Comments on Heterotic Flux Compactifications

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Purpose

Construct a realistic model of 4-dim. particle physics

- matter contents
- gauge symmetry and its breaking
- gravity, cosmology
- etc., etc.

Supergravity with fluxes has a long, and interesting story

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- **Freund-Rubin Ansatz in 11- and 10-dimensional supergravities**

  $F_p = |F| \times$ (vol.) generates a cosmological constant in AdS_q-space

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- **Freund-Rubin Ansatz in 11- and 10-dimensional supergravities**

  $F_p = |F| \times (\text{vol.})$ generates a cosmological constant in AdS_q-space

- **Gauge/Gravity Dualities in type II theories**

  F_p generates a superpotential in 4-dim. $\mathcal{N} = 1$ SYM $\left(\text{e.g.,}\ \ W = \int_{\mathrm{CY}_3} \Omega \wedge F_3\right)$

Both solutions have given us new insights in higher-dimensional theories

- **Flux can be a torsion on a (compactified) geometry**

$$\delta\psi_M \;=\; \left(\partial_M + (\omega_M{}^{AB} - H_M{}^{AB})\Gamma_{AB}\right)\xi$$

If $H_M{}^{AB} \neq 0$, the geometry looks no longer a Kähler manifold.

↓

G-structure manifold

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G-structure manifold

- **How does the flux work in heterotic theory?**

$$\begin{cases} \text{anomaly cancellation in a miraculous way} \\ \text{gauge symmetry } E_8 \times E_8 \text{ or } SO(32) \end{cases}$$

Contents

- $SU(3)$-structure manifold
- Heterotic theory on $SU(3)$-structure manifold
  - Vacuum configuration
  - Towards low energy effective theory: (zero mode eqs., gauge groups)
- Summary and Discussions

useful Refs. Becker, Becker, Dasgupta and Green [hep-th/0301161]
Cardoso, Curio, Dall'Agata and Lüst [hep-th/0306088]
Becker and Tseng [hep-th/0509131]
etc., etc..

$SU(3)$-structure Manifold

– mathematics –

G-structure group on an n-dim. manifold $\mathcal{M}$

$$\begin{cases} F(\mathcal{M}) \text{ frame bundle} : & \text{principal } GL(n) \text{ bundle} \\ G\text{-structure} : & \text{principal } G \text{ sub-bundle of } F(\mathcal{M}) \end{cases}$$

$\Leftrightarrow$ nowhere vanishing tensors on $\mathcal{M}$

tensors	G-structure	
η_{ab}	$O(n)$	
η_{ab} $\varepsilon_{a_1 \cdots a_n}$	$SO(n)$	
η_{ab} $J_a{}^b$	$U(m)$	$J^2 = -1$
η_{ab} $J_a{}^b$ $\Omega^{(m,0)}$	$SU(m)$	$(2m = n)$

6-dim. $SU(3)$-structure on manifold

Consider a geometry $\mathcal{K}_6$ with a Killing spinor equation including torsion

$$^{\exists}\xi \qquad \text{s.t.} \quad \nabla^{(T)}\xi = 0$$

This is a definition of the geometry with $SU(3)$-structure.

6-dim. $SU(3)$-structure on manifold

Consider a geometry $\mathcal{K}_6$ with a Killing spinor equation including torsion

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This is a definition of the geometry with $SU(3)$-structure.

Invariant p-forms on the $SU(3)$-structure manifold:

2-form in $SO(6)$ ➡ a real 2-form in $SU(3)$

$$_6C_2 = 15 \quad = \quad \mathbf{1} + \mathbf{3} + \overline{\mathbf{3}} + \mathbf{8} \quad : \quad J_{ab} = -i\xi^{\dagger}\Gamma_{ab}\xi$$

3-form in $SO(6)$ ➡ an (almost) complex 3-form in $SU(3)$

$$_6C_3 = 20 \quad = \quad \mathbf{1} + \mathbf{1} + \mathbf{3} + \overline{\mathbf{3}} + \mathbf{6} + \overline{\mathbf{6}} \quad : \quad \Omega_{abc} = \xi^{\mathrm{T}}\Gamma_{abc}\xi$$

Furthermore

$$J \wedge \Omega \ = \ 0 \, , \qquad J \wedge J \wedge J \ = \ \frac{3i}{4}\Omega \wedge \overline{\Omega} \ = \ 3! \times (\text{vol.})_{\mathcal{K}_6}$$

Heterotic Theory

– vacuum, gauge group and zero modes –

Supergravity

- **Bosonic part of the Lagrangian** (without fermion condensations)

$$\mathscr{L} = \frac{1}{4}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\Big[R(\omega) - \frac{1}{3}H_{MNP}H^{MNP} + 4(\nabla_M\Phi)^2 - \alpha'\Big\{\mathrm{tr}\big(F_{MN}F^{MN}\big)\Big\}\Big]$$

- **Bianchi identity** $[\omega]$

$$\mathrm{d}H = -\alpha'\Big[\mathrm{tr}\{F\wedge F\}\Big]$$

Chapline and Manton [Phys. Lett. B120 (1983) 105]

(supergravity)

Heterotic theory on $SU(3)$-structure manifold

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Green and Schwarz [Phys. Lett. B149 (1984) 117]

(anomaly cancellation)

(worldsheet 1-loop β-function)

Supergravity

QL

- **Bosonic part of the Lagrangian** (without fermion condensations)

$$\mathscr{L} = \frac{1}{4}\sqrt{-G}\, e^{-2\Phi}\Big[R(\omega) - \frac{1}{3} H_{MNP} H^{MNP} + 4(\nabla_M \Phi)^2 - \alpha' \Big\{ \mathrm{tr}\big(F_{MN} F^{MN}\big) - \mathrm{tr}\big(R_{MN}(\omega_+) R^{MN}(\omega_+)\big) \Big\} \Big]$$

- **Bianchi identity** $[\omega_+ = \omega + H]$

$$\mathrm{d}H = -\alpha' \Big[\mathrm{tr}\{F \wedge F\} - \mathrm{tr}\{R(\omega_+) \wedge R(\omega_+)\} \Big]$$

