DESY THEORY WORKSHOP “Quantum Field Theory: Developments and Perspectives”
Parallel Session 1B: Mathematical Physics
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Non-supersymmetric Extremal RN-AdS Black Holes in $\mathcal{N} = 2$ Gauged Supergravity

based on JHEP 09 (2010) 061 [arXiv:1005.4607]
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Motivation: Black Hole Solutions in 4D $\mathcal{N}=2$ SUGRA

✍ WHY $\mathcal{N}=2$ (8-SUSY charges)?

✓ Scalar fields living in highly symmetric spaces

✓ (Flux) compactifications in string/M-theory $\longleftarrow$ *I have mainly worked..*

✍ WHY Black Holes?

✓ Attractive solutions in 4D $\mathcal{N}=2$ SUGRA

✓ Application to "$\text{AdS}_4/\text{CFT}_3$ (or $\text{AdS}_4/\text{CMP}_3$) correspondence"

In the framework of 4D $\mathcal{N} = 2$ SUGRA,

(Extremal) RN-BH in asymptotically flat spacetime has been investigated.

We (have to) study BHs in asymptotically non-flat spacetime.

RN: Reissner-Nordström (i.e., charged and non-rotated)

Cosmological constant Λ is given as an expectation value of the scalar potential

(i.e., "mass deformations" of gravitini).

NOTICE: Naked Singularity appears in SUSY solution unless AdS-BH is rotating.

Romans [hep-th/9203018], Caldarelli and Klemm [hep-th/9808097] etc.

Questions

How can we obtain non-SUSY solutions with matter fields

in asymptotically non-flat spacetime?

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Multiplets in 4D $\mathcal{N}=2$ SUGRA:

1 graviton multiplet:	$\{g_{\mu\nu}, A^0_\mu, \psi_{A\mu}\}$	$\mu = 0,1,2,3$ (4D, curved) $A = 1,2$ ($SU(2)$ R-symmetry)
n_{V} vector multiplets:	$\{A^a_\mu, z^a, \lambda^{aA}\}$	$a = 1, \dots, n_{\mathsf{V}}$
	z^a in special Kähler geometry $\mathcal{SM}$	
$n_{\mathsf{H}}+1$ hypermultiplets:	$\{q^u, \zeta^\alpha\}$	$u = 1, \dots, 4n_{\mathsf{H}}+4$ $\alpha = 1, \dots, 2n_{\mathsf{H}}+2$
	q^u in quaternionic geometry $\mathcal{HM}$	

Gauging: *Promote* global isometry groups on scalar field spaces to local symmetries

Andrianopoli, Bertolini, Ceresole, D'Auria, Ferrara, Fré and Magri [hep-th/9605032]

Action (grav. const. κ; gauge coupling const. g; indices $\Lambda = 0, 1, \ldots, n_{\mathsf{V}}$):

$$\begin{aligned} S \;=\; \int \mathrm{d}^4 x \sqrt{-g} \Big\{ & \frac{1}{2\kappa^2} R - G_{a\bar{b}}(z,\bar{z}) \nabla_{\mu} z^a \nabla^{\mu} \bar{z}^{\bar{b}} - h_{uv}(q) \nabla_{\mu} q^u \nabla^{\mu} q^v \\ & + \frac{1}{4} \mu_{\Lambda\Sigma}(z,\bar{z}) F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} + \frac{1}{4} \nu_{\Lambda\Sigma}(z,\bar{z}) F^{\Lambda}_{\mu\nu} (*F^{\Sigma})^{\mu\nu} \\ & - \mathrm{g}^2 V(z,\bar{z},q) \\ & + \text{(fermionic terms)} \Big\} \end{aligned}$$

$\mu_{\Lambda\Sigma} = \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}$ (generalized $-1/\mathrm{g}^2$) , $\nu_{\Lambda\Sigma} = \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}$ (generalized θ-angle)

In this analysis...
- Set background fermionic fields to zero
- Reduce gauge symmetries to *abelian*
- Truncate hypermultiplets *by hand*

Equations of Motion (abbreviate κ and g):

$$g_{\mu\nu} : \quad \Big(R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} \Big) - 2 G_{a\bar{b}}\, \partial_{(\mu} z^a \partial_{\nu)} \overline{z}^{\bar{b}} + G_{a\bar{b}}\, \partial_\rho z^a \partial^\rho \overline{z}^{\bar{b}}\, g_{\mu\nu} \;=\; T_{\mu\nu} - V g_{\mu\nu}$$

$$T_{\mu\nu} \;=\; - \mu_{\Lambda\Sigma}\, F^\Lambda_{\mu\rho}\, F^\Sigma_{\nu\sigma}\, g^{\rho\sigma} + \frac{1}{4} \mu_{\Lambda\Sigma}\, F^\Lambda_{\rho\sigma}\, F^{\Sigma\rho\sigma}\, g_{\mu\nu}$$

