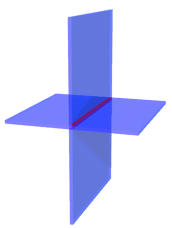
The University of Tokyo, Hongo: October 26, 2009

Yet Another Alternative to Compactification by Heterotic Five-branes

arXiv: 0905.2185 [hep-th]



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Introduction

A typical example: Calabi-Yau compactification in $E_8 \times E_8$ heterotic string theory

Standard Embedding: ${\omega_m}^{ab} \equiv A_m^{ab} \in SU(3) \quad \rightsquigarrow \quad E_8 \supset E_6 \times SU(3)$

$$\# \text{ of generations} \ = \ \frac{1}{2}|\chi(\text{CY})| \ = \ \left| h^{1,1} - h^{2,1} \right|$$

$h^{1,1}$ = # of Kähler moduli = # of $(\overline{\mathbf{27}}, \overline{\mathbf{3}})$ repr. (size of CY)

$h^{2,1}$ = # of complex structure moduli = # of $(\mathbf{27}, \mathbf{3})$ repr. (shape of CY)

M.B. Green, J.H. Schwarz and E. Witten: *Chapter 16.2*

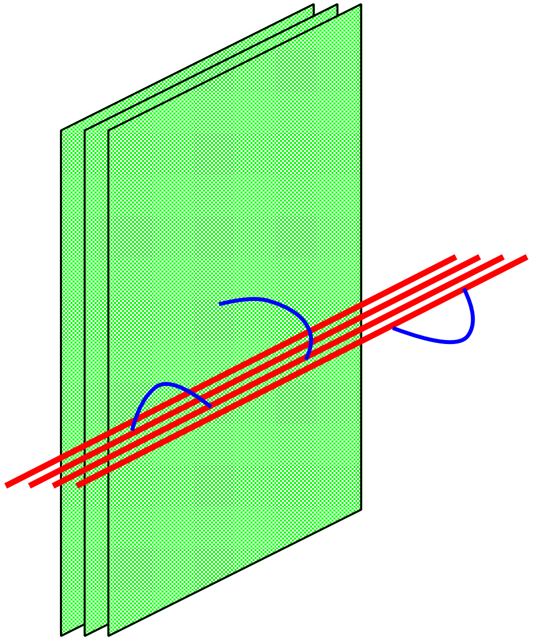
"Problems"

So many Calabi-Yau manifolds

So many massless modes appear in four-dimensional physics

$\rightsquigarrow$ flux compactification to yield potential energy

Another example: D-brane scenario in higher-dimensional theories



ex.)

N_f D7-branes (green planes)

N_c D3-branes (red lines)

$SU(N_c)$ gauge theory with N_f flavors

"Good Points"

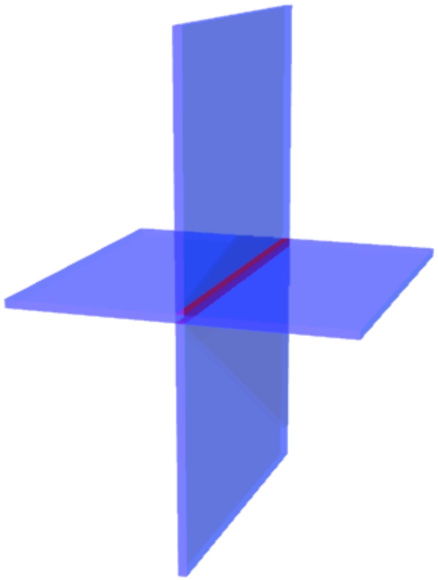
Simple and visible

"Problems"

a bit artificial setup

Yet Another Alternative: Intersecting 5-branes in hetertotic string theory

– Our Model –



	0	1	2	3	4	5	6	7	8	9
5-brane	○			○	○			○	○	○
5-brane'	○					○	○	○	○	○
our world	○							○	○	○

"Good Points"

Simple!

