Comments on Heterotic Flux Compactifications

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hep-th/0605247

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Seminar @ University of Tokyo, Komaba

July 19, 2006

Introduction

Purpose

Construct a realistic model of 4-dim. particle physics

- matter contents and their interactions
- gauge symmetry and its breaking
- gravity, cosmology
- etc., etc.

Supergravity with fluxes has a long, and interesting story

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▼ **Freund-Rubin Ansatz in** 11- **and** 10-**dimensional supergravities**

$$F_p = |F| \times (\text{vol.}) \text{ generates a cosmological constant in } AdS_q\text{-space}$$

Supergravity with fluxes has a long, and interesting story

▼ **Freund-Rubin Ansatz in 11- and 10-dimensional supergravities**

$$F_p = |F| \times (\textbf{vol.}) \textbf{ generates a cosmological constant in } AdS_q\textbf{-space}$$

▼ **Gauge/Gravity Dualities in type II theories**

$$F_p \textbf{ generates a superpotential in 4-dim. } \mathcal{N} = 1 \textbf{ SYM } \left(\textbf{e.g., } W = \int_{\mathrm{CY}_3} \Omega \wedge F_3\right)$$

Both solutions have given us new insights in higher-dimensional theories

▼ **Flux can be a torsion on a (compactified) geometry**

$$\delta\psi_M \;=\; \left(\partial_M + (\omega_M{}^{AB} - H_M{}^{AB})\Gamma_{AB} \right)\xi$$

If $H_M{}^{AB} \neq 0$, the geometry looks no longer a Kähler manifold.

$$\Downarrow$$

G-structure manifold

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▼ **How does the flux work in heterotic theory?**

$$\begin{cases} \text{anomaly cancellation in a miraculous way} \\ \text{gauge symmetry } E_8 \times E_8 \text{ or } SO(32) \end{cases}$$

Contents

▼ $SU(3)$-structure manifold

▼ Heterotic theory on $SU(3)$-structure manifold

- Vacuum configuration

- Towards low energy effective theory: (zero mode eqs., gauge groups)

▼ Summary and Discussions

useful Refs. Becker, Becker, Dasgupta and Green [hep-th/0301161]

Cardoso, Curio, Dall'Agata and Lüst [hep-th/0306088]

Becker and Tseng [hep-th/0509131]

etc., etc..

$SU(3)$-structure Manifold

G-structure group on an n-dim. manifold $\mathcal{M}$

$$\begin{cases} F(\mathcal{M}) \text{ frame bundle :} \quad \text{principal } GL(n) \text{ bundle} \\ G\text{-structure} \qquad\qquad\quad : \quad \text{principal } G \text{ sub-bundle of } F(\mathcal{M}) \end{cases}$$

$\Leftrightarrow$ **nowhere vanishing tensors on $\mathcal{M}$**

tensors			G-structure	
η_{ab}			$O(n)$	
η_{ab}	$\varepsilon_{a_1 \cdots a_n}$		$SO(n)$	
η_{ab}	$J_a{}^b$		$U(m)$	$J^2 = -1$
η_{ab}	$J_a{}^b$	$\Omega^{(m,0)}$	$SU(m)$	$(2m = n)$

6-dim. $SU(3)$-structure on manifold

Consider a geometry $\mathcal{K}_6$ with a Killing spinor equation including torsion

$$^\exists \xi \quad \text{s.t.} \quad \nabla^{(T)}\xi \;=\; 0$$

This is a definition of the geometry with $SU(3)$-structure.

6-dim. $SU(3)$-structure on manifold

Consider a geometry $\mathcal{K}_6$ with a Killing spinor equation including torsion

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This is a definition of the geometry with $SU(3)$-structure.

Invariant p-forms on the $SU(3)$-structure manifold:

2-form in $SO(6)$ ➡ **a real 2-form in $SU(3)$**

$$_6C_2 = 15 \qquad = \qquad 1 + 3 + \overline{3} + 8 \qquad : \quad J_{ab} = -i\,\xi^\dagger\,\Gamma_{ab}\,\xi$$

3-form in $SO(6)$ ➡ **an (almost) complex 3-form in $SU(3)$**

$$_6C_3 = 20 \qquad = \qquad 1 + 1 + 3 + \overline{3} + 6 + \overline{6} \; : \quad \Omega_{abc} = \xi^{\mathrm{T}}\,\Gamma_{abc}\,\xi$$

Furthermore

$$J \wedge \Omega \;=\; 0\,, \qquad\qquad J \wedge J \wedge J \;=\; \frac{3i}{4}\Omega \wedge \overline{\Omega} \;=\; 3! \times (\mathbf{vol.})_{\mathcal{K}_6}$$

Heterotic Theory

– vacuum, gauge group and zero modes –

Supergravity

▼ **Bosonic part of the Lagrangian** (without fermion condensations)

$$\mathscr{L} = \frac{1}{4}\sqrt{-G}\,e^{-2\Phi}\left[R(\omega) - \frac{1}{3}H_{MNP}H^{MNP} + 4(\nabla_M\Phi)^2 \right.$$

$$\left. -\alpha'\Big\{\mathrm{tr}\big(F_{MN}F^{MN}\big)\Big\}\right]$$

▼ **Bianchi identity** $\quad [\omega]$

$$\mathrm{d}H = -\alpha'\Big[\mathrm{tr}\{F \wedge F\}\Big]$$

Chapline and Manton [Phys. Lett. B120 (1983) 105]

(supergravity)

Supergravity

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▽ **Bianchi identity** $\quad[\omega]$

$$\mathrm{d}H \;=\; -\alpha'\Big[\mathrm{tr}\{F\wedge F\} - \mathrm{tr}\{R(\omega)\wedge R(\omega)\}\Big]$$

Green and Schwarz [Phys. Lett. B149 (1984) 117]

(anomaly cancellation)

(worldsheet 1-loop β-function)

Supergravity QL

▽ **Bosonic part of the Lagrangian** (without fermion condensations)

$$\mathscr{L} \;=\; \frac{1}{4}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\Bigg[R(\omega) - \frac{1}{3}H_{MNP}H^{MNP} + 4(\nabla_M\Phi)^2$$

$$-\,\alpha'\Big\{\mathrm{tr}\big(F_{MN}F^{MN}\big) - \mathrm{tr}\big(R_{MN}(\omega_+)R^{MN}(\omega_+)\big)\Big\}\Bigg]$$

▽ **Bianchi identity** $\quad [\omega_+ = \omega + H]$

$$\mathrm{d}H \;=\; -\alpha'\Big[\mathrm{tr}\big\{F \wedge F\big\} - \mathrm{tr}\big\{R(\omega_+) \wedge R(\omega_+)\big\}\Big]$$

Hull [Phys. Lett. B178 (1986) 357]

Bergshoeff and de Roo [Nucl. Phys. B328 (1989) 439]

(worldsheet 2-loop β-function)

Towards 4-dim. Physics...

