

## Realization of AdS Vacua in Attractor Mechanism on Generalized Geometries

arXiv:0810.0937 [hep-th]

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We are looking for the origin of 4D physics

### Physical information

- particle contents and spectra
- (broken) symmetries and interactions
- potential, vacuum and cosmological constant

4D  $\mathcal{N} = 1$  supergravity:

$$S = \int \left( \frac{1}{2} R * \mathbf{1} - \frac{1}{2} F^a \wedge * F^a - K_{\mathcal{M}\bar{\mathcal{N}}} \nabla \phi^{\mathcal{M}} \wedge * \nabla \bar{\phi}^{\bar{\mathcal{N}}} - V * \mathbf{1} \right)$$

$$V = e^K \left( K^{\mathcal{M}\bar{\mathcal{N}}} D_{\mathcal{M}} \mathcal{W} \overline{D_{\bar{\mathcal{N}}} \mathcal{W}} - 3 |\mathcal{W}|^2 \right) + \frac{1}{2} |D^a|^2$$

$K$ : Kähler potential

$\mathcal{W}$ : superpotential  $\leftarrow \delta\psi_{\mu} = \nabla_{\mu} \varepsilon - e^{\frac{K}{2}} \mathcal{W} \gamma_{\mu} \varepsilon^c$

$D^a$ : D-term  $\leftarrow \delta\chi^a = \text{Im} F_{\mu\nu}^a \gamma^{\mu\nu} \varepsilon + i D^a \varepsilon$

Search of vacua  $\partial_{\mathcal{P}} V|_{*} = 0$

$V_* > 0$  : de Sitter space

$V_* = 0$  : Minkowski space

$V_* < 0$  : Anti-de Sitter space

10D string theories could provide information  
via compactifications

$$10 = 4 + 6$$

A typical success:

$E_8 \times E_8$  heterotic string compactified on Calabi-Yau three-fold

- number of generations =  $|\chi(\text{CY}_3)|/2$
- $E_6$  gauge symmetry
- zero cosmological constant

P. Candelas, G.T. Horowitz, A. Strominger, E. Witten “*Vacuum configurations for superstrings,*” Nucl. Phys. B 258 (1985) 46

However, Calabi-Yau manifold is not sufficient  $\rightarrow$  Fluxes are strongly restricted

common	$H_3, \nabla\phi, \text{torsion}$
type IIA	$F_0, F_2, F_4, F_6$
type IIB	$F_1, F_3, F_5$

On Calabi-Yau three-fold:

{	type IIA:	No fluxes	(warped Calabi-Yau)
	type IIB:	$F_3 - \tau H$	
	heterotic:	No fluxes	

Decompose 10D type IIA SUSY parameters:

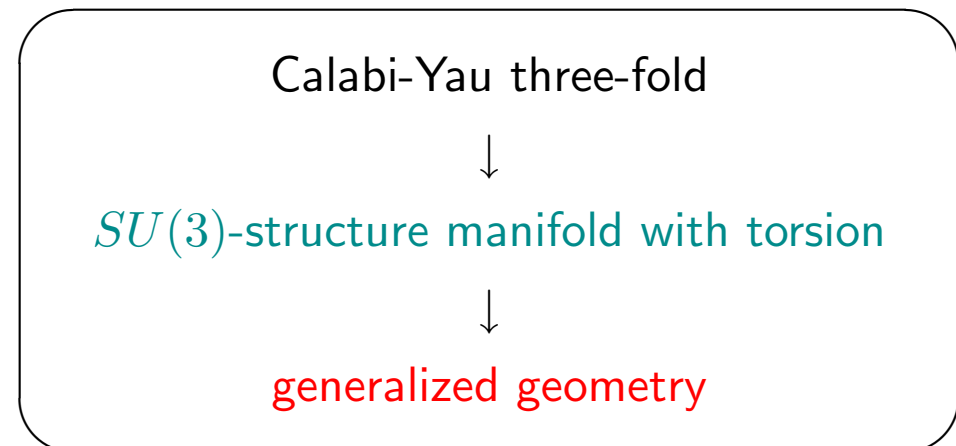
$$\epsilon^1 = \varepsilon_1 \otimes (a \eta_+^1) + \varepsilon_1^c \otimes (\bar{a} \eta_-^1), \quad \epsilon^2 = \varepsilon_2 \otimes (\bar{b} \eta_-^2) + \varepsilon_2^c \otimes (b \eta_+^2)$$

$\delta\psi_m^{\mathcal{A}} = 0$  gives the Killing spinor equation on the 6D compactified space  $\mathcal{M}$ :

$$\delta\psi_m^{\mathcal{A}} = \left( \partial_m + \frac{1}{4} \omega_{mab} \gamma^{ab} \right) \eta_+^{\mathcal{A}} + (\text{3-form fluxes} \cdot \eta)^{\mathcal{A}} + (\text{other fluxes} \cdot \eta)^{\mathcal{A}} = 0$$

Information of

6D  $SU(3)$  Killing spinors  $\eta_+^1, \eta_+^2$ :



► Calabi-Yau three-fold  $\rightarrow$  Fluxes are strongly restricted

$$\left\{ \begin{array}{ll} \text{type IIA :} & \text{No fluxes} \\ \text{type IIB :} & F_3 - \tau H \quad (\text{warped Calabi-Yau}) \\ \text{heterotic :} & \text{No fluxes} \end{array} \right.$$

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►  $SU(3)$ -structure manifold  $\rightarrow$  Some components of fluxes can be interpreted as torsion

$$\left. \begin{array}{l} \text{type IIA} \\ \text{type IIB} \\ \text{heterotic}^1 \end{array} \right\} \text{restricted fluxes are turned on}^2$$

1: Piljin Yi, TK “*Comments on heterotic flux compactifications,*” JHEP 0607 (2006) 030, [hep-th/0605247](https://arxiv.org/abs/hep-th/0605247)

2: TK “*Index theorems on torsional geometries,*” JHEP 0708 (2007) 048, [arXiv:0704.2111](https://arxiv.org/abs/0704.2111)



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- ▶ Generalized geometry  $\rightarrow$  Any types of fluxes can be included

**All the  $\mathcal{N} = 1$  SUSY solutions can be classified**

Consider the compactified space  $\mathcal{M}_6$

- Ordinary complex structure  $J^m_n$  lives in  $T\mathcal{M}$ :

$$J^2 = -\mathbf{1}_6, \quad J^m_n = -2i \eta_+^\dagger \gamma^m_n \eta_+$$

$\eta_+$ :  $SU(3)$  invariant spinor

- Generalized complex structures  $\mathcal{J}^\Lambda_\Sigma$  in  $T\mathcal{M} \oplus T^*\mathcal{M}$

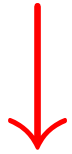
with basis  $\{dx^m \wedge, \iota_{\partial_n}\}$  and  $(6, 6)$ -signature

$$\mathcal{J}^2 = -\mathbf{1}_{12}, \quad \mathcal{J}^\Lambda_\Sigma = \langle \text{Re } \Phi_\pm, \Gamma^\Lambda_\Sigma \text{Re } \Phi_\pm \rangle$$

$\Phi_\pm$ :  $SU(3, 3)$  invariant spinors

$\Phi_\pm$  can be described by means of  $\eta_\pm^1$  and  $\eta_\pm^2$  in SUSY parameters

10D type IIA supergravity as a low energy theory of IIA string



compactifications on a certain compact space in the presence of fluxes

4D  $\mathcal{N} = 2$  supergravity



SUSY truncation

4D  $\mathcal{N} = 1$  supergravity

Classification of SUSY solutions on the  $SU(3)$  generalized geometries ( $\eta_+^1 = \eta_+^2$ ):

M. Graña, R. Minasian, M. Petrini, A. Tomasiello [hep-th/0407249](https://arxiv.org/abs/hep-th/0407249)

IIA	$a = 0$ or $b = 0$ (type A)	$a = b e^{i\beta}$ (type BC)
<b>1</b>	$\mathcal{W}_1 = H_3^{(1)} = 0$	
	$F_0^{(1)} = \mp F_2^{(1)} = F_4^{(1)} = \mp F_6^{(1)}$	$F_{2n}^{(1)} = 0$
<b>8</b>	$\mathcal{W}_2 = F_2^{(8)} = F_4^{(8)} = 0$	generic $\beta$
		$\beta = 0$
		$\text{Re}\mathcal{W}_2 = e^\phi F_2^{(8)}$
		$\text{Re}\mathcal{W}_2 = e^\phi F_2^{(8)} + e^\phi F_4^{(8)}$
		$\text{Im}\mathcal{W}_2 = 0$
		$\text{Im}\mathcal{W}_2 = 0$
<b>6</b>	$\mathcal{W}_3 = \mp *_6 H_3^{(6)}$	$\mathcal{W}_3 = H_3^{(6)} = 0$
<b>3</b>	$\bar{\mathcal{W}}_5 = 2\mathcal{W}_4 = \mp 2iH_3^{(3)} = \bar{\partial}\phi$	$F_2^{(3)} = 2i\bar{\mathcal{W}}_5 = -2i\bar{\partial}A = \frac{2i}{3}\bar{\partial}\phi$
	$\bar{\partial}A = \bar{\partial}a = 0$	$\mathcal{W}_4 = 0$

type A	NS-flux only (common to IIA, IIB, heterotic) $\mathcal{W}_1 = \mathcal{W}_2 = 0, \mathcal{W}_3 \neq 0$ : complex
type BC	RR-flux only $\mathcal{W}_1 = \text{Im}\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0, \text{Re}\mathcal{W}_2 \neq 0, \mathcal{W}_5 \neq 0$ : symplectic

For  $\mathcal{N} = 1$  AdS<sub>4</sub> vacua: [hep-th/0403049](https://arxiv.org/abs/hep-th/0403049), [hep-th/0407263](https://arxiv.org/abs/hep-th/0407263), [hep-th/0412250](https://arxiv.org/abs/hep-th/0412250), [hep-th/0502154](https://arxiv.org/abs/hep-th/0502154), [hep-th/0609124](https://arxiv.org/abs/hep-th/0609124), etc.

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$SU(3) \times SU(3)$  generalized geometries ( $\eta_+^1 \neq \eta_+^2$  at some points)

would complete the classification. (But, it's quite hard to find all solutions.)

