Realization of AdS Vacua in Attractor Mechanism on Generalized Geometry

arXiv:0810.0937 [hep-th]

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We are looking for the origin of 4D physics

Physical information

- particle contents and spectra
- (broken) symmetries and interactions
- potential, vacuum and cosmological constant

10D string theories could provide information

via compactifications

In the present stage, we have not understood yet

a dynamical compactification of the 10D spacetime

However, we can investigate physical data

of low energy effective theories reduced from string theories

under a set of assumptions.

ex.)

- ✓ $10 = 4 + 6$ with $4 =$ (A)dS or Minkowski, $6 =$ compact
- ✓ $\mathcal{N} = 1$ SUSY
- ✓ no non-trivial background fields

A typical success:

$E_8 \times E_8$ heterotic string compactified on Calabi-Yau three-fold

- number of generations $= |\chi(\mathrm{CY}_3)|/2$
- E_6 gauge symmetry
- zero cosmological constant

P. Candelas, G.T. Horowitz, A. Strominger, E. Witten "*Vacuum configurations for superstrings*," Nucl. Phys. B 258 (1985) 46

But, this vacuum is too simple.

There are no ways to truncate many redundant massless fields.

Relax the assumptions: introduce non-trivial background fields

RR-fluxes, NS-fluxes, fermion condensations, D-branes, etc.

$\downarrow$

warp factor, torsion, and/or cosmological constant are generated

How a suitable matter content and a vacuum are realized?

Study again 4D $\mathcal{N} = 1$ supergravity

$$S = \int \Big(\frac{1}{2} R * \mathbf{1} - \frac{1}{2} F^a \wedge * F^a - K_{\mathcal{M}\overline{\mathcal{N}}} \nabla \phi^{\mathcal{M}} \wedge * \nabla \overline{\phi}^{\overline{\mathcal{N}}} - V \Big)$$

$$V = \mathrm{e}^K \Big(K^{\mathcal{M}\overline{\mathcal{N}}} D_{\mathcal{M}} \mathcal{W} \, \overline{D_{\mathcal{N}} \mathcal{W}} - 3|\mathcal{W}|^2 \Big) + \frac{1}{2} |D^a|^2$$

K : Kähler potential

$\mathcal{W}$: superpotential ←-- $\delta\psi_\mu = \nabla_\mu \varepsilon - \mathrm{e}^{\frac{K}{2}} \mathcal{W} \gamma_\mu \varepsilon^c$

D^a : D-term ←-- $\delta\chi^a = \mathrm{Im} F^a_{\mu\nu} \gamma^{\mu\nu} \varepsilon + \mathrm{i} D^a \varepsilon$

Search of vacua $\partial_{\mathcal{P}} V \big|_* = 0$

$V_* > 0$: de Sitter space

$V_* = 0$: Minkowski space

$V_* < 0$: Anti-de Sitter space

Decompose 10D SUSY variations:

$$\epsilon^1 = \varepsilon_1 \otimes (a\,\eta^1_+) + \varepsilon_1^c \otimes (\overline{a}\,\eta^1_-) \qquad \epsilon^2 = \varepsilon_2 \otimes (\overline{b}\,\eta^2_-) + \varepsilon_2^c \otimes (b\,\eta^2_+)$$

$$\delta\Psi^{\mathcal{A}}_M = 0 \quad \Big\langle \begin{array}{lcl} \delta\psi_{\mathcal{A}\mu} = 0 & \dashrightarrow & \text{superpotential } \mathcal{W} \\ \delta\psi^{\mathcal{A}}_m = 0 & \dashrightarrow & \text{Kähler potential } K \end{array}$$

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$\delta\psi^{\mathcal{A}}_m = 0$ is nothing but the Killing spinor equation on compactified geometry $\mathcal{M}$:

$$\delta\psi^{\mathcal{A}}_m \;=\; \Big(\partial_m + \frac{1}{4}\omega_{mab}\,\gamma^{ab}\Big)\eta^{\mathcal{A}}_+ + \big(\text{3-form fluxes}\cdot\eta\big)^{\mathcal{A}} + \big(\text{other fluxes}\cdot\eta\big)^{\mathcal{A}} \;=\; 0$$

Information of

6D $SU(3)$ Killing spinors $\eta^{\mathcal{A}}_+$

Calabi-Yau three-fold

↓

$SU(3)$-structure manifold with torsion

↓

generalized geometry

- Calabi-Yau three-fold

- $SU(3)$-structure manifold

- Generalized geometry

- Calabi-Yau three-fold $\dashrightarrow$ Fluxes are strongly restricted

$$\begin{cases} \text{type IIA}: & \text{No fluxes} \\ \text{type IIB}: & F_3 - \tau H \qquad \text{(warped Calabi-Yau)} \\ \text{heterotic}: & \text{No fluxes} \end{cases}$$

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- $SU(3)$-structure manifold $\dashrightarrow$ Some components of fluxes can be interpreted as torsion

$$\left.\begin{matrix} \text{type IIA} \\ \text{type IIB} \\ \text{heterotic}^{1} \end{matrix}\right\} \qquad \text{restricted fluxes are turned on}^{2}$$

1: Piljin Yi, TK "*Comments on heterotic flux compactifications*," JHEP 0607 (2006) 030, hep-th/0605247

2: TK "*Index theorems on torsional geometries*," JHEP 0708 (2007) 048, arXiv:0704.2111

- Generalized geometry

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- Generalized geometry ⇢ Any types of fluxes can be included

Definition of almost complex structures is extended

10D type IIA supergravity as a low energy theory of IIA string

$\downarrow$ compactifications on a generalized geometry in the presence of fluxes

4D $\mathcal{N} = 2$ supergravity

$\downarrow$ SUSY truncation (via orientifold projections)

4D $\mathcal{N} = 1$ supergravity

- **Moduli stabilization**

  SUSY AdS or Minkowski vacua emerge on the attractor points

- **Mathematical feature**

  Attractor points are governed by discriminants of the $\mathcal{N}=1$ superpotentials

- **Stringy effects**

  Some α' corrections are included in certain configurations

  as the back reactions of fluxes on the compactified geometry

Contents

Data from generalized geometry

Extend the definition of the almost complex structure

$J \in T\mathcal{M}$ w/ $Spin(6)$ group $\dashrightarrow$ $\mathcal{J} \in T\mathcal{M} \oplus T^*\mathcal{M}$ w/ $Spin(6,6)$ group

Generalized almost complex structures are described by $SU(3,3)$ Weyl spinors $\Phi_\pm$:

$$\mathcal{J}_{\pm\Lambda\Sigma} = \big\langle \mathrm{Re}\Phi_\pm, \Gamma_{\Lambda\Sigma}\,\mathrm{Re}\Phi_\pm \big\rangle \qquad \text{cf.} \quad J_{mn} = -2\mathrm{i}\,\eta_\pm^\dagger\,\gamma_{mn}\,\eta_\pm$$

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Weyl spinors on $T\mathcal{M} \oplus T^*\mathcal{M}$	$\longleftrightarrow$	differential even/odd-forms on $T^*\mathcal{M}$
Φ_+	$\longleftrightarrow$	even-forms
Φ_-	$\longleftrightarrow$	odd-forms

Connect between $SU(3)$ spinors $\eta_\pm^{\mathcal{A}}$ and $SU(3,3)$ spinors $\Phi_\pm$:

$$\Phi_\pm \equiv 8\, \mathrm{e}^{-B}\, \eta_+^1 \otimes \eta_\pm^{2\dagger} \equiv \mathrm{e}^{-B} \sum_{k=0}^{6} \frac{1}{k!} \big(\eta_\pm^{2\dagger} \gamma_{m_1\cdots m_k} \eta_+^1\big)\, \gamma^{m_1\cdots m_k}$$

$$\equiv \sum_{k=0}^{6} \frac{1}{k!}\, \Phi_{m_1\cdots m_k}\, \mathrm{d}x^{m_1} \wedge \cdots \wedge \mathrm{d}x^{m_k}$$

The spaces of the $SU(3,3)$ spinors $\Phi_\pm$ are the Hodge-Kähler geometries

Kähler potentials, prepotentials, projective coordinates

$\dashrightarrow$ Building blocks of 4D $\mathcal{N}=2$ supergravity

M. Graña, J. Louis, D. Waldram hep-th/0505264

$$K_+ \;=\; -\log \mathrm{i} \int_{\mathcal{M}} \langle \Phi_+, \overline{\Phi}_+ \rangle \;=\; -\log \mathrm{i} \big(\overline{X}^A \mathcal{F}_A - X^A \overline{\mathcal{F}}_A \big)$$

$$K_- \;=\; -\log \mathrm{i} \int_{\mathcal{M}} \langle \Phi_-, \overline{\Phi}_- \rangle \;=\; -\log \mathrm{i} \big(\overline{Z}^I \mathcal{G}_I - Z^I \overline{\mathcal{G}}_I \big)$$

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Expand the even/odd-forms $\Phi_\pm$ by the basis forms:

$$\Phi_+ \;=\; X^A \omega_A - \mathcal{F}_A \widetilde{\omega}^A \,, \qquad \omega_A = (1, \omega_a)\,, \qquad \widetilde{\omega}^A = (\widetilde{\omega}^a, \mathrm{vol}(\mathcal{M})) \qquad : \quad \text{0,2,4,6-forms}$$

$$\Phi_- \;=\; Z^I \alpha_I - \mathcal{G}_I \beta^I \,, \qquad \alpha_I = (\alpha_0, \alpha_i)\,, \qquad \beta^I = (\beta^i, \beta^0) \qquad : \quad \text{1,3,5-forms}$$

$$\int_{\mathcal{M}} \langle \omega_A, \omega_B \rangle \;=\; 0\,, \quad \int_{\mathcal{M}} \langle \omega_A, \widetilde{\omega}^B \rangle \;=\; \delta_A{}^B\,, \quad \int_{\mathcal{M}} \langle \alpha_I, \alpha_J \rangle \;=\; 0\,, \quad \int_{\mathcal{M}} \langle \alpha_I, \beta^J \rangle \;=\; \delta_I{}^J$$

Basis forms are no longer closed:

$$\begin{aligned} \mathrm{d}_H \omega_A &= m_A{}^I \alpha_I - e_{IA} \beta^I & \qquad \mathrm{d}_H \widetilde{\omega}^A &= 0 \\ \mathrm{d}_H \alpha_I &= e_{IA} \widetilde{\omega}^A & \qquad \mathrm{d}_H \beta^I &= m_A{}^I \widetilde{\omega}^A \end{aligned}$$

where NS three-form H deforms the differential operator:

$$\mathrm{d}H = 0 \,, \qquad H = H^{\text{fl}} + \mathrm{d}B \,, \qquad H^{\text{fl}} = m_0{}^I \alpha_I - e_{I0} \beta^I$$

$$\mathrm{d}_H \equiv \mathrm{d} - H^{\text{fl}} \wedge$$

$$(\mathrm{d}_H)^2 = 0 \quad \rightarrow \quad m_A{}^I e_{IB} - e_{IA} m_B{}^I = 0$$

background	charges
NS-flux charges	e_{I0} $m_0{}^I$
torsion	e_{Ia} $m_a{}^I$

Furthermore, extend to the generalized differential operator $\mathcal{D}$:

$$\mathrm{d}_H = \mathrm{d} - H^{\mathsf{fl}} \wedge \quad \to \quad \mathcal{D} \equiv \mathrm{d} - H^{\mathsf{fl}} \wedge - Q \cdot - R \llcorner$$

$$\mathcal{D}\omega_A \sim m_A{}^I \alpha_I - e_{IA} \beta^I , \qquad \mathcal{D}\widetilde{\omega}^A \sim -q^{IA} \alpha_I + p_I{}^A \beta^I$$

$$\mathcal{D}\alpha_I \sim p_I{}^A \omega_A + e_{IA} \widetilde{\omega}^A , \qquad \mathcal{D}\beta^I \sim q^{IA} \omega_A + m_A{}^I \widetilde{\omega}^A$$

Necessary to introduce new fluxes Q and R to make a consistent algebra...

