

Comments on Heterotic Flux Compactifications

木村 哲士

IAS → YITP

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in collaboration with Piljin Yi (KIAS)

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Purpose

Construct a realistic model of 4-dim. particle physics

- **matter contents and their interactions**
- **gauge symmetry and its breaking**
- **gravity, cosmology**
- **etc., etc.**

We take an approach from string theories

▼ NS H -flux appears in the SUSY variation of the gravitino:

$$\delta\psi_M = \left(\partial_M + (\omega_M^{AB} - H_M^{AB})\Gamma_{AB} \right) \xi$$

$\delta\psi_M = 0$ gives us information of (compactified) geometry.

If $H_M^{AB} \neq 0$, the geometry is no longer a Kähler manifold.



G -structure manifold

Let us consider an effective theory of heterotic string.

▼ Why **heterotic** string theory?

- { contains only NS fields
- { includes Yang-Mills gauge symmetry
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▼ Why **heterotic** string theory?

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$$\text{S-duality: } \text{II}/\text{CY}_3 \leftrightarrow \text{HE}/K3 \times T^2$$

▼ a new insight on inflation scenario (?)

(cf.) KKLT model in type II string with fluxes

Contents

- ▼ G -structure manifold
- ▼ Heterotic theory on $SU(3)$ -structure manifold
 - Vacuum configuration
 - Towards low energy effective theory: (zero mode eqs., gauge groups)
- ▼ Summary and Discussions

G -structure Manifolds

G -structure group on an n -dim. manifold \mathcal{M}

\exists nowhere vanishing tensors X on \mathcal{M}
with satisfying $D_m(\omega)X = 0$:

tensors	G -structure
η_{ab}	$O(n)$
$\eta_{ab} \quad \varepsilon_{a_1 \dots a_n}$	$SO(n)$
$\eta_{ab} \quad J_a^b$	$U(m)$
$\eta_{ab} \quad J_a^b \quad \Omega^{(m)}$	$SU(m)$

$$J^2 = -1$$

$$(2m = n)$$

6-dim. $SU(3)$ -structure on manifold

Consider a geometry \mathcal{K}_6 with a Killing spinor equation including torsion

$$\exists \xi \quad \text{s.t.} \quad \nabla^{(T)} \xi = 0$$

This is a definition of the geometry with $SU(3)$ -structure.

6-dim. $SU(3)$ -structure on manifold

Consider a geometry \mathcal{K}_6 with a Killing spinor equation including torsion

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This is a definition of the geometry with $SU(3)$ -structure.

Invariant p -forms on the $SU(3)$ -structure manifold:

2-form in $SO(6)$ \Rightarrow a real 2-form in $SU(3)$

$${}_6C_2 = 15 = \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{8} \quad : \quad J_{ab} = i \xi^\dagger \Gamma_{ab} \xi$$

3-form in $SO(6)$ \Rightarrow an (almost) complex 3-form in $SU(3)$

$${}_6C_3 = 20 = \mathbf{1} + \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{6} + \bar{\mathbf{6}} \quad : \quad \Omega_{abc} = \xi^T \Gamma_{abc} \xi$$

Furthermore

$$J \wedge \Omega = 0, \quad J \wedge J \wedge J = -\frac{3i}{4} \Omega \wedge \bar{\Omega} = -3! \times (\text{vol.})_{\mathcal{K}_6}$$

Heterotic Theory

The story started from

A. Strominger, “*Superstrings with torsion*,” [Nucl. Phys. B274 (1986) 253]

Heterotic theory on $SU(3)$ -structure manifold

Supergravity with anti-hermitian Yang-Mills field

▼ Bosonic part of the Lagrangian (without fermion condensations)

$$\mathcal{L} = \frac{1}{4} \sqrt{-G} e^{-2\Phi} \left[R(\omega) - \frac{1}{3} H_{MNP} H^{MNP} + 4(\nabla_M \Phi)^2 + \alpha' \left\{ \text{tr}(F_{MN} F^{MN}) \right\} \right]$$

▼ Bianchi identity $[\omega]$

$$dH = +\alpha' \left[\text{tr}\{F \wedge F\} \right]$$

Chapline and Manton [Phys. Lett. B120 (1983) 105]

(supergravity)

Heterotic theory on $SU(3)$ -structure manifold

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Green and Schwarz [Phys. Lett. B149 (1984) 117]

(anomaly cancellation)

(worldsheet 1-loop β -function)

Heterotic theory on $SU(3)$ -structure manifold

Supergravity with anti-hermitian Yang-Mills field QL

▼ Bosonic part of the Lagrangian (without fermion condensations)

$$\mathcal{L} = \frac{1}{4} \sqrt{-G} e^{-2\Phi} \left[R(\omega) - \frac{1}{3} H_{MNP} H^{MNP} + 4(\nabla_M \Phi)^2 + \alpha' \left\{ \text{tr}(F_{MN} F^{MN}) - \text{tr}(R_{MN}(\omega_+) R^{MN}(\omega_+)) \right\} \right]$$

▼ Bianchi identity $[\omega_+ = \omega + H]$

$$dH = +\alpha' \left[\text{tr}\{F \wedge F\} - \text{tr}\{R(\omega_+) \wedge R(\omega_+)\} \right]$$

Hull [Phys. Lett. B178 (1986) 357]

Bergshoeff and de Roo [Nucl. Phys. B328 (1989) 439]

(worldsheet 2-loop β -function)

Towards 4-dim. Physics...

