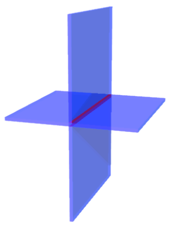
Okayama Institute for Quantum Physics: June 26, 2009

Yet Another Alternative to Compactification

— Heterotic five-branes explain why three generations in Nature

arXiv: 0905.2185 [hep-th]



Tetsuji KIMURA (KEK)

with Shun'ya Mizoguchi (KEK, SOKENDAI)

Introduction

Three Generations in Nature

Necessary condition of CP-violation: Kobayashi-Maskawa

No sufficient conditions in the Standard Model

Tried to embed this problem in GUTs and String Theories

Particles in the Standard Model

	particles	quantum number $SU(3)_C$	$SU(2)_W$	$U(1)_Y$	spin
quark	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	$1/6$	$1/2$
	u_R u_R u_R	3	1	$2/3$	
	d_R d_R d_R	3	1	$-1/3$	
lepton	$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$	1	2	$-1/2$	$1/2$
	e_R	1	1	-1	
	ν_{e_R}	1	1	0	
gauge field	A_μ^a	8	1	0	1
	$W_\mu^\pm$ W_μ^3	1	3	0	
	B_μ	1	1	0	
Higgs	$\begin{pmatrix} h^0 \\ h^- \end{pmatrix}$	1	2	$-1/2$	0

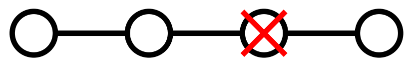
(only one generation in fermions)

$SU(5)$ Grand Unified Model

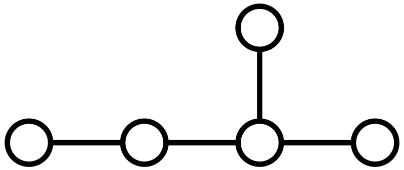
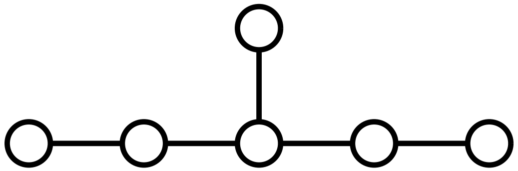
$$\begin{pmatrix} d^c_{\mathsf{R}} \\ d^c_{\mathsf{R}} \\ d^c_{\mathsf{R}} \\ \nu_{e_{\mathsf{L}}} \\ e_{\mathsf{L}} \end{pmatrix} \qquad \begin{pmatrix} 0 & u^c_{\mathsf{R}} & -u^c_{\mathsf{R}} & -u_{\mathsf{L}} & -d_{\mathsf{L}} \\ -u^c_{\mathsf{R}} & 0 & u^c_{\mathsf{R}} & -u_{\mathsf{L}} & -d_{\mathsf{L}} \\ u^c_{\mathsf{R}} & -u^c_{\mathsf{R}} & 0 & -u_{\mathsf{L}} & -d_{\mathsf{L}} \\ u_{\mathsf{L}} & u_{\mathsf{L}} & u_{\mathsf{L}} & 0 & -e^c_{\mathsf{R}} \\ d_{\mathsf{L}} & d_{\mathsf{L}} & d_{\mathsf{L}} & e^c_{\mathsf{R}} & 0 \end{pmatrix} \qquad \nu_{e_{\mathsf{R}}}$$

$$\overline{\mathbf{5}} \qquad\qquad \mathbf{10} \qquad\qquad \mathbf{1}$$

$$U(1)_Y : \left(\begin{array}{ccc|cc} +\frac{1}{3} & & & & \\ & +\frac{1}{3} & & & \\ & & +\frac{1}{3} & & \\ \hline & & & -\frac{1}{2} & \\ & & & & -\frac{1}{2} \end{array}\right) \in SU(5)$$



Further GUTs

		matter	Higgs	
$SU(5)$		$\overline{\mathbf{5}} \oplus \mathbf{10} \oplus \mathbf{1}$	$\mathbf{5} \oplus \overline{\mathbf{5}}$	$\mathbf{1}$
$SO(10)$		$\mathbf{16}$	$\mathbf{10}$	$\mathbf{1}$
E_6			$\mathbf{27}$	

