad + 2|\sigma|^2\Big\{\sum_{i=1}^{N}|s_i|^2 + \ell^2|p_1|^2 + (N-\ell)^2|p_2|^2\Big\} \\ &\quad + \big|G_\ell(s_i)\big|^2 + |p_1|^2 \cdot \sum_{i=1}^{N}\big|\partial_i G_\ell(s_j)\big|^2\end{aligned}$$

Let us analyze classical SUSY vacuum manifold $\mathcal{U}=0$ and perturbation theories

▼ $\ell = 1$ case: already well-known model

GLSM reduces to $\begin{cases} r \gg 0 : \mathcal{O}(-N+1) \text{ bundle on } \mathbb{C}\mathrm{P}^{N-2} \\ r \ll 0 : \mathbb{C}^{N-1}/\mathbb{Z}_{N-1} \text{ orbifold theory} \end{cases}$

$2 \leq \ell \leq N-1$ cases are nontrivial

- There appear many classical vacua in $r \ll 0$ phase
- Some of them are not well-known.

▼ classical SUSY vacuum manifold $\mathcal{M}_{\rm CY}$ in $r \gg 0$ phase $(2 \le \ell \le N-1)$:

$$\mathcal{M}_{\rm CY} \ = \ \Big\{ (S_i, P_2) \in \mathbb{C}^{N+1} \, \Big| \, r = \sum_{i=1}^{N} |S_i|^2 - (N-\ell)|P_2|^2, \ \ G_\ell(S_i) = 0 \Big\} \Big/ U(1)$$

➡ $\mathcal{O}(-N+\ell)$ bundle on $\mathbb{C}\mathrm{P}^{N-1}[\ell]$

ゲージ場及び $\mathcal{M}_{\rm CY}$ に沿わない場は質量 $m^2 \propto e^2 r$ を獲得する

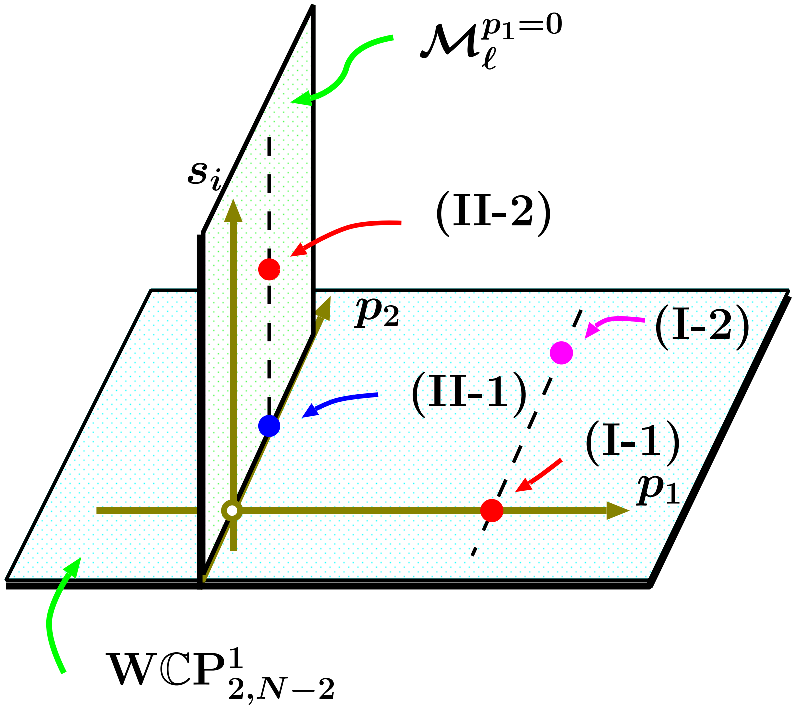
(Higgs mechanism)

IR limit ($e \to \infty$) ではこれらの有質量場はダイナミクスを持たない

ゼロ質量のみで構成される effective theory は

$\mathcal{N} = (2,2)$ SUSY nonlinear sigma model on $\mathcal{M}_{\rm CY}$

▼ classical SUSY vacuum manifold in $r \ll 0$ phase $(\ell = 2)$:



(I-1) : at $\langle p_1 \rangle \neq 0$, $\langle p_2 \rangle = 0$, $\langle s_i \rangle = 0$, massless DOF = $N+1$
sigma model on $\mathbb{C}^1/\mathbb{Z}_2$

