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Mirror Symmetry and Two-dimensional Field Theory

木村 哲士

高エネルギー加速器研究機構 素粒子原子核研究所

大阪大学大学院 理学研究科 物理学専攻 素粒子論研究室

Our Previous Work (K.Higashijima, TK and M.Nitta, 2001, 2002)

Metric on Non-compact Calabi-Yau

non-compact CY = line bundle on $[\otimes_a (G_a/H_a)]$

Kähler potential $\mathcal{K}(X)$ $\quad X \equiv \log|\rho|^2 + \sum_a h_a K_a$

$K_a = r_a \log(1 + f(\varphi, \overline{\varphi}))$: Kähler potential of base manifold

r_a: radius of G_a/H_a $\quad h_a$: parameter ($\sim$ dual Coxeter number of G_a)

Solution ($D = \dim_{\mathbb{C}}(\otimes_a G_a/H_a) + 1$):

$$\frac{d\mathcal{K}}{dX} = \left(\lambda e^X + b\right)^{1/D}$$

Example: line bundle on hermitian symmetric space G/H $(h = h_{\mathfrak{g}}/r)$

G/H	$h_{\mathfrak{g}}$	D
$\mathbb{C}\mathrm{P}^{N-1} = \frac{SU(N)}{SU(N-1)\times U(1)}$	N	$(N-1)+1$
$\mathrm{Q}^{N-2} = \frac{SO(N)}{SO(N-2)\times U(1)}$	$N-2$	$(N-2)+1$
$E_6/[SO(10)\times U(1)]$	12	$16+1$
$E_7/[E_6\times U(1)]$	18	$27+1$
$\mathrm{G}_{N,M} = \frac{U(N)}{U(N-M)\times U(M)}$	N	$M(N-M)+1$
$SO(2N)/U(N)$	$N-1$	$\frac{1}{2}N(N-1)+1$
$Sp(N)/U(N)$	$N+1$	$\frac{1}{2}N(N+1)+1$

Next Work

- study the LG descriptions of these non-compact CY (CY/LG correspondence)
- construct **Mirror Pairs** of these CY

$\Downarrow$

Let us study Gauged Linear Sigma Models and its T-duality

(in progress...)

Gauged Linear Sigma Model

$$\mathcal{L} = \int d^4\theta \Big\{ -\sum_{a,b=1}^{k} \frac{1}{e_{a,b}^2}\overline{\Sigma}_a\Sigma_b + \sum_{i=1}^{N} \overline{\Phi}^i \, e^{2Q_i^a V_a}\, \Phi^i \Big\}$$
$$+ \Big(\int d^2\theta \, W + (h.c.) \Big) + \Big(\frac{1}{\sqrt{2}} \int d^2\widetilde{\theta} \sum_a (-t_a\Sigma_a) + (h.c.) \Big)$$

$$t_a = r_a - i\frac{\vartheta}{2\pi}$$

$$U = \sum_{a,b} \frac{e_{a,b}^2}{2}\Big(\sum_i Q_i^a |\phi^i|^2 - r_a \Big)\Big(\sum_j Q_j^b |\phi^j|^2 - r_b \Big)$$
$$+ \sum_i |Q_i^a \,\sigma_a|^2 |\phi^i|^2 + \sum_i \Big| \frac{\partial W}{\partial \phi^i} \Big|^2$$

T-dualized Model

twisted chiral: $Y_i \sim Y_i + 2\pi i, \qquad Y_i + \overline{Y}_i = 2\,\overline{\Phi}^i\,\mathrm{e}^{2Q_i^a V_a}\,\Phi^i$

