Zero-mode Spectrum of Eleven-dimensional Theory on the Plane-wave Background

木村 哲士

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Introduction

— *Introduction* —

Towards quantum theory of Gravity

→ higher dim. Supergravity and theories of Extended Objects

- 10-dimensions:

  type I, IIA, IIB, heterotic Supergravities and Superstrings

- 11-dimensions:

  Supergravity and Supermembrane

— *Introduction* —

Towards quantum theory of Gravity

$\rightarrow$ higher dim. <u>Supergravity</u> and theories of <u>Extended Objects</u>

- 10-dimensions:

  type I, IIA, IIB, heterotic Supergravities and Superstrings

- 11-dimensions:

  Supergravity and Supermembrane

difficulties =

supergravities	:	classical theories
superstrings	:	more than one consistent theories
supermembrane	:	unstable

11-dim.

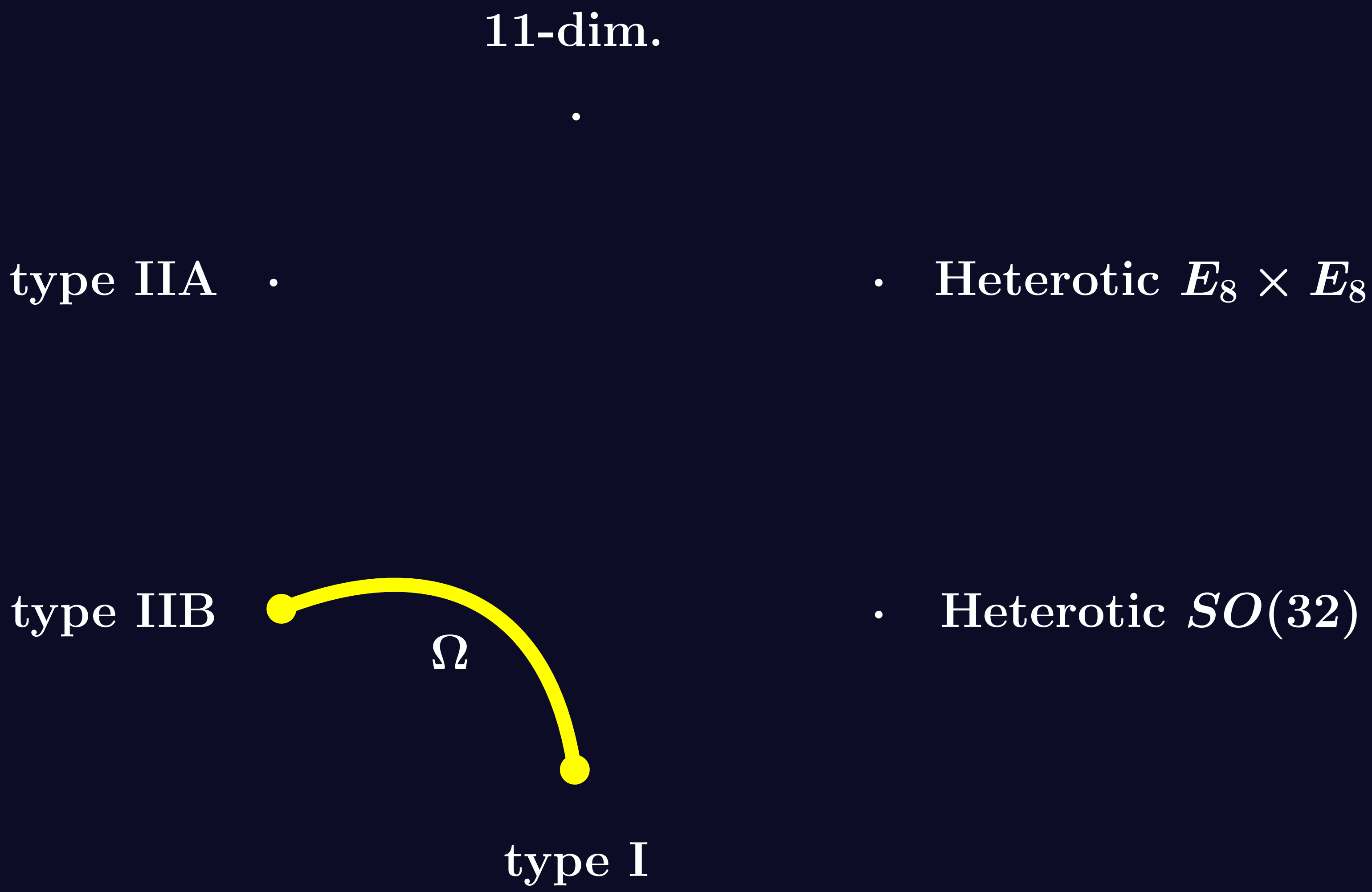
type IIA

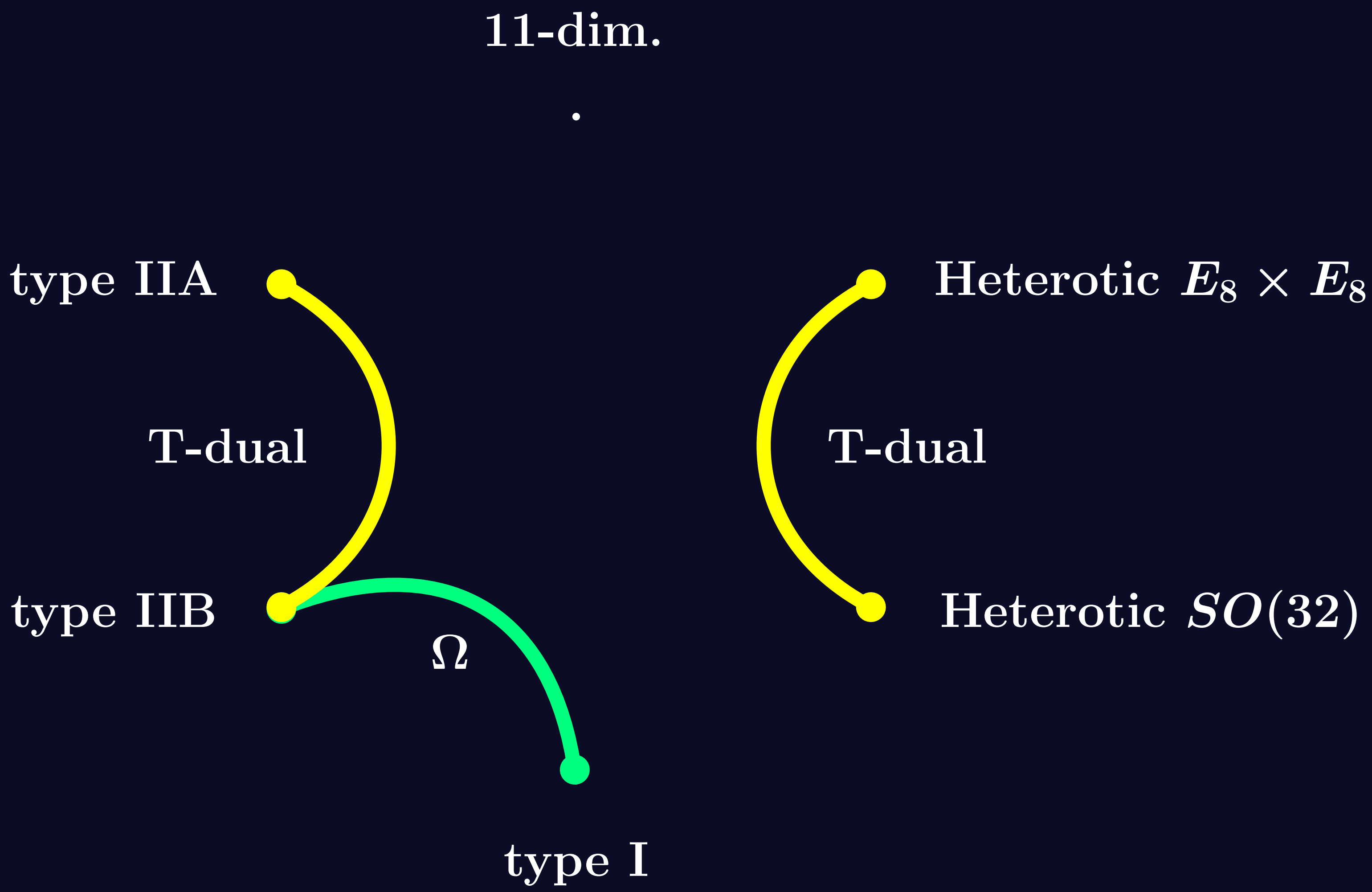
Heterotic $E_8 \times E_8$

type IIB

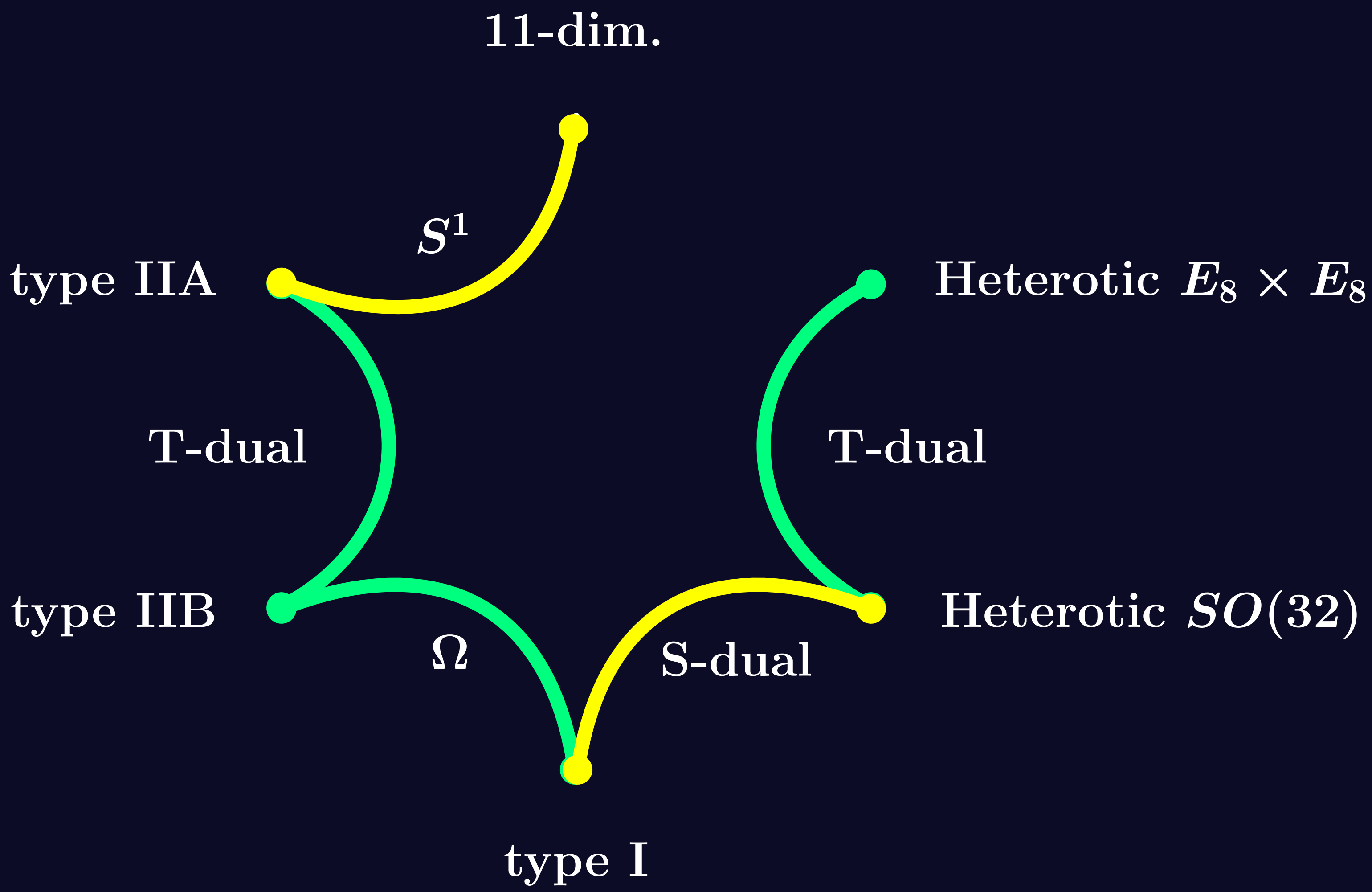
Heterotic $SO(32)$

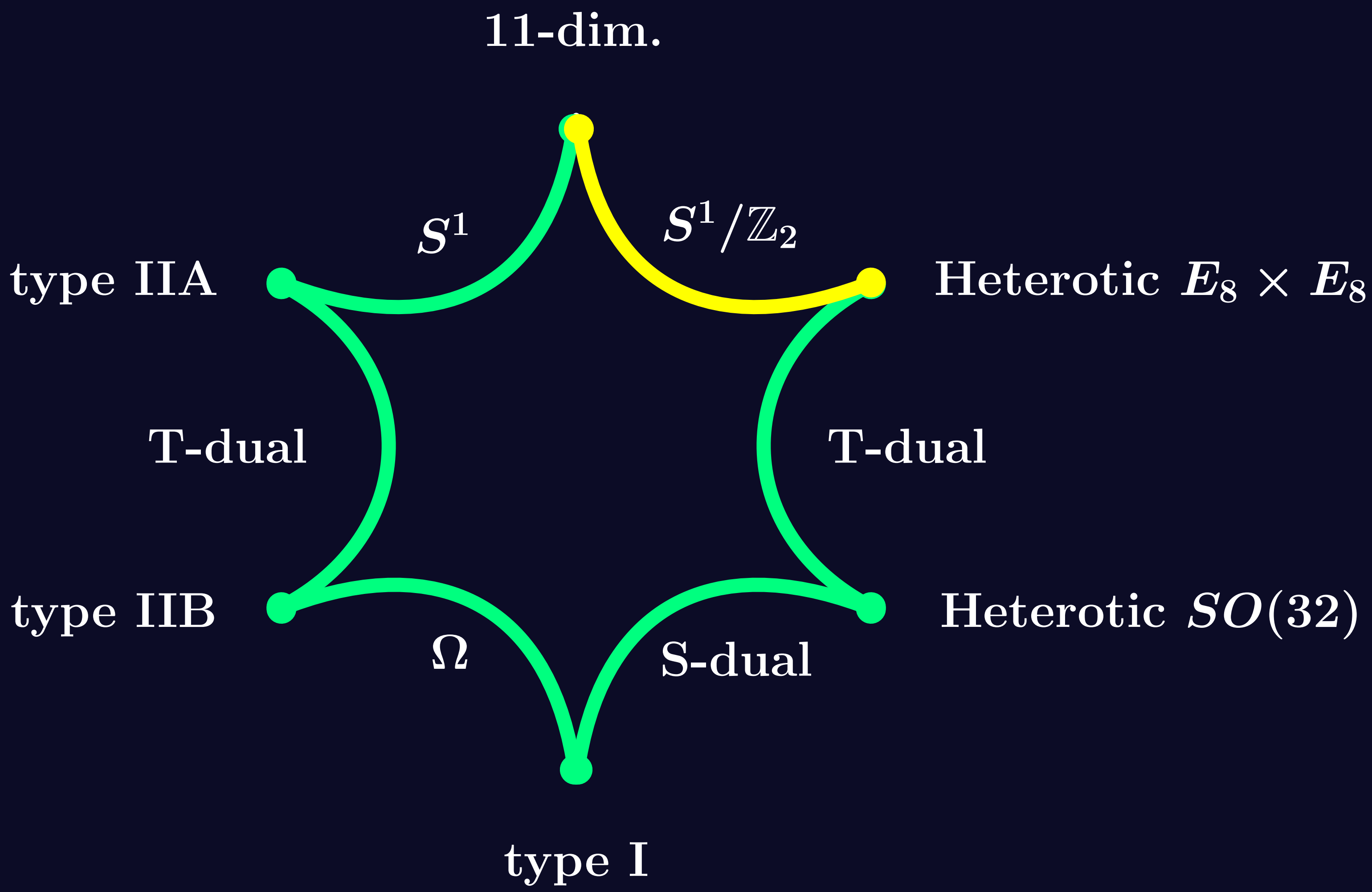
type I

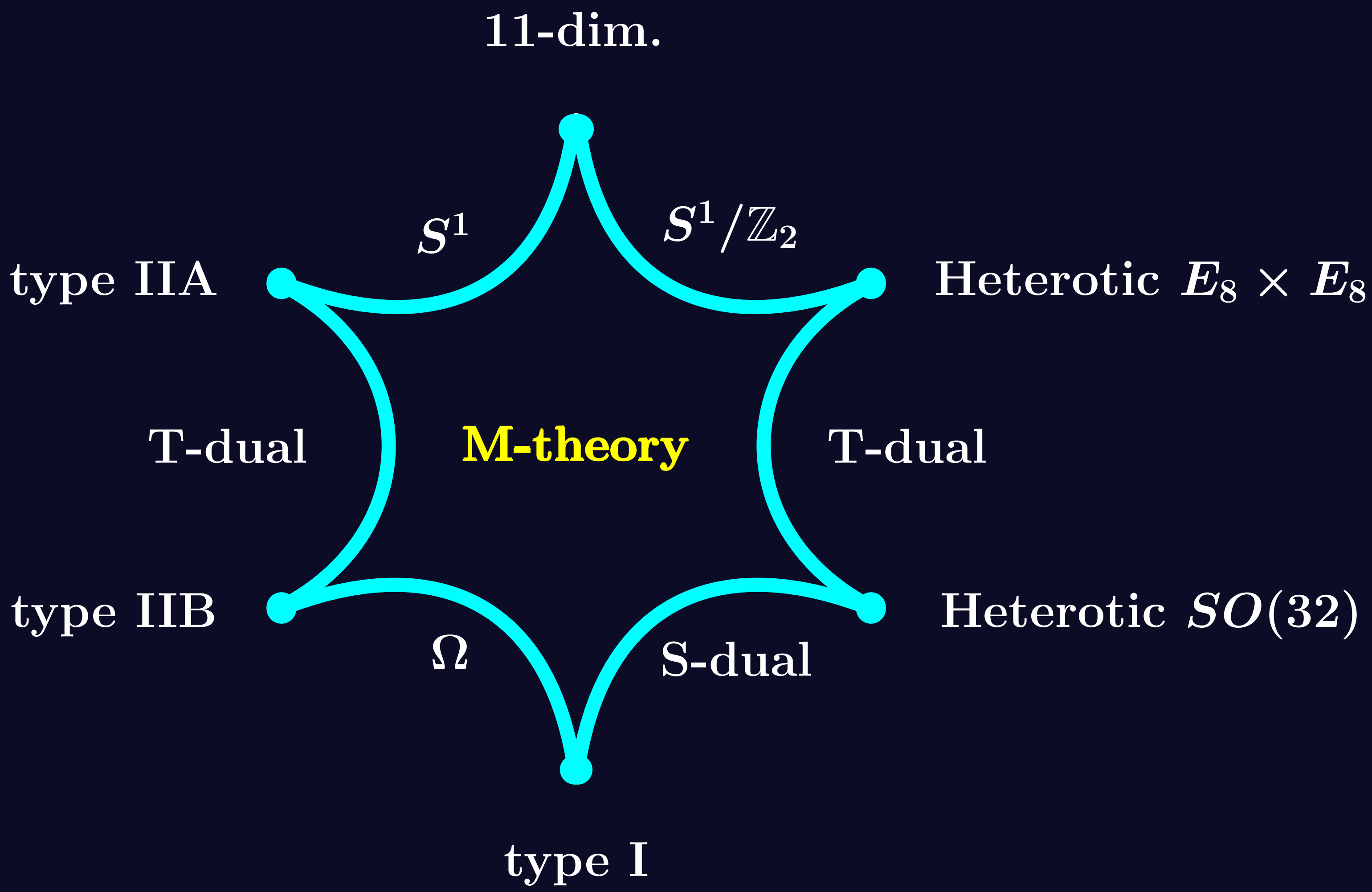

11-dim.
type IIA
Heterotic $E_8 \times E_8$
type IIB
Ω
Heterotic $SO(32)$
type I



11-dim.
type IIA
Heterotic $E_8 \times E_8$
T-dual
T-dual
type IIB
Heterotic $SO(32)$
Ω
type I