Hull [Phys. Lett. B178 (1986) 357]

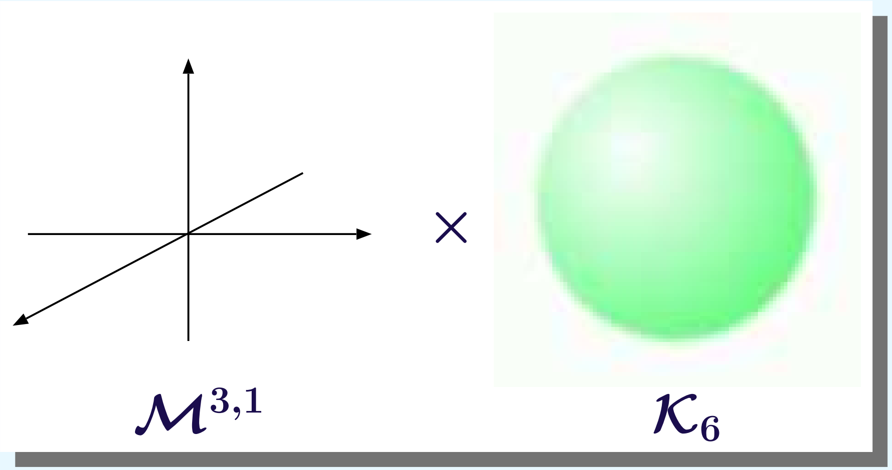
Bergshoeff and de Roo [Nucl. Phys. B328 (1989) 439]

(worldsheet 2-loop β-function)

Towards 4-dim. Physics...

- **Study vacuum configuration**
  - SUSY variations $\longrightarrow$ geometry with $SU(3)$-structure
- **Investigate low energy effective theory**
  - Gauge symmetry
  - Zero mode equations
  - Norm of fields

Vacuum Configuration

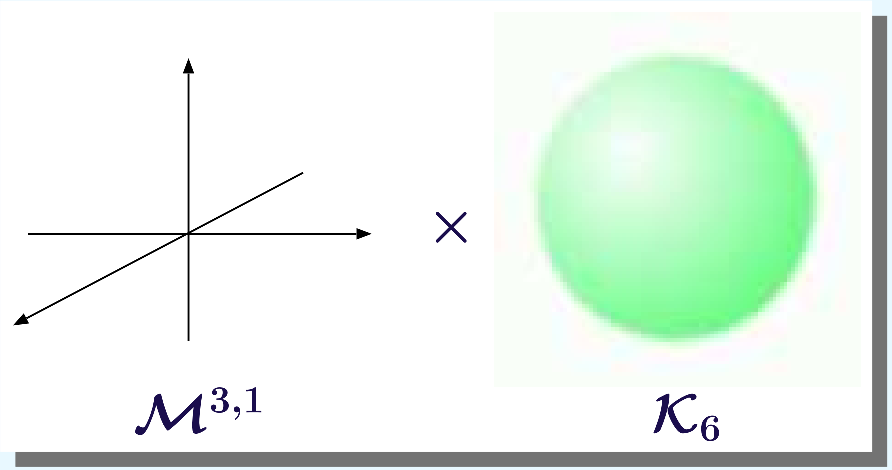


$$G_{MN}\,\mathrm{d}x^M\mathrm{d}x^N = \eta_{\mu\nu}\,\mathrm{d}x^\mu\mathrm{d}x^\nu + g_{mn}\,\mathrm{d}y^m\mathrm{d}y^n$$

$$G^{\mathsf{E}}_{MN}\,\mathrm{d}x^M\mathrm{d}x^N = \mathrm{e}^{-\widehat{D}/2}\Big(\widehat{\eta}_{\mu\nu}\,\mathrm{d}x^\mu\mathrm{d}x^\nu + \widehat{g}_{mn}\,\mathrm{d}y^m\mathrm{d}y^n\Big)$$

$$G_{MN} = \mathrm{e}^{\Phi/2}\,G^{\mathsf{E}}_{MN}$$

Vacuum Configuration



$$G_{MN}\, dx^M dx^N = \eta_{\mu\nu}\, dx^\mu dx^\nu + g_{mn}\, dy^m dy^n$$

$$G^{\mathsf{E}}_{MN}\, dx^M dx^N = e^{-\widehat{D}/2}\Big(\widehat{\eta}_{\mu\nu}\, dx^\mu dx^\nu + \widehat{g}_{mn}\, dy^m dy^n\Big)$$

$$G_{MN} = e^{\Phi/2}\, G^{\mathsf{E}}_{MN}$$

$$\boxed{Spin(9,1) \;\to\; SL(2,\mathbb{C}) \times SU(4)}$$

$$\mathbf{16} = (\mathbf{2},\mathbf{4}) + (\overline{\mathbf{2}},\overline{\mathbf{4}}) \;:\qquad \epsilon_+ = \eta_+ \otimes \xi_+ + \eta_- \otimes \xi_-$$

$\mathcal{N} = 1$ SUSY on $\mathcal{M}^{3,1}$ ⟷ **1 Killing spinor ξ_+ on $\mathcal{K}_6$** ⟷ **$SU(3)$-structure on $\mathcal{K}_6$**

SUSY variations

$0 \equiv \delta\psi_m = D_m(\omega_-)\xi_+ \leftarrow$ **Killing spinor eq.** $[\omega_- = \omega - H]$

$$\begin{aligned} J_{ab} &= -i\xi_+^\dagger \Gamma_{ab} \xi_+ &:& \qquad D_m(\omega_-)J_{ab} = 0 \\ \Omega_{abc} &= \xi_+^{\mathrm{T}} \Gamma_{abc} \xi_+ &:& \qquad D_m(\omega_-)\Omega_{abc} = 0 \end{aligned}$$

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$$J_{ab} = -i\xi_+^\dagger \Gamma_{ab} \xi_+ \quad : \quad D_m(\omega_-)J_{ab} = 0$$

$$\Omega_{abc} = \xi_+^{\rm T} \Gamma_{abc} \xi_+ \quad : \quad D_m(\omega_-)\Omega_{abc} = 0$$