Search 4D SUSY vacua in type IIA theory compactified on generalized geometries

- **Moduli stabilization**

We find SUSY AdS (or Minkowski) vacua

- **Mathematical feature**

We obtain a powerful rule to evaluate vacua:

**Discriminant** of the superpotential governs the cosmological constant

- **Stringy effects**

We see that  $\alpha'$  **corrections** are included in certain configurations

# Contents

- $\mathcal{N} = 1$  scalar potential from generalized geometry
- Search of SUSY vacua
- Summary and discussions

$\mathcal{N} = 1$  scalar potential from generalized geometry



Consider a compact space  $\mathcal{M}_6$

- Ordinary complex structure  $J^m_n$  in  $T\mathcal{M}$  is given by  $SU(3)$  invariant Weyl spinor  $\eta_+$ :

$$J^2 = -\mathbf{1}_6, \quad J^m_n = -2i \eta_+^\dagger \gamma^m_n \eta_+$$

- Generalized complex structures  $\mathcal{J}^\Lambda_\Sigma$  in  $T\mathcal{M} \oplus T^*\mathcal{M}$ : basis  $\{dx^m \wedge, \iota_{\partial_n}\}$ , (6, 6)-signature

$$\mathcal{J}_\pm^2 = -\mathbf{1}_{12}, \quad \mathcal{J}_{\pm\Sigma}^\Lambda = \langle \text{Re } \Phi_\pm, \Gamma^\Lambda_\Sigma \text{Re } \Phi_\pm \rangle$$

$\Phi_\pm$  :  $SU(3, 3)$  invariant Weyl spinors

$\Gamma^\Lambda$  :  $\text{Cliff}(6, 6)$  gamma matrix

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$\Phi_\pm$  :  $SU(3,3)$  invariant Weyl spinors *isomorphic* to even/odd-forms on  $T^*\mathcal{M}$

$\Gamma^\Lambda$  : Cliff(6,6) gamma matrix (repr. =  $(dx^m \wedge, \iota_{\partial_n})$ )

Mukai pairing:

$$\begin{aligned} \text{even forms: } & \langle \Psi_+, \Phi_+ \rangle = \Psi_6 \wedge \Phi_0 - \Psi_4 \wedge \Phi_2 + \Psi_2 \wedge \Phi_4 - \Psi_0 \wedge \Phi_6 \\ \text{odd forms: } & \langle \Psi_-, \Phi_- \rangle = \Psi_5 \wedge \Phi_1 - \Psi_3 \wedge \Phi_3 + \Psi_1 \wedge \Phi_5 \end{aligned}$$

There are two  $SU(3)$  spinors on  $\mathcal{M}$ :  $\eta_+^1, \eta_+^2$  with

$$\eta_+^2 = c_{\parallel}(y)\eta_+^1 + c_{\perp}(y)(v + iv')^m \gamma_m \eta_+^1, \quad (v - iv')^m = \eta_+^{1\dagger} \gamma^m \eta_+^2$$

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► On the  $SU(3)$  generalized geometry ( $\eta_+^1 = \eta_+^2$ ):

$$\begin{aligned} \Phi_+ &= e^{-B-iJ}, & \Phi_- &= e^{-B}\Omega \\ J_{mn} &= -2i \eta_+^{\dagger} \gamma_{mn} \eta_+, & \Omega_{mnp} &= -2i \eta_+^{\dagger} \gamma_{mnp} \eta_+ \end{aligned}$$

► On the  $SU(3) \times SU(3)$  generalized geometry ( $\eta_+^1 \neq \eta_+^2$  at some points  $y$ ):

$$\begin{aligned} \Phi_+ &= e^{-B}(\bar{c}_{\parallel}e^{-ij} - i\bar{c}_{\perp}w) \wedge e^{-iv \wedge v'}, & \Phi_- &= e^{-B}(c_{\parallel}e^{-ij} + ic_{\perp}w) \wedge (v + iv') \\ J^{\mathcal{A}} &= j \pm v \wedge v', & \Omega^{\mathcal{A}} &= w \wedge (v \pm iv') \end{aligned}$$

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Such a space  $\mathcal{M}$  has two moduli spaces: **special Kähler geometries of local type**  
**Kähler potentials, prepotentials, projective coordinates**

--> similar to 4D  $\mathcal{N} = 2$  supergravity by Calabi-Yau compactifications

Moduli spaces of  $\mathcal{M}$  are the special Kähler geometries of local type  
 Kähler potentials, prepotentials, projective coordinates

$$K_+ = -\log i \int_{\mathcal{M}} \langle \Phi_+, \bar{\Phi}_+ \rangle = -\log i (\bar{X}^A \mathcal{F}_A - X^A \bar{\mathcal{F}}_A)$$

$$K_- = -\log i \int_{\mathcal{M}} \langle \Phi_-, \bar{\Phi}_- \rangle = -\log i (\bar{Z}^I \mathcal{G}_I - Z^I \bar{\mathcal{G}}_I)$$

Expand the even/odd-forms  $\Phi_{\pm}$  by the basis forms:

$$\Phi_+ = X^A \omega_A - \mathcal{F}_A \tilde{\omega}^A, \quad \omega_A = (1, \omega_a), \quad \tilde{\omega}^A = (\tilde{\omega}^a, \text{vol}(\mathcal{M})) \quad : \quad 0,2,4,6\text{-forms}$$

$$\Phi_- = Z^I \alpha_I - \mathcal{G}_I \beta^I, \quad \alpha_I = (\alpha_0, \alpha_i), \quad \beta^I = (\beta^i, \beta^0) \quad : \quad 1,3,5\text{-forms}$$

$$\int_{\mathcal{M}} \langle \omega_A, \omega_B \rangle = 0, \quad \int_{\mathcal{M}} \langle \omega_A, \tilde{\omega}^B \rangle = \delta_A^B, \quad \int_{\mathcal{M}} \langle \alpha_I, \alpha_J \rangle = 0, \quad \int_{\mathcal{M}} \langle \alpha_I, \beta^J \rangle = \delta_I^J$$

On the  $SU(3)$  geometry ( $\eta_+^1 = \eta_+^2$ ):

$$\begin{aligned} d_H \omega_A &= m_A^I \alpha_I - e_{IA} \beta^I & d_H \tilde{\omega}^A &= 0 \\ d_H \alpha_I &= e_{IA} \tilde{\omega}^A & d_H \beta^I &= m_A^I \tilde{\omega}^A \end{aligned}$$

where NS three-form  $H$  deforms the differential operator:

$$dH = 0, \quad H = H^{\text{fl}} + dB, \quad H^{\text{fl}} = m_0^I \alpha_I - e_{I0} \beta^I$$

$$d_H \equiv d - H^{\text{fl}} \wedge$$

background	charges	
NS-flux charges	$e_{I0}$	$m_0^I$
torsion	$e_{Ia}$	$m_a^I$

On the  $SU(3) \times SU(3)$  geometry ( $\eta_+^1 \neq \eta_+^2$  at some points):

Extend to the generalized differential operator  $\mathcal{D}$ :

$$d_H = d - H^{\text{fl}} \wedge \quad \rightarrow \quad \mathcal{D} \equiv d - H^{\text{fl}} \wedge - Q \cdot - R \lrcorner$$

$$\mathcal{D}\omega_A \sim m_A^I \alpha_I - e_{IA} \beta^I \quad \mathcal{D}\tilde{\omega}^A \sim -q^{IA} \alpha_I + p_I^A \beta^I$$

$$\mathcal{D}\alpha_I \sim p_I^A \omega_A + e_{IA} \tilde{\omega}^A \quad \mathcal{D}\beta^I \sim q^{IA} \omega_A + m_A^I \tilde{\omega}^A$$

Necessary to introduce new fluxes  $Q$  and  $R$  to make a consistent algebra...



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But the compactified geometry becomes **nongeometric**:

$$\begin{aligned} (Q \cdot C)_{m_1 \dots m_{k-1}} &\equiv Q^{ab} [m_1 C_{|ab| m_2 \dots m_{k-1}}] && \text{feature of T-fold} \\ (R \lrcorner C)_{m_1 \dots m_{k-3}} &\equiv R^{abc} C_{abcm_1 \dots m_{k-3}} && \text{locally nongeometric background} \end{aligned}$$

Structure group contains Diffeo. + **Duality trsf.**  $\dashrightarrow$  *Doubled formalism*<sup>3</sup>

3: C. Albertsson, R.A. Reid-Edwards, TK “*D-branes and doubled geometry*,” [arXiv:0806.1783](https://arxiv.org/abs/0806.1783)

RR-fluxes  $F^{\text{even}} = e^B G$  without localized sources on the  $SU(3)$  geometry:

$$\begin{aligned} G &= G_0 + G_2 + G_4 + G_6 = G^{\text{fl}} + d_H A \\ F_n^{\text{even}} &= dC_{n-1} - H \wedge C_{n-3}, \quad C = e^B A \\ d_H F^{\text{even}} &= 0 \end{aligned}$$

Extension of RR-fluxes on the  $SU(3) \times SU(3)$  geometry:

$$\begin{aligned} G &= G^{\text{fl}} + \mathcal{D}A, \quad \mathcal{D}G = 0 \\ G^{\text{fl}} &= \sqrt{2}(m_{\text{RR}}^A \omega_A - e_{\text{RR}A} \tilde{\omega}^A), \quad A = \sqrt{2}(\xi^I \alpha_I - \tilde{\xi}_I \beta^I) \end{aligned}$$

↓

$$G \sim G^A \omega_A - \tilde{G}_A \tilde{\omega}^A$$

$$G^A \sim \sqrt{2}(m_{\text{RR}}^A + \xi^I p_I^A - \tilde{\xi}_I q^{IA}), \quad \tilde{G}_A \sim \sqrt{2}(e_{\text{RR}A} - \xi^I e_{IA} + \tilde{\xi}_I m_A^I)$$

fluxes	charges	
NS three-form $H$	$e_{I0}$	$m_0^I$
torsion	$e_{Ia}$	$m_a^I$
nongeometric fluxes	$p_I^A$	$q^{IA}$
RR-fluxes	$e_{RRA}$	$m_{RR}^A$

backgrounds	flux charges			
Calabi-Yau	—			
Calabi-Yau with $H$	$e_{I0}$	$m_0^I$		
$SU(3)$ geometry	$e_{IA}$	$m_A^I$		
$SU(3) \times SU(3)$ geometry	$e_{IA}$	$m_A^I$	$p_I^A$	$q^{IA}$

Note:  $SU(3)$  generalized geometry without RR-fluxes  $\sim SU(3)$ -structure manifold

All the information of the compactified space is translated into the (non)geometric flux charges and the RR-flux charges.

