Heterotirc String Compactification with Neveu-Schwarz Fluxes

in collaboration with Piljin Yi (KIAS)

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One of the significant purposes for particle theorists is...

Construct a realistic model in $(3+1)$-dim. spacetime

- matter contents and their interactions
- gauge symmetry and its breaking
- gravity, cosmology
- etc., etc.

An approach from String/Supergravity Theories

assumption: (minimal) Supersymmetry

▼ **SUSY $\Leftrightarrow$ Killing spinor(s):**

$$\left(\partial_M + (\omega_M{}^{AB} - H_M{}^{AB})\Gamma_{AB}\right)\xi \;=\; 0$$

The Killing spinor equation gives us information of (compactified) geometry:

movie gif

on the web "VISUALIZATION" maintained by Jeff Bryant

Let us consider an effective theory from HETEROTIC STRING.

What is heterotic string theory?

$$\begin{cases} \text{contains only NS fields : } G_{MN}, B_{MN}, \Phi \\ \text{includes Yang-Mills gauge symmetry : } E_8 \times E_8 \text{ or } SO(32) \\ \text{realizes anomaly cancellation in a miraculous way} \end{cases}$$

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A good issue to understand string dualities at deeper levels

$$\textbf{duality:}\ \mathrm{II}/\mathrm{CY}_3 \leftrightarrow \mathrm{HE}/[\mathrm{K3} \times T^2]$$

- **What is heterotic string theory?**

$$\begin{cases} \text{contains only NS fields} : G_{MN}, B_{MN}, \Phi \\ \text{includes Yang-Mills gauge symmetry} : E_8 \times E_8 \text{ or } SO(32) \\ \text{realizes anomaly cancellation in a miraculous way} \end{cases}$$

- **A good issue to understand string dualities at deeper levels**

duality: $\mathrm{II}/\mathrm{CY}_3 \leftrightarrow \mathrm{HE}/[\mathrm{K3} \times T^2]$

- **A new insight on inflation scenario** (?)

KKLT **model in type** II **string with fluxes**

Contents

References

A.Strominger: [NPB274 (1986) 253]

C.M.Hull: [PLB178 (1986) 357]

E.A.Bergshoeff and M.de Roo: [NPB328 (1989) 439]

S.Ivanov and G.Papadopoulos: [hep-th/0008232]

G.L.Cardoso, G.Curio, G.Dall'Agata and D.Lüst: [hep-th/0306088]

A.R.Frey and M.Lippert: [hep-th/0507202]

K.Becker, M.Becker, J.X.Fu, L.-S.Tseng and S.-T.Yau: [hep-th/0604137]

P.Yi and TK: [hep-th/0605247]

M.Becker, L.-S.Tseng and S.-T.Yau: [hep-th/0612290]

and more..

movie gif

G-structure Manfolds

G-structure group on an n-dim. manifold $\mathcal{M}$

$^\exists$nowhere vanishing tensors X on $\mathcal{M}$

with satisfying $D_m(\omega)X = 0$:

tensors	G-structure	
η_{ab}	$O(n)$	
η_{ab} $\mathcal{E}_{a_1 \cdots a_n}$	$SO(n)$	
η_{ab} $J_a{}^b$	$U(m)$	$J^2 = -1$
η_{ab} $J_a{}^b$ $\Omega^{(m)}$	$SU(m)$	$(2m = n)$

6-dim. $SU(3)$-structure on manifold

Consider a geometry $\mathcal{K}_6$ with one Killing spinor equation including torsion

$$ {}^{\exists}\text{complex Weyl } \xi \qquad \text{s.t. } \nabla^{(T)}\xi = 0 $$

This is a definition of geometry with $SU(3)$-structure.

6-dim. $SU(3)$-structure on manifold

Consider a geometry $\mathcal{K}_6$ with one Killing spinor equation including torsion

$$^{\exists}\text{complex Weyl } \xi \qquad \text{s.t.} \quad \nabla^{(T)}\xi \ = \ 0$$

This is a definition of geometry with $SU(3)$-structure.

2-form in $SO(6)$ ➡ a real 2-form in $SU(3)$

$${}_6C_2 = 15 \quad = \quad 1 + 3 + \overline{3} + 8 \quad : \quad J_{ab} = i\xi^{\dagger}\,\Gamma_{ab}\,\xi$$

3-form in $SO(6)$ ➡ an (almost) complex 3-form in $SU(3)$

$${}_6C_3 = 20 \quad = \quad 1 + 1 + 3 + \overline{3} + 6 + \overline{6} \ : \quad \Omega_{abc} = \xi^{\mathrm{T}}\,\Gamma_{abc}\,\xi$$

movie gif

Heterotic Theory

The story starts from

A. Strominger, "*Superstrings with torsion*" [Nucl. Phys. B274 (1986) 253]

Heterotic theory on $SU(3)$-structure manifold

Supergravity with anti-hermitian Yang-Mills field

- **Bosonic part of the Lagrangian** (without fermion condensations)

$$\mathscr{L} = \frac{1}{4}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\Big[R(\omega) - \frac{1}{3}H_{MNP}H^{MNP} + 4(\nabla_M\Phi)^2 + \alpha'\Big\{\mathrm{tr}\big(F_{MN}F^{MN}\big)\Big\}\Big]$$

- **Bianchi identity** $[\omega_{mab}]$

$$\mathrm{d}H = +\alpha'\Big[\mathrm{tr}\big\{F \wedge F\big\}\Big]$$

Chapline and Manton [Phys. Lett. B120 (1983) 105]

(supergravity)

Heterotic theory on $SU(3)$-structure manifold

Supergravity with anti-hermitian $\mathrm{Yang\text{-}Mills}$ field

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$$\mathrm{d}H = +\alpha'\Big[\mathrm{tr}\{F\wedge F\} - \mathrm{tr}\{R(\omega)\wedge R(\omega)\}\Big]$$

Green and Schwarz [Phys. Lett. B149 (1984) 117]

(anomaly cancellation)

(worldsheet 1-loop β-function)

Heterotic theory on $SU(3)$-structure manifold

Supergravity with anti-hermitian Yang-Mills field QL

- **Bosonic part of the Lagrangian** (without fermion condensations)

$$\mathscr{L} = \frac{1}{4}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\Big[R(\omega) - \frac{1}{3}H_{MNP}H^{MNP} + 4(\nabla_M\Phi)^2 \\ +\alpha'\Big\{\mathrm{tr}\big(F_{MN}F^{MN}\big) - \mathrm{tr}\big(R_{MN}(\omega_+)R^{MN}(\omega_+)\big)\Big\}\Big]$$