E_6 gauge symmetry appears

Naturally obtain E_6-charged multiplets in four dimensions as "Nambu-Goldstone" modes

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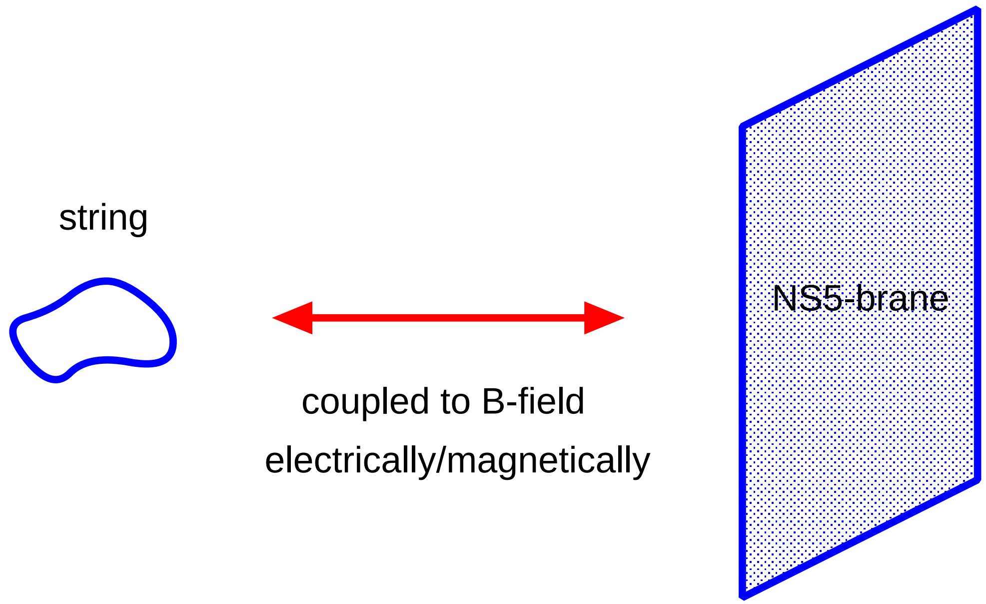
Heterotic String Theory in Ten Dimensions

Objects:



16 Supersymmetry Charges

$E_8 \times E_8$ or $SO(32)$ gauge symmetries

Effective action in the string frame is given as

$$S_{\text{boson}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-g}\, \mathrm{e}^{-2\phi} \Big\{ R + 4(\nabla_M \phi)^2 - \frac{1}{3} H_{MNP}^2 - \frac{\alpha'}{30} \mathrm{Tr} F_{MN}^2 + \ldots \Big\}$$

Supersymmetry transformations:

$$\begin{aligned} \text{gravitino} \qquad \delta\psi_M &= \Big(\partial_M + \frac{1}{4}\omega_{-M}{}^{AB}\,\Gamma_{AB}\Big)\epsilon \\ \text{gaugino} \qquad \delta\chi &= -\frac{1}{4}F_{MN}\Gamma^{MN}\,\epsilon \\ \text{dilatino} \qquad \delta\lambda &= -\frac{1}{4}\Big(\Gamma^M\partial_M\phi - \frac{1}{6}H_{MNP}\Gamma^{MNP}\Big)\epsilon \end{aligned}$$

Bianchi identity (via anomaly cancellation)

$$\mathrm{d}H = \alpha'\Big[\frac{1}{30}\mathrm{Tr}(F\wedge F) - \mathrm{tr}(R(\omega_+)\wedge R(\omega_+))\Big]$$

with $\omega_{\pm M}{}^{AB} := \omega_M{}^{AB} \pm H_M{}^{AB}$

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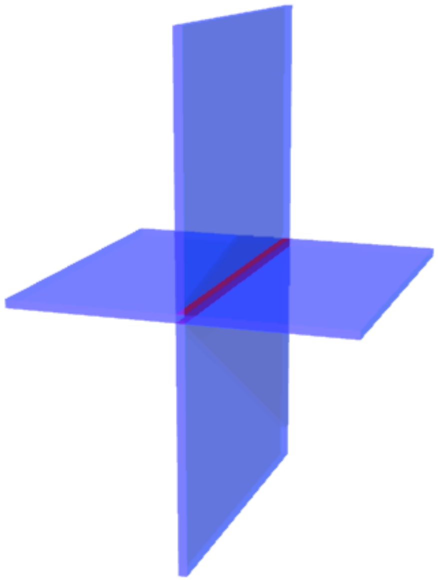
Intersection Rule for p-branes