(energy-momentum tensor)

$$z^a : \quad - \frac{G_{a\bar{b}}}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g} g^{\mu\nu} \partial_\nu \overline{z}^{\bar{b}} \Big) - \frac{\partial G_{a\bar{b}}}{\partial \overline{z}^{\bar{c}}}\, \partial_\rho \overline{z}^{\bar{b}}\, \partial^\rho \overline{z}^{\bar{c}}$$
$$= \frac{1}{4} \frac{\partial \mu_{\Lambda\Sigma}}{\partial z^a} F^\Lambda_{\mu\nu} F^{\Sigma\mu\nu} + \frac{1}{4} \frac{\partial \nu_{\Lambda\Sigma}}{\partial z^a} F^\Lambda_{\mu\nu} (*F^\Sigma)^{\mu\nu} - \frac{\partial V}{\partial z^a}$$

$$A^\Lambda_\mu : \quad \varepsilon^{\mu\nu\rho\sigma} \partial_\nu G_{\Lambda\rho\sigma} \;=\; 0 \,, \qquad G_{\Lambda\rho\sigma} \;=\; \nu_{\Lambda\Sigma} F^\Sigma_{\rho\sigma} - \mu_{\Lambda\Sigma} (*F^\Sigma)_{\rho\sigma}$$

electric charge $q_\Lambda \;\equiv\; \dfrac{1}{4\pi} \displaystyle\int_{S^2} G_\Lambda \,,$ magnetic charge $p^\Lambda \;\equiv\; \dfrac{1}{4\pi} \displaystyle\int_{S^2} F^\Lambda$

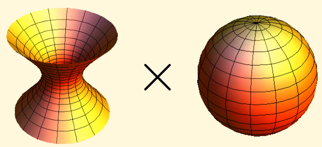
Introduce a metric ansatz for RN(-AdS) BH: “charged”, “static”, “spherically symmetric”

$$ds^2 = -\,e^{2A(r)}dt^2 + e^{2B(r)}dr^2 + e^{2C(r)}r^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big)$$

Near horizon geometry: $\mathrm{AdS}_2 \times S^2$ (radii: r_{A} and r_{H})

$$A(r) = \log\frac{r - r_{\mathsf{H}}}{r_{\mathsf{A}}}\,, \qquad B(r) = -A(r)\,, \qquad C(r) = \log\frac{r_{\mathsf{H}}}{r}$$

$$R(\mathrm{AdS}_2 \times S^2) = 2\Big(-\frac{1}{r_{\mathsf{A}}^2} + \frac{1}{r_{\mathsf{H}}^2}\Big)$$



$$\rightarrow \quad ds^2(\text{near horizon}) = -\left(\frac{r - r_{\mathsf{H}}}{r_{\mathsf{A}}}\right)^2 dt^2 + \left(\frac{r_{\mathsf{A}}}{r - r_{\mathsf{H}}}\right)^2 dr^2 + r_{\mathsf{H}}^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big)$$

$$= -\frac{e^{2\tau}}{r_{\mathsf{A}}^2}dt^2 + r_{\mathsf{A}}^2 d\tau^2 + r_{\mathsf{H}}^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big) \qquad (\tau = \log(r - r_{\mathsf{H}}))$$

If the attractor mechanism works (via extremality), the scalar fields are subject to

$$\frac{\mathrm{d}}{\mathrm{d}r} z^a \Big|_{\text{horizon}} = 0 \,, \qquad \left(\frac{\mathrm{d}}{\mathrm{d}r}\right)^2 z^a \Big|_{\text{horizon}} = 0$$

