Seminar at Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut)
Potsdam-Golm, GERMANY
October 25, 2010

Non-SUSY Solutions of RN-AdS Black Holes in 4D $\mathcal{N} = 2$ Gauged Supergravity

based on JHEP 09 (2010) 061 [arXiv:1005.4607]

Tetsuji KIMURA (KEK, JAPAN)

HIGH ENERGY ACCELERATOR RESEARCH ORGANIZATION

Motivation: Black Hole Solutions in 4D $\mathcal{N} = 2$ SUGRA

✍ WHY $\mathcal{N} = 2$ (8-SUSY charges)?

✓ Scalar fields living in highly symmetric spaces

✓ (Flux) compactifications in string/M-theory $\longleftarrow$ *I have mainly worked*

✍ WHY Black Holes?

✓ Attractive solutions in 4D $\mathcal{N} = 2$ SUGRA $\subset$ Einstein-Yang–Mills-Matters

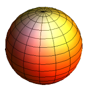
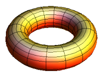
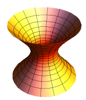
✓ Application to "$\text{AdS}_4/\text{CMP}_3$" correspondence $\longleftarrow$ *quite far from current stage*

Static, charged black holes in asymptotically non-flat 4D spacetime:

$$\mathrm{d}s^2 = -V(r)\,\mathrm{d}t^2 + \frac{1}{V(r)}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\sigma_{X_2}^2$$

$$V(r) = \kappa - \frac{2\eta}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\,, \quad Q^2 = \underset{\text{(elect.)}}{q^2} + \underset{\text{(magnet.)}}{p^2}\,, \quad \Lambda = \text{(cosmological const.)}$$

$$\Lambda = -\frac{3}{\ell^2}\,, \qquad \eta_0 = \frac{\ell}{3\sqrt{6}}\Big(\sqrt{\kappa^2 + 12\,Q^2\,\ell^{-2}} + 2\kappa\Big)\Big(\sqrt{\kappa^2 + 12\,Q^2\,\ell^{-2}} - \kappa\Big)^{1/2}$$

topology of X_2 w/ genus g		metric	ADM mass M
$\kappa = 1$ ($g = 0$; S^2)		: $\mathrm{d}\sigma_{X_2}^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2$	η
$\kappa = 0$ ($g = 1$; flat)		: $\mathrm{d}\sigma_{X_2}^2 = \mathrm{d}x^2 + 2\mathrm{Re}\tau\,\mathrm{d}x\,\mathrm{d}y + \lvert\tau\rvert^2\mathrm{d}y^2\,;\quad x, y \in [0,1]$	$\frac{\eta\lvert\mathrm{Im}\tau\rvert}{4\pi}$
$\kappa = -1$ ($g > 1$; H^2)		: $\mathrm{d}\sigma_{X_2}^2 = \mathrm{d}\theta^2 + \sinh^2\theta\,\mathrm{d}\phi^2$	$(\eta - \eta_0)(g - 1)$

e.g., Caldarelli and Klemm [hep-th/9808097]

Static, charged black holes in asymptotically non-flat 4D spacetime with $\kappa = 1$:

$$ds^2 = -V(r)\,dt^2 + \frac{1}{V(r)}\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)$$

$$V(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}, \quad Q^2 = \underset{\text{(elect.)}}{q^2} + \underset{\text{(magnet.)}}{p^2}, \quad \Lambda = \text{(cosmological const.)}$$

$$\Lambda = -\frac{3}{\ell^2}, \qquad \eta_0 = \frac{\ell}{3\sqrt{6}}\left(\sqrt{1 + 12\,Q^2\,\ell^{-2}} + 2\right)\left(\sqrt{1 + 12\,Q^2\,\ell^{-2}} - 1\right)^{1/2}$$

Various (non-rotating) AdS black holes:

Schwarzschild-AdS : $M \neq 0,\ Q = 0,\ \Lambda < 0$

Reissner-Nordström AdS (RN-AdS) : $M \neq 0,\ Q \neq 0,\ \Lambda < 0$

Condition for regular event horizon: $M \geq \eta_0$

Extremality: $M = \eta_0$

e.g., Caldarelli and Klemm [hep-th/9808097]

- In the framework of 4D $\mathcal{N} = 2$ ungauged SUGRA,

  (Extremal) RN-BH in asymptotically flat spacetime has been investigated.
  (Hypermultiplets are decoupled from the system.)

- We (have to) study BHs in asymptotically non-flat spacetime in $\mathcal{N} = 2$ system.

  The cosmological constant Λ is given as expectation value of scalar potential V.
  (viz "mass deformations" of gravitini)

Remark: Naked Singularity appears in SUSY RN-AdS solution.

- ☞ Pure AdS SUGRA (only gravitational multiplet):
  Romans [hep-th/9203018], Caldarelli and Klemm [hep-th/9808097], etc.
- ☞ Gauged SUGRA with vector multiplets (without hypermultiplets):
  Sabra, et.al. (electric charges [hep-th/9903143], magnetic/dyonic charges [hep-th/0003213]), etc.

There exist SUSY solutions of rotating AdS black holes with regular horizons.

Question

How can we obtain non-SUSY solutions with matter fields
in asymptotically non-flat spacetime?

— Steps —

- Vector-multiplets + electric FI — (studied)*
- Vector-multiplets + electric/magnetic FI (as electric/magnetic charges) — This Work
- Vector- + Hypermultiplets — ?
- Vector- + (massive) Scalar–Tensor-multiplets — ?†
- Non-abelianized versions — ?‡

$\star$ ex.) Bellucci, Ferrara, Marrani and Yeranyan [arXiv:0802.0141], etc.
$\dagger$ Directly related to (non)geometric flux backgrounds via [hep-th/0409097], [hep-th/0701247], etc.
$\ddagger$ Deform Heisenberg algebra, e.g. [hep-th/0410290], etc.

Question

How can we obtain non-SUSY solutions with matter fields in asymptotically non-flat spacetime?

— **Setup** and **Result** —

- Abelian gauged SUGRA with scalar potential given by electric/magnetic charges
- Truncate out all hypermultiplets by hand (artificial)
- Import the Attractor Mechanism
- Find a new attractor equation
- Obtain a non-SUSY solution with small $|\Lambda|$ in T^3-model
- Argue "generic descriptions of moduli" in T^3-model and in STU-model

Question

How can we obtain non-SUSY solutions with matter fields
in asymptotically non-flat spacetime?