R. Argurio, F. Englert and L. Houart, *Phys.Lett.B 398 (1997) 61*

$$\bar{q} + 1 \ = \ \frac{(p_A + 1)(p_B + 1)}{D - 2} - \frac{1}{2}(\varepsilon_A a_A)(\varepsilon_B a_B)$$



D : total spacetime dimensions

$\bar{q}$: intersecting dimensions

p_A : spatial dimensions of p_A-brane

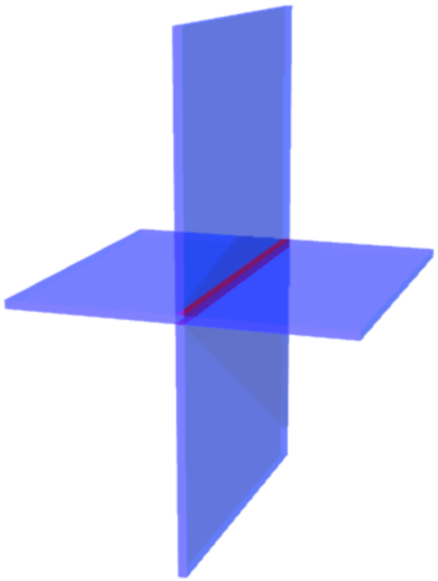
a_A : -1 (NSNS B-field), $\frac{1}{2}(5 - p_A)$ (RR $(p_A + 1)$-form)

ε_A : $+1$ (electric), -1 (magnetic)

Now, we consider two intersecting NS5-branes in heterotic string:

$$\bar{q} + 1 \ = \ \frac{(5+1)(5+1)}{10-2} - \frac{1}{2}(-1)(-1)(-1)(-1) \quad \rightarrow \quad \bar{q} \ = \ 3$$

$\rightsquigarrow$ good dimensions to consider four-dimensional spacetime



	0	1	2	3	4	5	6	7	8	9
5-brane	◯			◯	◯			◯	◯	◯
5-brane'	◯					◯	◯	◯	◯	◯
our world	◯							◯	◯	◯

The string frame metric and the solution are given as

$$ds^2 = \sum_{\mu,\nu=0,7,8,9} \eta_{\mu\nu}\, dx^\mu dx^\nu + h^2 \sum_{m=1}^{2} (dx^m)^2 + h \sum_{m=3}^{6} (dx^m)^2$$

$$h = 1 + N|x^1| = e^\phi\,, \qquad H_{234} = H_{256} = \frac{N}{2h}|x^1|'$$

K. Ohta and T. Yokono, *JHEP 02 (2000) 023* + *our new idea*

Spin connections $(\omega_{\pm})_m{}^a{}_b \equiv (\omega \pm H)_m{}^a{}_b$ are expressed as

$$
(\omega_{\pm})_1 \;=\; 0\,, \qquad
(\omega_{\pm})_2 \;=\; \frac{h'}{2h}\left(\begin{array}{cc|cc|cc}
 & -2 & & & & \\
+2 & & & & & \\ \hline
 & & & \pm 1 & & \\
 & & \mp 1 & & & \\ \hline
 & & & & & \pm 1 \\
 & & & & \mp 1 &
\end{array}\right)
$$

$$
(\omega_{\pm})_3 \;=\; \frac{h'}{2h^{\frac{3}{2}}}\left(\begin{array}{cc|cc|cc}
 & & -1 & & & \\
 & & & \mp 1 & & \\ \hline
+1 & & & & & \\
 & \pm 1 & & & & \\ \hline
 & & & & & \\
 & & & & &
\end{array}\right), \qquad
(\omega_{\pm})_4 \;=\; \frac{h'}{2h^{\frac{3}{2}}}\left(\begin{array}{cc|cc|cc}
 & & & -1 & & \\
 & & \pm 1 & & & \\ \hline
 & \mp 1 & & & & \\
+1 & & & & & \\ \hline
 & & & & & \\
 & & & & &
\end{array}\right)
$$