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Necessary to introduce new fluxes Q and R to make a consistent algebra...

But the compactified geometry becomes nongeometric:

$(Q \cdot C)_{m_1 \cdots m_{k-1}}$	$\equiv$	$Q^{ab}{}_{[m_1} C_{\lvert ab \rvert m_2 \cdots m_{k-1}]}$	feature of T-fold
$(R \llcorner C)_{m_1 \cdots m_{k-3}}$	$\equiv$	$R^{abc} C_{abc m_1 \cdots m_{k-3}}$	locally nongeometric background

Structure group contains Diffeo. + Duality trsf. $\dashrightarrow$ *Doubled formalism*[3]

3: C. Albertsson, R.A. Reid-Edwards, TK "*D-branes and doubled geometry*," arXiv:0806.1783

RR-fluxes $F^{\text{even}} = \mathrm{e}^B G$ without localized sources:

$$\begin{aligned} G &= G_0 + G_2 + G_4 + G_6 = G^{\text{fl}} + \mathrm{d}_H A \\ F_n^{\text{even}} &= \mathrm{d}C_{n-1} - H \wedge C_{n-3}\,, \quad C = \mathrm{e}^B A \\ \mathrm{d}_H F^{\text{even}} &= 0 \end{aligned}$$

Formal extension of RR-fluxes on generalized geometry:

$$G = G^{\text{fl}} + \mathcal{D} A\,, \qquad \mathcal{D} G = 0$$

$$G^{\text{fl}} = \sqrt{2}\big(m_{\text{RR}}^A\, \omega_A - e_{\text{RR}A}\, \widetilde{\omega}^A\big)\,, \qquad A = \sqrt{2}\big(\xi^I\, \alpha_I - \widetilde{\xi}_I\, \beta^I\big)$$

$$\Downarrow$$

$$G \sim G^A\, \omega_A - \widetilde{G}_A\, \widetilde{\omega}^A$$

$$G^A \sim \sqrt{2}\big(m_{\text{RR}}^A + \xi^I\, p_I{}^A - \widetilde{\xi}_I\, q^{IA}\big)\,, \qquad \widetilde{G}_A \sim \sqrt{2}\big(e_{\text{RR}A} - \xi^I\, e_{IA} + \widetilde{\xi}_I\, m_A{}^I\big)$$

fluxes	charges	
NS three-form H	e_{I0}	${m_0}^I$
torsion	e_{Ia}	${m_a}^I$
nongeometric fluxes	${p_I}^A$	q^{IA}
RR-fluxes	$e_{\text{RR}A}$	m_{RR}^A

backgrounds	flux charges			
Calabi-Yau	—			
Calabi-Yau with H	e_{I0}	${m_0}^I$		
$SU(3)$-structure manifold	e_{IA}	${m_A}^I$		
Generalized geometry	e_{IA}	${m_A}^I$	${p_I}^A$	q^{IA}

All the information of the compactified geometry is translated into

the (non)geometric flux charges and the RR-flux charges.

4D $\mathcal{N} = 2$ theory comes out by the compactification: $\varepsilon_1, \varepsilon_2$

NEXT STEP

Introduce the flux charges into 4D $\mathcal{N} = 1$ physics via various functionals:

K, $\mathcal{W}$, D^a

Setup in $\mathcal{N} = 1$ theory

Functionals are given by two Kähler potentials on two Hodge-Kähler geometries of $\Phi_\pm$:

$$\begin{aligned} K &= K_+ + 4\varphi \\ K_+ &= -\log \mathrm{i}\big(\overline{X}^A \mathcal{F}_A - X^A \overline{\mathcal{F}}_A\big) \\ K_- &= -\log \mathrm{i}\big(\overline{Z}^I \mathcal{G}_I - Z^I \overline{\mathcal{G}}_I\big) \end{aligned}$$

Introduce the compensator $\mathcal{C} = \sqrt{2} ab\, \mathrm{e}^{-\phi^{(10)}} = 4ab\, \mathrm{e}^{\frac{K_-}{2} - \varphi}$

$$\therefore \quad \mathrm{e}^{-2\varphi} = \frac{|\mathcal{C}|^2}{16|a|^2|b|^2}\mathrm{e}^{-K_-} = \frac{\mathrm{i}}{16|a|^2|b|^2}\int_{\mathcal{M}} \big\langle \mathcal{C}\Phi_-, \overline{\mathcal{C}\Phi}_- \big\rangle$$

See the SUSY variation of 4D $\mathcal{N} = 2$ gravitinos:

$$\begin{aligned} \delta\psi_{\mathcal{A}\mu} &= \nabla_\mu \varepsilon_{\mathcal{A}} - S_{\mathcal{A}\mathcal{B}}\, \gamma_\mu\, \varepsilon^{\mathcal{B}} + \dots \\ S_{\mathcal{A}\mathcal{B}} &= \frac{\mathrm{i}}{2} \mathrm{e}^{\frac{K_+}{2}} \begin{pmatrix} \mathcal{P}^1 - \mathrm{i}\mathcal{P}^2 & -\mathcal{P}^3 \\ -\mathcal{P}^3 & -\mathcal{P}^1 - \mathrm{i}\mathcal{P}^2 \end{pmatrix}_{\mathcal{A}\mathcal{B}} \end{aligned}$$

The components are also written by $\Phi_\pm$:

$$\mathcal{P}^1 - \mathrm{i}\mathcal{P}^2 \;=\; 2\,\mathrm{e}^{\frac{K_-}{2}+\varphi} \int_{\mathcal{M}} \big\langle \Phi_+, \mathcal{D}\Phi_- \big\rangle \,, \qquad \mathcal{P}^1 + \mathrm{i}\mathcal{P}^2 \;=\; 2\,\mathrm{e}^{\frac{K_-}{2}+\varphi} \int_{\mathcal{M}} \big\langle \Phi_+, \mathcal{D}\overline{\Phi}_- \big\rangle$$

$$\mathcal{P}^3 \;=\; -\frac{1}{\sqrt{2}}\, \mathrm{e}^{2\varphi} \int_{\mathcal{M}} \big\langle \Phi_+, G \big\rangle$$

4D $\mathcal{N}=1$ fermions given by the SUSY truncation from 4D $\mathcal{N}=2$ system:

$$
\begin{aligned}
\text{SUSY parameter :} \quad \varepsilon &\equiv \overline{n}^{\mathcal{A}} \varepsilon_{\mathcal{A}} = a\,\varepsilon_1 + \bar{b}\,\varepsilon_2 \\
\text{gravitino :} \quad \psi_\mu &\equiv \overline{n}^{\mathcal{A}} \psi_{\mathcal{A}\mu} = a\,\psi_{1\mu} + \bar{b}\,\psi_{2\mu} \\
\text{gauginos :} \quad \chi^A &\equiv -2\,\mathrm{e}^{\frac{K_+}{2}} D_b X^A \big(\overline{n}^{\mathcal{C}}\,\epsilon_{\mathcal{C}\mathcal{E}}\,\chi^{\mathcal{E}b}\big) \\
\text{dilatino :} \quad \lambda &\equiv \overline{n}^{\mathcal{A}} \lambda_{\mathcal{A}}
\end{aligned}
$$

$$
\text{where} \quad \overline{n}^{\mathcal{A}} = (a\,,\,\bar{b})\,, \qquad \epsilon_{\mathcal{A}\mathcal{B}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}
$$

SUSY variations yield the superpotential and the D-term:

$$\delta\psi_\mu = \nabla_\mu\varepsilon - \overline{n}^{\mathcal{A}}\, S_{\mathcal{AB}}\, n^{*\mathcal{B}}\, \gamma_\mu\, \varepsilon^c \equiv \nabla_\mu\varepsilon - \mathrm{e}^{\frac{K}{2}}\, \mathcal{W}\, \gamma_\mu\, \varepsilon^c$$

$$\delta\chi^A = \mathrm{Im}F^A_{\mu\nu}\, \gamma^{\mu\nu}\, \varepsilon + \mathrm{i}\, D^A\, \varepsilon$$

$$\mathcal{W} = \frac{\mathrm{i}}{4\overline{a}b}\Big[4\mathrm{i}\, \mathrm{e}^{\frac{K_-}{2}-\varphi} \int_{\mathcal{M}} \big\langle \Phi_+, \mathcal{D}\mathrm{Im}(ab\Phi_-)\big\rangle + \frac{1}{\sqrt{2}} \int_{\mathcal{M}} \big\langle \Phi_+, G\big\rangle\Big]$$

$$\equiv \mathcal{W}^{\mathsf{RR}} + U^I\, \mathcal{W}_I^{\mathbb{Q}} + \widetilde{U}_I\, \widetilde{\mathcal{W}}^I_{\mathbb{Q}}$$

$$\mathcal{W}^{\mathsf{RR}} = -\frac{\mathrm{i}}{4\overline{a}b}\Big[X^A\, e_{\mathsf{RR}A} - \mathcal{F}_A\, m^A_{\mathsf{RR}}\Big]$$

$$\mathcal{W}_I^{\mathbb{Q}} = \frac{\mathrm{i}}{4\overline{a}b}\Big[X^A\, e_{IA} + \mathcal{F}_A\, p_I{}^A\Big], \qquad \widetilde{\mathcal{W}}^I_{\mathbb{Q}} = -\frac{\mathrm{i}}{4\overline{a}b}\Big[X^A\, m_A{}^I + \mathcal{F}_A\, q^{IA}\Big]$$

$$D^A = 2\, \mathrm{e}^{K_+} (K_+)^{c\overline{d}}\, D_c X^A\, \overline{D_d X^B}\, \big[\overline{n}^{\mathcal{C}} (\sigma_x)_{\mathcal{C}}{}^{\mathcal{B}} n_{\mathcal{B}}\big] \Big(\mathcal{P}^x_B - \mathcal{N}_{BC} \widetilde{\mathcal{P}}^{xC}\Big)$$

