日本物理学会第64回年次大会 at 立教学院池袋キャンパス (2009年3月30日)

Realization of AdS Vacua in Attractor Mechanism on Generalized Geometry

arXiv:0810.0937 [hep-th]

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いきなり結論

(拡張された)幾何でコンパクト化された超弦理論

↓

4 次元$\mathcal{N} = 1$超重力の Superpotential (moduliの関数)の判別式が

正：SUSY AdS 真空　　負：SUSY 平坦真空

拡張された幾何とは？

何故そんなものを考えるのか？

ご利益

全ての超重力理論を超弦理論から導出できる可能性

NS-NS, R-R fluxes を自然に取り込んだ真空解が得られる

Superpotential を自然に導出できる

余分なゼロ質量場を追い出す事ができる

高次元理論を通常の幾何でコンパクト化しても
登場しない超重力理論がある(らしい)

↓

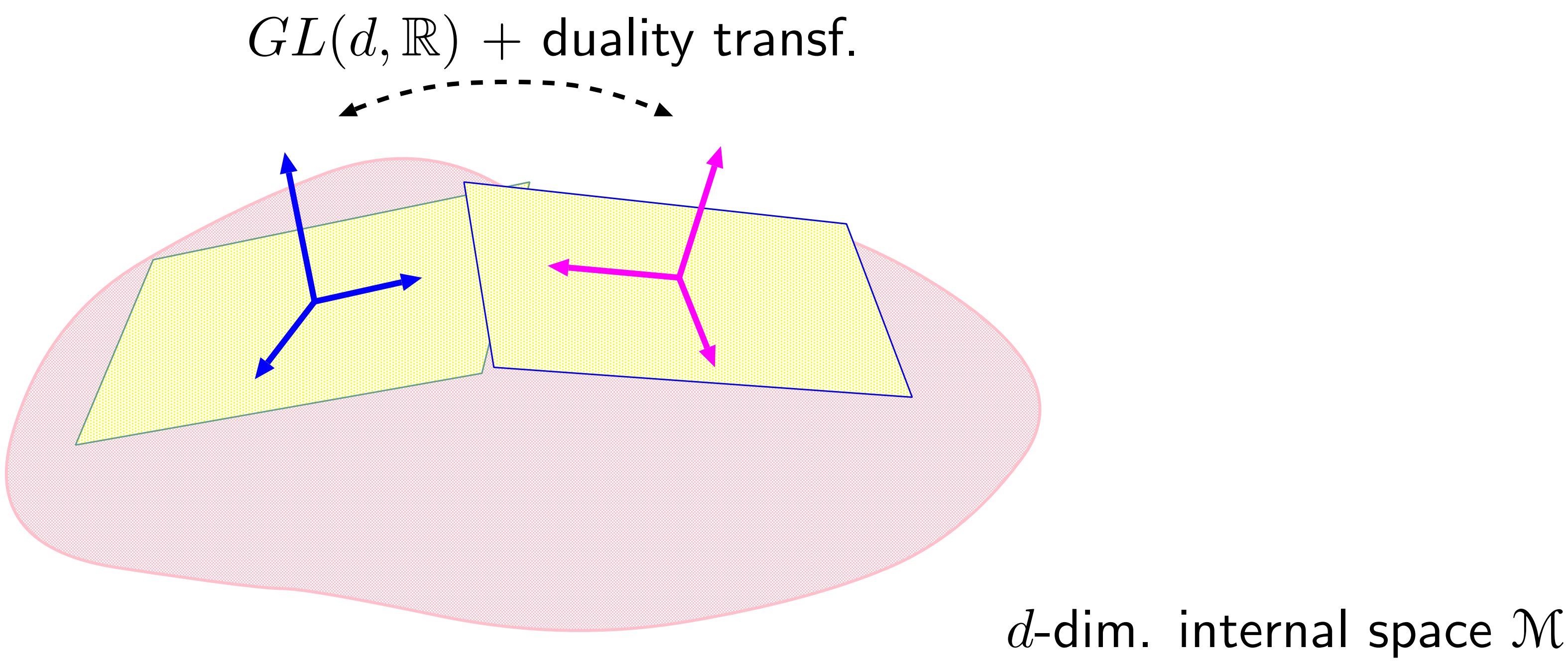
全ての超重力理論は超弦理論に由来すると考えるならば
超弦理論を通常でない幾何(Nongeometric Background)で
コンパクト化したと考えてみよう

質問: Nongeometric Background (拡張された幾何)とは?

$$\text{構造群} \;=\; \text{Diffeo群}\ (GL(d,\mathbb{R})) + \underbrace{\text{双対変換群}\,(O(d,d;\mathbb{Z}),\ \text{S-双対変換},\ \text{U-双対変換},\ \text{etc.})}$$