Furthermore "$0 \equiv \delta(\text{fermions})$" **indicates**

$$0 = R^{ab}{}_{mn}(\omega_-)J_{ab} \quad : \quad c_1(R_-) \text{ vanishes}$$

$$0 = N_{mn}{}^p \quad : \quad \mathcal{K}_6 \text{ is complex}$$

T

$$H_{mnp} = H^0_{mnp} + \widehat{H}_{mnp}$$

$$H^{(3,0)} = 0 = H^{(0,3)} \qquad 0 = H^0_{mnp}J^{np} \qquad \widehat{H}_{mnp} = \frac{3}{2}J_{[mn}J_{p]}{}^q\nabla_q\Phi$$

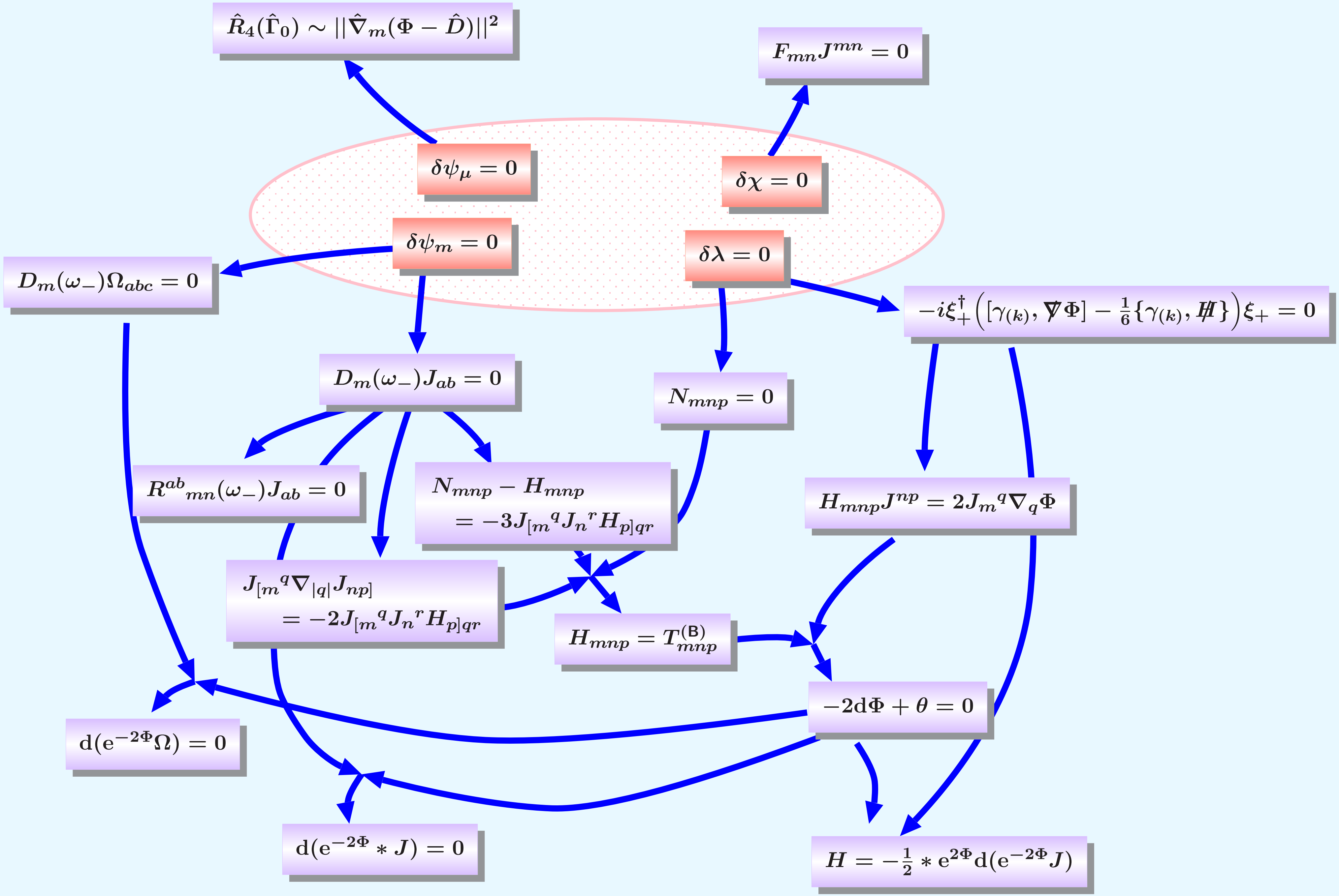
$$F^{(2,0)} = 0 = F^{(0,2)} \qquad 0 = F_{mn}J^{mn}$$

$$*(J \wedge {\rm d}H) = -\nabla_m^2\Phi + (\nabla_m\Phi)^2 - \frac{1}{3}(H^0_{mnp})^2$$

$$R(\omega) = -\frac{1}{3}(H^0_{mnp})^2 - 6\nabla_m^2\Phi + 7(\nabla_m\Phi)^2$$

${\rm d}H = 0$

${\rm d}H \neq 0$


$\hat{R}_4(\hat{\Gamma}_0) \sim ||\hat{\nabla}_m(\Phi - \hat{D})||^2$
$F_{mn}J^{mn} = 0$
$\delta\psi_\mu = 0$
$\delta\chi = 0$
$\delta\psi_m = 0$
$\delta\lambda = 0$
$D_m(\omega_-)\Omega_{abc} = 0$
$-i\xi_+^\dagger\left([\gamma_{(k)}, \nabla\!\!\!/ \Phi] - \frac{1}{6}\{\gamma_{(k)}, H\!\!\!/\}\right)\xi_+ = 0$
$D_m(\omega_-)J_{ab} = 0$
$N_{mnp} = 0$
$R^{ab}{}_{mn}(\omega_-)J_{ab} = 0$
$N_{mnp} - H_{mnp} = -3J_{[m}{}^q J_n{}^r H_{p]qr}$
$H_{mnp}J^{np} = 2J_m{}^q\nabla_q\Phi$
$J_{[m}{}^q\nabla_{|q|}J_{np]} = -2J_{[m}{}^q J_n{}^r H_{p]qr}$
$H_{mnp} = T^{(\mathrm{B})}_{mnp}$
$-2\mathrm{d}\Phi + \theta = 0$
$\mathrm{d}(\mathrm{e}^{-2\Phi}\Omega) = 0$
$\mathrm{d}(\mathrm{e}^{-2\Phi} * J) = 0$
$H = -\frac{1}{2} * \mathrm{e}^{2\Phi}\mathrm{d}(\mathrm{e}^{-2\Phi}J)$