$\mathcal{N}=2$ multiplets:

$(t^a = X^a/X^0,\ z^i = Z^i/Z^0)$

gravity multiplet	$g_{\mu\nu},\ A^0_\mu$	
vector multiplets	$A^a_\mu,\ t^a = b^a + \mathrm{i} v^a$	$a = 1, \ldots, b^+$
hypermultiplets	$z^i,\ \xi^i,\ \widetilde{\xi}_i$	$i = 1, \ldots, b^-$
tensor multiplet	$B_{\mu\nu},\ \varphi,\ \xi^0,\ \widetilde{\xi}_0$	

orientifold projection: $\mathcal{O} \equiv \Omega_{\mathsf{WS}} (-1)^{F_L} \sigma$

$\mathcal{N}=1$ multiplets:

gravity multiplet	$g_{\mu\nu}$	
vector multiplets	$A^{\hat{a}}_\mu$	$\hat{a} = 1, \ldots, \hat{n}_v = b^+ - n_{ch}$
chiral multiplets	$t^{\check{a}} = b^{\check{a}} + \mathrm{i} v^{\check{a}}$	$\check{a} = 1, \ldots, n_{ch}$
chiral/linear multiplets	$U^{\check{I}} = \xi^{\check{I}} + \mathrm{i}\,\mathrm{Im}(\mathcal{C} Z^{\check{I}})$ $\widetilde{U}_{\hat{I}} = \widetilde{\xi}_{\hat{I}} + \mathrm{i}\,\mathrm{Im}(\mathcal{C} \mathcal{G}_{\hat{I}})$	$I = (\check{I}, \hat{I}) = 0, 1, \ldots, b^-$
(projected out)	$B_{\mu\nu},\ A^0_\mu,\ A^{\check{a}}_\mu,\ t^{\hat{a}},\ U^{\hat{I}},\ \widetilde{U}_{\check{I}}$	

Parameters are restricted as $a = \bar{b}\, \mathrm{e}^{i\theta}$ and $|a|^2 = |b|^2 = \frac{1}{2}$

We are ready to search SUSY vacua in 4D $\mathcal{N} = 1$ supergravity.

Consider three typical situations given by

- generalized geometry with RR-flux charges

  $e_{IA},\ m_A{}^I,\ p_I{}^A,\ q^{IA},\ e_{\text{RR}A},\ m_{\text{RR}}^A$

- generalized geometry without RR-flux charges

  $e_{IA},\ m_A{}^I,\ p_I{}^A,\ q^{IA}$

- $SU(3)$-structure manifold without RR-flux charges

  $e_{IA},\ m_A{}^I$

Notice: 4D physics given by Calabi-Yau three-fold with RR-fluxes is forbidden.

RR-fluxes induce the non-zero NS-fluxes as well as torsion classes in SUSY solutions.

D. Lüst, D. Tsimpis hep-th/0412250

Search of SUSY vacua: flux vacua attractors

$$V = \mathrm{e}^K \Big(K^{\mathcal{M}\overline{\mathcal{N}}} D_{\mathcal{M}} \mathcal{W} \, \overline{D_{\mathcal{N}} \mathcal{W}} - 3|\mathcal{W}|^2 \Big) + \frac{1}{2} |D^{\hat{a}}|^2$$

$$\equiv V_{\mathcal{W}} + V_D$$

Search of vacua $\partial_{\mathcal{P}} V \big|_* = 0$

$V_* > 0$	:	de Sitter space
$V_* = 0$	:	Minkowski space
$V_* < 0$	:	Anti-de Sitter space

$$0 = \partial_{\mathcal{P}} V_{\mathcal{W}} = \mathrm{e}^K \Big\{ K^{\mathcal{M}\overline{\mathcal{N}}} D_{\mathcal{P}} D_{\mathcal{M}} \mathcal{W} \overline{D_{\mathcal{N}} \mathcal{W}} + \partial_{\mathcal{P}} K^{\mathcal{M}\overline{\mathcal{N}}} D_{\mathcal{M}} \mathcal{W} \overline{D_{\mathcal{N}} \mathcal{W}} - 2 \overline{\mathcal{W}} D_{\mathcal{P}} \mathcal{W} \Big\}$$

$$0 = \partial_{\mathcal{P}} V_D \quad \dashrightarrow \quad D^{\hat{a}} = 0$$

Consider the SUSY condition $D_{\mathcal{P}} \mathcal{W} \equiv \big(\partial_{\mathcal{P}} + \partial_{\mathcal{P}} K \big) \mathcal{W} = 0$ in various cases.

1. Set a simple prepotential: $\mathcal{F} = D_{abc} \frac{X^a X^b X^c}{X^0}$

2. Consider the (1,1)-moduli model: $t^{\check{a}} \equiv t,\ U^{\check{I}} \equiv U,\ \widetilde{U}_{\hat{I}} = 0 \quad (D_{abc} = D \equiv 1)$

Derivatives of the Kähler potential are

$$\partial_t K = -\frac{3}{t - \bar{t}} \qquad \partial_U K = -\frac{2}{U - \overline{U}}$$

The superpotential is reduced to

$$\begin{aligned} \mathcal{W} &= \mathcal{W}^{\mathsf{RR}} + U\,\mathcal{W}^{\mathbb{Q}} \\ \mathcal{W}^{\mathsf{RR}} &= m^0_{\mathsf{RR}}\, t^3 - 3\, m_{\mathsf{RR}}\, t^2 + e_{\mathsf{RR}}\, t + e_{\mathsf{RR}0} \\ \mathcal{W}^{\mathbb{Q}} &= p_0{}^0\, t^3 - 3\, p_0\, t^2 - e_0\, t - e_{00} \end{aligned}$$

Consider the SUSY condition $D_{\mathcal{P}}\mathcal{W} \equiv \big(\partial_{\mathcal{P}} + \partial_{\mathcal{P}} K\big)\mathcal{W} = 0$:

$$D_t \mathcal{W} = 0 \dashrightarrow 0 = D_t \mathcal{W}^{\mathsf{RR}} + U\, D_t \mathcal{W}^{\mathbb{Q}}$$

$$D_U \mathcal{W} = 0 \dashrightarrow 0 = \frac{\mathrm{i}}{\mathrm{Im} U}\Big(\mathcal{W}^{\mathsf{RR}} + \mathrm{Re} U\, \mathcal{W}^{\mathbb{Q}}\Big)$$

Note: $\mathrm{Im} U \neq 0$ to avoid curvature singularity

The discriminant of the superpotential $\mathcal{W}^{\mathsf{RR}}$ (and $\mathcal{W}^{\mathbb{Q}}$) governs the SUSY solutions.

▶ *Discriminant of cubic equation*

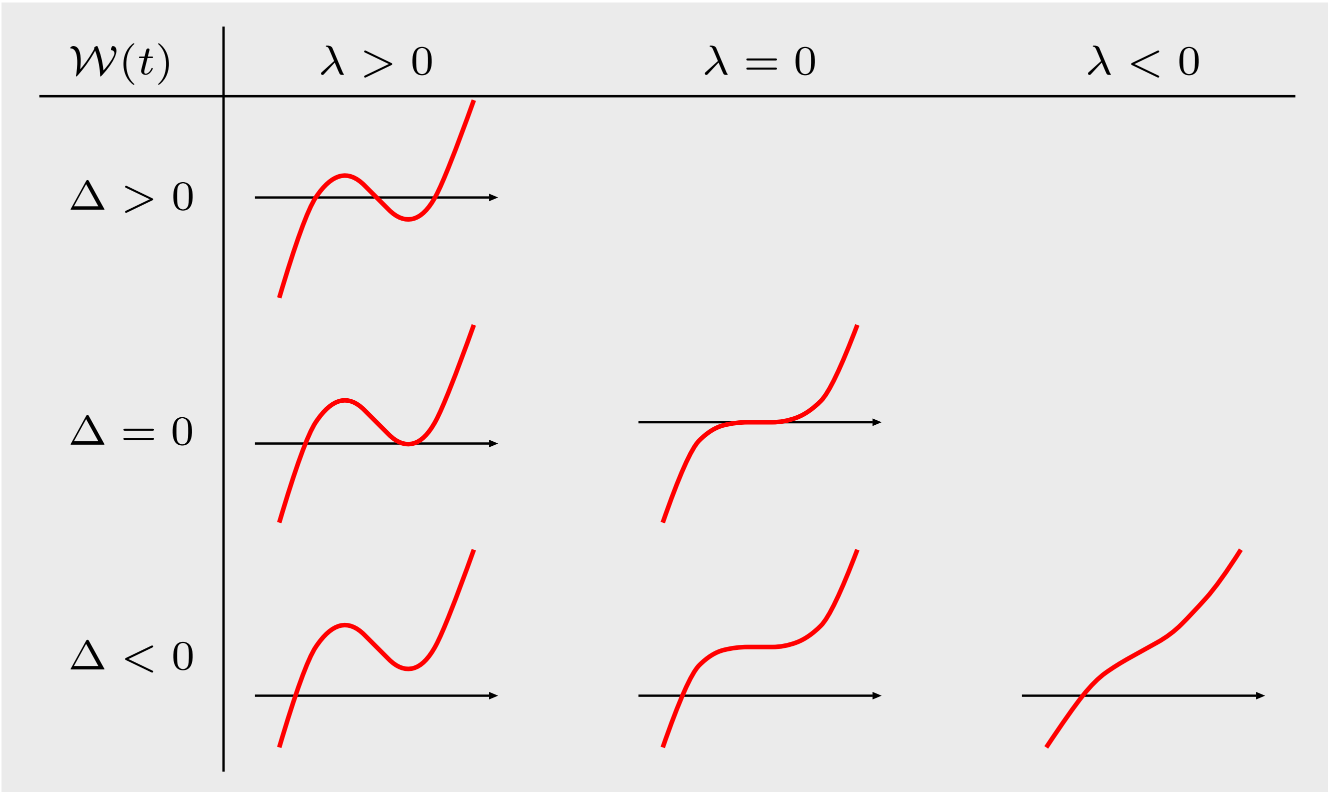
Consider a cubic function and its derivative:

$$\begin{cases} \mathcal{W}(t) &= a\,t^3 + b\,t^2 + c\,t + d \\ \partial_t \mathcal{W}(t) &= 3a\,t^2 + 2b\,t + c \end{cases}$$

