Talk at Universität Hamburg/DESY: Hamburg, GERMANY

November 01, 2010

NON-SUSY SOLUTIONS OF RN-AdS BLACK HOLES IN 4D $\mathcal{N} = 2$ GAUGED SUPERGRAVITY

HIGH ENERGY ACCELERATOR RESEARCH ORGANIZATION

based on JHEP 09 (2010) 061 [arXiv:1005.4607]

Tetsuji KIMURA (KEK, JAPAN)

Research Interest: String Compactification beyond Calabi-Yau

Fluxes and/or D-branes on "CY" $\longrightarrow$ back reactions to CY?

My Previous Works

- Heterotic string with H, $\partial\phi$ and torsion
  No-go theorem [hep-th/0605247], Index theorems [arXiv:0704.2111], Intersecting five-branes [arXiv:0912.1334]
- Type II strings with (non)geometric fluxes
  $\mathcal{N}=1$ AdS/Minkowski Vacua via generalized geometry [arXiv:0810.0937], T-duality on doubled geometry with D-branes [arXiv:0806.1783]

— Look at Lower-dimensional Physics —

Is it possible to understand "all" SUGRAs from reductions/truncations of String Theories?

(We hope so, then we study them!)

Current Interest: AdS Black Hole Solutions in 4D $\mathcal{N} = 2$ SUGRA

- WHY $\mathcal{N} = 2$ (8-SUSY charges)?
  - ✓ Scalar fields living in highly symmetric spaces
  - ✓ (Flux) compactification scenarios in string/M-theory
- WHY AdS Black Holes?
  - ✓ Attractive: 4D $\mathcal{N} = 2$ SUGRA $\subset$ Einstein-Yang–Mills-Matters
  - ✓ Non-trivial: the existence of the cosmological constant with matter fields

BH gives rise to a perspective on gauged SUGRA with magnetic charges and matters.
Magnetic charges carry information of string dualities:

Some magnetic charges come from nongeometric fluxes on internal space
whose transition function contains T-duality $SO(6,6)$ beyond diffeomorphism $GL(6,\mathbb{R})$.

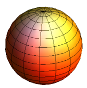
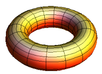
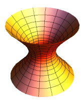
e.g., Graña, Louis and Waldram [hep-th/0612237], D'Auria, Ferrara and Trigiante [hep-th/0701247]

Static, charged AdS black hole in 4D spacetime:

$$\mathrm{d}s^2 \;=\; -V(r)\,\mathrm{d}t^2 + \frac{1}{V(r)}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\sigma^2_{X_2}$$

$$V(r) \;=\; \kappa - \frac{2\eta}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\,, \quad Q^2 \;=\; \underbrace{q^2}_{\text{(elect.)}} + \underbrace{p^2}_{\text{(magnet.)}}\,, \quad \Lambda \;=\; \text{(cosmological const.)}$$

$$\Lambda \;=\; -\frac{3}{\ell^2}\,, \qquad \eta_0 \;=\; \frac{\ell}{3\sqrt{6}}\Big(\sqrt{\kappa^2 + 12\,Q^2\,\ell^{-2}} + 2\kappa\Big)\Big(\sqrt{\kappa^2 + 12\,Q^2\,\ell^{-2}} - \kappa\Big)^{1/2}$$

topology of X_2 w/ genus g		metric	ADM mass M
$\kappa = 1$ ($g = 0$; S^2)		: $\mathrm{d}\sigma^2_{X_2} = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2$	η
$\kappa = 0$ ($g = 1$; flat)		: $\mathrm{d}\sigma^2_{X_2} = \mathrm{d}x^2 + 2\mathrm{Re}\tau\,\mathrm{d}x\,\mathrm{d}y + \lvert\tau\rvert^2\mathrm{d}y^2\,;\;\; x, y \in [0,1]$	$\dfrac{\eta\lvert\mathrm{Im}\tau\rvert}{4\pi}$
$\kappa = -1$ ($g > 1$; H^2)		: $\mathrm{d}\sigma^2_{X_2} = \mathrm{d}\theta^2 + \sinh^2\theta\,\mathrm{d}\phi^2$	$(\eta - \eta_0)(g - 1)$

e.g., Caldarelli and Klemm [hep-th/9808097]

Static, charged AdS black hole in 4D spacetime with $\kappa = 1$:

$$\mathrm{d}s^2 = -V(r)\,\mathrm{d}t^2 + \frac{1}{V(r)}\,\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$

$$V(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\,, \quad Q^2 = \underset{\text{(elect.)}}{q^2} + \underset{\text{(magnet.)}}{p^2}\,, \quad \Lambda = \text{(cosmological const.)}$$

$$\Lambda = -\frac{3}{\ell^2}\,, \qquad \eta_0 = \frac{\ell}{3\sqrt{6}}\left(\sqrt{1 + 12\,Q^2\,\ell^{-2}} + 2\right)\left(\sqrt{1 + 12\,Q^2\,\ell^{-2}} - 1\right)^{1/2}$$

Various (non-rotating) AdS black holes:

Schwarzschild-AdS : $M \neq 0,\ \ Q = 0,\ \ \Lambda < 0$

Reissner-Nordström AdS (RN-AdS) : $M \neq 0,\ \ Q \neq 0,\ \ \Lambda < 0$

Condition for regular event horizon: $M \geq \eta_0$

Extremality: $M = \eta_0$

e.g., Caldarelli, et.al. [hep-th/9808097]

- In the framework of 4D $\mathcal{N}=2$ ungauged SUGRA,

  (Extremal) RN-BH in asymptotically flat spacetime has been investigated.

  BH charges from D-branes wrapped on CY: "D0-D4", "D2-D6", "D0-D2-D6", etc.

  (Hypermultiplets are decoupled from the system.)

- We (have to) study AdS BH in 4D $\mathcal{N}=2$ system.

  The cosmological constant Λ is given as expectation value of scalar potential V.

  ("mass deformations" of gravitini)

Remark: Naked Singularity appears in SUSY RN-AdS solution.

- ☞ Pure AdS SUGRA (only gravitational multiplet):

  Romans [hep-th/9203018], Caldarelli, et.al. [hep-th/9808097], etc.

  There exist SUSY solutions of rotating AdS black holes with regular horizons.

- ☞ Gauged SUGRA with vector multiplets (without hyper-sector):

  Sabra, et.al. (electric charges [hep-th/9903143], magnetic/dyonic charges [hep-th/0003213]), etc.

Question

How can we obtain non-SUSY solutions with matter fields in asymptotically AdS spacetime?