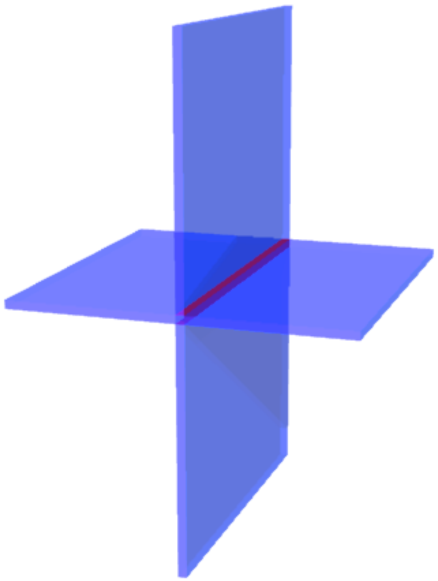
$$
(\omega_{\pm})_5 \;=\; \frac{h'}{2h^{\frac{3}{2}}}\left(\begin{array}{cc|cc|cc}
 & & & & -1 & \\
 & & & & & \mp 1 \\ \hline
 & & & & & \\
 & & & & & \\ \hline
+1 & & & & & \\
 & \pm 1 & & & &
\end{array}\right), \qquad
(\omega_{\pm})_6 \;=\; \frac{h'}{2h^{\frac{3}{2}}}\left(\begin{array}{cc|cc|cc}
 & & & & & -1 \\
 & & & & \pm 1 & \\ \hline
 & & & & & \\
 & & & & & \\ \hline
 & \mp 1 & & & & \\
+1 & & & & &
\end{array}\right)
$$

Each spin connection belongs to $SU(3)$ group

However, ω_+ does not correspond to ω_- by any similarity transformations

Embedding $\omega_{+m} \equiv A_m$, the Bianchi identity is given by

$$
\mathrm{d}H \;=\; \alpha' \left[\frac{1}{30} \mathrm{Tr}(F \wedge F) - \mathrm{tr}(R(\omega_+) \wedge R(\omega_+)) \right] \;=\; 0
$$



4 SUSY preserved (12 broken)

ω_{+m}: an $SU(3)$ holonomy connection in 6 directions

ω_{-m}: another $SU(3)$ holonomy connection in 6 directions

We can embed ω_{+m} into A_m $\rightsquigarrow$ $SU(3)_{\text{spin}} \equiv SU(3)_{\text{gauge}} =: SU(3)_{\text{frozen}}$

$\downarrow$

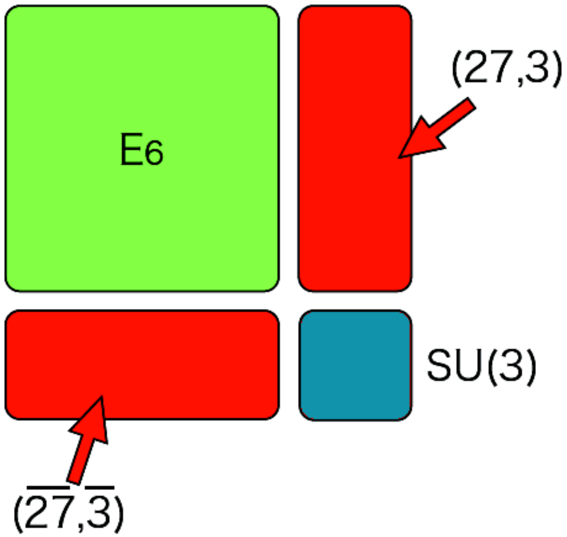
E_8 gauge symmetry is broken to $E_6 \times SU(3)_{\text{frozen}}$