Discriminants $\Delta(\mathcal{W})$ and $\Delta(\partial_t \mathcal{W})$ are

$$\Delta(\mathcal{W}) \equiv \Delta = -4b^3 d + b^2 c^2 - 4ac^3 + 18abcd - 27a^2 d^2$$

$$\Delta(\partial_t \mathcal{W}) \equiv \lambda = 4(b^2 - 3ac)$$



$\Delta(\mathcal{W}^{\mathsf{RR}}) \equiv \Delta^{\mathsf{RR}} > 0$ case: always $\lambda^{\mathsf{RR}} > 0$, and exists a zero point: $D_t \mathcal{W}^{\mathsf{RR}} = 0$

$$
\begin{aligned}
D_t \mathcal{W}^{\mathsf{RR}}|_* &= 0 \\
t_*^{\mathsf{RR}} &= \frac{6\,(3\, m_{\mathsf{RR}}^0\, e_{\mathsf{RR}0} + m_{\mathsf{RR}}\, e_{\mathsf{RR}})}{\lambda^{\mathsf{RR}}} - 2\mathrm{i}\, \frac{\sqrt{3\,\Delta^{\mathsf{RR}}}}{\lambda^{\mathsf{RR}}} \\
\mathcal{W}_*^{\mathsf{RR}} &= -\frac{24\,\Delta^{\mathsf{RR}}}{(\lambda^{\mathsf{RR}})^3} \Big(36\,(m_{\mathsf{RR}})^3 + 36\,(m_{\mathsf{RR}}^0)^2 e_{\mathsf{RR}0} - 3\, m_{\mathsf{RR}} \lambda^{\mathsf{RR}} - 4\mathrm{i}\, m_{\mathsf{RR}}^0 \sqrt{3\,\Delta^{\mathsf{RR}}} \Big)
\end{aligned}
$$

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$\Delta^{\mathsf{RR}} < 0$ case: only $\lambda^{\mathsf{RR}} < 0$ is (physically) allowed, and exists a zero point: $\mathcal{W}^{\mathsf{RR}} = 0$

$$\begin{aligned} \mathcal{W}_*^{\mathsf{RR}} &= m^0_{\mathsf{RR}}(t_* - e)(t_* - \alpha)(t_* - \overline{\alpha}) \ = \ 0\,, \qquad t_* \ = \ \alpha^{\mathsf{RR}} \ = \ \alpha_1 + \mathrm{i}\,\alpha_2 \\ \alpha_1 &= \frac{\lambda^{\mathsf{RR}} + F^{2/3} + 12\,m_{\mathsf{RR}}\,F^{1/3}}{12\,m^0_{\mathsf{RR}}\,F^{1/3}} \\ (\alpha_2)^2 &= \frac{1}{m^0_{\mathsf{RR}}}\Big(e_{\mathsf{RR}} - 6\,m_{\mathsf{RR}}\,\alpha_1 + 3\,m^0_{\mathsf{RR}}\,(\alpha_1)^2\Big) \\ e &= -\frac{1}{m^0_{\mathsf{RR}}}\Big(-3\,m_{\mathsf{RR}} + 2\,m^0_{\mathsf{RR}}\,\alpha_1\Big) \\ F &= 108\,(m^0_{\mathsf{RR}})^2 e_{\mathsf{RR0}} + 12\,m^0_{\mathsf{RR}}\sqrt{-3\Delta^{\mathsf{RR}}} + 108\,(m_{\mathsf{RR}})^3 - 9\,m_{\mathsf{RR}}\,\lambda^{\mathsf{RR}} \\ D_t \mathcal{W}^{\mathsf{RR}}|_* &= 2\mathrm{i}\,m^0_{\mathsf{RR}}(e - \alpha^{\mathsf{RR}})\alpha_2 \end{aligned}$$

... Analysis of $\mathcal{W}^{\mathbb{Q}}$ is also discussed.

Three types of solutions:

- SUSY AdS vacuum: attractor point

$$\Delta^{\mathsf{RR}} > 0\,, \qquad \Delta^{\mathbb{Q}} > 0\,; \qquad D_t \mathcal{W}^{\mathsf{RR}}|_* = 0 = D_t \mathcal{W}^{\mathbb{Q}}|_*$$

$$t_*^{\mathsf{RR}} = t_*^{\mathbb{Q}}\,, \qquad \mathrm{Re}\, U_* = -\frac{\mathcal{W}_*^{\mathsf{RR}}}{\mathcal{W}_*^{\mathbb{Q}}}$$

$$V_* = -3\, \mathrm{e}^K |\mathcal{W}_*|^2 = -\frac{4}{[\mathrm{Re}(\mathcal{C}\mathcal{G}_0)]^2} \sqrt{\frac{\Delta^{\mathbb{Q}}}{3}}$$

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- SUSY Minkowski vacuum: attractor point

$$\Delta^{\mathsf{RR}} < 0 \,, \quad \Delta^{\mathbb{Q}} < 0 \,; \quad \mathcal{W}_*^{\mathsf{RR}} = 0 = \mathcal{W}_*^{\mathbb{Q}}$$

$$\alpha^{\mathsf{RR}} = \alpha^{\mathbb{Q}} \,, \quad U_* = -\frac{D_t \mathcal{W}^{\mathsf{RR}}|_*}{D_t \mathcal{W}^{\mathbb{Q}}|_*} \neq 0$$

$$V_* = 0$$

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$$\alpha^{\mathsf{RR}} = \alpha^{\mathbb{Q}}\,, \qquad U_* = -\frac{D_t \mathcal{W}^{\mathsf{RR}}|_*}{D_t \mathcal{W}^{\mathbb{Q}}|_*} \neq 0$$

$$V_* = 0$$

- SUSY AdS vacua, but moduli t and U are not fixed: non attractor point

$$U = -\frac{D_t \mathcal{W}^{\mathsf{RR}}(t)}{D_t \mathcal{W}^{\mathbb{Q}}(t)}\,, \qquad \mathrm{Re} U = -\frac{\mathcal{W}^{\mathsf{RR}}(t)}{\mathcal{W}^{\mathbb{Q}}(t)}$$

1. Set $e_{\mathsf{RR}A} = 0 = m^A_{\mathsf{RR}}$
2. Set a simple prepotential: $\mathcal{F} = D_{abc} \dfrac{X^a X^b X^c}{X^0}$
3. Consider the (1,1)-moduli model: $t^{\check{a}} \equiv t$, $U^{\check{I}} \equiv U$, $\widetilde{U}_{\hat{I}} = 0 \quad (D_{abc} = D \equiv 1)$

The SUSY conditions on $\mathcal{W} = U\, \mathcal{W}^{\mathbb{Q}}$ are

$$D_t \mathcal{W} = 0 \dashrightarrow 0 = D_t \mathcal{W}^{\mathbb{Q}}$$

$$D_U \mathcal{W} = 0 \dashrightarrow 0 = \mathrm{Re} U\, \mathcal{W}^{\mathbb{Q}}$$

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The solution is given only when $\Delta^{\mathbb{Q}} > 0$, and the AdS vacuum emerges:

$$t_*^{\mathbb{Q}} = -\frac{6\,(3\,p_0{}^0\, e_{00} + p_0\, e_0)}{\lambda^{\mathbb{Q}}} - 2\mathrm{i}\,\frac{\sqrt{3\,\Delta^{\mathbb{Q}}}}{\lambda^{\mathbb{Q}}}, \qquad \mathrm{Re}\, U_* = 0$$

$$V_* = -3\, \mathrm{e}^K |\mathcal{W}_*|^2 = -\frac{4}{[\mathrm{Re}(\mathcal{C}\mathcal{G}_0)]^2} \sqrt{\frac{\Delta^{\mathbb{Q}}}{3}}$$

1. Set $e_{\mathsf{RR}A} = 0 = m^A_{\mathsf{RR}}$ and $p_I{}^A = 0 = q^{IA}$
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Functions are reduced to

$$\mathcal{W} \;=\; U\mathcal{W}^{\mathbb{Q}} \;=\; U(-e_{00} - e_0\, t)$$

$$D_t\mathcal{W} \;=\; \frac{U}{t-\bar{t}}\Big(e_0(2t+\bar{t}) + 3\, e_{00}\Big)\,, \qquad D_U\mathcal{W} \;=\; \mathrm{i}\,\frac{\mathrm{Re}U}{\mathrm{Im}U}\,\mathcal{W}^{\mathbb{Q}}$$

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There are neither SUSY solutions under the conditions $D_t \mathcal{W} = 0 = D_U \mathcal{W}$

nor non-SUSY solutions satisfying $\partial_{\mathcal{P}} V = 0\,$!

Ansatz 2. "Neglecting all α' corrections on the compactified gemetry" is too strong!

2'. Set a deformed prepotential: $\mathcal{F} = \frac{(X^t)^3}{X^0} + \sum_n N_n \frac{(X^t)^{n+3}}{(X^0)^{n+1}}$

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Consider a simple case as $N_1 \neq 0$, otherwise $N_n = 0$:

$$\begin{aligned} \partial_t K &= -\frac{3(t-\bar{t})^2 - \partial_t P}{(t-\bar{t})^3 - P} \\ D_t \mathcal{W}^{\mathbb{Q}} &= -e_{00} + \frac{3(t-\bar{t})^2 - \partial_t P}{(t-\bar{t})^3 - P} \Big(e_{00} + e_0 \, t \Big) \\ P &\equiv -2 \Big(N_1 \, t^4 - \overline{N}_1 \, \bar{t}^4 - 2N_1 \, t^3 \bar{t} + 2\overline{N}_1 \, t\bar{t}^3 \Big) \end{aligned}$$

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D_t \mathcal{W}^{\mathbb{Q}} &= -e_{00} + \frac{3(t-\bar{t})^2 - \partial_t P}{(t-\bar{t})^3 - P}\Big(e_{00} + e_0\, t\Big) \\
P &\equiv -2\Big(N_1\, t^4 - \overline{N}_1\, \bar{t}^4 - 2N_1\, t^3\bar{t} + 2\overline{N}_1\, t\bar{t}^3\Big)
\end{aligned}$$

SUSY AdS solution appears under the conditions $D_t\mathcal{W} = 0$ and $D_U\mathcal{W} = 0$:

$$t_*^{\mathbb{Q}} = -\frac{2\, e_{00}}{e_0}\,, \qquad \mathrm{Re}\, U_* = 0$$

$$\mathcal{W}_*^{\mathbb{Q}} = e_{00}\,, \qquad \mathrm{Im} N_1 < 0$$

$$V_* = -3\, \mathrm{e}^K |\mathcal{W}_*|^2 = \frac{1}{[\mathrm{Re}(\mathcal{C}\mathcal{G}_0)]^2} \frac{3\,(e_0)^4}{16\,(e_{00})^2\, \mathrm{Im} N_1}$$

Summary and Discussions

Summary

- Generalized geometry and nongeometric fluxes
- SUSY AdS vacua compactified on generalized geometry
- Application to compactification on $SU(3)$-structure manifold without RR-fluxes

Discussions

- Complete stabilization via nonperturbative corrections
- Duality transformations
- Understanding the physical interpretation of nongeometric fluxes
- Connection to doubled formalism

de Sitter vacua?