(I-2) : at $\langle p_1 \rangle \neq 0$, $\langle p_2 \rangle \neq 0$, $\langle s_i \rangle = 0$, massless DOF = $N+1$
sigma model on $\mathbb{C}^1/\mathbb{Z}_\alpha$, $\quad \alpha = \mathrm{GCM}\{2, N-2\}$

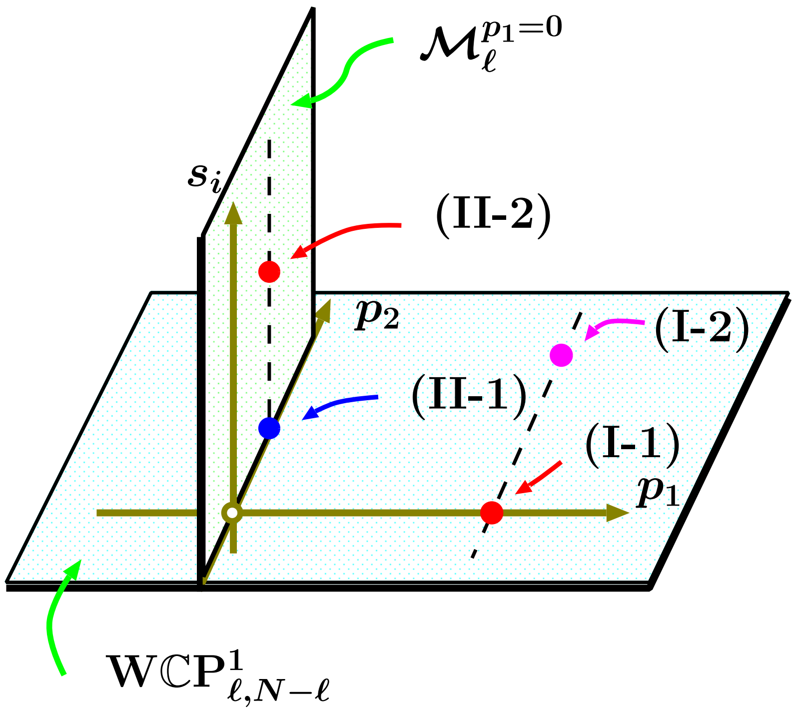
(II-1) : at $\langle p_1 \rangle = 0$, $\langle p_2 \rangle \neq 0$, $\langle s_i \rangle = 0$, massless DOF = $N+1$
$\{$LG theory with $W_{\mathrm{LG}} = P_1 \cdot G_2(S)\}/\mathbb{Z}_{N-2}$

(II-2) : at $\langle p_1 \rangle = 0$, $\langle p_2 \rangle \neq 0$, $\langle s_i \rangle \neq 0$, massless DOF = $N-1$
sigma model on $\mathcal{M}_2^{p_1=0}$

$$\mathrm{W}\mathbb{C}\mathrm{P}^1_{2,N-2} \;=\; \Big\{ (p_1, p_2) \in \mathbb{C}^2 \,\Big|\, -r = \ell |p_1|^2 + (N-2)|p_2|^2 \Big\} \;\simeq\; \mathbb{C}\mathrm{P}^1$$

$$\mathcal{M}_{\ell=2}^{p_1=0} \;=\; \Big\{ (p_2, s_i) \in \mathbb{C}^* \times \mathbb{C}^N \,\Big|\, r - \sum_{i=1}^N |s_i|^2 + (N-2)|p_2|^2 = 0 \,,\; G_2(s_i) = 0 \Big\}$$

classical SUSY vacuum manifold in $r \ll 0$ phase $(3 \leq \ell \leq N-1)$:



(I-1) : at $\langle p_1\rangle \neq 0$, $\langle p_2\rangle = 0$, $\langle s_i\rangle = 0$, massless DOF $= N+1$

$$\left\{ \begin{array}{l} (\text{sigma model on } \mathbb{C}^1) \\ \otimes\ (\text{LG theory with } W_{\mathrm{LG}} = \langle p_1\rangle G_\ell(S)) \end{array} \right\} \Big/ \mathbb{Z}_\ell$$

(I-2) : at $\langle p_1\rangle \neq 0$, $\langle p_2\rangle \neq 0$, $\langle s_i\rangle = 0$, massless DOF $= N+1$

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$\alpha = \mathrm{GCM}\{\ell, N-\ell\}$