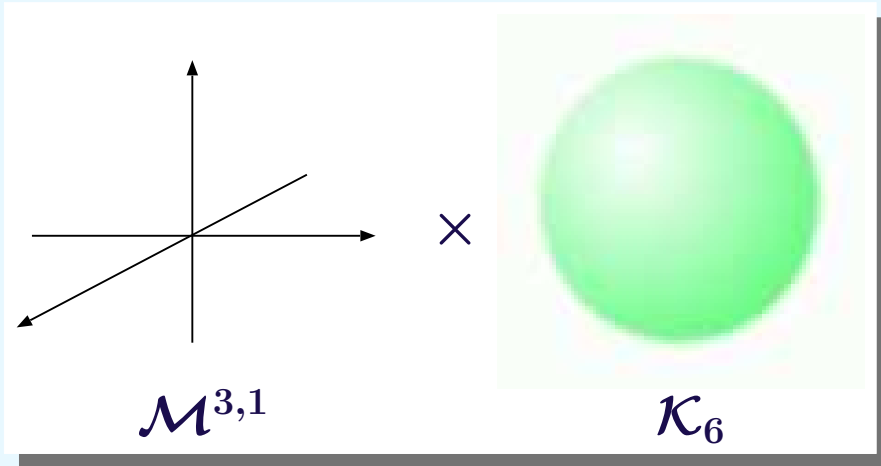
▼ Study vacuum configuration

- SUSY variations → geometry with $SU(3)$ -structure

▼ Investigate low energy effective theory

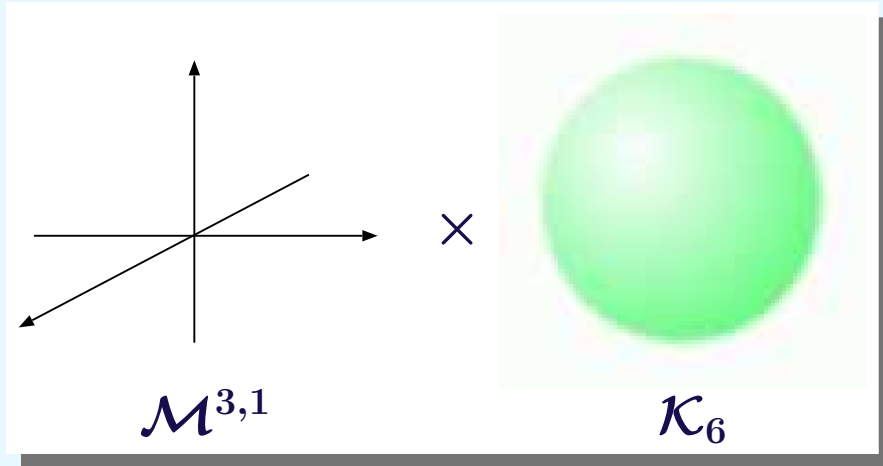
- Gauge symmetry
- Zero mode equations
- Norm of fields

Vacuum Configuration



$$G_{MN} dx^M dx^N = e^{(\Phi - \hat{D})/2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + g_{mn} dy^m dy^n \right)$$

Vacuum Configuration



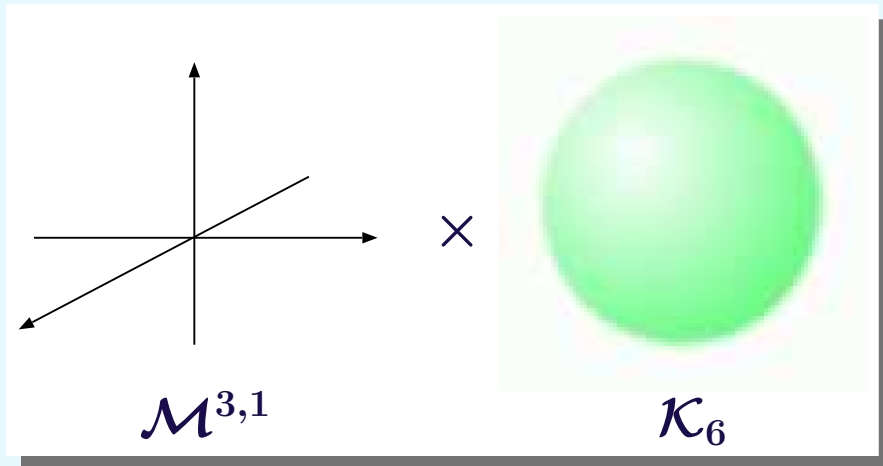
$$G_{MN} dx^M dx^N = e^{(\Phi - \hat{D})/2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + g_{mn} dy^m dy^n \right)$$

$$Spin(9, 1) \rightarrow SL(2, \mathbb{C}) \times SU(4)$$

$$16 = (2, 4) + (\bar{2}, \bar{4}) : \quad \epsilon_+ = \eta_+ \otimes \xi_+ + \eta_- \otimes \xi_-$$

