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# NON-SUSY SOLUTIONS OF RN-AdS BLACK HOLES IN 4D $\mathcal{N} = 2$ GAUGED SUPERGRAVITY

Research Interest: String Compactification beyond Calabi-Yau

Fluxes and/or D-branes on “CY”  $\longrightarrow$  back reactions to CY?

— My Previous Works —


- Heterotic string with  $H$ ,  $\partial\phi$  and torsion [No-go theorem \[hep-th/0605247\]](#), [Index theorems \[arXiv:0704.2111\]](#), [Intersecting five-branes \[arXiv:0912.1334\]](#)
- Type II strings with (non)geometric fluxes  [\$\mathcal{N} = 1\$  AdS/Minkowski Vacua via generalized geometry \[arXiv:0810.0937\]](#), [T-duality on doubled geometry with D-branes \[arXiv:0806.1783\]](#)

— Look at Lower-dimensional Physics —

Is it possible to understand “all” SUGRAs from reductions/truncations of String Theories?

*(We hope so, then we study them!)*

Current Interest: AdS Black Hole Solutions in 4D  $\mathcal{N} = 2$  SUGRA

 WHY  $\mathcal{N} = 2$  (8-SUSY charges)?

- ✓ Scalar fields living in highly symmetric spaces
- ✓ (Flux) compactification scenarios in string/M-theory

 WHY AdS Black Holes?




- ✓ Attractive: 4D  $\mathcal{N} = 2$  SUGRA  $\subset$  Einstein-Yang-Mills-Matters
- ✓ Non-trivial: the existence of the cosmological constant with matter fields

Static, charged AdS black hole in 4D spacetime:

$$ds^2 = -V(r) dt^2 + \frac{1}{V(r)} dr^2 + r^2 d\sigma_{X_2}^2$$

$$V(r) = \kappa - \frac{2\eta}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}, \quad Q^2 = \underbrace{q^2}_{(\text{elect.})} + \underbrace{p^2}_{(\text{magnet.})}, \quad \Lambda = (\text{cosmological const.})$$

$$\Lambda = -\frac{3}{\ell^2}, \quad \eta_0 = \frac{\ell}{3\sqrt{6}} \left( \sqrt{\kappa^2 + 12 Q^2 \ell^{-2}} + 2\kappa \right) \left( \sqrt{\kappa^2 + 12 Q^2 \ell^{-2}} - \kappa \right)^{1/2}$$

topology of $X_2$ w/ genus $g$	metric	ADM mass $M$
$\kappa = 1$ ( $g = 0$ ; $S^2$ ) 	$d\sigma_{X_2}^2 = d\theta^2 + \sin^2 \theta d\phi^2$	$\eta$
$\kappa = 0$ ( $g = 1$ ; flat) 	$d\sigma_{X_2}^2 = dx^2 + 2\text{Re}\tau dx dy +  \tau ^2 dy^2; \quad x, y \in [0, 1]$	$\frac{\eta  \text{Im}\tau }{4\pi}$
$\kappa = -1$ ( $g > 1$ ; $H^2$ ) 	$d\sigma_{X_2}^2 = d\theta^2 + \sinh^2 \theta d\phi^2$	$(\eta - \eta_0)(g - 1)$

e.g., Caldarelli and Klemm [[hep-th/9808097](https://arxiv.org/abs/hep-th/9808097)]

Static, charged AdS black hole in 4D spacetime with  $\kappa = 1$ :

$$ds^2 = -V(r) dt^2 + \frac{1}{V(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$V(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}, \quad Q^2 = \underbrace{q^2}_{(\text{elect.})} + \underbrace{p^2}_{(\text{magnet.})}, \quad \Lambda = (\text{cosmological const.})$$

$$\Lambda = -\frac{3}{\ell^2}, \quad \eta_0 = \frac{\ell}{3\sqrt{6}} \left( \sqrt{1 + 12 Q^2 \ell^{-2}} + 2 \right) \left( \sqrt{1 + 12 Q^2 \ell^{-2}} - 1 \right)^{1/2}$$

Various (non-rotating) AdS black holes:

$$\text{Schwarzschild-AdS : } M \neq 0, \quad Q = 0, \quad \Lambda < 0$$

$$\text{Reissner-Nordström AdS (RN-AdS) : } M \neq 0, \quad Q \neq 0, \quad \Lambda < 0$$

$$\text{Condition for regular event horizon: } M \geq \eta_0$$

$$\text{Extremality : } M = \eta_0$$

e.g., Caldarelli, et.al. [[hep-th/9808097](#)]

 In the framework of 4D  $\mathcal{N} = 2$  ungauged SUGRA,

(Extremal) RN-BH in asymptotically flat spacetime has been investigated.  
 BH charges from D-branes wrapped on CY: “D0-D4”, “D2-D6”, “D0-D2-D6”, etc.  
 (Hypermultiplets are decoupled from the system.)