- **Bianchi identity** $[\omega_{+mab} = \omega_{mab} + H_{mab}]$

$$\mathrm{d}H = +\alpha'\Big[\mathrm{tr}\{F \wedge F\} - \mathrm{tr}\{R(\omega_+) \wedge R(\omega_+)\}\Big]$$

Hull [Phys. Lett. B178 (1986) 357]

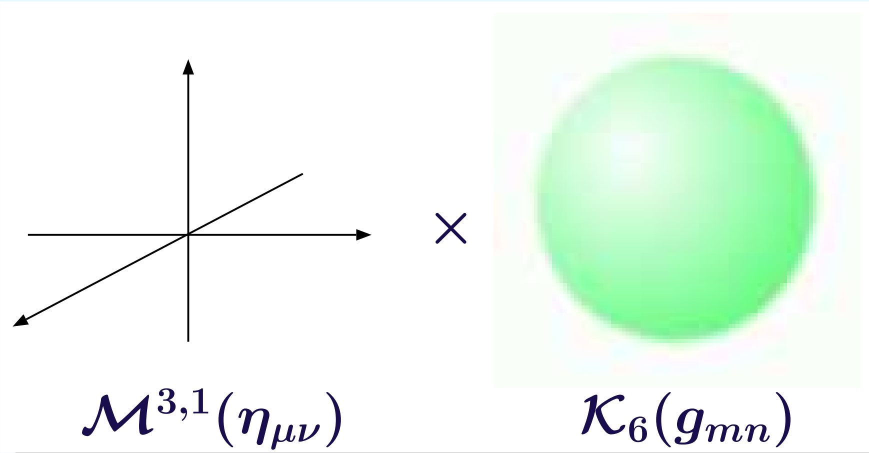
Bergshoeff and de Roo [Nucl. Phys. B328 (1989) 439]

(worldsheet 2-loop β-function)

Towards 4-dim. Physics...

- **Study vacuum configuration**
  - SUSY variations $\longrightarrow$ geometry with $SU(3)$-structure
- **Investigate low energy effective theory**
  - Gauge symmetry
  - Evaluations
  - Zero mode equations

Vacuum configuration

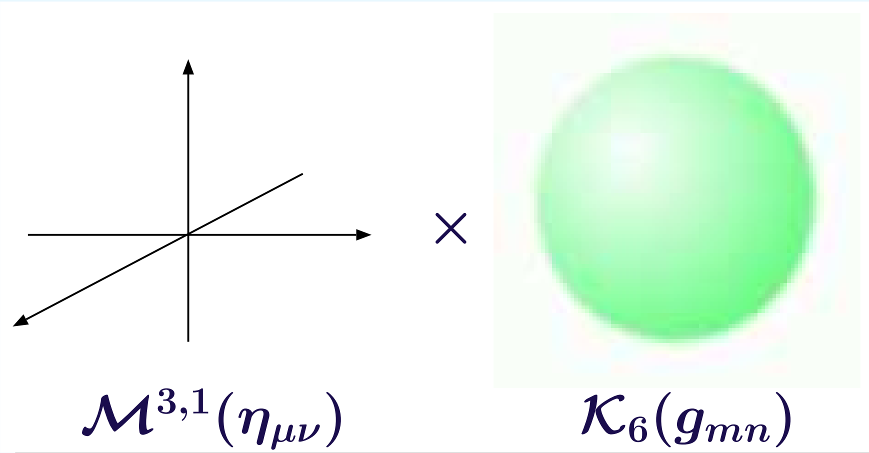


Ansatz:

$$G_{MN}\,\mathrm{d}x^M\mathrm{d}x^N = \mathrm{e}^{(\Phi-\widehat{D})/2}\Big(\eta_{\mu\nu}\,\mathrm{d}x^\mu\mathrm{d}x^\nu + g_{mn}\,\mathrm{d}y^m\mathrm{d}y^n\Big)$$

$\mathcal{M}^{3,1}$: maximally symmetric

Vacuum configuration



Ansatz:

$$\begin{aligned} G_{MN}\,\mathrm{d}x^M \mathrm{d}x^N & \\ &= \mathrm{e}^{(\Phi-\widehat{D})/2}\Big(\eta_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}\,\mathrm{d}y^m \mathrm{d}y^n\Big) \end{aligned}$$

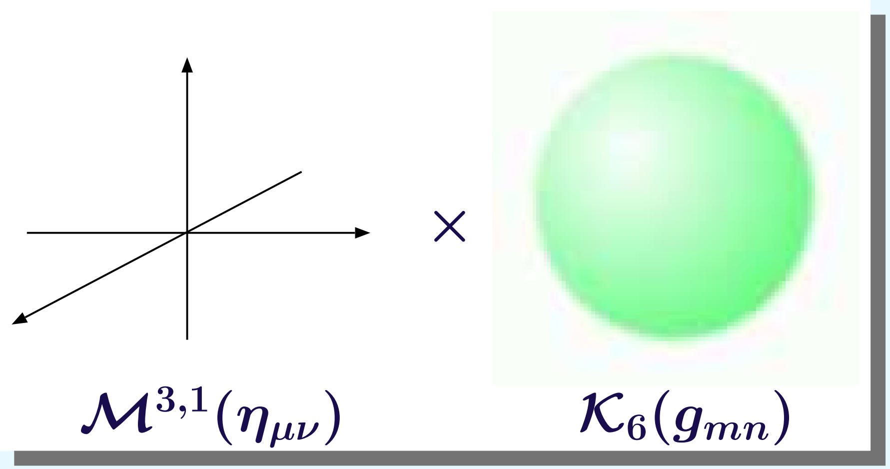
$\mathcal{M}^{3,1}$: maximally symmetric

$$\boxed{Spin(9,1) \;\rightarrow\; SL(2,\mathbb{C}) \times SU(4)}$$

$$\mathbf{16} \;=\; (\mathbf{2},\mathbf{4}) + (\overline{\mathbf{2}},\overline{\mathbf{4}}) : \qquad \epsilon_+ \;=\; \eta_+ \otimes \xi_+ + \eta_- \otimes \xi_-$$