— Open Problems —

- Strange behaviors of Λ and event horizon
- Hyper-sector? (axions become massive tensors in the presence of "magnetic" charges)

  scalar field space: $\text{SQG} \to \underbrace{[\text{SKG} + \text{dilaton}]}_{\text{contribute to } V} + \underbrace{\underbrace{[\text{NS-axion}]}_{\text{massive tensor}} + \underbrace{[\text{RR-axions}]}_{\text{massive or flat}}}_{\text{not contribute to } V(?)}$

- Is the attractor mechanism still available?

Contents

Contents

Supersymmetric multiplets in 4D $\mathcal{N} = 2$ SUGRA:

1 gravitational multiplet: $\{g_{\mu\nu}, A_\mu^0, \psi_{A\mu}\}$ — $\mu = 0, 1, 2, 3$ (4D, curved); $A = 1, 2$ ($SU(2)$ R-symmetry)

n_{V} vector multiplets: $\{A_\mu^a, z^a, \lambda^{aA}\}$ — $a = 1, \ldots, n_{\mathsf{V}}$

z^a in special Kähler geometry $\mathcal{SM}$

$n_{\mathsf{H}} + 1$ hypermultiplets: $\{q^u, \zeta_\alpha\}$ — $u = 1, \ldots, 4n_{\mathsf{H}} + 4$; $\alpha = 1, \ldots, 2n_{\mathsf{H}} + 2$

q^u in quaternionic geometry $\mathcal{HM}$

Gauging: *Promote* global isometry groups on scalar field spaces to local symmetries

"Standard": Andrianopoli, Bertolini, Ceresole, D'Auria, Ferrara, Fré and Magri [hep-th/9605032]

"Magnetically coupled": D'Auria, Sommovigo and Vaulà [hep-th/0409097]

Action (gravitational const. κ; gauge coupling const. g; index $\Lambda = 0, 1, \ldots, n_{\mathsf{V}}$):

$$S = \int \mathrm{d}^4 x \sqrt{-g} \Big\{ \frac{1}{2\kappa^2} R - G_{a\bar{b}}(z, \bar{z}) \nabla_\mu z^a \nabla^\mu \bar{z}^{\bar{b}} - h_{uv}(q) \nabla_\mu q^u \nabla^\mu q^v + \frac{1}{4} \mu_{\Lambda\Sigma}(z, \bar{z}) F^\Lambda_{\mu\nu} F^{\Sigma\mu\nu} + \frac{1}{4} \nu_{\Lambda\Sigma}(z, \bar{z}) F^\Lambda_{\mu\nu} (*F^\Sigma)^{\mu\nu} - \mathrm{g}^2 V(z, \bar{z}, q) + \text{(fermionic terms)} \Big\}$$

$\mu_{\Lambda\Sigma} = \mathrm{Im}\, \mathcal{N}_{\Lambda\Sigma}$ (generalized $-1/\mathrm{g}^2$) , $\nu_{\Lambda\Sigma} = \mathrm{Re}\, \mathcal{N}_{\Lambda\Sigma}$ (generalized θ-angle)

In this analysis...

- Set background fermionic fields to zero
- Reduce gauge symmetries to *abelian*
- Truncate hypermultiplets *by hand*

Appendix

CLICK!!

Equations of Motion (abbreviate κ and g):

$$\delta g^{\mu\nu} : \quad \Big(R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} \Big) - 2 G_{a\overline{b}} \, \partial_{(\mu} z^a \partial_{\nu)} \overline{z}^{\overline{b}} + G_{a\overline{b}} \, \partial_\rho z^a \partial^\rho \overline{z}^{\overline{b}} \, g_{\mu\nu} \ = \ T_{\mu\nu} - V g_{\mu\nu}$$

$$T_{\mu\nu} \ = \ - \mu_{\Lambda\Sigma} \, F_{\mu\rho}^{\Lambda} \, F_{\nu\sigma}^{\Sigma} \, g^{\rho\sigma} + \frac{1}{4} \mu_{\Lambda\Sigma} \, F_{\rho\sigma}^{\Lambda} \, F^{\Sigma\rho\sigma} \, g_{\mu\nu}$$

(energy-momentum tensor)

$$\delta z^a : \quad - \frac{G_{a\overline{b}}}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g} g^{\mu\nu} \partial_\nu \overline{z}^{\overline{b}} \Big) - \frac{\partial G_{a\overline{b}}}{\partial \overline{z}^{\overline{c}}} \, \partial_\rho \overline{z}^{\overline{b}} \, \partial^\rho \overline{z}^{\overline{c}}$$
$$= \ \frac{1}{4} \frac{\partial \mu_{\Lambda\Sigma}}{\partial z^a} F_{\mu\nu}^{\Lambda} F^{\Sigma\mu\nu} + \frac{1}{4} \frac{\partial \nu_{\Lambda\Sigma}}{\partial z^a} F_{\mu\nu}^{\Lambda} (* F^{\Sigma})^{\mu\nu} - \frac{\partial V}{\partial z^a}$$

$$\delta A_\mu^\Lambda : \quad \varepsilon^{\mu\nu\rho\sigma} \partial_\nu G_{\Lambda\rho\sigma} \ = \ 0 \, , \qquad G_{\Lambda\rho\sigma} \ = \ \nu_{\Lambda\Sigma} F_{\rho\sigma}^{\Sigma} - \mu_{\Lambda\Sigma} (* F^{\Sigma})_{\rho\sigma}$$

electric charge $q_\Lambda \ \equiv \ \dfrac{1}{4\pi} \displaystyle\int_{S^2} G_{2\Lambda} \, ,$ magnetic charge $p^\Lambda \ \equiv \ \dfrac{1}{4\pi} \displaystyle\int_{S^2} F_2^\Lambda$

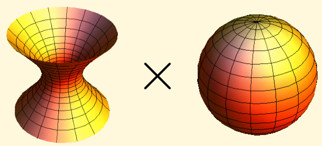
Introduce ansatz for RN(-AdS) BH: "Extremal," "static", "charged", "spherically symmetric"

$$ds^2 = -\,e^{2A(r)}dt^2 + e^{2B(r)}dr^2 + e^{2C(r)}r^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big)$$