↑

弦理論の双対性に起因すると考える



この分野における私の貢献

IIA型超弦理論を

6D $SU(3) \times SU(3)$ Generalized geometry でコンパクト化

$\downarrow$

4次元 $\mathcal{N} = 1$ 超重力の Superpotential (moduli の関数) の判別式が

正 : SUSY AdS 真空　　負 : SUSY 平坦真空

判別式を見るだけで宇宙項の大きさを議論できる

$$
\begin{aligned}
V &= \mathrm{e}^{K}\Big(K^{\mathcal{M}\overline{\mathcal{N}}} D_{\mathcal{M}}\mathcal{W}\,\overline{D_{\mathcal{N}}\mathcal{W}} - 3|\mathcal{W}|^2\Big) + \frac{1}{2}|D^a|^2 \\
&\equiv V_{\mathcal{W}} + V_D
\end{aligned}
$$

Search of vacua $\partial_{\mathcal{P}} V\big|_* = 0$

$V_* > 0$:	de Sitter space (non-SUSY)
$V_* = 0$:	Minkowski space
$V_* < 0$:	Anti-de Sitter space

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$$0 = \partial_{\mathcal{P}} V_{\mathcal{W}} = \mathrm{e}^K \Big\{ K^{\mathcal{M}\overline{\mathcal{N}}} D_{\mathcal{P}} D_{\mathcal{M}} \mathcal{W} \overline{D_{\mathcal{N}} \mathcal{W}} + \partial_{\mathcal{P}} K^{\mathcal{M}\overline{\mathcal{N}}} D_{\mathcal{M}} \mathcal{W} \overline{D_{\mathcal{N}} \mathcal{W}} - 2 \overline{\mathcal{W}} D_{\mathcal{P}} \mathcal{W} \Big\}$$

$$0 = \partial_{\mathcal{P}} V_D \quad \dashrightarrow \quad D^a = 0$$

Consider the SUSY condition $D_{\mathcal{P}} \mathcal{W} \equiv \big(\partial_{\mathcal{P}} + \partial_{\mathcal{P}} K \big) \mathcal{W} = 0$ in various cases.

1. *Set a simple prepotential:* $\mathcal{F} = D_{abc} \frac{X^a X^b X^c}{X^0}$

2. *Consider the simplest model:* single modulus t of Φ_+ (and U of Φ_-)

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The superpotential is reduced to

$$\begin{aligned}
\mathcal{W} &= \mathcal{W}^{\text{RR}} + U\,\mathcal{W}^{\mathbb{Q}} \\
\mathcal{W}^{\text{RR}} &= m_{\text{RR}}^0\,t^3 - 3\,m_{\text{RR}}\,t^2 + e_{\text{RR}}\,t + e_{\text{RR}0} \\
\mathcal{W}^{\mathbb{Q}} &= p_0{}^0\,t^3 - 3\,p_0\,t^2 - e_0\,t - e_{00}
\end{aligned}$$

Consider the SUSY condition:

$$\begin{aligned}
D_t \mathcal{W} = 0 \quad \dashrightarrow \quad 0 &= D_t \mathcal{W}^{\text{RR}} + U\,D_t \mathcal{W}^{\mathbb{Q}} \\
D_U \mathcal{W} = 0 \quad \dashrightarrow \quad 0 &= \frac{\mathrm{i}}{\mathrm{Im}U}\Big(\mathcal{W}^{\text{RR}} + \mathrm{Re}U\,\mathcal{W}^{\mathbb{Q}}\Big)
\end{aligned}$$

The discriminant of the superpotential $\mathcal{W}^{\text{RR}}$ (and $\mathcal{W}^{\mathbb{Q}}$) governs the SUSY solutions.