The EoM are drastically reduced to

Bellucci, Ferrara, Marrani and Yeranyan [arXiv:0802.0141]

$$g_{tt}, g_{rr} : \quad \frac{1}{r_{\mathsf{H}}^2} = \frac{1}{r_{\mathsf{H}}^4} I_1 + V \Big|_{\text{horizon}} \quad \Rightarrow \quad r_{\mathsf{H}}^2 = \frac{1 - \sqrt{1 - 4 I_1 V}}{2V} \Big|_{\text{horizon}}$$

$$g_{\theta\theta}, g_{\phi\phi} : \quad \frac{1}{r_{\mathsf{A}}^2} = \frac{1}{r_{\mathsf{H}}^4} I_1 - V \Big|_{\text{horizon}} \quad \Rightarrow \quad r_{\mathsf{A}}^2 = \frac{r_{\mathsf{H}}^2}{\sqrt{1 - 4 I_1 V}} \Big|_{\text{horizon}}$$

$$z^a : \quad 0 = \frac{1}{r_{\mathsf{H}}^4} \frac{\partial I_1}{\partial z^a} - \frac{\partial V}{\partial z^a} \Big|_{\text{horizon}} \quad \Rightarrow \quad 0 = \frac{1}{r_{\mathsf{H}}^4} (1 - 2 r_{\mathsf{H}}^2 V) \frac{\partial}{\partial z^a} r_{\mathsf{H}}^2 \Big|_{\text{horizon}}$$

1st Symplectic Invariant:

$$I_1(z, \overline{z}, p, q) = -\frac{1}{2} \begin{pmatrix} p^{\Lambda} & q_{\Lambda} \end{pmatrix} \begin{pmatrix} \mu_{\Lambda\Sigma} + \nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Delta} \nu_{\Delta\Sigma} & -\nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Sigma} \\ -(\mu^{-1})^{\Lambda\Gamma} \nu_{\Gamma\Sigma} & (\mu^{-1})^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} p^{\Sigma} \\ q_{\Sigma} \end{pmatrix} \equiv -\frac{1}{2} \Gamma^{\mathrm{T}} \mathbb{M} \Gamma$$

with $\quad T_t{}^t = T_r{}^r = -T_\theta{}^\theta = -T_\phi{}^\phi = -\frac{\mathrm{e}^{-4C}}{r^4} I_1$

BH Entropy (and the effective potential) are given as the Area of the event horizon:

$$S_{\mathsf{BH}}(p,q) \;=\; \frac{A_{\mathsf{H}}}{4\pi} \;=\; r_{\mathsf{H}}^2\Big|_{\mathsf{horizon}} \;\equiv\; V_{\mathsf{eff}}(z,\overline{z},p,q)\Big|_{\mathsf{horizon}}$$

$$V_{\mathsf{eff}}(z,\overline{z},p,q) \;=\; \frac{1-\sqrt{1-4I_1 V}}{2V} \;=\; I_1 + (I_1)^2 V + 2(I_1)^3 V^2 + \mathcal{O}((I_1)^4 V^3)$$

We read the "cosmological constant Λ" from the scalar curvature:

$$R(\mathrm{AdS}_2 \times S^2) \;=\; 2\Big(-\frac{1}{r_{\mathsf{A}}^2} + \frac{1}{r_{\mathsf{H}}^2}\Big) \;=\; 4V$$

$$V\Big|_{\mathsf{horizon}} \;\equiv\; \Lambda(\text{"cosmological constant"})$$

ATTRACTOR EQUATION

$$0 \;=\; \frac{1}{r_{\mathsf{H}}^4}\,(1-2r_{\mathsf{H}}^2 V)\,\frac{\partial}{\partial z^a} V_{\mathsf{eff}}\Big|_{\mathsf{horizon}} \qquad \rightarrow \qquad 0 \;=\; \frac{\partial}{\partial z^a} V_{\mathsf{eff}}(z,\overline{z},p,q)\Big|_{\mathsf{horizon}}$$

If $r_{\mathsf{H}} < \infty$ and $V\Big|_{\mathsf{horizon}} \leq 0$

The "ATTRACTOR EQUATION" which we have to solve is

$$0 = \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \overline{z}, p, q)\Big|_{\text{horizon}}$$

$$= \frac{1}{2V^2\sqrt{1 - 4I_1 V}} \left\{ 2V^2 \frac{\partial I_1}{\partial z^a} - \left(\sqrt{1 - 4I_1 V} + 2I_1 V - 1\right) \frac{\partial V}{\partial z^a} \right\} \Bigg|_{\text{horizon}}$$

Evaluate I_1 and V described in terms of the central charge Z

Useful when we consider (non-)SUSY solutions

SUSY variation of gravitini carries def. of Z and more..