▼ **Study vacuum configuration**

     • **SUSY** variations $\longrightarrow$ **geometry with $SU(3)$-structure**

▼ **Investigate low energy effective theory**

     • **Gauge symmetry**

     • **Zero mode equations**

     • **Norm of fields**

Equations of Motion

$$\Phi \; : \quad 0 \; = \; -R(\omega) + \frac{1}{3} H_{MNP} H^{MNP} + 4(\nabla_M \Phi)^2 - 4\nabla_M^2 \Phi$$

$$+ \alpha' \Big[\mathrm{tr}(F_{MN} F^{MN}) - \mathrm{tr}\{R_{MN}(\omega_+) R^{MN}(\omega_+)\} \Big]$$

$$G_{MN} \; : \quad 0 \; = \; R_{MN}(\omega) - H_{MPQ} H_N{}^{PQ} + 2\nabla_M \nabla_N \Phi$$

$$- \frac{1}{2} G_{MN} \Big[R(\omega) - \frac{1}{3} H_{PQR} H^{PQR} - 4(\nabla_P \Phi)^2 + 4\nabla_P^2 \Phi \Big]$$

$$+ \frac{\alpha'}{2} G_{MN} \Big[\mathrm{tr}(F_{MN} F^{MN}) - \mathrm{tr}\{R_{MN}(\omega_+) R^{MN}(\omega_+)\} \Big]$$

$$- 2\alpha' \Big[\mathrm{tr}(F_{MP} F_N{}^P) - \mathrm{tr}\{R_{MP}(\omega_+) R_N{}^P(\omega_+)\} \Big]$$

$$- 2\alpha' \mathrm{e}^{2\Phi} \Big[2\nabla^P \nabla^Q_{(+)} \{\mathrm{e}^{-2\Phi} R_{MPNQ}(\omega_+)\} - \nabla^Q_{(+)} \{\mathrm{e}^{-2\Phi} R_{MPQR}(\omega_+)\} H_N{}^{PR}$$

$$- 2\nabla^P \{\mathrm{e}^{-2\Phi} R_{MPQR}(\omega_+) H_N{}^{QR}\} - 2\mathrm{e}^{-2\Phi} R_{MPQR}(\omega_+) H_N{}^{PS} H_S{}^{QR}$$

$$- \nabla^P \nabla^Q_{(+)} \{\mathrm{e}^{-2\Phi} R_{MNPQ}(\omega_+)\} + \nabla^P \{\mathrm{e}^{-2\Phi} R_{MNQR}(\omega_+) H_P{}^{QR}\} \Big]$$

$$B_{MN} \; : \quad 0 \; = \; \nabla^M (\mathrm{e}^{-2\Phi} H_{MNP})$$

$$\chi \; : \quad 0 \; = \; \slashed{D}(\omega - \tfrac{1}{3}H, A)\chi - \Gamma^M \chi \nabla_M \Phi + \frac{3}{2} \Gamma^M \Gamma^{AB} (F_{AB} + \hat{F}_{AB}) \Big(\psi_M + \frac{2}{3} \Gamma_M \lambda \Big)$$

Vacuum Configuration

$\mathcal{M}^{3,1} \times \mathcal{K}_6$
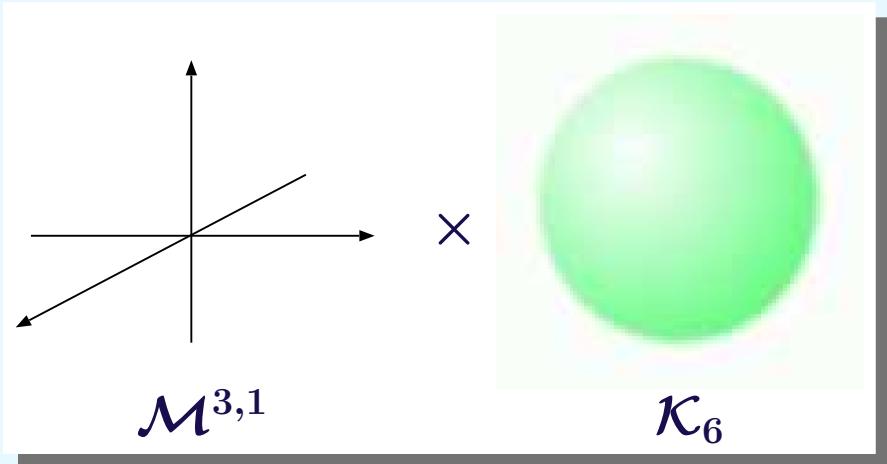
$$G_{MN}\,\mathrm{d}x^M\mathrm{d}x^N$$
$$= \mathrm{e}^{(\Phi-\widehat{D})/2}\Big(\eta_{\mu\nu}\,\mathrm{d}x^\mu\mathrm{d}x^\nu + g_{mn}\,\mathrm{d}y^m\mathrm{d}y^n\Big)$$

Vacuum Configuration
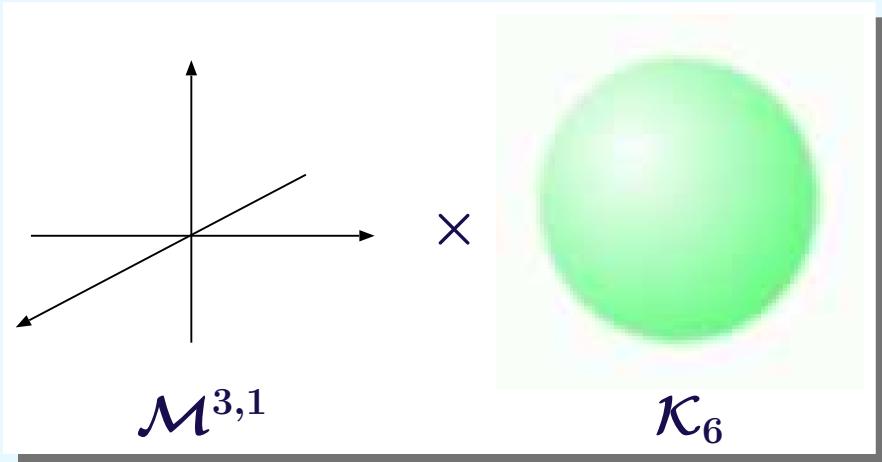
$$G_{MN}\, \mathrm{d}x^M \mathrm{d}x^N$$
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$$\boxed{Spin(9,1) \;\longrightarrow\; SL(2,\mathbb{C}) \times SU(4)}$$

$$16 \;=\; (2,4) + (\overline{2},\overline{4}) \;:\qquad \epsilon_+ \;=\; \eta_+ \otimes \xi_+ + \eta_- \otimes \xi_-$$