Gauge Symmetry Breaking

gauge algebra $\mathcal{G} = \mathcal{F} \oplus \mathcal{F}_\perp \,, \qquad \mathcal{F}_\perp = \mathcal{H} \oplus \mathcal{Q} \,, \qquad [\mathcal{H}, \mathcal{F}] = 0$

with F_{mn} **taking a value in** $\mathcal{F}$

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with F_{mn} taking a value in $\mathcal{F}$

- $E_8 \longrightarrow E_6 \times \underline{SU(3)}$: $\quad (\mathcal{G} = E_8, \quad \mathcal{F} = SU(3), \quad \mathcal{H} = E_6)$

$$\mathbf{248} = (\mathbf{78}, \mathbf{1}) + (\mathbf{1}, \mathbf{8}) + (\mathbf{27}, \mathbf{3}) + (\overline{\mathbf{27}}, \overline{\mathbf{3}})$$

- $E_8 \longrightarrow SO(16)$

  $\longrightarrow SO(10) \times \underline{SO(6)}$: $\quad (\mathcal{G} = E_8, \quad \mathcal{F} = SO(6), \quad \mathcal{H} = SO(10))$

$$\begin{aligned} \mathbf{248} &= \mathbf{120}_{SO(16)\ \text{adj.}} + \mathbf{128}_{SO(16)\ \text{spinor}} \\ &= (\mathbf{45}, \mathbf{1}) + (\mathbf{1}, \mathbf{15}) + (\mathbf{10}, \mathbf{6}) + (\mathbf{16}, \mathbf{4}) + (\overline{\mathbf{16}}, \overline{\mathbf{4}}) \end{aligned}$$

Each breaking scenario deeply depends on the way
of embedding the holonomy group into the gauge groups.

Candidates:

$$A \leftrightarrow \begin{cases} \omega_- \quad : \quad SU(3) \text{ holonomy} \\ \omega_+ \quad : \quad SO(6) \text{ holonomy} \qquad\qquad [\omega_\pm = \omega \pm H] \\ \quad \text{etc.} \end{cases}$$

Candidates:

$$A \;\leftrightarrow\; \begin{cases} \omega_- \;:\; & SU(3) \text{ holonomy} \\ \omega_+ \;:\; & SO(6) \text{ holonomy} \\ \text{etc.} & \end{cases} \qquad [\omega_\pm = \omega \pm H]$$

with following constraints

$$\begin{aligned} R_{mnpq}(\omega_+) &= R_{pqmn}(\omega_-) + (\mathrm{d}H)_{pqmn} \\ \mathrm{d}H &= -\alpha'\Big[\mathrm{tr}\big(F \wedge F\big) - \mathrm{tr}\big\{R(\omega_+) \wedge R(\omega_+)\big\}\Big] \end{aligned}$$

$\mathrm{d}H = 0$

$\mathrm{d}H \neq 0$

$R^{ab}{}_{mn}(\omega_-)$:	type $(1,1)$ w/ indices a, b
F :	$(1,1)$-form
$\mathrm{d}H$:	$(2,2)$-form, higher order in α'
$R(\omega_+)$:	$(1,1)$-form + higher order in α'

Mainly we consider $E_8 \longrightarrow SO(10) \times SO(6)$ breaking: $A = \omega_+$

Zero mode equations

# of massless modes on $\mathcal{M}^{3,1}$	$\Leftrightarrow$	# of zero modes on $\mathcal{K}_6$

Zero mode eq. for gaugino:

$$\begin{aligned} 0 &= \not{D}(\omega, A)\chi^0 - \frac{1}{12} H_{mnp} \Gamma^{mnp} \chi^0 \\ &= \not{D}(\hat{\omega}, A)\chi^0 \qquad \left[\hat{\omega} \equiv \omega - \tfrac{1}{3} H \right] \end{aligned}$$

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Zero mode eq. for gaugino:

$$
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&= \not{D}(\hat{\omega}, A)\chi^0 \qquad\qquad \left[\hat{\omega} \equiv \omega - \tfrac{1}{3} H \right]
\end{aligned}
$$

Decompose χ^0 into

$$
0 = \not{D}(\hat{\omega})\chi^0_{\mathcal{H}} \quad \text{and} \quad 0 = \not{D}(\hat{\omega}, A_{\mathcal{Q}})\chi^0_{\mathcal{Q}} \, ,
$$

which correspond to the neutral and charged matter fermions, respectively.

$$
[\text{\# of zero mode } \chi^0_{\mathcal{H}}] = \lim_{\beta \to 0} \mathrm{Tr}\, \Gamma_{(6+1)} \mathrm{e}^{-\beta \Delta_{\mathcal{H}}} \equiv \mathrm{index}\, \not{D}(\hat{\omega})
$$

Evaluate the square of the Dirac operators with $\hat{\omega} = \omega - \frac{1}{3}H$:

$$\Delta_{\mathcal{H}} \equiv -[\not{D}(\hat{\omega})]^2 = D_m(\omega_-)^\dagger D^m(\omega_-) + V$$

$$\Delta_{\mathcal{Q}} \equiv -[\not{D}(\hat{\omega}, A_{\mathcal{Q}})]^2 = D_m(\omega_-)^\dagger D^m(\omega_-) + V + \frac{i}{2} F^{\mathcal{Q}}_{mn} \Gamma^{mn}$$

$$V = \frac{1}{4}\left[R(\omega) - \frac{1}{3} H_{mnp} H^{mnp} + \frac{1}{12}(\mathrm{d}H)_{mnpq}\Gamma^{mnpq}\right]$$

$\mathrm{d}H = 0$

$\mathrm{d}H \neq 0$

The "potential" V plays a crucial role in

- the zero mode equations of Klein-Gordon type
- the Atiyah-(Patodi)-Singer index density
- etc.