In order to build (stable) de Sitter vacua perturbatively in type IIA*,*

in addition to the usual RR and NSNS fluxes and O6/D6 sources,

one must minimally have geometric fluxes and non-zero Romans' mass parameter.

S.S. Haque, G. Shiu, B. Underwood, T. Van Riet arXiv:0810.5328

Romans' mass parameter $\sim \; G_0$

- Search a (meta)stable de Sitter vacuum in this formulation

Appendix: compactifications in type II strings

Moduli spaces in $\mathcal{N} = 2$ supergravity are

vector multiplets:	Hodge-Kähler geometry
hypermultiplets:	quaternionic geometry

We look for the origin of the moduli spaces in 10D string theories

Decomposition of vector bundle on 10D spacetime:

$$TM_{1,9} = T_{1,3} \oplus F$$

$$\begin{cases} T_{1,3}: & \text{a real } SO(1,3) \text{ vector bundle} \\ F: & \text{an } SO(6) \text{ vector bundle which admits a pair of } SU(3) \text{ structures} \end{cases}$$

10D spacetime itself is not decomposed yet, i.e., do not yet consider truncation of modes.

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Decomposition of Lorentz symmetry:

$$Spin(1,9) \to Spin(1,3) \times Spin(6) = SL(2,\mathbb{C}) \times SU(4)$$

$$\mathbf{16} = (\mathbf{2},\mathbf{4}) \oplus (\overline{\mathbf{2}},\overline{\mathbf{4}}) \qquad \mathbf{16} = (\mathbf{2},\overline{\mathbf{4}}) \oplus (\overline{\mathbf{2}},\mathbf{4})$$

Decomposition of supersymmetry parameters (with $a, b \in \mathbb{C}$):

$$\begin{cases} \epsilon^1_{\mathrm{IIA}} &= \varepsilon_1 \otimes (a\eta^1_+) + \varepsilon_1^c \otimes (\overline{a}\eta^1_-) \\ \epsilon^2_{\mathrm{IIA}} &= \varepsilon_2 \otimes (\overline{b}\eta^2_-) + \varepsilon_2^c \otimes (b\eta^2_+) \end{cases} \qquad \begin{cases} \epsilon^1_{\mathrm{IIB}} &= \varepsilon_1 \otimes (a\eta^1_+) + \varepsilon_1^c \otimes (\overline{a}\eta^1_-) \\ \epsilon^2_{\mathrm{IIB}} &= \varepsilon_2 \otimes (b\eta^2_+) + \varepsilon_2^c \otimes (\overline{b}\eta^2_-) \end{cases}$$

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Set $SU(3)$ invariant spinor $\eta^{\mathcal{A}}_+$ s.t. $\nabla^{(T)}\eta^{\mathcal{A}}_+ = 0$ $(\mathcal{A} = 1,2)$

a pair of $SU(3)$ on F (η^1_+, η^2_+) $\longleftrightarrow$ a single $SU(3)$ on F $(\eta^1_+ = \eta^2_+ = \eta_+)$

Requirement that we have a pair of $SU(3)$ structures means there is a sub-supermanifold

$$\mathcal{N}^{1,9|4+4} \subset \mathcal{M}^{1,9|16+16}$$

$$\begin{pmatrix} (1,9): & \text{bosonic degrees} \\ 4+4: & \text{eight Grassmann variables as spinors of } Spin(1,3) \text{ and singlet of } SU(3)\text{s} \end{pmatrix}$$

Equivalence such as

eight SUSY theory reformulation of type II supergravity

$\Updownarrow$

a pair of $SU(3)$ structures on vector bundle F

$\Updownarrow$

an $SU(3) \times SU(3)$ structure on extended $F \oplus F^*$

10D spinors in type IIA in Einstein frame

$$\delta\Psi_m^{\mathcal{A}} = \nabla_m \epsilon^{\mathcal{A}} - \frac{1}{96} \mathrm{e}^{-\phi} \Big(\Gamma_m{}^{PQR} H_{PQR} - 9 \Gamma^{PQ} H_{mPQ} \Big) \Gamma_{(11)} \epsilon^{\mathcal{A}}$$
$$- \sum_{n=0,2,\ldots,8} \frac{1}{64 n!} \mathrm{e}^{\frac{5-n}{4}\phi} \Big[(n-1) \Gamma_m{}^{N_1 \cdots N_n} - n(9-n) \delta_m{}^{N_1} \Gamma^{N_2 \cdots N_n} \Big] F_{N_1 \cdots N_n} (\Gamma_{(11)})^{n/2} (\sigma^1 \epsilon)^{\mathcal{A}}$$

$$\epsilon^1 = \varepsilon_1 \otimes (a \eta^1_+) + \varepsilon_1^c \otimes (\overline{a} \eta^1_-) \qquad \epsilon^2 = \varepsilon_2 \otimes (\overline{b} \eta^2_-) + \varepsilon_2^c \otimes (b \eta^2_+)$$

$$0 \equiv \delta\psi_m^{\mathcal{A}} = \nabla_m \eta_+^{\mathcal{A}} + (\text{NS-fluxes} \cdot \eta)^{\mathcal{A}} + (\text{RR-fluxes} \cdot \eta)^{\mathcal{A}}$$

Information of

6D $SU(3)$ Killing spinors $\eta_+^{\mathcal{A}}$

Calabi-Yau three-fold

$\downarrow$

$SU(3)$-structure manifold with torsion

$\downarrow$

generalized geometry

- on a single $SU(3)$:

a real two-form	$J_{mn} = \mp 2\mathrm{i}\, \eta_\pm^\dagger \, \gamma_{mn}\, \eta_\pm$
a complex three-form	$\Omega_{mnp} = -2\mathrm{i}\, \eta_-^\dagger \, \gamma_{mnp}\, \eta_+$

- on a pair of $SU(3)$:

two real vectors	$(v - \mathrm{i}v')^m = \eta_+^{1\dagger} \gamma^m \, \eta_-^2$
(J^A, Ω^A)	$J^1 = j + v \wedge v' \quad \Omega^1 = w \wedge (v + \mathrm{i}v')$ $J^2 = j - v \wedge v' \quad \Omega^2 = w \wedge (v - \mathrm{i}v')$ (j, w): local $SU(2)$-invariant forms

If $\eta_+^1 \neq \eta_+^2$ globally, a pair of $SU(3)$ is reduced to global single $SU(2)$ w/ (j, w, v, v')

If $\eta_+^1 = \eta_+^2$ globally, a pair of $SU(3)$ is reduced to a single global $SU(3)$ w/ (J, Ω)

$$\eta_+^2 \;=\; c_\parallel \eta_+^1 + c_\perp (v + \mathrm{i}v')^m \, \gamma_m \, \eta_-^1 \,, \qquad\qquad |c_\parallel|^2 + |c_\perp|^2 = 1$$

a pair of $SU(3)$ on $T\mathcal{M}$ $\sim$ an $SU(3) \times SU(3)$ on $T\mathcal{M} \oplus T^*\mathcal{M}$

Appendix: Calabi-Yau compactifications

One can embed 4D $\mathcal{N} = 2$ theory into 10D type II theory

compactified on Calabi-Yau three-fold

	vector multiplets	hypermultiplets
generic	coord. of Hodge-Kähler	coord. of quaternionic
IIA on Calabi-Yau	Kähler moduli	complex moduli $+$ RR
IIB on Calabi-Yau	complex moduli	Kähler moduli $+$ RR

NS-NS fields in ten-dimensional spacetime are expanded as

$$\begin{aligned}\phi(x,y) &= \varphi(x) \\ G_{m\overline{n}}(x,y) &= \mathrm{i}\, v^a(x)(\omega_a)_{m\overline{n}}(y), \quad G_{mn}(x,y) = \mathrm{i}\, \overline{z}^k(x) \left(\frac{(\overline{\chi}_k)_{m\overline{pq}} \Omega^{\overline{pq}}{}_n}{||\Omega||^2} \right)(y) \\ B_2(x,y) &= B_2(x) + b^a(x)\omega_a(y)\end{aligned}$$

RR fields in type IIA are

$$\begin{aligned}C_1(x,y) &= C_1^0(x) \\ C_3(x,y) &= C_1^a(x)\omega_a(y) + \xi^K(x)\alpha_K(y) - \widetilde{\xi}_K(x)\beta^K(y)\end{aligned}$$

RR fields in type IIB are

$$\begin{aligned}C_0(x,y) &= C_0(x) \\ C_2(x,y) &= C_2(x) + c^a(x)\omega_a(y) \\ C_4(x,y) &= V_1^K(x)\alpha_K(y) + \rho_a(x)\widetilde{\omega}^a(y)\end{aligned}$$

cohomology class	basis	
$H^{(1,1)}$	ω_a	$a = 1, \ldots, h^{(1,1)}$
$H^{(0)} \oplus H^{(1,1)}$	$\omega_A = (1, \omega_a)$	$A = 0, 1, \ldots, h^{(1,1)}$
$H^{(2,2)}$	$\widetilde{\omega}^a$	$a = 1, \ldots, h^{(1,1)}$
$H^{(2,1)}$	χ_k	$k = 1, \ldots, h^{(2,1)}$
$H^{(3)}$	(α_K, β^K)	$K = 0, 1, \ldots, h^{(2,1)}$

4D type IIA $\mathcal{N} = 2$ ungauged supergravity action of bosonic fields is

$$S^{(4)}_{\rm IIA} = \int_{\mathcal{M}_{1,3}} \Big(-\frac{1}{2} R * \mathbf{1} + \frac{1}{2} {\rm Re}\mathcal{N}_{AB} F^A \wedge F^B + \frac{1}{2} {\rm Im}\mathcal{N}_{AB} F^A \wedge * F^B - G_{a\bar{b}}\, {\rm d}t^a \wedge * {\rm d}\bar{t}^{\bar{b}} - h_{uv}\, {\rm d}q^u \wedge * {\rm d}q^v \Big)$$

gravity multiplet	$g_{\mu\nu}\,,\ C_1^0$	1
vector multiplet	$C_1^a\,,\ v^a\,,\ b^a$	$a = 1, \ldots, h^{(1,1)}$
hypermultiplet	$z^k\,,\ \xi^k\,,\ \widetilde{\xi}_k$	$k = 1, \ldots, h^{(2,1)}$
tensor multiplet	$B_2\,,\ \varphi\,,\ \xi^0\,,\ \widetilde{\xi}_0$	1