11-dim.
S^1
type IIA
Heterotic $E_8 \times E_8$
T-dual
T-dual
type IIB
Heterotic $SO(32)$
Ω
S-dual
type I


11-dim.
S^1
$S^1/\mathbb{Z}_2$
type IIA
Heterotic $E_8 \times E_8$
T-dual
T-dual
type IIB
Heterotic $SO(32)$
Ω
S-dual
type I


11-dim.
S^1
$S^1/\mathbb{Z}_2$
type IIA
Heterotic $E_8 \times E_8$
T-dual
M-theory
T-dual
type IIB
Heterotic $SO(32)$
Ω
S-dual
type I

M(atrix) Theory conjecture

M-theory compactified on circle of radius R

$\longrightarrow$ type IIA string (M-theory conjecture)

N D0-branes $\longleftarrow$ $(0+1)$-dim. reduction of 10D $U(N)$ SYM

(fields $= N \times N$ matrix variables)

In flat background,

- **Hamiltonian corresponds to that of supermembrane**
- **new interpretation for supermembrane instability**
- **graviton amplitudes agree with those of 11-dim. supergravity**
- *etc.*

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- **Hamiltonian corresponds to that of supermembrane**
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How about on curved background??

(Model should be well-defined on any background.)

On general curved background,

it is difficult or impossible

to solve the dynamics in terms of today's technique.

$\because$ **highly non-trivial interactions**

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to solve the dynamics in terms of today's technique.

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But,

Plane-wave background is a useful spacetime!

- Maximally Supersymmetric (32 supercharges)
- Penrose Limit of $AdS_p \times S^q$ (considering around a null geodesic)
- IIB string is solved as a massive free model (beyond $\mathrm{AdS}_5/\mathrm{CFT}_4$)

Matrix Model ⇒ **Berenstein, Maldacena and Nastase**

Supermembrane ⇒
- **Dasgupta, Sheikh-Jabbari and Van Raamsdonk**
- **Sugiyama and Yoshida**

Supergravity ⇒ **UNKNOWN YET!**

It is important to study supergravity on the plane-wave because of the lack of investigation whether BMN matrix model includes the supergraviton modes or not.

Contents

- **Geometry of the Plane-wave**
- **Matrix Theory**
- **Supergravity**
- **Conclusion**

In this talk only the zero-mode spectrum will be considered.

— *Background Geometry* —

Metric

$$ds^2 = -2\,dx^+ dx^- + G_{++} \cdot (dx^+)^2 + \sum_{I=1}^{9} (dx^I)^2$$

$$G_{++} = -\left(\frac{\mu}{3}\right)^2 \sum_{\tilde{I}=1}^{3} (x^{\tilde{I}})^2 - \left(\frac{\mu}{6}\right)^2 \sum_{I'=4}^{9} (x^{I'})^2$$

Freund-Rubin Ansatz

$$F_{123+} = \mu > 0$$

Vielbeins, Spin Connections, etc.

$$e_+{}^+ = e_-{}^- = 1 \qquad e_+{}^- = -\frac{1}{2}G_{++}$$

$$E_+{}^+ = E_-{}^- = 1 \qquad E_+{}^- = \frac{1}{2}G_{++}$$

$$\omega_+{}^{I-} = \frac{1}{2}\partial_I G_{++}$$

$$\Gamma^I_{++} = \Gamma^-_{+I} = -\frac{1}{2}\partial_I G_{++}$$

$$R^I{}_{+J+} = -\frac{1}{2}\partial_I\partial_J G_{++} \qquad \mathcal{R}_{++} = \frac{1}{2}\mu^2 \qquad \mathcal{R} = 0$$

— *Matrix Theory on the Plane-wave* —

D. Berenstein, J.M. Maldacena and H. Nastase:

"*Strings in flat space and pp-wave from $\mathcal{N}$=4 super Yang-Mills*"

JHEP 0204 (2002) 013, hep-th/0202021.

Lagrangian

N D0-branes' effective action

arising from the dimensional reduction of 10-dim. $U(N)$ SYM

	$+$	1	2	$\cdots$	8	9	$-$
M-theory	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
type IIA	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	S^1
D0-branes	$\bullet$	$-$	$-$	$-$	$-$	$-$	(S^1)

N D0-branes $\quad (x^- \sim x^- + 2\pi R)$

$$L = \mathrm{Tr}\left\{ \frac{1}{2R} D_\tau X^I \, D_\tau X^I + \frac{R}{4}[X^I, X^J]^2 + \frac{i}{2}\Psi^\dagger D_\tau \Psi + \frac{1}{2}\Psi^\dagger \gamma^I [X^I, \Psi] \right\}$$

N D0-branes on the plane-wave $(x^- \sim x^- + 2\pi R)$

$$
\begin{aligned}
L \;=\; & \mathrm{Tr}\Big\{ \frac{1}{2R} D_\tau X^I \, D_\tau X^I + \frac{R}{4}[X^I, X^J]^2 + \frac{i}{2}\Psi^\dagger D_\tau \Psi + \frac{1}{2}\Psi^\dagger \gamma^I [X^I, \Psi] \Big\} \\
& + R\,\mathrm{Tr}\Big\{ -\frac{1}{2}\Big[\Big(\frac{\mu}{3R}\Big)^2 (X^{\widetilde{I}})^2 + \Big(\frac{\mu}{6R}\Big)^2 (X^{I'})^2 \Big] - \frac{i\,\mu}{8R}\,\Psi^\dagger \gamma^{123} \Psi \Big\} \\
& + R\,\mathrm{Tr}\Big\{ -\frac{i\,\mu}{3R}\,\epsilon_{\widetilde{I}\widetilde{J}\widetilde{K}}\, X^{\widetilde{I}} X^{\widetilde{J}} X^{\widetilde{K}} \Big\}
\end{aligned}
$$

Additional terms are derived from

- supervielbeins on the plane-wave
- Myers' effect: Dielectric Phenomenon

Hamiltonian

$$
\begin{aligned}
H \;=\; & R\,\mathrm{Tr}\Big\{\frac{1}{2}P_I^2 - \frac{1}{4}[X^I, X^J]^2 - \frac{1}{2}\Psi^\dagger\gamma^I[X^I,\Psi] \\
& + \frac{1}{2}\Big[\Big(\frac{\mu}{3R}\Big)^2 (X^{\widetilde{I}})^2 + \Big(\frac{\mu}{6R}\Big)^2 (X^{I'})^2\Big] + \frac{i\,\mu}{8R}\,\Psi^\dagger\gamma^{123}\Psi \\
& + \frac{i\,\mu}{3R}\,\epsilon_{\widetilde{I}\widetilde{J}\widetilde{K}}\, X^{\widetilde{I}} X^{\widetilde{J}} X^{\widetilde{K}}\Big\}
\end{aligned}
$$

This corresponds to the Light-cone Hamiltonian

of matrix regularized supermembrane on the plane-wave.

Lagrangian has 32 supersymmetries:

- **16 dynamical SUSY: Q**

  nonlinearly realized

- **16 kinematical SUSY: q**

  linearly realized

16 Dynamical Supersymmetry

$$\delta_\epsilon X^I = \sqrt{R}\Psi^\dagger\gamma^I\epsilon(\tau) \qquad \delta_\epsilon\omega = \sqrt{R}\Psi^\dagger\epsilon(\tau)$$

$$\begin{aligned}\delta_\epsilon\Psi &= \sqrt{R}\Big(-\frac{i}{R}D_\tau X^I\gamma^I\epsilon(\tau) + \frac{1}{2}[X^I, X^J]\gamma^{IJ}\epsilon(\tau) \\ &\quad - \frac{i\,\mu}{3R}X^{\tilde{I}}\gamma^{\tilde{I}}\gamma^{123}\epsilon(\tau) + \frac{i\,\mu}{6R}X^{I'}\gamma^{I'}\gamma^{123}\epsilon(\tau)\Big) \\ \epsilon(\tau) &= \exp\Big(-\frac{\mu}{12}\gamma^{123}\tau\Big)\,\epsilon_0\end{aligned}$$

$$\begin{aligned}Q &= \sqrt{R}\,\mathrm{Tr}\Big\{P^I\gamma^I\Psi - \frac{i}{2}[X^I, X^J]\gamma^{IJ}\Psi \\ &\quad - \frac{\mu}{3R}X^{\tilde{I}}\gamma^{\tilde{I}}\gamma^{123}\Psi - \frac{\mu}{6R}X^{I'}\gamma^{I'}\gamma^{123}\Psi\Big\}\end{aligned}$$

This supersymmetry acts on the $SU(N)$ interaction part of the theory.