4D  $\mathcal{N} = 2$  theory comes out by the compactification:  $\varepsilon_1, \varepsilon_2$

### NEXT STEP

Introduce the flux charges into 4D  $\mathcal{N} = 1$  physics via various functionals:

$$K, \mathcal{W}, D^a$$

Setup in  $\mathcal{N} = 1$  theory

Functionals are given by two Kähler potentials on two Hodge-Kähler geometries of  $\Phi_{\pm}$ :

$$K = K_+ + 4\varphi$$

$$K_+ = -\log i \int_{\mathcal{M}} \langle \Phi_+, \bar{\Phi}_+ \rangle = -\log i (\bar{X}^A \mathcal{F}_A - X^A \bar{\mathcal{F}}_A)$$

$$K_- = -\log i \int_{\mathcal{M}} \langle \Phi_-, \bar{\Phi}_- \rangle = -\log i (\bar{Z}^I \mathcal{G}_I - Z^I \bar{\mathcal{G}}_I)$$

Introduce  $\mathcal{C} = \sqrt{2}ab e^{-\phi^{(10)}} = 4ab e^{\frac{K_-}{2} - \varphi}$

$$\begin{aligned} \therefore e^{-2\varphi} &= \frac{|\mathcal{C}|^2}{16|a|^2|b|^2} e^{-K_-} = \frac{i}{16|a|^2|b|^2} \int_{\mathcal{M}} \langle \mathcal{C}\Phi_-, \bar{\mathcal{C}}\bar{\Phi}_- \rangle \\ &= \frac{1}{8|a|^2|b|^2} \left[ \text{Im}(\mathcal{C}Z^I) \text{Re}(\mathcal{C}\mathcal{G}_I) - \text{Re}(\mathcal{C}Z^I) \text{Im}(\mathcal{C}\mathcal{G}_I) \right] \end{aligned}$$

See the SUSY variation of 4D  $\mathcal{N} = 2$  gravitinos:

$$\delta\psi_{\mathcal{A}\mu} = \nabla_{\mu}\varepsilon_{\mathcal{A}} - S_{\mathcal{A}\mathcal{B}}\gamma_{\mu}\varepsilon^{\mathcal{B}} + \dots$$

$$S_{\mathcal{A}\mathcal{B}} = \frac{i}{2}e^{\frac{K_{+}}{2}} \begin{pmatrix} \mathcal{P}^1 - i\mathcal{P}^2 & -\mathcal{P}^3 \\ -\mathcal{P}^3 & -\mathcal{P}^1 - i\mathcal{P}^2 \end{pmatrix}_{\mathcal{A}\mathcal{B}}$$

The components are also written by  $\Phi_{\pm}$ :

$$\mathcal{P}^1 - i\mathcal{P}^2 = 2e^{\frac{K_{-}}{2} + \varphi} \int_{\mathcal{M}} \langle \Phi_{+}, \mathcal{D}\Phi_{-} \rangle, \quad \mathcal{P}^1 + i\mathcal{P}^2 = 2e^{\frac{K_{-}}{2} + \varphi} \int_{\mathcal{M}} \langle \Phi_{+}, \mathcal{D}\bar{\Phi}_{-} \rangle$$

$$\mathcal{P}^3 = -\frac{1}{\sqrt{2}}e^{2\varphi} \int_{\mathcal{M}} \langle \Phi_{+}, G \rangle$$

Note:  $\hat{\Psi}_{\mathcal{A}\mu} = \Psi_{\mathcal{A}\mu} + \frac{1}{2}\Gamma_{\mu}{}^m\Psi_m^{\mathcal{A}} = \psi_{\mathcal{A}\mu\pm} \otimes \eta_{+} + \psi_{\mathcal{A}\mu\mp} \otimes \eta_{-} + \dots$

4D  $\mathcal{N} = 1$  fermions given by the SUSY truncation from 4D  $\mathcal{N} = 2$  system:

SUSY parameter :  $\varepsilon \equiv \bar{n}^{\mathcal{A}} \varepsilon_{\mathcal{A}} = a \varepsilon_1 + \bar{b} \varepsilon_2$

gravitino :  $\psi_{\mu} \equiv \bar{n}^{\mathcal{A}} \psi_{\mathcal{A}\mu} = a \psi_{1\mu} + \bar{b} \psi_{2\mu}, \quad \tilde{\psi}_{\mu} \equiv (b \psi_{1\mu} - \bar{a} \psi_{2\mu})$

gauginos :  $\chi^A \equiv -2 e^{\frac{K_+}{2}} D_b X^A (\bar{n}^{\mathcal{C}} \epsilon_{\mathcal{C}\mathcal{E}} \chi^{\mathcal{E}b})$

where  $\bar{n}^{\mathcal{A}} = (a, \bar{b}), \quad \epsilon_{\mathcal{A}\mathcal{B}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$



SUSY variations yield the superpotential and the D-term:

$$\delta\psi_\mu = \nabla_\mu \varepsilon - \bar{n}^A S_{AB} n^{*B} \gamma_\mu \varepsilon^c \equiv \nabla_\mu \varepsilon - e^{\frac{K}{2}} \mathcal{W} \gamma_\mu \varepsilon^c$$

$$\delta\tilde{\psi}_\mu = 0$$

$$\delta\chi^A = \text{Im}F_{\mu\nu}^A \gamma^{\mu\nu} \varepsilon + i D^A \varepsilon$$

$$\mathcal{W} = \frac{i}{4\bar{a}b} \left[ 4i e^{\frac{K_-}{2} - \varphi} \int_{\mathcal{M}} \langle \Phi_+, \mathcal{D}\text{Im}(ab\Phi_-) \rangle + \frac{1}{\sqrt{2}} \int_{\mathcal{M}} \langle \Phi_+, G \rangle \right]$$

$$\equiv \mathcal{W}^{\text{RR}} + U^I \mathcal{W}_I^{\text{Q}} + \tilde{U}_I \tilde{\mathcal{W}}_I^{\text{Q}}$$

$$\mathcal{W}^{\text{RR}} = -\frac{i}{4\bar{a}b} \left[ X^A e_{\text{RRA}} - \mathcal{F}_A m_{\text{RR}}^A \right]$$

$$\mathcal{W}_I^{\text{Q}} = \frac{i}{4\bar{a}b} \left[ X^A e_{IA} + \mathcal{F}_A p_I^A \right], \quad \tilde{\mathcal{W}}_I^{\text{Q}} = -\frac{i}{4\bar{a}b} \left[ X^A m_A^I + \mathcal{F}_A q^{IA} \right]$$

$$D^A = 2e^{K_+} (K_+)^{cd} D_c X^A \overline{D_d X^B} [\bar{n}^C (\sigma_x)_C^B n_B] \left( \mathcal{P}_B^x - \mathcal{N}_{BC} \tilde{\mathcal{P}}^{xC} \right)$$

$\mathcal{N} = 2$  multiplets:

$(t^a = X^a/X^0, z^i = Z^i/Z^0)$

gravity multiplet	$g_{\mu\nu}, A_\mu^0$	
vector multiplets	$A_\mu^a, t^a = b^a + iv^a$	$a = 1, \dots, b^+$
hypermultiplets	$z^i, \xi^i, \tilde{\xi}_i$	$i = 1, \dots, b^-$
tensor multiplet	$B_{\mu\nu}, \varphi, \xi^0, \tilde{\xi}_0$	


 orientifold projection:  $\mathcal{O} \equiv \Omega_{\text{WS}} (-1)^{FL} \sigma$ 
 $\mathcal{N} = 1$  multiplets:

gravity multiplet	$g_{\mu\nu}$	
vector multiplets	$A_\mu^{\hat{a}}$	$\hat{a} = 1, \dots, \hat{n}_v = b^+ - n_{ch}$
chiral multiplets	$t^{\check{a}} = b^{\check{a}} + iv^{\check{a}}$	$\check{a} = 1, \dots, n_{ch}$
chiral/linear multiplets	$U^{\check{I}} = \xi^{\check{I}} + i \text{Im}(\mathcal{C}Z^{\check{I}})$ $\tilde{U}_{\hat{I}} = \tilde{\xi}_{\hat{I}} + i \text{Im}(\mathcal{C}\mathcal{G}_{\hat{I}})$	$I = (\check{I}, \hat{I}) = 0, 1, \dots, b^-$
(projected out)	$B_{\mu\nu}, A_\mu^0, A_\mu^{\check{a}}, t^{\hat{a}}, U^{\hat{I}}, \tilde{U}_{\check{I}}$	

 Parameters are restricted as  $a = \bar{b} e^{i\theta}$  and  $|a|^2 = |b|^2 = \frac{1}{2}$

We are ready to search SUSY vacua in 4D  $\mathcal{N} = 1$  supergravity.

Consider **three** typical situations given by

- generalized geometry with RR-flux charges

$$e_{IA}, m_A^I, p_I^A, q^{IA}, e_{\text{RR}A}, m_{\text{RR}}^A$$

- generalized geometry without RR-flux charges

$$e_{IA}, m_A^I, p_I^A, q^{IA}$$

- $SU(3)$ -structure manifold

$$e_{IA}, m_A^I$$

Notice: 4D physics given by Calabi-Yau three-fold with RR-fluxes is **forbidden**.

RR-fluxes induce the non-zero NS-fluxes as well as torsion classes in SUSY solutions.

Search of SUSY vacua: flux vacua attractors

$$V = e^K \left( K^{\mathcal{M}\bar{\mathcal{N}}} D_{\mathcal{M}} \mathcal{W} \overline{D_{\bar{\mathcal{N}}} \mathcal{W}} - 3|\mathcal{W}|^2 \right) + \frac{1}{2} |D^a|^2$$

$$\equiv V_{\mathcal{W}} + V_D$$

Search of vacua  $\partial_{\mathcal{P}} V|_* = 0$

$V_* > 0$  : de Sitter space

$V_* = 0$  : Minkowski space

$V_* < 0$  : Anti-de Sitter space

$$0 = \partial_{\mathcal{P}} V_{\mathcal{W}} = e^K \left\{ K^{\mathcal{M}\bar{\mathcal{N}}} D_{\mathcal{P}} D_{\mathcal{M}} \mathcal{W} \overline{D_{\bar{\mathcal{N}}} \mathcal{W}} + \partial_{\mathcal{P}} K^{\mathcal{M}\bar{\mathcal{N}}} D_{\mathcal{M}} \mathcal{W} \overline{D_{\bar{\mathcal{N}}} \mathcal{W}} - 2\overline{\mathcal{W}} D_{\mathcal{P}} \mathcal{W} \right\}$$

$$0 = \partial_{\mathcal{P}} V_D \quad \rightarrow \quad D^a = 0$$

Consider the SUSY condition  $D_{\mathcal{P}} \mathcal{W} \equiv (\partial_{\mathcal{P}} + \partial_{\mathcal{P}} K) \mathcal{W} = 0$  in various cases.