(II-1) : at $\langle p_1\rangle = 0$, $\langle p_2\rangle \neq 0$, $\langle s_i\rangle = 0$, massless DOF $= N+1$

$\{\text{LG theory with } W_{\mathrm{LG}} = P_1 \cdot G_\ell(S)\}/\mathbb{Z}_{N-\ell}$

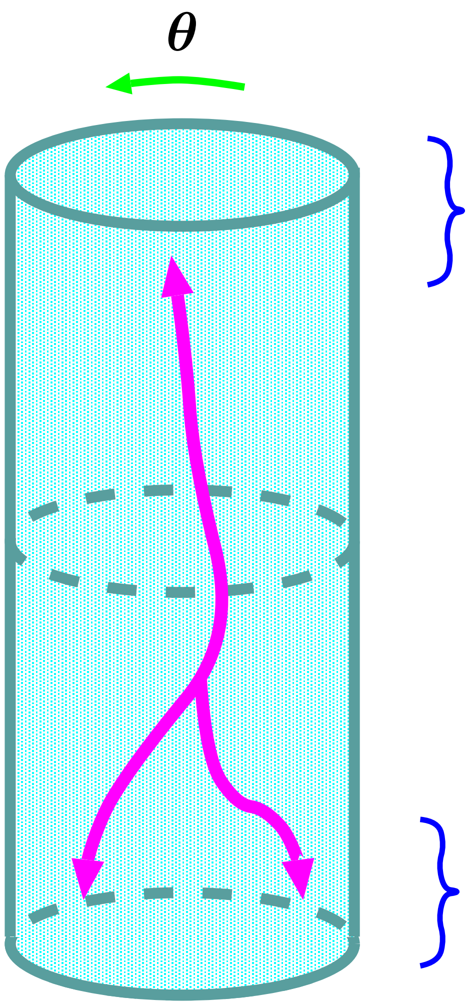
(II-2) : at $\langle p_1\rangle = 0$, $\langle p_2\rangle \neq 0$, $\langle s_i\rangle \neq 0$, massless DOF $= N-1$

sigma model on $\mathcal{M}_\ell^{p_1=0}$

$$\mathrm{W}\mathbb{C}\mathrm{P}^1_{\ell,N-\ell} = \left\{ (p_1, p_2) \in \mathbb{C}^2 \,\middle|\, r = -\ell|p_1|^2 - (N-\ell)|p_2|^2 \right\} \simeq \mathbb{C}\mathrm{P}^1$$

$$\mathcal{M}_\ell^{p_1=0} = \left\{ (p_2, s_i) \in \mathbb{C}^* \times \mathbb{C}^N \,\middle|\, r - \sum_{i=1}^N |s_i|^2 + (N-\ell)|p_2|^2 = 0\,,\ G_\ell(s_i) = 0 \right\}$$

CY/LG correspondence



CY sigma model on

$\mathcal{O}(-N+\ell)$ **bundle on** $\mathbb{CP}^{N-1}[\ell]$

classically, not one-to-one (except for $\ell = 1, N$)

one-to-one correspondence via quantum effects?

duality?

(now in progress)

massless effective theories on

(I-1), (I-2), (II-1), (II-2)

T-dualized Theory

$$\mathscr{L} = \int d^4\theta \Big\{ -\frac{1}{e^2}\overline{\Sigma}\Sigma - \sum_a \Big(\frac{1}{2}(Y_a + \overline{Y}_a)\log(Y_a + \overline{Y}_a)\Big)\Big\} + \Big(\frac{1}{\sqrt{2}}\int d^2\widetilde{\theta}\,\widetilde{W} + c.c.\Big)$$

$$\widetilde{W} = \Sigma\Big(\sum_{i=1}^{N} Y_i - \ell Y_{P_1} - (N-\ell)Y_{P_2} - t\Big) + \sum_{i=1}^{N} e^{-Y_i} + e^{-Y_{P_1}} + e^{-Y_{P_2}}$$

chiral superfield	Φ_a	S_1	S_2	$\cdots$	S_N	P_1	P_2
$U(1)$ charge	Q_a	1	1	$\cdots$	1	$-\ell$	$-N+\ell$
twisted chiral	Y_a	Y_1	Y_2	$\cdots$	Y_N	Y_{P_1}	Y_{P_2}

relation between chiral superfields $\{\Phi_a\}$ and twisted chiral superfields $\{Y_a\}$:

$$2\overline{\Phi}_a e^{2Q_a V}\Phi_a = Y_a + \overline{Y}_a$$

$U(1)$ phase rotation symmetry on Φ_a ➡ shift symmetry on Y_a:

$$Y_a \to Y_a + 2\pi i$$

A powerful tool = "period integral":

$$\widehat{\Pi} \;\equiv\; \int \mathrm{d}\Sigma \prod_{i=1}^{N} \mathrm{d}Y_i\, \mathrm{d}Y_{P_1}\, \mathrm{d}Y_{P_2}\, (\ell\Sigma)\; \exp\big(- \widetilde{W}\big)$$

topological A-twisted sector のみを追う量

一言でいうと「partition function of topological A-model with $W_{\mathrm{GLSM}} = P_1 \cdot G_\ell(S_i)$」

Hori-Vafa の "proof of mirror symmetry" の不十分な点

しかし $G_\ell(S_i)$ に余分な条件がなければこの量で十分

$$\ell\Sigma \;\longrightarrow\; \begin{cases} \ell \dfrac{\partial}{\partial t} : \text{ twisted LG theory を追う} \\[2ex] \dfrac{\partial}{\partial Y_{P_1}} : \text{ T-dualized geometry を追う} \end{cases}$$

$\mathbb{Z}_\ell$ orbifold LG theory

IR limit $(e \to \infty)$ では Σ が freeze $\to Y_{P_1}$ を他の場で書き直す:

$$Y_{P_1} = \frac{1}{\ell}\Big\{t - \sum_{i=1}^{N} Y_i + (N-\ell)Y_{P_2}\Big\}$$

"period integral" $\widehat{\Pi}$ が canonical measure を保つ様に場を再定義:

$$X_i \equiv \mathrm{e}^{-\frac{1}{\ell}Y_i}\,, \quad X_{P_2} \equiv \mathrm{e}^{\frac{N-\ell}{\ell}Y_{P_2}}\,, \quad X_i \to \omega_i X_i\,, \quad X_{P_2} \to \omega_{P_2} X_{P_2}\,,$$ $(\mathbb{Z}_\ell)^N$ symmetry

すると twisted chiral で記述される LG superpotential が得られる:

$$\Big\{\widetilde{W}_\ell = X_1^\ell + \cdots + X_N^\ell + X_{P_2}^{-\frac{\ell}{N-\ell}} + \mathrm{e}^{t/\ell} X_1 \cdots X_N X_{P_2}\Big\}\Big/(\mathbb{Z}_\ell)^N$$

負ベキの項は「$\mathcal{N}=2$ Liouville potential」と同定可能:

$$\frac{\ell}{N-\ell} = k = \frac{2}{Q^2}$$

よって LG superpotential は次の様に見做される

$$\mathcal{N}=2 \text{ Liouville potential} \otimes \Big\{\widetilde{W}_\ell = \sum_{i=1}^{N} X_i^\ell + \mathrm{e}^{t/\ell} X_1 \cdots X_N \cdot \mathrm{e}^{\frac{N-\ell}{\ell}Y_{P_2}}\Big\}\Big/(\mathbb{Z}_\ell)^{N-1}$$

$\mathbb{Z}_\ell$ orbifold geometry

導出が複雑なので結果のみ記す

$$\widetilde{\mathcal{M}}_\ell = \Big\{\{\mathcal{F}(Z_i) = 0\}/\mathbb{C}^*\,,\ \mathcal{G}(Z_b;u,v) = 0\Big\}\Big/(\mathbb{Z}_\ell)^{N-2}$$

