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D-branes and Doubled Geometry

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Hull's doubled space formalism (as a sigma model)

- ✓ has a doubled target space: $\mathcal{M}_d \to \mathcal{M}_{2d} \sim \mathcal{M}_d \times \widetilde{\mathcal{M}}_d$.
- ✓ can contain non-trivial B-field.
- ✓ has manifest T-duality symmetry in the language of $O(d,d;\mathbb{Z})$ linear transformation.
- ✓ traces non-geometric string backgrounds.

Our purpose is to establish the consistent introduction of D-branes on the doubled space.

$$S = \frac{1}{4}\int_\Sigma \mathcal{M}_{MN}\,\mathcal{P}^M \wedge *\mathcal{P}^N + \frac{1}{12}\int_V t_{MNP}\,\mathcal{P}^M \wedge \mathcal{P}^N \wedge \mathcal{P}^P - \int_D \omega$$

- Scalar fields of doubled coordinates and doubled vielbeins:

$$\mathbb{Y}^I = \begin{pmatrix} Y^i \\ \widetilde{Y}_i \end{pmatrix} \; ; \quad \mathcal{P} = \mathfrak{g}^{-1}\mathrm{d}\mathfrak{g} = \mathcal{P}^M{}_I\,(r\,T_M)\,\mathrm{d}\mathbb{Y}^I \;, \quad \text{w/ } \mathfrak{g} \in G \subset O(d,d)$$

$$\mathrm{d}\mathcal{P}^M = -\frac{r}{2} t_{NP}{}^M\,\mathcal{P}^N \wedge \mathcal{P}^P$$

- Self-duality constraint (to go back to conventional system):

$$\mathcal{P}^M = L^{MN}\mathcal{M}_{NP} * \mathcal{P}^P \qquad \text{w/ } O(d,d) \text{ invariant metric } L$$

Global symmetry of the sigma model:

$$\rho \in O(d,d); \quad \begin{cases} \mathbb{Y}^I & \to \quad \mathbb{Y}'^I = \rho^I{}_J \, \mathbb{Y}^J \\ \mathcal{P}^M{}_I(\mathbb{Y}) & \to \quad \mathcal{P}'^M{}_I(\mathbb{Y}') = \rho^P{}_Q \, \mathcal{P}^Q{}_J(\mathbb{Y}') \, \rho^J{}_I \end{cases}$$

$O(d,d;\mathbb{Z})$ sub-group: T-duality transformation (ex. $d=3$ case):

$$\rho_i = \begin{pmatrix} \mathbb{1}_3 - \mathrm{T}_i & \mathrm{T}_i \\ \mathrm{T}_i & \mathbb{1}_3 - \mathrm{T}_i \end{pmatrix} \in O(3,3;\mathbb{Z})$$

$$\mathrm{T}_1 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}, \quad \mathrm{T}_2 = \begin{pmatrix} 0 & & \\ & 1 & \\ & & 0 \end{pmatrix}, \quad \mathrm{T}_3 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}$$

This action exchanges physical coordinates Y^i with dual coordinates $\widetilde{Y}_i$

- Bulk

$$0 = \mathrm{d} * \mathcal{M}_{MN}\mathcal{P}^N + \mathcal{M}_{NP}\, t_{MQ}{}^P\, \mathcal{P}^Q \wedge *\mathcal{P}^N - \frac{1}{2} t_{MNP}\, \mathcal{P}^N \wedge \mathcal{P}^P$$

- Boundary

$$0 = \widetilde{\Xi}^I{}_K \delta\mathbb{Y}^K = \widetilde{\Xi}^N{}_M\, \mathcal{P}^M{}_I\, \partial_\tau \mathbb{Y}^I \qquad \text{Dirichlet}$$

$$0 = \Xi^I{}_K \Big(- \frac{1}{2} \mathcal{P}_I{}^M\, \mathcal{M}_{MN}\, \mathcal{P}^N{}_J \partial_\sigma \mathbb{Y}^J + \omega_{IJ} \partial_\tau \mathbb{Y}^J \Big) \qquad \text{Neumann}$$

Introduce the Neumann and the Dirichlet Projectors

$$\widetilde{\Xi} + \Xi = \mathbf{1}\,, \qquad \widetilde{\Xi}^I{}_J\, \Xi^J{}_K = 0\,, \qquad \widetilde{\Xi}^I{}_J\, \widetilde{\Xi}^J{}_K = \Xi^I{}_J\, \Xi^J{}_K = \delta^I{}_K$$

Null condition: $\Xi_K{}^I\, L_{IJ}\, \Xi^J{}_L \;=\; 0 \;=\; \widetilde{\Xi}_K{}^I\, L_{IJ}\, \widetilde{\Xi}^J{}_L$

Structure constant: $\Xi^{I'}{}_{[I}\Xi^{J'}{}_J\Xi^{K'}{}_{K]}\, t_{I'J'K'} \;=\; 0$