$\mathcal{N} = 1$ **SUSY**
on $\mathcal{M}^{3,1}$

Vacuum configuration



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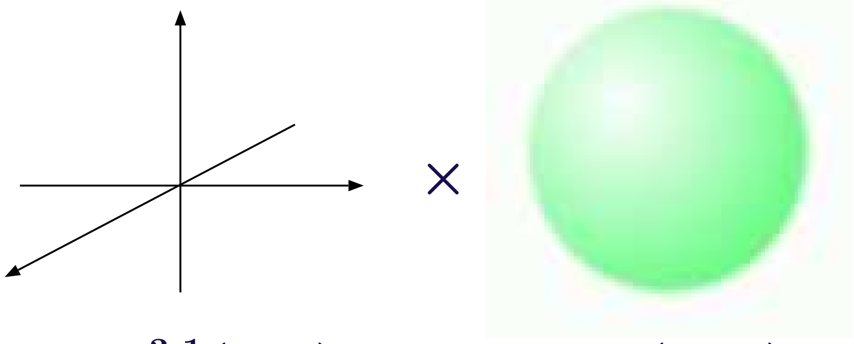
$\mathcal{N} = 1$ SUSY on $\mathcal{M}^{3,1}$ $\Longleftrightarrow$ 1 Killing spinor ξ_+ on $\mathcal{K}_6$

Vacuum configuration



$\mathcal{M}^{3,1}(\eta_{\mu\nu})$ $\quad\times\quad$ $\mathcal{K}_6(g_{mn})$

Ansatz:

$$G_{MN}\,\mathrm{d}x^M\mathrm{d}x^N = \mathrm{e}^{(\Phi-\widehat{D})/2}\Big(\eta_{\mu\nu}\,\mathrm{d}x^\mu\mathrm{d}x^\nu + g_{mn}\,\mathrm{d}y^m\mathrm{d}y^n\Big)$$

$\mathcal{M}^{3,1}$: maximally symmetric

$$\boxed{Spin(9,1) \;\rightarrow\; SL(2,\mathbb{C})\times SU(4)}$$

$$\mathbf{16} \;=\; (\mathbf{2},\mathbf{4})+(\overline{\mathbf{2}},\overline{\mathbf{4}}):\qquad \epsilon_+ \;=\; \eta_+\otimes\xi_+ + \eta_-\otimes\xi_-$$

$\mathcal{N}=1$ **SUSY** on $\mathcal{M}^{3,1}$	$\Leftrightarrow$	1 **Killing spinor** ξ_+ on $\mathcal{K}_6$	$\Leftrightarrow$	$SU(3)$**-structure** on $\mathcal{K}_6$

SUSY variations

$0 \equiv \delta\psi_m = D_m(\omega_-)\xi_+ \leftarrow$ **Killing spinor eq.** $\qquad [\omega_{-mab} = \omega_{mab} - H_{mab}]$

$$\begin{aligned} J_{ab} &= i\xi_+^\dagger \Gamma_{ab}\, \xi_+ \quad : \qquad D_m(\omega_-)J_{ab} &= 0 \\ \Omega_{abc} &= \xi_+^{\mathrm{T}} \Gamma_{abc}\, \xi_+ \quad : \qquad D_m(\omega_-)\Omega_{abc} &= 0 \end{aligned}$$

SUSY variations

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Furthermore "$0 \equiv \delta(\mathrm{fermions})$" **indicates**

$$\begin{aligned} R^{ab}{}_{mn}(\omega_-)J_{ab} &= 0 \quad : \quad c_1(R_-) \text{ vanishes} \\ N_{mn}{}^p &= 0 \quad : \quad \mathcal{K}_6 \text{ is complex} \\ F^{(2,0)} = F^{(0,2)} = F_{mn}J^{mn} &= 0 \quad : \end{aligned}$$

T

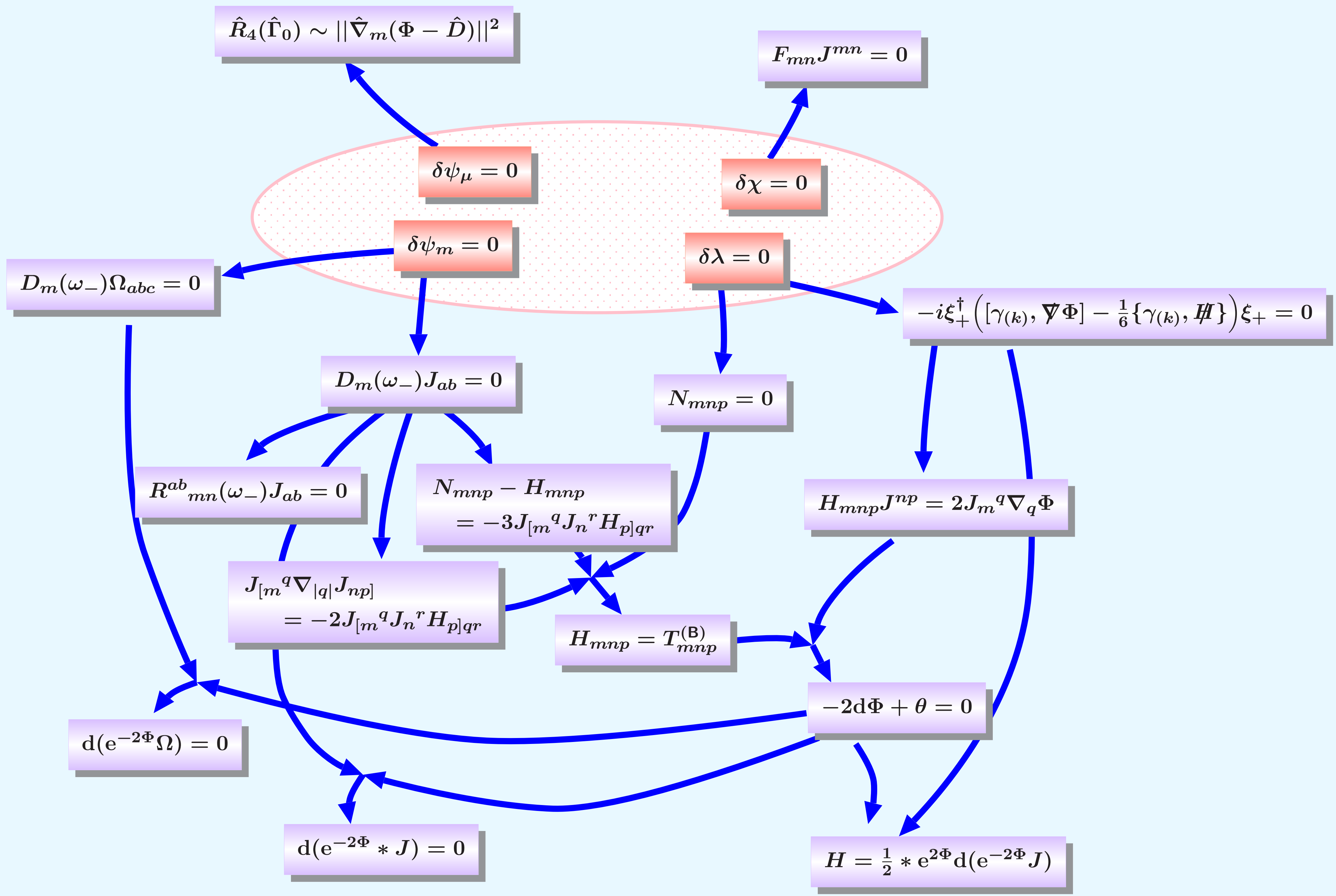
$$H = \frac{i}{2}(\partial - \overline{\partial})J \,, \qquad \mathrm{d}H = -i\partial\overline{\partial}J \,, \qquad \mathrm{d}\Phi = \frac{1}{2}J \lrcorner \mathrm{d}J$$