$$\delta\psi_{A\mu} = D_\mu \varepsilon_A + \epsilon_{AB} T^-_{\mu\nu} \gamma^\nu \varepsilon^B + i\, \mathrm{g}\, \mathcal{S}_{AB} \gamma_\mu \varepsilon^B$$

$$Z = -\frac{1}{2}\left(\frac{1}{4\pi}\int_{S^2} T^-\right), \qquad \mathcal{S}_{AB} = \frac{i}{2}\sum_{x=1}^{3} (\sigma^x)_{AB} \mathcal{P}_x$$

Use the property of scalar field spaces $\mathcal{SM} \times \mathcal{HM}$

Write down Z, I_1 and V in terms of $(L^\Lambda, M_\Lambda) = \mathrm{e}^{K/2}(X^\Lambda, \mathcal{F}_\Lambda)$ on $\mathcal{SM}$:

$$
\begin{aligned}
Z &= L^\Lambda q_\Lambda - M_\Lambda p^\Lambda \\
I_1 &= |Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z} \\
V &= \sum_{x=1}^{3} \Big(-3|\mathcal{P}_x|^2 + G^{a\bar{b}} D_a \mathcal{P}_x \overline{D_b \mathcal{P}_x} \Big) + 4 h_{uv} k^u \overline{k}^v
\end{aligned}
$$

$\mathcal{P}_x$: $SU(2)$ triplet of Killing prepotentials in $\mathcal{N} = 2$ SUGRA

(In general, both vector moduli and hyper moduli contribute to $\mathcal{P}_x$)

"Truncate" hypermultiplets $\rightarrow$ only $\mathcal{P}_3 = \mathcal{P}_{3,\Lambda} L^\Lambda - \widetilde{\mathcal{P}}_3^\Lambda M_\Lambda$ remains in the scalar potential

Further, Identify $(\mathcal{P}_{3,\Lambda}, \widetilde{\mathcal{P}}_3^\Lambda) = (q_\Lambda, p^\Lambda) \rightsquigarrow \mathcal{P}_3 \equiv Z$

Cassani, Ferrara, Marrani, Morales and Samtleben [arXiv:0911.2708]

$$
V = -3|Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}
$$

The "ATTRACTOR EQUATION" (with a non-trivial factor $-1 \le G_{\mathsf{V}} \equiv \frac{1 - V_{\text{eff}}^2}{1 + V_{\text{eff}}^2} \le 1$)

$$\begin{aligned} 0 &= \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \overline{z}, p, q) \Big|_{\text{horizon}} \\ &= \frac{1}{2V^2 \sqrt{1 - 4I_1 V}} \Big\{ 2V^2 \frac{\partial I_1}{\partial z^a} - \big(\sqrt{1 - 4I_1 V} + 2I_1 V - 1 \big) \frac{\partial V}{\partial z^a} \Big\} \Big|_{\text{horizon}} \\ &= \frac{1 + V_{\text{eff}}^2}{\sqrt{1 - 4I_1 V}} \Big\{ 2\, G_{\mathsf{V}}\, \overline{Z} D_a Z + i C_{abc} G^{b\overline{b}} G^{c\overline{c}}\, \overline{D_b Z}\, \overline{D_c Z} \Big\} \Big|_{\text{horizon}} \end{aligned}$$

Solve the equation $0 = 2G_{\mathsf{V}} \overline{Z} D_a Z + i C_{abc} G^{b\overline{b}} G^{c\overline{c}}\, \overline{D_b Z}\, \overline{D_c Z} \Big|_{\text{horizon}}$

under the condition $V < 0$, $1 - 4I_1 V > 0$, $\partial_a I_1 \neq 0$, $\partial_a V \neq 0$, $D_a Z \neq 0$

- ✍ If $V < 0$ and $D_a Z = 0$ (SUSY) $\rightarrow$ Naked Singularity $\rightarrow$ Search non-SUSY sol. $D_a Z \neq 0$
- ✍ If $\partial_a I_1 = 0$ $(G_{\mathsf{V}} = 1)$ or $\partial_a V = 0$ $(G_{\mathsf{V}} = -1)$ $\rightarrow$ asymptotically flat $V = 0$ or Empty Hole $Z = 0$

Contents

- Consider the Single Modulus Model of $\Gamma = (0, p, 0, q_0)$ ("D0-D4" system)

with prepotential $\mathcal{F} = \frac{(X^1)^3}{X^0}$, $t = \frac{X^1}{X^0}$ $\longrightarrow$ Kähler potential $\mathrm{e}^K = \frac{i}{(t - \bar{t}\,)^3}$

(as the "large volume limit of Calabi-Yau")