$\mathcal{N} = 1$ SUSY
on $\mathcal{M}^{3,1}$

Vacuum Configuration



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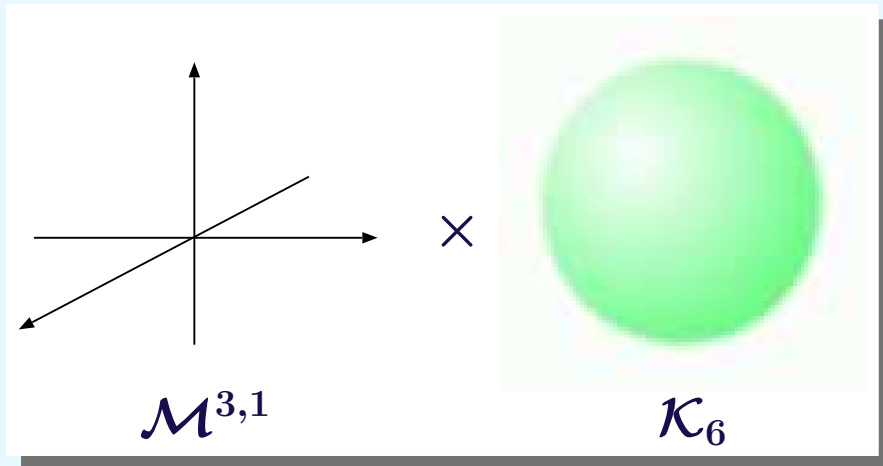
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1 Killing spinor ξ_+
on \mathcal{K}_6

Vacuum Configuration



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1 Killing spinor ξ_+
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$SU(3)$ -structure
on \mathcal{K}_6

▼ SUSY variations

$$0 \equiv \delta\psi_m = D_m(\omega_-)\xi_+ \leftarrow \text{Killing spinor eq.} \quad [\omega_- = \omega - H]$$

$$J_{ab} = i\xi_+^\dagger \Gamma_{ab} \xi_+ \quad : \quad D_m(\omega_-)J_{ab} = 0$$

$$\Omega_{abc} = \xi_+^\dagger \Gamma_{abc} \xi_+ \quad : \quad D_m(\omega_-)\Omega_{abc} = 0$$

▼ SUSY variations

$$0 \equiv \delta\psi_m = D_m(\omega_-)\xi_+ \leftarrow \text{Killing spinor eq.} \quad [\omega_- = \omega - H]$$

$$J_{ab} = i\xi_+^\dagger \Gamma_{ab} \xi_+ \quad : \quad D_m(\omega_-)J_{ab} = 0$$

$$\Omega_{abc} = \xi_+^T \Gamma_{abc} \xi_+ \quad : \quad D_m(\omega_-)\Omega_{abc} = 0$$

Furthermore “ $0 \equiv \delta(\text{fermions})$ ” indicates

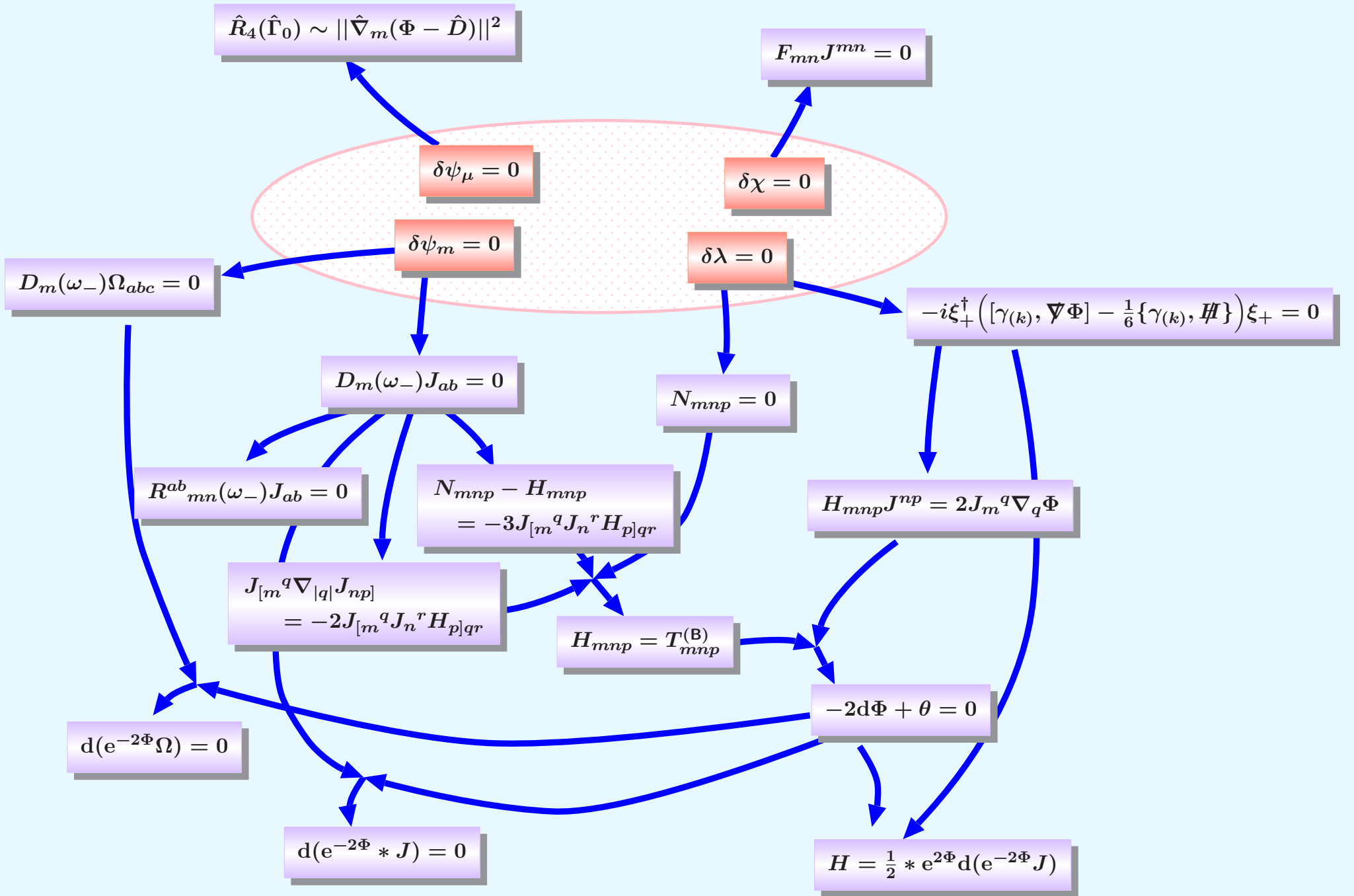
$$R^{ab}{}_{mn}(\omega_-)J_{ab} = 0 \quad : \quad c_1(R_-) \text{ vanishes}$$