Orthogonality: $\Xi_K{}^I\, \mathcal{M}_{IJ}\, \widetilde{\Xi}^J{}_L \;=\; 0$

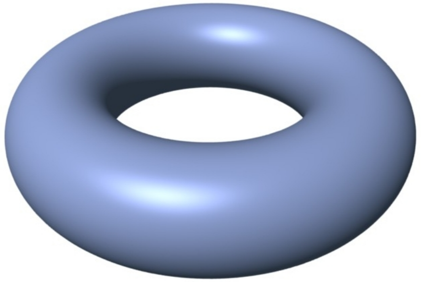
Integrability (D-brane is locally smooth): $\Xi^{I'}{}_I\Xi^{J'}{}_J\partial_{[I'}\Xi^{K'}{}_{J']} \;=\; 0$

A generic solution of $\widetilde{\Xi}$ is

$$\widetilde{\Xi} \;=\; \begin{pmatrix} a & b \\ c & \mathbf{1} - a^T \end{pmatrix} \qquad \text{with} \quad \begin{cases} b^T \;=\; -b\,, & c^T \;=\; -c \\ ab + (ab)^T \;=\; 0\,, & ca + (ca)^T \;=\; 0\,, \\ bc \;=\; a(\mathbf{1} - a) & \end{cases}$$

Apply the Neumann/Dirichlet projectors to a sigma model on a flat 3-torus with H-flux:

Four consistent configurations of $\widetilde{\Xi}^I{}_J$:

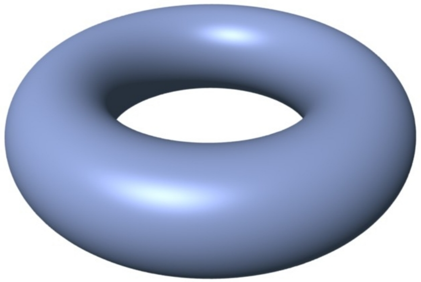


$$H = m\, \mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z$$

$$k = m/3$$

$\widetilde{\Xi}_1$	$a = \mathbf{1},\ b = \mathbf{0},\ c = B$
$\widetilde{\Xi}_2$	$b = \mathbf{0},\ a_{11} = 1,\ c_{23} = -kx$
$\widetilde{\Xi}_3(a_{33})$	$a_{22} = 1 - a_{33},\ a_{23}^2 = a_{33}(1 - a_{33})$ $c_{12} = kz(1 - a_{33}) - kya_{23},\ c_{13} = kza_{23} - kya_{33}$
$\widetilde{\Xi}_4(a_{33})$	$a_{23} = a_{33},\ a_{21} = -kyb_{23},\ a_{13} = -kzb_{23}$ $c_{12} = kza_{33},\ c_{13} = -kya_{33},\ c_{23} = a_{33}(a_{33} - 1)/b_{23}$ $4a_{33}(a_{33} - 1) \leq (kx)^2$ $b_{23} = \frac{kx(2a_{33}-1)\pm\sqrt{(kx)^2-4a_{33}(a_{33}-1)}}{2(1+(kx)^2)} \neq 0$

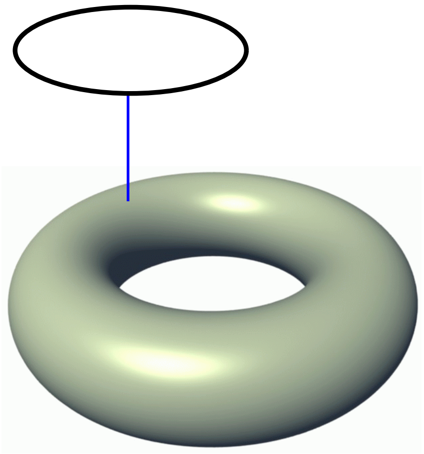
Apply the Neumann/Dirichlet projectors to a sigma model on a flat 3-torus with H-flux:



$$H = m\,\mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z$$

Dirichlet projector	type of brane	x	y	z	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$
$\widetilde{\Xi}_1$	D0	-	-	-	$\odot$	$\odot$	$\odot$
$\widetilde{\Xi}_2,\ \widetilde{\Xi}_4(a_{33}=1)$	D2	-	/	\	$\odot$	\	/
$\widetilde{\Xi}_3(a_{33}=0)$	D1	-	-	$\odot$	$\odot$	$\odot$	-
$\widetilde{\Xi}_3(a_{33}=1)$	D1	-	$\odot$	-	$\odot$	-	$\odot$
$\widetilde{\Xi}_4(a_{33}=0)$	D2	-	$\odot$	$\odot$	$\odot$	-	-