— Steps —

- Vector-multiplets + electric FI (studied)*
- Vector-multiplets + electric/magnetic FI (as electric/magnetic charges) This Work
- Vector- + Hypermultiplets ?†
- Vector- + (massive) Scalar–Tensor-multiplets ?$^{\natural}$
- Non-abelianization ?$^{\sharp}$

$\star$ ex.) Bellucci, Ferrara, Marrani and Yeranyan [arXiv:0802.0141], etc.
$\dagger$ ex.) 1 vector + 2 hypers in terms of Sasaki-Einstein$_7$ in M-theory [arXiv:1009.3805], etc.
$\natural$ Directly related to (non)geometric flux backgrounds via [hep-th/0409097], [hep-th/0701247], etc.
$\sharp$ Deform Heisenberg algebra(?), e.g. [hep-th/0410290], etc.

Question

How can we obtain non-SUSY solutions with matter fields
in asymptotically AdS spacetime?

— **Setup** and **Result** —

- 4D $\mathcal{N} = 2$ Abelian gauged SUGRA with mutually local electric/magnetic charges
- Truncate out hyper-sector by hand (artificial)
- Import the Attractor Mechanism
- Find a new attractor equation
- Obtain a non-SUSY solution with small $|\Lambda|$ in T^3-model
- Argue "generic descriptions of moduli" in T^3-model and in STU-model

Question

How can we obtain non-SUSY solutions with matter fields in asymptotically AdS spacetime?

— **Open Problems** —

- Strange behaviors of Λ and size of the event horizon
- Hyper-sector: axions become massive tensors if "magnetic" charges are present

$$\text{SQG} \longrightarrow \underbrace{[\text{SKG} + \text{dilaton}]}_{\text{contribute to } V} + \underbrace{\underbrace{[\text{NS-axion}]}_{\text{massive tensor}} + \underbrace{[\text{RR-axions}]}_{\text{massive or flat}}}_{\text{not contribute to } V(?)}$$

(Property of internal space emerges in hyper-sector via magnetic charges.)

- Is the attractor mechanism still available?

Contents

Contents

Supersymmetric multiplets in 4D $\mathcal{N} = 2$ SUGRA:

1 gravitational multiplet: $\{g_{\mu\nu}, A_\mu^0, \psi_{A\mu}\}$ $\mu = 0, 1, 2, 3$ (4D, curved) $A = 1, 2$ ($SU(2)$ R-symmetry)

n_{V} vector multiplets: $\{A_\mu^a, z^a, \lambda^{aA}\}$ $a = 1, \ldots, n_{\mathsf{V}}$

z^a in special Kähler geometry $\mathcal{SM}$

$n_{\mathsf{H}} + 1$ hypermultiplets: $\{q^u, \zeta_\alpha\}$ $u = 1, \ldots, 4n_{\mathsf{H}} + 4$ $\alpha = 1, \ldots, 2n_{\mathsf{H}} + 2$

q^u in quaternionic geometry $\mathcal{HM}$

Gauging: *Promote* global isometry groups on scalar field spaces to local symmetries

"Standard": Andrianopoli, Bertolini, Ceresole, D'Auria, Ferrara, Fré and Magri [hep-th/9605032]

"Magnetically dualized": D'Auria, Sommovigo and Vaulà [hep-th/0409097]

Action (gravitational const. κ; gauge coupling const. g; index $\Lambda = 0, 1, \ldots, n_{\mathsf{V}}$):

$$
\begin{aligned}
S \;=\; \int \mathrm{d}^4 x \sqrt{-g} \Big\{ \frac{1}{2\kappa^2} R - G_{a\bar{b}}(z,\bar{z}) \nabla_\mu z^a \nabla^\mu \bar{z}^{\bar{b}} - h_{uv}(q) \nabla_\mu q^u \nabla^\mu q^v \\
+ \frac{1}{4} \mu_{\Lambda\Sigma}(z,\bar{z}) F^\Lambda_{\mu\nu} F^{\Sigma\mu\nu} + \frac{1}{4} \nu_{\Lambda\Sigma}(z,\bar{z}) F^\Lambda_{\mu\nu} (*F^\Sigma)^{\mu\nu} \\
- \mathrm{g}^2 V(z,\bar{z},q) \\
+ \text{(fermionic terms)} \Big\}
\end{aligned}
$$

$$\mu_{\Lambda\Sigma} = \mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma} \quad \text{(generalized } -1/\mathrm{g}^2\text{)}\,, \qquad \nu_{\Lambda\Sigma} = \mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma} \quad \text{(generalized } \theta\text{-angle)}$$

In this analysis...

- Set background fermionic fields to zero
- Reduce gauge symmetries to *abelian*: $\nabla_\mu z^a \to \partial_\mu z^a$
- Truncate hyper-sector *by hand*, *after the formulation*

Appendix CLICK!!

Equations of Motion (abbreviate κ and g, and hyper-sector):

$$\delta g^{\mu\nu} : \quad \Big(R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} \Big) - 2 G_{a\bar{b}} \, \partial_{(\mu} z^a \partial_{\nu)} \overline{z}^{\bar{b}} + G_{a\bar{b}} \, \partial_\rho z^a \partial^\rho \overline{z}^{\bar{b}} \, g_{\mu\nu} \ = \ T_{\mu\nu} - V g_{\mu\nu}$$

$$T_{\mu\nu} \ = \ - \mu_{\Lambda\Sigma} \, F^{\Lambda}_{\mu\rho} \, F^{\Sigma}_{\nu\sigma} \, g^{\rho\sigma} + \frac{1}{4} \mu_{\Lambda\Sigma} \, F^{\Lambda}_{\rho\sigma} \, F^{\Sigma\rho\sigma} \, g_{\mu\nu} \qquad \text{(energy-momentum tensor)}$$

$$\delta z^a : \quad - \frac{G_{a\bar{b}}}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g} g^{\mu\nu} \partial_\nu \overline{z}^{\bar{b}} \Big) - \frac{\partial G_{a\bar{b}}}{\partial \overline{z}^{\bar{c}}} \, \partial_\rho \overline{z}^{\bar{b}} \, \partial^\rho \overline{z}^{\bar{c}}$$

$$= \ \frac{1}{4} \frac{\partial \mu_{\Lambda\Sigma}}{\partial z^a} F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} + \frac{1}{4} \frac{\partial \nu_{\Lambda\Sigma}}{\partial z^a} F^{\Lambda}_{\mu\nu} (* F^{\Sigma})^{\mu\nu} - \frac{\partial V}{\partial z^a}$$

$$\delta A^{\Lambda}_{\mu} : \quad \varepsilon^{\mu\nu\rho\sigma} \partial_\nu G_{\Lambda\rho\sigma} \ = \ 0 \, , \qquad G_{\Lambda\rho\sigma} \ = \ \nu_{\Lambda\Sigma} F^{\Sigma}_{\rho\sigma} - \mu_{\Lambda\Sigma} (* F^{\Sigma})_{\rho\sigma}$$

electric charge $q_{\Lambda} \equiv \dfrac{1}{4\pi} \displaystyle\int_{S^2} G_{2\Lambda} \, ,$ magnetic charge $p^{\Lambda} \equiv \dfrac{1}{4\pi} \displaystyle\int_{S^2} F^{\Lambda}_{2}$