$$N_{mn}{}^p = 0 \quad : \quad \mathcal{K}_6 \text{ is complex } \boxed{\text{T}}$$

$$H = \frac{i}{2}(\partial - \bar{\partial})J, \quad dH = -i\partial\bar{\partial}J, \quad d\Phi = \frac{1}{2}J \lrcorner dJ$$

$$H = H^0 + \widehat{H}, \quad J \lrcorner H^0 = 0, \quad \widehat{H}_{mnp} = \frac{3}{2}J_{[mn}J_p]^q \nabla_q \Phi$$

$$*(J \wedge dH) = \nabla_m^2 \Phi - (\nabla_m \Phi)^2 + \frac{1}{3}(H_{mnp}^0)^2 \quad \begin{array}{l} \boxed{dH = 0} \\ \boxed{dH \neq 0} \end{array}$$



Gauge Symmetry Breaking

$$\nabla E_8 \rightarrow E_6 \times \underline{SU(3)}$$

$$248 = (78, 1) + (1, 8) + (27, 3) + (\overline{27}, \overline{3})$$

$$\nabla E_8 \rightarrow SO(16)$$

$$\rightarrow SO(10) \times \underline{SO(6)}$$

$$248 = 120_{SO(16) \text{ adj.}} + 128_{SO(16) \text{ spinor}}$$

$$= (45, 1) + (1, 15) + (10, 6) + (16, 4) + (\overline{16}, \overline{4})$$

Each breaking scenario deeply depends on the way
of embedding the holonomy group into the gauge groups.

$$\text{Embedding } A \leftrightarrow \begin{cases} \omega_- : & SU(3) \text{ holonomy} \\ \omega_+ : & SO(6) \text{ holonomy} \\ & \text{etc.} \end{cases} \quad [\omega_{\pm} = \omega \pm H]$$

$$\text{Embedding } A \leftrightarrow \begin{cases} \omega_- : SU(3) \text{ holonomy} \\ \omega_+ : SO(6) \text{ holonomy} \\ \text{etc.} \end{cases} \quad [\omega_{\pm} = \omega \pm H]$$

with following constraints

$$R_{mnpq}(\omega_+) = R_{pqmn}(\omega_-) + (dH)_{pqmn}$$

$$dH = +\alpha' \left[\text{tr}(F \wedge F) - \text{tr}\{R(\omega_+) \wedge R(\omega_+)\} \right] \quad \begin{array}{l} dH = 0 \\ dH \neq 0 \end{array}$$

$R_{pqmn}(\omega_-)$: type (1, 1) w/ indices p, q

F : (1, 1)-form

dH : (2, 2)-form, higher order in α'

$R_{mnpq}(\omega_+)$: (1, 1)-form w/ indices p, q + higher order in α'

$dH = 0$ case: $A \equiv \omega_+ \Rightarrow E_8 \rightarrow SO(10) \times \underline{SO(6)}$

$dH \neq 0$ case: $A = \text{others..} \Rightarrow E_8 \rightarrow \mathcal{G} \times \underline{\mathcal{H}}$

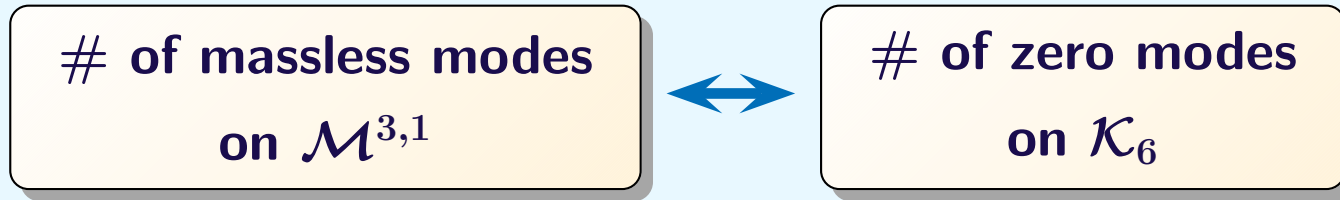
Zero mode equations

of massless modes
on $\mathcal{M}^{3,1}$



of zero modes
on \mathcal{K}_6

Zero mode equations



Zero mode eq. for gaugino: $[\hat{\omega} \equiv \omega - \frac{1}{3}H]$

$$0 = \mathcal{D}(\omega, A)\chi^0 - \frac{1}{12}H_{mnp}\Gamma^{mnp}\chi^0 \equiv \mathcal{D}(\hat{\omega}, A)\chi^0$$