$$H = H^0 + \widehat{H} \,, \qquad J \lrcorner H^0 = 0 \,, \qquad \widehat{H}_{mnp} = \frac{3}{2}J_{[mn}J_{p]}{}^q\nabla_q\Phi$$

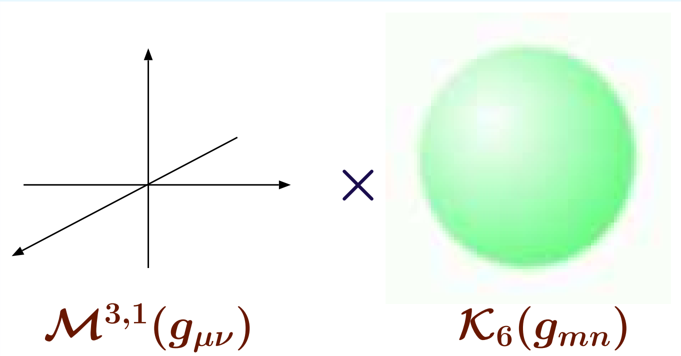
$$*(J \wedge \mathrm{d}H) = \nabla_m^2\Phi - (\nabla_m\Phi)^2 + \frac{1}{3}(H^0_{mnp})^2$$

$\mathrm{d}H = 0$

$\mathrm{d}H \neq 0$


$\hat{R}_4(\hat{\Gamma}_0) \sim ||\hat{\nabla}_m(\Phi - \hat{D})||^2$
$F_{mn}J^{mn} = 0$
$\delta\psi_\mu = 0$
$\delta\chi = 0$
$\delta\psi_m = 0$
$\delta\lambda = 0$
$D_m(\omega_-)\Omega_{abc} = 0$
$-i\xi_+^\dagger\Big([\gamma_{(k)}, \nabla\!\!\!\!/\,\Phi] - \frac{1}{6}\{\gamma_{(k)}, H\!\!\!\!/\}\Big)\xi_+ = 0$
$D_m(\omega_-)J_{ab} = 0$
$N_{mnp} = 0$
$R^{ab}{}_{mn}(\omega_-)J_{ab} = 0$
$N_{mnp} - H_{mnp}$
$= -3J_{[m}{}^qJ_n{}^rH_{p]qr}$
$H_{mnp}J^{np} = 2J_m{}^q\nabla_q\Phi$
$J_{[m}{}^q\nabla_{|q|}J_{np]}$
$= -2J_{[m}{}^qJ_n{}^rH_{p]qr}$
$H_{mnp} = T^{(\mathrm{B})}_{mnp}$
$-2\mathrm{d}\Phi + \theta = 0$
$\mathrm{d}(\mathrm{e}^{-2\Phi}\Omega) = 0$
$\mathrm{d}(\mathrm{e}^{-2\Phi} * J) = 0$
$H = \frac{1}{2} * \mathrm{e}^{2\Phi}\mathrm{d}(\mathrm{e}^{-2\Phi}J)$

Appearing spacetime geometry in Heterotic flux compactification



$$
\begin{aligned}
& g_{MN}\, dx^M dx^N \\
& \qquad = e^{(\Phi - \widehat{D})/2}\Big(g_{\mu\nu}\, dx^\mu dx^\nu + g_{mn}\, dy^m dy^n\Big)
\end{aligned}
$$

$$
\begin{aligned}
0 &= R(g_{\mu\nu}) & \qquad \mathcal{M}^{3,1} &= \text{Minkowski} \;\; \& \;\; \Phi(y) = \widehat{D}(y) \\
0 &= d\big(e^{-2\Phi} J \wedge J\big) & \mathcal{K}_6 &= \text{conformally balanced}
\end{aligned}
$$

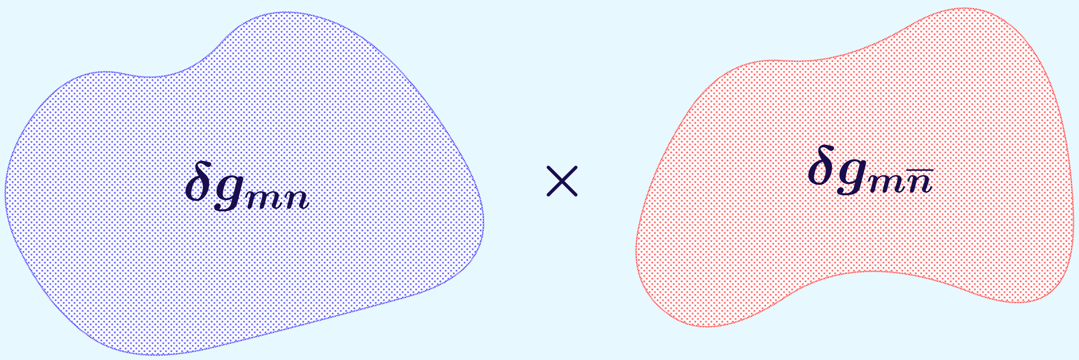
Furthermore, $\mathcal{K}_6$ can be reduced to...

if $\partial_m \Phi = 0$	:	$J \lrcorner dJ = 0$	balanced
if $H^0_{mnp} = 0$, $\partial_m \Phi \neq 0$	:	$d\big(e^{-\Phi} J\big) = 0$	conformally Kähler
if $dH = 0$	:		strong Kähler with torsion
if $H_{mnp} = \partial_m \Phi = 0$	:		Calabi–Yau

Moduli space of Calabi-Yau manifolds

movie gif

P.Candelas and X.C.de la Ossa [NPB355 (1991) 455]



$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{1,1} & & 0 & \\
1 & & h^{2,1} & & h^{1,2} & & 1 \\
 & 0 & & h^{2,2} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

$$\dim(\delta g_{mn}) = h^{2,1} : \quad \text{\# of complex moduli}$$
$$\dim(\delta g_{m\bar{n}}) = h^{1,1} : \quad \text{\# of Kähler moduli}$$

Moduli space of conformally balanced manifolds

M.Becker, L.-S.Tseng and S.-T.Yau [hep-th/0612290]

There are a number of interesting open questions.