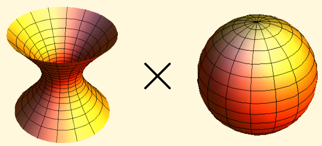
Introduce ansatz for RN(-AdS) BH: Extremal, static, charged, spherically symmetric

$$\boxed{\mathrm{d}s^2 \;=\; -\,\mathrm{e}^{2A(r)}\mathrm{d}t^2 + \mathrm{e}^{2B(r)}\mathrm{d}r^2 + \mathrm{e}^{2C(r)}r^2\big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\big)}$$

Near horizon geometry: $\mathrm{AdS}_2 \times S^2$ (radii: r_{A} and r_{H})

$$A(r) \;=\; \log\frac{r - r_{\mathsf{H}}}{r_{\mathsf{A}}}\,, \qquad B(r) \;=\; -A(r)\,, \qquad C(r) \;=\; \log\frac{r_{\mathsf{H}}}{r}$$

$$R(\mathrm{AdS}_2 \times S^2) \;=\; 2\Big(-\frac{1}{r_{\mathsf{A}}^2} + \frac{1}{r_{\mathsf{H}}^2}\Big)$$



$$\begin{aligned}
\to \quad \mathrm{d}s^2_{\text{(near horizon)}} \;&=\; -\left(\frac{r - r_{\mathsf{H}}}{r_{\mathsf{A}}}\right)^2 \mathrm{d}t^2 + \left(\frac{r_{\mathsf{A}}}{r - r_{\mathsf{H}}}\right)^2 \mathrm{d}r^2 + r_{\mathsf{H}}^2\big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\big) \\
&=\; -\frac{\mathrm{e}^{2\tau}}{r_{\mathsf{A}}^2}\mathrm{d}t^2 + r_{\mathsf{A}}^2\mathrm{d}\tau^2 + r_{\mathsf{H}}^2\big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\big) \qquad (\tau = \log(r - r_{\mathsf{H}}))
\end{aligned}$$

If the attractor mechanism works (via extremality), the scalar fields are subject to

$$\frac{\mathrm{d}}{\mathrm{d}r} z^a \Big|_{\text{horizon}} = 0 \,, \qquad \Big(\frac{\mathrm{d}}{\mathrm{d}r} \Big)^2 z^a \Big|_{\text{horizon}} = 0$$

The EoM are drastically reduced to Bellucci, et.al. [arXiv:0802.0141]

$$\begin{aligned}
\delta g^{tt}, \, \delta g^{rr} : \quad \frac{1}{r_{\mathsf{H}}^2} &= \frac{1}{r_{\mathsf{H}}^4} I_S + V \Big|_{\text{horizon}} & \longrightarrow \quad r_{\mathsf{H}}^2 &= \frac{1 - \sqrt{1 - 4 I_S V}}{2V} \Big|_{\text{horizon}} \\
\delta g^{\theta\theta}, \, \delta g^{\phi\phi} : \quad \frac{1}{r_{\mathsf{A}}^2} &= \frac{1}{r_{\mathsf{H}}^4} I_S - V \Big|_{\text{horizon}} & \longrightarrow \quad r_{\mathsf{A}}^2 &= \frac{r_{\mathsf{H}}^2}{\sqrt{1 - 4 I_S V}} \Big|_{\text{horizon}} \\
\delta z^a : \quad 0 &= \frac{1}{r_{\mathsf{H}}^4} \frac{\partial I_S}{\partial z^a} - \frac{\partial V}{\partial z^a} \Big|_{\text{horizon}} & \longrightarrow \quad 0 &= \frac{1}{r_{\mathsf{H}}^4} (1 - 2 r_{\mathsf{H}}^2 V) \frac{\partial}{\partial z^a} r_{\mathsf{H}}^2 \Big|_{\text{horizon}}
\end{aligned}$$

(1st) Symplectic Invariant:

$$I_S(z, \overline{z}, p, q) = -\frac{1}{2} \big(p^{\Lambda} \ q_{\Lambda} \big) \begin{pmatrix} \mu_{\Lambda\Sigma} + \nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Delta} \nu_{\Delta\Sigma} & -\nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Sigma} \\ -(\mu^{-1})^{\Lambda\Gamma} \nu_{\Gamma\Sigma} & (\mu^{-1})^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} p^{\Sigma} \\ q_{\Sigma} \end{pmatrix} \equiv -\frac{1}{2} \Gamma^{\mathrm{T}} \mathbb{M} \Gamma$$

with $\quad T_t{}^t = T_r{}^r = -T_{\theta}{}^{\theta} = -T_{\phi}{}^{\phi} = -\frac{\mathrm{e}^{-4C}}{r^4} I_S$

BH entropy (and the effective potential) is given as the Area of the event horizon:

$$S_{\text{BH}}(p,q) \;=\; \frac{A_{\text{H}}}{4\pi} \;=\; r_{\text{H}}^2\Big|_{\text{horizon}} \;\equiv\; V_{\text{eff}}(z,\overline{z},p,q)\Big|_{\text{horizon}}$$

$$\boxed{V_{\text{eff}}(z,\overline{z},p,q) \;=\; \frac{1-\sqrt{1-4I_S V}}{2V} \;=\; I_S + (I_S)^2 V + 2(I_S)^3 V^2 + \mathcal{O}((I_S)^4 V^3)}$$

We read the "cosmological constant Λ" from the scalar curvature:

$$R(\text{AdS}_2 \times S^2) \;=\; 2\Big(-\frac{1}{r_{\text{A}}^2} + \frac{1}{r_{\text{H}}^2}\Big) \;=\; 4V$$

$$V\Big|_{\text{horizon}} \;\equiv\; \Lambda(\text{“cosmological constant”})$$

Attractor Equation

$$0 \;=\; \frac{1}{r_{\text{H}}^4}\,(1-2r_{\text{H}}^2 V)\,\frac{\partial}{\partial z^a} V_{\text{eff}}\Big|_{\text{horizon}} \qquad \rightarrow \qquad 0 \;=\; \frac{\partial}{\partial z^a} V_{\text{eff}}(z,\overline{z},p,q)\Big|_{\text{horizon}}$$