$\mathcal{N}=1$ **SUSY** on $\mathcal{M}^{3,1}$	$\longleftrightarrow$	1 **Killing spinor** ξ_+ on $\mathcal{K}_6$	$\longleftrightarrow$	$SU(3)$-**structure** on $\mathcal{K}_6$
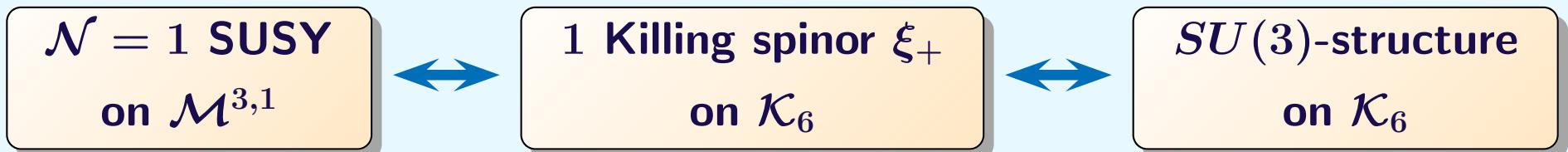
▽ SUSY variations

$$0 \equiv \delta\psi_m = D_m(\omega_-)\xi_+ \leftarrow \text{ Killing spinor eq.} \qquad [\omega_- = \omega - H]$$

$$J_{ab} = -i\xi_+^\dagger\,\Gamma_{ab}\,\xi_+ \quad : \qquad D_m(\omega_-)J_{ab} = 0$$

$$\Omega_{abc} = \xi_+^{\mathrm{T}}\,\Gamma_{abc}\,\xi_+ \qquad : \qquad D_m(\omega_-)\Omega_{abc} = 0$$

▽ SUSY variations

$0 \equiv \delta\psi_m = D_m(\omega_-)\xi_+ \leftarrow$ **Killing spinor eq.** $\qquad [\omega_- = \omega - H]$

$$J_{ab} = -i\xi_+^\dagger \Gamma_{ab} \xi_+ \quad : \qquad D_m(\omega_-)J_{ab} = 0$$

$$\Omega_{abc} = \xi_+^T \Gamma_{abc} \xi_+ \quad : \qquad D_m(\omega_-)\Omega_{abc} = 0$$

Furthermore "$0 \equiv \delta(\text{fermions})$" **indicates**

$$0 = R^{ab}{}_{mn}(\omega_-)J_{ab} \qquad : \quad c_1(R_-) \text{ vanishes}$$

$$0 = N_{mn}{}^p \qquad : \quad \mathcal{K}_6 \text{ is complex} \boxed{\text{T}}$$

$$H_{mnp} = H^0_{mnp} + \widehat{H}_{mnp}$$

$$H^{(3,0)} = 0 = H^{(0,3)} , \qquad 0 = H^0_{mnp}J^{np} , \qquad \widehat{H}_{mnp} = \frac{3}{2}J_{[mn}J_{p]}{}^q\nabla_q\Phi$$

$$F^{(2,0)} = 0 = F^{(0,2)} , \qquad 0 = F_{mn}J^{mn}$$

$$*(J \wedge dH) = -\nabla_m^2\Phi + (\nabla_m\Phi)^2 - \frac{1}{3}(H^0_{mnp})^2$$

$$\Phi = -\frac{1}{2}\log|\widetilde{\Omega}| + \Phi_0 , \qquad \widetilde{\Omega} \equiv e^{-2\Phi}\Omega , \qquad \overline{\partial}\widetilde{\Omega} = 0$$

$\boxed{dH = 0}$
$\boxed{dH \neq 0}$

$$\hat{R}_4(\hat{\Gamma}_0) \sim ||\hat{\nabla}_m(\Phi - \hat{D})||^2$$

$$F_{mn}J^{mn} = 0$$

$$\delta\psi_\mu = 0$$

$$\delta\chi = 0$$

$$\delta\psi_m = 0$$

$$\delta\lambda = 0$$

$$D_m(\omega_-)\Omega_{abc} = 0$$

$$-i\xi_+^\dagger\left([\gamma_{(k)}, \slashed{\nabla}\Phi] - \tfrac{1}{6}\{\gamma_{(k)}, \slashed{H}\}\right)\xi_+ = 0$$