$\Downarrow$

4-dim'l $\mathcal{N} = 1$ with E_6-gauge symmetry on the intersecting spacetime

Counting E_6-charged chiral multiplets in 4-dim'l theory

$$E_8 \supset E_6 \times SU(3) \qquad \mathbf{248} = (\mathbf{78},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8}) \oplus (\mathbf{27},\mathbf{3}) \oplus (\overline{\mathbf{27}},\overline{\mathbf{3}})$$



Focus only on E_6 fundamental: $(27 \times 3) + (\overline{27} \times \overline{3})$

$(\mathbf{27},\mathbf{3})$ and $(\overline{\mathbf{27}},\overline{\mathbf{3}})$ are complex conjugate

$\{(27 \times 3) + (\overline{27} \times \overline{3})\}/2 = 27 \times 3$ complex bosons

$=$ 3 complex bosons of E_6 fundamental repr.

cf) C.W. Bernard, N.H. Christ, A.H. Guth and E.J. Weinberg, *Phys. Rev. D16 (1977) 2967*

$\mathcal{N} = 1$ SUSY implies the existence of 3 Weyl fermions

The three bosons and fermions are combined into

3 complex chiral multiplets of E_6 fundamental repr.!

3 complex chiral multiplets $=$ 3 generations??

Remark This intersecting five-brane configuration is T-dual to deformed conifold

(non-compact CY with $h^{2,1} = 1$, $h^{1,1} = 0$)

$\Downarrow$

# of generations is just one!? (from "ordinary" viewpoint)

The Answer: one generation

See the Dirac equation of gaugino in 10D

$$0 \;=\; \not{D}(\omega - \tfrac{1}{3}H, A)\chi - \Gamma^M \chi \cdot \partial_M \Phi + \frac{1}{4}\Gamma^M \Gamma^{AB} F_{AB}\Big(\psi_M + \frac{2}{3}\Gamma_M \lambda\Big)$$

with background $\psi_M = 0 = \lambda$ and $\chi' = \mathrm{e}^{-\Phi}\chi$:

$$\downarrow$$

$$0 \;=\; \not{D}(\omega - \tfrac{1}{3}H, A)\chi'$$

$$\downarrow$$

$$0 \;=\; \Gamma^\mu \partial_\mu \chi' + \Gamma^m D_m(\omega - \tfrac{1}{3}H, A)\chi'$$

Focus only on the fermionic modes χ' belonging to $(\mathbf{27}, \mathbf{3})$ or $(\overline{\mathbf{27}}, \overline{\mathbf{3}})$.

Factorize $\chi' = \chi_{\text{4D}} \otimes \psi_{\text{6D}}$ and evaluate the second term as the “mass term” of χ_{4D}.

Assume that χ' depends only on x^1 as a smeared configuration (cf. the metric)

$$0 \;=\; \Gamma^a e_a{}^m \frac{\partial}{\partial x^m} \chi'(x^1) + \Gamma^a e_a{}^i \Big(\frac{1}{4} (\omega - \tfrac{1}{3} H)_m{}^{ab} \Gamma_{ab} + A_m \Big) \chi'$$

with

$$e_a{}^m = \begin{pmatrix} h^{-1} & & & & & \\ & h^{-1} & & & & \\ & & h^{-\frac{1}{2}} & & & \\ & & & h^{-\frac{1}{2}} & & \\ & & & & h^{-\frac{1}{2}} & \\ & & & & & h^{-\frac{1}{2}} \end{pmatrix}, \qquad h = 1 + N|x^1|$$

where

$$\Gamma^1 = \widetilde{\gamma}_{\#} \otimes \gamma_1 \,, \qquad \Gamma^2 = \widetilde{\gamma}_{\#} \otimes \gamma_2 \,, \qquad \Gamma^3 = \widetilde{\gamma}_{\#} \otimes \gamma_3 \,,$$

$$\Gamma^4 = \widetilde{\gamma}_{\#} \otimes \gamma_4 \,, \qquad \Gamma^5 = \widetilde{\gamma}_{\#} \otimes \gamma_5 \,, \qquad \Gamma^6 = \widetilde{\gamma}_{\#} \otimes \gamma_6$$