If $r_{\text{H}} < \infty$ and $V\Big|_{\text{horizon}} \leq 0$

The "ATTRACTOR EQUATION" which we have to solve is

$$0 \;=\; \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \overline{z}, p, q)\Big|_{\text{horizon}}$$

$$=\; \frac{1}{2V^2\sqrt{1 - 4I_S V}} \left\{ 2V^2 \frac{\partial I_S}{\partial z^a} - \left(\sqrt{1 - 4I_S V} + 2I_S V - 1\right) \frac{\partial V}{\partial z^a} \right\} \Big|_{\text{horizon}}$$

Evaluate I_S and V described in terms of the central charge Z

Useful when we consider (non-)SUSY solutions

SUSY variation of gravitini carries definition of Z *and more..*

$$\delta\psi_{A\mu} \;=\; D_\mu \varepsilon_A + \epsilon_{AB}\, T^-_{\mu\nu}\, \gamma^\nu\, \varepsilon^B + i\, \mathrm{g}\, \mathcal{S}_{AB}\, \gamma_\mu\, \varepsilon^B$$

$$Z \;=\; -\frac{1}{2} \left(\frac{1}{4\pi} \int_{S^2} T^-_2 \right), \qquad \mathcal{S}_{AB} \;=\; \frac{i}{2} \sum_{x=1}^{3} (\sigma^x)_{AB} \mathcal{P}_x$$

Describe Z, I_S and V in terms of $(L^\Lambda, M_\Lambda) = \mathrm{e}^{K/2}(X^\Lambda, \mathcal{F}_\Lambda)$ on $\mathcal{SM}$:

$$\begin{aligned} Z &= L^\Lambda q_\Lambda - M_\Lambda p^\Lambda \\ I_S &= |Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z} \\ V &= \sum_{x=1}^{3} \Big(-3|\mathcal{P}_x|^2 + G^{a\bar{b}} D_a \mathcal{P}_x \overline{D_b \mathcal{P}_x} \Big) + 4h_{uv} k^u \overline{k}^v \end{aligned}$$

$\mathcal{P}_x$: $SU(2)$ triplet of Killing prepotentials in $\mathcal{N} = 2$ SUGRA

(In general, both vector moduli and hyper moduli contribute to $\mathcal{P}_x$) Appendix

Hyper-sector is "truncated" $\rightarrow$ $\mathcal{P}_3 = \mathcal{P}_{3,\Lambda} L^\Lambda - \widetilde{\mathcal{P}}_3^\Lambda M_\Lambda$ still remains in V

Further, Identify electric/magnetic FI with charges $(\mathcal{P}_{3,\Lambda}, \widetilde{\mathcal{P}}_3^\Lambda) = (q_\Lambda, p^\Lambda) \rightsquigarrow \mathcal{P}_3 \equiv Z$

Cassani, Ferrara, Marrani, Morales and Samtleben [arXiv:0911.2708]

$$\boxed{V \equiv -3|Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}}$$

The "ATTRACTOR EQUATION" (with a non-trivial factor $-1 \leq G_{\mathsf{V}} \equiv \dfrac{1 - V_{\text{eff}}^2}{1 + V_{\text{eff}}^2} \leq 1$)

$$
\begin{aligned}
0 &= \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \overline{z}, p, q) \Big|_{\text{horizon}} \\
&= \frac{1}{2V^2 \sqrt{1 - 4 I_S V}} \left\{ 2V^2 \frac{\partial I_S}{\partial z^a} - \big(\sqrt{1 - 4 I_S V} + 2 I_S V - 1 \big) \frac{\partial V}{\partial z^a} \right\} \Big|_{\text{horizon}} \\
&= \frac{1 + V_{\text{eff}}^2}{\sqrt{1 - 4 I_S V}} \Big\{ 2\, G_{\mathsf{V}}\, \overline{Z} D_a Z + i\, C_{abc}\, G^{b\overline{b}} G^{c\overline{c}}\, \overline{D_b Z}\, \overline{D_c Z} \Big\} \Big|_{\text{horizon}}
\end{aligned}
$$

Appendix

Solve the equation $0 = 2\, G_{\mathsf{V}}\, \overline{Z} D_a Z + i\, C_{abc}\, G^{b\overline{b}} G^{c\overline{c}}\, \overline{D_b Z}\, \overline{D_c Z} \Big|_{\text{horizon}}$

under the condition $V < 0,\ \ 1 - 4 I_S V > 0,\ \ \partial_a I_S \neq 0,\ \ \partial_a V \neq 0,\ \ D_a Z \neq 0$

- If $V < 0$ and $D_a Z = 0$ (SUSY) $\rightarrow$ Naked Singularity $\rightarrow$ Search non-SUSY solution $D_a Z \neq 0$
- If $\partial_a I_S = 0$ $(G_{\mathsf{V}} = 1)$ or $\partial_a V = 0$ $(G_{\mathsf{V}} = -1)$ $\rightarrow$ asymptotically flat $V = 0$ or Empty Hole $Z = 0$

Contents

- Consider the Single Modulus Model of $\Gamma = (0, p, 0, q_0)$ (analogous to "D0-D4" system)

with cubic prepotential $\mathcal{F} = \frac{(X^1)^3}{X^0}$, $t = \frac{X^1}{X^0}$ $\longrightarrow$ Kähler potential $\mathrm{e}^K = \frac{i}{(t - \bar{t}\,)^3}$

(as the "large volume limit of Calabi-Yau")

The Attractor Equation and its solution ($t = 0 + iy$, $y < 0$):

$$p(y^2)^3 + (q_0 - 18p^3 q_0^2)(y^2)^2 - 12p^2 q_0^3 (y^2) - 2pq_0^4 \;=\; 0$$

with $p \neq 0\,, \quad q_0 \neq 0\,, \quad pq_0 < 0$

$$\begin{aligned}
y^2 &= \mathfrak{A} + \mathfrak{B} \quad \text{or} \quad \mathfrak{A} + \omega^{\pm}\mathfrak{B} \qquad (\omega^3 = 1) \\
\mathfrak{A} &= \frac{q_0}{3p}(18\,p^3 q_0 - 1)\,, \qquad \mathfrak{B} \;=\; \frac{1}{3p}\left(\mathfrak{C}^{1/3} + \frac{q_0^2}{4}\frac{1 + (18\,p^3 q_0)^2}{\mathfrak{C}^{1/3}}\right) \\
\mathfrak{C} &= -q_0^3\left[1 - 27\,p^3 q_0 - (18\,p^3 q_0)^3 - 3\sqrt{3}\sqrt{-2\,p^3 q_0 - 9(p^3 q_0)^2 - 432(p^3 q_0)^3}\,\right]
\end{aligned}$$

Various values at the event horizon:

$$Z\Big|_{\text{horizon}} = \frac{q_0 + 3p\,y^2}{2}\sqrt{-\frac{1}{2y^3}} \neq \mathbf{0}\,, \qquad D_t Z\Big|_{\text{horizon}} = \frac{3i(q_0 - p\,y^2)}{4y}\sqrt{-\frac{1}{2y^3}} \neq \mathbf{0}$$

$$I_S\Big|_{\text{horizon}} = \frac{q_0^2 + 3p^2 y^4}{-2y^3} > \mathbf{0}$$

$$\Lambda = \frac{6(pq_0)^2(q_0 + 3p\,y^2)^2}{y^5} < \mathbf{0}$$

$$S_{\text{BH}} = \frac{-y}{12(pq_0)^2(q_0 + 3p\,y^2)^2}\Big\{-y^4 + \sqrt{y^8 + 12(pq_0)^2(q_0 + 3p\,y^2)^2(q_0^2 + 3p^2\,y^4)}\Big\} > \mathbf{0}$$

their asymptotic behaviors?