Near horizon geometry: $\mathrm{AdS}_2 \times S^2$ (radii: r_{A} and r_{H})

$$A(r) = \log\frac{r - r_{\mathsf{H}}}{r_{\mathsf{A}}}, \qquad B(r) = -A(r), \qquad C(r) = \log\frac{r_{\mathsf{H}}}{r}$$

$$R(\mathrm{AdS}_2 \times S^2) = 2\Big(-\frac{1}{r_{\mathsf{A}}^2} + \frac{1}{r_{\mathsf{H}}^2}\Big)$$



$$\to \quad ds^2(\text{near horizon}) = -\Big(\frac{r - r_{\mathsf{H}}}{r_{\mathsf{A}}}\Big)^2 dt^2 + \Big(\frac{r_{\mathsf{A}}}{r - r_{\mathsf{H}}}\Big)^2 dr^2 + r_{\mathsf{H}}^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big)$$

$$= -\frac{e^{2\tau}}{r_{\mathsf{A}}^2}dt^2 + r_{\mathsf{A}}^2 d\tau^2 + r_{\mathsf{H}}^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big) \qquad (\tau = \log(r - r_{\mathsf{H}}))$$

If the attractor mechanism works (via extremality), the scalar fields are subject to

$$\frac{\mathrm{d}}{\mathrm{d}r} z^a \Big|_{\text{horizon}} = 0 \,, \qquad \Big(\frac{\mathrm{d}}{\mathrm{d}r}\Big)^2 z^a \Big|_{\text{horizon}} = 0$$

The EoM are drastically reduced to Bellucci, et.al. [arXiv:0802.0141]

$$\delta g^{tt}, \delta g^{rr} : \quad \frac{1}{r_{\mathsf{H}}^2} = \frac{1}{r_{\mathsf{H}}^4} I_S + V \Big|_{\text{horizon}} \quad \Rightarrow \quad r_{\mathsf{H}}^2 = \frac{1 - \sqrt{1 - 4 I_S V}}{2V} \Big|_{\text{horizon}}$$

$$\delta g^{\theta\theta}, \delta g^{\phi\phi} : \quad \frac{1}{r_{\mathsf{A}}^2} = \frac{1}{r_{\mathsf{H}}^4} I_S - V \Big|_{\text{horizon}} \quad \Rightarrow \quad r_{\mathsf{A}}^2 = \frac{r_{\mathsf{H}}^2}{\sqrt{1 - 4 I_S V}} \Big|_{\text{horizon}}$$

$$\delta z^a : \quad 0 = \frac{1}{r_{\mathsf{H}}^4} \frac{\partial I_S}{\partial z^a} - \frac{\partial V}{\partial z^a} \Big|_{\text{horizon}} \quad \Rightarrow \quad 0 = \frac{1}{r_{\mathsf{H}}^4} (1 - 2 r_{\mathsf{H}}^2 V) \frac{\partial}{\partial z^a} r_{\mathsf{H}}^2 \Big|_{\text{horizon}}$$

(1st) Symplectic Invariant:

$$I_S(z, \overline{z}, p, q) = -\frac{1}{2} \begin{pmatrix} p^{\Lambda} & q_{\Lambda} \end{pmatrix} \begin{pmatrix} \mu_{\Lambda\Sigma} + \nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Delta} \nu_{\Delta\Sigma} & -\nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Sigma} \\ -(\mu^{-1})^{\Lambda\Gamma} \nu_{\Gamma\Sigma} & (\mu^{-1})^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} p^{\Sigma} \\ q_{\Sigma} \end{pmatrix} \equiv -\frac{1}{2} \Gamma^{\mathrm{T}} \mathbb{M} \Gamma$$

$$\text{with} \quad T_t{}^t = T_r{}^r = -T_\theta{}^\theta = -T_\phi{}^\phi = -\frac{\mathrm{e}^{-4C}}{r^4} I_S$$

BH Entropy (and the effective potential) are given as the Area of the event horizon:

$$S_{\text{BH}}(p,q) \;=\; \frac{A_{\text{H}}}{4\pi} \;=\; r_{\text{H}}^2\Big|_{\text{horizon}} \;\equiv\; V_{\text{eff}}(z,\overline{z},p,q)\Big|_{\text{horizon}}$$

$$V_{\text{eff}}(z,\overline{z},p,q) \;=\; \frac{1-\sqrt{1-4I_S V}}{2V} \;=\; I_S + (I_S)^2 V + 2(I_S)^3 V^2 + \mathcal{O}((I_S)^4 V^3)$$

We read the "cosmological constant Λ" from the scalar curvature:

$$R(\text{AdS}_2 \times S^2) \;=\; 2\Big(-\frac{1}{r_{\text{A}}^2} + \frac{1}{r_{\text{H}}^2}\Big) \;=\; 4V$$

$$V\Big|_{\text{horizon}} \;\equiv\; \Lambda(\text{"cosmological constant"})$$

Attractor Equation

$$0 \;=\; \frac{1}{r_{\text{H}}^4}\left(1-2r_{\text{H}}^2 V\right)\frac{\partial}{\partial z^a}V_{\text{eff}}\Big|_{\text{horizon}} \qquad \rightarrow \qquad 0 \;=\; \frac{\partial}{\partial z^a}V_{\text{eff}}(z,\overline{z},p,q)\Big|_{\text{horizon}}$$

If $r_{\text{H}} < \infty$ and $V\Big|_{\text{horizon}} \leq 0$

The "ATTRACTOR EQUATION" which we have to solve is

$$0 = \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \overline{z}, p, q)\Big|_{\text{horizon}}$$

$$= \frac{1}{2V^2\sqrt{1-4I_S V}} \left\{ 2V^2 \frac{\partial I_S}{\partial z^a} - \left(\sqrt{1-4I_S V} + 2I_S V - 1\right) \frac{\partial V}{\partial z^a} \right\}\Bigg|_{\text{horizon}}$$

Evaluate I_S and V described in terms of the central charge Z

Useful when we consider (non-)SUSY solutions

SUSY variation of gravitini carries definition of Z *and more..*

$$\delta\psi_{A\mu} = D_\mu \varepsilon_A + \epsilon_{AB} T^-_{\mu\nu} \gamma^\nu \varepsilon^B + i\, \mathrm{g}\, \mathcal{S}_{AB} \gamma_\mu \varepsilon^B$$

$$Z = -\frac{1}{2}\left(\frac{1}{4\pi}\int_{S^2} T^-_2\right), \qquad \mathcal{S}_{AB} = \frac{i}{2}\sum_{x=1}^{3} (\sigma^x)_{AB} \mathcal{P}_x$$