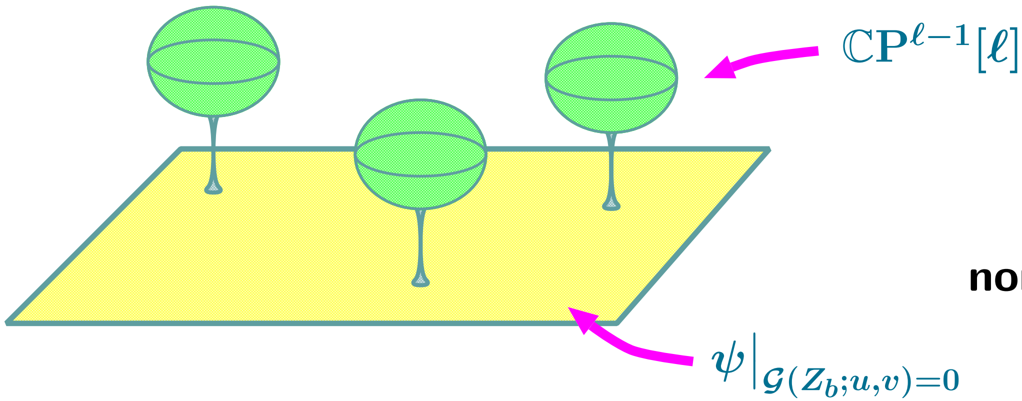
$$\mathcal{F}(Z_i) = Z_1^\ell + \cdots + Z_\ell^\ell + \psi Z_1\cdots Z_\ell\,,\qquad \psi = \mathrm{e}^{t/\ell} Z_{\ell+1}\cdots Z_N$$

$$\mathcal{G}(Z_b;u,v) = Z_{\ell+1}^\ell + \cdots + Z_N^\ell + 1 - uv$$

$Z_a \mapsto \lambda\,\omega_a\,Z_a$ for $a = 1,\cdots,\ell$ (homogeneous coordinates of $\mathbb{C}\mathrm{P}^{\ell-1}[\ell]$)

$Z_b \mapsto \omega_b\,Z_b$ for $b = \ell+1,\cdots,N$ (homogeneous coordinates of $\mathbb{C}^{N-\ell}$)

$$\omega_a^\ell = \omega_b^\ell = \omega_1\cdots\omega_N = 1\,,\qquad \lambda : \mathbb{C}^*\text{-value}$$



total dim.: $(N+2)-1-1-1 = N-1$

compact 方向: $\ell - 2$

noncompact 方向: $N - \ell + 1$

$\mathbb{Z}_{N-\ell}$ orbifold LG theory

Solve Y_{P_2} by using the constraint derived from integrating out Σ:

$$Y_{P_2} = \frac{1}{N-\ell}\Big\{t - \sum_{i=1}^{N} Y_i + \ell Y_{P_1}\Big\}$$

Field re-definition preserving canonical measure in $\widehat{\Pi}$:

$$X_i \equiv e^{-\frac{1}{N-\ell}Y_i}\,, \quad X_{P_1} \equiv e^{\frac{\ell}{N-\ell}Y_{P_1}}\,, \quad X_i \to \omega_i X_i\,, \quad X_{P_1} \to \omega_{P_1} X_{P_1}\,, \quad (\mathbb{Z}_{N-\ell})^N \text{ symmetry}$$

Thus we obtain the twisted LG superpotential:

$$\Big\{\widetilde{W}_{N-\ell} = X_1^{N-\ell} + \cdots + X_N^{N-\ell} + X_{P_1}^{-\frac{N-\ell}{\ell}} + e^{t/\ell} X_1 \cdots X_N X_{P_1}\Big\}\Big/(\mathbb{Z}_{N-\ell})^N$$

negative power term = interpreted to $\mathcal{N}=2$ Liouville potential with

$$\frac{N-\ell}{\ell} = k = \frac{2}{Q^2}$$

Thus the IR effective superpotential becomes

$$\mathcal{N}=2 \text{ Liouville potential} \otimes \Big\{\widetilde{W}_{N-\ell} = \sum_{i=1}^{N} X_i^{N-\ell} + e^{t/(N-\ell)} X_1 \cdots X_N \cdot e^{\frac{\ell}{N-\ell}Y_{P_1}}\Big\}\Big/(\mathbb{Z}_{N-\ell})^{N-1}$$

$\mathbb{Z}_{N-\ell}$ orbifold geometry

導出が複雑なので結果のみ記す

$$\widetilde{\mathcal{M}}_{N-\ell} = \left\{F(Z_a) = 0\,,\ \{G(Z_i;u,v) = 0\}/\mathbb{C}^*\right\}\Big/(\mathbb{Z}_{N-\ell})^{N-2}$$