 We (have to) study AdS BH in 4D  $\mathcal{N} = 2$  system.

The cosmological constant  $\Lambda$  is given as expectation value of scalar potential  $V$ .  
 (“mass deformations” of gravitini)

**Remark: Naked Singularity appears in SUSY RN-AdS solution.**

 Pure AdS SUGRA (only gravitational multiplet):

Romans [[hep-th/9203018](#)], Caldarelli, et.al. [[hep-th/9808097](#)], etc.

*There exists a SUSY solution of rotating AdS black hole with regular horizon.*

 Gauged SUGRA with vector multiplets (without hyper-sector):

Sabra, et.al. (electric charges [[hep-th/9903143](#)], magnetic/dyonic charges [[hep-th/0003213](#)]), etc.

## Question

How can we obtain **non-SUSY** solutions **with** matter fields  
in asymptotically **AdS** spacetime?

## — Steps —

- Vector-multiplets + electric FI (studied)\*
- Vector-multiplets + electric/magnetic FI (as electric/magnetic charges) **This Work**
- Vector- + Hypermultiplets ?†
- Vector- + (massive) Scalar–Tensor-multiplets ?‡
- Non-abelianization ?‡

\* ex.) Bellucci, Ferrara, Marrani and Yeranyan [[arXiv:0802.0141](#)], etc.

† ex.) 1 vector + 2 hypers in terms of Sasaki-Einstein<sub>7</sub> in M-theory [[arXiv:1009.3805](#)], etc.

‡ Directly related to (non)geometric flux backgrounds via [[hep-th/0409097](#)], [[hep-th/0701247](#)], etc.

‡ Deform Heisenberg algebra(?), e.g. [[hep-th/0410290](#)], etc.

### Question

How can we obtain **non-SUSY** solutions **with** matter fields  
in asymptotically **AdS** spacetime?

### — Setup and Result —

- 4D  $\mathcal{N} = 2$  Abelian gauged SUGRA with mutually local electric/magnetic charges
- Truncate out hyper-sector **by hand** (artificial)
- Import the Attractor Mechanism
- Find a new attractor equation
- Obtain a non-SUSY solution with small  $|\Lambda|$  in  $T^3$ -model
- Argue “generic descriptions of moduli” in  $T^3$ -model and in STU-model



## Question

How can we obtain **non-SUSY** solutions **with** matter fields  
in asymptotically **AdS** spacetime?

## — Open Problems —

- Strange behaviors of  $\Lambda$  and size of the event horizon
- Hyper-sector: axions become massive tensors if *magnetic* charges are present

$$\text{SQG} \longrightarrow \underbrace{[\text{SKG} + \text{dilaton}]}_{\text{contribute to } V} + \underbrace{[\text{NS-axion}]}_{\text{massive tensor}} + \underbrace{[\text{RR-axions}]}_{\text{massive or flat}}$$

not contribute to  $V$ (?)

BH gives rise to a perspective on gauged SUGRA with **magnetic** charges and matters.

*Some magnetic charges* come from **nongeometric fluxes** on internal space  
whose transition function contains **T-duality**  $SO(6, 6)$  beyond diffeomorphism  $GL(6, \mathbb{R})$ .

e.g., Graña, Louis and Waldram [[hep-th/0612237](#)], D'Auria, Ferrara and Trigiante [[hep-th/0701247](#)]

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- $\mathcal{N} = 2$  Gauged SUGRA
  - Effective Black Hole Potential
  - Attractor Equation
- Single Modulus Model
- Discussions
- Appendix: hyper-sector in the presence of magnetic charges (*now learning..*)
- Appendix: equations and identity without hyper-sector

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Supersymmetric multiplets in 4D  $\mathcal{N} = 2$  SUGRA:

1 gravitational multiplet:  $\{g_{\mu\nu}, A_\mu^0, \psi_{A\mu}\}$   $\mu = 0, 1, 2, 3$  (4D, curved)  
 $A = 1, 2$  ( $SU(2)$  R-symmetry)

$n_V$  vector multiplets:  $\{A_\mu^a, z^a, \lambda^{aA}\}$   $a = 1, \dots, n_V$