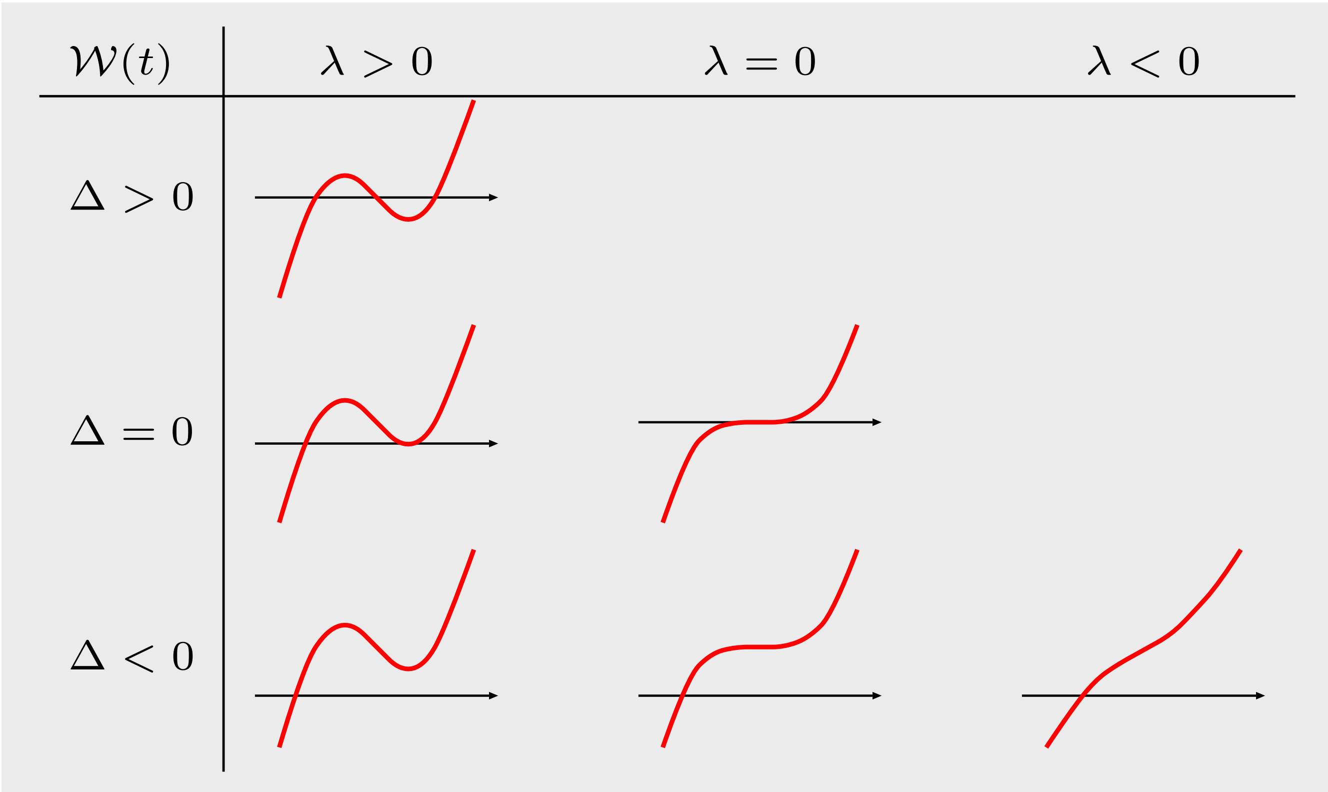
▶ Discriminant of cubic equation

Consider a cubic function and its derivative:
$$\begin{cases} \mathcal{W}(t) &= a\,t^3 + b\,t^2 + c\,t + d \\ \partial_t \mathcal{W}(t) &= 3a\,t^2 + 2b\,t + c \end{cases}$$

Discriminants $\Delta(\mathcal{W})$ and $\Delta(\partial_t \mathcal{W})$ are

$$\Delta(\mathcal{W}) \equiv \Delta = -4b^3 d + b^2 c^2 - 4ac^3 + 18abcd - 27a^2 d^2$$

$$\Delta(\partial_t \mathcal{W}) \equiv \lambda = 4(b^2 - 3ac)$$



$\Delta^{\mathsf{RR}} > 0$ case: always $\lambda^{\mathsf{RR}} > 0$, and exists a zero point: $D_t \mathcal{W}^{\mathsf{RR}} = 0$

$$
\begin{aligned}
D_t \mathcal{W}^{\mathsf{RR}}|_* &= 0 \\
t_*^{\mathsf{RR}} &= \frac{6\,(3\,m_{\mathsf{RR}}^0\,e_{\mathsf{RR}0} + m_{\mathsf{RR}}\,e_{\mathsf{RR}})}{\lambda^{\mathsf{RR}}} - 2\mathrm{i}\,\frac{\sqrt{3\,\Delta^{\mathsf{RR}}}}{\lambda^{\mathsf{RR}}} \\
\mathcal{W}_*^{\mathsf{RR}} &= -\frac{24\,\Delta^{\mathsf{RR}}}{(\lambda^{\mathsf{RR}})^3}\Big(36\,(m_{\mathsf{RR}})^3 + 36\,(m_{\mathsf{RR}}^0)^2 e_{\mathsf{RR}0} - 3\,m_{\mathsf{RR}}\lambda^{\mathsf{RR}} - 4\mathrm{i}\,m_{\mathsf{RR}}^0\sqrt{3\,\Delta^{\mathsf{RR}}}\Big)
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\end{aligned}
$$