A typical example: Calabi-Yau compactification in $E_8 \times E_8$ heterotic string theory

Standard Embedding: $\omega_m{}^{ab} \equiv A_m^{ab} \in SU(3) \quad \rightsquigarrow \quad E_8 \supset E_6 \times SU(3)$

$$\# \text{ of generations} \; = \; \frac{1}{2}|\chi(\text{CY})| \; = \; \big| h^{1,1} - h^{2,1} \big|$$

$h^{1,1} \; = \; \#$ of Kähler moduli $\; = \; \#$ of $(\overline{\mathbf{27}}, \overline{\mathbf{3}})$ repr. (size of CY)

$h^{2,1} \; = \; \#$ of complex structure moduli $\; = \; \#$ of $(\mathbf{27}, \mathbf{3})$ repr. (shape of CY)

M.B. Green, J.H. Schwarz and E. Witten: *Chapter 16.2*

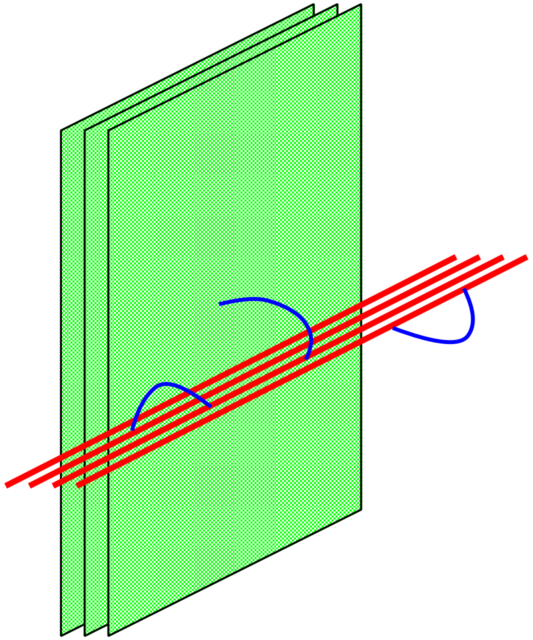
"Problems"

So many Calabi-Yau manifolds

So many massless modes appear in four-dimensional physics

$\rightsquigarrow$ flux compactification to yield potential energy

Another example: D-brane scenario in higher-dimensional theories



ex.)

N_f D7-branes (green planes)

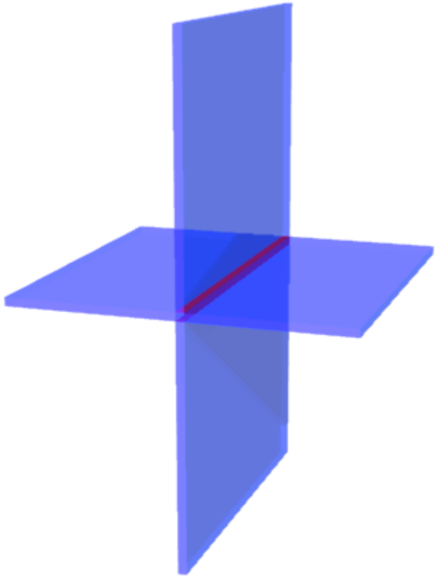
N_c D3-branes (red lines)

$SU(N_c)$ gauge theory with N_f flavors

"Good Points"	"Problems"
Simple and visible	a bit artificial setup

Yet Another Alternative: Intersecting 5-branes in hetertotic string theory

– Our Model –



	0	1	2	3	4	5	6	7	8	9
5-brane	○	○	○	○	○	○				
5-brane'	○	○	○	○			○	○		
our world	○	○	○	○						

"Good Points"

Simple!