16 Kinematical Supersymmetry

$$\delta_\eta X^I = 0 \qquad \delta_\eta \omega = 0 \qquad \delta_\eta \Psi = \frac{1}{\sqrt{R}}\eta(\tau)$$

$$\eta(\tau) = \exp\left(\frac{\mu}{4}\gamma^{123}\tau\right)\eta_0$$

$$q = \frac{1}{\sqrt{R}}\mathrm{Tr}(\Psi)$$

This supercharge acts only $U(1)$ part of the theory.

(center of mass)

From now on we consider only the $U(1)$ part of the theory.

Commutation Relations

$$[H, q_\alpha] \; = \; -\frac{\mu}{4}\left(i\gamma^{123} q\right)_\alpha$$

$SU(4) \times SU(2)$ Decomposition

$$q \; = \; \begin{pmatrix} q_{i\alpha} \\ \epsilon_{\alpha\beta}\, q^{\dagger i\beta} \end{pmatrix} \qquad\qquad i\gamma^{123} \; = \; \begin{pmatrix} \mathbf{1}_8 & 0 \\ 0 & -\mathbf{1}_8 \end{pmatrix}$$

Thus the above commutation relation reduces to

$$\left[H, q_{i\alpha}\right] \; = \; -\frac{\mu}{4}\, q_{i\alpha} \qquad\qquad \left[H, q^{\dagger i\alpha}\right] \; = \; \frac{\mu}{4}\, q^{\dagger i\alpha}$$

Construction of Zero-mode Supermultiplet

$$q_{i\alpha}|\,\Lambda\,\rangle \;=\; 0 \qquad \text{for } {}^{\forall}i\;,\;{}^{\forall}\alpha$$

"Fermionic" Raising Operator

$$q^{\dagger i\alpha} \;\sim\; \left(\Box\,,\,\Box\right)$$

We will construct the ground state supermultiplet.

free $U(1)$ part of the matrix model

degrees of center of mass = superparticles

Floor 0 (ground state):

$$|\boldsymbol{\Lambda}\rangle = \left|\,\mathbf{1}\,,\,\mathbf{1}\,\right\rangle$$

Floor 1:

$$\left(\square\,,\,\square\right)\otimes\left|\,\mathbf{1}\,,\,\mathbf{1}\,\right\rangle = \left|\,\square\,,\,\square\,\right\rangle$$

Floor 2:

$$\left(\square\,,\,\square\right)\otimes\left|\,\square\,,\,\square\,\right\rangle$$

$$= \left|\begin{array}{c}\square\\\square\end{array},\begin{array}{c}\square\\\square\end{array}\right\rangle \oplus \left|\begin{array}{c}\square\\\square\end{array},\square\!\square\right\rangle \oplus \left|\square\!\square\,,\begin{array}{c}\square\\\square\end{array}\right\rangle \oplus \left|\square\!\square\,,\,\square\!\square\right\rangle$$

Floor 0 (ground state):

$$|\,\mathbf{\Lambda}\,\rangle = \left|\,\mathbf{1}\,,\,\mathbf{1}\,\right\rangle$$

Floor 1:

$$\left(\ydiagram{1}\,,\ydiagram{1}\right)\otimes\left|\,\mathbf{1}\,,\,\mathbf{1}\,\right\rangle = \left|\,\ydiagram{1}\,,\ydiagram{1}\,\right\rangle$$

Floor 2:

$$\left(\ydiagram{1}\,,\ydiagram{1}\right)\otimes\left|\,\ydiagram{1}\,,\ydiagram{1}\,\right\rangle$$

$$= \left|\,\ydiagram{1,1}\,,\ydiagram{1,1}\,\right\rangle \oplus \left|\,\ydiagram{1,1}\,,\ydiagram{2}\,\right\rangle \oplus \left|\,\ydiagram{2}\,,\ydiagram{1,1}\,\right\rangle \oplus \left|\,\ydiagram{2}\,,\ydiagram{2}\,\right\rangle$$

Floor 3:

$$\left(\ydiagram{1}\,,\ydiagram{1}\right)\otimes\left\{\left|\ydiagram{1,1}\,,\ydiagram{2}\right\rangle\oplus\left|\ydiagram{2}\,,\ydiagram{1,1}\right\rangle\right\}$$

$$=\left|\ydiagram{1,1,1}\,,\ydiagram{2,1}\right\rangle\oplus\left|\ydiagram{1,1,1}\,,\ydiagram{3}\right\rangle\oplus\left|\ydiagram{2,1}\,,\ydiagram{2,1}\right\rangle$$

$$\oplus\left|\ydiagram{2,1}\,,\ydiagram{3}\right\rangle\oplus\left|\ydiagram{2,1}\,,\ydiagram{2,1}\right\rangle\oplus\left|\ydiagram{3}\,,\ydiagram{2,1}\right\rangle$$

Floor 3:

$$\left(\ydiagram{1}\,,\ \ydiagram{1}\right) \otimes \left\{ \left|\ydiagram{1,1}\,,\ \ydiagram{2}\right\rangle \oplus \left|\ydiagram{2}\,,\ \ydiagram{1,1}\right\rangle \right\}$$

$$= \left|\ydiagram{1,1,1}\,,\ \ydiagram{2,1}\right\rangle \oplus \left|\ydiagram{1,1,1}\,,\ \ydiagram{3}\right\rangle \oplus \left|\ydiagram{2,1}\,,\ \ydiagram{2,1}\right\rangle$$

$$\oplus \left|\ydiagram{2,1}\,,\ \ydiagram{3}\right\rangle \oplus \left|\ydiagram{2,1}\,,\ \ydiagram{2,1}\right\rangle \oplus \left|\ydiagram{3}\,,\ \ydiagram{2,1}\right\rangle$$

Floor 4:

$$\left(\ydiagram{1}\,,\ \ydiagram{1}\right) \otimes \left\{ \left| \ydiagram{1,1,1}\,,\ \ydiagram{3} \right\rangle \oplus \left| \ydiagram{2,1}\,,\ \ydiagram{2,1} \right\rangle \right\}$$

$$= \left| \ydiagram{1,1,1,1}\,,\ \ydiagram{3,1} \right\rangle \oplus \left| \ydiagram{1,1,1,1}\,,\ \ydiagram{4} \right\rangle \oplus \left| \ydiagram{2,1,1}\,,\ \ydiagram{3,1} \right\rangle \oplus \left| \ydiagram{2,1,1}\,,\ \ydiagram{4} \right\rangle$$

$$\oplus \left| \ydiagram{2,1,1}\,,\ \ydiagram{2,2} \right\rangle \oplus \left| \ydiagram{2,1,1}\,,\ \ydiagram{3,1} \right\rangle \oplus \left| \ydiagram{2,2}\,,\ \ydiagram{2,2} \right\rangle \oplus \left| \ydiagram{2,2}\,,\ \ydiagram{3,1} \right\rangle$$

$$\oplus \left| \ydiagram{3,1}\,,\ \ydiagram{2,2} \right\rangle \oplus \left| \ydiagram{3,1}\,,\ \ydiagram{3,1} \right\rangle$$

Floor 5:

$$\left(\ydiagram{1}\,,\ \ydiagram{1}\right) \otimes \left\{ \left|\ydiagram{1,1,1,1}\,,\ \ydiagram{4}\right\rangle \oplus \left|\ydiagram{2,1,1}\,,\ \ydiagram{3,1}\right\rangle \oplus \left|\ydiagram{2,2}\,,\ \ydiagram{2,2}\right\rangle \right\}$$

$$= \left|\ydiagram{2,1,1,1}\,,\ \ydiagram{4,1}\right\rangle \oplus \left|\ydiagram{2,1,1,1}\,,\ \ydiagram{5}\right\rangle \oplus \left|\ydiagram{2,1,1,1}\,,\ \ydiagram{3,2}\right\rangle$$

$$\oplus \left|\ydiagram{2,1,1,1}\,,\ \ydiagram{4,1}\right\rangle \oplus \left|\ydiagram{2,2,1}\,,\ \ydiagram{3,2}\right\rangle \oplus \left|\ydiagram{2,2,1}\,,\ \ydiagram{4,1}\right\rangle$$

$$\oplus \left|\ydiagram{3,1,1}\,,\ \ydiagram{3,2}\right\rangle \oplus \left|\ydiagram{3,1,1}\,,\ \ydiagram{4,1}\right\rangle \oplus \left|\ydiagram{2,2,1}\,,\ \ydiagram{3,2}\right\rangle$$

$$\oplus \left|\ydiagram{3,2}\,,\ \ydiagram{3,2}\right\rangle$$

Floor 6:

$$\left(\ydiagram{1}\,,\,\ydiagram{1}\right)\otimes\left\{\left|\ydiagram{2,1,1,1}\,,\,\ydiagram{4,1}\right\rangle\oplus\left|\ydiagram{2,2,1}\,,\,\ydiagram{3,2}\right\rangle\right\}$$

$$\begin{aligned}
=\;&\left|\ydiagram{2,2,1,1}\,,\,\ydiagram{4,2}\right\rangle\oplus\left|\ydiagram{2,2,1,1}\,,\,\ydiagram{5,1}\right\rangle\oplus\left|\ydiagram{3,1,1,1}\,,\,\ydiagram{4,2}\right\rangle\\
&\oplus\left|\ydiagram{3,1,1,1}\,,\,\ydiagram{5,1}\right\rangle\oplus\left|\ydiagram{2,2,1,1}\,,\,\ydiagram{3,3}\right\rangle\oplus\left|\ydiagram{2,2,1,1}\,,\,\ydiagram{4,2}\right\rangle\\
&\oplus\left|\ydiagram{2,2,2}\,,\,\ydiagram{3,3}\right\rangle\oplus\left|\ydiagram{2,2,2}\,,\,\ydiagram{4,2}\right\rangle\oplus\left|\ydiagram{3,2,1}\,,\,\ydiagram{3,3}\right\rangle\\
&\oplus\left|\ydiagram{3,2,1}\,,\,\ydiagram{4,2}\right\rangle
\end{aligned}$$

