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Gauged Linear Sigma Models for Noncompact Calabi-Yau Varieties

[邦題] 2次元超対称ゲージ場の量子論を用いた非コンパクトなカラビヤウ多様体の解析

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Gauged linear sigma model E. Witten (1993), K. Hori and C. Vafa (2000)

$\mathcal{N}=(2,2)$ SUSY gauge theory with matters $(\mathrm{FI}:\ t \equiv r - i\theta)$

$$\mathscr{L} = \int d^4\theta \Big\{ -\frac{1}{e^2}\overline{\Sigma}\Sigma + \sum_a \overline{\Phi}_a \, e^{2Q_a V} \Phi_a \Big\} + \Big(\frac{1}{\sqrt{2}} \int d^2\widetilde{\theta} \, (-\Sigma \, t) + c.c. \Big) + \Big(\int d^2\theta \, W_{\mathrm{GLSM}}(\Phi_a) + c.c. \Big)$$

- Φ_a : charged chiral superfield, $\overline{D}_{\pm}\Phi_a = 0$
  Σ : twisted chiral superfield, $\overline{D}_{+}\Sigma = D_{-}\Sigma = 0$, $\Sigma = \frac{1}{\sqrt{2}}\overline{D}_{+}D_{-}V$
- There exist at least two phases:

  $\mathrm{FI} \gg 0$: differential-geometric phase $\longrightarrow$ SUSY NLSM

  $\mathrm{FI} \ll 0$: algebro-gemetric phase $\longrightarrow$ LG, orbifold, SCFT

- Calabi-Yau/Landau-Ginzburg correspondence

  harmonic forms $\longleftrightarrow$ NS-NS chiral primary states

- "Mirror geometry" appears in the T-dual theory in terms of twisted chiral superfields Y_a

$$Y_a + \overline{Y}_a \equiv 2\overline{\Phi}_a \, e^{2Q_a V} \Phi_a$$

Effective theories

The potential energy density is given by

$$\mathcal{U}(\phi,\sigma) = \frac{e^2}{2}\mathcal{D}^2 + \sum_a |F_a|^2 + \mathcal{U}_\sigma(\phi,\sigma)$$

$$\mathcal{D} = \frac{1}{e^2}D = r - \sum_a Q_a|\phi_a|^2 \, , \quad \overline{F}_a = -\frac{\partial}{\partial\phi_a}W_{\mathrm{GLSM}}(\phi) \, , \quad \mathcal{U}_\sigma(\phi,\sigma) = 2|\sigma|^2 \sum_a Q_a^2|\phi_a|^2$$

The supersymmetric vacuum manifold $\mathcal{M}$ is defined by

$$\mathcal{M} = \Big\{(\phi_a,\sigma) \in \mathbb{C}^n \,\Big|\, \mathcal{D} = F_a = \mathcal{U}_\sigma = 0\Big\}\Big/U(1)$$

In the IR limit $e \to \infty$, there appears the supersymmetric NLSM on $\mathcal{M}$ whose coupling is

$$r = \frac{1}{g^2}$$

Renormalization of the FI parameter is

$$\boxed{r_0 = r_R + s\cdot\log\Big(\frac{\Lambda_{\mathrm{UV}}}{\mu}\Big), \quad s = \sum_a Q_a}$$

Thus we find that

$$s > 0 \quad \longrightarrow \quad \text{the theory is asymptotic free}$$

$$s = 0 \quad \longrightarrow \quad \text{the theory is conformal}$$

$$s < 0 \quad \longrightarrow \quad \text{the theory is infrared free}$$

Gauged Linear Sigma Model for $\mathcal{O}(-N+\ell)$ bundle on $\mathbb{CP}^{N-1}[\ell]$

chiral superfield	S_1	$\cdots$	S_N	P_1	P_2
$U(1)$ charge	1	$\cdots$	1	$-\ell$	$-N+\ell$

$$W_{\text{GLSM}} = P_1 \cdot G_\ell(S_i)$$

$G_\ell(S_i)$: homogeneous polynomial of degree ℓ

potential energy density:

$$\mathcal{U} = \frac{e^2}{2}\mathcal{D}^2 + \big|G_\ell(s)\big|^2 + \sum_{i=1}^{N} \big|p_1 \partial_i G_\ell(s)\big|^2 + \mathcal{U}_\sigma$$