E_6 gauge symmetry appears

Naturally obtain 3 E_6-charged multiplets in four dimensions as Nambu-Goldstone modes

(different counting of E_6-charged matter fields from CY)

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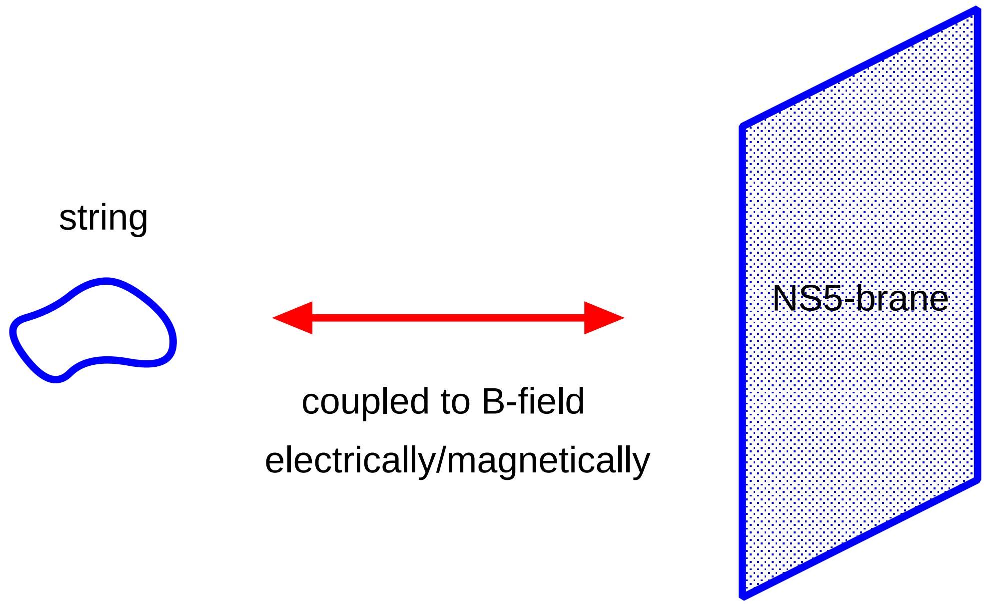
Heterotic String Theory in Ten Dimensions

Objects:



16 Supersymmetry Charges

$E_8 \times E_8$ or $SO(32)$ gauge symmetries

Effective action in the string frame is given as

$$S_{\text{boson}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-g}\, \mathrm{e}^{-2\phi} \Big\{ R + 4(\nabla_M \phi)^2 - \frac{1}{3} H_{MNP}^2 - \frac{\alpha'}{30} \mathrm{Tr} F_{MN}^2 + \ldots \Big\}$$

Supersymmetry transformations:

$$\begin{aligned} \text{gravitino} \qquad \delta\psi_M &= \Big(\partial_M + \frac{1}{4}\omega_{-M}{}^{AB}\,\Gamma_{AB}\Big)\epsilon \\ \text{gaugino} \qquad \delta\chi &= -\frac{1}{4}F_{MN}\Gamma^{MN}\,\epsilon \\ \text{dilatino} \qquad \delta\lambda &= -\frac{1}{4}\Big(\Gamma^M\partial_M\phi - \frac{1}{6}H_{MNP}\Gamma^{MNP}\Big)\epsilon \end{aligned}$$

Bianchi identity (via anomaly cancellation)

$$\mathrm{d}H = \alpha'\left[\frac{1}{30}\mathrm{Tr}(F\wedge F) - \mathrm{tr}(R(\omega_+)\wedge R(\omega_+))\right]$$

with $\omega_{\pm M}{}^{AB} := \omega_M{}^{AB} \pm H_M{}^{AB}$

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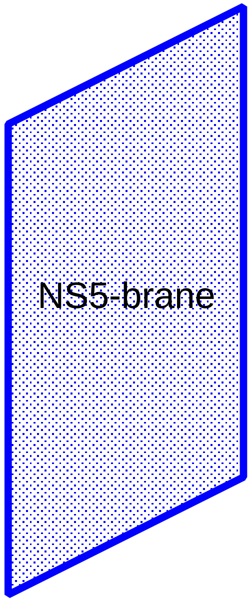
A Five-brane Solution: "Symmetric solution"