Floor 7:

$$\left(\ydiagram{1}\,,\ \ydiagram{1}\right) \otimes \left\{ \left| \ydiagram{2,2,1,1}\,,\ \ydiagram{4,2} \right\rangle \oplus \left| \ydiagram{2,2,2}\,,\ \ydiagram{3,3} \right\rangle \right\}$$

$$= \left| \ydiagram{2,2,2,1}\,,\ \ydiagram{4,3} \right\rangle \oplus \left| \ydiagram{2,2,2,1}\,,\ \ydiagram{5,2} \right\rangle \oplus \left| \ydiagram{3,2,1,1}\,,\ \ydiagram{4,3} \right\rangle$$

$$\oplus \left| \ydiagram{3,2,1,1}\,,\ \ydiagram{5,2} \right\rangle \oplus \left| \ydiagram{2,2,2,1}\,,\ \ydiagram{4,3} \right\rangle \oplus \left| \ydiagram{3,2,2}\,,\ \ydiagram{4,3} \right\rangle$$

Floor 8:

$$\left(\ydiagram{1}\,,\,\ydiagram{1}\right) \otimes \left|\,\ydiagram{2,2,2,1}\,,\,\ydiagram{4,3}\right\rangle$$

$$= \left|\,\ydiagram{2,2,2,2}\,,\,\ydiagram{4,4}\right\rangle \oplus \left|\,\ydiagram{2,2,2,2}\,,\,\ydiagram{5,3}\right\rangle \oplus \left|\,\ydiagram{3,2,2,1}\,,\,\ydiagram{4,4}\right\rangle$$

$$\oplus \left|\,\ydiagram{3,2,2,1}\,,\,\ydiagram{5,3}\right\rangle$$

Floor N	$SU(4) \times SU(2)$ Representation			Energy
8		$(\mathbf{1}, \mathbf{1})$		2μ
7		$(\bar{\mathbf{4}}, \mathbf{2})$		$7\mu/4$
6	$(\bar{\mathbf{6}}, \mathbf{3})$		$(\overline{\mathbf{10}}, \mathbf{1})$	$3\mu/2$
5	$(\mathbf{4}, \mathbf{4})$		$(\overline{\mathbf{20}}, \mathbf{2})$	$5\mu/4$
4	$(\mathbf{1}, \mathbf{5})$	$(\mathbf{15}, \mathbf{3})$	$(\mathbf{20}', \mathbf{1})$	μ
3	$(\bar{\mathbf{4}}, \mathbf{4})$		$(\mathbf{20}, \mathbf{2})$	$3\mu/4$
2	$(\mathbf{6}, \mathbf{3})$		$(\mathbf{10}, \mathbf{1})$	$\mu/2$
1		$(\mathbf{4}, \mathbf{2})$		$\mu/4$
0		$(\mathbf{1}, \mathbf{1})$		0

The multiplet grouped into irreducible representations of $SU(4) \times SU(2)$ on each floor of equal energies.

— *Supergravity on the Plane-wave* —

T.K. and K. Yoshida:

"*Spectrum of eleven-dimensional supergravity on a pp-wave background*"

Phys. Rev. D68 (2003) 125007, `hep-th/0307193`.

Field Contents

$$e_M{}^A \;:\; \text{vielbein}\;, \qquad E_A{}^M \;:\; \text{inverse vielbein}$$

$$\Psi_M \;:\; \text{gravitino (vectorial Majorana spinor)}$$

$$C_{MNP} \;:\; \text{three-form gauge field}$$

$$\omega_M{}^{AB} \;:\; \text{spin connection}$$

Lagrangian

$$\mathcal{L} = e\,\mathcal{R} - \frac{1}{2} e\,\overline{\Psi}_M \Gamma^{MNP} D_N \Psi_P - \frac{1}{48} e\, F_{MNPQ} F^{MNPQ} - \frac{1}{192} e\, \overline{\Psi}_M \widetilde{\Gamma}^{MNPQRS} \Psi_N F_{PQRS} - \frac{1}{(144)^2} \varepsilon^{MNPQRSUVWXY} F_{MNPQ} F_{RSUV} C_{WXY}$$

Light-cone Hamiltonian $H = i\partial_+$:

$$
\begin{aligned}
H &= \frac{1}{-2p_-}\Big\{(p_I)^2 - \widetilde{G}_{++}\cdot(p_-)^2 - \alpha\,\mu p_-\Big\} \\
&= \frac{1}{3}\mu\sum_{\widetilde{I}}\bar{a}^{\widetilde{I}}a^{\widetilde{I}} + \frac{1}{6}\mu\sum_{I'}\bar{a}^{I'}a^{I'} + \frac{1}{2}\mu\,(2+\alpha)
\end{aligned}
$$

$$
a^{\widetilde{I}} \equiv \frac{1}{\sqrt{2\widetilde{m}}}\{p_{\widetilde{I}} + \widetilde{m}\partial_{p_{\widetilde{I}}}\} \qquad a^{I'} \equiv \frac{1}{\sqrt{2m'}}\{p_{I'} + m'\partial_{p_{I'}}\} \qquad [a^I, \bar{a}^J] = \delta^{IJ}
$$

$$
\widetilde{m} \equiv -\frac{1}{3}\mu\,p_- \qquad m' \equiv -\frac{1}{6}\mu\,p_-
$$

Zero Point Energy

$$
\big(\Box + \alpha\,\mu\,i\partial_-\big)\phi = 0 \quad \longrightarrow \quad E_0 = \frac{1}{2}\mu\,\big(2+\alpha\big)
$$

Classical Field Equations

$$\begin{aligned}
\mathcal{R}_{MN} &= -\frac{1}{144} g_{MN} F_{PQRS} F^{PQRS} + \frac{1}{12} F_{MPQR} F_N{}^{PQR} \\
0 &= \Gamma^{MNP} D_N \Psi_P + \frac{1}{96} \widetilde{\Gamma}^{MNPQRS} \Psi_N F_{PQRS} \\
0 &= \nabla^Q \{ e F_{QMNP} \} \\
&\quad - \frac{18}{(144)^2} \varepsilon_{MNP}{}^{QRSUVWXY} F_{QRSU} F_{VWXY}
\end{aligned}$$

Fluctuations

$$g_{MN} \longrightarrow g_{MN} + h_{MN} \qquad\qquad \Psi_M \longrightarrow 0 + \psi_M$$

$$F_{MNPQ} \longrightarrow F_{MNPQ} + \mathcal{F}_{MNPQ}$$

Light-cone Gauge Fixing

$$h_{-M} = 0 \qquad \mathcal{C}_{-MN} = 0 \qquad \psi_{-} = 0$$

Light-cone Gauge Fixing and Derived Constraints:

$$h_{-M} = 0 \qquad \mathcal{C}_{-MN} = 0 \qquad \psi_{-} = 0$$

Constraints for Bosons:

$$h_M{}^M = 0 \qquad \partial^M h_{IM} = 0 \qquad \partial^M h_{+M} = \frac{1}{3}\mu\,\mathcal{C}_{123}$$

$$\partial^M \mathcal{C}_{IJM} = 0 \qquad \partial^M \mathcal{C}_{+IM} = 0$$

Constraints for Fermions:

$$\Gamma^M \psi_M = 0 \qquad \partial^M \psi_M = 0$$

$$\Gamma^{+}\Gamma^{M} D_M \psi_I = 0$$

Non-trivial Equations for Bosonic Fields

$$
\begin{aligned}
0 &= \Box\, h_{\tilde{I}\tilde{J}} - \frac{2}{3}\,\mu\,\delta_{\tilde{I}\tilde{J}}\,\partial_{-}\mathcal{C} \\
0 &= \Box\, h_{\tilde{I}J'} - \mu\,\partial_{-}\mathcal{C}_{\tilde{I}J'} \\
0 &= \Box\, h_{I'J'} + \frac{1}{3}\,\mu\,\delta_{I'J'}\partial_{-}\mathcal{C} \\
0 &= \Box\,\mathcal{C} + 2\,\mu\,\partial_{-}h_{\tilde{I}\tilde{I}} \\
0 &= \Box\,\mathcal{C}_{\tilde{I}J'} + \mu\,\partial_{-}h_{\tilde{I}J'} \\
0 &= \Box\,\mathcal{C}_{\tilde{I}J'K'} \\
0 &= \Box\,\mathcal{C}_{I'J'K'} - \frac{1}{6}\,\mu\,\varepsilon^{I'J'K'W'X'Y'}\partial_{-}\mathcal{C}_{W'X'Y'}
\end{aligned}
$$

$$\mathcal{C}_{\tilde{I}J'} \equiv \frac{1}{2}\,\epsilon_{\tilde{I}\tilde{K}\tilde{L}}\mathcal{C}_{\tilde{K}\tilde{L}J'} \qquad \mathcal{C} \equiv 2\,\mathcal{C}_{123}$$