$$D_m(\omega_-)J_{ab} = 0$$

$$N_{mnp} = 0$$

$$R^{ab}{}_{mn}(\omega_-)J_{ab} = 0$$

$$N_{mnp} - H_{mnp} = -3J_{[m}{}^q J_n{}^r H_{p]qr}$$

$$H_{mnp}J^{np} = 2J_m{}^q\nabla_q\Phi$$

$$J_{[m}{}^q\nabla_{|q|}J_{np]} = -2J_{[m}{}^q J_n{}^r H_{p]qr}$$

$$H_{mnp} = T^{(\mathsf{B})}_{mnp}$$

$$-2\mathrm{d}\Phi + \theta = 0$$

$$\mathrm{d}(\mathrm{e}^{-2\Phi}\Omega) = 0$$

$$\mathrm{d}(\mathrm{e}^{-2\Phi} * J) = 0$$

$$H = -\tfrac{1}{2} * \mathrm{e}^{2\Phi}\mathrm{d}(\mathrm{e}^{-2\Phi}J)$$
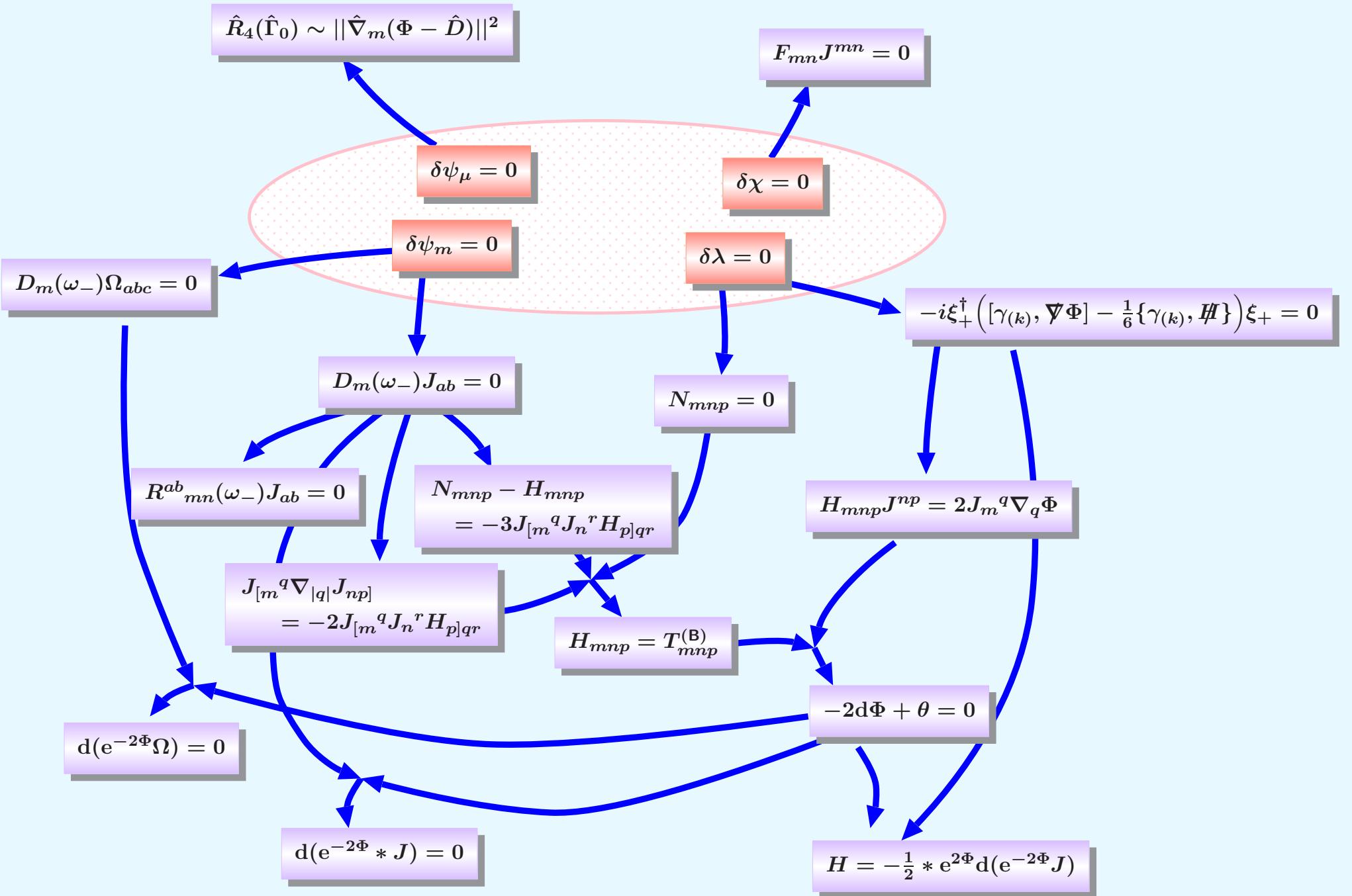
Gauge Symmetry Breaking

gauge algebra $\quad \mathcal{G} = \mathcal{F} \oplus \mathcal{F}_\perp \,, \qquad \mathcal{F}_\perp = \mathcal{H} \oplus \mathcal{Q} \,, \qquad [\mathcal{H}, \mathcal{F}] = 0$

with F_{mn} taking a value in $\mathcal{F}$

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with F_{mn} taking a value in $\mathcal{F}$

▼ $E_8 \rightarrow E_6 \times \underline{SU(3)} :$ $\qquad (\mathcal{G} = E_8, \quad \mathcal{F} = SU(3), \quad \mathcal{H} = E_6)$

$$248 = (78, 1) + (1, 8) + (27, 3) + (\overline{27}, \overline{3})$$

▼ $E_8 \rightarrow SO(16)$

$\quad \rightarrow SO(10) \times \underline{SO(6)} : \quad (\mathcal{G} = E_8, \quad \mathcal{F} = SO(6), \quad \mathcal{H} = SO(10))$

$$248 = 120_{SO(16) \text{ adj.}} + 128_{SO(16) \text{ spinor}}$$

$$= (45, 1) + (1, 15) + (10, 6) + (16, 4) + (\overline{16}, \overline{4})$$

Each breaking scenario deeply depends on the way

of embedding the holonomy group into the gauge groups.

Candidates:

$$
A \;\leftrightarrow\; \begin{cases} \omega_- \;:\; & SU(3) \text{ \textbf{holonomy}} \\[4pt] \omega_+ \;:\; & SO(6) \text{ \textbf{holonomy}} \qquad [\omega_\pm = \omega \pm H] \\[4pt] \quad \textbf{etc.} \end{cases}
$$

Candidates:

$$A \leftrightarrow \begin{cases} \omega_- & : \quad SU(3) \text{ holonomy} \\ \omega_+ & : \quad SO(6) \text{ holonomy} \qquad [\omega_\pm = \omega \pm H] \\ \quad \text{etc.} \end{cases}$$

with following constraints

$$R_{mnpq}(\omega_+) = R_{pqmn}(\omega_-) + (\mathrm{d}H)_{pqmn}$$
$$\mathrm{d}H = -\alpha'\Big[\mathrm{tr}(F \wedge F) - \mathrm{tr}\{R(\omega_+) \wedge R(\omega_+)\}\Big] \qquad \boxed{\begin{array}{l} \mathrm{d}H = 0 \\ \hline \mathrm{d}H \neq 0 \end{array}}$$

$$R_{pqmn}(\omega_-) : \qquad \text{type } (1,1) \text{ w/ indices } p, q$$
$$F : \qquad (1,1)\text{-form}$$
$$\mathrm{d}H : \qquad (2,2)\text{-form, higher order in } \alpha'$$
$$R_{mnpq}(\omega_+) : \qquad (1,1)\text{-form w/ indices } p, q + \text{higher order in } \alpha'$$

Mainly we consider $E_8 \rightarrow SO(10) \times SO(6)$ breaking: $A = \omega_+$

Zero mode equations

| # of massless modes on $\mathcal{M}^{3,1}$	$\longleftrightarrow$	# of zero modes on $\mathcal{K}_6$
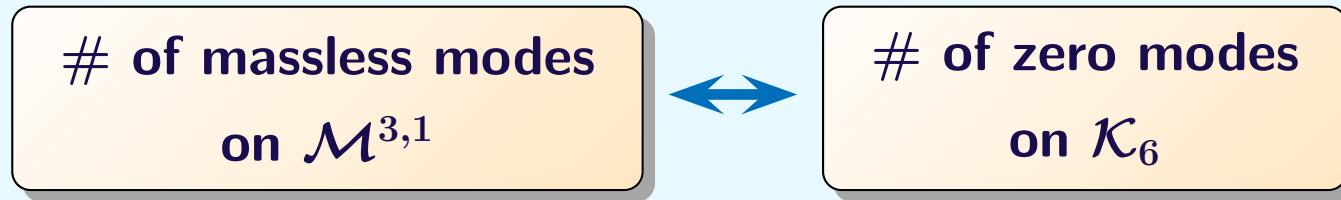
Zero mode eq. for gaugino:

$$
\begin{aligned}
0 &= \not{D}(\omega, A)\chi^0 - \frac{1}{12}H_{mnp}\,\Gamma^{mnp}\,\chi^0 \\
&= \not{D}(\hat{\omega}, A)\chi^0 \qquad \left[\,\hat{\omega} \equiv \omega - \tfrac{1}{3}H\,\right]
\end{aligned}
$$