The ATTRACTOR EQUATION and its solution ($t = 0 + iy$, $y < 0$):

$$p(y^2)^3 + (q_0 - 18p^3 q_0^2)(y^2)^2 - 12p^2 q_0^3 (y^2) - 2pq_0^4 \;=\; 0$$

with $\quad p \neq 0\,, \quad q_0 \neq 0\,, \quad pq_0 < 0$

$$y^2 \;=\; A + B \quad \text{or} \quad A + \omega^{\pm} B \qquad (\omega^3 = 1)$$

$$A \;=\; \frac{q_0}{3p}(18\,p^3 q_0 - 1)\,, \qquad B \;=\; \frac{1}{3p}\left(C^{1/3} + \frac{q_0^2}{4}\frac{1 + (18\,p^3 q_0)^2}{C^{1/3}}\right)$$

$$C \;=\; -q_0^3 \left[1 - 27\,p^3 q_0 - (18\,p^3 q_0)^3 - 3\sqrt{3}\sqrt{-2\,p^3 q_0 - 9(p^3 q_0)^2 - 432(p^3 q_0)^3}\,\right]$$

Various values at the event horizon:

$$Z\Big|_{\text{horizon}} = \frac{q_0 + 3p\,y^2}{2}\sqrt{-\frac{1}{2y^3}} \neq \mathbf{0}\,, \qquad D_t Z\Big|_{\text{horizon}} = \frac{3i(q_0 - p\,y^2)}{4y}\sqrt{-\frac{1}{2y^3}} \neq \mathbf{0}$$

$$I_1 = \frac{q_0^2 + 3p^2 y^4}{-2y^3} > \mathbf{0}$$

$$\Lambda = \frac{6(pq_0)^2 (q_0 + 3p\,y^2)^2}{y^5} < \mathbf{0}$$

$$S_{\text{BH}} = \frac{-y}{12(pq_0)^2(q_0 + 3p\,y^2)^2}\Big\{-y^4 + \sqrt{y^8 + 12(pq_0)^2(q_0 + 3p\,y^2)^2(q_0^2 + 3p^2\,y^4)}\Big\} > \mathbf{0}$$

their asymptotic behaviors?

Look at the Small q_0 limit:

The dominant part of the Modulus $t = 0 + iy$ $(y < 0)$ is

$$y \sim -\sqrt{-\frac{q_0}{p}} + \text{(sub-leading orders)}$$

The dominant parts of various values are

$$Z\Big|_{\text{horizon}} \sim -q_0 \left(-\frac{p^3}{q_0^3}\right)^{1/4} + \ldots , \qquad D_t Z\Big|_{\text{horizon}} \sim ip \left(-\frac{p}{q_0}\right)^{1/4} + \ldots$$

$$I_1 \sim \sqrt{-p^3 q_0} + \ldots$$

$$\Lambda \sim -\sqrt{(-p^3 q_0)^{\mathbf{3}}} + \ldots$$

$$S_{\text{BH}} \sim \sqrt{-p^3 q_0} + \ldots$$

$$I_1 \sim S_{\text{BH}} \gg -\Lambda > 0$$

very small $|\Lambda|$ compared to others: similar to the non-BPS RN-BH,
but Never connected to RN-BH with $\Lambda = 0$!

Comparison: values in the case of non-BPS RN-BH with $\Lambda = 0$:

$$t = 0 + iy \,, \qquad y = -\sqrt{-\frac{q_0}{p}}$$

$$Z\Big|_{\text{horizon}} = -\frac{q_0}{\sqrt{2}}\Big(-\frac{p^3}{q_0^3}\Big)^{1/4} \neq 0 \,, \qquad D_t Z\Big|_{\text{horizon}} = -3ip\Big(-\frac{p}{q_0}\Big)^{1/4} \neq 0$$

$$S_{\text{BH}} = I_1 = |Z|^2 + G^{t\bar{t}} D_t Z \overline{D_t Z} = 4|Z|^2 = \sqrt{-4p^3 q_0} > 0$$

Argue the Large q_0 limit:

The dominant part ot the Modulus $t = 0 + iy$ $(y < 0)$ is

$$y \sim pq_0 + \text{(sub-leading orders)}$$

The dominant parts of various values are

$$Z\Big|_{\text{horizon}} \sim \sqrt{-p^3 q_0} + \ldots \neq 0\,, \qquad D_t Z\Big|_{\text{horizon}} \sim \frac{-i}{pq_0}\sqrt{-p^3 q_0} + \ldots \neq 0$$

$$I_1 \sim -p^3 q_0 + \ldots > 0$$

$$\Lambda \sim p^3 q_0 + \ldots < 0 \qquad \text{(same magnitude to } I_1 \text{ !)}$$

$$S_{\text{BH}} \sim \sqrt{\frac{5}{6}} + \ldots > 0 \qquad \text{(Why constant ??)}$$

Strange behaviors of Λ and S_{BH}: incorrect expansions?