$z^a$  in special Kähler geometry  $\mathcal{SM}$

$n_H + 1$  hypermultiplets:  $\{q^u, \zeta_\alpha\}$   $u = 1, \dots, 4n_H + 4$   
 $\alpha = 1, \dots, 2n_H + 2$

$q^u$  in quaternionic geometry  $\mathcal{HM}$

Gauging: *Promote* global isometry groups on scalar field spaces  
to local symmetries

“Standard”: Andrianopoli, Bertolini, Ceresole, D’Auria, Ferrara, Fré and Magri [[hep-th/9605032](#)]

“Magnetically dualized”: D’Auria, Sommovigo and Vaulà [[hep-th/0409097](#)]

Action (gravitational const.  $\kappa$ ; gauge coupling const.  $g$ ; index  $\Lambda = 0, 1, \dots, n_V$ ):

$$\begin{aligned}
 S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - G_{a\bar{b}}(z, \bar{z}) \nabla_\mu z^a \nabla^\mu \bar{z}^{\bar{b}} - h_{uv}(q) \nabla_\mu q^u \nabla^\mu q^v \right. \\
 + \frac{1}{4} \mu_{\Lambda\Sigma}(z, \bar{z}) F_{\mu\nu}^\Lambda F^{\Sigma\mu\nu} + \frac{1}{4} \nu_{\Lambda\Sigma}(z, \bar{z}) F_{\mu\nu}^\Lambda (*F^\Sigma)^{\mu\nu} \\
 - g^2 V(z, \bar{z}, q) \\
 \left. + (\text{fermionic terms}) \right\}
 \end{aligned}$$

$$\mu_{\Lambda\Sigma} = \text{Im} \mathcal{N}_{\Lambda\Sigma} \quad (\text{generalized } -1/g^2), \quad \nu_{\Lambda\Sigma} = \text{Re} \mathcal{N}_{\Lambda\Sigma} \quad (\text{generalized } \theta\text{-angle})$$

In this analysis...  $\left[ \begin{array}{l} \text{Set background fermionic fields to zero} \\ \text{Reduce gauge symmetries to } \textit{abelian}: \nabla_\mu z^a \rightarrow \partial_\mu z^a \\ \text{Truncate hyper-sector } \textit{by hand, after the formulation} \quad (\textit{later}) \end{array} \right.$

Equations of Motion (abbreviate  $\kappa$  and  $g$ , and hyper-sector):

$$\delta g^{\mu\nu} : \quad \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) - 2G_{a\bar{b}} \partial_{(\mu} z^a \partial_{\nu)} \bar{z}^{\bar{b}} + G_{a\bar{b}} \partial_{\rho} z^a \partial^{\rho} \bar{z}^{\bar{b}} g_{\mu\nu} = T_{\mu\nu} - V g_{\mu\nu}$$

$$T_{\mu\nu} = -\mu_{\Lambda\Sigma} F_{\mu\rho}^{\Lambda} F_{\nu\sigma}^{\Sigma} g^{\rho\sigma} + \frac{1}{4} \mu_{\Lambda\Sigma} F_{\rho\sigma}^{\Lambda} F^{\Sigma\rho\sigma} g_{\mu\nu} \quad (\text{energy-momentum tensor})$$

$$\begin{aligned} \delta z^a : \quad & -\frac{G_{a\bar{b}}}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} g^{\mu\nu} \partial_{\nu} \bar{z}^{\bar{b}} \right) - \frac{\partial G_{a\bar{b}}}{\partial \bar{z}^{\bar{c}}} \partial_{\rho} \bar{z}^{\bar{b}} \partial^{\rho} \bar{z}^{\bar{c}} \\ & = \frac{1}{4} \frac{\partial \mu_{\Lambda\Sigma}}{\partial z^a} F_{\mu\nu}^{\Lambda} F^{\Sigma\mu\nu} + \frac{1}{4} \frac{\partial \nu_{\Lambda\Sigma}}{\partial z^a} F_{\mu\nu}^{\Lambda} (*F^{\Sigma})^{\mu\nu} - \frac{\partial V}{\partial z^a} \end{aligned}$$

$$\delta A_{\mu}^{\Lambda} : \quad \varepsilon^{\mu\nu\rho\sigma} \partial_{\nu} G_{\Lambda\rho\sigma} = 0, \quad G_{\Lambda\rho\sigma} = \nu_{\Lambda\Sigma} F_{\rho\sigma}^{\Sigma} - \mu_{\Lambda\Sigma} (*F^{\Sigma})_{\rho\sigma}$$