Minimal embedding: $\mathrm{d}H = 0$

SUSY A V

Zero mode equation tells us $[V = \frac{1}{3}(H^0_{mnp})^2]$

$$0 = \left[D_m(\omega_-)^\dagger D^m(\omega_-) + \frac{1}{3}(H^0_{mnp})^2 \right] \chi^0_{\mathcal{H}}$$

$$\therefore \quad 0 = \int_{\mathcal{K}_6} \left[\left| D_m(\omega_-)\chi^0_{\mathcal{H}} \right|^2 + \frac{1}{3} \left| H^0_{mnp} \right|^2 \left| \chi^0_{\mathcal{H}} \right|^2 \right]$$

If no boundaries/singularities ⇒ $\begin{cases} H^0 = 0 \text{ or} \\ \chi^0_{\mathcal{H}} = 0 \end{cases}$ ⇒ no massless modes

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If no boundaries/singularities ➡ $\begin{cases} H^0 = 0 \text{ or} \\ \chi^0_{\mathcal{H}} = 0 \end{cases}$ ➡ no massless modes

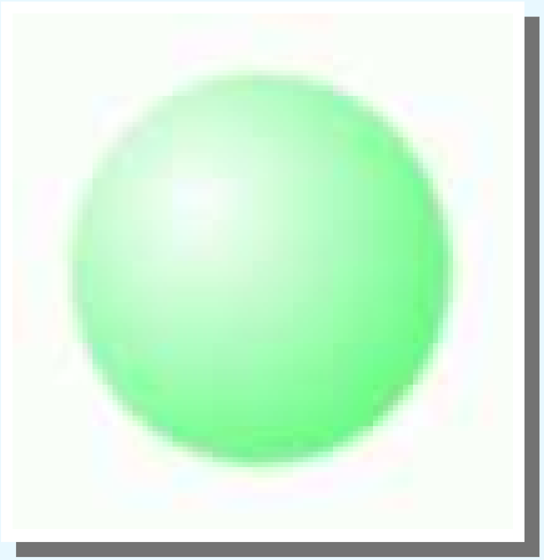
The condition $*(J \wedge \mathrm{d}H) = 0$ denotes

$$\frac{1}{2} \nabla^2_m \mathrm{e}^{-\Phi} = \frac{1}{3} \mathrm{e}^{-\Phi} (H^0_{mnp})^2$$

If there are no boundaries/singularities on $\mathcal{K}_6$, then

$$\frac{1}{3} \int_{\mathcal{K}_6} \mathrm{e}^{-\Phi} \left| H^0_{mnp} \right|^2 = \frac{1}{2} \int_{\mathcal{K}_6} \nabla^2_m \mathrm{e}^{-\Phi} = 0$$

This means $H^0 = 0$ ➡ $\Phi = \text{const.}$ ➡ $\mathcal{K}_6 = \mathrm{CY}_3$

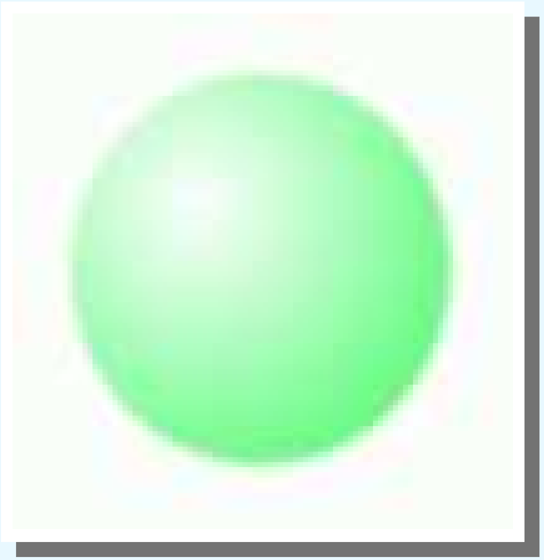


Without boundaries/singularities on $\mathcal{K}_6$:

- **all fluxes are trivial** $H = \mathrm{d}\Phi = 0$
- $\mathcal{K}_6 = \mathrm{CY}_3$
- $\omega_+ = \omega_- = \omega$, $E_8 \longrightarrow E_6 \times SU(3)$
- **$\#$ of zero modes — AS index theorem**

Candelas, Horowitz, Strominger and Witten

[Nucl. Phys. B258 (1985) 46]



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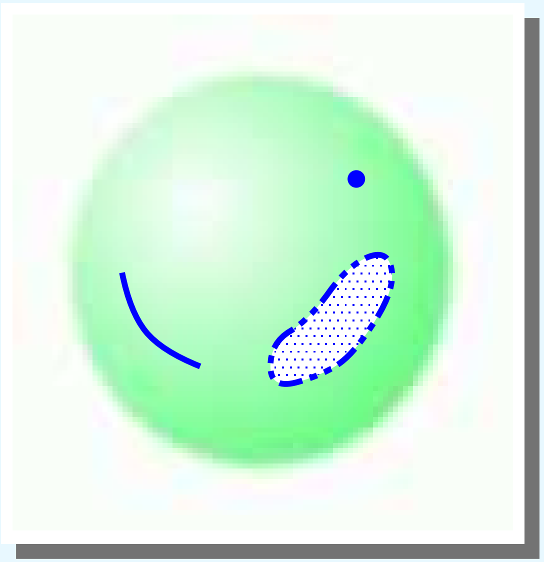

With boundaries/singularities on $\mathcal{K}_6$:

- non-trivial fluxes can exist

  $\partial_m \Phi = 0$, $H^0_{mnp} \neq 0$: conformally balanced T
- $E_8 \to SO(10) \times SO(6)$
- $\chi^0_{\mathcal{H}}$ lives in the boundaries
- $\#$ of zero modes — APS index theorem