$$[\# \text{ of zero mode } \chi^0] = \lim_{\beta \rightarrow 0} \text{Tr } \Gamma_{(6+1)} e^{-\beta\Delta} \equiv \text{index } \mathcal{D}(\hat{\omega}, A)$$

$$\begin{aligned} \Delta &\equiv -[\mathcal{D}(\hat{\omega}, A)]^2 \\ &= -\frac{1}{\sqrt{|g_6|}} D_m(\omega_-, A) \left\{ g^{mn} \sqrt{|g_6|} D_n(\omega_-, A) \right\} + \frac{1}{4}R(\omega) + \frac{i}{2}F_{ab}\Gamma^{ab} \\ &\quad + \frac{1}{4} \left\{ -\frac{1}{3}H_{abc}H^{abc} + \frac{1}{12}(dH)_{abcd}\Gamma^{abcd} \right\} \end{aligned}$$

$dH = 0$

$dH \neq 0$

Minimal embedding: $dH = 0$

SUSY A

The condition $*(J \wedge dH) = 0$ denotes

$$0 = \nabla_m^2 \Phi - (\nabla_m \Phi)^2 + \frac{1}{3} (H_{mnp}^0)^2$$

If there are no boundaries/singularities on \mathcal{K}_6 , then

$$\int_{\mathcal{K}_6} \nabla_m^2 e^{-\Phi} = \frac{1}{3} \int_{\mathcal{K}_6} e^{-\Phi} \|H_{mnp}^0\|^2$$

$dH = 0$ without any boundaries $\Rightarrow H^0 = 0 \Rightarrow \Phi = \text{const.}$

\Rightarrow smooth $\mathcal{K}_6 = \text{CY}_3$

No-go theorem on smooth compactification with fluxes



Without boundaries/singularities on \mathcal{K}_6 :

- all fluxes are trivial $H = d\Phi = 0$
- $\mathcal{K}_6 = \text{CY}_3$
- $\omega_+ = \omega_- = \omega$, $E_8 \rightarrow E_6 \times SU(3)$
- # of zero modes — AS index theorem

Candelas, Horowitz, Strominger and Witten

[Nucl. Phys. B258 (1985) 46]

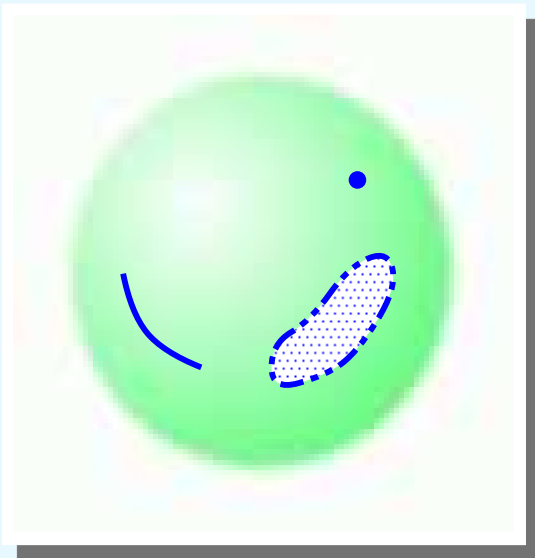


Without boundaries/singularities on \mathcal{K}_6 :

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- # of zero modes — AS index theorem

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With boundaries/singularities on \mathcal{K}_6 :

- non-trivial fluxes can exist

$$\partial_m \Phi \neq 0, H_{mnp}^0 \neq 0 \quad \boxed{\text{T}}$$

- $E_8 \rightarrow SO(10) \times SO(6)$
- $\chi_{\mathcal{H}}^0$ lives in the boundaries
- # of zero modes — APS index theorem Dirac

Non-minimal embedding: $dH \neq 0$

SUSY A

Combining equations of motion and SUSY conditions, we obtain

$$0 = \frac{1}{2} \nabla_m^2 e^{-2\Phi} - \frac{1}{3} e^{-2\Phi} (H_{mnp})^2 + e^{-2\Phi} * (J \wedge dH)$$

$$\text{tr}(R_{mn}R^{mn}) - \text{tr}(F_{mn}F^{mn}) = -2 * \left[J \wedge \left(\text{tr}(R \wedge R) - \text{tr}(F \wedge F) \right) \right] + \mathcal{O}(\alpha')$$

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Combining equations of motion and SUSY conditions, we obtain

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Then, within the linear order in α' , we find

$$\nabla_m^2 e^{-2\Phi} = e^{-2\Phi} \left[\frac{2}{3} \|H_{mnp}\|^2 + \alpha' (\text{tr} \|F_{mn}\|^2 - \text{tr} \|R_{mn}\|^2) \right]$$

Integral on a **smooth manifold** \mathcal{K}_6 :

$$\int_{\mathcal{K}_6} e^{-2\Phi} \left[\frac{2}{3} \|H_{mnp}\|^2 + \alpha' \text{tr} \|F_{mn}\|^2 \right] = \int_{\mathcal{K}_6} e^{-2\Phi} \left[\alpha' \text{tr} \|R_{mn}\|^2 \right]$$

$$\text{with} \quad \text{tr} \|F_{mn}\|^2 \neq \text{tr} \|R_{mn}\|^2$$

Smooth compactification scenario is possible!