Use the property of scalar field spaces $\mathcal{SM} \times \mathcal{HM}$

Describe Z, I_S and V in terms of $(L^\Lambda, M_\Lambda) = e^{K/2}(X^\Lambda, \mathcal{F}_\Lambda)$ on $\mathcal{SM}$:

$$
\begin{aligned}
Z &= L^\Lambda q_\Lambda - M_\Lambda p^\Lambda \\
I_S &= |Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z} \\
V &= \sum_{x=1}^{3} \Big(-3|\mathcal{P}_x|^2 + G^{a\bar{b}} D_a \mathcal{P}_x \overline{D_b \mathcal{P}_x} \Big) + 4 h_{uv} k^u \overline{k}^v
\end{aligned}
$$

Appendix

$\mathcal{P}_x$: $SU(2)$ triplet of Killing prepotentials in $\mathcal{N}=2$ SUGRA

(In general, both vector moduli and hyper moduli contribute to $\mathcal{P}_x$) Appendix

"Truncate" hypermultiplets $\rightarrow$ only $\mathcal{P}_3 = \mathcal{P}_{3,\Lambda} L^\Lambda - \widetilde{\mathcal{P}}_3^\Lambda M_\Lambda$ remains in the scalar potential

Further, Identify $(\mathcal{P}_{3,\Lambda}, \widetilde{\mathcal{P}}_3^\Lambda) = (q_\Lambda, p^\Lambda) \rightsquigarrow \mathcal{P}_3 \equiv Z$

Cassani, Ferrara, Marrani, Morales and Samtleben [arXiv:0911.2708]

$$
V = -3|Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}
$$

The "ATTRACTOR EQUATION" (with a non-trivial factor $-1 \leq G_{\mathsf{V}} \equiv \frac{1 - V_{\text{eff}}^2}{1 + V_{\text{eff}}^2} \leq 1$)

$$\begin{aligned} 0 &= \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \overline{z}, p, q) \Big|_{\text{horizon}} \\ &= \frac{1}{2V^2 \sqrt{1 - 4 I_S V}} \Big\{ 2V^2 \frac{\partial I_S}{\partial z^a} - \big(\sqrt{1 - 4 I_S V} + 2 I_S V - 1 \big) \frac{\partial V}{\partial z^a} \Big\} \Big|_{\text{horizon}} \\ &= \frac{1 + V_{\text{eff}}^2}{\sqrt{1 - 4 I_S V}} \Big\{ 2\, G_{\mathsf{V}}\, \overline{Z} D_a Z + i C_{abc} G^{b\overline{b}} G^{c\overline{c}}\, \overline{D_b Z}\, \overline{D_c Z} \Big\} \Big|_{\text{horizon}} \end{aligned}$$

Appendix

Solve the equation $0 = 2\, G_{\mathsf{V}}\, \overline{Z} D_a Z + i\, C_{abc}\, G^{b\overline{b}} G^{c\overline{c}}\, \overline{D_b Z}\, \overline{D_c Z} \Big|_{\text{horizon}}$

under the condition $V < 0, \ \ 1 - 4 I_S V > 0, \ \ \partial_a I_S \neq 0, \ \ \partial_a V \neq 0, \ \ D_a Z \neq 0$

- If $V < 0$ and $D_a Z = 0$ (SUSY) $\rightarrow$ Naked Singularity $\rightarrow$ Search non-SUSY solution $D_a Z \neq 0$
- If $\partial_a I_S = 0$ $(G_{\mathsf{V}} = 1)$ or $\partial_a V = 0$ $(G_{\mathsf{V}} = -1)$ $\rightarrow$ asymptotically flat $V = 0$ or Empty Hole $Z = 0$

Contents

- Consider the Single Modulus Model of $\Gamma = (0, p, 0, q_0)$ ("D0-D4" system)

  with cubic prepotential $\mathcal{F} = \dfrac{(X^1)^3}{X^0}$, $t = \dfrac{X^1}{X^0}$ $\longrightarrow$ Kähler potential $\mathrm{e}^K = \dfrac{i}{(t - \bar{t}\,)^3}$

  (as the "large volume limit of Calabi-Yau")

The Attractor Equation and its solution ($t = 0 + iy$, $y < 0$):

$$p(y^2)^3 + (q_0 - 18p^3 q_0^2)(y^2)^2 - 12p^2 q_0^3 (y^2) - 2pq_0^4 \;=\; 0$$

with $p \neq 0\,, \quad q_0 \neq 0\,, \quad pq_0 < 0$

$$y^2 \;=\; \mathfrak{A} + \mathfrak{B} \quad \text{or} \quad \mathfrak{A} + \omega^{\pm}\mathfrak{B} \qquad (\omega^3 = 1)$$

$$\mathfrak{A} \;=\; \frac{q_0}{3p}(18\,p^3 q_0 - 1)\,, \qquad \mathfrak{B} \;=\; \frac{1}{3p}\left(\mathfrak{C}^{1/3} + \frac{q_0^2}{4}\frac{1 + (18\,p^3 q_0)^2}{\mathfrak{C}^{1/3}}\right)$$

$$\mathfrak{C} \;=\; -q_0^3\left[1 - 27\,p^3 q_0 - (18\,p^3 q_0)^3 - 3\sqrt{3}\sqrt{-2\,p^3 q_0 - 9(p^3 q_0)^2 - 432(p^3 q_0)^3}\right]$$

Various values at the event horizon:

$$Z\Big|_{\text{horizon}} = \frac{q_0 + 3p\, y^2}{2}\sqrt{-\frac{1}{2y^3}} \neq 0\,, \qquad D_t Z\Big|_{\text{horizon}} = \frac{3i(q_0 - p\, y^2)}{4y}\sqrt{-\frac{1}{2y^3}} \neq 0$$

$$I_S\Big|_{\text{horizon}} = \frac{q_0^2 + 3p^2 y^4}{-2y^3} > 0$$

$$\Lambda = \frac{6(pq_0)^2(q_0 + 3p\, y^2)^2}{y^5} < 0$$

$$S_{\text{BH}} = \frac{-y}{12(pq_0)^2(q_0 + 3p\, y^2)^2}\Big\{-y^4 + \sqrt{y^8 + 12(pq_0)^2(q_0 + 3p\, y^2)^2(q_0^2 + 3p^2\, y^4)}\Big\} > 0$$

their asymptotic behaviors?