1. Set a simple prepotential:  $\mathcal{F} = D_{abc} \frac{X^a X^b X^c}{X^0}$
2. Consider the simplest model: single modulus  $t$  of  $\Phi_+$  (and  $U$  of  $\Phi_-$ )

Derivatives of the Kähler potential are

$$\partial_t K = -\frac{3}{t - \bar{t}} \quad \partial_U K = -\frac{2}{U - \bar{U}}$$

The superpotential is reduced to

$$\begin{aligned} \mathcal{W} &= \mathcal{W}^{\text{RR}} + U \mathcal{W}^{\text{Q}} \\ \mathcal{W}^{\text{RR}} &= m_{\text{RR}}^0 t^3 - 3 m_{\text{RR}} t^2 + e_{\text{RR}} t + e_{\text{RR}0} \\ \mathcal{W}^{\text{Q}} &= p_0^0 t^3 - 3 p_0 t^2 - e_0 t - e_{00} \end{aligned}$$

Consider the SUSY condition  $D_{\mathcal{P}}\mathcal{W} \equiv (\partial_{\mathcal{P}} + \partial_{\mathcal{P}}K)\mathcal{W} = 0$ :

$$D_t\mathcal{W} = 0 \quad \rightarrow \quad 0 = D_t\mathcal{W}^{\text{RR}} + U D_t\mathcal{W}^{\text{Q}}$$

$$D_U\mathcal{W} = 0 \quad \rightarrow \quad 0 = \frac{i}{\text{Im}U} \left( \mathcal{W}^{\text{RR}} + \text{Re}U \mathcal{W}^{\text{Q}} \right)$$

Note:  $\text{Im}U \neq 0$  to avoid curvature singularity

The discriminant of the superpotential  $\mathcal{W}^{\text{RR}}$  (and  $\mathcal{W}^{\text{Q}}$ ) governs the SUSY solutions.

$$\Delta(\mathcal{W}^{\text{RR}}) = -27 (m_{\text{RR}}^0 e_{\text{RR}0})^2 - 54 m_{\text{RR}}^0 e_{\text{RR}0} m_{\text{RR}} e_{\text{RR}} + 9 (m_{\text{RR}} e_{\text{RR}})^2$$

$$+ 108 (m_{\text{RR}})^3 e_{\text{RR}0} - 4 m_{\text{RR}}^0 (e_{\text{RR}})^3$$

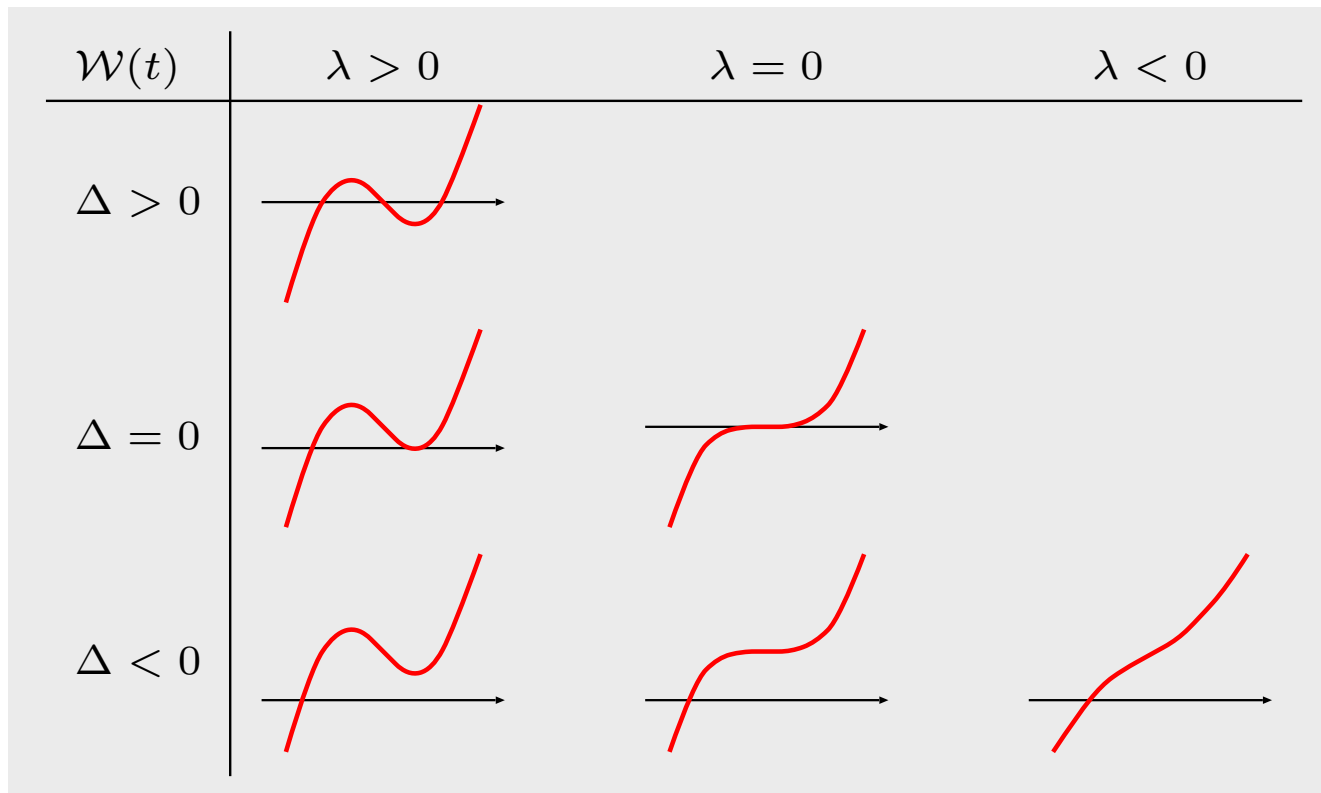
► *Discriminant of cubic equation*

Consider a cubic function and its derivative: 
$$\begin{cases} \mathcal{W}(t) = at^3 + bt^2 + ct + d \\ \partial_t \mathcal{W}(t) = 3at^2 + 2bt + c \end{cases}$$

Discriminants  $\Delta(\mathcal{W})$  and  $\Delta(\partial_t \mathcal{W})$  are

$$\Delta(\mathcal{W}) \equiv \Delta = -4b^3d + b^2c^2 - 4ac^3 + 18abcd - 27a^2d^2$$

$$\Delta(\partial_t \mathcal{W}) \equiv \lambda = 4(b^2 - 3ac)$$





$\Delta^{\text{RR}} > 0$  case: always  $\lambda^{\text{RR}} > 0$ , and exists a zero point:  $D_t \mathcal{W}^{\text{RR}} = 0$

$$D_t \mathcal{W}^{\text{RR}}|_* = 0$$

$$t_*^{\text{RR}} = \frac{6(3m_{\text{RR}}^0 e_{\text{RR}0} + m_{\text{RR}} e_{\text{RR}})}{\lambda^{\text{RR}}} - 2i \frac{\sqrt{3\Delta^{\text{RR}}}}{\lambda^{\text{RR}}}$$

$$\mathcal{W}_*^{\text{RR}} = -\frac{24\Delta^{\text{RR}}}{(\lambda^{\text{RR}})^3} \left( 36(m_{\text{RR}})^3 + 36(m_{\text{RR}}^0)^2 e_{\text{RR}0} - 3m_{\text{RR}}\lambda^{\text{RR}} - 4im_{\text{RR}}^0 \sqrt{3\Delta^{\text{RR}}} \right)$$

$\Delta^{\text{RR}} > 0$  case: always  $\lambda^{\text{RR}} > 0$ , and exists a zero point:  $D_t \mathcal{W}^{\text{RR}} = 0$

$$\begin{aligned}
 D_t \mathcal{W}^{\text{RR}}|_* &= 0 \\
 t_*^{\text{RR}} &= \frac{6(3m_{\text{RR}}^0 e_{\text{RR}0} + m_{\text{RR}} e_{\text{RR}})}{\lambda^{\text{RR}}} - 2i \frac{\sqrt{3\Delta^{\text{RR}}}}{\lambda^{\text{RR}}} \\
 \mathcal{W}_*^{\text{RR}} &= -\frac{24\Delta^{\text{RR}}}{(\lambda^{\text{RR}})^3} \left( 36(m_{\text{RR}})^3 + 36(m_{\text{RR}}^0)^2 e_{\text{RR}0} - 3m_{\text{RR}} \lambda^{\text{RR}} - 4i m_{\text{RR}}^0 \sqrt{3\Delta^{\text{RR}}} \right)
 \end{aligned}$$

$\Delta^{\text{RR}} < 0$  case: only  $\lambda^{\text{RR}} < 0$  is physically allowed, and exists a zero point:  $\mathcal{W}^{\text{RR}} = 0$

$$\begin{aligned}
 \mathcal{W}_*^{\text{RR}} &= m_{\text{RR}}^0 (t_* - e)(t_* - \alpha)(t_* - \bar{\alpha}) = 0, \quad t_* = \alpha^{\text{RR}} = \alpha_1 + i\alpha_2 \\
 \alpha_1 &= \frac{\lambda^{\text{RR}} + F^{2/3} + 12m_{\text{RR}} F^{1/3}}{12m_{\text{RR}}^0 F^{1/3}} \\
 (\alpha_2)^2 &= \frac{1}{m_{\text{RR}}^0} \left( e_{\text{RR}} - 6m_{\text{RR}} \alpha_1 + 3m_{\text{RR}}^0 (\alpha_1)^2 \right) \\
 e &= -\frac{1}{m_{\text{RR}}^0} \left( -3m_{\text{RR}} + 2m_{\text{RR}}^0 \alpha_1 \right) \\
 F &= 108(m_{\text{RR}}^0)^2 e_{\text{RR}0} + 12m_{\text{RR}}^0 \sqrt{-3\Delta^{\text{RR}}} + 108(m_{\text{RR}})^3 - 9m_{\text{RR}} \lambda^{\text{RR}} \\
 D_t \mathcal{W}^{\text{RR}}|_* &= 2i m_{\text{RR}}^0 (e - \alpha^{\text{RR}}) \alpha_2
 \end{aligned}$$

... Analysis of  $\mathcal{W}^{\text{Q}}$  is also discussed.

Three types of solutions to satisfy  $0 = D_t \mathcal{W}^{\text{RR}} + U D_t \mathcal{W}^{\text{Q}}$  and  $0 = \mathcal{W}^{\text{RR}} + \text{Re}U \mathcal{W}^{\text{Q}}$ :

● SUSY AdS vacuum: **attractor point**

$$\Delta^{\text{RR}} > 0, \quad \Delta^{\text{Q}} > 0; \quad D_t \mathcal{W}^{\text{RR}}|_* = 0 = D_t \mathcal{W}^{\text{Q}}|_*$$

$$t_*^{\text{RR}} = t_*^{\text{Q}}, \quad \text{Re}U_* = -\frac{\mathcal{W}_*^{\text{RR}}}{\mathcal{W}_*^{\text{Q}}}$$

$$V_* = -3e^K |\mathcal{W}_*|^2 = -\frac{4}{[\text{Re}(\mathcal{C}\mathcal{G}_0)]^2} \sqrt{\frac{\Delta^{\text{Q}}}{3}} \ll 1$$

● SUSY Minkowski vacuum: **attractor point**

$$\Delta^{\text{RR}} < 0, \quad \Delta^{\text{Q}} < 0; \quad \mathcal{W}_*^{\text{RR}} = 0 = \mathcal{W}_*^{\text{Q}}$$

$$\alpha^{\text{RR}} = \alpha^{\text{Q}}, \quad U_* = -\frac{D_t \mathcal{W}^{\text{RR}}|_*}{D_t \mathcal{W}^{\text{Q}}|_*} \neq 0$$

$$V_* = 0$$

● SUSY AdS vacua, but moduli  $t$  and  $U$  are not fixed: **non attractor point**

$$U = -\frac{D_t \mathcal{W}^{\text{RR}}(t)}{D_t \mathcal{W}^{\text{Q}}(t)}, \quad \text{Re}U = -\frac{\mathcal{W}^{\text{RR}}(t)}{\mathcal{W}^{\text{Q}}(t)}$$

1. Set  $e_{RRA} = 0 = m_{RR}^A$
2. Set a simple prepotential:  $\mathcal{F} = D_{abc} \frac{X^a X^b X^c}{X^0}$
3. Consider the simplest model: single modulus  $t$  of  $\Phi_+$  (and  $U$  of  $\Phi_-$ )

The SUSY conditions on  $\mathcal{W} = U \mathcal{W}^{\mathbb{Q}}$  are

$$D_t \mathcal{W} = 0 \quad \dashrightarrow \quad 0 = D_t \mathcal{W}^{\mathbb{Q}}$$

$$D_U \mathcal{W} = 0 \quad \dashrightarrow \quad 0 = \text{Re} U \mathcal{W}^{\mathbb{Q}}$$