Look at the Small q_0 limit:

The dominant part of the Modulus $t = 0 + iy$ $(y < 0)$ is

$$y \sim -\sqrt{-\frac{q_0}{p}} + \text{(sub-leading orders)}$$

The dominant parts of various values are

$$Z\Big|_{\text{horizon}} \sim -q_0 \left(-\frac{p^3}{q_0^3}\right)^{1/4} + \ldots, \qquad D_t Z\Big|_{\text{horizon}} \sim ip \left(-\frac{p}{q_0}\right)^{1/4} + \ldots$$

$$I_S\Big|_{\text{horizon}} \sim \sqrt{-p^3 q_0} + \ldots$$

$$\Lambda \sim -\sqrt{(-p^3 q_0)^{\mathbf{3}}} + \ldots$$

$$\boxed{I_S \sim S_{\text{BH}} \gg -\Lambda > 0}$$

$$S_{\text{BH}} \sim \sqrt{-p^3 q_0} + \ldots$$

very small $|\Lambda|$ compared to others: similar to the non-BPS RN-BH,

but Never connected to RN-BH with $\Lambda = 0$!

Compare to values in the case of non-BPS RN-BH with $\Lambda = 0$:

$$t \;=\; 0 + iy\,, \qquad y \;=\; -\sqrt{-\frac{q_0}{p}}$$

$$Z\Big|_{\text{horizon}} \;=\; -\frac{q_0}{\sqrt{2}}\Big(-\frac{p^3}{q_0^3}\Big)^{1/4} \;\neq\; 0\,, \qquad D_t Z\Big|_{\text{horizon}} \;=\; -3ip\Big(-\frac{p}{q_0}\Big)^{1/4} \;\neq\; 0$$

$$S_{\text{BH}} \;=\; I_S\Big|_{\text{horizon}} \;=\; |Z|^2 + G^{t\bar{t}} D_t Z \overline{D_t Z}\Big|_{\text{horizon}} \;=\; 4|Z|^2\Big|_{\text{horizon}} \;=\; \sqrt{-4p^3 q_0} \;>\; 0$$

Argue the Large q_0 limit:

The dominant part of the Modulus $t = 0 + iy$ $(y < 0)$ is

$$y \ \sim \ pq_0 + (\text{sub-leading orders})$$

The dominant parts of various values are

$$Z\Big|_{\text{horizon}} \ \sim \ \sqrt{-p^3 q_0} + \ldots \ \neq \ 0 \,, \qquad D_t Z\Big|_{\text{horizon}} \sim \ \frac{-i}{pq_0}\sqrt{-p^3 q_0} + \ldots \ \neq \ 0$$

$$I_S\Big|_{\text{horizon}} \ \sim \ -p^3 q_0 + \ldots \ > \ 0$$

$$\Lambda \ \sim \ p^3 q_0 + \ldots \ < \ 0 \qquad \text{(same magnitude to } I_S \text{ !)}$$

$$S_{\text{BH}} \ \sim \ \sqrt{\frac{5}{6}} + \ldots \ > \ 0 \qquad \text{(Why constant ??)}$$

Pathological behaviors of Λ and S_{BH}: *not yet completely understood the reason*
incorrect expansions? and/or ignorance of hyper-sector?

Contents

☑ Studied Extremal RN-AdS Black Hole solution in Abelian gauged SUGRA

☑ Obtained the analytic description of non-SUSY solution in T^3-model

❗ Different behaviors from the ones of non-BPS RN black hole in asympt.-flat spacetime:

- ✓ Should we expand the solution in terms of "Λ" rather than "q_0"?
- ✓ Consider the contribution of (electrically/magnetically coupled) hyper-sector! Appendix

  D'Auria, et.al. [hep-th/0409097] (massive tensors)

  D'Auria, et.al. [hep-th/0701247], Cassani, et.al. [arXiv:0911.2708] (generalized geometry)

  Hristov, Looyestijn and Vandoren [arXiv:1005.3650] (gauged SUGRA to ungauged SUGRA via Higgs mechanism)

- ✓ Is the "Attractor Mechanism" still available?

☞ Argue more general solutions in more general setups (rotating, non-extremal, etc..)

Consider the contribution of (electrically/magnetically coupled) hyper-sector!

D'Auria, et.al. [hep-th/0409097], Gauntlett, Kim, Varela and Waldram [arXiv:0901.0676] (massive tensors)

D'Auria, et.al. [hep-th/0701247], Cassani, et.al. [arXiv:0911.2708] (generalized geometry)

Hristov, et.al. [arXiv:1005.3650] (gauged SUGRA to ungauged SUGRA via Higgs mechanism)

Donos, Gauntlett, Kim and Varela [arXiv:1009.3805] (1 vector + 2 hypers in consistent truncation of M-theory)

Practice:

- ✓ Which vector fields are eaten by two-form fields via anti-Higgs mechanism?
  EoM for all (massive) fields should be re-analyzed.
- ✓ Where do super-partners of the vector fields and of the massive two-form fields go?
  We have to check the number of physical DOFs and the preservation of $\mathcal{N} = 2$ local SUSY.
- ✓ Does the "No-go theorem" of SUSY AdS-BH solution work even with hyper-sector?
  We shall investigate the SUSY variation of the gravitini.
- ✓ Which level is the mass scale of the massive tensor multiplets at?
  (This depends on the viewpoints: "purely 4D SUGRA" vs "10D Effective Theory")
- ✓ How is the gauge symmetry non-abelianized?
  Everything should be reconsidered from the beginning..

Thank you for your attention.