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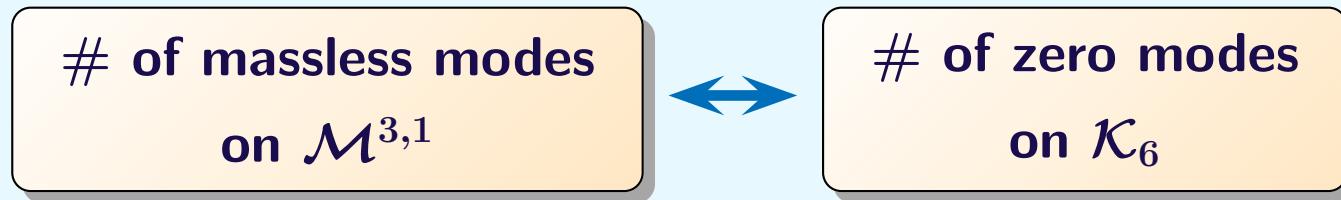
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&= \slashed{D}(\hat\omega, A)\chi^0 \qquad \left[\,\hat\omega \;\equiv\; \omega - \tfrac{1}{3}H\,\right]
\end{aligned}
$$

Decompose χ^0 into

$$
0 \;=\; \slashed{D}(\hat\omega)\chi^0_{\mathcal{H}} \quad \text{and} \quad 0 \;=\; \slashed{D}(\hat\omega, A_{\mathcal{Q}})\chi^0_{\mathcal{Q}}\,,
$$

which correspond to the neutral and charged matter fermions, respectively.

$$
[\text{\# of zero mode } \chi^0_{\mathcal{H}}] \;=\; \lim_{\beta\to 0}\mathrm{Tr}\,\Gamma_{(6+1)}\mathrm{e}^{-\beta\Delta_{\mathcal{H}}} \;\equiv\; \mathrm{index}\,\slashed{D}(\hat\omega)
$$

Evaluate the square of the Dirac operators with $\hat{\omega} = \omega - \frac{1}{3}H$:

$$\Delta_{\mathcal{H}} \equiv -[\slashed{D}(\hat{\omega})]^2$$
$$= D_m(\omega_-)^\dagger D^m(\omega_-) + V$$

$$\Delta_{\mathcal{Q}} \equiv -[\slashed{D}(\hat{\omega}, A_{\mathcal{Q}})]^2$$
$$= D_m(\omega_-)^\dagger D^m(\omega_-) + V + \frac{i}{2}F^{\mathcal{Q}}_{mn}\Gamma^{mn}$$

$$V = \frac{1}{4}\left[R(\omega) - \frac{1}{3}H_{mnp}H^{mnp} + \frac{1}{12}(\mathrm{d}H)_{mnpq}\Gamma^{mnpq}\right] \quad \boxed{\begin{array}{l}\mathrm{d}H = 0 \\ \mathrm{d}H \neq 0\end{array}}$$

The "potential" V plays a crucial role in

- **the zero mode equations of Klein-Gordon type**
- **the Atiyah-(Patodi)-Singer index density**
- **etc.**

Minimal embedding: $\mathrm{d}H = 0$

Zero mode equation tells us $\quad [V = \frac{1}{3}(H^0_{mnp})^2]$

$$0 = \left[D_m(\omega_-)^\dagger D^m(\omega_-) + \frac{1}{3}(H^0_{mnp})^2 \right] \chi^0_{\mathcal{H}}$$

$$\therefore \quad 0 = \int_{\mathcal{K}_6} \left[\left| D_m(\omega_-)\chi^0_{\mathcal{H}} \right|^2 + \frac{1}{3} \left| H^0_{mnp} \right|^2 \left| \chi^0_{\mathcal{H}} \right|^2 \right]$$

If no boundaries/singularities ⮕
$$\begin{cases} H^0 = 0 \quad \text{or} \\ \chi^0_{\mathcal{H}} = 0 \quad ⮕ \quad \text{no massless modes} \end{cases}$$

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If no boundaries/singularities $\Rightarrow$ $\begin{cases} H^0 = 0 \quad \text{or} \\ \chi^0_{\mathcal{H}} = 0 \quad \Rightarrow \quad \text{no massless modes} \end{cases}$

The condition $*(J \wedge \mathrm{d}H) = 0$ **denotes**

$$\frac{1}{2} \nabla^2_m \mathrm{e}^{-\Phi} = \frac{1}{3} \mathrm{e}^{-\Phi} (H^0_{mnp})^2$$

If there are no boundaries/singularities on $\mathcal{K}_6$, **then**

$$\frac{1}{3} \int_{\mathcal{K}_6} \mathrm{e}^{-\Phi} \left| H^0_{mnp} \right|^2 = \frac{1}{2} \int_{\mathcal{K}_6} \nabla^2_m \mathrm{e}^{-\Phi} = 0$$

This means $H^0 = 0$ $\Rightarrow$ $\Phi = \text{const.}$ $\Rightarrow$ $\mathcal{K}_6 = \mathrm{CY}_3$
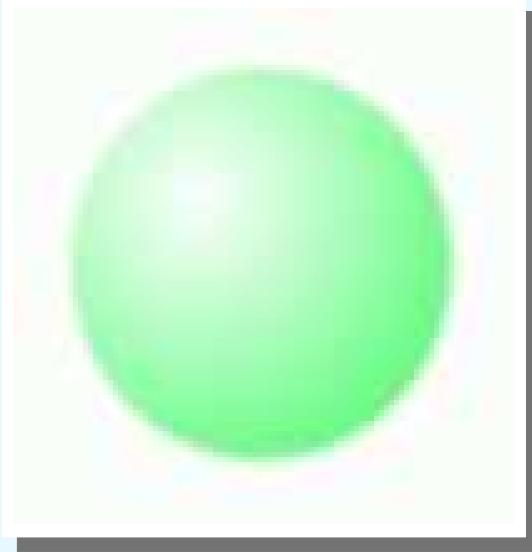
Without boundaries/singularities on $\mathcal{K}_6$:

- all fluxes are trivial $H = \mathrm{d}\Phi = 0$
- $\mathcal{K}_6 = \mathrm{CY}_3$
- $\omega_+ = \omega_- = \omega$, $E_8 \to E_6 \times SU(3)$
- # of zero modes — AS index theorem

Candelas, Horowitz, Strominger and Witten

[Nucl. Phys. B258 (1985) 46]
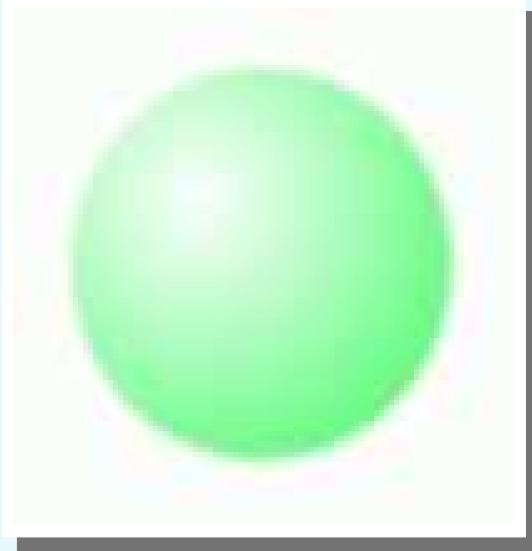
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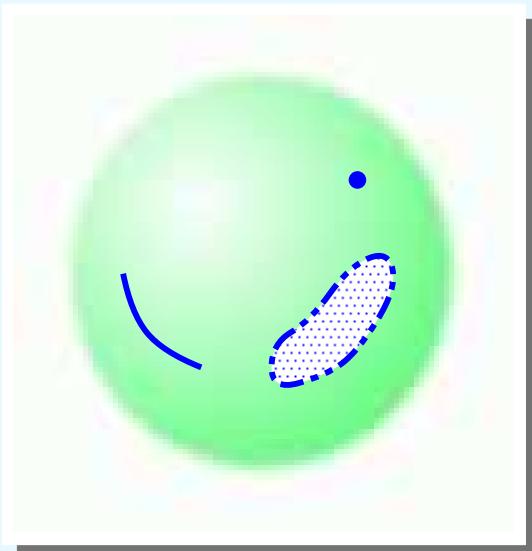
With boundaries/singularities on $\mathcal{K}_6$:

- non-trivial fluxes can exist

  $\partial_m \Phi = 0$, $H^0_{mnp} \neq 0$: conformally balanced ⊤
- $E_8 \to SO(10) \times SO(6)$
- $\chi^0_{\mathcal{H}}$ lives in the boundaries
- \# of zero modes — APS index theorem

Non-minimal embedding: $\mathrm{d}H \neq 0$ SUSY A V

In this case we should notice the α'-ordering in the Lagrangian with keeping

$$\frac{\alpha'}{L^2} \ll 1 \qquad L = \text{linear size of } \mathcal{K}_6$$

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Notice that the scaling orders of variables:

$$F_{mn} \sim R^p{}_{qmn}(\omega) \sim \frac{1}{L^2}$$

$$(\nabla_m \Phi)^2 \sim \mathrm{d}H \sim (H_{mnp})^2 \sim \frac{\alpha'}{L^4}$$

$$R^p{}_{qmn}(\omega_+) = R^p{}_{qmn}(\omega) + 2\nabla_{[m}H^p{}_{|q|n]} + 2H^p{}_{r[m}H^r{}_{|q|n]}$$

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Then, in the leading order in α',

we replace $R_{abmn}(\omega_+)$ to $R_{abmn}(\omega)$ in the Lagrangian and the Bianchi identity

$$\rightarrow \quad \mathrm{d}H = -\alpha'\Big[\mathrm{tr}(F \wedge F) - \mathrm{tr}\big\{R(\omega) \wedge R(\omega)\big\}\Big]$$

Combining equations of motion and SUSY conditions, we obtain

$$\text{tr}\left(R_{mn}R^{mn}\right) - \text{tr}(F_{mn}F^{mn}) \; = \; -2 * \left[J \wedge \left(\text{tr}(R \wedge R) - \text{tr}(F \wedge F)\right)\right] + \mathcal{O}(\alpha')$$

$$0 \; = \; \frac{1}{2}\nabla_m^2 \mathrm{e}^{-2\Phi} - \frac{1}{3}\mathrm{e}^{-2\Phi}(H_{mnp})^2 - \mathrm{e}^{-2\Phi} * (J \wedge \mathrm{d}H) + \mathcal{O}(\alpha'^2)$$

Then, within the linear order in α', we find

$$\nabla_m^2 \mathrm{e}^{-2\Phi} \; = \; \mathrm{e}^{-2\Phi}\left[\frac{2}{3}\big|H_{mnp}\big|^2 + \alpha'\big(\text{tr}\big|F_{mn}\big|^2 - \text{tr}\big|R_{mn}\big|^2\big)\right]$$

Integral on a smooth manifold $\mathcal{K}_6$:

$$\int_{\mathcal{K}_6}\mathrm{e}^{-2\Phi}\left[\frac{2}{3}\big|H_{mnp}\big|^2 + \alpha'\text{tr}\big|F_{mn}\big|^2\right] \; = \; \int_{\mathcal{K}_6}\mathrm{e}^{-2\Phi}\left[\alpha'\text{tr}\big|R_{mn}\big|^2\right]$$

$$\text{with} \quad \text{tr}\big|F_{mn}\big|^2 \; \neq \; \text{tr}\big|R_{mn}\big|^2$$

Smooth compactification scenario is possible!

Summary and Discussions

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▼ **Vacuum configuration of the flux compactifications in heterotic theory**

▼ **No-go theorem on smooth manifolds with $H \neq 0$ and $\mathrm{d}H = 0$**

▼ **Possibility of smooth compactifications with $H \neq 0$ and $\mathrm{d}H \neq 0$**

Summary and Discussions

▼ **Vacuum configuration of the flux compactifications in heterotic theory**

▼ **No-go theorem on smooth manifolds with $H \neq 0$ and $\mathrm{d}H = 0$**

▼ **Possibility of smooth compactifications with $H \neq 0$ and $\mathrm{d}H \neq 0$**

▼**?** **# of zero modes under the condition $\mathrm{d}H \neq 0$**

modification of the Atiyah-(Patodi)-Singer index theorem

▼ **other possibilities of gauge symmetry breaking**

▼ **compactifications on non-complex geometries** SUSY

Frey and Lippert [hep-th/0507202]

Manousselis, Prezas and Zoupanos [hep-th/0511122]