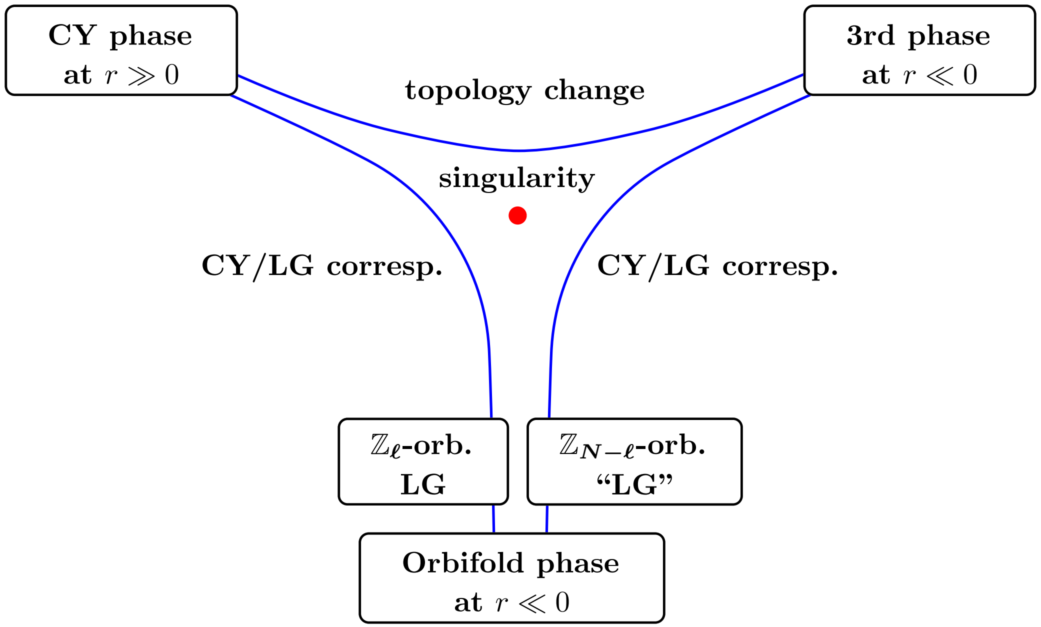
$$\mathcal{D} = r - \sum_{i=1}^{N} |s_i|^2 + \ell|p_1|^2 + (N-\ell)|p_2|^2$$

$$\mathcal{U}_\sigma = +2|\sigma|^2 \Big\{ \sum_{i=1}^{N} |s_i|^2 + \ell^2|p_1|^2 + (N-\ell)^2|p_2|^2 \Big\}$$

Let us analyze SUSY vacuum manifold $\mathcal{U}=0$ and massless effective theories

CY/LG correspondence and topology change

The four theories are related to each other via CY/LG correspondence and topology change:



Notice that the above relation is just a conjectured one because we still have no mathematical techniques to check the topological aspects on the noncompact CY.

We also notice that we have obtained various massless effective theories by decomposing all massive modes. Thus they are just approximate descriptions. However the T-dual theory of the GLSM is so powerful to obtain the exact effective theories. Analyzing them exact theories we will re-investigate the massless effective theories in the original GLSM.

T-dual Theory

$$\mathscr{L} = \int d^4\theta \Big\{ -\frac{1}{e^2}\overline{\Sigma}\Sigma - \sum_a \Big(\frac{1}{2}(Y_a + \overline{Y}_a)\log(Y_a + \overline{Y}_a)\Big)\Big\} + \Big(\frac{1}{\sqrt{2}}\int d^2\widetilde{\theta}\,\widetilde{W} + c.c.\Big)$$

$$\widetilde{W} = \Sigma\Big(\sum_{i=1}^N Y_i - \ell Y_{P_1} - (N-\ell)Y_{P_2} - t\Big) + \sum_{i=1}^N e^{-Y_i} + e^{-Y_{P_1}} + e^{-Y_{P_2}}$$

Period integral : $\widehat{\Pi} \equiv \int d\Sigma \prod_{i=1}^N dY_i\, dY_{P_1}\, dY_{P_2}\, (\ell\Sigma)\, \exp\big(-\widetilde{W}\big)$

chiral superfield	S_1	S_2	$\cdots$	S_N	P_1	P_2
$U(1)$ charge	1	1	$\cdots$	1	$-\ell$	$-N+\ell$
twisted chiral	Y_1	Y_2	$\cdots$	Y_N	Y_{P_1}	Y_{P_2}

$$2\overline{\Phi}_a\, e^{2Q_aV}\,\Phi_a = Y_a + \overline{Y}_a$$

$U(1)$ phase rotation symmetry on Φ_a ➔ shift symmetry on Y_a: $Y_a \equiv Y_a + 2\pi i$

In the IR limit $e \to \infty$, the gauge field Σ is no longer dynamical and should be integrated out.

in order to obtain LG theory or geometry, we replace Σ to $\Sigma \to \frac{\partial}{\partial t}$ or $\Sigma \to \frac{\partial}{\partial Y_P}$