1. Set  $e_{RRA} = 0 = m_{RR}^A$
2. Set a simple prepotential:  $\mathcal{F} = D_{abc} \frac{X^a X^b X^c}{X^0}$
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The SUSY conditions on  $\mathcal{W} = U \mathcal{W}^{\mathbb{Q}}$  are

$$D_t \mathcal{W} = 0 \quad \dashrightarrow \quad 0 = D_t \mathcal{W}^{\mathbb{Q}}$$

$$D_U \mathcal{W} = 0 \quad \dashrightarrow \quad 0 = \text{Re} U \mathcal{W}^{\mathbb{Q}}$$

The solution is given only when  $\Delta^{\mathbb{Q}} > 0$ , and the AdS vacuum emerges:

$$t_*^{\mathbb{Q}} = -\frac{6(3p_0^0 e_{00} + p_0 e_0)}{\lambda^{\mathbb{Q}}} - 2i \frac{\sqrt{3\Delta^{\mathbb{Q}}}}{\lambda^{\mathbb{Q}}}, \quad \text{Re} U_* = 0$$

$$V_* = -3e^K |\mathcal{W}_*|^2 = -\frac{4}{[\text{Re}(\mathcal{C}\mathcal{G}_0)]^2} \sqrt{\frac{\Delta^{\mathbb{Q}}}{3}} \ll 1$$

1. Set  $e_{RRA} = 0 = m_{RR}^A$  and  $p_I^A = 0 = q^{IA}$
2. Set a simple prepotential:  $\mathcal{F} = D_{abc} \frac{X^a X^b X^c}{X^0}$
3. Consider the simplest model: single modulus  $t$  of  $\Phi_+$  (and  $U$  of  $\Phi_-$ )

Functions are reduced to

$$D_t \mathcal{W} = \frac{U}{t - \bar{t}} \left( e_0(2t + \bar{t}) + 3e_{00} \right), \quad D_U \mathcal{W} = i \frac{\text{Re}U}{\text{Im}U} \mathcal{W}^{\mathbb{Q}}$$

1. Set  $e_{RRA} = 0 = m_{RR}^A$  and  $p_I^A = 0 = q^{IA}$
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There are **neither** SUSY solutions under the conditions  $D_t \mathcal{W} = 0 = D_U \mathcal{W}$   
**nor** non-SUSY solutions satisfying  $\partial_{\mathcal{P}} V = 0!$

Ansatz 2. “Neglecting all  $\alpha'$  corrections on the compactified space” is **too strong!**

2'. Set a deformed prepotential:  $\mathcal{F} = \frac{(X^t)^3}{X^0} + \sum_n N_n \frac{(X^t)^{n+3}}{(X^0)^{n+1}}$

Consider a simple case as  $N_1 \neq 0$ , otherwise  $N_n = 0$ :

$$D_t \mathcal{W}^{\mathbb{Q}} = -e_{00} + \frac{3(t - \bar{t})^2 - \partial_t P}{(t - \bar{t})^3 - P} (e_{00} + e_0 t)$$

$$P \equiv -2 \left( N_1 t^4 - \bar{N}_1 \bar{t}^4 - 2N_1 t^3 \bar{t} + 2\bar{N}_1 t \bar{t}^3 \right)$$



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$$P \equiv -2(N_1 t^4 - \bar{N}_1 \bar{t}^4 - 2N_1 t^3 \bar{t} + 2\bar{N}_1 t \bar{t}^3)$$

SUSY AdS solution appears under the conditions  $D_t \mathcal{W} = 0$  and  $D_U \mathcal{W} = 0$ :

$$t_*^{\mathbb{Q}} = -\frac{2e_{00}}{e_0}, \quad \text{Re } U_* = 0$$

$$\mathcal{W}_*^{\mathbb{Q}} = e_{00}, \quad \text{Im } N_1 < 0$$

$$V_* = -3e^K |\mathcal{W}_*|^2 = \frac{1}{[\text{Re}(\mathcal{CG}_0)]^2} \frac{3(e_0)^4}{16(e_{00})^2 \text{Im } N_1}$$

This is also given by the [heterotic](#) string compactifications on  $SU(3)$ -structure manifolds [with torsion](#), which carries  $\alpha'$  corrections.

Summary and Discussions

## Summary

- We found SUSY AdS (or Minkowski) vacuum on an **attractor** point
- We obtained a powerful rule to evaluate the attractor points: **Discriminants**
- We confirmed that  $\alpha'$  **corrections** are included in certain configurations

## Discussions

- Complete stabilization via nonperturbative corrections
- Duality transformations
- Understanding the physical interpretation of nongeometric fluxes
- Connection to doubled formalism

## de Sitter vacua?

*In order to build (stable) de Sitter vacua perturbatively in type IIA, in addition to the usual RR and NSNS fluxes and O6/D6 sources, one must minimally have geometric fluxes and non-zero Romans' mass parameter.*

S.S. Haque, G. Shiu, B. Underwood, T. Van Riet [arXiv:0810.5328](https://arxiv.org/abs/0810.5328)

Romans' mass parameter  $\sim G_0$

🔴 Search a (meta)stable de Sitter vacuum in this formulation

Appendix: compactifications in type II strings

Moduli spaces in  $\mathcal{N} = 2$  supergravity are

vector multiplets: Hodge-Kähler geometry  
hypermultiplets: quaternionic geometry

We look for the origin of the moduli spaces in 10D string theories

Decomposition of vector bundle on 10D spacetime:

$$T\mathcal{M}_{1,9} = T_{1,3} \oplus F$$

$$\left\{ \begin{array}{l} T_{1,3} : \text{ a real } SO(1,3) \text{ vector bundle} \\ F : \text{ an } SO(6) \text{ vector bundle which admits a pair of } SU(3) \text{ structures} \end{array} \right.$$

10D spacetime itself is not decomposed yet, i.e., do not yet consider truncation of modes.

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Decomposition of Lorentz symmetry:

$$Spin(1,9) \rightarrow Spin(1,3) \times Spin(6) = SL(2, \mathbb{C}) \times SU(4)$$

$$\mathbf{16} = (\mathbf{2}, \mathbf{4}) \oplus (\bar{\mathbf{2}}, \bar{\mathbf{4}}) \quad \mathbf{16} = (\mathbf{2}, \bar{\mathbf{4}}) \oplus (\bar{\mathbf{2}}, \mathbf{4})$$

Decomposition of supersymmetry parameters (with  $a, b \in \mathbb{C}$ ):

$$\begin{cases} \epsilon_{\text{IIA}}^1 = \varepsilon_1 \otimes (a\eta_+^1) + \varepsilon_1^c \otimes (\bar{a}\eta_-^1) \\ \epsilon_{\text{IIA}}^2 = \varepsilon_2 \otimes (\bar{b}\eta_-^2) + \varepsilon_2^c \otimes (b\eta_+^2) \end{cases} \quad \begin{cases} \epsilon_{\text{IIB}}^1 = \varepsilon_1 \otimes (a\eta_+^1) + \varepsilon_1^c \otimes (\bar{a}\eta_-^1) \\ \epsilon_{\text{IIB}}^2 = \varepsilon_2 \otimes (b\eta_+^2) + \varepsilon_2^c \otimes (\bar{b}\eta_-^2) \end{cases}$$



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Decomposition of supersymmetry parameters (with  $a, b \in \mathbb{C}$ ):

$$\begin{cases} \epsilon_{\text{IIA}}^1 = \varepsilon_1 \otimes (a\eta_+^1) + \varepsilon_1^c \otimes (\bar{a}\eta_-^1) \\ \epsilon_{\text{IIA}}^2 = \varepsilon_2 \otimes (\bar{b}\eta_-^2) + \varepsilon_2^c \otimes (b\eta_+^2) \end{cases} \quad \begin{cases} \epsilon_{\text{IIB}}^1 = \varepsilon_1 \otimes (a\eta_+^1) + \varepsilon_1^c \otimes (\bar{a}\eta_-^1) \\ \epsilon_{\text{IIB}}^2 = \varepsilon_2 \otimes (b\eta_+^2) + \varepsilon_2^c \otimes (\bar{b}\eta_-^2) \end{cases}$$

Set  $SU(3)$  invariant spinor  $\eta_+^{\mathcal{A}}$  s.t.  $\nabla^{(T)}\eta_+^{\mathcal{A}} = 0$  ( $\mathcal{A} = 1, 2$ )

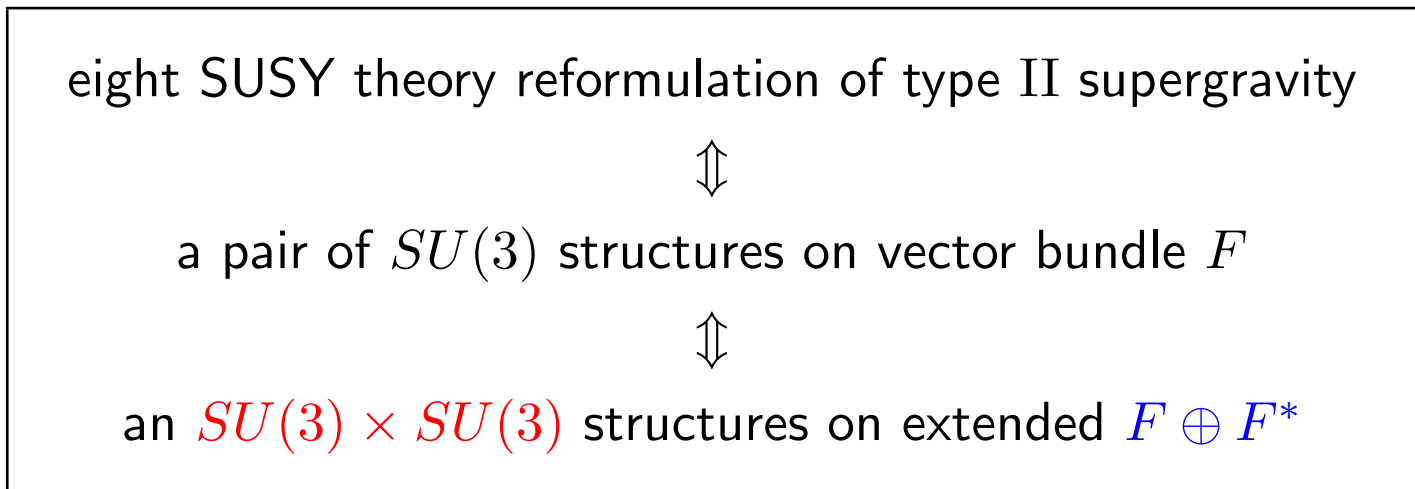
a pair of  $SU(3)$  on  $F$   $(\eta_+^1, \eta_+^2)$   $\longleftrightarrow$  a single  $SU(3)$  on  $F$   $(\eta_+^1 = \eta_+^2 = \eta_+)$