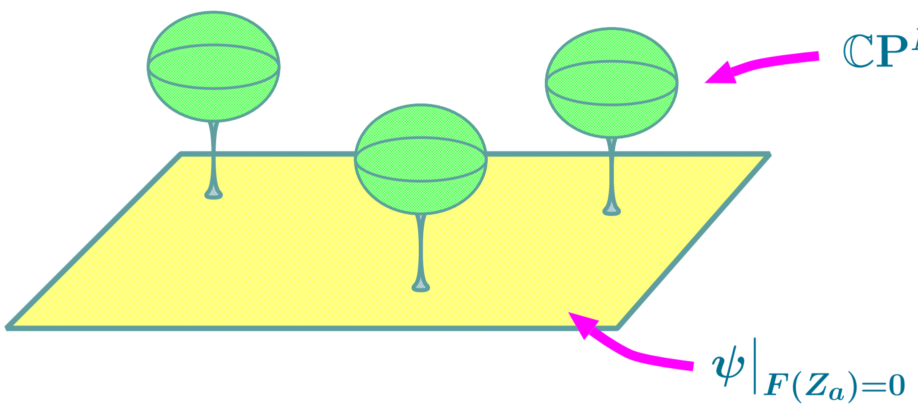
$$F(Z_a) = Z_1^{N-\ell} + \cdots + Z_\ell^{N-\ell} + 1$$

$$G(Z_i;u,v) = Z_{\ell+1}^{N-\ell} + \cdots + Z_N^{N-\ell} + \psi Z_{\ell+1}\cdots Z_N\,, \quad \psi = (\mathrm{e}^{t/(N-\ell)} - uv) Z_1 \cdots Z_\ell$$

$Z_a \mapsto \omega_a Z_a$ for $a = 1, \cdots, \ell$ (homogeneous coordinates of $\mathbb{C}^\ell$)

$Z_b \mapsto \lambda\, \omega_b Z_b$ for $b = \ell+1, \cdots, N$ (homogeneous coordinates of $\mathbb{CP}^{N-\ell-1}[N-\ell]$)

$$\omega_a^{N-\ell} = \omega_b^{N-\ell} = \omega_1 \cdots \omega_N = 1\,, \qquad \lambda : \mathbb{C}^*\text{-value}$$



total dim.: $(N+2) - 1 - 1 - 1 = N - 1$

compact 方向: $N - \ell - 2$

noncompact 方向: $\ell + 1$

mirror CY_2 $(N=3)$		$\ell=1$	$\ell=2$
$\mathbb{Z}_\ell$	cpt. dim.	–	0
	noncpt. dim.	–	2
$\mathbb{Z}_{N-\ell}$	cpt. dim.	0	–
	noncpt. dim.	2	–

mirror CY_3 $(N=4)$		$\ell=1$	$\ell=2$	$\ell=3$
$\mathbb{Z}_\ell$	cpt. dim.	–	0	1
	noncpt. dim.	–	3	2
$\mathbb{Z}_{N-\ell}$	cpt. dim.	1	0	–
	noncpt. dim.	2	3	–

mirror CY_4 $(N=5)$		$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$
$\mathbb{Z}_\ell$	cpt. dim.	–	0	1	2
	noncpt. dim.	–	4	3	2
$\mathbb{Z}_{N-\ell}$	cpt. dim.	2	1	0	–
	noncpt. dim.	2	3	4	–

Summary

- Calabi-Yau 多様体 ($\mathcal{O}(-N+\ell)$ bundle on $\mathbb{C}\mathrm{P}^{N-1}[\ell]$) に対する GLSM から出発
  - CY phase では supersymmetric NLSM を再現
  - orbifold phase では SUSY vacua が 4 種類も登場

    それらの関係は現時点では不明 (quantum effect で持ち上がり? duality?)

- GLSM と T-dual な Lagrangian を表示
  - $\mathbb{Z}_\ell$ orbifold solution と $\mathbb{Z}_{N-\ell}$ orbifold solution が存在する
  - LG twisted superpotential には Liouville-type potential が存在
  - mirror geometry は ℓ によって振舞いが異なる

Discussions

- GLSM 側の orbifold phase に存在する 4 種類の SUSY vacua それぞれの関係は?
- mirror geometry が殆ど開いている理由は?
- topological charge などを読み取る作業が未だ不明