Twisted Landau-Ginzburg theory:

Solve Y_{P_1} by using the constraint derived from integrating out Σ:

$$Y_{P_1} = \frac{1}{\ell}\Big\{ t - \sum_{i=1}^{N} Y_i + (N-\ell) Y_{P_2} \Big\}$$

Field re-definition preserving canonical measure in $\widehat{\Pi}$:

$$X_i \equiv \mathrm{e}^{-\frac{1}{\ell}Y_i}\,, \quad X_{P_2} \equiv \mathrm{e}^{\frac{N-\ell}{\ell}Y_{P_2}}\,, \quad X_i \to \omega_i X_i\,, \quad X_{P_2} \to \omega_{P_2} X_{P_2}\,, \quad (\mathbb{Z}_\ell)^N \text{ symmetry}$$

Thus we obtain the twisted LG superpotential:

$$\Big\{ \widetilde{W}_\ell = X_1^\ell + \cdots + X_N^\ell + X_{P_2}^{-\frac{\ell}{N-\ell}} + \mathrm{e}^{t/\ell} X_1 \cdots X_N X_{P_2} \Big\} \Big/ (\mathbb{Z}_\ell)^N$$

The negative power term describes $\mathcal{N}=2$ Kazama-Suzuki model on $SL(2,\mathbb{R})_k/U(1)$:

$$\frac{\ell}{N-\ell} = k = \frac{2}{Q^2}$$

Thus we argue that

this effective theory is the LG minimal model coupled to the KS model with $(\mathbb{Z}_\ell)^N$ orbifold symmetry

Return to the original GLSM

Recall the following two arguments:

- $\mathcal{N}=2$ SCFT on $SL(2,\mathbb{R})_k/U(1)$ is equivalent to $\mathcal{N}=2$ Liouville theory via T-duality
- If a CFT $\mathcal{C}$ has an abelian discrete symmetry group Γ, the orbifold CFT $\mathcal{C}'=\mathcal{C}/\Gamma$ has a symmetry group Γ' which is isomorphic to Γ. Furthermore a new orbifold CFT $\mathcal{C}'/\Gamma'$ is identical to the original CFT $\mathcal{C}$.

Thus we insist that

"$\{$CFT on $\mathbb{C}^1 \otimes$ LG with $W_{\mathrm{LG}}=\langle p_1\rangle G_\ell(S)\}/\mathbb{Z}_\ell$" in the original GLSM

is described by

$\{\mathcal{N}=2$ Liouville theory coupled to the LG minimal model with $W_{\mathrm{LG}}\}/\mathbb{Z}_\ell$

as an exact effective theory

Summary

- **We found three non-trivial phases and four effective theories in the GLSM**

  two CY sigma models

  two orbifolded LG theories coulped to 1-dim. SCFT

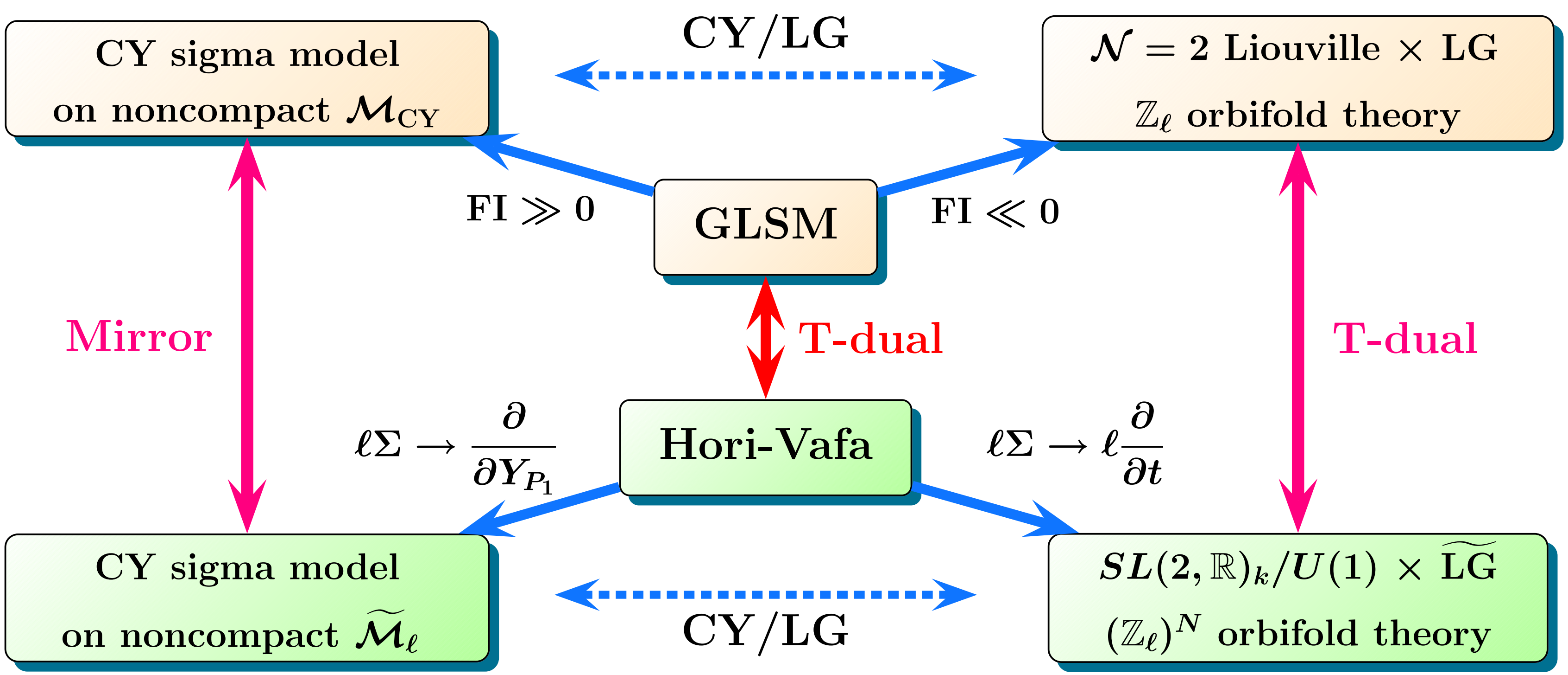
- **We constructed four exact effective theories in the T-dual theory**