Summary and Discussions

Summary and Discussions

- ▼ Vacuum configuration of the flux compactifications in heterotic theory
- ▼ No-go theorem on smooth manifolds with $H \neq 0$ and $dH = 0$
- ▼ Possibility of smooth compactifications with $H \neq 0$ and $dH \neq 0$

Summary and Discussions

- ▼ Vacuum configuration of the flux compactifications in heterotic theory
- ▼ No-go theorem on smooth manifolds with $H \neq 0$ and $dH = 0$
- ▼ Possibility of smooth compactifications with $H \neq 0$ and $dH \neq 0$
- ▼ # of zero modes under the condition $dH \neq 0$ Dirac
modification of the Atiyah-(Patodi)-Singer index theorem
- ▼ other possibilities of gauge symmetry breaking
- ▼ compactifications on non-complex geometries SUSY

Frey and Lippert [hep-th/0507202]

Manousselis, Prezas and Zoupanos [hep-th/0511122]

Appendix

Appendix: Quartic effective Lagrangian

$$\mathcal{L}_{\text{total}} = \mathcal{L}_0(\mathbf{R}) + \mathcal{L}_\beta(\mathbf{F}^2) + \mathcal{L}_\alpha(\mathbf{R}^2) \quad \square$$

$$\begin{aligned} \mathcal{L}_0(\mathbf{R}) = & \frac{1}{2\kappa_{10}^2} \sqrt{-G} e^{-2\Phi} \left[R(\omega) - \frac{1}{3} H_{MNP} H^{MNP} + 4(\nabla_M \Phi)^2 - \bar{\psi}_M \Gamma^{MNP} D_N(\omega) \psi_P + 16 \bar{\lambda} \mathcal{D}(\omega) \lambda \right. \\ & + 8 \bar{\lambda} \Gamma^{MN} D_M(\omega) \psi_N + 8 \bar{\psi}_M \Gamma^N \Gamma^M \lambda (\nabla_N \Phi) - 2 \bar{\psi}_M \Gamma^M \psi_N (\nabla^N \Phi) \\ & + \frac{1}{12} H^{PQR} \left\{ \bar{\psi}_M \Gamma^{[M} \Gamma_{PQR} \Gamma^{N]} \psi_N + 8 \bar{\psi}_M \Gamma^M \Gamma_{PQR} \lambda - 16 \bar{\lambda} \Gamma_{PQR} \lambda \right\} \\ & \left. + \frac{1}{48} \bar{\psi}^M \Gamma^{ABC} \psi_M \left\{ 2 \bar{\lambda} \Gamma_{ABC} \lambda + \bar{\lambda} \Gamma_{ABC} \Gamma^N \psi_N - \frac{1}{4} \bar{\psi}^N \Gamma_{ABC} \psi_N - \frac{1}{8} \bar{\psi}^N \Gamma_N \Gamma_{ABC} \Gamma^P \psi_P \right\} \right] \end{aligned}$$

$$\begin{aligned} \mathcal{L}_\beta(\mathbf{F}^2) = & -\frac{1}{2\kappa_{10}^2} \frac{\kappa_{10}^2}{2g_{10}^2} \sqrt{-G} e^{-2\Phi} \left[-\text{tr}(F_{MN} F^{MN}) - 2 \text{tr}\{\bar{\chi} \mathcal{D}(\omega, A) \chi\} + \frac{1}{6} \text{tr}(\bar{\chi} \Gamma^{ABC} \chi) \hat{H}_{ABC} \right. \\ & - \frac{1}{2} \text{tr}\{\bar{\chi} \Gamma^M \Gamma^{AB} (F_{AB} + \hat{F}_{AB})\} \left(\psi_M + \frac{2}{3} \Gamma_M \lambda \right) - \frac{1}{48} \text{tr}(\bar{\chi} \Gamma^{ABC} \chi) \bar{\psi}_M (4 \Gamma_{ABC} \Gamma^M + 3 \Gamma^M \Gamma_{ABC}) \lambda \\ & \left. + \frac{1}{12} \text{tr}(\bar{\chi} \Gamma^{ABC} \chi) \bar{\lambda} \Gamma_{ABC} \lambda - \frac{\beta}{96} \text{tr}(\bar{\chi} \Gamma^{ABC} \chi) \text{tr}(\bar{\chi} \Gamma_{ABC} \chi) \right] \end{aligned}$$

$$\begin{aligned} \mathcal{L}_\alpha(\mathbf{R}^2) = & -\frac{1}{2\kappa_{10}^2} \frac{\kappa_{10}^2}{2g_{10}^2} \sqrt{-G} e^{-2\Phi} \left[-R_{ABMN}(\omega_+) R^{ABMN}(\omega_+) - 2 \bar{\psi}^{AB} \mathcal{D}(\omega(e, \psi), \omega_+) \psi_{AB} + \frac{1}{6} \bar{\psi}^{AB} \Gamma^{MNP} \psi_{AB} \hat{H}_{MNP} \right. \\ & + \frac{1}{2} \bar{\psi}_{AB} \Gamma^M \Gamma^{NP} \{ R^{AB}_{NP}(\omega_+) + \hat{R}^{AB}_{NP}(\omega_+) \} \left(\psi_M + \frac{2}{3} \Gamma_M \lambda \right) \\ & - \frac{1}{48} \bar{\psi}_{AB} \Gamma^{CDE} \psi_{AB} \cdot \bar{\psi}_M (4 \Gamma_{CDE} \Gamma^M + 3 \Gamma^M \Gamma_{CDE}) \lambda \\ & \left. + \frac{1}{12} \bar{\psi}^{AB} \Gamma^{CDE} \psi_{AB} (\bar{\lambda} \Gamma_{CDE} \lambda) - \frac{\alpha}{96} \bar{\psi}^{AB} \Gamma^{FGH} \psi_{AB} (\bar{\psi}^{CD} \Gamma_{FGH} \psi_{CD}) \right] \end{aligned}$$