$\widetilde{\gamma}_{\#}$: chirality operator in 4D, γ_a: gamma matrix in 6D

$$\gamma_1 = \sigma_2 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \quad \gamma_2 = \sigma_1 \otimes \sigma_1 \otimes \mathbf{1}_2 \quad \gamma_3 = \sigma_1 \otimes \sigma_2 \otimes \mathbf{1}_2$$

$$\gamma_4 = \sigma_1 \otimes \sigma_3 \otimes \sigma_1 \quad \gamma_5 = \sigma_1 \otimes \sigma_3 \otimes \sigma_2 \quad \gamma_6 = \sigma_1 \otimes \sigma_3 \otimes \sigma_3$$

Spin connections $(\omega - \frac{1}{3}H)_m{}^a{}_b \equiv \widehat{\omega}_m{}^a{}_b$ are expressed as

$$\widehat{\omega}_1 \;=\; 0\,,$$

$$\widehat{\omega}_2 \;=\; \frac{h'}{2h}\left(\begin{array}{cc|cc|cc} & -2 & & & & \\ +2 & & & & & \\ \hline & & & -\frac{1}{3} & & \\ & & \frac{1}{3} & & & \\ \hline & & & & & -\frac{1}{3} \\ & & & & \frac{1}{3} & \end{array}\right)$$

$$\widehat{\omega}_3 \;=\; \frac{h'}{2h^{\frac{3}{2}}}\left(\begin{array}{cc|cc|cc} & & -1 & & & \\ & & & \frac{1}{3} & & \\ \hline +1 & & & & & \\ & -\frac{1}{3} & & & & \\ \hline & & & & & \\ & & & & & \end{array}\right),$$

$$\widehat{\omega}_4 \;=\; \frac{h'}{2h^{\frac{3}{2}}}\left(\begin{array}{cc|cc|cc} & & & -1 & & \\ & & -\frac{1}{3} & & & \\ \hline & \frac{1}{3} & & & & \\ +1 & & & & & \\ \hline & & & & & \\ & & & & & \end{array}\right)$$

$$\widehat{\omega}_5 \;=\; \frac{h'}{2h^{\frac{3}{2}}}\left(\begin{array}{cc|cc|cc} & & & & -1 & \\ & & & & & \frac{1}{3} \\ \hline & & & & & \\ & & & & & \\ \hline +1 & & & & & \\ & -\frac{1}{3} & & & & \end{array}\right),$$

$$\widehat{\omega}_6 \;=\; \frac{h'}{2h^{\frac{3}{2}}}\left(\begin{array}{cc|cc|cc} & & & & & -1 \\ & & & & -\frac{1}{3} & \\ \hline & & & & & \\ & & & & & \\ \hline & \frac{1}{3} & & & & \\ +1 & & & & & \end{array}\right)$$

$h' = \partial h / \partial x^1 = |x^1|'$: step function

Dirac equation of 6D part is given as

$$
\begin{aligned}
0 \;&=\; \gamma^1 e_1{}^1 \frac{\mathrm{d}}{\mathrm{d}x^1}\psi_{\text{6D}} + \gamma^a e_a{}^i \Big(\frac{1}{4}(\omega - \tfrac{1}{3}H)_m{}^{ab}\gamma_{ab} + A_m \Big)\psi_{\text{6D}} \\
&\equiv\; \begin{pmatrix} 0 & -ih^{-1}\mathbf{1}_2 \otimes \mathbf{1}_2 \\ ih^{-1}\mathbf{1}_2 \otimes \mathbf{1}_2 & 0 \end{pmatrix} \frac{\mathrm{d}}{\mathrm{d}x^1}\psi_{\text{6D}} + \begin{pmatrix} 0 & M_1 \\ M_2 & 0 \end{pmatrix} \psi_{\text{6D}}
\end{aligned}
$$