...not completely understood yet

Contents

- ☑ Studied Extremal RN-AdS Black Hole solutions in Abelian gauged SUGRA
- ☑ Described the non-SUSY solution of the D0-D4 system in the T^3-model

- ❗ Different behavior of the modulus, BH entropy, etc.

- ☞ Description in all region in the asymptotically non-flat spacetime?
- ☞ Include (charged) hypermultiplets?

  Hristov, Looyestijn and Vandoren [arXiv:1005.3650] (constant sol. of Behrndt-Lüst-Sabra–type, etc.)

  Cassani et.al. [arXiv:0911.2708] (nongeometric flux compactifications)

Thanks

I would like to enjoy Hamburg (and more) until November 18, 2010.

Appendix

Mainly we use the followings (The basic variables are X^Λ and $\mathcal{F}_\Lambda$):

$$\mathcal{F}_\Lambda = \frac{\partial \mathcal{F}}{\partial X^\Lambda}, \qquad z^a = \frac{X^a}{X^0}$$

$$K = -\log\left[i(\overline{X}^\Lambda \mathcal{F}_\Lambda - X^\Lambda \overline{\mathcal{F}}_\Lambda)\right], \qquad G_{a\bar{b}} = \frac{\partial}{\partial z^a}\frac{\partial}{\partial \overline{z}^{\bar{b}}} K$$

$$\Pi = \mathrm{e}^{K/2}\begin{pmatrix} X^\Lambda \\ \mathcal{F}_\Lambda \end{pmatrix} = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \qquad D_a \Pi = \left(\frac{\partial}{\partial z^a} + \frac{1}{2}\frac{\partial K}{\partial z^a}\right)\Pi = \begin{pmatrix} f_a^\Lambda \\ h_{\Lambda a} \end{pmatrix}$$

$$M_\Lambda = \mathcal{N}_{\Lambda\Sigma} L^\Sigma, \qquad h_{\Lambda a} = \overline{\mathcal{N}}_{\Lambda\Sigma} f_a^\Sigma, \qquad G^{a\bar{b}} f_a^\Lambda f_{\bar{b}}^\Sigma = -\frac{1}{2}\mathrm{Im}(\mathcal{N}^{-1})^{\Lambda\Sigma} - \overline{L}^\Lambda L^\Sigma$$

Hypermultiplets

Action including hypermultiplets:

$$
\begin{aligned}
S \;=\; \int \mathrm{d}^4 x \sqrt{-g} \Big\{ & \frac{1}{2\kappa^2} R - G_{a\bar{b}}(z,\bar{z}) \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} - h_{uv}(q) \nabla_\mu q^u \nabla^\mu q^v \\
& + \frac{1}{4} \mu_{\Lambda\Sigma}(z,\bar{z}) F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} + \frac{1}{4} \nu_{\Lambda\Sigma}(z,\bar{z}) F^{\Lambda}_{\mu\nu} (*F^{\Sigma})^{\mu\nu} \\
& - \mathrm{g}^2 V(z,\bar{z},q) \\
& + (\text{fermionic terms}) \Big\}
\end{aligned}
$$

Moduli space of hypermultiplets = quaternionic geometry

We borrow the description in (non)geometric flux compactifications scenarios

arXiv:0911.2708 etc.

$$
\underset{4n_{\mathsf{H}}+4}{\{q^u\}} \;=\; \underset{2n_{\mathsf{H}}(\mathsf{SKG})}{\{\mathsf{z}^i, \bar{\mathsf{z}}^{\bar{\jmath}}\}} + \underset{2n_{\mathsf{H}}}{\{\xi^i, \widetilde{\xi}_i\}} + \underset{4\,(\text{universal})}{\{\varphi, a, \xi^0, \widetilde{\xi}_0\}}
$$

(special quaternionic geometry)