$$\begin{aligned}
\delta_0 e_M{}^A &= \frac{1}{2} \bar{\epsilon} \Gamma^A \psi_M \\
\delta_0 \psi_M &= \left(\partial_M + \frac{1}{4} \omega_{-M}{}^{AB} \Gamma_{AB} \right) \epsilon + \left\{ \epsilon (\bar{\psi}_M \lambda) - \psi_M (\bar{\epsilon} \lambda) + \Gamma^A \lambda (\bar{\psi}_M \Gamma_A \epsilon) \right\} \\
\delta_0 B_{MN} &= \bar{\epsilon} \Gamma_{[M} \psi_{N]} \\
\delta_0 \lambda &= -\frac{1}{4} \not{D} \Phi \epsilon + \frac{1}{24} \Gamma^{ABC} \epsilon \left(\hat{H}_{ABC} - \frac{1}{4} \bar{\lambda} \Gamma_{ABC} \lambda \right) \\
\delta_0 \Phi &= -\bar{\epsilon} \lambda \\
\delta_0 A_M &= \frac{1}{2} \bar{\epsilon} \Gamma_M \chi \\
\delta_0 \chi &= -\frac{1}{4} \Gamma^{AB} \epsilon \hat{F}_{AB} + \left\{ \epsilon (\bar{\chi} \lambda) - \chi (\bar{\epsilon} \lambda) + \Gamma^A \lambda (\bar{\chi} \Gamma_A \epsilon) \right\} \\
\delta_\beta \psi_M &= \frac{\beta}{192} \Gamma^{ABC} \Gamma_M \epsilon \operatorname{tr}(\bar{\chi} \Gamma_{ABC} \chi) \\
\delta_\beta B_{MN} &= -\beta \operatorname{tr} \{ A_{[M} \delta_0 A_{N]} \} \\
\delta_\beta \lambda &= \frac{\beta}{384} \Gamma^{ABC} \epsilon \operatorname{tr}(\bar{\chi} \Gamma_{ABC} \chi) \\
\delta_\alpha \psi_M &= \frac{\alpha}{192} \Gamma^{CDE} \Gamma_M \epsilon \bar{\psi}^{AB} \Gamma_{CDE} \psi_{AB} \\
\delta_\alpha B_{MN} &= -\alpha \omega_{+[M}{}^{AB} \delta_0 \omega_{+N]}{}^{AB} \\
\delta_\alpha \lambda &= \frac{\alpha}{384} \Gamma^{CDE} \Gamma_M \epsilon \bar{\psi}^{AB} \Gamma_{CDE} \psi_{AB}
\end{aligned}$$

Bergshoeff and de Roo [Nucl. Phys. B328 (1989) 439]

Equations of Motion

$$\Phi : 0 = -R(\omega) + \frac{1}{3}H_{MNP}H^{MNP} + 4(\nabla_M\Phi)^2 - 4\nabla_M^2\Phi \\ - \alpha' \left[\text{tr}(F_{MN}F^{MN}) - \text{tr}\{R_{MN}(\omega_+)R^{MN}(\omega_+)\} \right]$$

$$G_{MN} : 0 = R_{MN}(\omega) - H_{MPQ}H_N{}^{PQ} + 2\nabla_M\nabla_N\Phi \\ - \frac{1}{2}G_{MN} \left[R(\omega) - \frac{1}{3}H_{PQR}H^{PQR} - 4(\nabla_P\Phi)^2 + 4\nabla_P^2\Phi \right] \\ - \frac{\alpha'}{2}G_{MN} \left[\text{tr}(F_{MN}F^{MN}) - \text{tr}\{R_{MN}(\omega_+)R^{MN}(\omega_+)\} \right] \\ + 2\alpha' \left[\text{tr}(F_{MP}F_N{}^P) - \text{tr}\{R_{MP}(\omega_+)R_N{}^P(\omega_+)\} \right] \\ + 2\alpha'e^{2\Phi} \left[2\nabla^P\nabla_{(+)}^Q \{e^{-2\Phi}R_{MPNQ}(\omega_+)\} - \nabla_{(+)}^Q \{e^{-2\Phi}R_{MPQR}(\omega_+)\}H_N{}^{PR} \right. \\ \left. - 2\nabla^P \{e^{-2\Phi}R_{MPQR}(\omega_+)\}H_N{}^{QR} \} - 2e^{-2\Phi}R_{MPQR}(\omega_+)H_N{}^{PS}H_S{}^{QR} \right. \\ \left. - \nabla^P\nabla_{(+)}^Q \{e^{-2\Phi}R_{MNPQ}(\omega_+)\} + \nabla^P \{e^{-2\Phi}R_{MNQR}(\omega_+)\}H_P{}^{QR} \} \right]$$

$$B_{MN} : 0 = \nabla^M(e^{-2\Phi}H_{MNP})$$

$$\chi : 0 = \mathcal{D}(\omega - \frac{1}{3}H, A)\chi - \Gamma^M\chi\nabla_M\Phi + \frac{3}{2}\Gamma^M\Gamma^{AB}(F_{AB} + \hat{F}_{AB})\left(\psi_M + \frac{2}{3}\Gamma_M\lambda\right)$$