$$0 = \left(\Box - \mu i \partial_-\right) H_{\widetilde{I}J'} \qquad 0 = \left(\Box + \mu i \partial_-\right) \overline{H}_{\widetilde{I}J'}$$

$$0 = \Box h^{\perp}_{\widetilde{I}\widetilde{J}} \qquad 0 = \Box h^{\perp}_{I'J'}$$

$$0 = \left(\Box - 2\mu i \partial_-\right) h \qquad 0 = \left(\Box + 2\mu i \partial_-\right) \overline{h}$$

$$0 = \left(\Box - \mu i \partial_-\right) \mathcal{C}^{\oplus}_{I'J'K'} \qquad 0 = \left(\Box + \mu i \partial_-\right) \mathcal{C}^{\ominus}_{I'J'K'}$$

$$H_{\widetilde{I}J'} = h_{\widetilde{I}J'} + i\mathcal{C}_{\widetilde{I}J'} \qquad \overline{H}_{\widetilde{I}J'} = h_{\widetilde{I}J'} - i\mathcal{C}_{\widetilde{I}J'}$$

$$h^{\perp}_{\widetilde{I}\widetilde{J}} = h_{\widetilde{I}\widetilde{J}} - \frac{1}{3}\delta_{\widetilde{I}\widetilde{J}} h_{\widetilde{K}\widetilde{K}} \qquad h^{\perp}_{I'J'} = h_{I'J'} - \frac{1}{6}\delta_{I'J'} h_{K'K'}$$

$$h = h_{\widetilde{K}\widetilde{K}} + i\mathcal{C} \qquad \overline{h} = h_{\widetilde{K}\widetilde{K}} - i\mathcal{C}$$

$$\mathcal{C}^{\oplus}_{I'J'K'} = \frac{i}{6}\varepsilon^{I'J'K'W'X'Y'}\mathcal{C}^{\oplus}_{W'X'Y'}$$

$$\mathcal{C}^{\ominus}_{I'J'K'} = -\frac{i}{6}\varepsilon^{I'J'K'W'X'Y'}\mathcal{C}^{\ominus}_{W'X'Y'}$$

Field Equations for Fermions

$$0 = \left(\Box + \frac{1}{2}\mu\, i\partial_-\right)\psi^{\oplus\perp}_{\widetilde{I}\mathrm{R}} \qquad 0 = \left(\Box - \frac{1}{2}\mu\, i\partial_-\right)\psi^{\oplus\perp}_{\widetilde{I}\mathrm{L}}$$

$$0 = \left(\Box - \frac{1}{2}\mu\, i\partial_-\right)\psi^{\oplus\perp}_{I'\mathrm{R}} \qquad 0 = \left(\Box + \frac{1}{2}\mu\, i\partial_-\right)\psi^{\oplus\perp}_{I'\mathrm{L}}$$

$$0 = \left(\Box - \frac{3}{2}\mu\, i\partial_-\right)\psi^{\oplus\parallel}_{\mathrm{R}} \qquad 0 = \left(\Box + \frac{3}{2}\mu\, i\partial_-\right)\psi^{\oplus\parallel}_{\mathrm{L}}$$

$$\psi_I^{\oplus} = -\frac{1}{2}\Gamma^{-}\Gamma^{+}\psi_I \qquad \psi_I^{\ominus} = -\frac{1}{2}\Gamma^{+}\Gamma^{-}\psi_I$$

$$\psi_{\tilde{I}}^{\oplus\perp} = \left(\delta_{\tilde{I}\tilde{J}} - \frac{1}{3}\Gamma_{\tilde{I}}\Gamma_{\tilde{J}}\right)\psi_{\tilde{J}}^{\oplus} \qquad \psi_1^{\oplus\|} = \Gamma^{\tilde{I}}\psi_{\tilde{I}}^{\oplus}$$

$$\psi_{I'}^{\oplus\perp} = \left(\delta_{I'J'} - \frac{1}{6}\Gamma_{I'}\Gamma_{J'}\right)\psi_{J'}^{\oplus} \qquad \psi_2^{\oplus\|} = \Gamma^{I'}\psi_{I'}^{\oplus}$$

$$\psi_{\tilde{I}\mathrm{R}}^{\oplus\perp} = \frac{1 + i\Gamma^{123}}{2}\psi_{\tilde{I}}^{\oplus\perp} \qquad \psi_{\tilde{I}\mathrm{L}}^{\oplus\perp} = \frac{1 - i\Gamma^{123}}{2}\psi_{\tilde{I}}^{\oplus\perp}$$

$$\psi_{I'\mathrm{R}}^{\oplus\perp} = \frac{1 + i\Gamma^{123}}{2}\psi_{I'}^{\oplus\perp} \qquad \psi_{I'\mathrm{L}}^{\oplus\perp} = \frac{1 - i\Gamma^{123}}{2}\psi_{I'}^{\oplus\perp}$$

$$\psi_{1\mathrm{R}}^{\oplus\|} = \frac{1 + i\Gamma^{123}}{2}\psi_1^{\oplus\|} \qquad \psi_{1\mathrm{L}}^{\oplus\|} = \frac{1 - i\Gamma^{123}}{2}\psi_1^{\oplus\|}$$

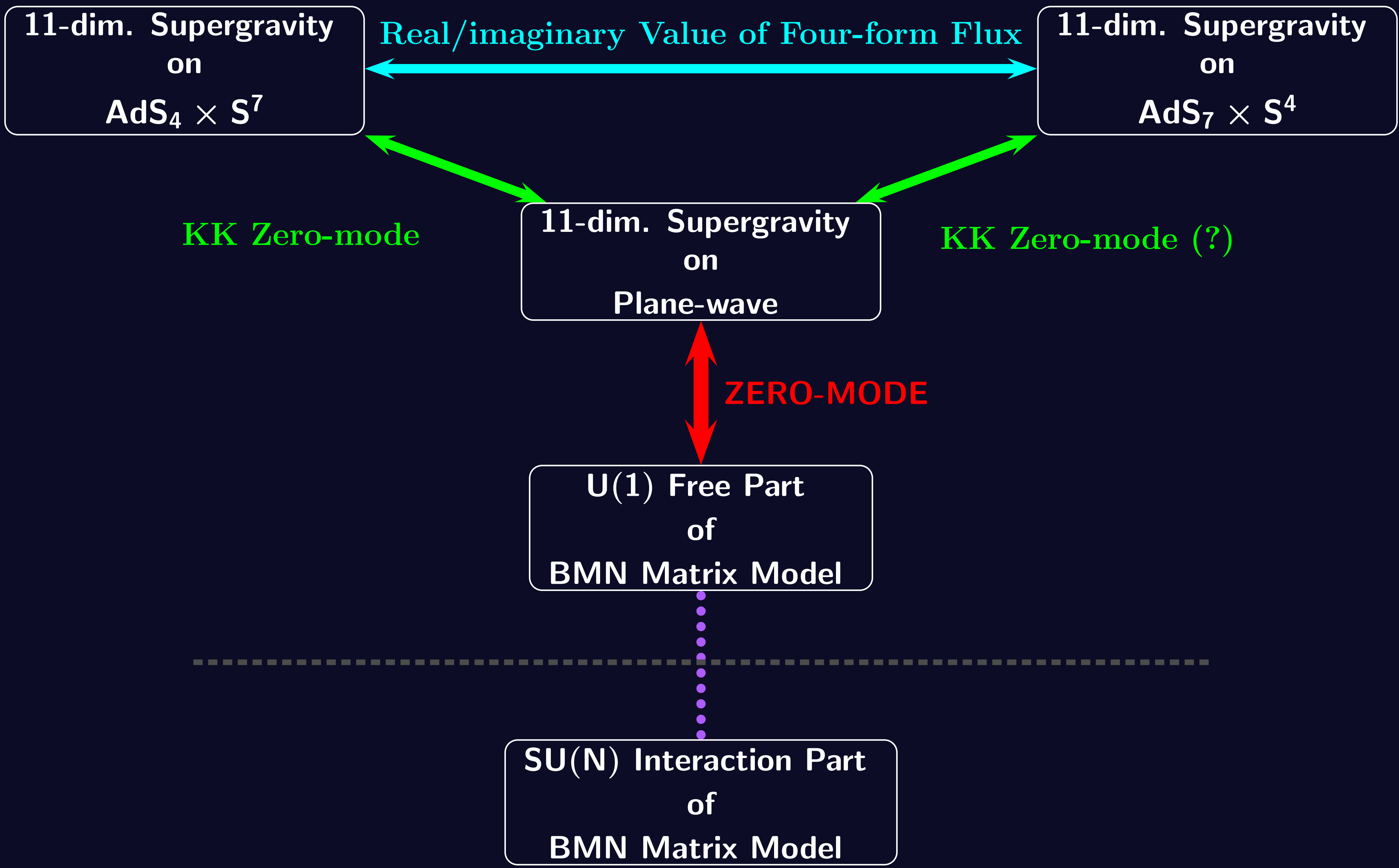
$$\psi_{2\mathrm{R}}^{\oplus\|} = \frac{1 + i\Gamma^{123}}{2}\psi_2^{\oplus\|} \qquad \psi_{2\mathrm{L}}^{\oplus\|} = \frac{1 - i\Gamma^{123}}{2}\psi_2^{\oplus\|}$$

$$\psi_{\mathrm{R}}^{\oplus\|} = \frac{2}{5}\psi_{1\mathrm{R}}^{\oplus\|} - \psi_{2\mathrm{R}}^{\oplus\|} \qquad \psi_{\mathrm{L}}^{\oplus\|} = \frac{2}{5}\psi_{1\mathrm{L}}^{\oplus\|} - \psi_{2\mathrm{L}}^{\oplus\|}$$

energy E_0	bosonic/fermionic fields	degrees of freedom
2μ	h	1
$7\mu/4$	$\psi^{\oplus\parallel}_{\mathrm{L}}$	8
$3\mu/2$	$H_{\tilde{I}J'}$ $\quad$ $\mathcal{C}^{\ominus}_{I'J'K'}$	$18+10$
$5\mu/4$	$\psi^{\oplus\perp}_{\tilde{I}\mathrm{R}}$ $\quad$ $\psi^{\oplus\perp}_{I'\mathrm{L}}$	$16+40$
μ	$h^{\perp}_{\tilde{I}\tilde{J}}$ $\quad$ $\mathcal{C}_{\tilde{I}J'K'}$ $\quad$ $h^{\perp}_{I'J'}$	$5+45+20$
$3\mu/4$	$\psi^{\oplus\perp}_{\tilde{I}\mathrm{L}}$ $\quad$ $\psi^{\oplus\perp}_{I'\mathrm{R}}$	$16+40$
$\mu/2$	$\overline{H}_{\tilde{I}J'}$ $\quad$ $\mathcal{C}^{\oplus}_{I'J'K'}$	$18+10$
$\mu/4$	$\psi^{\oplus\parallel}_{\mathrm{R}}$	8
0	$\overline{h}$	1