Evaluate the eigenvalues $\lambda \frac{h'}{h^2}$ of the second term and solve the equation:

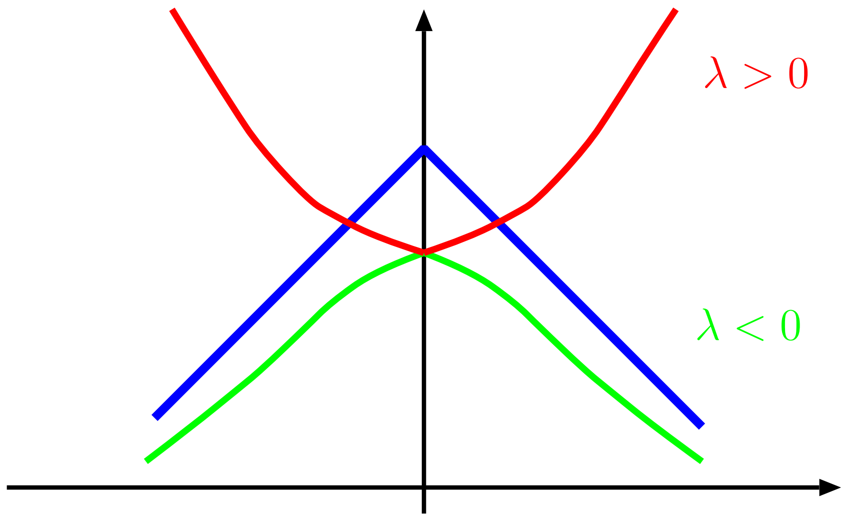
$$
0 \;=\; \frac{i}{h}\frac{\mathrm{d}}{\mathrm{d}x^1}\psi + i\lambda \frac{h'}{h^2}\psi
$$

$$
\therefore \quad \psi \;=\; \text{(const.)} \times h^{-\lambda} \;=\; \text{(const.)} \times (1 + N|x^1|)^{-\lambda}
$$

Note that $h = 1 + N|x^1| = e^{\phi}$ $\dashrightarrow$ $\begin{cases} N > 0 : & \text{negative tension} \\ N < 0 : & \text{positive tension} \end{cases}$

Then only the negative eigenvalues $\lambda < 0$ imply the normalizable modes.

$h(x^1)$ with $N < 0$



Result two left-chiral normalizable modes and one right-chiral normalizable mode

$\dashrightarrow$ $2 - 1 = 1$ generation

consistent with the information from deformed conifold (T-dual of this setup)

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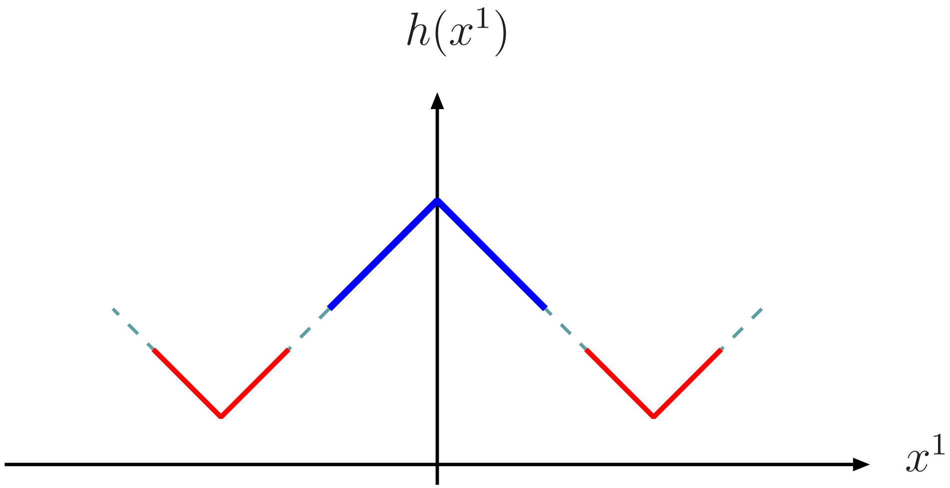
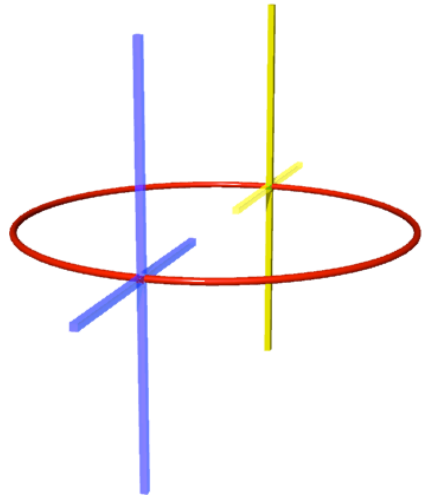
Yet Another Alternative to Compactification (cf. Randall-Sundrum 1 model)