Various functions in the actions:

$$B + {\rm i}J = (b^a + {\rm i}v^a)\, \omega_a = t^a \omega_a$$

$$K^{\sf KS} = -\log\Big(\tfrac{4}{3} \int_{\mathcal{M}_6} J \wedge J \wedge J \Big)$$

$$\mathcal{K}_{abc} = \int_{\mathcal{M}_6} \omega_a \wedge \omega_b \wedge \omega_c$$

$$\mathcal{K}_{ab} = \int_{\mathcal{M}_6} \omega_a \wedge \omega_b \wedge J = \mathcal{K}_{abc} v^c$$

$$\mathcal{K}_a = \int_{\mathcal{M}_6} \omega_a \wedge J \wedge J = \mathcal{K}_{abc} v^b v^c$$

$$\mathcal{K} = \int_{\mathcal{M}_6} J \wedge J \wedge J = \mathcal{K}_{abc} v^a v^b v^c$$

$${\rm Re}\mathcal{N}_{AB} = \begin{pmatrix} -\frac{1}{3}\mathcal{K}_{abc} b^a b^b b^c & \frac{1}{2}\mathcal{K}_{abc} b^b b^c \\ \frac{1}{2}\mathcal{K}_{abc} b^b b^c & -\mathcal{K}_{abc} b^c \end{pmatrix}$$

$${\rm Im}\mathcal{N}_{AB} = -\frac{\mathcal{K}}{6} \begin{pmatrix} 1 + 4G_{ab} b^a b^b & -4G_{ab} b^b \\ -4G_{ab} b^b & 4G_{ab} \end{pmatrix}$$

$$G_{a\bar{b}} = \frac{3}{2} \frac{\int \omega_a \wedge * \omega_b}{\int J \wedge J \wedge J} = \partial_{t^a} \overline{\partial}_{\bar{t}^{\bar{b}}} K^{\sf KS}$$

4D type IIB $\mathcal{N} = 2$ ungauged supergravity action of bosonic fields is

$$S_{\rm IIB}^{(4)} = \int_{\mathcal{M}_{1,3}} \Big(-\frac{1}{2} R * \mathbf{1} + \frac{1}{2} {\rm Re}\mathcal{M}_{KL} F^K \wedge F^L + \frac{1}{2} {\rm Im}\mathcal{M}_{KL} F^K \wedge *F^L - G_{k\bar{l}}\, {\rm d}z^k \wedge * {\rm d}\bar{z}^{\bar{l}} - h_{pq}\, {\rm d}\tilde{q}^p \wedge * {\rm d}\tilde{q}^q \Big)$$

gravity multiplet	$g_{\mu\nu}\,,\ V_1^0$	1
vector multiplet	$V_1^k\,,\ z^k$	$k = 1, \ldots, h^{(2,1)}$
hypermultiplet	$v^a\,,\ b^a\,,\ c^a\,,\ \rho_a$	$a = 1, \ldots, h^{(1,1)}$
tensor multiplet	$B_2\,,\ C_2\,,\ \varphi\,,\ C_0$	1

Various functions in the actions:

$$\Omega = Z^K \alpha_K - \mathcal{G}_K \beta^K \qquad z^k = Z^K / Z^0 \qquad \mathcal{G}_{KL} = \partial_L \mathcal{G}_K$$

$$K^{\sf CS} = -\log \Big({\rm i} \int_{\mathcal{M}_6} \Omega \wedge \overline{\Omega} \Big) \qquad G_{k\bar{l}} = - \frac{\int \chi_k \wedge \overline{\chi}_{\bar{l}}}{\int \Omega \wedge \overline{\Omega}} = \partial_{z^k} \overline{\partial}_{\bar{z}^{\bar{l}}} K^{\sf CS}$$

$$\mathcal{M}_{KL} = \overline{\mathcal{G}}_{KL} + 2{\rm i} \frac{({\rm Im}\mathcal{G})_{KM} Z^M ({\rm Im}\mathcal{G})_{LN} Z^N}{Z^N ({\rm Im}\mathcal{G})_{NM} Z^M}$$

Appendix: $SU(3)$-structure manifold with torsion

Information from Killing spinor eqs. with torsion $D^{(T)}\eta_{\pm} = 0$ ($^{\exists}$complex Weyl $\eta_{\pm}$)

- Invariant p-forms on $SU(3)$-structure manifold:

a real two-form $J_{mn} = \mp 2\mathrm{i}\, \eta_{\pm}^{\dagger}\, \gamma_{mn}\, \eta_{\pm}$

a holomorphic three-form $\Omega_{mnp} = -2\mathrm{i}\, \eta_{-}^{\dagger}\, \gamma_{mnp}\, \eta_{+}$

$$\mathrm{d}J = \frac{3}{2}\,\mathrm{Im}(\overline{\mathcal{W}}_1 \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

- Five classes of (intrinsic) torsion are given as

components	interpretations	$SU(3)$-representations
$\mathcal{W}_1$	$J \wedge \mathrm{d}\Omega$ or $\Omega \wedge \mathrm{d}J$	$\mathbf{1} \oplus \mathbf{1}$
$\mathcal{W}_2$	$(\mathrm{d}\Omega)_0^{2,2}$	$\mathbf{8} \oplus \mathbf{8}$
$\mathcal{W}_3$	$(\mathrm{d}J)_0^{2,1} + (\mathrm{d}J)_0^{1,2}$	$\mathbf{6} \oplus \overline{\mathbf{6}}$
$\mathcal{W}_4$	$J \wedge \mathrm{d}J$	$\mathbf{3} \oplus \overline{\mathbf{3}}$
$\mathcal{W}_5$	$(\mathrm{d}\Omega)^{3,1}$	$\mathbf{3} \oplus \overline{\mathbf{3}}$

- Vanishing torsion classes in $SU(3)$-structure manifolds:

complex	hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = 0$
	balanced	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = 0$
	special hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	Kähler	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	Calabi-Yau	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	conformally Calabi-Yau	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 + 2\mathcal{W}_5 = 0$
almost complex	symplectic	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	nearly Kähler	$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	almost Kähler	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	quasi Kähler	$\mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	semi Kähler	$\mathcal{W}_4 = \mathcal{W}_5 = 0$
	half-flat	$\mathrm{Im}\mathcal{W}_1 = \mathrm{Im}\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$

Appendix: generalized geometry

Introduce a generalized almost complex structure $\mathcal{J}$ on $T\mathcal{M}_d \oplus T^*\mathcal{M}_d$ s.t.

$$\mathcal{J} : \; T\mathcal{M}_d \oplus T^*\mathcal{M}_d \;\longrightarrow\; T\mathcal{M}_d \oplus T^*\mathcal{M}_d$$

$$\mathcal{J}^2 \;=\; -\mathbb{1}_{2d}$$

$$^{\exists}\; O(d,d) \text{ invariant metric } L\text{, s.t. } \; \mathcal{J}^T L \mathcal{J} \;=\; L$$

Structure group on $T\mathcal{M}_d \oplus T^*\mathcal{M}_d$:

$^{\exists}L$	$GL(2d)$	$\dashrightarrow$	$O(d,d)$
$\mathcal{J}^2 = -\mathbb{1}_{2d}$	$O(d,d)$	$\dashrightarrow$	$U(d/2,d/2)$
$\mathcal{J}_1, \mathcal{J}_2$	$U_1(d/2,d/2) \cap U_2(d/2,d/2)$	$\dashrightarrow$	$U(d/2) \times U(d/2)$
integrable $\mathcal{J}_{1,2}$	$U(d/2) \times U(d/2)$	$\dashrightarrow$	$SU(d/2) \times SU(d/2)$

- Integrability is discussed by "(0,1)" part of the complexified $T\mathcal{M}_d \oplus T^*\mathcal{M}_d$:

$$\Pi \equiv \frac{1}{2}(\mathbb{1}_{2d} - \mathrm{i}\mathcal{J})$$

$$\Pi A = A \quad \text{where } A = v + \zeta \text{ is a section of } T\mathcal{M}_d \oplus T^*\mathcal{M}_d$$

We call this A i-eigenbundle $L_{\mathcal{J}}$, whose dimension is $\dim L_{\mathcal{J}} = d$.

Integrability condition of $\mathcal{J}$ is

$$\overline{\Pi}\big[\Pi(v+\zeta), \Pi(w+\eta)\big]_{\mathrm{C}} = 0 \qquad v, w \in T\mathcal{M}_d \qquad \zeta, \eta \in T^*\mathcal{M}_d$$

$$[v+\zeta, w+\eta]_{\mathrm{C}} = [v,w] + \mathcal{L}_v \eta - \mathcal{L}_w \zeta - \frac{1}{2}\mathrm{d}(\iota_v \eta - \iota_w \zeta) : \text{ Courant bracket}$$

- Two typical examples of generalized almost complex structures:

$$\mathcal{J}_1 = \begin{pmatrix} J & \mathbf{0} \\ \mathbf{0} & -J^T \end{pmatrix} \qquad \text{w/ } J^2 = -\mathbb{1}_d\text{: almost complex structure}$$

$$\mathcal{J}_2 = \begin{pmatrix} \mathbf{0} & -\omega^{-1} \\ \omega & \mathbf{0} \end{pmatrix} \qquad \text{w/ } \omega\text{: almost symplectic form}$$

$$\begin{aligned} \text{integrable } \mathcal{J}_1 \quad &\leftrightarrow \quad \text{integrable } J \\ \text{integrable } \mathcal{J}_2 \quad &\leftrightarrow \quad \text{integrable } \omega \end{aligned}$$

On a usual geometry, $J_{mn} = J_m{}^p g_{pn}$ is given by an $SU(3)$ invariant (pure) spinor η_+ as

$$J_{mn} = -2\mathrm{i}\, \eta_+^\dagger \gamma_{mn} \eta_+ \qquad \gamma^i \eta_+ = 0 \qquad \gamma^{\bar{\imath}} \eta_+ \neq 0$$

In a similar analogy, we want to find Cliff$(6,6)$ pure spinor(s) Φ.

∵) Compared to almost complex structures, (pure) spinors can be easily utilized in supergravity framework.