- In our analysis we have kept for simplicity the complex structure fixed. It is well known that for Calabi-Yau compactifications the moduli space is a direct product of complex structure and Kähler structure deformations. For non-Kähler manifolds with torsion, this likely is not the case and it would be interesting to allow for a simultaneous variation of the complex structure and the hermitian form.
- It would be interesting to analyze the geometry of the moduli space and to determine if powerful tools such as the well known ''special geometry'' of Calabi-Yau compactifications can be derived in this case.
- Counting techniques for moduli fields need to be developed and we expect that the number of moduli can be characterized in terms of an index or some topological invariants of the manifold.

Gauge symmetry breaking

- $E_8 \longrightarrow E_6 \times \underline{SU(3)}$

$$\mathbf{248} = (\mathbf{78}, \mathbf{1}) + (\mathbf{1}, \mathbf{8}) + (\mathbf{27}, \mathbf{3}) + (\overline{\mathbf{27}}, \overline{\mathbf{3}})$$

- $E_8 \longrightarrow SO(16)$
  $\longrightarrow SO(10) \times \underline{SO(6)}$

$$\begin{aligned}\mathbf{248} &= \mathbf{120}_{SO(16)\ \text{adj.}} + \mathbf{128}_{SO(16)\ \text{spinor}} \\ &= (\mathbf{45}, \mathbf{1}) + (\mathbf{1}, \mathbf{15}) + (\mathbf{10}, \mathbf{6}) + (\mathbf{16}, \mathbf{4}) + (\overline{\mathbf{16}}, \overline{\mathbf{4}})\end{aligned}$$

Each breaking scenario deeply depends on the way of embedding the holonomy group into the gauge groups.

$$\textbf{Embedding } \boldsymbol{A} \leftrightarrow \begin{cases} \omega_- \ : \ SU(3) \text{ holonomy} \\ \omega_+ \ : \ SO(6) \text{ holonomy} \\ \text{etc.} \end{cases} \qquad [\omega_\pm = \omega \pm \boldsymbol{H}]$$

$$\text{Embedding } A \leftrightarrow \begin{cases} \omega_- : & SU(3) \text{ holonomy} \\ \omega_+ : & SO(6) \text{ holonomy} \\ \text{etc.} & \end{cases} \qquad [\omega_\pm = \omega \pm H]$$

with following constraints

$$R_{mnpq}(\omega_+) = R_{pqmn}(\omega_-) + (\mathrm{d}H)_{pqmn}$$

$$\mathrm{d}H = +\alpha'\Big[\mathrm{tr}\big(F \wedge F\big) - \mathrm{tr}\big\{R(\omega_+) \wedge R(\omega_+)\big\}\Big] \qquad \boxed{\mathrm{d}H = 0} \ \boxed{\mathrm{d}H \neq 0}$$

$R_{pqmn}(\omega_-)$: type $(1,1)$ w/ indices p, q

F : $(1,1)$-form

$\mathrm{d}H$: $(2,2)$-form, higher order in α'

$R_{mnpq}(\omega_+)$: $(1,1)$-form w/ indices p, q + higher order in α'

$$\boxed{\begin{array}{lll} \mathrm{d}H = 0 \text{ case: } A \equiv \omega_+ & \Rightarrow & E_8 \to SO(10) \times \underline{SO(6)} \\ \mathrm{d}H \neq 0 \text{ case: } A = \text{others..} & \Rightarrow & E_8 \to \mathcal{G} \times \underline{\mathcal{H}} \end{array}}$$

Minimal embedding: $\mathrm{d}H = 0$

SUSY A

The condition $*(J \wedge \mathrm{d}H) = 0$ denotes

$$0 \;=\; \nabla_m^2 \Phi - (\nabla_m \Phi)^2 + \frac{1}{3}(H^0_{mnp})^2$$

If there are no boundaries/singularities on $\mathcal{K}_6$, then

$$\int_{\mathcal{K}_6} \nabla_m^2 \mathrm{e}^{-\Phi} \;=\; \frac{1}{3}\int_{\mathcal{K}_6} \mathrm{e}^{-\Phi} \big\| H^0_{mnp} \big\|^2$$

$\mathrm{d}H = 0$ without any boundaries ➡ $H^0 = 0$ ➡ $\Phi = \text{const.}$

➡ smooth $\mathcal{K}_6 = \mathrm{CY}_3$

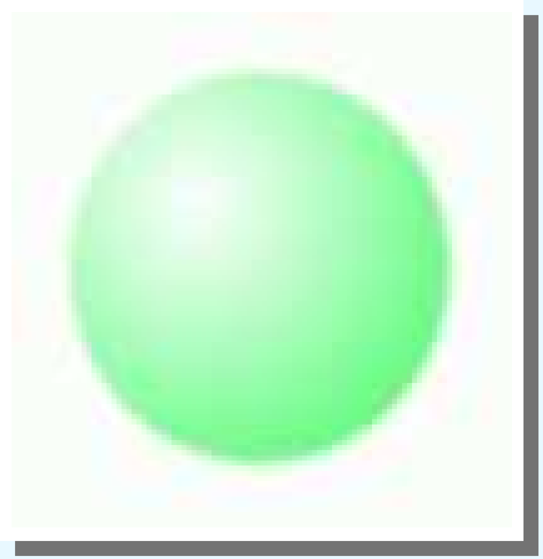
No-go theorem on smooth compactification with fluxes

Without boundaries/singularities on $\mathcal{K}_6$:

- all fluxes are trivial $H = \mathrm{d}\Phi = 0$
- $\mathcal{K}_6 = \mathrm{CY}_3$
- $\omega_+ = \omega_- = \omega$, $E_8 \longrightarrow E_6 \times SU(3)$
- $\#$ of zero modes — AS index theorem

Candelas, Horowitz, Strominger and Witten

[Nucl. Phys. B258 (1985) 46]



Without boundaries/singularities on $\mathcal{K}_6$:

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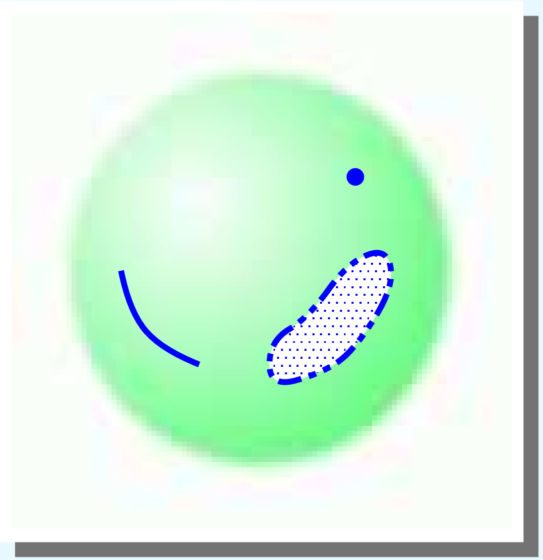

With boundaries/singularities on $\mathcal{K}_6$:

- non-trivial fluxes can exist

  $\partial_m \Phi \neq 0$, $H^0_{mnp} \neq 0$ T
- $E_8 \to SO(10) \times SO(6)$
- $\chi^0_{\mathcal{H}}$ lives in the boundaries
- $\#$ of zero modes — APS index theorem

Non-minimal embedding: $\mathrm{d}H \neq 0$

SUSY A

Combining equations of motion and SUSY conditions, we obtain

$$
\begin{aligned}
0 &= \frac{1}{2}\nabla_m^2 e^{-2\Phi} - \frac{1}{3} e^{-2\Phi}(H_{mnp})^2 + e^{-2\Phi} * (J \wedge \mathrm{d}H) \\
\mathrm{tr}\big(R_{mn}R^{mn}\big) - \mathrm{tr}(F_{mn}F^{mn}) &= -2 * \Big[J \wedge \Big(\mathrm{tr}(R \wedge R) - \mathrm{tr}(F \wedge F)\Big)\Big] + \mathcal{O}(\alpha')
\end{aligned}
$$

Non-minimal embedding: $\mathrm{d}H \neq 0$

SUSY A

Combining equations of motion and SUSY conditions, we obtain

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$$\mathrm{tr}\big(R_{mn}R^{mn}\big) - \mathrm{tr}(F_{mn}F^{mn}) = -2 * \Big[J \wedge \Big(\mathrm{tr}(R \wedge R) - \mathrm{tr}(F \wedge F)\Big)\Big] + \mathcal{O}(\alpha')$$

Then, within the linear order in α', we find

$$\nabla_m^2 e^{-2\Phi} = e^{-2\Phi}\left[\frac{2}{3}\big\|H_{mnp}\big\|^2 + \alpha'\big(\mathrm{tr}\big\|F_{mn}\big\|^2 - \mathrm{tr}\big\|R_{mn}\big\|^2\big)\right]$$

Integral on a smooth manifold $\mathcal{K}_6$:

$$\int_{\mathcal{K}_6} e^{-2\Phi}\left[\frac{2}{3}\big\|H_{mnp}\big\|^2 + \alpha'\mathrm{tr}\big\|F_{mn}\big\|^2\right] = \int_{\mathcal{K}_6} e^{-2\Phi}\left[\alpha'\mathrm{tr}\big\|R_{mn}\big\|^2\right]$$

with $\quad \mathrm{tr}\big\|F_{mn}\big\|^2 \neq \mathrm{tr}\big\|R_{mn}\big\|^2$

Smooth compactification scenario is possible!

Summary and Discussions

Summary and Discussions

- Vacuum configuration of the flux compactifications in heterotic theory
- No-go theorem on smooth manifolds with $H \neq 0$ and $\mathrm{d}H = 0$
- Possibility of smooth compactifications with $H \neq 0$ and $\mathrm{d}H \neq 0$

Summary and Discussions

- Vacuum configuration of the flux compactifications in heterotic theory
- No-go theorem on smooth manifolds with $H \neq 0$ and $\mathrm{d}H = 0$
- Possibility of smooth compactifications with $H \neq 0$ and $\mathrm{d}H \neq 0$

- Moduli space of conformally balanced manifolds
- $\#$ of zero modes under the condition $\mathrm{d}H \neq 0$

  modification of the Atiyah-(Patodi)-Singer index theorem

- compactifications on non-complex geometries SUSY

Frey and Lippert [hep-th/0507202]

Manousselis, Prezas and Zoupanos [hep-th/0511122]

Appendix

Appendix: Quartic effective Lagrangian

$$\mathscr{L}_{\text{total}} = \mathscr{L}_0(R) + \mathscr{L}_\beta(F^2) + \mathscr{L}_\alpha(R^2)$$