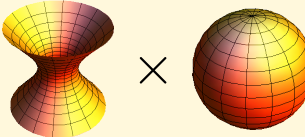
$$\text{electric charge } q_{\Lambda} \equiv \frac{1}{4\pi} \int_{S^2} G_{2\Lambda}, \quad \text{magnetic charge } p^{\Lambda} \equiv \frac{1}{4\pi} \int_{S^2} F_2^{\Lambda}$$

Introduce ansatz for RN(-AdS) BH: **Extremal**, static, charged, spherically symmetric

$$ds^2 = -e^{2A(r)} dt^2 + e^{2B(r)} dr^2 + e^{2C(r)} r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

Near horizon geometry:  $\text{AdS}_2 \times S^2$  (radii:  $r_A$  and  $r_H$ )

$$A(r) = \log \frac{r - r_H}{r_A}, \quad B(r) = -A(r), \quad C(r) = \log \frac{r_H}{r}$$

$$R(\text{AdS}_2 \times S^2) = 2 \left( -\frac{1}{r_A^2} + \frac{1}{r_H^2} \right) \quad \text{AdS}_2 \times S^2$$


$$\begin{aligned} \rightarrow ds^2(\text{near horizon}) &= - \left( \frac{r - r_H}{r_A} \right)^2 dt^2 + \left( \frac{r_A}{r - r_H} \right)^2 dr^2 + r_H^2 (d\theta^2 + \sin^2 \theta d\phi^2) \\ &= - \frac{e^{2\tau}}{r_A^2} dt^2 + r_A^2 d\tau^2 + r_H^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (\tau = \log(r - r_H)) \end{aligned}$$

If the attractor mechanism works (via extremality), the scalar fields are subject to

$$\left. \frac{d}{dr} z^a \right|_{\text{horizon}} = 0, \quad \left. \left( \frac{d}{dr} \right)^2 z^a \right|_{\text{horizon}} = 0$$

The EoM are drastically reduced to

Bellucci, et.al. [arXiv:0802.0141]

$$\begin{aligned} \delta g^{tt}, \delta g^{rr} : \quad \frac{1}{r_H^2} &= \frac{1}{r_H^4} I_S + V \Big|_{\text{horizon}} & \longrightarrow & \quad r_H^2 = \frac{1 - \sqrt{1 - 4I_S V}}{2V} \Big|_{\text{horizon}} \\ \delta g^{\theta\theta}, \delta g^{\phi\phi} : \quad \frac{1}{r_A^2} &= \frac{1}{r_H^4} I_S - V \Big|_{\text{horizon}} & \longrightarrow & \quad r_A^2 = \frac{r_H^2}{\sqrt{1 - 4I_S V}} \Big|_{\text{horizon}} \\ \delta z^a : \quad 0 &= \frac{1}{r_H^4} \frac{\partial I_S}{\partial z^a} - \frac{\partial V}{\partial z^a} \Big|_{\text{horizon}} & \longrightarrow & \quad 0 = \frac{1}{r_H^4} (1 - 2r_H^2 V) \frac{\partial}{\partial z^a} r_H^2 \Big|_{\text{horizon}} \end{aligned}$$

(1st) Symplectic Invariant:

$$I_S(z, \bar{z}, p, q) = -\frac{1}{2} \begin{pmatrix} p^\Lambda & q_\Lambda \end{pmatrix} \begin{pmatrix} \mu_{\Lambda\Sigma} + \nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Delta} \nu_{\Delta\Sigma} & -\nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Sigma} \\ -(\mu^{-1})^{\Lambda\Gamma} \nu_{\Gamma\Sigma} & (\mu^{-1})^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} p^\Sigma \\ q_\Sigma \end{pmatrix} \equiv -\frac{1}{2} \Gamma^T \mathbb{M} \Gamma$$

$$\text{with} \quad T_t^t = T_r^r = -T_\theta^\theta = -T_\phi^\phi = -\frac{e^{-4C}}{r^4} I_S$$



BH entropy (and the effective potential) is given as the Area of the event horizon:

$$S_{\text{BH}}(p, q) = \frac{A_{\text{H}}}{4\pi} = r_{\text{H}}^2 \Big|_{\text{horizon}} \equiv V_{\text{eff}}(z, \bar{z}, p, q) \Big|_{\text{horizon}}$$

$$V_{\text{eff}}(z, \bar{z}, p, q) = \frac{1 - \sqrt{1 - 4I_S V}}{2V} = I_S + (I_S)^2 V + 2(I_S)^3 V^2 + \mathcal{O}((I_S)^4 V^3)$$