On $T\mathcal{M}_6 \oplus T^*\mathcal{M}_6$, we can define Cliff$(6,6)$ algebra and $Spin(6,6)$ spinor Φ:

$$\{\Gamma^m, \Gamma^n\} = 0 \qquad \{\Gamma^m, \widetilde{\Gamma}_n\} = \delta^m_n \qquad \{\widetilde{\Gamma}_m, \widetilde{\Gamma}_n\} = 0$$

Irreducible repr. of $Spin(6,6)$ spinor is a Majorana-Weyl

$\rightarrow$ a generic $Spin(6,6)$ spinor bundle S splits to $S^{\pm}$ (Weyl)

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Weyl spinor bundles $S^{\pm}$ are isomorphic to bundles of forms on $T^*\mathcal{M}_6$:

$$S^+ \text{ on } T\mathcal{M}_6 \oplus T^*\mathcal{M}_6 \quad \sim \quad \wedge^{\text{even}} T^*\mathcal{M}_6$$

$$S^- \text{ on } T\mathcal{M}_6 \oplus T^*\mathcal{M}_6 \quad \sim \quad \wedge^{\text{odd}} T^*\mathcal{M}_6$$

Thus we often regard a Cliff$(6,6)$ spinor as a form on $\wedge^{\text{even/odd}} T^*\mathcal{M}_6$

A form-valued representation of the algebra

$$\Gamma^m = \mathrm{d}x^m \wedge \,, \qquad \widetilde{\Gamma}_n = \iota_n$$

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A form-valued representation of the algebra

$$\Gamma^m = \mathrm{d}x^m \wedge \,, \qquad \widetilde{\Gamma}_n = \iota_n$$

IF Φ is annihilated by half numbers of the Cliff$(6,6)$ generators:

$\rightarrow$ Φ is called a pure spinor

cf.) $SU(3)$ invariant spinor η_+ is a Cliff(6) pure spinor: $\gamma^i \eta_+ = 0$

An equivalent definition of a Cliff$(6,6)$ pure spinor is given by "Clifford action":

$$(v+\zeta)\cdot\Phi \;=\; v^m \iota_{\partial_m}\Phi + \zeta_n \, \mathrm{d}x^n \wedge \Phi \qquad \text{w/ } v\text{: vector} \qquad \zeta\text{: one-form}$$

Define the annihilator of a spinor as

$$L_\Phi \;\equiv\; \Big\{ v+\zeta \in T\mathcal{M}_6 \oplus T^*\mathcal{M}_6 \,\Big|\, (v+\zeta)\cdot\Phi = 0 \Big\}$$

$$\dim L_\Phi \;\leq\; d$$

If $\dim L_\Phi = 6$ (maximally isotropic) $\rightarrow$ Φ is a pure spinor

Correspondence between pure spinors and generalized almost complex structures:

$$\mathcal{J} \leftrightarrow \Phi \qquad \text{if} \;\; L_{\mathcal{J}} = L_{\Phi} \quad \text{with} \;\; \dim L_{\Phi} = 6$$

More precisely: $\mathcal{J} \leftrightarrow$ a line bundle of pure spinor Φ

$\because$) rescaling Φ does not change its annihilator L_{Φ}

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Then, we can rewrite the generalized almost complex structure as

$$\boxed{\mathcal{J}_{\pm\Pi\Sigma} \ = \ \big\langle \mathrm{Re}\Phi_{\pm}, \Gamma_{\Pi\Sigma}\, \mathrm{Re}\Phi_{\pm} \big\rangle}$$

w/ Mukai pairing:

even forms: $\big\langle \Psi_+, \Phi_+ \big\rangle = \Psi_6 \wedge \Phi_0 - \Psi_4 \wedge \Phi_2 + \Psi_2 \wedge \Phi_4 - \Psi_0 \wedge \Phi_6$

odd forms: $\big\langle \Psi_-, \Phi_- \big\rangle = \Psi_5 \wedge \Phi_1 - \Psi_3 \wedge \Phi_3 + \Psi_1 \wedge \Phi_5$

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odd forms: $\big\langle \Psi_-, \Phi_- \big\rangle = \Psi_5 \wedge \Phi_1 - \Psi_3 \wedge \Phi_3 + \Psi_1 \wedge \Phi_5$

$\mathcal{J}$ is integrable $\longleftrightarrow$ $^{\exists}$ vector v and one-form ζ s.t. $\mathrm{d}\Phi = (v \llcorner + \zeta \wedge)\Phi$

generalized CY $\longleftrightarrow$ $^{\exists}\Phi$ is pure s.t. $\mathrm{d}\Phi = 0$

"twisted" GCY $\longleftrightarrow$ $^{\exists}\Phi$ is pure, and H is closed s.t. $(\mathrm{d} - H\wedge)\Phi = 0$

A Cliff$(6,6)$ spinor can also be mapped to a bispinor:

$$C \ \equiv \ \sum_k \frac{1}{k!} C^{(k)}_{m_1 \cdots m_k} \, \mathrm{d}x^{m_1} \wedge \cdots \wedge \mathrm{d}x^{m_k} \qquad \longleftrightarrow \qquad \not{C} \ \equiv \ \sum_k \frac{1}{k!} C^{(k)}_{m_1 \cdots m_k} \, \gamma_{\alpha\beta}^{m_1 \cdots m_k}$$

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On a geometry of a single $SU(3)$-structure, the following two $SU(3,3)$ spinors:

$$\Phi_{0+} \;=\; \eta_+ \otimes \eta_+^\dagger \;=\; \frac{1}{4} \sum_{k=0}^{6} \frac{1}{k!} \eta_+^\dagger \gamma_{m_k \cdots m_1} \eta_+ \, \gamma^{m_1 \cdots m_k} \;=\; \frac{1}{8} \mathrm{e}^{-\mathrm{i}J}$$

$$\Phi_{0-} \;=\; \eta_+ \otimes \eta_-^\dagger \;=\; \frac{1}{4} \sum_{k=0}^{6} \frac{1}{k!} \eta_-^\dagger \gamma_{m_k \cdots m_1} \eta_+ \, \gamma^{m_1 \cdots m_k} \;=\; -\frac{\mathrm{i}}{8} \Omega$$

Check purity: $(\delta + \mathrm{i}J)_m{}^n \, \gamma_n \, \eta_+ \otimes \eta_\pm^\dagger \;=\; 0 \;=\; \eta_+ \otimes \eta_\pm^\dagger \, \gamma_n (\delta \mp \mathrm{i}J)^n{}_m$

One-to-one correspondence: $\Phi_{0-} \leftrightarrow \mathcal{J}_1, \quad \Phi_{0+} \leftrightarrow \mathcal{J}_2$

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One-to-one correspondence: $\Phi_{0-} \leftrightarrow \mathcal{J}_1, \quad \Phi_{0+} \leftrightarrow \mathcal{J}_2$

On a generic geometry of a pair of $SU(3)$-structure defined by (η_+^1, η_+^2)

$$\Phi_{0+} = \eta_+^1 \otimes \eta_+^{2\dagger} = \frac{1}{8} \big(\bar{c}_\parallel \mathrm{e}^{-\mathrm{i}j} - \mathrm{i}\bar{c}_\perp w \big) \wedge \mathrm{e}^{-\mathrm{i}v \wedge v'}$$

$$\Phi_{0-} = \eta_+^1 \otimes \eta_-^{2\dagger} = -\frac{1}{8} \big(c_\perp \mathrm{e}^{-\mathrm{i}j} + \mathrm{i} c_\parallel w \big) \wedge (v + \mathrm{i}v')$$

$$|c_\parallel|^2 + |c_\perp|^2 = 1$$

$$\Phi_\pm = \mathrm{e}^{-B} \Phi_{0\pm}$$

Each $\Phi_{\pm}$ defines an $SU(3,3)$ structure on E. Common structure is $SU(3) \times SU(3)$.

(F is extended to E by including e^{-B})

Compatibility requires

$$\big\langle \Phi_+, V \cdot \Phi_- \big\rangle \;=\; \big\langle \overline{\Phi}_+, V \cdot \Phi_- \big\rangle \;=\; 0 \qquad \text{for } \forall V = x + \xi$$

$$\big\langle \Phi_+, \overline{\Phi}_+ \big\rangle \;=\; \big\langle \Phi_-, \overline{\Phi}_- \big\rangle$$

Start with a real form $\chi_f \in \wedge^{\text{even/odd}} F^*$ (associated with a real $Spin(6,6)$ spinor χ_s)

Regard χ_f as a stable form satisfying

$$q(\chi_f) \;=\; -\frac{1}{4}\langle \chi_f, \Gamma_{\Pi\Sigma}\chi_f\rangle\langle \chi_f, \Gamma^{\Pi\Sigma}\chi_f\rangle \;\in\; \wedge^6 F^* \otimes \wedge^6 F^*$$
$$U \;=\; \big\{\chi_f \in \wedge^{\text{even/odd}} F^* \;:\; q(\chi_f) < 0\big\}$$

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$$\begin{aligned} q(\chi_f) &= -\frac{1}{4}\langle \chi_f, \Gamma_{\Pi\Sigma}\chi_f\rangle\langle \chi_f, \Gamma^{\Pi\Sigma}\chi_f\rangle \ \in \ \wedge^6 F^* \otimes \wedge^6 F^* \\ U &= \big\{\chi_f \in \wedge^{\text{even/odd}} F^* \ : \ q(\chi_f) < 0\big\} \end{aligned}$$

Define a Hitchin function

$$H(\chi_f) \equiv \sqrt{-\frac{1}{3}q(\chi_f)} \ \in \ \wedge^6 F^*$$

which gives an integrable complex structure on U

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$$U \;=\; \big\{ \chi_f \in \wedge^{\text{even/odd}} F^* \;:\; q(\chi_f) < 0 \big\}$$

Define a Hitchin function

$$H(\chi_f) \;\equiv\; \sqrt{-\frac{1}{3}q(\chi_f)} \;\in\; \wedge^6 F^*$$

which gives an integrable complex structure on U

Then we can get another real form $\hat{\chi}_f$ and a complex form Φ_f by Mukai pairing

$$\langle \hat{\chi}_f, \chi_f \rangle \;=\; -\mathrm{d}H(\chi_f) \qquad \text{i.e.,} \quad \hat{\chi}_f \;=\; -\frac{\partial H(\chi_f)}{\partial \chi_f}$$

$$\dashrightarrow \qquad \Phi_f \;\equiv\; \frac{1}{2}(\chi_f + \mathrm{i}\hat{\chi}_f) \qquad H(\Phi_f) \;=\; \mathrm{i}\langle \Phi_f, \overline{\Phi}_f \rangle$$

Hitchin showed: Φ_f is a (form corresponding to) pure spinor!

N.J. Hitchin math/0010054, math/0107101, math/0209099

Consider the space of pure spinors Φ ...