Lagrangian:

$$\widetilde{\mathcal{L}} = \int \mathrm{d}^4\theta \Big\{ -\sum_{a,b} \frac{1}{e_{a,b}^2} \overline{\Sigma}_a \Sigma_b - \sum_i \Big(\frac{1}{2} (Y_i + \overline{Y}_i) \log (Y_i + \overline{Y}_i) \Big) \Big\}$$
$$+ \Big\{ \frac{1}{\sqrt{2}} \int \mathrm{d}^2 \widetilde{\theta}\, \widetilde{W} + (h.c.) \Big\}$$

$$\widetilde{W} = \sum_a \Sigma_a \Big(\sum_i Q_i^a Y_i - \tau_a \Big) + \sum_i \mathrm{e}^{-Y_i}$$

IR limit $e_a \to \infty$: Σ_a becomes non-dynamical fields

$\Rightarrow$ the IR effective theory is written only in terms of

$$\widetilde{W} = \sum_i \mathrm{e}^{-Y_i} \qquad \text{with constraint} \quad \tau_a = \sum_i Q_i^a Y_i$$

Example: $\mathcal{O}(-N)$ bundle on $\mathbb{C}P^{N-1}$, CY hypersurface in $\mathbb{C}P^{N-1}$

Contents of chiral superfields and their $U(1)$ charges:

$$\begin{aligned} \Phi^i &= \left(S^1, S^2, \cdots, S^{N-1}, S^N, P \right) \\ Q_i &= \left(1, 1, \cdots, 1, 1, -N \right) \end{aligned}$$

superpotential $W = \lambda P \cdot G_N(S^i)$; (ex.) $G_N(S^i) = \sum_{i=1}^{N} (S^i)^N$

Potential

$$U(s,p,\sigma) = \frac{1}{2e^2} D^2 + 2|\sigma|^2 \Big(\sum_{i=1}^{N} |s^i|^2 + N^2 |p|^2 \Big) + \lambda^2 \left(|p|^2 \sum_{i=1}^{N} \Big| \frac{\partial G_N}{\partial S^i} \Big|^2 + \Big| \sum_{i=1}^{N} G_N \Big|^2 \right)$$

$$\frac{1}{e^2} D = r - \sum_{i=1}^{N} |s^i|^2 + N|p|^2$$

$\lambda = 0$:

— CY phase ($r > 0$) —

$^{\exists}\langle s^i \rangle \neq 0$

CY sigma model

on $\mathcal{O}(-N)$ bundle on $\mathbb{C}P^{N-1}$

S^i: $\mathbb{C}P^{N-1}$, P: fiber

— LG phase ($r < 0$) —

$\langle p \rangle \neq 0$

$\mathbb{C}^N/\mathbb{Z}_N$ orbifold model

S^i ($i = 1, \cdots, N$): chiral fields

$\lambda = 1$:

— CY phase ($r > 0$) —

$^{\exists}\langle s^i \rangle \neq 0$

CY sigma model

on hypersurface in $\mathbb{C}P^{N-1}$

S^i: $\mathbb{C}P^{N-1}$, $G_N(S^i) = 0$

— LG phase ($r < 0$) —

$\langle p \rangle \neq 0$

LG $\mathbb{Z}_N$ orbifold model

superpotential $W' = G_N(S^i)$

$\mathcal{O}(-N)$ bundle on $\mathbb{C}\mathrm{P}^{N-1}$

$$(\phi^0, \phi^1, \cdots, \phi^{N-1})$$

$$\swarrow \qquad\qquad \searrow$$

$$\phi^0(1, \varphi^1, \cdots, \varphi^{N-1})$$
$$\varphi^i = \phi^i/\phi^0$$
$$\rho \sim (\phi^0)^N$$

$$\phi^{N-1}(\widetilde{\varphi}^0, \widetilde{\varphi}^1, \cdots, \widetilde{\varphi}^{N-2}, 1)$$
$$\widetilde{\varphi}^i = \phi^i/\phi^{N-1}$$
$$\widetilde{\rho} \sim (\phi^{N-1})^N$$

φ^i and $\widetilde{\varphi}^i$ are local coordinates of the $\mathbb{C}\mathrm{P}^{N-1}$