We read the “cosmological constant  $\Lambda$ ” from the scalar curvature:

$$R(\text{AdS}_2 \times S^2) = 2 \left( -\frac{1}{r_{\text{A}}^2} + \frac{1}{r_{\text{H}}^2} \right) = 4V$$

$$V \Big|_{\text{horizon}} \equiv \Lambda(\text{“cosmological constant”})$$

### ATTRACTOR EQUATION

$$0 = \frac{1}{r_{\text{H}}^4} (1 - 2r_{\text{H}}^2 V) \frac{\partial}{\partial z^a} V_{\text{eff}} \Big|_{\text{horizon}} \rightarrow 0 = \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \bar{z}, p, q) \Big|_{\text{horizon}}$$

If  $r_{\text{H}} < \infty$  and  $V \Big|_{\text{horizon}} \leq 0$

The “ATTRACTOR EQUATION” which we have to solve is

$$\begin{aligned}
 0 &= \left. \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \bar{z}, p, q) \right|_{\text{horizon}} \\
 &= \frac{1}{2V^2 \sqrt{1 - 4I_S V}} \left\{ 2V^2 \frac{\partial I_S}{\partial z^a} - \left( \sqrt{1 - 4I_S V} + 2I_S V - 1 \right) \frac{\partial V}{\partial z^a} \right\} \Bigg|_{\text{horizon}}
 \end{aligned}$$

Evaluate  $I_S$  and  $V$  described in terms of the central charge  $Z$

Useful when we consider (non-)SUSY solutions

SUSY variation of gravitini carries definition of  $Z$  *and more..*

$$\delta\psi_{A\mu} = D_\mu \varepsilon_A + \epsilon_{AB} T_{\mu\nu}^- \gamma^\nu \varepsilon^B + i \mathbf{g} \mathcal{S}_{AB} \gamma_\mu \varepsilon^B$$

$$Z = -\frac{1}{2} \left( \frac{1}{4\pi} \int_{S^2} T_2^- \right), \quad \mathcal{S}_{AB} = \frac{i}{2} \sum_{x=1}^3 (\sigma^x)_{AB} \mathcal{P}_x$$

Describe  $Z$ ,  $I_S$  and  $V$  in terms of  $(L^\Lambda, M_\Lambda) = e^{K/2}(X^\Lambda, \mathcal{F}_\Lambda)$  on  $\mathcal{SM}$ :

$$Z = L^\Lambda q_\Lambda - M_\Lambda p^\Lambda$$

$$I_S = |Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}$$

$$V = \sum_{x=1}^3 \left( -3|\mathcal{P}_x|^2 + G^{a\bar{b}} D_a \mathcal{P}_x \overline{D_b \mathcal{P}_x} \right) + 4h_{uv} k^u \bar{k}^v$$

$\mathcal{P}_x$ :  $SU(2)$  triplet of Killing prepotentials in  $\mathcal{N} = 2$  SUGRA

(In general, both vector moduli and hyper moduli contribute to  $\mathcal{P}_x$ )

Appendix

Hyper-sector is “truncated”  $\rightarrow \mathcal{P}_3 = \mathcal{P}_{3,\Lambda} L^\Lambda - \tilde{\mathcal{P}}_3^\Lambda M_\Lambda$  still remains in  $V$

Further, **Identify** electric/magnetic FI with charges  $(\mathcal{P}_{3,\Lambda}, \tilde{\mathcal{P}}_3^\Lambda) = (q_\Lambda, p^\Lambda) \rightsquigarrow \mathcal{P}_3 \equiv Z$

Cassani, Ferrara, Marrani, Morales and Samtleben [arXiv:0911.2708]

$$V \equiv -3|Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}$$

The “ATTRACTOR EQUATION” (with a non-trivial factor  $-1 \leq G_V \equiv \frac{1 - V_{\text{eff}}^2}{1 + V_{\text{eff}}^2} \leq 1$ )

$$\begin{aligned}
 0 &= \left. \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \bar{z}, p, q) \right|_{\text{horizon}} \\
 &= \frac{1}{2V^2 \sqrt{1 - 4I_S V}} \left\{ 2V^2 \frac{\partial I_S}{\partial z^a} - (\sqrt{1 - 4I_S V} + 2I_S V - 1) \frac{\partial V}{\partial z^a} \right\} \Bigg|_{\text{horizon}} \quad \text{Appendix} \\
 &= \frac{1 + V_{\text{eff}}^2}{\sqrt{1 - 4I_S V}} \left\{ 2G_V \bar{Z} D_a Z + i C_{abc} G^{b\bar{b}} G^{c\bar{c}} \overline{D_b Z} \overline{D_c Z} \right\} \Bigg|_{\text{horizon}}
 \end{aligned}$$