Non-minimal embedding: $\mathrm{d}H \neq 0$

SUSY A V

In this case we should notice the α'-ordering in the Lagrangian with keeping

$$\frac{\alpha'}{L^2} \ll 1 \qquad L = \text{linear size of } \mathcal{K}_6$$

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SUSY A V

In this case we should notice the α'-ordering in the Lagrangian with keeping

$$\frac{\alpha'}{L^2} \ll 1 \qquad L = \text{linear size of } \mathcal{K}_6$$

Notice that the scaling orders of variables:

$$F_{mn} \sim R^p{}_{qmn}(\omega) \sim \frac{1}{L^2}$$

$$(\nabla_m \Phi)^2 \sim \mathrm{d}H \sim (H_{mnp})^2 \sim (Ric)_{mn}(\omega) \sim \frac{\alpha'}{L^4}$$

$$R^p{}_{qmn}(\omega_+) = R^p{}_{qmn}(\omega) + 2\nabla_{[m} H^p{}_{|q|n]} + 2H^p{}_{r[m} H^r{}_{|q|n]}$$

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$$R^p{}_{qmn}(\omega_+) = R^p{}_{qmn}(\omega) + 2\nabla_{[m} H^p{}_{|q|n]} + 2H^p{}_{r[m} H^r{}_{|q|n]}$$

Then, in the leading order in α',

we replace $R_{abmn}(\omega_+)$ to $R_{abmn}(\omega)$ in the Lagrangian and the Bianchi identity

$$\rightarrow \qquad \mathrm{d}H = -\alpha' \Big[\mathrm{tr}\big(F \wedge F\big) - \mathrm{tr}\big\{ R(\omega) \wedge R(\omega) \big\} \Big]$$

Combining equations of motion and SUSY conditions, we obtain

$$\mathrm{tr}\big(R_{mn}R^{mn}\big) - \mathrm{tr}(F_{mn}F^{mn}) \;=\; -2 * \Big[J \wedge \Big(\mathrm{tr}(R \wedge R) - \mathrm{tr}(F \wedge F)\Big)\Big] + \mathcal{O}(\alpha')$$

$$0 \;=\; \frac{1}{2}\nabla_m^2 \mathrm{e}^{-2\Phi} - \frac{1}{3}\mathrm{e}^{-2\Phi}(H_{mnp})^2 - \mathrm{e}^{-2\Phi} * (J \wedge \mathrm{d}H) + \mathcal{O}(\alpha'^2)$$

Then, within the linear order in α', we find

$$\nabla_m^2 \mathrm{e}^{-2\Phi} \;=\; \mathrm{e}^{-2\Phi}\left[\frac{2}{3}\big|H_{mnp}\big|^2 + \alpha'\big(\mathrm{tr}\big|F_{mn}\big|^2 - \mathrm{tr}\big|R_{mn}\big|^2\big)\right]$$

Integral on a smooth manifold $\mathcal{K}_6$:

$$\int_{\mathcal{K}_6} \mathrm{e}^{-2\Phi}\left[\frac{2}{3}\big|H_{mnp}\big|^2 + \alpha'\mathrm{tr}\big|F_{mn}\big|^2\right] \;=\; \int_{\mathcal{K}_6} \mathrm{e}^{-2\Phi}\left[\alpha'\mathrm{tr}\big|R_{mn}\big|^2\right]$$

with $\quad \mathrm{tr}\big|F_{mn}\big|^2 \;\neq\; \mathrm{tr}\big|R_{mn}\big|^2$

Smooth compactification scenario is possible!

Summary and Discussions

Summary and Discussions

- **Vacuum configuration of the flux compactifications in heterotic theory**
- **No-go theorem on smooth manifolds with $H \neq 0$ and $\mathrm{d}H = 0$**
- **Possibility of smooth compactifications with $H \neq 0$ and $\mathrm{d}H \neq 0$**

Summary and Discussions

- Vacuum configuration of the flux compactifications in heterotic theory
- No-go theorem on smooth manifolds with $H \neq 0$ and $\mathrm{d}H = 0$
- Possibility of smooth compactifications with $H \neq 0$ and $\mathrm{d}H \neq 0$

- ? $\#$ of zero modes under the condition $\mathrm{d}H \neq 0$

  modification of the Atiyah-(Patodi)-Singer index theorem
- other possibilities of gauge symmetry breaking
- compactifications on non-complex geometries SUSY

Frey and Lippert [hep-th/0507202]

Manousselis, Prezas and Zoupanos [hep-th/0511122]

Appendix

Appendix: Quartic effective Lagrangian

$$\mathscr{L}_{\text{total}} = \mathscr{L}_0(R) + \mathscr{L}_\beta(F^2) + \mathscr{L}_\alpha(R^2)$$ L