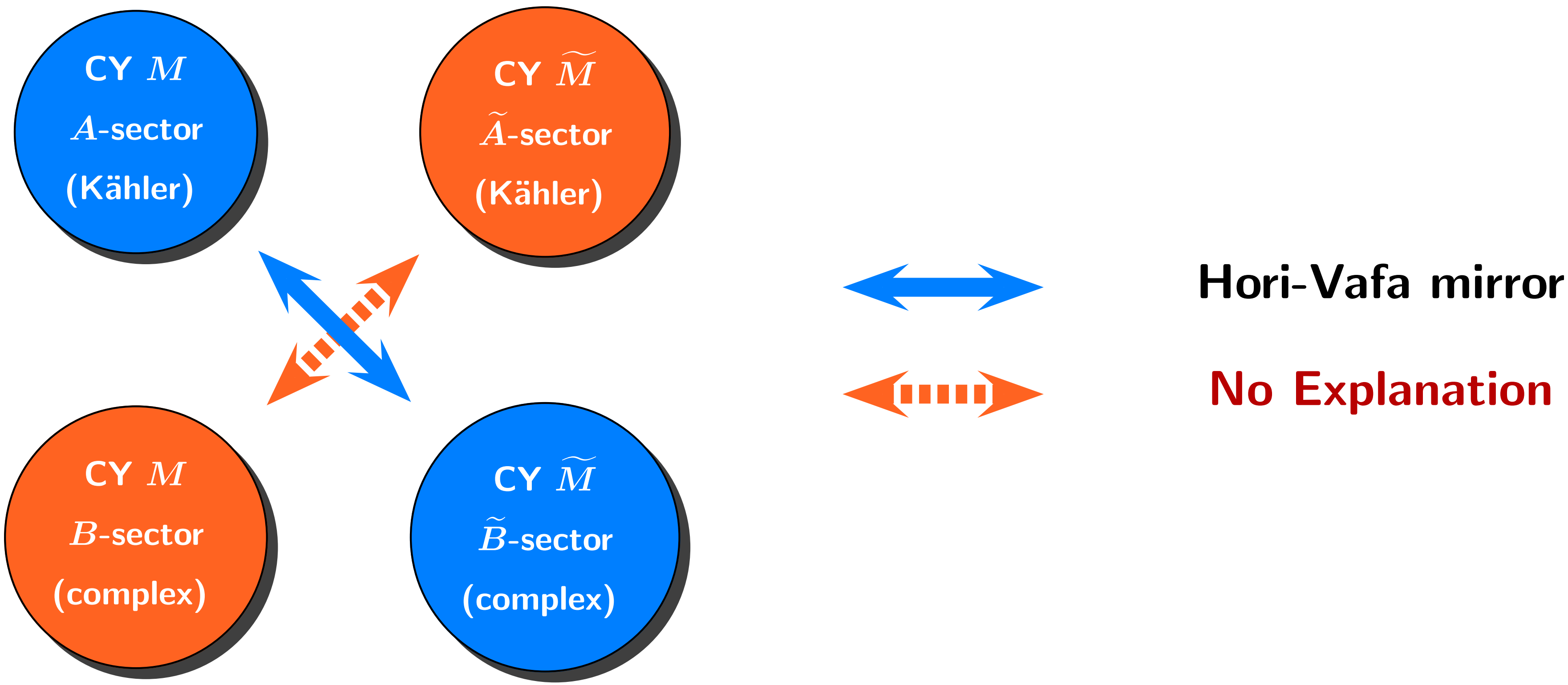
toric CY と呼ばれる物は最近 topological vertex で計算可能

- Hori-Vafa の「欠陥」を補う処方箋の開発が望まれる

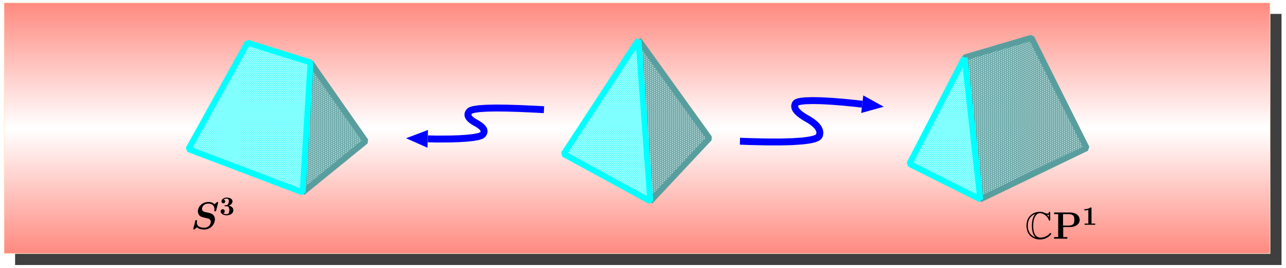
Higashijima-Nitta-TK の mirror geometry を探求するには必要不可欠

Hori-Vafa の「欠陥」とは . . .