Contents

Action including hypermultiplets: Action | Vector sector | Discussions

$$S = \int d^4x \sqrt{-g} \Big\{ \frac{1}{2\kappa^2} R - G_{a\bar{b}}(z,\bar{z}) \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} - h_{uv}(q) \nabla_\mu q^u \nabla^\mu q^v + \frac{1}{4} \mu_{\Lambda\Sigma}(z,\bar{z}) F^\Lambda_{\mu\nu} F^{\Sigma\mu\nu} + \frac{1}{4} \nu_{\Lambda\Sigma}(z,\bar{z}) F^\Lambda_{\mu\nu} (*F^\Sigma)^{\mu\nu} - \mathrm{g}^2 V(z,\bar{z},q) + (\text{fermionic terms}) \Big\}$$

Scalar field space of hypermultiplets = (special) quaternionic geometry

$$\underbrace{\{q^u\}}_{4n_{\mathsf{H}}+4} = \underbrace{\{\mathsf{z}^i, \bar{\mathsf{z}}^{\bar{\jmath}}\}}_{2n_{\mathsf{H}}(\text{SKG})} + \underbrace{\{\xi^i, \widetilde{\xi}_j\}}_{2n_{\mathsf{H}}} + \underbrace{\{\varphi, a, \xi^0, \widetilde{\xi}_0\}}_{4\,(\text{universal})} = \underbrace{\{\mathsf{z}^i, \bar{\mathsf{z}}^{\bar{\jmath}}, \varphi\}}_{\text{SKG+dilaton}} + \underbrace{\{a, \xi^I, \widetilde{\xi}_J\}}_{\text{“Heisenberg”}}$$

Kinematical term and Scalar potential:

$$h_{uv}\, dq^u\, dq^v = \underbrace{G_{i\bar{\jmath}}\, d\mathsf{z}^i\, d\bar{\mathsf{z}}^{\bar{\jmath}}}_{\text{SKG}_{\mathsf{H}}} + \underbrace{(d\varphi)^2}_{\text{4D dilaton}} + \frac{1}{4} e^{4\varphi} \big(\underbrace{da}_{\text{axion}} - \xi^{\mathrm{T}} \mathbb{C}_{\mathsf{H}}\, d\xi \big)^2 - \underbrace{\frac{1}{2} e^{2\varphi} d\xi^{\mathrm{T}}\, \mathbb{M}_{\mathsf{H}}\, d\xi}_{\text{RR-axions}}$$

$$\begin{aligned} V &= \sum_{x=1}^{3} \Big(G^{a\bar{b}} D_a \mathcal{P}_x \overline{D_b \mathcal{P}_x} - 3 |\mathcal{P}_x|^2 \Big) + 4 h_{uv} k^u \bar{k}^v \\ &= G^{a\bar{b}} D_a \mathcal{P}_+ \overline{D_b \mathcal{P}_+} + G^{a\bar{b}} D_a \mathcal{P}_3 \overline{D_b \mathcal{P}_3} - 3|\mathcal{P}_+|^2 - 3|\mathcal{P}_3|^2 + G^{i\bar{\jmath}} D_i \mathcal{P}_+ \overline{D_j \mathcal{P}_+} + |\mathcal{P}_+|^2 + 4|\mathcal{P}_3|^2 \end{aligned}$$

e.g., Cassani, et.al. [arXiv:0911.2708]

Contribution of hypermultiplets to the Killing prepotentials: Vector sector

$$\Pi_{\mathsf{V}} = \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2} (X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$$
$$z^a = X^a / X^0$$
$$a = 1, \ldots, n_{\mathsf{V}}$$
$\mathsf{SKG_V}$ in vector-moduli

$$\begin{aligned} \mathcal{P}_+ &\equiv \mathcal{P}_1 + i \mathcal{P}_2 = 2 \mathrm{e}^{\varphi} \, \Pi_{\mathsf{V}}^{\mathrm{T}} \, Q \, \mathbb{C}_{\mathsf{H}} \, \Pi_{\mathsf{H}} \\ \mathcal{P}_- &\equiv \mathcal{P}_1 - i \mathcal{P}_2 = 2 \mathrm{e}^{\varphi} \, \Pi_{\mathsf{V}}^{\mathrm{T}} \, Q \, \mathbb{C}_{\mathsf{H}} \, \overline{\Pi_{\mathsf{H}}} \\ \mathcal{P}_3 &= \mathrm{e}^{2\varphi} \, \Pi_{\mathsf{V}}^{\mathrm{T}} \, \mathbb{C}_{\mathsf{V}} (c + \widetilde{Q} \xi) \end{aligned}$$

$$\Pi_{\mathsf{H}} = \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2} (Z^I, \mathcal{G}_I)^{\mathrm{T}}$$
$$\mathsf{z}^i = Z^i / Z^0$$
$$i = 1, \ldots, n_{\mathsf{H}}$$
$\mathsf{SKG_H}$ in hyper-moduli

Functions of period matrices:

$$\mathbb{M}_{\mathsf{V,H}} \equiv \begin{pmatrix} 1 & -\nu \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mu & 0 \\ 0 & \mu^{-1} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\nu & 1 \end{pmatrix}_{\mathsf{V,H}} = \begin{pmatrix} \mu + \nu \mu^{-1} \nu & -\nu \mu^{-1} \\ -\mu^{-1} \nu & \mu^{-1} \end{pmatrix}_{\mathsf{V,H}} , \qquad \begin{aligned} \mu_{\mathsf{V,H}} &\equiv \mathrm{Im} \, \mathcal{N}_{\mathsf{V,H}} \\ \nu_{\mathsf{V,H}} &\equiv \mathrm{Re} \, \mathcal{N}_{\mathsf{V,H}} \end{aligned}$$

Electric/magnetic charges with constraints ($n_{\mathsf{V}} \leq n_{\mathsf{H}}$):

$$Q_{\mathbf{\Lambda}}{}^{\mathbb{I}} \equiv \begin{pmatrix} e_\Lambda{}^I & e_{\Lambda I} \\ m^{\Lambda I} & m^\Lambda{}_I \end{pmatrix} , \quad c \equiv \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} ; \quad \mathbb{C}_{\mathsf{V,H}} \equiv \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}_{\mathsf{V,H}} , \quad \widetilde{Q}^{\mathbf{\Lambda}}{}_{\mathbb{I}} \equiv \mathbb{C}_{\mathsf{V}}^{\mathrm{T}} \, Q \, \mathbb{C}_{\mathsf{H}}$$

$Q \, \mathbb{C}_{\mathsf{H}} \, Q^{\mathrm{T}} = 0 = c^{\mathrm{T}} Q$: Closure condition of massive tensor gauge transformation (tensors appear!)
Tadpole cancellation in 10D string theory,
or Nilpotency of exterior derivative on $SU(3) \times SU(3)$ generalized geometry

$Q^{\mathrm{T}} \, \mathbb{C}_{\mathsf{V}} \, Q = 0$: Abelianity condition (of Heisenberg algebra)
also from Nilpotency of exterior derivative $\longrightarrow$ possible to non-abelianize?