Let κ be a contorsion in $\nabla^{(T)}$ with acting on the $SU(3)$ Killing spinor ξ :

$$0 = \nabla^{(T)}\xi = (\nabla + \kappa^0 + \kappa^{\mathfrak{g}})\xi$$

where we decomposed $\kappa \equiv \kappa^0 + \kappa^{\mathfrak{g}}$ in such a way as $\kappa^{\mathfrak{g}}\xi = 0$ (where $\mathfrak{g} = \mathfrak{su}(3)$):

$$\xi = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hline * \end{pmatrix} \quad \kappa^{\mathfrak{g}} \equiv \begin{pmatrix} * & * & * & | & 0 \\ * & * & * & | & 0 \\ * & * & * & | & 0 \\ \hline 0 & 0 & 0 & | & 0 \end{pmatrix} \quad \kappa^0 \equiv \begin{pmatrix} 0 & 0 & 0 & | & * \\ 0 & 0 & 0 & | & * \\ 0 & 0 & 0 & | & * \\ \hline * & * & * & | & * \end{pmatrix}$$

Then, under the same structure group G we find

$$(\nabla^{(T_1)} - \nabla^{(T_2)})\xi \propto \kappa^{\mathfrak{g}}\xi = 0$$

So, from the group-theoretical viewpoint, κ^0 carries an **intrinsic** part of the contorsion when we consider the classification of the $SU(3)$ -structure manifolds!

Torsion $T_{mn} \equiv T^p{}_{mn} dx^p = \kappa^p{}_{[mn]} dx^p$ is given in the various representations:

$$T_{mn}^g = \kappa^g{}_{[mn]} \sim \mathfrak{su}(3), \quad T_{mn}^0 = \kappa^0{}_{[mn]} \sim \mathfrak{so}(6)/\mathfrak{su}(3) \equiv \mathfrak{su}(3)^\perp$$

$$\therefore (T^0)^p{}_{mn} \in \Lambda^1 \otimes \mathfrak{su}(3)^\perp \quad \text{on } \mathcal{K}_6$$

$$\Lambda^1 \sim 3 \oplus \bar{3}, \quad \mathfrak{su}(3) \sim 8, \quad \mathfrak{su}(3)^\perp = \mathfrak{so}(6)/\mathfrak{su}(3) \sim 1 \oplus 3 \oplus \bar{3}$$

Thus the **intrinsic torsion** T^0 can be decomposed

$$\begin{aligned} (T^0)^p{}_{mn} \in \Lambda^1 \otimes \mathfrak{su}(3)^\perp &= (3 \oplus \bar{3}) \otimes (1 \oplus 3 \oplus \bar{3}) \\ &= \underbrace{(1 \oplus 1)}_{W_1} \oplus \underbrace{(8 \oplus 8)}_{W_2} \oplus \underbrace{(6 \oplus \bar{6})}_{W_3} \oplus \underbrace{(3 \oplus \bar{3})}_{W_4} \oplus \underbrace{(3 \oplus \bar{3})'}_{W_5} \end{aligned}$$

where

W_1 : complex scalar in $(1 \oplus 1)$

W_2 : complex primitive 2-form in $(8 \oplus 8)$

W_3 : real primitive $(2, 1) \oplus (1, 2)$ -form in $(6 \oplus \bar{6})$

W_4 : real 1-form in $(3 \oplus \bar{3})$

W_5 : complex $(1, 0)$ -form in $(3 \oplus \bar{3})'$

▼ complex manifolds

SUSY $dH = 0$ $dH \neq 0$

$$W_1 = W_2 = 0$$

$$T^0 \in W_3 \oplus W_4 \oplus W_5$$

hermitian

$$W_1 = W_2 = W_4 = 0$$

$$T^0 \in W_3 \oplus W_5$$

balanced

$$W_1 = W_2 = W_4 = W_5 = 0$$

$$T^0 \in W_3$$

special-hermitian

$$W_1 = W_2 = W_3 = W_4 = 0$$

$$T^0 \in W_5$$

Kähler

$$W_1 = W_2 = W_3 = W_4 = W_5 = 0$$

$$T^0 = 0$$

Calabi-Yau

$$W_1 = W_2 = W_3 = 3W_4 + 2W_5 = 0$$

$$T^0 \in W_4 \oplus W_5$$

conformally Calabi-Yau

▼ non-complex manifolds

Summary

$$W_1 = W_3 = W_4 = 0$$

$$T^0 \in W_2 \oplus W_5$$

symplectic

$$W_2 = W_3 = W_4 = W_5 = 0$$

$$T^0 \in W_1$$

nearly-Kähler

$$W_1 = W_3 = W_4 = W_5 = 0$$

$$T^0 \in W_2$$

almost-Kähler

$$W_3 = W_4 = W_5 = 0$$

$$T^0 \in W_1 \oplus W_2$$

quasi-Kähler

$$W_4 = W_5 = 0$$

$$T^0 \in W_1 \oplus W_2 \oplus W_3$$

semi-Kähler

$$W_1^- = W_2^- = W_4 = W_5 = 0$$

$$T^0 \in W_1^+ \oplus W_2^+ \oplus W_3$$

half-flat