Contribution of hypermultiplets to the kinematics and potential:

$$h_{uv}\,\mathrm{d}q^u\,\mathrm{d}q^v \;=\; \underbrace{G_{i\bar{\jmath}}\,\mathrm{dz}^i\,\mathrm{d}\bar{\mathrm{z}}^{\bar{\jmath}}}_{\text{SKG}_\text{H}} + \underbrace{(\mathrm{d}\varphi)^2}_{\text{4D dilaton}} + \frac{1}{4}\mathrm{e}^{4\varphi}\big(\underbrace{\mathrm{d}a}_{\text{axion}} - \xi^{\mathrm{T}}\,\mathbb{C}_\mathsf{H}\,\mathrm{d}\xi\big)^2 - \frac{1}{2}\mathrm{e}^{2\varphi}\underbrace{\mathrm{d}\xi^{\mathrm{T}}\,\mathbb{M}_\mathsf{H}\,\mathrm{d}\xi}_{\text{scalars from RR}}$$

$$\nabla_\mu q^u \;=\; \partial_\mu q^u + \mathrm{g}\,k^u_\Lambda A^\Lambda_\mu\,, \qquad k_\Lambda \;=\; -\big[2q_\Lambda + e_\Lambda{}^{\mathbb{I}}(\mathbb{C}_\mathsf{H}\xi)_{\mathbb{I}}\big]\frac{\partial}{\partial a} - e_\Lambda{}^{\mathbb{I}}\frac{\partial}{\partial \xi^{\mathbb{I}}}$$

$$\mathcal{P}_+ \;\equiv\; \mathcal{P}_1 + i\mathcal{P}_2 \;=\; 2\mathrm{e}^{\varphi}\,\Pi_\mathsf{V}^{\mathrm{T}}\,Q\,\mathbb{C}_\mathsf{H}\,\Pi_\mathsf{H}$$

$$\mathcal{P}_- \;\equiv\; \mathcal{P}_1 - i\mathcal{P}_2 \;=\; 2\mathrm{e}^{\varphi}\,\Pi_\mathsf{V}^{\mathrm{T}}\,Q\,\mathbb{C}_\mathsf{H}\,\overline{\Pi_\mathsf{H}}$$

$$\mathcal{P}_3 \;=\; \mathrm{e}^{2\varphi}\,\Pi_\mathsf{V}^{\mathrm{T}}\,\mathbb{C}_\mathsf{V}(c + \widetilde{Q}\xi)$$

$$\mathbb{M}_{\mathsf{V,H}} \;=\; \begin{pmatrix} \mu + \nu\mu^{-1}\nu & -\nu\mu^{-1} \\ -\mu^{-1}\nu & \mu^{-1} \end{pmatrix}_{\mathsf{V,H}}\,, \qquad Q_{\boldsymbol{\Lambda}}{}^{\mathbb{I}} \;=\; \begin{pmatrix} e_\Lambda{}^I & e_{\Lambda I} \\ m^{\Lambda I} & m^\Lambda{}_I \end{pmatrix}, \qquad \mathbb{C}_{\mathsf{V,H}} \;=\; \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}$$

$$\mu_{\mathsf{V,H}} \;=\; \mathrm{Im}\mathcal{N}_{\mathsf{V,H}}\,, \quad \nu_{\mathsf{V,H}} \;=\; \mathrm{Re}\mathcal{N}_{\mathsf{V,H}} \qquad\qquad \widetilde{Q}^{\boldsymbol{\Lambda}}{}_{\mathbb{I}} \;=\; \mathbb{C}_\mathsf{V}^{\mathrm{T}}\,Q\,\mathbb{C}_\mathsf{H}$$

$\Pi_\mathsf{H} = \mathrm{e}^{\mathcal{K}_\mathsf{H}/2}(Z^I, \mathcal{G}_I)^{\mathrm{T}}$, $\mathrm{z}^i = Z^i/Z^0$: SKG variables in hypermoduli

$\Pi_\mathsf{V} = \mathrm{e}^{\mathcal{K}_\mathsf{V}/2}(X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$: SKG variables in vector moduli