Appendix

$$\mathscr{L}_{\text{total}} = \mathscr{L}_0(R) + \mathscr{L}_\beta(F^2) + \mathscr{L}_\alpha(R^2) \quad \boxed{\text{L}}$$

$$
\begin{aligned}
\mathscr{L}_0(R) &= \frac{1}{2\kappa_{10}^2}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\bigg[R(\omega) - \frac{1}{3}H_{MNP}H^{MNP} + 4(\nabla_M\Phi)^2 - \overline{\psi}_M\Gamma^{MNP}D_N(\omega)\psi_P + 16\,\overline{\lambda}\slashed{D}(\omega)\lambda \\
&\quad + 8\,\overline{\lambda}\Gamma^{MN}D_M(\omega)\psi_N + 8\,\overline{\psi}_M\Gamma^N\Gamma^M\lambda\,(\nabla_N\Phi) - 2\,\overline{\psi}_M\Gamma^M\psi_N\,(\nabla^N\Phi) \\
&\quad + \frac{1}{12}H^{PQR}\Big\{\overline{\psi}_M\Gamma^{[M}\Gamma_{PQR}\Gamma^{N]}\psi_N + 8\,\overline{\psi}_M\Gamma^M{}_{PQR}\lambda - 16\,\overline{\lambda}\Gamma_{PQR}\lambda\Big\} \\
&\quad + \frac{1}{48}\overline{\psi}^M\Gamma^{ABC}\psi_M\Big\{2\overline{\lambda}\Gamma_{ABC}\lambda + \overline{\lambda}\Gamma_{ABC}\Gamma^N\psi_N - \frac{1}{4}\overline{\psi}^N\Gamma_{ABC}\psi_N - \frac{1}{8}\overline{\psi}^N\Gamma_N\Gamma_{ABC}\Gamma^P\psi_P\Big\}\bigg] \\[6pt]
\mathscr{L}_\beta(F^2) &= \frac{1}{2\kappa_{10}^2}\frac{\kappa_{10}^2}{2g_{10}^2}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\bigg[-\mathrm{tr}(F_{MN}F^{MN}) - 2\,\mathrm{tr}\big\{\overline{\chi}\slashed{D}(\omega,A)\chi\big\} + \frac{1}{6}\mathrm{tr}\big(\overline{\chi}\Gamma^{ABC}\chi\big)\hat{H}_{ABC} \\
&\quad - \frac{1}{2}\mathrm{tr}\big\{\overline{\chi}\Gamma^M\Gamma^{AB}(F_{AB}+\hat{F}_{AB})\big\}\Big(\psi_M + \frac{2}{3}\Gamma_M\lambda\Big) - \frac{1}{48}\mathrm{tr}(\overline{\chi}\Gamma^{ABC}\chi)\,\overline{\psi}_M\big(4\Gamma_{ABC}\Gamma^M + 3\Gamma^M\Gamma_{ABC}\big)\lambda \\
&\quad + \frac{1}{12}\mathrm{tr}(\overline{\chi}\Gamma^{ABC}\chi)\overline{\lambda}\Gamma_{ABC}\lambda - \frac{\beta}{96}\mathrm{tr}(\overline{\chi}\Gamma^{ABC}\chi)\,\mathrm{tr}(\overline{\chi}\Gamma_{ABC}\chi)\bigg] \\[6pt]
\mathscr{L}_\alpha(R^2) &= \frac{1}{2\kappa_{10}^2}\frac{\kappa_{10}^2}{2g_{10}^2}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\bigg[-R_{ABMN}(\omega_+)R^{ABMN}(\omega_+) - 2\overline{\psi}^{AB}\slashed{D}(\omega(e,\psi),\omega_+)\psi_{AB} + \frac{1}{6}\overline{\psi}^{AB}\Gamma^{MNP}\psi_{AB}\hat{H}_{MNP} \\
&\quad + \frac{1}{2}\overline{\psi}_{AB}\Gamma^M\Gamma^{NP}\big\{R^{AB}{}_{NP}(\omega_+) + \hat{R}^{AB}{}_{NP}(\omega_+)\big\}\Big(\psi_M + \frac{2}{3}\Gamma_M\lambda\Big) \\
&\quad - \frac{1}{48}\overline{\psi}_{AB}\Gamma^{CDE}\psi_{AB}\cdot\overline{\psi}_M\big(4\Gamma_{CDE}\Gamma^M + 3\Gamma^M\Gamma_{CDE}\big)\lambda \\
&\quad + \frac{1}{12}\overline{\psi}^{AB}\Gamma^{CDE}\psi_{AB}\big(\overline{\lambda}\Gamma_{CDE}\lambda\big) - \frac{\alpha}{96}\overline{\psi}^{AB}\Gamma^{FGH}\psi_{AB}\big(\overline{\psi}^{CD}\Gamma_{FGH}\psi_{CD}\big)\bigg]
\end{aligned}
$$

$$\delta_0 e_M{}^A = \frac{1}{2}\bar{\epsilon}\Gamma^A\psi_M$$

$$\delta_0\psi_M = \left(\partial_M + \frac{1}{4}\omega_{-M}{}^{AB}\Gamma_{AB}\right)\epsilon + \left\{\epsilon(\overline{\psi}_M\lambda) - \psi_M(\bar{\epsilon}\lambda) + \Gamma^A\lambda(\overline{\psi}_M\Gamma_A\epsilon)\right\}$$

$$\delta_0 B_{MN} = \bar{\epsilon}\Gamma_{[M}\psi_{N]}$$

$$\delta_0\lambda = -\frac{1}{4}\not{D}\Phi\,\epsilon + \frac{1}{24}\Gamma^{ABC}\epsilon\left(\hat{H}_{ABC} - \frac{1}{4}\overline{\lambda}\Gamma_{ABC}\lambda\right)$$

$$\delta_0\Phi = -\bar{\epsilon}\lambda$$

$$\delta_0 A_M = \frac{1}{2}\bar{\epsilon}\Gamma_M\chi$$

$$\delta_0\chi = -\frac{1}{4}\Gamma^{AB}\epsilon\,\hat{F}_{AB} + \left\{\epsilon(\overline{\chi}\lambda) - \chi(\bar{\epsilon}\lambda) + \Gamma^A\lambda(\overline{\chi}\Gamma_A\epsilon)\right\}$$

$$\delta_\beta\psi_M = \frac{\beta}{192}\Gamma^{ABC}\Gamma_M\epsilon\,\mathrm{tr}\left(\overline{\chi}\Gamma_{ABC}\chi\right)$$

$$\delta_\beta B_{MN} = -\beta\,\mathrm{tr}\left\{A_{[M}\delta_0 A_{N]}\right\}$$

$$\delta_\beta\lambda = \frac{\beta}{384}\Gamma^{ABC}\epsilon\,\mathrm{tr}\left(\overline{\chi}\Gamma_{ABC}\chi\right)$$

$$\delta_\alpha\psi_M = \frac{\alpha}{192}\Gamma^{CDE}\Gamma_M\epsilon\,\overline{\psi}^{AB}\Gamma_{CDE}\psi_{AB}$$

$$\delta_\alpha B_{MN} = -\alpha\,\omega_{+[M}{}^{AB}\,\delta_0\omega_{+N]}{}^{AB}$$

$$\delta_\alpha\lambda = \frac{\alpha}{384}\Gamma^{CDE}\Gamma_M\epsilon\,\overline{\psi}^{AB}\Gamma_{CDE}\psi_{AB}$$