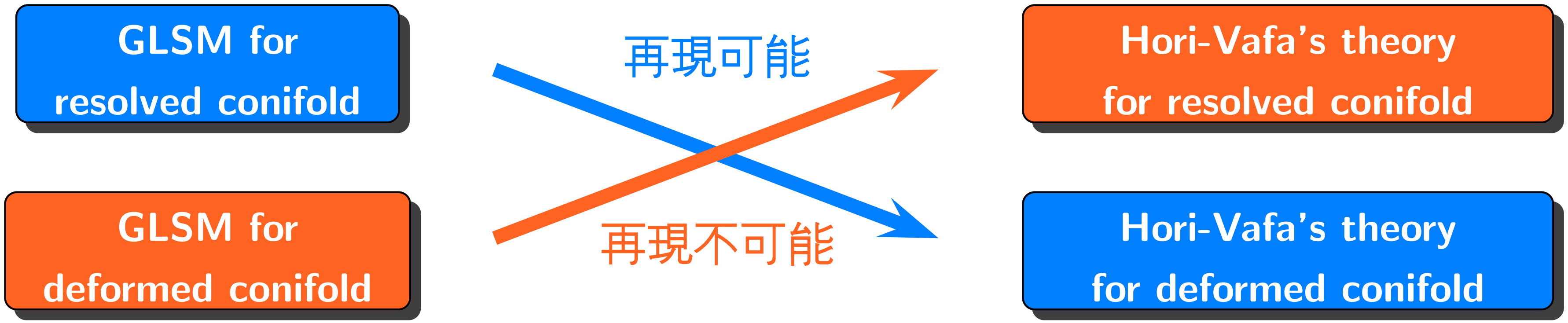
topological A-twisted sector only → How about B-sector?

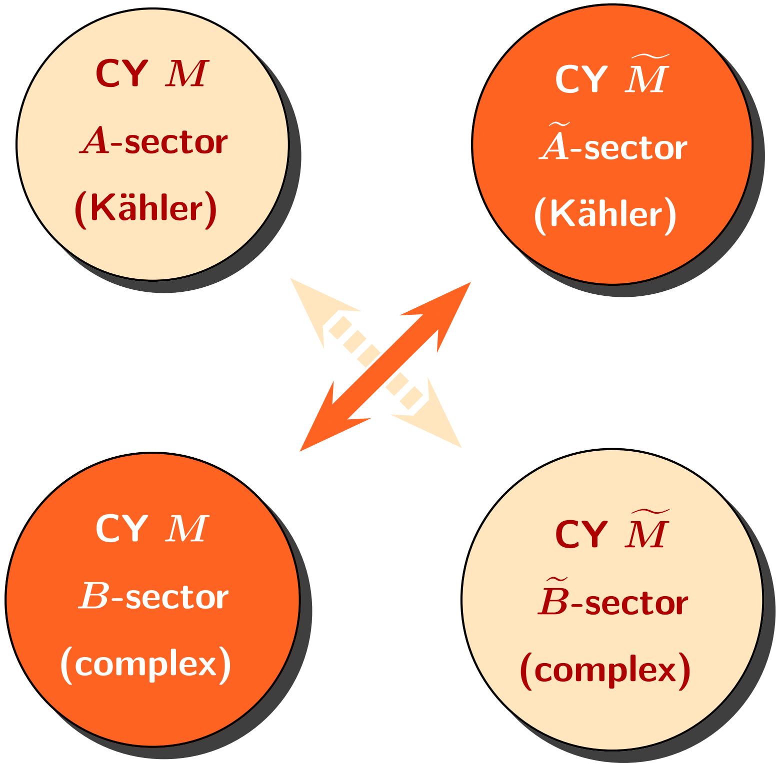


Hori-Vafa の未解決部分の例: resolved/deformed conifold



- deformed conifold: deformation of complex moduli
- resolved conifold: deformation of Kähler moduli





$W_{\text{GLSM}} = P \cdot G_\ell(S)$ に **additional** な対称性

example: sigma model on quadric surface

and its line bundle

$$\left(\mathbb{C} \ltimes\right) \frac{SO(N)}{SO(N-2) \times U(1)}$$

$$\Downarrow$$

$SO(N)$ **symmetry on** $G_{\ell=2}(S)$