$$
\begin{aligned}
\mathrm{d}s^2 &= \sum_{\mu,\nu=0}^{5} \eta_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{e}^{2\phi} \sum_{m,n=6}^{9} \delta_{mn}\,\mathrm{d}x^m \mathrm{d}x^n \\
A_m &= -2\rho_N^2\,\Sigma_{mn}\,\frac{x^n}{x^2(x^2+\rho_N^2)}\,, \qquad \rho_N^2 \;:=\; N\alpha'\,\mathrm{e}^{-2\phi}\,, \qquad \Sigma_{mn} \;:=\; +\frac{1}{2}\epsilon_{mn}{}^{pq}\,\Sigma_{pq} \\
\mathrm{e}^{2\phi} &= \mathrm{e}^{2\phi_0} + \frac{N\alpha'}{x^2}\,, \qquad x^2 \;:=\; \sum_{m=6}^{9} (x^m)^2 \\
H_{mnp} &= -\epsilon_{mnp}{}^{q}\,\nabla_q \phi
\end{aligned}
$$

$$
\begin{aligned}
\omega_{+m}{}^{ab} &= \omega_m{}^{ab} + H_m{}^{ab} \;=\; 2\sigma_{mn}{}^{ab}\,\nabla^n \phi \\
\sigma_{mn}{}^{ab} &:= \delta_{mn}{}^{ab} - \frac{1}{2}\epsilon_{mn}{}^{ab}\,, \qquad \sigma_{mn}{}^{ab} \;=\; +\frac{1}{2}\epsilon_{mn}{}^{pq}\,\sigma_{pq}{}^{ab}
\end{aligned}
$$

C.G. Callan, J.A. Harvey and A. Strominger, *Nucl.Phys.B 359 (1991) 611*



8 SUSY preserved (8 broken)

A_m: $SU(2)$ self-dual instanton in 4 transverse directions

ω_{+m}: $SU(2)$ self-dual

We can embed ω_{+m} into A_m $\rightsquigarrow$ $SU(2)_{\text{spin}} \equiv SU(2)_{\text{gauge}} =: SU(2)_{\text{frozen}}$

$\Downarrow$

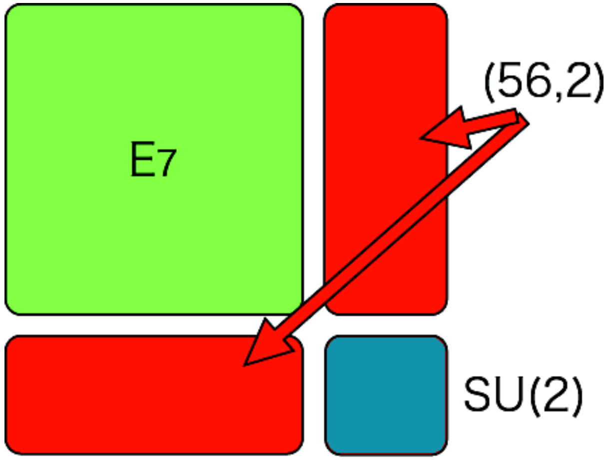
E_8 gauge symmetry is broken to $E_7 \times SU(2)_{\text{frozen}}$

120 bosonic moduli $=$ 30 hypermultiplets in 6-dim'l theory

28 $SU(2)$-Nambu-Goldstone hypermultiplets

Counting hypermultiplets in 6-dim'l theory (exercise to E_6)

$$E_8 \supset E_7 \times SU(2) \qquad \mathbf{248} = (\mathbf{133}, \mathbf{1}) \oplus \underbrace{(\mathbf{1}, \mathbf{3}) \oplus (\mathbf{56}, \mathbf{2})}_{SU(2)\text{-Nambu-Goldstone}}$$



$SU(2)$ instanton moduli in 4-transverse space:

4 translations

1 scale of instanton

$56 \times 2 + 1 \times 3 = 115$

$\rightarrow 4 + 1 + 115 = 120$

$120/4 = 30$ hypermultiplets in 6-dim'l theory

$((56 \times 2)/4 = 28$ of them from $SU(2)$-Nambu-Goldstone)