  two NLSMs on mirror CY geometries

  two orbifolded LG theories including a term with negative power $-k$

  This term represents a gauged WZW model on $SL(2,\mathbb{R})_k/U(1)$ at level k

- **We argue that the LG theories in the orignal GLSM can be interpreted as $\mathcal{N}=2$ Liouville theories coupled to LG minimal models**


CY sigma model
on noncompact $\mathcal{M}_{\rm CY}$
CY/LG
$\mathcal{N}=2$ Liouville × LG
$\mathbb{Z}_\ell$ orbifold theory
FI ≫ 0
GLSM
FI ≪ 0
Mirror
T-dual
T-dual
$\ell\Sigma \to \frac{\partial}{\partial Y_{P_1}}$
Hori-Vafa
$\ell\Sigma \to \ell\frac{\partial}{\partial t}$
CY sigma model
on noncompact $\widetilde{\mathcal{M}}_\ell$
CY/LG
$SL(2,\mathbb{R})_k/U(1) \times \widetilde{\rm LG}$
$(\mathbb{Z}_\ell)^N$ orbifold theory

linear dilaton CFT and Liouville theory

$$\mathbb{R}^{9,1} = \underbrace{\mathbb{R}^{d-1,1}}_{\text{free SCFT}} \times \underbrace{X^{2n}}_{\text{singular CY}} \sim \mathbb{R}^{d-1,1} \times \underbrace{\mathbb{R}_\phi \times S^1}_{\text{linear dilaton SCFT}} \times \underbrace{\mathcal{M}/U(1)}_{\mathcal{N}=2 \text{ Landau-Ginzburg}}$$

linear dilaton: $\Phi = -\frac{Q}{2}\phi$

Landau-Ginzburg: $W_{\rm LG} = F(Z_a)$, $F(\lambda^{r_a} Z_a) = \lambda F(Z_a)$

$$c_{\rm total} = c_d + c_{\rm dilaton} + c_{\rm LG} \quad \rightarrow \quad 15 = \frac{3}{2}d + \left(\frac{3}{2} + 3Q^2\right) + 3\sum_{a=1}^{n+1}(1 - 2r_a)$$

$$\mathcal{N}=2 \text{ “LG” on } \mathbb{R}_\phi \times S^1 \times \mathcal{M}/U(1)\text{:} \quad W = -\mu Z_0^{-k} + F(Z_a)$$

$$k = \frac{1}{r_\Omega} = \frac{2}{Q^2}, \qquad r_\Omega \equiv \textstyle\sum_a r_a - 1$$

linear dilaton SCFT on $\mathbb{R}_\phi \times S^1$ $\equiv$ “LG” with 「$W = -\mu Z_0^{-k}$」

$\equiv$ Kazama-Suzuki model on $SL(2,\mathbb{R})_k/U(1)$

$\overset{\text{T-dual}}{\equiv}$ Liouville theory of charge Q