- If p^Λ exists, the axion a is dualized (back) to NS B_2-field (magnetically coupled).

  Gauge field strength is replaced: $F_2^\Lambda \to \widehat{F}_2^\Lambda = F_2^\Lambda + p^\Lambda B_2$ and B_2 becomes massive.

  e.g., D'Auria, et.al. [hep-th/0409097]

- In addition, if non-vanishing $(m^{\Lambda I}, m^\Lambda{}_J)$ are incorporated,

$$\longrightarrow \begin{cases} n_{\mathsf{V}} + 1 (\leq n_{\mathsf{H}} + 1) \text{ RR-axions in } (\xi^I, \widetilde{\xi}_J) \text{ become massive tensors } (C_{2\widehat{I}}, C_2^{\widehat{J}}) \\ n_{\mathsf{V}} + 1 (\leq n_{\mathsf{H}} + 1) \text{ RR-axions in } (\xi^I, \widetilde{\xi}_J) \text{ become massive scalars} \\ \text{Gauge field strength is replaced: } F_2^\Lambda \to \widehat{F}_2^\Lambda = F_2^\Lambda + m^{\Lambda \widehat{I}} C_{2\widehat{I}} + m^\Lambda{}_{\widehat{J}} C_2^{\widehat{J}} + p^\Lambda B_2 \\ \text{Remaining } 2(n_{\mathsf{H}} - n_{\mathsf{V}}) \text{ RR-axions are flat directions for } V \end{cases}$$

$I = 0, \ldots, n_{\mathsf{H}}$; $\widehat{I} = 0, \ldots, n_{\mathsf{V}}$ with $n_{\mathsf{V}} \leq n_{\mathsf{H}}$.

We can choose $\xi^{\widehat{I}}$ to dualized $C_{2\widehat{I}}$, while $\widetilde{\xi}_{\widehat{I}}$ to massive scalars.

> When the massive fields are integrated out under the conditions,
> the scalar potential V carries only Π_{V}, Π_{H} and φ.

e.g., D'Auria, et.al. [hep-th/0701247], Cassani [arXiv:0804.0595]

Contents

δg^{tt} :

$$-\mathrm{e}^{-2B}\Big[\frac{1}{r^2}\big(1-\mathrm{e}^{2(B-C)}\big)+C'(3C'-2B')+\frac{2}{r}(3C'-B')+2C''\Big]+\mathrm{e}^{-2B}G_{a\bar{b}}z^{a\prime}\overline{z}^{\bar{b}\prime}$$
$$=\frac{\mathrm{e}^{-4C}}{r^4}I_S+V$$

δg^{rr} :

$$-\mathrm{e}^{-2B}\Big[\frac{1}{r^2}\big(1-\mathrm{e}^{2(B-C)}\big)+C'(C'+2A')+\frac{2}{r}(C'+A')\Big]+\mathrm{e}^{-2B}G_{a\bar{b}}z^{a\prime}\overline{z}^{\bar{b}\prime}$$
$$=\frac{\mathrm{e}^{-4C}}{r^4}I_S+V$$

$\delta g^{\theta\theta},\ \delta g^{\phi\phi}$:

$$-\mathrm{e}^{-2B}\Big[A''+C''+A'(A'-B')+C'(A'-B'+C')+\frac{1}{r}(A'-B'+2C')\Big]-\mathrm{e}^{-2B}G_{a\bar{b}}z^{a\prime}\overline{z}^{\bar{b}\prime}$$
$$=-\frac{\mathrm{e}^{-4C}}{r^4}I_S+V$$

δz^a :

$$\mathrm{e}^{-2B}\Big[G_{a\bar{b}}\overline{z}^{\bar{b}\prime\prime}+\frac{\partial G_{a\bar{b}}}{\partial\overline{z}^{\bar{c}}}\overline{z}^{\bar{b}\prime}\overline{z}^{\bar{c}\prime}+G_{a\bar{b}}\overline{z}^{\bar{b}\prime}\big(A'-B'+2C'+\frac{2}{r}\big)\Big]\ =\ \frac{\mathrm{e}^{-4C}}{r^4}\frac{\partial I_S}{\partial z^a}+\frac{\partial V}{\partial z^a}$$

Bellucci, et.al. [arXiv:0802.0141]

Comment: We have already known $I_S(z,\bar{z},p,q)$, $V(z,\bar{z},p,q)$ and their derivatives as

Attractor Equations CLICK

- $\partial_a I_S = 0$ appears in the search of extremal RN-BH in $\mathcal{N}=2$ SUGRA

$$I_S \;=\; |Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}\,, \qquad \partial_a I_S \;=\; 2\overline{Z} D_a Z + i C_{abc} G^{b\bar{b}} G^{c\bar{c}}\, \overline{D_b Z}\, \overline{D_c Z}$$

- $\partial_a V = 0$ has the same form which appears in the search of (flux) vacua of $\mathcal{N}=1$ SUGRA

$$V \;=\; -3|Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}\,, \qquad \partial_a V \;=\; -2\overline{Z} D_a Z + i C_{abc} G^{b\bar{b}} G^{c\bar{c}}\, \overline{D_b Z}\, \overline{D_c Z}$$

Z in $\mathcal{N}=1$ SUGRA is not the central charge but the superpotential

$$C_{abc} \equiv \partial_a X^{\Lambda} \partial_b X^{\Sigma} \partial_c X^{\Gamma} \frac{\partial^3 \mathcal{F}}{\partial X^{\Lambda} \partial X^{\Sigma} \partial X^{\Gamma}}$$

A useful formula among the BH charges $\Gamma = (p^\Lambda, q_\Lambda)^{\mathrm{T}}$ and the invariant $I_S(z, \overline{z}, p, q)$

$$\boxed{\Gamma^{\mathrm{T}} + i \frac{\partial I_S}{\partial \widetilde{\Gamma}} \;=\; 2i \overline{Z} \Pi^{\mathrm{T}} + 2i G^{a\bar{b}} D_a Z \overline{D_b \Pi}^{\mathrm{T}}}$$

$$\widetilde{\Gamma} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} \Gamma \, , \qquad \Pi = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix} , \qquad Z = L^\Lambda q_\Lambda - M_\Lambda p^\Lambda = \widetilde{\Gamma}^{\mathrm{T}} \Pi$$

This does not (explicitly) depend on the scalar potential $-\mathrm{g}^2 V$.

This can be applied to any points in the spacetime.