C.G. Callan, J.A. Harvey and A. Strominger, *Nucl.Phys.B 367 (1991) 60*

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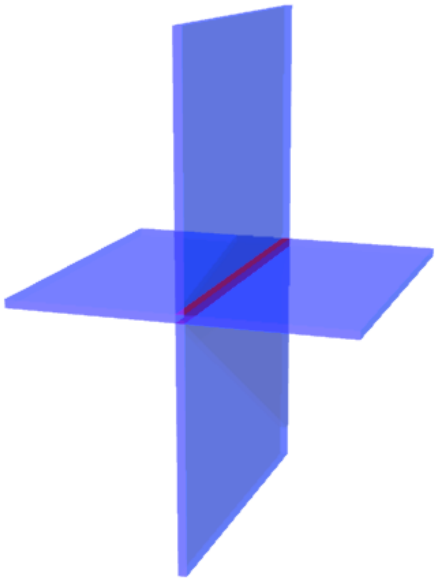
Intersection Rule for p-branes

R. Argurio, F. Englert and L. Houart, *Phys.Lett.B 398 (1997) 61*

$$\bar{q} + 1 \;=\; \frac{(p_A + 1)(p_B + 1)}{D - 2} - \frac{1}{2}(\varepsilon_A a_A)(\varepsilon_B a_B)$$



D : total spacetime dimensions

$\bar{q}$: intersecting dimensions

p_A : spatial dimensions of p_A-brane

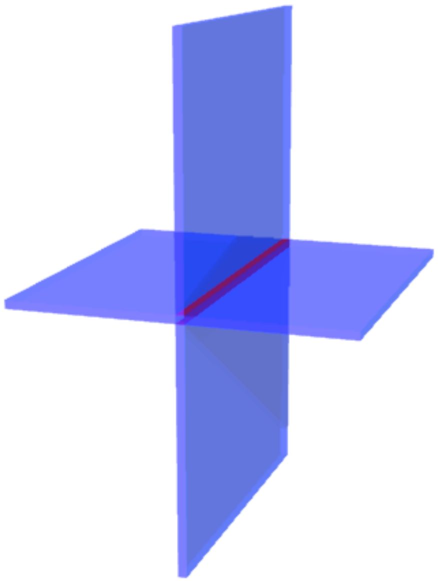
a_A : -1 (NSNS B-field), $\frac{1}{2}(5 - p_A)$ (RR $(p_A + 1)$-form)

ε_A : $+1$ (electric), -1 (magnetic)

Now, we consider two intersecting NS5-branes in heterotic string:

$$\bar{q} + 1 \;=\; \frac{(5+1)(5+1)}{10-2} - \frac{1}{2}(-1)(-1)(-1)(-1) \quad \rightarrow \quad \bar{q} \;=\; 3$$

$\rightsquigarrow$ good dimensions to consider four-dimensional spacetime



	0	1	2	3	4	5	6	7	8	9
5-brane	◯	◯	◯	◯	◯	◯				
5-brane'	◯	◯	◯	◯			◯	◯		
our world	◯	◯	◯	◯						

The string frame metric and the solution are given as

$$\mathrm{d}s^2 \;=\; \sum_{\mu,\nu=0}^{3} \eta_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + h \sum_{m=4}^{7} (\mathrm{d}x^m)^2 + h^2 \sum_{m=8}^{9} (\mathrm{d}x^m)^2$$

$$h \;=\; 1 + N|x^8|\,, \qquad h^2 \;=\; \mathrm{e}^{2\phi}\,, \qquad H_{459} \;=\; H_{679} \;=\; \frac{N}{2}|x^8|'$$

K. Ohta and T. Yokono, *JHEP 02 (2000) 023 + our new idea*

Spin connections $(\omega_{\pm})_m{}^{ab}$ are expressed as

$$
(\omega_{\pm})_4 = \frac{N|x^8|'}{2h^{\frac{3}{2}}} \left(\begin{array}{cc|cc|cc} & & -1 & & & \\ & & & \mp 1 & & \\ \hline +1 & & & & & \\ & \pm 1 & & & & \\ \hline & & & & & \\ & & & & & \end{array}\right), \qquad
(\omega_{\pm})_5 = \frac{N|x^8|'}{2h^{\frac{3}{2}}} \left(\begin{array}{cc|cc|cc} & & & -1 & & \\ & & \pm 1 & & & \\ \hline & \mp 1 & & & & \\ +1 & & & & & \\ \hline & & & & & \\ & & & & & \end{array}\right)
$$