L

$$\begin{aligned}
\mathscr{L}_0(R) \;=\;& \frac{1}{2\kappa_{10}^2}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\Big[R(\omega) - \frac{1}{3}H_{MNP}H^{MNP} + 4(\nabla_M\Phi)^2 - \overline{\psi}_M\Gamma^{MNP}D_N(\omega)\psi_P + 16\,\overline{\lambda}\not{D}(\omega)\lambda \\
& + 8\,\overline{\lambda}\Gamma^{MN}D_M(\omega)\psi_N + 8\,\overline{\psi}_M\Gamma^N\Gamma^M\lambda\,(\nabla_N\Phi) - 2\,\overline{\psi}_M\Gamma^M\psi_N\,(\nabla^N\Phi) \\
& + \frac{1}{12}\,H^{PQR}\Big\{\overline{\psi}_M\Gamma^{[M}\Gamma_{PQR}\Gamma^{N]}\psi_N + 8\,\overline{\psi}_M\Gamma^M{}_{PQR}\lambda - 16\,\overline{\lambda}\Gamma_{PQR}\lambda\Big\} \\
& + \frac{1}{48}\,\overline{\psi}^M\Gamma^{ABC}\psi_M\Big\{2\overline{\lambda}\Gamma_{ABC}\lambda + \overline{\lambda}\Gamma_{ABC}\Gamma^N\psi_N - \frac{1}{4}\,\overline{\psi}^N\Gamma_{ABC}\psi_N - \frac{1}{8}\,\overline{\psi}^N\Gamma_N\Gamma_{ABC}\Gamma^P\psi_P\Big\}\Big]
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}_\beta(F^2) \;=\;& -\frac{1}{2\kappa_{10}^2}\frac{\kappa_{10}^2}{2g_{10}^2}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\Big[-\,\mathrm{tr}(F_{MN}F^{MN}) - 2\,\mathrm{tr}\{\overline{\chi}\not{D}(\omega,A)\chi\} + \frac{1}{6}\,\mathrm{tr}\big(\overline{\chi}\Gamma^{ABC}\chi\big)\hat{H}_{ABC} \\
& - \frac{1}{2}\,\mathrm{tr}\{\overline{\chi}\Gamma^M\Gamma^{AB}(F_{AB}+\hat{F}_{AB})\}\Big(\psi_M + \frac{2}{3}\Gamma_M\lambda\Big) - \frac{1}{48}\,\mathrm{tr}(\overline{\chi}\Gamma^{ABC}\chi)\,\overline{\psi}_M\big(4\Gamma_{ABC}\Gamma^M + 3\Gamma^M\Gamma_{ABC}\big)\lambda \\
& + \frac{1}{12}\,\mathrm{tr}(\overline{\chi}\Gamma^{ABC}\chi)\overline{\lambda}\Gamma_{ABC}\lambda - \frac{\beta}{96}\,\mathrm{tr}(\overline{\chi}\Gamma^{ABC}\chi)\,\mathrm{tr}(\overline{\chi}\Gamma_{ABC}\chi)\Big]
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}_\alpha(R^2) \;=\;& -\frac{1}{2\kappa_{10}^2}\frac{\kappa_{10}^2}{2g_{10}^2}\sqrt{-G}\,\mathrm{e}^{-2\Phi}\Big[-R_{ABMN}(\omega_+)R^{ABMN}(\omega_+) - 2\overline{\psi}^{AB}\not{D}(\omega(e,\psi),\omega_+)\psi_{AB} + \frac{1}{6}\overline{\psi}^{AB}\Gamma^{MNP}\psi_{AB}\hat{H}_{MNP} \\
& + \frac{1}{2}\overline{\psi}_{AB}\Gamma^M\Gamma^{NP}\{R^{AB}{}_{NP}(\omega_+) + \hat{R}^{AB}{}_{NP}(\omega_+)\}\Big(\psi_M + \frac{2}{3}\Gamma_M\lambda\Big) \\
& - \frac{1}{48}\,\overline{\psi}_{AB}\Gamma^{CDE}\psi_{AB}\cdot\overline{\psi}_M\big(4\Gamma_{CDE}\Gamma^M + 3\Gamma^M\Gamma_{CDE}\big)\lambda \\
& + \frac{1}{12}\overline{\psi}^{AB}\Gamma^{CDE}\psi_{AB}\big(\overline{\lambda}\Gamma_{CDE}\lambda\big) - \frac{\alpha}{96}\overline{\psi}^{AB}\Gamma^{FGH}\psi_{AB}\big(\overline{\psi}^{CD}\Gamma_{FGH}\psi_{CD}\big)\Big]
\end{aligned}$$

$$
\begin{aligned}
\delta_0 e_M{}^A &= \frac{1}{2}\bar{\epsilon}\Gamma^A\psi_M \\
\delta_0 \psi_M &= \Big(\partial_M + \frac{1}{4}\omega_{-M}{}^{AB}\Gamma_{AB}\Big)\epsilon + \Big\{\epsilon\big(\overline{\psi}_M\lambda\big) - \psi_M\big(\bar{\epsilon}\lambda\big) + \Gamma^A\lambda\big(\overline{\psi}_M\Gamma_A\epsilon\big)\Big\} \\
\delta_0 B_{MN} &= \bar{\epsilon}\Gamma_{[M}\psi_{N]} \\
\delta_0 \lambda &= -\frac{1}{4}\not{D}\Phi\,\epsilon + \frac{1}{24}\Gamma^{ABC}\epsilon\Big(\hat{H}_{ABC} - \frac{1}{4}\bar{\lambda}\Gamma_{ABC}\lambda\Big) \\
\delta_0 \Phi &= -\bar{\epsilon}\lambda \\
\delta_0 A_M &= \frac{1}{2}\bar{\epsilon}\Gamma_M\chi \\
\delta_0 \chi &= -\frac{1}{4}\Gamma^{AB}\epsilon\,\hat{F}_{AB} + \Big\{\epsilon\big(\overline{\chi}\lambda\big) - \chi\big(\bar{\epsilon}\lambda\big) + \Gamma^A\lambda\big(\overline{\chi}\Gamma_A\epsilon\big)\Big\} \\
\delta_\beta \psi_M &= \frac{\beta}{192}\,\Gamma^{ABC}\Gamma_M\epsilon\,\mathrm{tr}\big(\overline{\chi}\Gamma_{ABC}\chi\big) \\
\delta_\beta B_{MN} &= -\beta\,\mathrm{tr}\big\{A_{[M}\delta_0 A_{N]}\big\} \\
\delta_\beta \lambda &= \frac{\beta}{384}\,\Gamma^{ABC}\epsilon\,\mathrm{tr}\big(\overline{\chi}\Gamma_{ABC}\chi\big) \\
\delta_\alpha \psi_M &= \frac{\alpha}{192}\Gamma^{CDE}\Gamma_M\epsilon\,\overline{\psi}^{AB}\Gamma_{CDE}\psi_{AB} \\
\delta_\alpha B_{MN} &= -\alpha\,\omega_{+[M}{}^{AB}\,\delta_0\omega_{+N]}{}^{AB} \\
\delta_\alpha \lambda &= \frac{\alpha}{384}\,\Gamma^{CDE}\Gamma_M\epsilon\,\overline{\psi}^{AB}\Gamma_{CDE}\psi_{AB}
\end{aligned}
$$

Bergshoeff and de Roo [Nucl. Phys. B328 (1989) 439]