Look at the Small q_0 limit:

The dominant part of the Modulus $t = 0 + iy$ $(y < 0)$ is

$$y \sim -\sqrt{-\frac{q_0}{p}} + \text{(sub-leading orders)}$$

The dominant parts of various values are

$$Z\Big|_{\text{horizon}} \sim -q_0 \left(-\frac{p^3}{q_0^3}\right)^{1/4} + \ldots, \qquad D_t Z\Big|_{\text{horizon}} \sim ip\left(-\frac{p}{q_0}\right)^{1/4} + \ldots$$

$$I_S\Big|_{\text{horizon}} \sim \sqrt{-p^3 q_0} + \ldots$$

$$\Lambda \sim -\sqrt{(-p^3 q_0)^{\mathbf{3}}} + \ldots$$

$$S_{\text{BH}} \sim \sqrt{-p^3 q_0} + \ldots$$

$$I_S \sim S_{\text{BH}} \gg -\Lambda > 0$$

very small $|\Lambda|$ compared to others: similar to the non-BPS RN-BH,

but Never connected to RN-BH with $\Lambda = 0$*!*

Compare to values in the case of non-BPS RN-BH with $\Lambda = 0$:

$$t \;=\; 0 + iy\,, \qquad y \;=\; -\sqrt{-\frac{q_0}{p}}$$

$$Z\Big|_{\text{horizon}} \;=\; -\frac{q_0}{\sqrt{2}}\Big(-\frac{p^3}{q_0^3}\Big)^{1/4} \;\neq\; 0\,, \qquad D_t Z\Big|_{\text{horizon}} \;=\; -3ip\Big(-\frac{p}{q_0}\Big)^{1/4} \;\neq\; 0$$

$$S_{\text{BH}} \;=\; I_S\Big|_{\text{horizon}} \;=\; |Z|^2 + G^{t\bar{t}} D_t Z \overline{D_t Z}\Big|_{\text{horizon}} \;=\; 4|Z|^2\Big|_{\text{horizon}} \;=\; \sqrt{-4p^3 q_0} \;>\; 0$$

Argue the Large q_0 limit:

The dominant part of the Modulus $t = 0 + iy$ $(y < 0)$ is

$$y \sim pq_0 + \text{(sub-leading orders)}$$

The dominant parts of various values are

$$Z\Big|_{\text{horizon}} \sim \sqrt{-p^3 q_0} + \ldots \neq 0\,, \qquad D_t Z\Big|_{\text{horizon}} \sim \frac{-i}{pq_0}\sqrt{-p^3 q_0} + \ldots \neq 0$$

$$I_S\Big|_{\text{horizon}} \sim -p^3 q_0 + \ldots > 0$$

$$\Lambda \sim p^3 q_0 + \ldots < 0 \qquad \text{(same magnitude to } I_S \text{ !)}$$

$$S_{\text{BH}} \sim \sqrt{\frac{5}{6}} + \ldots > 0 \qquad \text{(Why constant ??)}$$

"Disastrous" behaviors of Λ and S_{BH}: incorrect expansions?

...not completely understood yet

Contents

- ☑ Studied Extremal RN-AdS Black Hole solutions in Abelian gauged SUGRA

- ☑ Described the non-SUSY solution of the D0-D4 system in the T^3-model

- ❗ Different behaviors from the ones of non-BPS RN black hole in asympt.-flat spacetime:

  - ✓ Should we expand the solution in terms of "Λ" rather than "q_0"?
  - ✓ Consider the contribution of (electrically/magnetically coupled) hyper-sector! Appendix

    D'Auria, et.al. [hep-th/0409097] (massive tensors)

    D'Auria, Ferrara and Trigiante [hep-th/0701247], Cassani, et.al. [arXiv:0911.2708] (generalized geometry)

    Hristov, Looyestijn and Vandoren [arXiv:1005.3650] (gauged SUGRA to ungauged SUGRA via Higgs mechanism)

  - ✓ Is the "Attractor Mechanism" still available?

- ☞ Argue more general solutions in more general setups (rotating, non-extremal, etc..)

Consider the contribution of (electrically/magnetically coupled) hyper-sector!

D'Auria, et.al. [hep-th/0409097] (massive tensors)

D'Auria, et.al. [hep-th/0701247], Cassani, et.al. [arXiv:0911.2708] (generalized geometry)

Hristov, et.al. [arXiv:1005.3650] (gauged SUGRA to ungauged SUGRA via Higgs mechanism)

Keypoints:

✓ Are all vector gauge fields eaten by two-form fields via anti-Higgs mechanism?

EoM for all (massive) fields should be re-analyzed.

✓ Where do superpartners of the gauge fields and of the massive two-form fields go?

We have to check the number of physical DOFs and the preservation of $\mathcal{N} = 2$ local SUSY.

✓ Does the "No-go theorem" of SUSY AdS-BH solution work even in this situation?

We shall investigate the SUSY variation of the gravitini.

✓ Does the mass scale correspond to the scale of string compactification?

(This deeply depends on the viewpoints : "purely 4D SUGRA" vs "10D Effective Theory")

✓ How is the gauge symmetry non-abelianized?

Everything should be reconsidered from the beginning..

Thank you for your attention.

Contents

Action including hypermultiplets: Action | Vector sector | Discussions

$$S = \int d^4x\sqrt{-g}\Big\{\frac{1}{2\kappa^2}R - G_{a\bar{b}}(z,\bar{z})\partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} - h_{uv}(q)\nabla_\mu q^u \nabla^\mu q^v + \frac{1}{4}\mu_{\Lambda\Sigma}(z,\bar{z})F^\Lambda_{\mu\nu}F^{\Sigma\mu\nu} + \frac{1}{4}\nu_{\Lambda\Sigma}(z,\bar{z})F^\Lambda_{\mu\nu}(*F^\Sigma)^{\mu\nu} - g^2 V(z,\bar{z},q) + (\text{fermionic terms})\Big\}$$