$$
(\omega_{\pm})_6 = \frac{N|x^8|'}{2h^{\frac{3}{2}}} \left(\begin{array}{cc|cc|cc} & & & & -1 & \\ & & & & & \mp 1 \\ \hline & & & & & \\ & & & & & \\ \hline +1 & & & & & \\ & \pm 1 & & & & \end{array}\right), \qquad
(\omega_{\pm})_7 = \frac{N|x^8|'}{2h^{\frac{3}{2}}} \left(\begin{array}{cc|cc|cc} & & & & & -1 \\ & & & & \pm 1 & \\ \hline & & & & & \\ & & & & & \\ \hline & \mp 1 & & & & \\ +1 & & & & & \end{array}\right)
$$

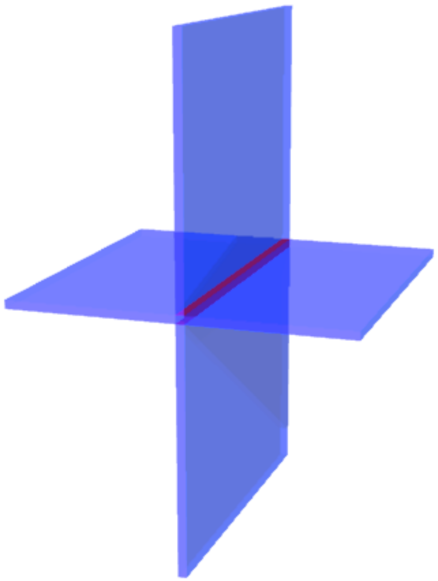
$$
(\omega_{\pm})_8 = 0 \, , \qquad
(\omega_{\pm})_9 = \frac{N|x^8|'}{2h} \left(\begin{array}{cc|cc|cc} & -2 & & & & \\ +2 & & & & & \\ \hline & & & \pm 1 & & \\ & & \mp 1 & & & \\ \hline & & & & & \pm 1 \\ & & & & \mp 1 & \end{array}\right)
$$

Each spin connection belongs to $SU(3)$ group

However, ω_+ does not correspond to ω_- by any similarity transformations

Embedding $\omega_{+m} \equiv A_m$, the Bianchi identity is given by

$$
\mathrm{d}H = \alpha' \left[\frac{1}{30} \mathrm{Tr}(F \wedge F) - \mathrm{tr}(R(\omega_+) \wedge R(\omega_+)) \right] = 0
$$



4 SUSY preserved (12 broken)

ω_{+m}: an $SU(3)$ holonomy connection in 6 directions

ω_{-m}: another $SU(3)$ holonomy connection in 6 directions

We can embed ω_{+m} into A_m $\rightsquigarrow$ $SU(3)_{\text{spin}} \equiv SU(3)_{\text{gauge}} =: SU(3)_{\text{frozen}}$