Bergshoeff and de Roo [Nucl. Phys. B328 (1989) 439]

Supergravity QL

▽ **Bosonic part of the Lagrangian** (without fermion condensations)

$$\mathscr{L} = \frac{1}{4}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\left[R(\omega) - \frac{1}{3}H_{MNP}H^{MNP} + 4(\nabla_M\Phi)^2\right.$$

$$\left. -\alpha'\Big\{\mathrm{tr}\big(F_{MN}F^{MN}\big) - \mathrm{tr}\big(R_{MN}(\widetilde{\omega})R^{MN}(\widetilde{\omega})\big)\Big\}\right]$$

▽ **Bianchi identity** $\quad [\omega_+ = \omega + H \to \widetilde{\omega}]$

$$\mathrm{d}H = -\alpha'\Big[\mathrm{tr}\{F \wedge F\} - \mathrm{tr}\{R(\widetilde{\omega}) \wedge R(\widetilde{\omega})\}\Big]$$

Hull [Phys. Lett. B167 (1986) 51]

(worldsheet 2-loop β-function)

Let κ be a contorsion in $\nabla^{(T)}$ with acting on the $SU(3)$ Killing spinor ξ:

$$0 \;=\; \nabla^{(T)}\xi \;=\; (\nabla + \kappa^0 + \kappa^{\mathfrak{g}})\xi$$

where we decomposed $\kappa \equiv \kappa^0 + \kappa^{\mathfrak{g}}$ in such a way as $\kappa^{\mathfrak{g}}\xi = 0$ (where $\mathfrak{g} = \mathfrak{su}(3)$):

$$\xi \;=\; \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hline * \end{pmatrix} \quad \kappa^{\mathfrak{g}} \equiv \left(\begin{array}{ccc|c} * & * & * & 0 \\ * & * & * & 0 \\ * & * & * & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right) \quad \kappa^0 \equiv \left(\begin{array}{ccc|c} 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \\ \hline * & * & * & * \end{array}\right)$$

Then, under the same structure group G we find

$$(\nabla^{(T_1)} - \nabla^{(T_2)})\xi \;\propto\; \kappa^{\mathfrak{g}}\xi \;=\; 0$$

So, from the group-theoretical viewpoint, κ^0 carries an **intrinsic** part of the contorsion when we consider the classification of the $SU(3)$-structure manifolds!

Torsion $T_{mn} \equiv T^p{}_{mn} \mathrm{d}x^p = \kappa^p{}_{[mn]} \mathrm{d}x^p$ **is given in the various representations:**

$$T^{\mathfrak{g}}_{mn} = \kappa^{\mathfrak{g}}{}_{[mn]} \sim \mathfrak{su}(3) , \qquad T^0_{mn} = \kappa^0_{[mn]} \sim \mathfrak{so}(6)/\mathfrak{su}(3) \equiv \mathfrak{su}(3)^{\perp}$$

$$\therefore \qquad (T^0)^p{}_{mn} \in \Lambda^1 \otimes \mathfrak{su}(3)^{\perp} \quad \text{on } \mathcal{K}_6$$

$$\Lambda^1 \sim 3 \oplus \overline{3} , \qquad \mathfrak{su}(3) \sim 8 , \qquad \mathfrak{su}(3)^{\perp} = \mathfrak{so}(6)/\mathfrak{su}(3) \sim 1 \oplus 3 \oplus \overline{3}$$

Thus the **intrinsic torsion** T^0 **can be decomposed**

$$
\begin{aligned}
(T^0)^p{}_{mn} \in \Lambda^1 \otimes \mathfrak{su}(3)^{\perp} &= (3 \oplus \overline{3}) \otimes (1 \oplus 3 \oplus \overline{3}) \\
&= \underset{W_1}{(1 \oplus 1)} \oplus \underset{W_2}{(8 \oplus 8)} \oplus \underset{W_3}{(6 \oplus \overline{6})} \oplus \underset{W_4}{(3 \oplus \overline{3})} \oplus \underset{W_5}{(3 \oplus \overline{3})'}
\end{aligned}
$$

where
$\quad W_1$: complex scalar in $(1 \oplus 1)$
$\quad W_2$: complex primitive 2-form in $(8 \oplus 8)$
$\quad W_3$: real primitive $(2,1) \oplus (1,2)$-form in $(6 \oplus \overline{6})$
$\quad W_4$: real 1-form in $(3 \oplus \overline{3})$
$\quad W_5$: complex $(1,0)$-form in $(3 \oplus \overline{3})'$

▽ complex manifolds [SUSY] [$dH = 0$] [$dH \neq 0$]

$W_1 = W_2 = 0$	$T^0 \in W_3 \oplus W_4 \oplus W_5$	**hermitian**
$W_1 = W_2 = W_4 = 0$	$T^0 \in W_3 \oplus W_5$	**balanced**
$W_1 = W_2 = W_4 = W_5 = 0$	$T^0 \in W_3$	**special-hermitian**
$W_1 = W_2 = W_3 = W_4 = 0$	$T^0 \in W_5$	**Kähler**
$W_1 = W_2 = W_3 = W_4 = W_5 = 0$	$T^0 = 0$	**Calabi-Yau**
$W_1 = W_2 = W_3 = 3W_4 + 2W_5 = 0$	$T^0 \in W_4 \oplus W_5$	**conformally Calabi-Yau**

▽ non-complex manifolds [Summary]

$W_1 = W_3 = W_4 = 0$	$T^0 \in W_2 \oplus W_5$	**symplectic**
$W_2 = W_3 = W_4 = W_5 = 0$	$T^0 \in W_1$	**nearly-Kähler**
$W_1 = W_3 = W_4 = W_5 = 0$	$T^0 \in W_2$	**almost-Kähler**
$W_3 = W_4 = W_5 = 0$	$T^0 \in W_1 \oplus W_2$	**quasi-Kähler**
$W_4 = W_5 = 0$	$T^0 \in W_1 \oplus W_2 \oplus W_3$	**semi-Kähler**
$W_1^- = W_2^- = W_4 = W_5 = 0$	$T^0 \in W_1^+ \oplus W_2^+ \oplus W_3$	**half-flat**