Scalar field space of hypermultiplets = (special) quaternionic geometry

$$\underset{4n_{\rm H}+4}{\{q^u\}} = \underset{2n_{\rm H}(\text{SKG})}{\{z^i,\bar{z}^{\bar{\jmath}}\}} + \underset{2n_{\rm H}}{\{\xi^i,\widetilde{\xi}_j\}} + \underset{4\,(\text{universal})}{\{\varphi,a,\xi^0,\widetilde{\xi}_0\}} = \underset{\text{SKG+dilaton}}{\{z^i,\bar{z}^{\bar{\jmath}},\varphi\}} + \underset{\text{“Heisenberg”}}{\{a,\xi^I,\widetilde{\xi}_J\}}$$

Kinematical term and Scalar potential:

$$h_{uv}\,dq^u\,dq^v = \underset{\text{SKG}_{\rm H}}{G_{i\bar{\jmath}}\,dz^i\,d\bar{z}^{\bar{\jmath}}} + \underset{\text{4D dilaton}}{(d\varphi)^2} + \frac{1}{4}e^{4\varphi}\big(\underset{\text{axion}}{da} - \xi^{\rm T}\,\mathbb{C}_{\rm H}\,d\xi\big)^2 - \frac{1}{2}e^{2\varphi}\underset{\text{RR-axions}}{d\xi^{\rm T}\,\mathbb{M}_{\rm H}\,d\xi}$$

$$\begin{aligned} V &= \sum_{x=1}^{3}\Big(G^{a\bar{b}}D_a\mathcal{P}_x\overline{D_b\mathcal{P}_x} - 3|\mathcal{P}_x|^2\Big) + 4h_{uv}k^u\bar{k}^v \\ &= G^{a\bar{b}}D_a\mathcal{P}_+\overline{D_b\mathcal{P}_+} + G^{a\bar{b}}D_a\mathcal{P}_3\overline{D_b\mathcal{P}_3} - 3|\mathcal{P}_+|^2 - 3|\mathcal{P}_3|^2 + G^{i\bar{\jmath}}D_i\mathcal{P}_+\overline{D_j\mathcal{P}_+} + |\mathcal{P}_+|^2 + 4|\mathcal{P}_3|^2 \end{aligned}$$

e.g., Cassani, et.al. [arXiv:0911.2708]

Contribution of hypermultiplets to the Killing prepotentials: Vector sector

$$\Pi_{\mathsf{V}} = \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2}(X^{\Lambda}, \mathcal{F}_{\Lambda})^{\mathrm{T}}$$
$$z^a = X^a / X^0$$
$$a = 1, \ldots, n_{\mathsf{V}}$$
SKG$_{\mathsf{V}}$ in vector-moduli

$$\begin{aligned} \mathcal{P}_+ &\equiv \mathcal{P}_1 + i\mathcal{P}_2 &= 2\mathrm{e}^{\varphi}\, \Pi_{\mathsf{V}}^{\mathrm{T}}\, Q\, \mathbb{C}_{\mathsf{H}}\, \Pi_{\mathsf{H}} \\ \mathcal{P}_- &\equiv \mathcal{P}_1 - i\mathcal{P}_2 &= 2\mathrm{e}^{\varphi}\, \Pi_{\mathsf{V}}^{\mathrm{T}}\, Q\, \mathbb{C}_{\mathsf{H}}\, \overline{\Pi_{\mathsf{H}}} \\ & \mathcal{P}_3 &= \mathrm{e}^{2\varphi}\, \Pi_{\mathsf{V}}^{\mathrm{T}}\, \mathbb{C}_{\mathsf{V}}(c + \widetilde{Q}\xi) \end{aligned}$$

$$\Pi_{\mathsf{H}} = \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2}(Z^{I}, \mathcal{G}_{I})^{\mathrm{T}}$$
$$\mathsf{z}^i = Z^i / Z^0$$
$$i = 1, \ldots, n_{\mathsf{H}}$$
SKG$_{\mathsf{H}}$ in hyper-moduli

Functions of period matrices:

$$\mathbb{M}_{\mathsf{V,H}} \equiv \begin{pmatrix} 1 & -\nu \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mu & 0 \\ 0 & \mu^{-1} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\nu & 1 \end{pmatrix}_{\mathsf{V,H}} = \begin{pmatrix} \mu + \nu\mu^{-1}\nu & -\nu\mu^{-1} \\ -\mu^{-1}\nu & \mu^{-1} \end{pmatrix}_{\mathsf{V,H}}, \quad \begin{aligned} \mu_{\mathsf{V,H}} &\equiv \mathrm{Im}\,\mathcal{N}_{\mathsf{V,H}} \\ \nu_{\mathsf{V,H}} &\equiv \mathrm{Re}\,\mathcal{N}_{\mathsf{V,H}} \end{aligned}$$

Electric/magnetic charges with constraints ($n_{\mathsf{V}} \leq n_{\mathsf{H}}$):

$$Q_{\mathbf{\Lambda}}{}^{\mathbb{I}} \equiv \begin{pmatrix} e_{\Lambda}{}^{I} & e_{\Lambda I} \\ m^{\Lambda I} & m^{\Lambda}{}_{I} \end{pmatrix}, \quad c \equiv \begin{pmatrix} p^{\Lambda} \\ q_{\Lambda} \end{pmatrix}; \quad \mathbb{C}_{\mathsf{V,H}} \equiv \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}_{\mathsf{V,H}}, \quad \widetilde{Q}^{\mathbf{\Lambda}}{}_{\mathbb{I}} \equiv \mathbb{C}_{\mathsf{V}}^{\mathrm{T}}\, Q\, \mathbb{C}_{\mathsf{H}}$$

$Q\,\mathbb{C}_{\mathsf{H}}\,Q^{\mathrm{T}} = 0 = c^{\mathrm{T}}Q$: Closure condition of massive tensor gauge transformation (tensors appear!)
Tadpole cancellation in 10D string theory,
or Nilpotency of exterior derivative on $SU(3) \times SU(3)$ generalized geometry

$Q^{\mathrm{T}}\,\mathbb{C}_{\mathsf{V}}\,Q = 0$: Abelianity condition (of Heisenberg algebra) $\longrightarrow$ possible to non-abelianize?
What is 10D origin?

- If p^Λ exists, the axion a is dualized (back) to NS B_2-field (magnetically coupled).