ρ and $\widetilde{\rho}$ are fiber coordinates

$$\varphi^i = \widetilde{\varphi}^i \cdot P \qquad\qquad P = \frac{\phi^{N-1}}{\phi^0}$$

$$\rho = \widetilde{\rho} \cdot P^{-N}$$

T-dual of line bundle on $\mathbb{CP}^{N-1}$, hypersurface in $\mathbb{CP}^{N-1}$

$$\widetilde{W} = \sum_{i=1}^{N} \mathrm{e}^{-Y_i} + \mathrm{e}^{-Y_P} \qquad \tau = \sum_{i=1}^{N} Y_i - N Y_P$$

solve the constraint:

$$X_i \equiv \mathrm{e}^{-Y_i/N} \quad (\text{for } i = 1, 2, \cdots, N) \qquad e^{-Y_P} \equiv \mathrm{e}^{\tau/N} X_1 X_2 \cdots X_N$$

$$\Longrightarrow \qquad \widetilde{W} = \sum_{i=1}^{N} (X_i)^N + \mathrm{e}^{\tau/N} \prod_{i=1}^{N} X_i$$

$$\text{periodicity: } Y_i \sim Y_i + 2\pi i \Longrightarrow X_i \sim X_i \mathrm{e}^{-2\pi i/N}$$

The theory is LG orbifold with twisted superpotential

$$\widetilde{W}//G \qquad G = (\mathbb{Z}_N)^{N-1}$$

— Mirror geometry —

for non-compact CY

$$\mathcal{O}(-N) \text{ on } \mathbb{C}\mathbf{P}^{N-1} \quad \Longleftrightarrow \quad xz = \widetilde{W}(X_i) \text{ with } (\mathbb{Z}_N)^{N-2} \text{ orbifolding}$$

for compact CY

$$\begin{array}{ccc} \mathrm{CY}_{N-2} : \mathbb{C}\mathbf{P}^{N-1}[N] & \longleftarrow \text{Mirror} \longrightarrow & \mathrm{CY}_{N-2} : \mathbb{C}\mathbf{P}^{N-1}[N]//(\mathbb{Z}_N)^{N-2} \\ \uparrow & & \uparrow \\ \mathrm{CY/LG} & & \mathrm{CY/LG} \\ \downarrow & & \downarrow \\ \mathrm{LG}_{cc} : W//\mathbb{Z}_N & \longleftarrow \text{Mirror} \longrightarrow & \mathrm{LG}_{ac} : \widetilde{W}//(\mathbb{Z}_N)^{N-1} \end{array}$$

NEW: $\mathcal{O}(-N+2)$ bundle on Q^{N-2}

Contents of chiral superfields and their $U(1)$ charges:

$$\begin{array}{rcl} \Phi^i &=& \big(S^1 , S^2 , \cdots , S^N , T , P \big) \\ Q_i &=& \big(1 , 1 , \cdots , 1 , -2 , -N+2 \big) \end{array}$$

with superpotential $$W = T \sum_{i=1}^{N} (S^i)^2$$

Potential

$$\begin{aligned} U(s,t,p,\sigma) &= \frac{1}{2e^2} D^2 + 2|\sigma|^2 \Big(\sum_{i=1}^{N} |s^i|^2 + 4|t|^2 + (N-2)^2 |p|^2 \Big) \\ &\quad + 4|t|^2 \sum_{i=1}^{N} |s^i|^2 + \Big| \sum_{i=1}^{N} (s^i)^2 \Big|^2 \end{aligned}$$

$$\frac{1}{e^2} D = r - \sum_{i=1}^{N} |s^i|^2 + 2|t|^2 + (N-2)|p|^2$$

— “CY phase” $(r > 0)$ —

$^{\exists}\langle s^i \rangle \neq 0$

line bundle on quadratic hypersurface in $\mathbb{C}\mathrm{P}^{N-1}$

$\left(\text{base: } \sum_{i=1}^{N} (S^i)^2 = 0 \text{ in } \mathbb{C}\mathrm{P}^{N-1}\right)$

$\rightarrow \mathcal{O}(-N+2)$ bundle on Q^{N-2}

— “LG phase” $(r < 0)$ —

$\langle p \rangle \neq 0$

“LG” $\mathbb{Z}_{N-2}$ orbifold model

with superpotential $W = T \sum_{i=1}^{N} (S^i)^2$

It’s a strange superpotential for LG!!