Solve the equation  $0 = 2G_V \bar{Z} D_a Z + i C_{abc} G^{b\bar{b}} G^{c\bar{c}} \overline{D_b Z} \overline{D_c Z} \Big|_{\text{horizon}}$

under the condition  $V < 0$ ,  $1 - 4I_S V > 0$ ,  $\partial_a I_S \neq 0$ ,  $\partial_a V \neq 0$ ,  $D_a Z \neq 0$

🔗 If  $V < 0$  and  $D_a Z = 0$  (SUSY)  $\rightarrow$  Naked Singularity  $\rightarrow$  Search non-SUSY solution  $D_a Z \neq 0$

🔗 If  $\partial_a I_S = 0$  or  $\partial_a V = 0$   $\rightarrow$  asymptotically flat  $V = 0$  or Empty Hole  $Z = 0$   
 ( $G_V = 1$ )      ( $G_V = -1$ )

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- Consider the Single Modulus Model of  $\Gamma = (0, p, 0, q_0)$  (analogous to “D0-D4” system)

with cubic prepotential  $\mathcal{F} = \frac{(X^1)^3}{X^0}$ ,  $t = \frac{X^1}{X^0} \longrightarrow$  Kähler potential  $e^K = \frac{i}{(t - \bar{t})^3}$

(as the “large volume limit of Calabi-Yau”)

The ATTRACTOR EQUATION and its solution ( $t = 0 + iy$ ,  $y < 0$ ):

$$p(y^2)^3 + (q_0 - 18p^3q_0^2)(y^2)^2 - 12p^2q_0^3(y^2) - 2pq_0^4 = 0$$

$$\text{with } p \neq 0, \quad q_0 \neq 0, \quad pq_0 < 0$$

$$y^2 = \mathfrak{A} + \mathfrak{B} \quad \text{or} \quad \mathfrak{A} + \omega^\pm \mathfrak{B} \quad (\omega^3 = 1)$$

$$\mathfrak{A} = \frac{q_0}{3p}(18p^3q_0 - 1), \quad \mathfrak{B} = \frac{1}{3p} \left( \mathfrak{e}^{1/3} + \frac{q_0^2}{4} \frac{1 + (18p^3q_0)^2}{\mathfrak{e}^{1/3}} \right)$$

$$\mathfrak{e} = -q_0^3 \left[ 1 - 27p^3q_0 - (18p^3q_0)^3 - 3\sqrt{3} \sqrt{-2p^3q_0 - 9(p^3q_0)^2 - 432(p^3q_0)^3} \right]$$

Various values at the event horizon:

$$Z|_{\text{horizon}} = \frac{q_0 + 3p y^2}{2} \sqrt{-\frac{1}{2y^3}} \neq 0, \quad D_t Z|_{\text{horizon}} = \frac{3i(q_0 - p y^2)}{4y} \sqrt{-\frac{1}{2y^3}} \neq 0$$

$$I_S|_{\text{horizon}} = \frac{q_0^2 + 3p^2 y^4}{-2y^3} > 0$$

$$\Lambda = \frac{6(pq_0)^2 (q_0 + 3p y^2)^2}{y^5} < 0$$

$$S_{\text{BH}} = \frac{-y}{12(pq_0)^2 (q_0 + 3p y^2)^2} \left\{ -y^4 + \sqrt{y^8 + 12(pq_0)^2 (q_0 + 3p y^2)^2 (q_0^2 + 3p^2 y^4)} \right\} > 0$$

their asymptotic behaviors?



Look at the Small  $q_0$  limit:

The dominant part of the Modulus  $t = 0 + iy$  ( $y < 0$ ) is

$$y \sim -\sqrt{-\frac{q_0}{p}} + (\text{sub-leading orders})$$

The dominant parts of various values are

$$Z|_{\text{horizon}} \sim -q_0 \left(-\frac{p^3}{q_0^3}\right)^{1/4} + \dots, \quad D_t Z|_{\text{horizon}} \sim ip \left(-\frac{p}{q_0}\right)^{1/4} + \dots$$

$$I_S|_{\text{horizon}} \sim \sqrt{-p^3 q_0} + \dots$$

$$\Lambda \sim -\sqrt{(-p^3 q_0)^3} + \dots$$

$$I_S \sim S_{\text{BH}} \gg -\Lambda > 0$$

$$S_{\text{BH}} \sim \sqrt{-p^3 q_0} + \dots$$

very small  $|\Lambda|$  compared to others: similar to the non-BPS RN-BH,  
 but *Never* connected to RN-BH with  $\Lambda = 0!$