$$\begin{aligned}
\mathscr{L}_0(R) \;=\; & \frac{1}{2\kappa_{10}^2}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\Big[R(\omega) - \frac{1}{3}H_{MNP}H^{MNP} + 4(\nabla_M\Phi)^2 - \overline{\psi}_M\Gamma^{MNP}D_N(\omega)\psi_P + 16\,\overline{\lambda}\not{D}(\omega)\lambda \\
& + 8\,\overline{\lambda}\Gamma^{MN}D_M(\omega)\psi_N + 8\,\overline{\psi}_M\Gamma^N\Gamma^M\lambda\,(\nabla_N\Phi) - 2\,\overline{\psi}_M\Gamma^M\psi_N\,(\nabla^N\Phi) \\
& + \frac{1}{12}H^{PQR}\Big\{\overline{\psi}_M\Gamma^{[M}\Gamma_{PQR}\Gamma^{N]}\psi_N + 8\,\overline{\psi}_M\Gamma^M{}_{PQR}\lambda - 16\,\overline{\lambda}\Gamma_{PQR}\lambda\Big\} \\
& + \frac{1}{48}\overline{\psi}^M\Gamma^{ABC}\psi_M\Big\{2\overline{\lambda}\Gamma_{ABC}\lambda + \overline{\lambda}\Gamma_{ABC}\Gamma^N\psi_N - \frac{1}{4}\overline{\psi}^N\Gamma_{ABC}\psi_N - \frac{1}{8}\overline{\psi}^N\Gamma_N\Gamma_{ABC}\Gamma^P\psi_P\Big\}\Big]
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}_\beta(F^2) \;=\; & \frac{1}{2\kappa_{10}^2}\frac{\kappa_{10}^2}{2g_{10}^2}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\Big[-\operatorname{tr}(F_{MN}F^{MN}) - 2\operatorname{tr}\{\overline{\chi}\not{D}(\omega,A)\chi\} + \frac{1}{6}\operatorname{tr}(\overline{\chi}\Gamma^{ABC}\chi)\hat{H}_{ABC} \\
& - \frac{1}{2}\operatorname{tr}\{\overline{\chi}\Gamma^M\Gamma^{AB}(F_{AB} + \hat{F}_{AB})\}\Big(\psi_M + \frac{2}{3}\Gamma_M\lambda\Big) - \frac{1}{48}\operatorname{tr}(\overline{\chi}\Gamma^{ABC}\chi)\,\overline{\psi}_M\big(4\Gamma_{ABC}\Gamma^M + 3\Gamma^M\Gamma_{ABC}\big)\lambda \\
& + \frac{1}{12}\operatorname{tr}(\overline{\chi}\Gamma^{ABC}\chi)\overline{\lambda}\Gamma_{ABC}\lambda - \frac{\beta}{96}\operatorname{tr}(\overline{\chi}\Gamma^{ABC}\chi)\operatorname{tr}(\overline{\chi}\Gamma_{ABC}\chi)\Big]
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}_\alpha(R^2) \;=\; & \frac{1}{2\kappa_{10}^2}\frac{\kappa_{10}^2}{2g_{10}^2}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\Big[-R_{ABMN}(\omega_+)R^{ABMN}(\omega_+) - 2\overline{\psi}^{AB}\not{D}(\omega(e,\psi),\omega_+)\psi_{AB} + \frac{1}{6}\overline{\psi}^{AB}\Gamma^{MNP}\psi_{AB}\hat{H}_{MNP} \\
& + \frac{1}{2}\overline{\psi}_{AB}\Gamma^M\Gamma^{NP}\{R^{AB}{}_{NP}(\omega_+) + \hat{R}^{AB}{}_{NP}(\omega_+)\}\Big(\psi_M + \frac{2}{3}\Gamma_M\lambda\Big) \\
& - \frac{1}{48}\overline{\psi}_{AB}\Gamma^{CDE}\psi_{AB}\cdot\overline{\psi}_M\big(4\Gamma_{CDE}\Gamma^M + 3\Gamma^M\Gamma_{CDE}\big)\lambda \\
& + \frac{1}{12}\overline{\psi}^{AB}\Gamma^{CDE}\psi_{AB}\big(\overline{\lambda}\Gamma_{CDE}\lambda\big) - \frac{\alpha}{96}\overline{\psi}^{AB}\Gamma^{FGH}\psi_{AB}\big(\overline{\psi}^{CD}\Gamma_{FGH}\psi_{CD}\big)\Big]
\end{aligned}$$

$$
\begin{aligned}
\delta_0 e_M{}^A &= \frac{1}{2}\bar{\epsilon}\Gamma^A\psi_M \\
\delta_0\psi_M &= \Big(\partial_M + \frac{1}{4}\omega_{-M}{}^{AB}\Gamma_{AB}\Big)\epsilon + \Big\{\epsilon\big(\overline{\psi}_M\lambda\big) - \psi_M\big(\bar{\epsilon}\lambda\big) + \Gamma^A\lambda\big(\overline{\psi}_M\Gamma_A\epsilon\big)\Big\} \\
\delta_0 B_{MN} &= \bar{\epsilon}\Gamma_{[M}\psi_{N]} \\
\delta_0\lambda &= -\frac{1}{4}\,\not{D}\Phi\,\epsilon + \frac{1}{24}\Gamma^{ABC}\epsilon\Big(\hat{H}_{ABC} - \frac{1}{4}\overline{\lambda}\Gamma_{ABC}\lambda\Big) \\
\delta_0\Phi &= -\bar{\epsilon}\lambda \\
\delta_0 A_M &= \frac{1}{2}\bar{\epsilon}\Gamma_M\chi \\
\delta_0\chi &= -\frac{1}{4}\Gamma^{AB}\epsilon\,\hat{F}_{AB} + \Big\{\epsilon\big(\overline{\chi}\lambda\big) - \chi\big(\bar{\epsilon}\lambda\big) + \Gamma^A\lambda\big(\overline{\chi}\Gamma_A\epsilon\big)\Big\} \\
\delta_\beta\psi_M &= \frac{\beta}{192}\,\Gamma^{ABC}\Gamma_M\epsilon\,\mathrm{tr}\big(\overline{\chi}\Gamma_{ABC}\chi\big) \\
\delta_\beta B_{MN} &= -\beta\,\mathrm{tr}\big\{A_{[M}\delta_0 A_{N]}\big\} \\
\delta_\beta\lambda &= \frac{\beta}{384}\,\Gamma^{ABC}\epsilon\,\mathrm{tr}\big(\overline{\chi}\Gamma_{ABC}\chi\big) \\
\delta_\alpha\psi_M &= \frac{\alpha}{192}\Gamma^{CDE}\Gamma_M\epsilon\,\overline{\psi}^{AB}\Gamma_{CDE}\psi_{AB} \\
\delta_\alpha B_{MN} &= -\alpha\,\omega_{+[M}{}^{AB}\,\delta_0\omega_{+N]}{}^{AB} \\
\delta_\alpha\lambda &= \frac{\alpha}{384}\,\Gamma^{CDE}\Gamma_M\epsilon\,\overline{\psi}^{AB}\Gamma_{CDE}\psi_{AB}
\end{aligned}
$$

Bergshoeff and de Roo [Nucl. Phys. B328 (1989) 439]