Kallosh, Sivanandam and Soroush [hep-th/0606263]

- Single modulus model ($a = 1$): $\mathcal{F} = \dfrac{(X^1)^3}{X^0}$

$$Z = \mathrm{e}^{K/2}\Big(q_0 + q\,t - 3p\,t^2 + p^0\,t^3\Big)\,, \qquad t = \frac{X^1}{X^0}$$

$$\mathrm{e}^K = \frac{i}{(t-\bar{t})^3}\,, \qquad G_{t\bar{t}} = -\frac{3}{(t-\bar{t})^2} \equiv e_t{}^{\hat{1}}\, e_{\bar{t}}{}^{\bar{\hat{1}}}\, \delta_{\hat{1}\bar{\hat{1}}}\,, \qquad C_{ttt} = \frac{6i}{(t-\bar{t})^3}$$

Search a solution with $V = -3|Z|^2 + |D_{\hat{1}}Z|^2 < 0 \to Z \neq 0$

Consider a non-SUSY solution $\to D_{\hat{1}}Z \neq 0$

$$\Downarrow$$

The generic forms of the central charge and its derivative:

$$Z \equiv -i\rho\,\mathrm{e}^{i(\alpha - 3\phi)}\,, \qquad D_{\hat{1}}Z \equiv \sigma\,\mathrm{e}^{-i\phi} \qquad (\rho, \sigma > 0)$$

Kallosh, et.al. [hep-th/0606263]

The generic forms: $Z \equiv -i\rho\,\mathrm{e}^{i(\alpha-3\phi)}\,, \qquad D_{\hat{1}}Z \equiv \sigma\,\mathrm{e}^{-i\phi} \qquad (\rho, \sigma > 0)$

The volume factors ρ and σ are related via the attractor equation.

$$\sigma = -\frac{\rho}{3}\,\mathrm{e}^{-i\alpha} G_{\mathsf{V}} \qquad (G_{\mathsf{V}} \neq 0)$$

The formula leads to the following two equations: $(\Gamma = (p^0, p, q, q_0)^{\mathrm{T}})$:

$$\begin{aligned} p + \frac{\partial I_S}{\partial q} &= -\frac{2\rho}{3\sqrt{3}}\,\mathrm{e}^{-i\alpha}\mathrm{e}^{K/2}\Big[\big(3\sqrt{3} - 2G_{\mathsf{V}}\big)\,t - G_{\mathsf{V}}\,\bar{t}\,\Big] \\ p^0 + \frac{\partial I_S}{\partial q_0} &= -\frac{2\rho}{3\sqrt{3}}\,\mathrm{e}^{-i\alpha}\mathrm{e}^{K/2}\big(3\sqrt{3} - G_{\mathsf{V}}\big) \end{aligned}$$

$$\rightarrow \qquad t = \frac{3\sqrt{3} - 2G_{\mathsf{V}}}{3\sqrt{3} - G_{\mathsf{V}}}\left[\frac{p + i\frac{\partial I_S}{\partial q}}{p^0 + i\frac{\partial I_S}{\partial q_0}}\right] + \frac{G_{\mathsf{V}}}{3\sqrt{3} - G_{\mathsf{V}}}\left[\frac{p - i\frac{\partial I_S}{\partial q}}{p^0 - i\frac{\partial I_S}{\partial q_0}}\right]$$

"generic solution"

Difficult to evaluate the explicit solution caused by the complicated functions G_{V} and I_S

- Three Moduli model called the STU-model: $\mathcal{F} = \dfrac{X^1 X^2 X^3}{X^0}$

  (Cartan part of 4D $\mathcal{N} = 8$ $SO(8)$ gauged SUGRA $\leftarrow$ IIA/IIB/Heterotic string triality)

$$Z = e^{K/2}\Big(q_0 + q_a z^a - p^1 z^2 z^3 - p^2 z^3 z^1 - p^3 z^1 z^2 + p^0 z^1 z^2 z^3 \Big) \,, \qquad z^a = \frac{X^a}{X^0}$$

$$K = -\log \Big[- i(z^1 - \overline{z}^1)(z^2 - \overline{z}^2)(z^3 - \overline{z}^3) \Big]$$

$$G_{a\overline{b}} = - \frac{\delta_{ab}}{(z^a - \overline{z}^{\overline{a}})^2} = e_a{}^{\widehat{a}} \, e_{\overline{b}}{}^{\overline{\widehat{b}}} \, \delta_{\widehat{a}\overline{\widehat{b}}} \,, \qquad C_{\widehat{1}\widehat{2}\widehat{3}} = 1$$

Search a solution with $V = -3|Z|^2 + |D_{\widehat{a}} Z|^2 < 0 \to Z \neq 0$

Consider a non-SUSY solution $\to D_{\widehat{a}} Z \neq 0$

$\Downarrow$

The generic forms: $Z \equiv -i\rho \, e^{i(\alpha - 3\phi)} \,, \qquad D_{\widehat{a}} Z \equiv \sigma \, e^{-i\phi} \qquad (\rho, \sigma > 0)$

Kallosh, et.al. [hep-th/0606263]

The generic forms: $Z \equiv -i\rho\,\mathrm{e}^{i(\alpha - 3\phi)}\,, \qquad D_{\widehat{a}} Z \equiv \sigma\,\mathrm{e}^{-i\phi} \qquad (\rho, \sigma > 0)$

The volume factors ρ and σ are related via the attractor equation.

$$\sigma = -\rho\,\mathrm{e}^{-i\alpha} G_{\mathsf{V}} \qquad (G_{\mathsf{V}} \neq 0)$$

The formula leads to the following two equations:

$$\begin{aligned} p^a + \frac{\partial I_S}{\partial q_a} &= -2\rho\,\mathrm{e}^{-i\alpha}\mathrm{e}^{K/2}\Big[\big(1 - G_{\mathsf{V}}\big)\, z^a - 2G_{\mathsf{V}}\,\overline{z}^{\overline{a}}\Big] \\ p^0 + \frac{\partial I_S}{\partial q_0} &= -2\rho\,\mathrm{e}^{-i\alpha}\mathrm{e}^{K/2}\big(1 - 3G_{\mathsf{V}}\big) \end{aligned}$$

$$\longrightarrow \qquad z^a = V_{\text{eff}}^2 \left[\frac{p^a + i\frac{\partial I_S}{\partial q_a}}{p^0 + i\frac{\partial I_S}{\partial q_0}}\right] + (1 - V_{\text{eff}}^2) \left[\frac{p^a - i\frac{\partial I_S}{\partial q_a}}{p^0 - i\frac{\partial I_S}{\partial q_0}}\right] \qquad \text{“generic solution”}$$

Neither $V_{\text{eff}} = 1$ nor $V_{\text{eff}} = 0$

Difficult to evaluate the explicit solution caused by the complicated functions G_{V} and I_S