*Compare* to values of non-BPS RN-BH with  $\Lambda = 0$  in  $\mathcal{N} = 2$  ungauged SUGRA:

$$t = 0 + iy, \quad y = -\sqrt{-\frac{q_0}{p}}$$

$$Z|_{\text{horizon}} = -\frac{q_0}{\sqrt{2}} \left(-\frac{p^3}{q_0^3}\right)^{1/4} \neq 0, \quad D_t Z|_{\text{horizon}} = -3ip \left(-\frac{p}{q_0}\right)^{1/4} \neq 0$$

$$S_{\text{BH}} = I_S|_{\text{horizon}} = |Z|^2 + G^{t\bar{t}} D_t Z \bar{D}_t \bar{Z}|_{\text{horizon}} = 4|Z|^2|_{\text{horizon}} = \sqrt{-4p^3 q_0} > 0$$



Argue the Large  $q_0$  limit:

The dominant part of the Modulus  $t = 0 + iy$  ( $y < 0$ ) is

$$y \sim pq_0 + (\text{sub-leading orders})$$

The dominant parts of various values are

$$Z \Big|_{\text{horizon}} \sim \sqrt{-p^3 q_0} + \dots \neq 0, \quad D_t Z \Big|_{\text{horizon}} \sim \frac{-i}{pq_0} \sqrt{-p^3 q_0} + \dots \neq 0$$

$$I_S \Big|_{\text{horizon}} \sim -p^3 q_0 + \dots > 0$$

$$\Lambda \sim p^3 q_0 + \dots < 0 \quad (\text{same magnitude to } I_S !)$$

$$S_{\text{BH}} \sim \sqrt{\frac{5}{6}} + \dots > 0 \quad (\text{Why constant ??})$$

**Pathological** behaviors of  $\Lambda$  and  $S_{\text{BH}}$ : *not yet completely understood the reason incorrect expansions? and/or breaking  $SU(2)$  by truncating hyper-sector?*

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- ☑ Studied Extremal RN-AdS Black Hole solution in Abelian gauged SUGRA
- ☑ Obtained the analytic description of non-SUSY solution in  $T^3$ -model
- ⚠ Different behaviors from the ones of non-BPS RN black hole in asympt.-flat spacetime:
  - ✓ Should we expand the solution in terms of “ $\Lambda$ ” rather than “ $q_0$ ”?
  - ✓ Consider the contribution of (electrically/magnetically coupled) **hyper-sector**!? Appendix
    - D’Auria, et.al. [[hep-th/0409097](#)] (massive tensors)
    - D’Auria, et.al. [[hep-th/0701247](#)], Cassani, et.al. [[arXiv:0911.2708](#)] (generalized geometry)
    - Hristov, Looyestijn and Vandoren [[arXiv:1005.3650](#)] (gauged SUGRA to ungauged SUGRA via Higgs mechanism)
  - ✓ Is the “Attractor Mechanism” still available?
- ☞ Argue more general solutions in more general setups (rotating, non-extremal, etc..)

❗ Consider the contribution of (electrically/magnetically coupled) **hyper-sector!**

D'Auria, et.al. [[hep-th/0409097](#)], Gauntlett, Kim, Varela and Waldram [[arXiv:0901.0676](#)] (massive tensors)

D'Auria, et.al. [[hep-th/0701247](#)], Cassani, et.al. [[arXiv:0911.2708](#)] (generalized geometry)

Hristov, et.al. [[arXiv:1005.3650](#)] (gauged SUGRA to ungauged SUGRA via Higgs mechanism)

Donos, Gauntlett, Kim and Varela [[arXiv:1009.3805](#)] (1 vector + 2 hyps in consistent truncation of M-theory)

Practice:

- ✓ Which vector fields are eaten by two-form fields via anti-Higgs mechanism?  
EoM for all (massive) fields should be re-analyzed.
- ✓ Where do super-partners of the vector fields and of the massive two-form fields go?  
We have to check the number of physical DOFs and the preservation of  $\mathcal{N} = 2$  local SUSY.
- ✓ Does the “No-go theorem” of SUSY AdS-BH solution work even with hyper-sector?  
We shall investigate the SUSY variation of the gravitini.
- ✓ Which level is the mass scale of the massive tensor multiplets at?  
(This depends on the viewpoints: “purely 4D SUGRA” vs “10D Effective Theory”)
- ✓ How is the gauge symmetry non-abelianized?  
*Everything should be reconsidered from the beginning..*

Thank you for your attention.