Supergravity QL

- **Bosonic part of the Lagrangian** (without fermion condensations)

$$\mathscr{L} = \frac{1}{4}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\Big[R(\omega) - \frac{1}{3}H_{MNP}H^{MNP} + 4(\nabla_M\Phi)^2 - \alpha'\Big\{\mathrm{tr}\big(F_{MN}F^{MN}\big) - \mathrm{tr}\big(R_{MN}(\widetilde{\omega})R^{MN}(\widetilde{\omega})\big)\Big\}\Big]$$

- **Bianchi identity** $[\omega_+ = \omega + H \longrightarrow \widetilde{\omega}]$

$$\mathrm{d}H = -\alpha'\Big[\mathrm{tr}\big\{F \wedge F\big\} - \mathrm{tr}\big\{R(\widetilde{\omega}) \wedge R(\widetilde{\omega})\big\}\Big]$$

Hull [Phys. Lett. B167 (1986) 51]

(worldsheet 2-loop β-function)

Appendix: Intrinsic torsion

Chiossi and Salamon [math.DG/0202282]

Let κ be a contorsion in $\nabla^{(T)}$ with acting on the $SU(3)$ Killing spinor ξ:

$$0 = \nabla^{(T)}\xi = (\nabla + \kappa^0 + \kappa^{\mathfrak{g}})\xi$$

where we decomposed $\kappa \equiv \kappa^0 + \kappa^{\mathfrak{g}}$ in such a way as $\kappa^{\mathfrak{g}}\xi = 0$ (where $\mathfrak{g} = \mathfrak{su}(3)$):

$$\xi = \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline * \end{array}\right) \qquad \kappa^{\mathfrak{g}} \equiv \left(\begin{array}{ccc|c} * & * & * & 0 \\ * & * & * & 0 \\ * & * & * & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right) \qquad \kappa^0 \equiv \left(\begin{array}{ccc|c} 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \\ \hline * & * & * & * \end{array}\right)$$

Then, under the same structure group G we find

$$(\nabla^{(T_1)} - \nabla^{(T_2)})\xi \propto \kappa^{\mathfrak{g}}\xi = 0$$

So, from the group-theoretical viewpoint, κ^0 carries an intrinsic part of the contorsion when we consider the classification of the $SU(3)$-structure manifolds!

Torsion $T_{mn} \equiv T^p{}_{mn} \mathrm{d}x^p = \kappa^p{}_{[mn]} \mathrm{d}x^p$ is given in the various representations:

$$T^{\mathfrak{g}}_{mn} \;=\; \kappa^{\mathfrak{g}}{}_{[mn]} \;\sim\; \mathfrak{su}(3)\,, \qquad T^0_{mn} \;=\; \kappa^0_{[mn]} \;\sim\; \mathfrak{so}(6)/\mathfrak{su}(3) \;\equiv\; \mathfrak{su}(3)^\perp$$

$$\therefore \qquad (T^0)^p{}_{mn} \in \Lambda^1 \otimes \mathfrak{su}(3)^\perp \quad \text{on } \mathcal{K}_6$$

$$\Lambda^1 \;\sim\; 3 \oplus \overline{3}\,, \qquad \mathfrak{su}(3) \;\sim\; 8\,, \qquad \mathfrak{su}(3)^\perp \;=\; \mathfrak{so}(6)/\mathfrak{su}(3) \;\sim\; 1 \oplus 3 \oplus \overline{3}$$

Thus the intrinsic torsion T^0 can be decomposed

$$\begin{aligned}
(T^0)^p{}_{mn} \;\in\; \Lambda^1 \otimes \mathfrak{su}(3)^\perp \;&=\; (3 \oplus \overline{3}) \otimes (1 \oplus 3 \oplus \overline{3}) \\
&=\; \underset{W_1}{(1 \oplus 1)} \oplus \underset{W_2}{(8 \oplus 8)} \oplus \underset{W_3}{(6 \oplus \overline{6})} \oplus \underset{W_4}{(3 \oplus \overline{3})} \oplus \underset{W_5}{(3 \oplus \overline{3})'}
\end{aligned}$$

where
- W_1 : complex scalar in $(1 \oplus 1)$
- W_2 : complex primitive 2-form in $(8 \oplus 8)$
- W_3 : real primitive $(2,1) \oplus (1,2)$-form in $(6 \oplus \overline{6})$
- W_4 : real 1-form in $(3 \oplus \overline{3})$
- W_5 : complex $(1,0)$-form in $(3 \oplus \overline{3})'$

complex manifolds

SUSY | $\mathrm{d}H = 0$ | $\mathrm{d}H \neq 0$

$W_1 = W_2 = 0$	$T^0 \in W_3 \oplus W_4 \oplus W_5$	hermitian
$W_1 = W_2 = W_4 = 0$	$T^0 \in W_3 \oplus W_5$	balanced
$W_1 = W_2 = W_4 = W_5 = 0$	$T^0 \in W_3$	special-hermitian
$W_1 = W_2 = W_3 = W_4 = 0$	$T^0 \in W_5$	Kähler
$W_1 = W_2 = W_3 = W_4 = W_5 = 0$	$T^0 = 0$	Calabi-Yau
$W_1 = W_2 = W_3 = 3W_4 + 2W_5 = 0$	$T^0 \in W_4 \oplus W_5$	conformally Calabi-Yau

non-complex manifolds

Summary

$W_1 = W_3 = W_4 = 0$	$T^0 \in W_2 \oplus W_5$	symplectic
$W_2 = W_3 = W_4 = W_5 = 0$	$T^0 \in W_1$	nearly-Kähler
$W_1 = W_3 = W_4 = W_5 = 0$	$T^0 \in W_2$	almost-Kähler
$W_3 = W_4 = W_5 = 0$	$T^0 \in W_1 \oplus W_2$	quasi-Kähler
$W_4 = W_5 = 0$	$T^0 \in W_1 \oplus W_2 \oplus W_3$	semi-Kähler
$W_1^- = W_2^- = W_4 = W_5 = 0$	$T^0 \in W_1^+ \oplus W_2^+ \oplus W_3$	half-flat