$\downarrow$

E_8 gauge symmetry is broken to $E_6 \times SU(3)_{\text{frozen}}$

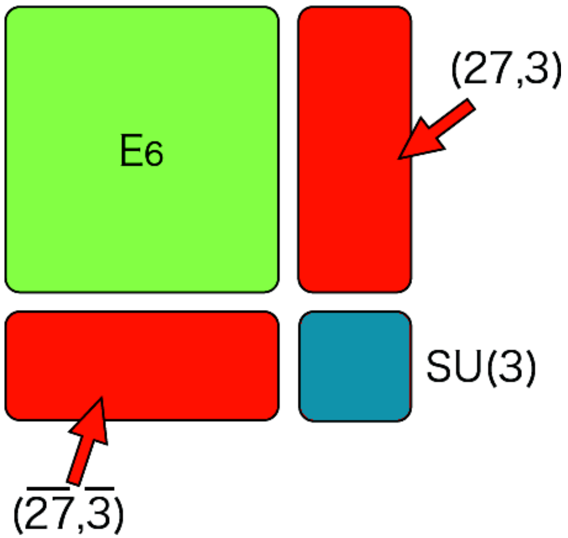
$\Downarrow$

4-dim'l $\mathcal{N} = 1$ E_6 on the intersecting spacetime

3 E_6-charged complex matter multiplets $=$ $SU(3)$-Nambu-Goldstone modes

Counting E_6-charged chiral multiplets in 4-dim'l theory

$$E_8 \supset E_6 \times SU(3) \qquad \mathbf{248} = (\mathbf{78},\mathbf{1}) \oplus \underbrace{(\mathbf{1},\mathbf{8}) \oplus (\mathbf{27},\mathbf{3}) \oplus (\overline{\mathbf{27}},\overline{\mathbf{3}})}_{SU(3)\text{-Nambu-Goldstone}}$$



Focus only on $SU(3)$-Nambu-Goldstone

E_6 fundamental: $(27 \times 3) + (\overline{27} \times \overline{3})$

$(\mathbf{27},\mathbf{3})$ and $(\overline{\mathbf{27}},\overline{\mathbf{3}})$ are complex conjugate

$\{(27 \times 3) + (\overline{27} \times \overline{3})\}/2 = 27 \times 3$ $SU(3)$-Nambu-Goldstone complex chiral bosons

$=$ 3 copies of complex chiral bosons of E_6 fundamental repr.

cf) C.W. Bernard, N.H. Christ, A.H. Guth and E.J. Weinberg, *Phys. Rev. D16 (1977) 2967*

Fermionic moduli via broken SUSY parameters:

Generically, 6-dim'l space has an $SO(6) \simeq SU(4)$ holonomy group

1/4 SUSY yields 1 Killing spinor ϵ_4 and 3 broken spinors ϵ_i in 6 dimensions $(x^4, \ldots, x^9)$

$$\delta\psi_m = \Big(\partial_m + \frac{1}{4}\omega_{-m}{}^{ab}\gamma_{ab}\Big)\epsilon = 0 \quad \rightsquigarrow \quad \underbrace{\begin{pmatrix} * & * & * & 0 \\ * & * & * & 0 \\ * & * & * & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{SU(3)\text{ holonomy}} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \end{pmatrix} = 0$$

Furthermore, we embed $\omega_{+m} \equiv A_m \quad \rightsquigarrow \quad SU(3)_{\text{spin}} \equiv SU(3)_{\text{gauge}}$

# of broken spinors = # of $SU(3)_{\text{gauge}}$ non-singlet fermions in 4 dimensions

They are $SU(3)$-Nambu-Goldstone fermions in 4-dim'l spacetime

and behave as 3 copies of complex moduli of $\mathbf{27}$ under E_6 transformations

The $SU(3)$-Nambu-Goldstone bosons and fermions are combined into

3 copies of complex chiral multiplets of E_6 fundamental repr.!

3 copies = 3 generations!

Remark This intersecting five-brane configuration is T-dual to deformed conifold (non-compact CY with $h^{2,1} = 1$, $h^{1,1} = 0$)

$\Downarrow$

$\#$ of generations is just one!? (from "ordinary" viewpoint)

Different counting of generations from CY compactification

- We counted broken SUSY parameters ϵ_i,
  which are three times as many as the Killing spinor ϵ_4.

- Dirac index is sensitive to $\#$ of set of $(\mathbf{27}, \mathbf{3})$,
  but insensitive to the way how these $\mathbf{27}$ are embedded into the original E_8 repr.

$\rightsquigarrow$ possibility of multiplication three by the Dirac index!

Then, the factor three differs from the counting in the CY compactification

deformed conifold ($\#$ is one) $\leftrightarrow$ intersecting five-brane ($\#$ is three)