Zero point energy spectrum of all the physical fields of the linearized supergravity on the plane-wave background.

— *Conclusion* —



11-dim. Supergravity on $AdS_4 \times S^7$
Real/imaginary Value of Four-form Flux
11-dim. Supergravity on $AdS_7 \times S^4$
KK Zero-mode
11-dim. Supergravity on Plane-wave
KK Zero-mode (?)
ZERO-MODE
U(1) Free Part of BMN Matrix Model
SU(N) Interaction Part of BMN Matrix Model

— *Discussions* —

- **(Vacua have already been investigated...)**
- **Graviton Amplitudes on the Plane-wave**
- **Non–zero-mode Interactions**
- **(Transverse) M5-brane Formulation**
- **Origin of Myers' Term — Nonabelian D-branes**

$SO(10,1)$ Dirac Gamma Matrix

$$\{\Gamma^A, \Gamma^B\} = 2\,\eta^{AB}$$

$$(\Gamma^A)^\dagger = \Gamma_A = -\Gamma^0\Gamma^A(\Gamma^0)^{-1}$$

$$C\Gamma^A C^{-1} = -(\Gamma^A)^T$$

$SO(9)$ Representation

$$\Gamma^0 = \begin{pmatrix} 0 & i\,\mathbf{1}_{16} \\ i\,\mathbf{1}_{16} & 0 \end{pmatrix} \qquad \Gamma^{10} = \begin{pmatrix} 0 & -i\,\mathbf{1}_{16} \\ i\,\mathbf{1}_{16} & 0 \end{pmatrix}$$

$$\Gamma^I = \begin{pmatrix} -(\gamma^I)^T & 0 \\ 0 & \gamma^I \end{pmatrix}$$

$$\{\gamma^I, \gamma^J\} = 2\,\delta^{IJ} \qquad (\gamma^I)^\dagger = \gamma^I$$

Myers' Effect (Dielectric Phenomenon)

The single Dp-brane action

$$
\begin{aligned}
S &= S_{\rm DBI} + S_{\rm WZ} \\
S_{\rm DBI} &= -T_p \int \mathrm{d}^{p+1}\sigma \left(\mathrm{e}^{-\phi} \sqrt{-\det \left\{ \mathrm{P}[G+B]_{ab} + \lambda F_{ab} \right\}} \right) \\
S_{\rm WZ} &= \mu_p \int \mathrm{P}\left[\sum C^{(n)}\, \mathrm{e}^{B} \right] \mathrm{e}^{\lambda F} \\
\lambda &= 2\pi\alpha'
\end{aligned}
$$

Wess-Zumino term should be added

in order for the consistency of T-duality.

N coincident Dp-branes' action

$$
\begin{aligned}
S_{\mathrm{DBI}} &= -T_p \int \mathrm{d}^{p+1}\sigma\, \mathrm{STr}\Big(\mathrm{e}^{-\phi}\sqrt{\det(Q^i{}_j)} \\
&\qquad \times \sqrt{-\det\left\{\mathrm{P}[E_{ab} + E_{ai}(Q^{-1}-\delta)^{ij}E_{jb}] + \lambda F_{ab}\right\}}\Big) \\
S_{\mathrm{WZ}} &= \mu_p \int \mathrm{STr}\Big(\mathrm{P}\Big[\mathrm{e}^{i\lambda \mathrm{i}_X \mathrm{i}_X}\big(\sum C^{(n)}\,\mathrm{e}^{B}\big)\Big]\,\mathrm{e}^{\lambda F}\Big)
\end{aligned}
$$

where

$$
\begin{aligned}
Q^i{}_j &= \delta^i_j + i\lambda\,[X^i, X^k]\,E_{kj} \\
E_{\mu\nu} &= G_{\mu\nu} + B_{\mu\nu} \\
\mathrm{i}_X \mathrm{i}_X C^{(n)} &= \frac{1}{2(n-2)!}\,[X^i, X^j]\,C_{ji\mu_3\cdots\mu_n}\,\mathrm{d}x^{\mu_3}\wedge\cdots\wedge \mathrm{d}x^{\mu_n}
\end{aligned}
$$

X^i: adjoint scalar matrix in the transverse direction to Dp-branes

WZ term for D0-branes:

$$
\begin{aligned}
S_{\rm WZ} &= i\lambda\mu_0 \int \mathrm{Tr}\Big(\mathrm{P}\Big[(\mathrm{i}_X \mathrm{i}_X)\, C^{(3)}\Big]\Big) \\
&= \frac{i\lambda}{2}\mu_0 \int \mathrm{d}t\, \mathrm{Tr}\Big(C_{tij}\,[X^i, X^j] + \lambda C_{ijk}\, D_t X^i\,[X^k, X^j]\Big)
\end{aligned}
$$

with $B = F = 0$ and $C^{(1)} = C^{(5)} = C^{(7)} = C^{(9)}$

Existence of non-vanishing constant RR field strength:

$$
F_{t\widetilde{I}\widetilde{J}\widetilde{K}} = -\mu\, \epsilon_{\widetilde{I}\widetilde{J}\widetilde{K}}
$$

Then, the WZ term reduces to

$$
S_{\rm WZ} = \frac{i}{3}\lambda^2\mu_0 \int \mathrm{d}t\, \mathrm{Tr}(X^{\widetilde{I}} X^{\widetilde{J}} X^{\widetilde{K}})\, F_{t\widetilde{I}\widetilde{J}\widetilde{K}}
$$

Relation

$$
\begin{array}{lclcl}
\textbf{11D} & : & \boldsymbol{F}^{(4)} = \mathrm{d}\boldsymbol{C}^{(3)} & \longleftarrow & \textbf{couples to M2-brane} \\
\downarrow & & & & \downarrow \\
\textbf{10D} & : & \boldsymbol{F}^{(4)}_{\mathrm{RR}} = \mathrm{d}\boldsymbol{C}^{(3)}_{\mathrm{RR}} & \longleftarrow & \textbf{couples to D2-brane}
\end{array}
$$

$SU(4) \times SU(2)$ Decomposition

$$\mathbf{16} \;\longrightarrow\; (\mathbf{4},\mathbf{2}) \oplus (\overline{\mathbf{4}},\mathbf{2}) \;\;: \qquad \Psi \;\longrightarrow\; \left\{ \psi_{i\alpha} \;,\; \psi^{\dagger i\alpha} \right\}$$

$$\Psi \;=\; \begin{pmatrix} \psi_{i\alpha} \\ \epsilon_{\alpha\beta}\, \psi^{\dagger i\beta} \end{pmatrix}$$

$$\gamma^{\widetilde{I}} \;=\; \begin{pmatrix} -\boldsymbol{\sigma}^{\widetilde{I}} \otimes \mathbf{1}_4 & 0 \\ 0 & \boldsymbol{\sigma}^{\widetilde{I}} \otimes \mathbf{1}_4 \end{pmatrix} \qquad \gamma^{I'} \;=\; \begin{pmatrix} 0 & \mathbf{1}_2 \otimes \mathbf{g}^{I'} \\ \mathbf{1}_2 \otimes (\mathbf{g}^{I'})^\dagger & 0 \end{pmatrix}$$

$$\{\boldsymbol{\sigma}^{\widetilde{I}}, \boldsymbol{\sigma}^{\widetilde{J}}\} \;=\; 2\delta^{\widetilde{I}\widetilde{J}} \cdot \mathbf{1}_2$$

$$\mathbf{g}^{I'}(\mathbf{g}^{J'})^\dagger + \mathbf{g}^{J'}(\mathbf{g}^{I'})^\dagger \;=\; 2\delta^{I'J'} \cdot \mathbf{1}_4$$