Requirement that we have a pair of  $SU(3)$  structures means there is a sub-supermanifold

$$\mathcal{N}^{1,9|4+4} \subset \mathcal{M}^{1,9|16+16}$$

$$\left( \begin{array}{l} (1, 9) : \text{ bosonic degrees} \\ 4 + 4 : \text{ eight Grassmann variables as spinors of } Spin(1, 3) \text{ and singlet of } SU(3)\text{s} \end{array} \right)$$

Equivalence such as



## 10D spinors in type IIA in Einstein frame

$$\begin{aligned} \delta\Psi_M^{\mathcal{A}} &= \nabla_M \epsilon^{\mathcal{A}} - \frac{1}{96} e^{-\phi} \left( \Gamma_M^{PQR} H_{PQR} - 9\Gamma^{PQ} H_{MPQ} \right) \Gamma_{(11)} \epsilon^{\mathcal{A}} \\ &\quad - \sum_{n=0,2,4,6,8} \frac{1}{64n!} e^{\frac{5-n}{4}\phi} \left[ (n-1)\Gamma_M^{N_1 \dots N_n} - n(9-n)\delta_M^{N_1} \Gamma^{N_2 \dots N_n} \right] F_{N_1 \dots N_n} (\Gamma_{(11)})^{n/2} (\sigma^1 \epsilon)^{\mathcal{A}} \end{aligned}$$

### Split spacetime 10 = 4 + 6

$$\epsilon^1 = \varepsilon_1 \otimes (a\eta_+^1) + \varepsilon_1^c \otimes (\bar{a}\eta_-^1), \quad \epsilon^2 = \varepsilon_2 \otimes (\bar{b}\eta_-^2) + \varepsilon_2^c \otimes (b\eta_+^2)$$

$$0 \equiv \delta\psi_m^{\mathcal{A}} = \nabla_m \eta_+^{\mathcal{A}} + (\text{NS-fluxes} \cdot \eta)^{\mathcal{A}} + (\text{RR-fluxes} \cdot \eta)^{\mathcal{A}}$$

Information of

6D  $SU(3)$  Killing spinors  $\eta_+^{\mathcal{A}}$

Calabi-Yau three-fold



$SU(3)$ -structure manifold with torsion



generalized geometry

▶ on a single $SU(3)$ :	a real two-form	$J_{mn} = \mp 2i \eta_{\pm}^{\dagger} \gamma_{mn} \eta_{\pm}$
	a complex three-form	$\Omega_{mnp} = -2i \eta_{-}^{\dagger} \gamma_{mnp} \eta_{+}$
▶ on a pair of $SU(3)$ :	two real vectors	$(v - iv')^m = \eta_{+}^{1\dagger} \gamma^m \eta_{-}^2$
	$(J^A, \Omega^A)$	$J^1 = j + v \wedge v' \quad \Omega^1 = w \wedge (v + iv')$
		$J^2 = j - v \wedge v' \quad \Omega^2 = w \wedge (v - iv')$
		$(j, w)$ : local $SU(2)$ -invariant forms

If  $\eta_{+}^1 \neq \eta_{+}^2$  globally, a pair of  $SU(3)$  is reduced to global single  $SU(2)$  w/  $(j, w, v, v')$

If  $\eta_{+}^1 = \eta_{+}^2$  globally, a pair of  $SU(3)$  is reduced to a single global  $SU(3)$  w/  $(J, \Omega)$

$$\eta_{+}^2 = c_{\parallel} \eta_{+}^1 + c_{\perp} (v + iv')^m \gamma_m \eta_{-}^1, \quad |c_{\parallel}|^2 + |c_{\perp}|^2 = 1$$

a pair of  $SU(3)$  on  $T\mathcal{M} \sim$  an  $SU(3) \times SU(3)$  on  $T\mathcal{M} \oplus T^*\mathcal{M}$

Appendix: Calabi-Yau compactifications

One can embed 4D  $\mathcal{N} = 2$  theory into 10D type II theory  
compactified on Calabi-Yau three-fold

	vector multiplets	hypermultiplets
generic	coord. of Hodge-Kähler	coord. of quaternionic
IIA on Calabi-Yau	Kähler moduli	complex moduli + RR
IIB on Calabi-Yau	complex moduli	Kähler moduli + RR

NS-NS fields in ten-dimensional spacetime are expanded as

$$\begin{aligned}\phi(x, y) &= \varphi(x) \\ G_{m\bar{n}}(x, y) &= i v^a(x) (\omega_a)_{m\bar{n}}(y), \quad G_{mn}(x, y) = i \bar{z}^k(x) \left( \frac{(\bar{\chi}_k)_{m\bar{p}\bar{q}} \Omega^{\bar{p}\bar{q}}{}_n}{\|\Omega\|^2} \right) (y) \\ B_2(x, y) &= B_2(x) + b^a(x) \omega_a(y)\end{aligned}$$

RR fields in type IIA are

$$\begin{aligned}C_1(x, y) &= C_1^0(x) \\ C_3(x, y) &= C_1^a(x) \omega_a(y) + \xi^K(x) \alpha_K(y) - \tilde{\xi}_K(x) \beta^K(y)\end{aligned}$$

RR fields in type IIB are

$$\begin{aligned}C_0(x, y) &= C_0(x) \\ C_2(x, y) &= C_2(x) + c^a(x) \omega_a(y) \\ C_4(x, y) &= V_1^K(x) \alpha_K(y) + \rho_a(x) \tilde{\omega}^a(y)\end{aligned}$$

cohomology class	basis	
$H^{(1,1)}$	$\omega_a$	$a = 1, \dots, h^{(1,1)}$
$H^{(0)} \oplus H^{(1,1)}$	$\omega_A = (1, \omega_a)$	$A = 0, 1, \dots, h^{(1,1)}$
$H^{(2,2)}$	$\tilde{\omega}^a$	$a = 1, \dots, h^{(1,1)}$
$H^{(2,1)}$	$\chi_k$	$k = 1, \dots, h^{(2,1)}$
$H^{(3)}$	$(\alpha_K, \beta^K)$	$K = 0, 1, \dots, h^{(2,1)}$

4D type IIA  $\mathcal{N} = 2$  ungauged supergravity action of bosonic fields is

$$S_{\text{IIA}}^{(4)} = \int_{\mathcal{M}_{1,3}} \left( -\frac{1}{2} R * \mathbf{1} + \frac{1}{2} \text{Re} \mathcal{N}_{AB} F^A \wedge F^B + \frac{1}{2} \text{Im} \mathcal{N}_{AB} F^A \wedge * F^B - G_{a\bar{b}} dt^a \wedge * d\bar{t}^{\bar{b}} - h_{uv} dq^u \wedge * dq^v \right)$$

gravity multiplet	$g_{\mu\nu}, C_1^0$	1
vector multiplet	$C_1^a, v^a, b^a$	$a = 1, \dots, h^{(1,1)}$
hypermultiplet	$z^k, \xi^k, \tilde{\xi}_k$	$k = 1, \dots, h^{(2,1)}$
tensor multiplet	$B_2, \varphi, \xi^0, \tilde{\xi}_0$	1

Various functions in the actions:

$$B + iJ = (b^a + iv^a) \omega_a = t^a \omega_a$$

$$K^{\text{KS}} = -\log \left( \frac{4}{3} \int_{\mathcal{M}_6} J \wedge J \wedge J \right)$$

$$\mathcal{K}_{abc} = \int_{\mathcal{M}_6} \omega_a \wedge \omega_b \wedge \omega_c$$

$$\mathcal{K}_{ab} = \int_{\mathcal{M}_6} \omega_a \wedge \omega_b \wedge J = \mathcal{K}_{abc} v^c$$

$$\mathcal{K}_a = \int_{\mathcal{M}_6} \omega_a \wedge J \wedge J = \mathcal{K}_{abc} v^b v^c$$

$$\mathcal{K} = \int_{\mathcal{M}_6} J \wedge J \wedge J = \mathcal{K}_{abc} v^a v^b v^c$$

$$\text{Re} \mathcal{N}_{AB} = \begin{pmatrix} -\frac{1}{3} \mathcal{K}_{abc} b^a b^b b^c & \frac{1}{2} \mathcal{K}_{abc} b^b b^c \\ \frac{1}{2} \mathcal{K}_{abc} b^b b^c & -\mathcal{K}_{abc} b^c \end{pmatrix}$$

$$\text{Im} \mathcal{N}_{AB} = -\frac{\mathcal{K}}{6} \begin{pmatrix} 1 + 4G_{ab} b^a b^b & -4G_{ab} b^b \\ -4G_{ab} b^b & 4G_{ab} \end{pmatrix}$$

$$G_{a\bar{b}} = \frac{3}{2} \frac{\int_{\mathcal{M}_6} \omega_a \wedge * \omega_b}{\int_{\mathcal{M}_6} J \wedge J \wedge J} = \partial_{t^a} \bar{\partial}_{\bar{t}^{\bar{b}}} K^{\text{KS}}$$



4D type IIB  $\mathcal{N} = 2$  ungauged supergravity action of bosonic fields is

$$S_{\text{IIB}}^{(4)} = \int_{\mathcal{M}_{1,3}} \left( -\frac{1}{2} R * \mathbf{1} + \frac{1}{2} \text{Re} \mathcal{M}_{KL} F^K \wedge F^L + \frac{1}{2} \text{Im} \mathcal{M}_{KL} F^K \wedge *F^L - G_{k\bar{l}} dz^k \wedge *d\bar{z}^{\bar{l}} - h_{pq} d\tilde{q}^p \wedge *d\tilde{q}^q \right)$$

---

gravity multiplet	$g_{\mu\nu}, V_1^0$	1
vector multiplet	$V_1^k, z^k$	$k = 1, \dots, h^{(2,1)}$
hypermultiplet	$v^a, b^a, c^a, \rho_a$	$a = 1, \dots, h^{(1,1)}$
tensor multiplet	$B_2, C_2, \varphi, C_0$	1

---

Various functions in the actions:

$$\Omega = Z^K \alpha_K - \mathcal{G}_K \beta^K \quad z^k = Z^K / Z^0 \quad \mathcal{G}_{KL} = \partial_L \mathcal{G}_K$$

$$K^{\text{CS}} = -\log \left( i \int_{\mathcal{M}_6} \Omega \wedge \bar{\Omega} \right) \quad G_{k\bar{l}} = -\frac{\int \chi_k \wedge \bar{\chi}_{\bar{l}}}{\int \Omega \wedge \bar{\Omega}} = \partial_{z^k} \bar{\partial}_{\bar{z}^{\bar{l}}} K^{\text{CS}}$$

$$\mathcal{M}_{KL} = \bar{\mathcal{G}}_{KL} + 2i \frac{(\text{Im} \mathcal{G})_{KM} Z^M (\text{Im} \mathcal{G})_{LN} Z^N}{Z^N (\text{Im} \mathcal{G})_{NM} Z^M}$$

Appendix:  $SU(3)$ -structure manifold with torsion

**i** Information from Killing spinor eqs. with torsion  $D^{(T)}\eta_{\pm} = 0$  ( $\exists$  complex Weyl  $\eta_{\pm}$ )