Equations of Motion

$$
\begin{aligned}
\Phi \quad : \quad 0 \;=\;& -R(\omega) + \frac{1}{3} H_{MNP} H^{MNP} + 4(\nabla_M \Phi)^2 - 4\nabla_M^2 \Phi \\
& - \alpha' \Big[\mathrm{tr}(F_{MN} F^{MN}) - \mathrm{tr}\{R_{MN}(\omega_+) R^{MN}(\omega_+)\} \Big] \\
G_{MN} \quad : \quad 0 \;=\;& R_{MN}(\omega) - H_{MPQ} H_N{}^{PQ} + 2\nabla_M \nabla_N \Phi \\
& - \frac{1}{2} G_{MN} \Big[R(\omega) - \frac{1}{3} H_{PQR} H^{PQR} - 4(\nabla_P \Phi)^2 + 4\nabla_P^2 \Phi \Big] \\
& - \frac{\alpha'}{2} G_{MN} \Big[\mathrm{tr}(F_{MN} F^{MN}) - \mathrm{tr}\{R_{MN}(\omega_+) R^{MN}(\omega_+)\} \Big] \\
& + 2\alpha' \Big[\mathrm{tr}(F_{MP} F_N{}^P) - \mathrm{tr}\{R_{MP}(\omega_+) R_N{}^P(\omega_+)\} \Big] \\
& + 2\alpha' \mathrm{e}^{2\Phi} \Big[2\nabla^P \nabla^Q_{(+)} \big\{ \mathrm{e}^{-2\Phi} R_{MPNQ}(\omega_+) \big\} - \nabla^Q_{(+)} \big\{ \mathrm{e}^{-2\Phi} R_{MPQR}(\omega_+) \big\} H_N{}^{PR} \\
& \qquad - 2\nabla^P \big\{ \mathrm{e}^{-2\Phi} R_{MPQR}(\omega_+) H_N{}^{QR} \big\} - 2\mathrm{e}^{-2\Phi} R_{MPQR}(\omega_+) H_N{}^{PS} H_S{}^{QR} \\
& \qquad - \nabla^P \nabla^Q_{(+)} \big\{ \mathrm{e}^{-2\Phi} R_{MNPQ}(\omega_+) \big\} + \nabla^P \big\{ \mathrm{e}^{-2\Phi} R_{MNQR}(\omega_+) H_P{}^{QR} \big\} \Big] \\
B_{MN} \quad : \quad 0 \;=\;& \nabla^M (\mathrm{e}^{-2\Phi} H_{MNP}) \\
\chi \quad : \quad 0 \;=\;& \not{D}(\omega - \tfrac{1}{3} H, A) \chi - \Gamma^M \chi \nabla_M \Phi + \frac{3}{2} \Gamma^M \Gamma^{AB} (F_{AB} + \hat{F}_{AB}) \Big(\psi_M + \frac{2}{3} \Gamma_M \lambda \Big)
\end{aligned}
$$

Appendix: Intrinsic torsion

Chiossi and Salamon [math.DG/0202282]

Let κ be a contorsion in $\nabla^{(T)}$ with acting on the $SU(3)$ Killing spinor ξ:

$$0 \;=\; \nabla^{(T)}\xi \;=\; (\nabla + \kappa^0 + \kappa^{\mathfrak{g}})\xi$$

where we decomposed $\kappa \equiv \kappa^0 + \kappa^{\mathfrak{g}}$ in such a way as $\kappa^{\mathfrak{g}}\,\xi = 0$ (where $\mathfrak{g} = \mathfrak{su}(3)$)**:**

$$\xi \;=\; \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline * \end{array}\right) \qquad \kappa^{\mathfrak{g}} \;\equiv\; \left(\begin{array}{ccc|c} * & * & * & 0 \\ * & * & * & 0 \\ * & * & * & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right) \qquad \kappa^0 \;\equiv\; \left(\begin{array}{ccc|c} 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \\ \hline * & * & * & * \end{array}\right)$$

Then, under the same structure group G we find

$$(\nabla^{(T_1)} - \nabla^{(T_2)})\xi \;\propto\; \kappa^{\mathfrak{g}}\xi \;=\; 0$$

So, from the group-theoretical viewpoint, κ^0 carries an intrinsic part of the contorsion when we consider the classification of the $SU(3)$-structure manifolds!

Torsion $T_{mn} \equiv T^p{}_{mn} \mathrm{d}x^p = \kappa^p{}_{[mn]} \mathrm{d}x^p$ is given in the various representations:

$$T^{\mathfrak{g}}_{mn} = \kappa^{\mathfrak{g}}{}_{[mn]} \sim \mathfrak{su}(3) , \qquad T^0_{mn} = \kappa^0_{[mn]} \sim \mathfrak{so}(6)/\mathfrak{su}(3) \equiv \mathfrak{su}(3)^\perp$$

$$\therefore \qquad (T^0)^p{}_{mn} \in \Lambda^1 \otimes \mathfrak{su}(3)^\perp \quad \text{on } \mathcal{K}_6$$

$$\Lambda^1 \sim 3 \oplus \overline{3} , \qquad \mathfrak{su}(3) \sim 8 , \qquad \mathfrak{su}(3)^\perp = \mathfrak{so}(6)/\mathfrak{su}(3) \sim 1 \oplus 3 \oplus \overline{3}$$

Thus the intrinsic torsion T^0 can be decomposed

$$\begin{aligned}(T^0)^p{}_{mn} \in \Lambda^1 \otimes \mathfrak{su}(3)^\perp &= (3 \oplus \overline{3}) \otimes (1 \oplus 3 \oplus \overline{3}) \\ &= \underset{W_1}{(1 \oplus 1)} \oplus \underset{W_2}{(8 \oplus 8)} \oplus \underset{W_3}{(6 \oplus \overline{6})} \oplus \underset{W_4}{(3 \oplus \overline{3})} \oplus \underset{W_5}{(3 \oplus \overline{3})'}\end{aligned}$$

where

W_1 : complex scalar in $(1 \oplus 1)$

W_2 : complex primitive 2-form in $(8 \oplus 8)$

W_3 : real primitive $(2,1) \oplus (1,2)$-form in $(6 \oplus \overline{6})$

W_4 : real 1-form in $(3 \oplus \overline{3})$

W_5 : complex $(1,0)$-form in $(3 \oplus \overline{3})'$

complex manifolds

SUSY $\mathrm{d}H = 0$ $\mathrm{d}H \neq 0$

$W_1 = W_2 = 0$	$T^0 \in W_3 \oplus W_4 \oplus W_5$	hermitian
$W_1 = W_2 = W_4 = 0$	$T^0 \in W_3 \oplus W_5$	balanced
$W_1 = W_2 = W_4 = W_5 = 0$	$T^0 \in W_3$	special-hermitian
$W_1 = W_2 = W_3 = W_4 = 0$	$T^0 \in W_5$	Kähler
$W_1 = W_2 = W_3 = W_4 = W_5 = 0$	$T^0 = 0$	Calabi-Yau
$W_1 = W_2 = W_3 = 3W_4 + 2W_5 = 0$	$T^0 \in W_4 \oplus W_5$	conformally Calabi-Yau

non-complex manifolds

Summary

$W_1 = W_3 = W_4 = 0$	$T^0 \in W_2 \oplus W_5$	symplectic
$W_2 = W_3 = W_4 = W_5 = 0$	$T^0 \in W_1$	nearly-Kähler
$W_1 = W_3 = W_4 = W_5 = 0$	$T^0 \in W_2$	almost-Kähler
$W_3 = W_4 = W_5 = 0$	$T^0 \in W_1 \oplus W_2$	quasi-Kähler
$W_4 = W_5 = 0$	$T^0 \in W_1 \oplus W_2 \oplus W_3$	semi-Kähler
$W_1^- = W_2^- = W_4 = W_5 = 0$	$T^0 \in W_1^+ \oplus W_2^+ \oplus W_3$	half-flat