Commutation Relations

$$\{Q^{\dagger i\alpha}, Q_{j\beta}\} = 2\delta^i_j\,\delta^\alpha_\beta\,H + \frac{\mu}{3}\epsilon^{\widetilde{I}\widetilde{J}\widetilde{K}}\,(\sigma^{\widetilde{K}})_\beta{}^\alpha\,\delta^i_j\,\Sigma^{\widetilde{I}\widetilde{J}} + \frac{i\,\mu}{6}\,\delta^\alpha_\beta\,(\mathrm{g}^{I'J'})_j{}^i\,\Sigma^{I'J'}$$

$$\{q_{i\alpha}, Q_{j\beta}\} = -i\sqrt{\frac{\mu}{3}}\,(\mathrm{g}^{I'})_{ij}\,\epsilon_{\alpha\beta}\,a^{I'}$$

$$\{q^{\dagger i\alpha}, Q_{j\beta}\} = -i\sqrt{\frac{2\mu}{3}}\,(\sigma^{\widetilde{I}})_\beta{}^\alpha\,\delta^i_j\,a^{\dagger\widetilde{I}}$$

$$\{q^{\dagger i\alpha}, q_{j\beta}\} = \delta^\alpha_\beta\,\delta^i_j\,P^+$$

$$
\begin{aligned}
P^{+} &= \frac{1}{R}\,\mathrm{Tr}(1) \\
a^{\widetilde{I}} &= \frac{1}{\sqrt{R}}\,\mathrm{Tr}\Big(\sqrt{\frac{\mu}{6R}}X^{\widetilde{I}} + i\sqrt{\frac{3R}{2\mu}}P^{\widetilde{I}}\Big) \\
a^{I'} &= \frac{1}{\sqrt{R}}\,\mathrm{Tr}\Big(\sqrt{\frac{\mu}{12R}}X^{I'} + i\sqrt{\frac{3R}{\mu}}P^{I'}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\Sigma^{\widetilde{I}\widetilde{J}} &= \mathrm{Tr}\Big(P^{\widetilde{I}}X^{\widetilde{J}} - P^{\widetilde{J}}X^{\widetilde{I}} - i\epsilon^{\widetilde{I}\widetilde{J}\widetilde{K}}\,\psi^{\dagger}\,\boldsymbol{\sigma}^{\widetilde{K}}\,\psi\Big) \\
\Sigma^{I'J'} &= \mathrm{Tr}\Big(P^{I'}X^{J'} - P^{J'}X^{I'} - \frac{1}{2}\psi^{\dagger}\,\mathbf{g}^{I'J'}\,\psi\Big)
\end{aligned}
$$

Supervielbeins on the Plane-wave

Isometry superalgebra on $AdS_{4(7)} \times S^{7(4)}$

$$F_{\widetilde{M}\widetilde{N}\widetilde{P}\widetilde{Q}} = f\,\varepsilon_{\widetilde{M}\widetilde{N}\widetilde{P}\widetilde{Q}} \qquad \text{Freund-Rubin Ansatz}$$

$$f \;:\; \text{real} \longrightarrow AdS_4 \times S^7 \qquad \text{imaginary} \longrightarrow AdS_7 \times S^4$$

$$[P_{\widetilde{A}}, P_{\widetilde{B}}] = \frac{i}{9} f^2\,\Sigma_{\widetilde{A}\widetilde{B}} \qquad [P_{A'}, P_{B'}] = -\frac{i}{36} f^2\,\Sigma_{A'B'}$$

$$[P_A, \Sigma_{CD}] = i\big(\eta_{AC}\,P_D - \eta_{AD}\,P_C\big)$$

$$i[\Sigma_{AB}, \Sigma_{CD}] = \eta_{AC}\,\Sigma_{BD} + \eta_{BD}\,\Sigma_{AC} - \eta_{AD}\,\Sigma_{BC} - \eta_{BC}\,\Sigma_{AD}$$

$$[P_A, \overline{Q}] = i\overline{Q}\,T_A{}^{BCDE}\,F_{BCDE}$$

$$[\Sigma_{AB}, \overline{Q}] = \frac{i}{2}\overline{Q}\,\Gamma_{AB}$$

$$\{Q, \overline{Q}\} = 2i\,\Gamma_A P^A - \frac{i}{144}\{\Gamma^{ABCDEF}\,F_{CDEF} + 24\,\Gamma_{CD}\,F^{ABCD}\}\Sigma_{AB}$$

Representative

$$L(Z) \;=\; \ell(x)\cdot \widehat{L}(\theta)$$

$$\ell(x) \;=\; \exp(i\,x^{A}P_{A}) \qquad\qquad \widehat{L}(\theta) \;=\; \exp(i\,\overline{\theta}Q)$$

Maurer-Cartan one-form

$$\alpha \;=\; i^{-1}\,L^{-1}\mathrm{d}L \;=\; \widehat{E}+\widehat{\Omega}$$

$$\widehat{E} \;=\; E^{A}\,P_{A}+\overline{Q}E \qquad\qquad \widehat{\Omega} \;=\; \frac{1}{2}\Omega^{AB}\,\Sigma_{AB}$$

$$\mathrm{d}\alpha + i\,\alpha\wedge\alpha \;=\; 0$$

$$
\begin{aligned}
E^{A}(x,\theta) &= e^{A} + 2\sum_{n=0}^{15}\frac{1}{(2n+2)!}\overline{\theta}\,\Gamma^{A}\mathcal{M}^{2n}\,D\theta \\
E(x,\theta) &= \sum_{n=0}^{16}\frac{1}{(2n+1)!}\mathcal{M}^{2n}D\theta \\
\Omega^{AB}(x,\theta) &= -\omega^{AB} \\
&\quad - \frac{1}{72}\sum_{n=0}^{15}\frac{1}{(2n+2)!}\overline{\theta}\{\Gamma^{ABCDEF}\,F_{CDEF} + 24\,\Gamma_{CD}\,F^{ABCD}\}\mathcal{M}^{2n}D\theta \\
D\theta &= \mathrm{d}\theta - e^{A}T_{A}{}^{BCDE}\theta\,F_{BCDE} + \frac{1}{4}\,\omega^{AB}\Gamma_{AB}\theta \\
\mathcal{M}^{2} &= -2\big(T_{A}{}^{BCDE}\,\theta\big)\,F_{BCDE}\,\big(\overline{\theta}\Gamma^{A}\big) \\
&\quad + \frac{1}{288}(\Gamma_{AB}\,\theta)\Big(\overline{\theta}\Big[\Gamma^{ABCDEF}\,F_{CDEF} + 24\,\Gamma_{CD}\,F^{ABCD}\Big]\Big)
\end{aligned}
$$

Penrose Limit

$$\mathrm{d}s^2_{AdS_4\times S^7} = R_A^2\Big\{-\cosh^2\rho\cdot\mathrm{d}t^2+\mathrm{d}\rho^2+\sinh^2\rho\cdot\mathrm{d}\Omega_2^2\Big\} + R_S^2\Big\{\cos^2\theta\cdot\mathrm{d}\varphi^2+\mathrm{d}\theta^2+\sin^2\theta\cdot\mathrm{d}\Omega_5'^{\,2}\Big\}$$

Choose a null geodesic

$$R_S = 2R_A \qquad t = 2\varphi \qquad \rho = \theta = 0$$

Take $R_A \longrightarrow \infty$ (*Penrose Limit*):

$$\mathrm{d}s^2 = -2\mathrm{d}x^+\mathrm{d}x^- + G_{++}\cdot(\mathrm{d}x^+)^2+\sum_{I=1}^{9}(\mathrm{d}x^I)^2$$

where

$$x = R_A\rho \qquad y = 2R_A\theta$$

$$x^+ = \frac{1}{2}(t+2\varphi)\cdot\frac{3}{\mu} \qquad x^- = R_A^2(t-2\varphi)\cdot\frac{\mu}{3}$$

Field Equations for Fluctuations

δg_{MN}:

$$\delta\mathcal{R}_{MN} = -\frac{1}{2}\Big\{\nabla_N\nabla_M h_P{}^P - \nabla_N\nabla^P h_{MP} - \nabla_M\nabla^P h_{NP}\Big\} + \frac{1}{2}\widehat{\Delta} h_{MN}$$

$$\widehat{\Delta} h_{MN} = -\nabla_P\nabla^P h_{MN} - 2\,R_{MPNQ}\,h^{PQ} + \mathcal{R}_M{}^P\,h_{PN} + \mathcal{R}_N{}^P\,h_{PM}$$

$\widehat{\Delta}$: Lichnerowicz operator

$$\delta\mathcal{R}_{MN} = -\frac{1}{144}\,h_{MN}\,F_{PQRS}\,F^{PQRS} - \frac{1}{72}\,g_{MN}\,F^{PQRS}\,\mathcal{F}_{PQRS} + \frac{1}{36}\,h^{PU}\,g_{MN}\,F_{PQRS}\,F_U{}^{QRS} - \frac{1}{4}\,h^{PU}\,F_{MPQR}\,F_{NU}{}^{QR} + \frac{1}{12}\Big(\mathcal{F}_{MPQR}\,F_N{}^{PQR} + \mathcal{F}_{NPQR}\,F_M{}^{PQR}\Big)$$

$\delta\Psi_M$:

$$0 \;=\; \Gamma^{MNP}\, D_N \psi_P + \frac{1}{96}\,\widetilde{\Gamma}^{MNPQRS}\, F_{PQRS}\, \psi_N$$

δC_{MNP}:

$$\begin{aligned} 0 \;=\; & e\, \nabla^Q \mathcal{F}_{QMNP} \\ & - e \left\{ F_{SMNP} \left(\nabla^Q h_Q{}^S - \frac{1}{2} \partial^S h_Q{}^Q \right) + F_{QSNP} \nabla^Q h_M{}^S \right. \\ & \qquad \left. + F_{QMSP} \nabla^Q h_N{}^S + F_{QMNS} \nabla^Q h_P{}^S \right\} \\ & - \frac{1}{576}\, \varepsilon_{MNP}{}^{QRSUVWXY}\, \mathcal{F}_{QRSU}\, F_{VWXY} \end{aligned}$$