► Invariant  $p$ -forms on  $SU(3)$ -structure manifold:

a real two-form  $J_{mn} = \mp 2i \eta_{\pm}^{\dagger} \gamma_{mn} \eta_{\pm}$

a holomorphic three-form  $\Omega_{mnp} = -2i \eta_{-}^{\dagger} \gamma_{mnp} \eta_{+}$

$$dJ = \frac{3}{2} \text{Im}(\overline{\mathcal{W}}_1 \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \quad d\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

► Five classes of (intrinsic) torsion are given as

components	interpretations	$SU(3)$ -representations
$\mathcal{W}_1$	$J \wedge d\Omega$ or $\Omega \wedge dJ$	$\mathbf{1} \oplus \mathbf{1}$
$\mathcal{W}_2$	$(d\Omega)_0^{2,2}$	$\mathbf{8} \oplus \mathbf{8}$
$\mathcal{W}_3$	$(dJ)_0^{2,1} + (dJ)_0^{1,2}$	$\mathbf{6} \oplus \overline{\mathbf{6}}$
$\mathcal{W}_4$	$J \wedge dJ$	$\mathbf{3} \oplus \overline{\mathbf{3}}$
$\mathcal{W}_5$	$(d\Omega)^{3,1}$	$\mathbf{3} \oplus \overline{\mathbf{3}}$

► Vanishing torsion classes in  $SU(3)$ -structure manifolds:

complex	hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = 0$
	balanced	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = 0$
	special hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	Kähler	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	Calabi-Yau	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	conformally Calabi-Yau	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 + 2\mathcal{W}_5 = 0$
almost complex	symplectic	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	nearly Kähler	$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	almost Kähler	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	quasi Kähler	$\mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	semi Kähler	$\mathcal{W}_4 = \mathcal{W}_5 = 0$
	half-flat	$\text{Im}\mathcal{W}_1 = \text{Im}\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$

Appendix: generalized geometry

Introduce a generalized almost complex structure  $\mathcal{J}$  on  $T\mathcal{M}_d \oplus T^*\mathcal{M}_d$  s.t.

$$\mathcal{J} : T\mathcal{M}_d \oplus T^*\mathcal{M}_d \longrightarrow T\mathcal{M}_d \oplus T^*\mathcal{M}_d$$

$$\mathcal{J}^2 = -\mathbf{1}_{2d}$$

$$\exists O(d, d) \text{ invariant metric } L, \text{ s.t. } \mathcal{J}^T L \mathcal{J} = L$$

Structure group on  $T\mathcal{M}_d \oplus T^*\mathcal{M}_d$ :

$\exists L$	$GL(2d)$	$\dashrightarrow$	$O(d, d)$
$\mathcal{J}^2 = -\mathbf{1}_{2d}$	$O(d, d)$	$\dashrightarrow$	$U(d/2, d/2)$
$\mathcal{J}_1, \mathcal{J}_2$	$U_1(d/2, d/2) \cap U_2(d/2, d/2)$	$\dashrightarrow$	$U(d/2) \times U(d/2)$
integrable $\mathcal{J}_{1,2}$	$U(d/2) \times U(d/2)$	$\dashrightarrow$	$SU(d/2) \times SU(d/2)$

► Integrability is discussed by “(0, 1)” part of the complexified  $T\mathcal{M}_d \oplus T^*\mathcal{M}_d$ :

$$\Pi \equiv \frac{1}{2}(\mathbf{1}_{2d} - i\mathcal{J})$$

$$\Pi A = A \quad \text{where } A = v + \zeta \text{ is a section of } T\mathcal{M}_d \oplus T^*\mathcal{M}_d$$

We call this  $A$  **i-eigenbundle**  $L_{\mathcal{J}}$ , whose dimension is  $\dim L_{\mathcal{J}} = d$ .

Integrability condition of  $\mathcal{J}$  is

$$\bar{\Pi}[\Pi(v + \zeta), \Pi(w + \eta)]_{\mathbb{C}} = 0 \quad v, w \in T\mathcal{M}_d \quad \zeta, \eta \in T^*\mathcal{M}_d$$

$$[v + \zeta, w + \eta]_{\mathbb{C}} = [v, w] + \mathcal{L}_v\eta - \mathcal{L}_w\zeta - \frac{1}{2}d(\iota_v\eta - \iota_w\zeta) : \text{ Courant bracket}$$

- ▶ Two typical examples of generalized almost complex structures:

$$\mathcal{J}_- = \begin{pmatrix} I & \mathbf{0} \\ \mathbf{0} & -I^T \end{pmatrix} \quad \text{w/ } I^2 = -\mathbf{1}_d: \text{ almost complex structure}$$

$$\mathcal{J}_+ = \begin{pmatrix} \mathbf{0} & -J^{-1} \\ J & \mathbf{0} \end{pmatrix} \quad \text{w/ } J: \text{ almost symplectic form}$$

$$\text{integrable } \mathcal{J}_- \quad \leftrightarrow \quad \text{integrable } I$$

$$\text{integrable } \mathcal{J}_+ \quad \leftrightarrow \quad \text{integrable } J$$

On a usual geometry,  $J_{mn} = I_m^p g_{pn}$  is given by an  $SU(3)$  invariant (pure) spinor  $\eta_+$  as

$$J_{mn} = -2i \eta_+^\dagger \gamma_{mn} \eta_+ \quad \gamma^i \eta_+ = 0 \quad \gamma^{\bar{i}} \eta_+ \neq 0$$

In a similar analogy, we want to find  $\text{Cliff}(6, 6)$  pure spinor(s)  $\Phi$ .

∴ Compared to almost complex structures, (pure) spinors can be easily utilized in supergravity framework.



On  $T\mathcal{M}_6 \oplus T^*\mathcal{M}_6$ , we can define Cliff(6, 6) algebra and  $Spin(6, 6)$  spinor  $\Phi$ :

$$\{\Gamma^m, \Gamma^n\} = 0 \quad \{\Gamma^m, \tilde{\Gamma}_n\} = \delta_n^m \quad \{\tilde{\Gamma}_m, \tilde{\Gamma}_n\} = 0$$

Irreducible repr. of  $Spin(6, 6)$  spinor is a Majorana-**Weyl**

→ a generic  $Spin(6, 6)$  spinor bundle  $S$  splits to  $S^\pm$  (Weyl)

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Weyl spinor bundles  $S^\pm$  are isomorphic to bundles of forms on  $T^*\mathcal{M}_6$ :

$$S^+ \text{ on } T\mathcal{M}_6 \oplus T^*\mathcal{M}_6 \sim \wedge^{\text{even}} T^*\mathcal{M}_6$$

$$S^- \text{ on } T\mathcal{M}_6 \oplus T^*\mathcal{M}_6 \sim \wedge^{\text{odd}} T^*\mathcal{M}_6$$

Thus we often regard a Cliff(6, 6) spinor as a form on  $\wedge^{\text{even/odd}} T^*\mathcal{M}_6$

A form-valued representation of the algebra

$$\Gamma^m = dx^m \wedge, \quad \tilde{\Gamma}_n = \iota_{\partial_n}$$

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IF  $\Phi$  is annihilated by half numbers of the Cliff(6, 6) generators:

→  $\Phi$  is called a **pure spinor**

cf.)  $SU(3)$  invariant spinor  $\eta_+$  is a Cliff(6) pure spinor:  $\gamma^i \eta_+ = 0$

An equivalent definition of a  $\text{Cliff}(6, 6)$  pure spinor is given by “Clifford action”:

$$(v + \zeta) \cdot \Phi = v^m \iota_{\partial_m} \Phi + \zeta_n dx^n \wedge \Phi \quad \text{w/ } v: \text{ vector} \quad \zeta: \text{ one-form}$$

Define the annihilator of a spinor as

$$L_\Phi \equiv \{v + \zeta \in T\mathcal{M}_6 \oplus T^*\mathcal{M}_6 \mid (v + \zeta) \cdot \Phi = 0\}$$

$$\dim L_\Phi \leq d$$

If  $\dim L_\Phi = 6$  (maximally isotropic)  $\rightarrow \Phi$  is a **pure spinor**

Correspondence between pure spinors and generalized almost complex structures:

$$\mathcal{J} \leftrightarrow \Phi \quad \text{if } L_{\mathcal{J}} = L_{\Phi} \quad \text{with } \dim L_{\Phi} = 6$$

More precisely:  $\mathcal{J} \leftrightarrow$  a line bundle of pure spinor  $\Phi$

$\therefore$ ) rescaling  $\Phi$  does not change its annihilator  $L_{\Phi}$

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Then, we can rewrite the generalized almost complex structure as

$$\mathcal{J}_{\pm\Pi\Sigma} = \langle \text{Re}\Phi_{\pm}, \Gamma_{\Pi\Sigma} \text{Re}\Phi_{\pm} \rangle$$

w/ Mukai pairing:

$$\text{even forms: } \langle \Psi_+, \Phi_+ \rangle = \Psi_6 \wedge \Phi_0 - \Psi_4 \wedge \Phi_2 + \Psi_2 \wedge \Phi_4 - \Psi_0 \wedge \Phi_6$$

$$\text{odd forms: } \langle \Psi_-, \Phi_- \rangle = \Psi_5 \wedge \Phi_1 - \Psi_3 \wedge \Phi_3 + \Psi_1 \wedge \Phi_5$$

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$$\text{odd forms: } \langle \Psi_-, \Phi_- \rangle = \Psi_5 \wedge \Phi_1 - \Psi_3 \wedge \Phi_3 + \Psi_1 \wedge \Phi_5$$

$$\mathcal{J} \text{ is integrable} \iff \exists \text{ vector } v \text{ and one-form } \zeta \text{ s.t. } d\Phi = (v_{\perp} + \zeta \wedge)\Phi$$

$$\text{generalized CY} \iff \exists \Phi \text{ is pure s.t. } d\Phi = 0$$

$$\text{"twisted" GCY} \iff \exists \Phi \text{ is pure, and } H \text{ is closed s.t. } (d - H \wedge)\Phi = 0$$

A  $\text{Cliff}(6, 6)$  spinor can also be mapped to a bispinor:

$$C \equiv \sum_k \frac{1}{k!} C_{m_1 \dots m_k}^{(k)} dx^{m_1} \wedge \dots \wedge dx^{m_k} \quad \longleftrightarrow \quad \mathcal{C} \equiv \sum_k \frac{1}{k!} C_{m_1 \dots m_k}^{(k)} \gamma_{\alpha\beta}^{m_1 \dots m_k}$$



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On a geometry of a **single**  $SU(3)$ -structure, the following two  $SU(3, 3)$  spinors:

$$\begin{aligned} \Phi_{0+} &= \eta_+ \otimes \eta_+^\dagger = \frac{1}{4} \sum_{k=0}^6 \frac{1}{k!} \eta_+^\dagger \gamma_{m_k \dots m_1} \eta_+ \gamma^{m_1 \dots m_k} = \frac{1}{8} e^{-iJ} \\ \Phi_{0-} &= \eta_+ \otimes \eta_-^\dagger = \frac{1}{4} \sum_{k=0}^6 \frac{1}{k!} \eta_-^\dagger \gamma_{m_k \dots m_1} \eta_+ \gamma^{m_1 \dots m_k} = -\frac{i}{8} \Omega \end{aligned}$$

Check purity:  $(\delta + iJ)_m{}^n \gamma_n \eta_+ \otimes \eta_\pm^\dagger = 0 = \eta_+ \otimes \eta_\pm^\dagger \gamma_n (\delta \mp iJ)^n{}_m$

One-to-one correspondence:  $\Phi_{0-} \leftrightarrow \mathcal{J}_1, \quad \Phi_{0+} \leftrightarrow \mathcal{J}_2$

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One-to-one correspondence:  $\Phi_{0-} \leftrightarrow \mathcal{J}_1$ ,  $\Phi_{0+} \leftrightarrow \mathcal{J}_2$

On a generic geometry of a **pair** of  $SU(3)$ -structure defined by  $(\eta_+^1, \eta_+^2)$

$$\begin{aligned} \Phi_{0+} &= \eta_+^1 \otimes \eta_+^{2\dagger} = \frac{1}{8} (\bar{c}_\parallel e^{-ij} - i\bar{c}_\perp w) \wedge e^{-iv \wedge v'} \\ \Phi_{0-} &= \eta_+^1 \otimes \eta_-^{2\dagger} = -\frac{1}{8} (c_\perp e^{-ij} + ic_\parallel w) \wedge (v + iv') \\ &\quad \Phi_\pm = e^{-B} \Phi_{0\pm} \end{aligned} \quad |c_\parallel|^2 + |c_\perp|^2 = 1$$

Each  $\Phi_{\pm}$  defines an  $SU(3, 3)$  structure on  $E$ . Common structure is  $SU(3) \times SU(3)$ .