  Gauge field strength is replaced: $F_2^\Lambda \to \widehat{F}_2^\Lambda = F_2^\Lambda + p^\Lambda B_2$ and B_2 becomes massive.

  e.g., D'Auria, et.al. [hep-th/0409097]

- In addition, if non-vanishing $(m^{\Lambda I}, m^\Lambda{}_J)$ are incorporated,

$$\longrightarrow \begin{cases} n_\mathsf{V} + 1 (\leq n_\mathsf{H} + 1) \text{ RR-axions in } (\xi^I, \widetilde{\xi}_J) \text{ become massive tensors } (C_{2\widehat{I}}, C_2^{\widehat{J}}) \\ n_\mathsf{V} + 1 (\leq n_\mathsf{H} + 1) \text{ RR-axions in } (\xi^I, \widetilde{\xi}_J) \text{ become massive scalars} \\ \text{Gauge field strength is replaced: } F_2^\Lambda \to \widehat{F}_2^\Lambda = F_2^\Lambda + m^{\Lambda \widehat{I}} C_{2\widehat{I}} + m^\Lambda{}_{\widehat{J}} C_2^{\widehat{J}} + p^\Lambda B_2 \\ \text{Remaining } 2(n_\mathsf{H} - n_\mathsf{V}) \text{ RR-axions are flat directions for } V \end{cases}$$

$I = 0, \ldots, n_\mathsf{H}$; $\widehat{I} = 0, \ldots, n_\mathsf{V}$ with $n_\mathsf{V} \leq n_\mathsf{H}$.

We can choose $\xi^{\widehat{I}}$ to dualized $C_{2\widehat{I}}$, while $\widetilde{\xi}_{\widehat{I}}$ to massive scalars.

> When the massive fields are integrated out under the conditions,
> the scalar potential V carries only Π_V, Π_H and φ.

e.g., D'Auria, et.al. [hep-th/0701247], Cassani [arXiv:0804.0595]

Contents

δg^{tt} :

$$-\mathrm{e}^{-2B}\Big[\frac{1}{r^2}\big(1-\mathrm{e}^{2(B-C)}\big)+C'(3C'-2B')+\frac{2}{r}(3C'-B')+2C''\Big]+\mathrm{e}^{-2B}G_{a\bar{b}}z^{a\prime}\bar{z}^{\bar{b}\prime} = \frac{\mathrm{e}^{-4C}}{r^4}I_S+V$$

δg^{rr} :

$$-\mathrm{e}^{-2B}\Big[\frac{1}{r^2}\big(1-\mathrm{e}^{2(B-C)}\big)+C'(C'+2A')+\frac{2}{r}(C'+A')\Big]+\mathrm{e}^{-2B}G_{a\bar{b}}z^{a\prime}\bar{z}^{\bar{b}\prime} = \frac{\mathrm{e}^{-4C}}{r^4}I_S+V$$

$\delta g^{\theta\theta},\ \delta g^{\phi\phi}$:

$$-\mathrm{e}^{-2B}\Big[A''+C''+A'(A'-B')+C'(A'-B'+C')+\frac{1}{r}(A'-B'+2C')\Big]-\mathrm{e}^{-2B}G_{a\bar{b}}z^{a\prime}\bar{z}^{\bar{b}\prime} = -\frac{\mathrm{e}^{-4C}}{r^4}I_S+V$$

δz^a :

$$\mathrm{e}^{-2B}\Big[G_{a\bar{b}}\bar{z}^{\bar{b}\prime\prime}+\frac{\partial G_{a\bar{b}}}{\partial\bar{z}^{\bar{c}}}\bar{z}^{\bar{b}\prime}\bar{z}^{\bar{c}\prime}+G_{a\bar{b}}\bar{z}^{\bar{b}\prime}\big(A'-B'+2C'+\frac{2}{r}\big)\Big] = \frac{\mathrm{e}^{-4C}}{r^4}\frac{\partial I_S}{\partial z^a}+\frac{\partial V}{\partial z^a}$$

Bellucci, et.al. [arXiv:0802.0141]

Comment: We have already known $I_S(z, \overline{z}, p, q)$, $V(z, \overline{z}, p, q)$ and their derivatives as

Attractor Equations CLICK

- $\partial_a I_S = 0$ appears in the search of extremal RN-BH in $\mathcal{N} = 2$ SUGRA

$$I_S = |Z|^2 + G^{a\overline{b}} D_a Z \overline{D_b Z} \,, \qquad \partial_a I_S = 2\overline{Z} D_a Z + i C_{abc} G^{b\overline{b}} G^{c\overline{c}} \, \overline{D_b Z} \, \overline{D_c Z}$$

- $\partial_a V = 0$ has the same form which appears in the search of (flux) vacua of $\mathcal{N} = 1$ SUGRA

$$V = -3|Z|^2 + G^{a\overline{b}} D_a Z \overline{D_b Z} \,, \qquad \partial_a V = -2\overline{Z} D_a Z + i C_{abc} G^{b\overline{b}} G^{c\overline{c}} \, \overline{D_b Z} \, \overline{D_c Z}$$

Z in $\mathcal{N} = 1$ SUGRA is not the central charge but the superpotential

$C_{abc} = \partial_a X^\Lambda \partial_b X^\Sigma \partial_c X^\Gamma \mathcal{F}_{\Lambda\Sigma\Gamma}$

A useful formula among the BH charges $\Gamma = (p^\Lambda, q_\Lambda)^{\mathrm{T}}$ and the invariant $I_S(z, \overline{z}, p, q)$

$$\Gamma^{\mathrm{T}} + i\frac{\partial I_S}{\partial \widetilde{\Gamma}} \;=\; 2i\overline{Z}\Pi^{\mathrm{T}} + 2iG^{a\bar{b}} D_a Z \overline{D_b \Pi}^{\mathrm{T}}$$

$$\widetilde{\Gamma} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} \Gamma \,, \qquad \Pi = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix} \,, \qquad Z = L^\Lambda q_\Lambda - M_\Lambda p^\Lambda = \widetilde{\Gamma}^{\mathrm{T}} \Pi$$

This does not (explicitly) depend on the scalar potential $-\mathrm{g}^2 V$.

This can be applied to any points in the spacetime.