$c = (p^\Lambda, q_\Lambda)^{\mathrm{T}}$ can also be regarded as the BH charges

Truncate SKG part in hypermultiplets: Set $\Pi_{\mathsf{H}} = 0 = \xi^i = \widetilde{\xi}_i$, i.e., no SKG$_{\mathsf{H}}$ DOF

$$h_{uv}\, \mathrm{d}q^u\, \mathrm{d}q^v = \underbrace{G_{i\bar{\jmath}}\, \mathrm{dz}^i\, \mathrm{d}\bar{\mathrm{z}}^{\bar{\jmath}}}_{\text{SKG}_{\mathsf{H}}} + \underbrace{(\mathrm{d}\varphi)^2}_{\text{4D dilaton}} + \frac{1}{4}\mathrm{e}^{4\varphi}\big(\underbrace{\mathrm{d}a}_{\text{axion}} - \xi^{\mathrm{T}}\, \mathbb{C}_{\mathsf{H}}\, \mathrm{d}\xi\big)^2 - \frac{1}{2}\mathrm{e}^{2\varphi}\underbrace{\mathrm{d}\xi^{\mathrm{T}}\, \mathbb{M}_{\mathsf{H}}\, \mathrm{d}\xi}_{\text{scalars from RR}}$$

$$\nabla_\mu q^u = \partial_\mu q^u + \mathrm{g}\, k_\Lambda^u A_\mu^\Lambda\,, \qquad k_\Lambda = -\big[2q_\Lambda + e_\Lambda{}^{\mathbb{I}}(\mathbb{C}_{\mathsf{H}}\xi)_{\mathbb{I}}\big]\frac{\partial}{\partial a} - e_\Lambda{}^{\mathbb{I}}\frac{\partial}{\partial \xi^{\mathbb{I}}}$$

$$\mathcal{P}_+ \equiv \mathcal{P}_1 + i\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\, \Pi_{\mathsf{V}}^{\mathrm{T}}\, Q\, \mathbb{C}_{\mathsf{H}}\, \Pi_{\mathsf{H}}$$

$$\mathcal{P}_- \equiv \mathcal{P}_1 - i\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\, \Pi_{\mathsf{V}}^{\mathrm{T}}\, Q\, \mathbb{C}_{\mathsf{H}}\, \overline{\Pi_{\mathsf{H}}}$$

$$\mathcal{P}_3 = \mathrm{e}^{2\varphi}\, \Pi_{\mathsf{V}}^{\mathrm{T}}\, \mathbb{C}_{\mathsf{V}}(c + \widetilde{Q}\xi)$$

$\Pi_{\mathsf{H}} = \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2}(Z^I, \mathcal{G}_I)^{\mathrm{T}}$, $\mathrm{z}^i = Z^i/Z^0$: SKG variables in hypermoduli
$\Pi_{\mathsf{V}} = \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2}(X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$: SKG variables in vector moduli
$c = (p^\Lambda, q_\Lambda)^{\mathrm{T}}$ can also be regarded as the BH charges

Contribution of the universal hypermultiplet to the Lagrangian:

$$h_{uv}\nabla_\mu q^u \nabla^\mu q^v = (\partial_\mu \varphi)^2 + \frac{1}{4}\mathrm{e}^{4\varphi}\big(\nabla_\mu a - \xi^0 \nabla_\mu \widetilde{\xi}_0 + \widetilde{\xi}^0 \nabla_\mu \xi^0\big)^2$$

$$\nabla_\mu a = \partial_\mu a - \mathrm{g}(2q_\Lambda + e_\Lambda{}^0\, \widetilde{\xi}_0 - e_{\Lambda 0}\, \xi^0) A_\mu^\Lambda$$

$$\nabla_\mu \xi^0 = \partial_\mu \xi^0 - \mathrm{g}(e_\Lambda{}^0) A_\mu^\Lambda \,, \quad \nabla_\mu \widetilde{\xi}_0 = \partial_\mu \widetilde{\xi}_0 - \mathrm{g}(e_{\Lambda 0}) A_\mu^\Lambda$$

$$V(z, \overline{z}, q) = G^{a\overline{b}} D_a \mathcal{P}_3 \overline{D_b \mathcal{P}_3} - 3|\mathcal{P}_3|^2 \,, \quad \mathcal{P}_3 = \mathrm{e}^{2\varphi}\big(Z + Z_\xi\big)$$

$$Z \equiv L^\Lambda q_\Lambda - M_\Lambda p^\Lambda \,, \quad Z_\xi \equiv L^\Lambda (e_\Lambda{}^0\, \widetilde{\xi}_0 - e_{\Lambda 0}\, \xi^0) - M_\Lambda (m^\Lambda{}_0\, \xi^0 - m^{\Lambda 0}\, \widetilde{\xi}_0)$$

Still complicated even when we focus only on the Universal hypermultiplet

compared to the system only with Vector multiplets

How is non-SUSY RN(-AdS) BH-sol. in the presence of Universal hypermoduli?

$\longrightarrow$ *work in progress*