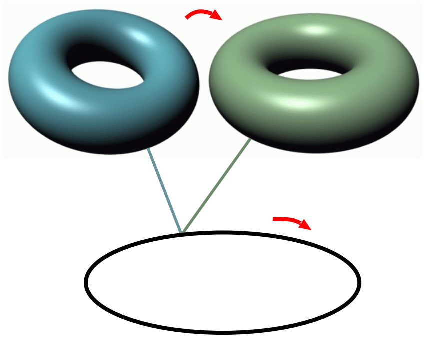
T-duality transform along z-direction:



nilmanifold

Dirichlet projector	type of brane	x	y	z	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$
$\widetilde{\Xi}_1$	D1	-	-	$\odot$	$\odot$	$\odot$	-
$\widetilde{\Xi}_2$, $\widetilde{\Xi}_4(a_{33} = 1)$	D1	-	$/$	$/$	$\odot$	$\backslash$	$\backslash$
$\widetilde{\Xi}_3(a_{33} = 0)$	D0	-	-	-	$\odot$	$\odot$	$\odot$
$\widetilde{\Xi}_3(a_{33} = 1)$	D2	-	$\odot$	$\odot$	$\odot$	-	-
$\widetilde{\Xi}_4(a_{33} = 0)$	D1	-	$\odot$	-	$\odot$	-	$\odot$

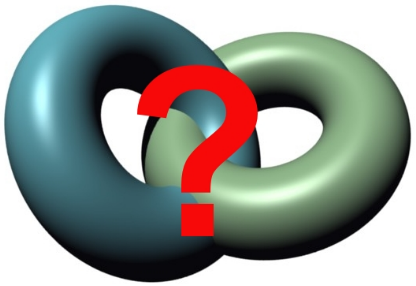
T-duality transform along (y, z)-direction from the flat torus:



T-fold

Dirichlet projector	type of brane	x	y	z	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$
$\widetilde{\Xi}_1$	D2	-	$\odot$	$\odot$	$\odot$	-	-
$\widetilde{\Xi}_2$, $\widetilde{\Xi}_4(a_{33} = 1)$	D2	-	$\backslash$	/	$\odot$	/	$\backslash$
$\widetilde{\Xi}_3(a_{33} = 0)$	D1	-	$\odot$	-	$\odot$	-	$\odot$
$\widetilde{\Xi}_3(a_{33} = 1)$	D1	-	-	$\odot$	$\odot$	$\odot$	-
$\widetilde{\Xi}_4(a_{33} = 0)$	D0	-	-	-	$\odot$	$\odot$	$\odot$

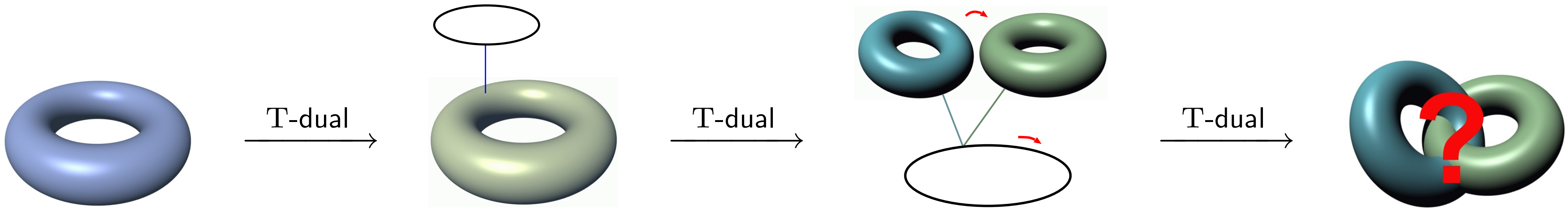
T-duality transform along all (x, y, z)-direction from the flat torus:



R-flux background

Dirichlet projector	type of brane	x	y	z	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$
$\widetilde{\Xi}_1$	D3	$\odot$	$\odot$	$\odot$	-	-	-
$\widetilde{\Xi}_2$, $\widetilde{\Xi}_4(a_{33}=1)$	D3	$\odot$	$\backslash$	$/$	-	$/$	$\backslash$
$\widetilde{\Xi}_3(a_{33}=0)$	D2	$\odot$	$\odot$	-	-	-	$\odot$
$\widetilde{\Xi}_3(a_{33}=1)$	D2	$\odot$	-	$\odot$	-	$\odot$	-
$\widetilde{\Xi}_4(a_{33}=0)$	D1	$\odot$	-	-	-	$\odot$	$\odot$

Summary



h_{mnp}	$f^{m}{}_{np}$	$Q^{mn}{}_{p}$	R^{mnp}
flat-torus	nilmanifold	T-fold	nongeometric
		stringy	stringy

T-duality in the presence of B-field generates geometric/nongeometric backgrounds.

D-brane distributions and T-duality transformations.

- Introduce doubled space $\mathcal{M}_{2d}$ induced by B-field
- Perform T-duality transformations
- Analyze consistent Dirichlet boundary conditions and D-brane distributions

Extend to U-fold endowed with U-duality transformation (hidden symmetry)

? Supersymmetry on doubled geometry ?

? Investigate quantum aspects of the doubled sigma model ?