( $F$  is extended to  $E$  by including  $e^{-B}$ )

Compatibility requires

$$\begin{aligned}\langle \Phi_+, V \cdot \Phi_- \rangle &= \langle \bar{\Phi}_+, V \cdot \Phi_- \rangle = 0 \quad \text{for } \forall V = x + \xi \\ \langle \Phi_+, \bar{\Phi}_+ \rangle &= \langle \Phi_-, \bar{\Phi}_- \rangle\end{aligned}$$

Start with a real form  $\chi_f \in \wedge^{\text{even/odd}} F^*$  (associated with a real  $Spin(6,6)$  spinor  $\chi_s$ )

Regard  $\chi_f$  as a stable form satisfying

$$q(\chi_f) = -\frac{1}{4} \langle \chi_f, \Gamma_{\Pi\Sigma} \chi_f \rangle \langle \chi_f, \Gamma^{\Pi\Sigma} \chi_f \rangle \in \wedge^6 F^* \otimes \wedge^6 F^*$$

$$U = \{ \chi_f \in \wedge^{\text{even/odd}} F^* : q(\chi_f) < 0 \}$$

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$$H(\chi_f) \equiv \sqrt{-\frac{1}{3}q(\chi_f)} \in \wedge^6 F^*$$

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Then we can get another real form  $\hat{\chi}_f$  and a complex form  $\Phi_f$  by Mukai pairing

$$\begin{aligned} \langle \hat{\chi}_f, \chi_f \rangle &= -dH(\chi_f) \quad \text{i.e.,} \quad \hat{\chi}_f = -\frac{\partial H(\chi_f)}{\partial \chi_f} \\ \dashrightarrow \quad \Phi_f &\equiv \frac{1}{2}(\chi_f + i\hat{\chi}_f) \quad H(\Phi_f) = i\langle \Phi_f, \bar{\Phi}_f \rangle \end{aligned}$$

Hitchin showed:  $\Phi_f$  is a (form corresponding to) **pure spinor!**

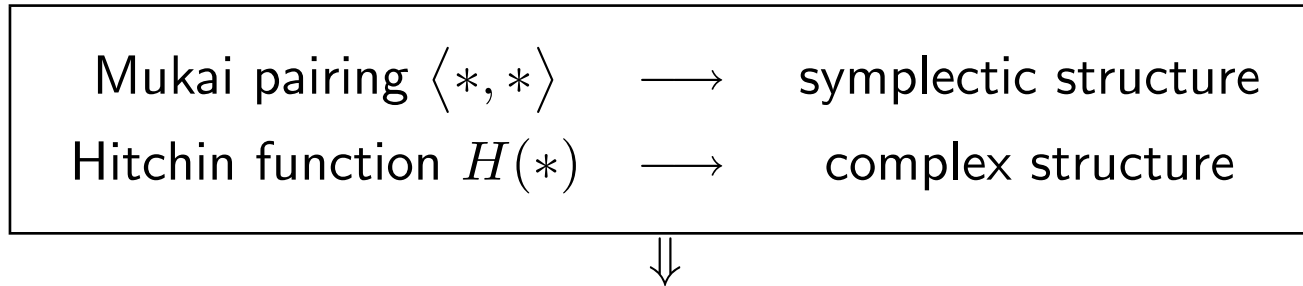
Consider the space of pure spinors  $\Phi$  ...

Mukai pairing $\langle *, * \rangle$	$\longrightarrow$	symplectic structure
Hitchin function $H(*)$	$\longrightarrow$	complex structure



The space of pure spinor is Kähler

Consider the space of pure spinors  $\Phi$  ...



The space of pure spinor is Kähler

Quotienting this space by the  $\mathbb{C}^*$  action  $\Phi \rightarrow \lambda\Phi$  for  $\lambda \in \mathbb{C}^*$

--> The space becomes a **local** special Kähler geometry with Kähler potential  $K$ :

$$e^{-K} = H(\Phi) = i\langle \Phi, \bar{\Phi} \rangle = i(\bar{X}^A \mathcal{F}_A - X^A \bar{\mathcal{F}}_A) \in \wedge^6 F^*$$

$X^A$  : holomorphic projective coordinates

$\mathcal{F}_A$  : derivative of prepotential  $\mathcal{F}$ , i.e.,  $\mathcal{F}_A = \partial\mathcal{F}/\partial X^A$

These are nothing but objects which we want to introduce in  $\mathcal{N} = 2$  supergravity!



Spaces of pure spinors  $\Phi_{\pm}$  on  $F \oplus F^*$  with  $SU(3) \times SU(3)$  structures

||

special Kähler geometries of local type = Hodge-Kähler geometries

For the single  $SU(3)$ -structure case:

$$\begin{aligned}\Phi_+ &= \frac{1}{8} e^{-B-iJ} & K_+ &= -\log \left( \frac{1}{48} J \wedge J \wedge J \right) \\ \Phi_- &= -\frac{i}{8} e^{-B} \Omega & K_- &= -\log \left( \frac{i}{64} \Omega \wedge \bar{\Omega} \right)\end{aligned}$$

Structure of forms is exactly same as the one in the case of Calabi-Yau compactification!

We should truncate Kaluza-Klein massive modes from these forms to obtain 4D supergravity.

Appendix: generalization of the differential operator

M. Graña, J. Louis, D. Waldram [hep-th/0612237](https://arxiv.org/abs/hep-th/0612237)

Recall that  $\Phi_{\pm}$  are expanded in terms of truncation bases  $\Sigma_{+}$  and  $\Sigma_{-}$ .

Whenever  $c_{\parallel} \neq 0$ , the structure  $\Phi_{+}$  contains a scalar. This implies that at least one of the forms in the basis  $\Sigma_{+}$  contains a *scalar*. Let us call this element  $\Sigma_{+}^1$ , and take the simple case where the only non-zero elements of  $\mathbb{Q}$  are those of the form  $\mathbb{Q}_{\hat{I}}^1$  (where  $\hat{I} = 1, \dots, 2b^{-} + 2$ ).

Thus  $d(\Sigma_{-})_{\hat{I}} = \mathbb{Q}_{\hat{I}}^1 \Sigma_{+}^1$  and so if  $\mathbb{Q}_{\hat{I}}^1 \neq 0$  then  $(d\Sigma_{-})_{\hat{I}}$  contains a *scalar*.

But this is *not possible* if  $d$  is an honest exterior derivative, acting as  $d : \Lambda^p \rightarrow \Lambda^{p+1}$ .

The same is true if  $c_{\parallel}$  is zero. In this case, there may be no scalars in any of the even forms  $\Sigma_{-}$ , and for an “honest”  $d$  operator, there should be then *no one-forms* in  $d\Sigma_{-}$ . But we again see from that  $\Phi_{-}$  contains a *one-form*, and as a consequence so do some of the elements in  $\Sigma_{-}$ .

One way to generate a completely general charge matrix  $\mathbb{Q}$  in this picture is to consider a modified operator  $d$  which is now a generic map  $d : U^+ \rightarrow U^-$  which satisfies  $d^2 = 0$  but does not transform the degree of a form properly.

In particular, the operator  $d$  can map a  $p$ -form to a  $(p - 1)$ -form.  
 Of course, this  $d$  does *not* act this way in *conventional* geometrical compactifications.

One is thus led to conjecture that to obtain a generic  $\mathbb{Q}$  we must consider non-geometrical compactifications. One can still use the structures

$$d\Sigma_- \sim \mathbb{Q}\Sigma_+, \quad d\Sigma_+ \sim \mathcal{S}_+ \mathbb{Q}^T (\mathcal{S}_-)^{-1} \Sigma_-$$

to derive sensible effective actions, expanding in bases  $\Sigma_+$  and  $\Sigma_-$  with a generalised  $d$  operator, but there is of course now *no interpretation* in terms of differential forms and the exterior derivative.

--> introduce generalized fluxes  
 (not only geometrical  $H$ - and  $f$ -fluxes, but also  $Q$ - and  $R$ -fluxes)

For a geometrical background it is natural to consider forms of the type

$$\omega = e^{-B} \omega_{m_1 \dots m_p} e^{m_1} \wedge \dots \wedge e^{m_p} \quad \text{w/ } \omega_{m_1 \dots m_p} \text{ constant}$$

Action of  $d$  on  $\omega$  is

$$d\omega = -H^{\text{fl}} \wedge \omega + f \cdot \omega, \quad (f \cdot \omega)_{m_1 \dots m_{p+1}} = f^a_{[m_1 m_2} \omega_a{}_{m_3 \dots m_{p+1}]}$$

The natural **nongeometric extension** is then to an operator  $\mathcal{D}$  such that

$$\mathcal{D} := d - H^{\text{fl}} \wedge -f \cdot -Q \cdot -R_{\perp}$$

$$(Q \cdot \omega)_{m_1 \dots m_{p-1}} = Q^{ab}{}_{[m_1} \omega_{|ab|}{}_{m_2 \dots m_{p-1}]}, \quad (R_{\perp} \omega)_{m_1 \dots m_{p-3}} = R^{abc} \omega_{abcm_1 \dots m_{p-3}}$$

Requiring  $\mathcal{D}^2 = 0$  implies that same conditions on fluxes as arose from the Jacobi identities for the extended Lie algebra

$$\begin{aligned} [Z_a, Z_b] &= f_{ab}{}^c Z_c + H_{abc} X^c \\ [X^a, X^b] &= Q^{ab}{}^c X^c + R^{abc} Z_c \\ [X^a, Z_b] &= f^a{}_{bc} X^c - Q^{ac}{}_b Z_c \end{aligned}$$

We can see nongeometric information in  $\mathbb{Q}$  as contribution from  $Q$  and  $R$ .

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and more...