(**No** isolated singularity)

Can we treat this as **minimal model (CFT)**?

I have not found the CY/LG correspondence yet.

study in progress...

T-dual of line bundle on Q^{N-2}

$$
\begin{aligned}
\widetilde{W} &= \sum_{i=1}^{N}(X_i)^{N-2} + (X_T)^{N-2} + e^{\tau/(N-2)}\Big(\prod_{i=1}^{N} X_i\Big)/(X_T)^2 \\
X_i &= e^{-Y_i/(N-2)} \quad \text{for } i = 1, 2, \cdots, N, T \\
e^{-Y_P} &= e^{\tau/(N-2)} X_1 X_2 \cdots X_N/(X_T)^2
\end{aligned}
$$

The effective theory is LG orbifold model with twisted superpotential

$$
\widetilde{W}//G \qquad \text{where} \quad G = (\mathbb{Z}_{N-2})^{N+1}
$$

しかしこれはまだ推測の域を出ていない 精密な議論を経ていない

Mirror pair を見出せていない

Discussions

問題は山積みである

- **LG orbifold model が持つ "タチのよい" 性質を持っていない**

  (isolated singularity, unique highest charged state, etc.)

- **IR limit での constraint の解に不定性が存在する**

  (何故 X_T ではなく X_P について解いたのか, etc.)

- **LG model の chiral-ring が定まらない**

  (non-compact CY であるために無限次元?)

Appendix: Landau-Ginzburg Theory

"Well-studied" LG theory is dictated by the following properties:

$$\mathcal{L} = \int d^4\theta\, K(\Phi, \overline{\Phi}) + \Big(\int d^2\theta\, W(\Phi) + (h.c.) \Big)$$

$$\int d^4\theta\, K \quad \text{is an irrelevant operator}$$

$$W|_{\Phi=0} = 0 \qquad \frac{\partial W}{\partial \Phi^i}\Big|_{\Phi=0} = 0$$

$$\det_{i,j}\Big(\frac{\partial^2 W}{\partial\Phi^i \partial\Phi^j}\Big)\Big|_{\Phi=0} = 0$$

$$W(\lambda^{\omega_i}\Phi^i) = \lambda^{+1} W(\Phi^i) \quad \text{where} \quad h_i = \overline{h}_i = \frac{\omega_i}{2} > 0$$

where

$$z \to \lambda^{-1} z\,, \qquad dz \to \lambda^{-1} z\,, \qquad \theta \to \lambda^{-1/2}\theta\,, \qquad d\theta \to \lambda^{+1/2} d\theta$$

$$d^2z d^2\theta \to \lambda^{-2+1} d^2z d^2\theta$$

— Relation to the "Singularity Theory" —

chiral ring:

$$\mathcal{R} \;=\; \frac{\mathbb{C}(\Phi)}{[\partial_i W(\Phi)]} \qquad\qquad \mu \;=\; \dim \mathcal{R} \;\; (\text{multiplicity of } W)$$

singularity index β and central charge c of CFT:

$$\beta \;=\; \sum_i \Big(\frac{1}{2} - \omega_i\Big) \qquad\qquad c \;=\; 6\beta$$

Poincaré polynomial ($\omega_i = p_j/N$):

$$P(t) \;=\; \sum_{k=0}^{M} b_k t^k \;=\; \prod_i \frac{1 - t^{N-p_i}}{1 - t^{p_i}} \qquad\qquad M \;=\; 2N\beta$$

Poincaré duality:

$$P(t) \;=\; t^M P(1/t) \qquad \longrightarrow \qquad b_k \;=\; b_{M-k}$$