Kallosh, Sivanandam and Soroush [hep-th/0606263]

- Single modulus model ($a = 1$): $\mathcal{F} = \dfrac{(X^1)^3}{X^0}$

$$Z \;=\; \mathrm{e}^{K/2}\Big(q_0 + q\,t - 3p\,t^2 + p^0\,t^3\Big)\,, \qquad t \;=\; \frac{X^1}{X^0}$$

$$\mathrm{e}^K \;=\; \frac{i}{(t-\bar{t})^3}\,, \qquad G_{t\bar{t}} \;=\; -\frac{3}{(t-\bar{t})^2} \;\equiv\; e_t{}^{\hat{1}}\,e_{\bar{t}}{}^{\bar{\hat{1}}}\,\delta_{\hat{1}\bar{\hat{1}}}\,, \qquad C_{ttt} \;=\; \frac{6i}{(t-\bar{t})^3}$$

Search a solution with $V = -3|Z|^2 + |D_{\hat{1}}Z|^2 < 0 \to Z \neq 0$

Consider a non-SUSY solution $\to D_{\hat{1}}Z \neq 0$

$\Downarrow$

The generic forms of the central charge and its derivative:

$$Z \;\equiv\; -i\rho\,\mathrm{e}^{i(\alpha-3\phi)}\,, \qquad D_{\hat{1}}Z \;\equiv\; \sigma\,\mathrm{e}^{-i\phi} \qquad (\rho, \sigma > 0)$$

Kallosh, et.al. [hep-th/0606263]

The generic forms: $Z \equiv -i\rho\, \mathrm{e}^{i(\alpha - 3\phi)}\,, \qquad D_{\hat{1}} Z \equiv \sigma\, \mathrm{e}^{-i\phi} \qquad (\rho, \sigma > 0)$

The volume factors ρ and σ are related via the attractor equation.

$$\sigma = -\frac{\rho}{3}\, \mathrm{e}^{-i\alpha} G_{\mathsf{V}} \qquad (G_{\mathsf{V}} \neq 0)$$

The formula leads to the following two equations: $(\Gamma = (p^0, p, q, q_0)^{\mathrm{T}})$:

$$p + \frac{\partial I_S}{\partial q} = -\frac{2\rho}{3\sqrt{3}}\, \mathrm{e}^{-i\alpha} \mathrm{e}^{K/2} \Big[\big(3\sqrt{3} - 2G_{\mathsf{V}}\big)\, t - G_{\mathsf{V}}\, \bar{t}\, \Big]$$

$$p^0 + \frac{\partial I_S}{\partial q_0} = -\frac{2\rho}{3\sqrt{3}}\, \mathrm{e}^{-i\alpha} \mathrm{e}^{K/2} \big(3\sqrt{3} - G_{\mathsf{V}}\big)$$

$$\rightarrow \qquad t = \frac{3\sqrt{3} - 2G_{\mathsf{V}}}{3\sqrt{3} - G_{\mathsf{V}}} \left[\frac{p + i\frac{\partial I_S}{\partial q}}{p^0 + i\frac{\partial I_S}{\partial q_0}} \right] + \frac{G_{\mathsf{V}}}{3\sqrt{3} - G_{\mathsf{V}}} \left[\frac{p - i\frac{\partial I_S}{\partial q}}{p^0 - i\frac{\partial I_S}{\partial q_0}} \right]$$

"generic solution"

Difficult to evaluate the explicit solution caused by the complicated functions G_{V} and I_S

- Three Moduli model called the STU-model: $\mathcal{F} = \dfrac{X^1 X^2 X^3}{X^0}$

(Cartan part of 4D $\mathcal{N} = 8$ $SO(8)$ gauged SUGRA $\leftarrow$ IIA/IIB/Heterotic string triality)

$$Z = e^{K/2}\Big(q_0 + q_a z^a - p^1 z^2 z^3 - p^2 z^3 z^1 - p^3 z^1 z^2 + p^0 z^1 z^2 z^3 \Big) \, , \qquad z^a = \frac{X^a}{X^0}$$

$$K = -\log \Big[-i(z^1 - \overline{z}^1)(z^2 - \overline{z}^2)(z^3 - \overline{z}^3) \Big]$$

$$G_{a\overline{b}} = -\frac{\delta_{ab}}{(z^a - \overline{z}^{\overline{a}})^2} = e_a{}^{\widehat{a}} \, e_{\overline{b}}{}^{\overline{\widehat{b}}} \, \delta_{\widehat{a}\overline{\widehat{b}}} \, , \qquad C_{\widehat{1}\widehat{2}\widehat{3}} = 1$$

Search a solution with $V = -3|Z|^2 + |D_{\widehat{a}} Z|^2 < 0 \to Z \neq 0$

Consider a non-SUSY solution $\to D_{\widehat{a}} Z \neq 0$

$\Downarrow$

The generic forms: $Z \equiv -i\rho \, e^{i(\alpha - 3\phi)} \, , \qquad D_{\widehat{a}} Z \equiv \sigma \, e^{-i\phi} \qquad (\rho, \sigma > 0)$

Kallosh, et.al. [hep-th/0606263]

The generic forms: $Z \equiv -i\rho\, \mathrm{e}^{i(\alpha - 3\phi)}\,, \qquad D_{\widehat{a}} Z \equiv \sigma\, \mathrm{e}^{-i\phi} \qquad (\rho, \sigma > 0)$

The volume factors ρ and σ are related via the attractor equation.

$$\sigma = -\rho\, \mathrm{e}^{-i\alpha} G_{\mathsf{V}} \qquad (G_{\mathsf{V}} \neq 0)$$

The formula leads to the following two equations:

$$p^a + \frac{\partial I_S}{\partial q_a} = -2\rho\, \mathrm{e}^{-i\alpha} \mathrm{e}^{K/2} \Big[\big(1 - G_{\mathsf{V}}\big)\, z^a - 2 G_{\mathsf{V}}\, \overline{z}^{\overline{a}} \Big]$$

$$p^0 + \frac{\partial I_S}{\partial q_0} = -2\rho\, \mathrm{e}^{-i\alpha} \mathrm{e}^{K/2} \big(1 - 3G_{\mathsf{V}}\big)$$

$$\longrightarrow \qquad z^a = V_{\mathsf{eff}}^2 \left[\frac{p^a + i \frac{\partial I_S}{\partial q_a}}{p^0 + i \frac{\partial I_S}{\partial q_0}} \right] + (1 - V_{\mathsf{eff}}^2) \left[\frac{p^a - i \frac{\partial I_S}{\partial q_a}}{p^0 - i \frac{\partial I_S}{\partial q_0}} \right]$$

"generic solution"

Neither $V_{\mathsf{eff}} = 1$ nor $V_{\mathsf{eff}} = 0$

Difficult to evaluate the explicit solution caused by the complicated functions G_{V} and I_S