Compactify all extra directions $(x^1, \ldots, x^6)$ to "six-torus" $(T^5 \times (S^1/\mathbb{Z}_2))$





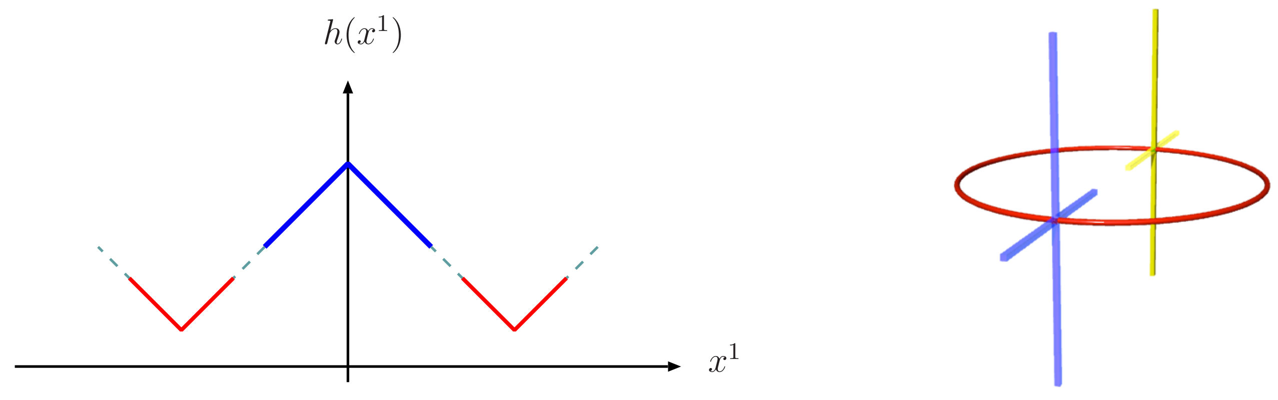
Notice:

In order to introduce objects which absorb/emit NS charges,

we have to put another intersecting 5-branes with negative tension in the x^1 direction

$\rightarrow$ Then, we modify the function h to

$$h(x^1) \;=\; h_0 + N|x^1 - 2\pi k r_c| \,, \qquad k \in \mathbb{Z}$$



This setup has the following features:

- Cosmological constant vanishes
- Warp factor is milder (linear) than that of RS1 (exponential)
- Supersymmetry is broken

$\Rightarrow$ We obtain 4-dim'l non-SUSY model with E_6 gauge symmetry

(under the vanishing limit of h_0)

Contents

Summary

- Studied NS5-brane and its intersection in heterotic string
- Obtained a simple model to yield a chiral model in four dimensions
- Applied it to consider a non-SUSY model via torus compactification

Discussions

- Much clearer (or direct) description of massless modes

  Nambu-Goldstone, Higgs, gauge bosons, SUSY $\in$ effective action
- Connecting to bottom-up model-building
- More understanding intersecting five-branes
- Comparison to type II and F-theoretical configurations via string dualities

Thank You