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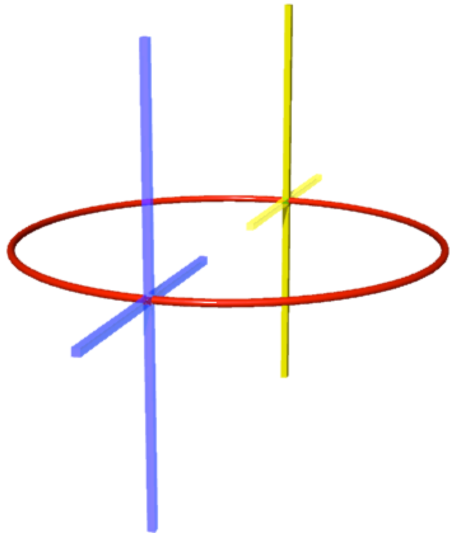
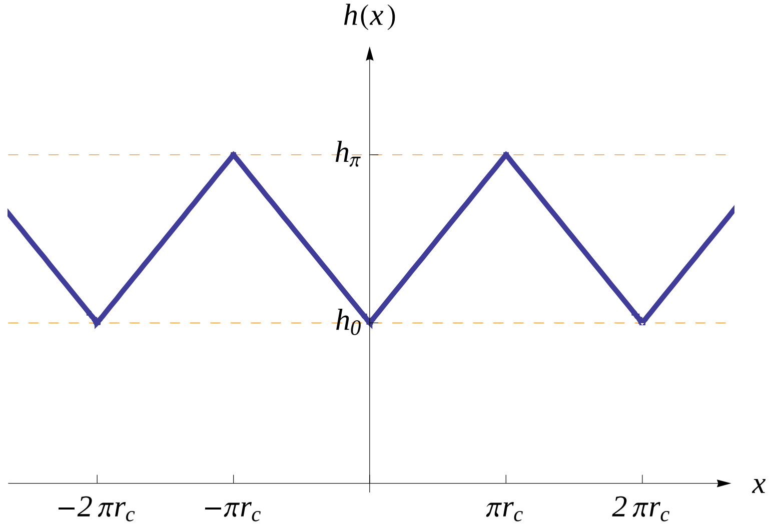
Yet Another Alternative to Compactification (cf. Randall-Sundrum 1 model)

Compactify all extra directions $(x^4, \ldots, x^9)$ to "six-torus" $(T^5 \times (S^1/\mathbb{Z}_2))$





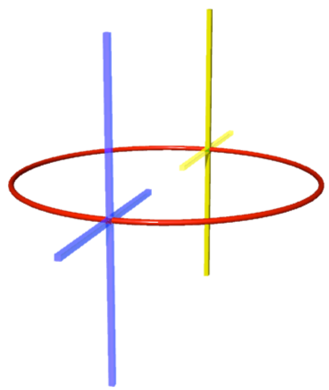
Notice:

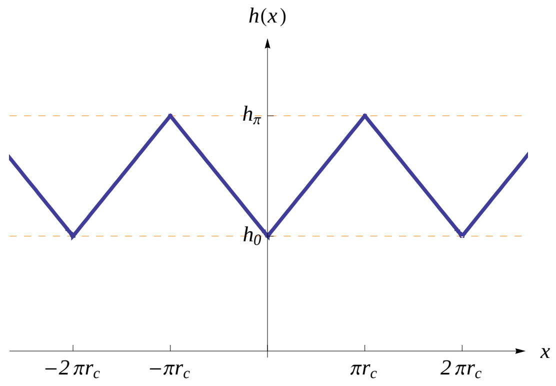
In order to introduce objects which absorb/emit NS charges,

we have to put another intersecting 5-branes with negative tension in the x^8 direction

$\longrightarrow$ Then, we modify the function h to

$$h(x^8) \;=\; h_0 + N|x^8 - 2\pi k r_c| \,, \qquad k \in \mathbb{Z}$$





This setup has the following features:

- Cosmological constant vanishes
- Warp factor is milder (linear) than that of RS1 (exponential)
- Supersymmetry is broken completely!

  $\Rightarrow$ We obtain 4-dim'l non-SUSY model with E_6 gauge symmetry and 3 generations!

  (under the vanishing limit of h_0)

Contents

Summary

- Studied NS5-brane and its intersection in heterotic string
- Obtained a simple model to yield 3 generations in four dimensions
- Applied it to consider a non-SUSY model via torus compactification
- Unified "Nambu" and "Kobayashi-Maskawa" *Nobel Prize in Physics 2008*

Discussions

- Connecting to bottom-up model-building (cf. Maekawa et al.)
- More understanding intersecting NS5-branes
- Comparison to type II and F-theoretical configurations via string dualities

Thank You