Mukai pairing $\langle *, * \rangle$	$\longrightarrow$	symplectic structure
Hitchin function $H(*)$	$\longrightarrow$	complex structure

$$\Downarrow$$

The space of pure spinor is Kähler

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Mukai pairing $\langle *, * \rangle$	$\longrightarrow$	symplectic structure
Hitchin function $H(*)$	$\longrightarrow$	complex structure

$$\Downarrow$$

The space of pure spinor is Kähler

Quotienting this space by the $\mathbb{C}^*$ action $\Phi \to \lambda\Phi$ for $\lambda\mathbb{C}^*$

--→ The space becomes a local special Kähler geometry with Kähler potential K:

$$\mathrm{e}^{-K} \;=\; H(\Phi) \;=\; \mathrm{i}\langle \Phi, \overline{\Phi} \rangle \;=\; \mathrm{i}\big(\overline{X}^A \mathcal{F}_A - X^A \overline{\mathcal{F}}_A\big) \;\in\; \wedge^6 F^*$$

X^A : holomorphic projective coordinates

$\mathcal{F}_A$: derivative of prepotential $\mathcal{F}$, i.e., $\mathcal{F}_A = \partial\mathcal{F}/\partial X^A$

These are nothing but objects which we want to introduce in $\mathcal{N} = 2$ supergravity!

Spaces of pure spinors $\Phi_\pm$ on $F \oplus F^*$ with $SU(3) \times SU(3)$ structure

$\|$

special Kähler geometries of local type $=$ Hodge-Kähler geometries

For a single $SU(3)$-structure case:

$$\Phi_+ \; = \; -\frac{1}{8}\,\mathrm{e}^{-B-\mathrm{i}J} \qquad\qquad K_+ \; = \; -\log\left(\frac{1}{48}\,J\wedge J\wedge J\right)$$

$$\Phi_- \; = \; -\frac{\mathrm{i}}{8}\,\mathrm{e}^{-B}\Omega \qquad\qquad K_- \; = \; -\log\left(\frac{\mathrm{i}}{64}\,\Omega\wedge\overline{\Omega}\right)$$

Structure of forms is exactly same as the one in the case of Calabi-Yau compactification!

We should truncate Kaluza-Klein massive modes from these forms to obtain 4D supergravity.

Appendix: puzzle on conventional differential forms

M. Graña, J. Louis, D. Waldram hep-th/0612237

Recall that $\Phi_\pm$ *are expanded in terms of truncation bases* Σ_+ *and* Σ_-.

Whenever $c_{\|} \neq 0$*, the structure* Φ_+ *contains a scalar. This implies that at least one of the forms in the basis* Σ_+ *contains a scalar. Let us call this element* Σ_+^1*, and take the simple case where the only non-zero elements of* $\mathbb{Q}$ *are those of the form* ${\mathbb{Q}_{\hat{I}}}^1$ *(where* $\hat{I} = 1, \ldots, 2b^- + 2$*).*

Thus $\mathrm{d}(\Sigma_-)_{\hat{I}} = {\mathbb{Q}_{\hat{I}}}^1 \Sigma_+^1$ *and so if* ${\mathbb{Q}_{\hat{I}}}^1 \neq 0$ *then* $(\mathrm{d}\Sigma_-)_{\hat{I}}$ *contains a scalar.*

But this is not possible if d *is an honest exterior derivative, acting as* $\mathrm{d} : \Lambda^p \longrightarrow \Lambda^{p+1}$.

The same is true if $c_{\|}$ *is zero. In this case, there may be no scalars in any of the even forms* Σ_-*, and for an "honest"* d *operator, there should be then no one-forms in* $\mathrm{d}\Sigma_-$*. But we again see from that* Φ_- *contains a one-form, and as a consequence so do some of the elements in* Σ_-.

One way to generate a completely general charge matrix $\mathbb{Q}$ in this picture is to consider a modified operator d which is now a generic map $\mathrm{d} : U^+ \longrightarrow U^-$ which satisfies $\mathrm{d}^2 = 0$ but does not transform the degree of a form properly.

In particular, the operator d can map a p-form to a $(p-1)$-form.
Of course, this d does not act this way in conventional geometrical compactifications.

One is thus led to conjecture that to obtain a generic $\mathbb{Q}$ we must consider non-geometrical compactifications. One can still use the structures

$$\mathrm{d}\Sigma_- \ \sim \ \mathbb{Q}\Sigma_+ \,, \qquad \mathrm{d}\Sigma_+ \ \sim \ \mathcal{S}_+ \mathbb{Q}^T (\mathcal{S}_-)^{-1} \Sigma_-$$

to derive sensible effective actions, expanding in bases Σ_+ and Σ_- with a generalised d operator, but there is of course now no interpretation in terms of differential forms and the exterior derivative.

$\dashrightarrow$ introduce generalized fluxes
(not only geometrical H- and f-fluxes, but also Q- and R-fluxes)

For a geometrical background it is natural to consider forms of the type

$$\omega \ = \ \mathrm{e}^{-B}\omega_{m_1\cdots m_p}\, e^{m_1}\wedge\cdots\wedge e^{m_p} \qquad \text{w/ } \omega_{m_1\cdots m_p} \text{ constant}$$

Action of d on ω is

$$\mathrm{d}\omega \ = \ -H^{\text{fl}}\wedge\omega + f\cdot\omega\,, \qquad (f\cdot\omega)_{m_1\cdots m_{p+1}} \ = \ f^a{}_{[m_1 m_2|}\omega_{a|m_3\cdots m_{p+1}]}$$

The natural nongeometrical extension is then to an operator $\mathcal{D}$ such that

$$\mathcal{D} \ := \ \mathrm{d} - H^{\text{fl}}\wedge - f\cdot - Q\cdot - R\llcorner$$

$$(Q\cdot\omega)_{m_1\cdots m_{p-1}} \ = \ Q^{ab}{}_{[m_1}\omega_{|ab|m_2\cdots m_{p-1}]}\,, \qquad (R\llcorner\omega)_{m_1\cdots m_{p-3}} \ = \ R^{abc}\omega_{abcm_1\cdots m_{p-3}}$$

Requiring $\mathcal{D}^2 = 0$ implies that same conditions on fluxes as arose from the Jacobi identities for the extended Lie algebra

$$\begin{aligned} [Z_a, Z_b] \ &= \ f_{ab}{}^c\, Z_c + H_{abc}\, X^c \\ [X^a, X^b] \ &= \ Q^{ab}{}_c\, X^c + R^{abc}\, Z_c \\ [X^a, Z_b] \ &= \ f^a{}_{bc}\, X^c - Q^{ac}{}_b\, Z_c \end{aligned}$$

We can see nongeometrical information in $\mathbb{Q}$ as contribution from Q and R.

Appendix: $\mathcal{N} = 1$ Minkowski vacua

M. Graña, R. Minasian, M. Petrini, A. Tomasiello hep-th/0407249 M. Graña hep-th/0509003

IIA	$a=0$ or $b=0$ (type A)	$a=b\,\mathrm{e}^{\mathrm{i}\beta}$ (type BC)	
1	$\mathcal{W}_1 = H_3^{(1)} = 0$		
1	$F_0^{(1)} = \mp F_2^{(1)} = F_4^{(1)} = \mp F_6^{(1)}$	$F_{2n}^{(1)} = 0$	
8		generic β	$\beta = 0$
8	$\mathcal{W}_2 = F_2^{(8)} = F_4^{(8)} = 0$	$\mathrm{Re}\mathcal{W}_2 = \mathrm{e}^{\phi} F_2^{(8)}$ $\mathrm{Im}\mathcal{W}_2 = 0$	$\mathrm{Re}\mathcal{W}_2 = \mathrm{e}^{\phi} F_2^{(8)} + \mathrm{e}^{\phi} F_4^{(8)}$ $\mathrm{Im}\mathcal{W}_2 = 0$
6	$\mathcal{W}_3 = \mp *_6\, H_3^{(6)}$	$\mathcal{W}_3 = H_3^{(6)}$	
3	$\overline{\mathcal{W}}_5 = 2\mathcal{W}_4 = \mp 2\mathrm{i} H_3^{(3)} = \bar\partial\phi$ $\bar\partial A = \bar\partial a = 0$	$F_2^{(3)} = 2\mathrm{i}\overline{\mathcal{W}}_5 = -2\mathrm{i}\bar\partial A = \frac{2\mathrm{i}}{3}\bar\partial\phi$ $\mathcal{W}_4 = 0$	

type A	NS-flux only (common to IIA, IIB, heterotic) $\mathcal{W}_1 = \mathcal{W}_2 = 0$, $\mathcal{W}_3 \neq 0$: complex
type BC	RR-flux only $\mathcal{W}_1 = \mathrm{Im}\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$, $\mathrm{Re}\mathcal{W}_2 \neq 0$, $\mathcal{W}_5 \neq 0$: symplectic

For $\mathcal{N}=1$ AdS_4 vacua: hep-th/0403049, hep-th/0407263, hep-th/0412250, hep-th/0502154, hep-th/0609124, etc.

IIB	$a = 0$ or $b = 0$ (type A)	$a = \pm ib$ (type B)	$a = \pm b$ (type C)	(type ABC)
1	$\mathcal{W}_1 = F_3^{(1)} = H_3^{(1)} = 0$			
8	$\mathcal{W}_2 = 0$			
6	$F_3^{(6)} = 0$ $\mathcal{W}_3 = \pm * H_3^{(6)}$	$\mathcal{W}_3 = 0$ $e^{\phi} F_3^{(6)} = \mp * H_3^{(6)}$	$H_3^{(6)} = 0$ $\mathcal{W}_3 = \pm e^{\phi} * F_3^{(6)}$	$(***)$
3	$\overline{\mathcal{W}}_5 = 2\mathcal{W}_4 = \mp 2i H_3^{(3)} = 2\overline{\partial}\phi$ $\overline{\partial} A = \overline{\partial} a = 0$	$e^{\phi} F_5^{(3)} = \frac{2i}{3}\overline{\mathcal{W}}_5 = i\mathcal{W}_4$ $= -2i\overline{\partial} A = -4i\overline{\partial}\log a$ $\overline{\partial}\phi = 0$ F: $e^{\phi} F_1^{(3)} = 2e^{\phi} F_5^{(3)}$ $= i\overline{\mathcal{W}}_5 = i\mathcal{W}_4 = i\overline{\partial}\phi$	$e^{\phi} F_3^{(3)} = 2i\overline{\mathcal{W}}_5 = -2i\overline{\partial} A$ $= -4i\overline{\partial}\log a = -i\overline{\partial}\phi$	$(***)$

type A	NS-flux only (common to IIA, IIB, heterotic) $dJ \pm i H_3$ is (2,1)-primitive $\mathcal{W}_1 = \mathcal{W}_2 = 0$: complex
type B	both NS- and RR-flux $G_3 = F_3 - i e^{-\phi} H_3 = -i *_6 G_3$ is (2,1)-primitive $\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$, $2\mathcal{W}_5 = 3\mathcal{W}_4 = -6\overline{\partial} A$: conformally CY
type C	RR-flux only (S-dual of type A) $d(e^{-\phi} J) \pm i F_3$ is (2,1)-primitive $\mathcal{W}_1 = \mathcal{W}_2 = 0$: complex
type ABC	(skip detail...)

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and more...