$\Delta^{\text{RR}} < 0$ case: only $\lambda^{\text{RR}} < 0$ is physically allowed, and exists a zero point: $\mathcal{W}^{\text{RR}} = 0$

$$
\begin{aligned}
\mathcal{W}_*^{\text{RR}} &= m_{\text{RR}}^0 (t_* - e)(t_* - \alpha)(t_* - \overline{\alpha}) \;=\; 0\,, \qquad t_* \;=\; \alpha^{\text{RR}} \;=\; \alpha_1 + \text{i}\,\alpha_2 \\
\alpha_1 &= \frac{\lambda^{\text{RR}} + F^{2/3} + 12\,m_{\text{RR}}\,F^{1/3}}{12\,m_{\text{RR}}^0\,F^{1/3}} \\
(\alpha_2)^2 &= \frac{1}{m_{\text{RR}}^0}\Big(e_{\text{RR}} - 6\,m_{\text{RR}}\,\alpha_1 + 3\,m_{\text{RR}}^0\,(\alpha_1)^2\Big) \\
e &= -\frac{1}{m_{\text{RR}}^0}\Big(-3\,m_{\text{RR}} + 2\,m_{\text{RR}}^0\,\alpha_1\Big) \\
F &= 108\,(m_{\text{RR}}^0)^2 e_{\text{RR}0} + 12\,m_{\text{RR}}^0\sqrt{-3\Delta^{\text{RR}}} + 108\,(m_{\text{RR}})^3 - 9\,m_{\text{RR}}\,\lambda^{\text{RR}} \\
D_t \mathcal{W}^{\text{RR}}|_* &= 2\text{i}\,m_{\text{RR}}^0 (e - \alpha^{\text{RR}})\alpha_2
\end{aligned}
$$

... Analysis of $\mathcal{W}^{\mathbb{Q}}$ is also discussed.

Three types of solutions to satisfy $0 = D_t \mathcal{W}^{\mathsf{RR}} + U D_t \mathcal{W}^{\mathbb{Q}}$ and $0 = \mathcal{W}^{\mathsf{RR}} + \mathrm{Re} U \, \mathcal{W}^{\mathbb{Q}}$:

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- SUSY AdS vacuum: moduli are (mostly) stabilized

$$\Delta^{\mathsf{RR}} > 0 \,, \qquad \Delta^{\mathbb{Q}} > 0 \,; \qquad D_t \mathcal{W}^{\mathsf{RR}}|_* = 0 = D_t \mathcal{W}^{\mathbb{Q}}|_*$$

$$t_*^{\mathsf{RR}} = t_*^{\mathbb{Q}} \,, \qquad \mathrm{Re}\, U_* = -\frac{\mathcal{W}_*^{\mathsf{RR}}}{\mathcal{W}_*^{\mathbb{Q}}}$$

$$V_* = -3 \, \mathrm{e}^K |\mathcal{W}_*|^2 = -\frac{4}{[\mathrm{Re}(\mathcal{C}\mathcal{G}_0)]^2} \sqrt{\frac{\Delta^{\mathbb{Q}}}{3}} \ll \mathcal{O}(1)$$

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- SUSY Minkowski vacuum: moduli are stabilized

$$\Delta^{\mathsf{RR}} < 0 \, , \quad \Delta^{\mathbb{Q}} < 0 \, ; \quad \mathcal{W}_*^{\mathsf{RR}} = 0 = \mathcal{W}_*^{\mathbb{Q}}$$

$$\alpha^{\mathsf{RR}} = \alpha^{\mathbb{Q}} \, , \quad U_* = -\frac{D_t \mathcal{W}^{\mathsf{RR}}|_*}{D_t \mathcal{W}^{\mathbb{Q}}|_*} \neq 0$$

$$V_* = 0$$

Three types of solutions to satisfy $0 = D_t \mathcal{W}^{\mathsf{RR}} + U D_t \mathcal{W}^{\mathbb{Q}}$ and $0 = \mathcal{W}^{\mathsf{RR}} + \mathrm{Re} U \, \mathcal{W}^{\mathbb{Q}}$:

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- SUSY Minkowski vacuum: moduli are stabilized

$$\Delta^{\mathsf{RR}} < 0 \,, \qquad \Delta^{\mathbb{Q}} < 0 \,; \qquad \mathcal{W}_*^{\mathsf{RR}} = 0 = \mathcal{W}_*^{\mathbb{Q}}$$

$$\alpha^{\mathsf{RR}} = \alpha^{\mathbb{Q}} \,, \qquad U_* = -\frac{D_t \mathcal{W}^{\mathsf{RR}}|_*}{D_t \mathcal{W}^{\mathbb{Q}}|_*} \neq 0$$

$$V_* = 0$$

- SUSY AdS vacua, but moduli t and U are not fixed: non-stabilized point

$$U = -\frac{D_t \mathcal{W}^{\mathsf{RR}}(t)}{D_t \mathcal{W}^{\mathbb{Q}}(t)} \,, \qquad \mathrm{Re}\, U = -\frac{\mathcal{W}^{\mathsf{RR}}(t)}{\mathcal{W}^{\mathbb{Q}}(t)}$$