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Action including hypermultiplets: [Vector sector](#) [Discussions](#)

$$\begin{aligned}
 S = \int d^4x \sqrt{-g} & \left\{ \frac{1}{2\kappa^2} R - G_{a\bar{b}}(z, \bar{z}) \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} - h_{uv}(q) \nabla_\mu q^u \nabla^\mu q^v \right. \\
 & + \frac{1}{4} \mu_{\Lambda\Sigma}(z, \bar{z}) F_{\mu\nu}^\Lambda F^{\Sigma\mu\nu} + \frac{1}{4} \nu_{\Lambda\Sigma}(z, \bar{z}) F_{\mu\nu}^\Lambda (*F^\Sigma)^{\mu\nu} \\
 & \left. - g^2 V(z, \bar{z}, q) + (\text{fermionic terms}) \right\}
 \end{aligned}$$

Scalar field space of hypermultiplets = (special) quaternionic geometry

$$\underbrace{\{q^u\}}_{4n_H + 4} = \underbrace{\{z^i, \bar{z}^{\bar{j}}\}}_{2n_H(\text{SKG})} + \underbrace{\{\xi^i, \tilde{\xi}_j\}}_{2n_H} + \underbrace{\{\varphi, a, \xi^0, \tilde{\xi}_0\}}_{4(\text{universal})} = \underbrace{\{z^i, \bar{z}^{\bar{j}}, \varphi\}}_{\text{SKG+dilaton}} + \underbrace{\{a, \xi^I, \tilde{\xi}_J\}}_{\text{"Heisenberg"}}$$

Kinetic term and Scalar potential:

$$h_{uv} dq^u dq^v = \underbrace{G_{i\bar{j}} dz^i d\bar{z}^{\bar{j}}}_{\text{SKG}_H} + \underbrace{(d\varphi)^2}_{\text{4D dilaton}} + \underbrace{\frac{1}{4} e^{4\varphi} (da - \xi^T \mathbb{C}_H d\xi)^2}_{\text{axion}} - \underbrace{\frac{1}{2} e^{2\varphi} d\xi^T \mathbb{M}_H d\xi}_{\text{RR-axions}}$$

$$\begin{aligned}
 V &= \sum_{x=1}^3 \left( G^{a\bar{b}} D_a \mathcal{P}_x \overline{D_b \mathcal{P}_x} - 3|\mathcal{P}_x|^2 \right) + 4h_{uv} k^u \bar{k}^v \\
 &= G^{a\bar{b}} D_a \mathcal{P}_+ \overline{D_b \mathcal{P}_+} + G^{a\bar{b}} D_a \mathcal{P}_3 \overline{D_b \mathcal{P}_3} - 3|\mathcal{P}_+|^2 - 3|\mathcal{P}_3|^2 + G^{i\bar{j}} D_i \mathcal{P}_+ \overline{D_j \mathcal{P}_+} + |\mathcal{P}_+|^2 + 4|\mathcal{P}_3|^2
 \end{aligned}$$

e.g., Cassani, et.al. [arXiv:0911.2708]



Contribution of hypermultiplets to the Killing prepotentials:

Vector sector

$$\Pi_V = e^{\mathcal{K}_V/2} (X^\Lambda, \mathcal{F}_\Lambda)^T$$

$$z^a = X^a / X^0$$

$$a = 1, \dots, n_V$$

SKG<sub>V</sub> in vector-moduli

$$\mathcal{P}_+ \equiv \mathcal{P}_1 + i\mathcal{P}_2 = 2e^\varphi \Pi_V^T Q \mathbb{C}_H \Pi_H$$

$$\mathcal{P}_- \equiv \mathcal{P}_1 - i\mathcal{P}_2 = 2e^\varphi \Pi_V^T Q \mathbb{C}_H \bar{\Pi}_H$$

$$\mathcal{P}_3 = e^{2\varphi} \Pi_V^T \mathbb{C}_V (c + \tilde{Q}\xi)$$

$$\Pi_H = e^{\mathcal{K}_H/2} (Z^I, \mathcal{G}_I)^T$$

$$z^i = Z^i / Z^0$$

$$i = 1, \dots, n_H$$

SKG<sub>H</sub> in hyper-moduli

Functions of period matrices:

$$\mathbb{M}_{V,H} \equiv \begin{pmatrix} 1 & -\nu \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mu & 0 \\ 0 & \mu^{-1} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\nu & 1 \end{pmatrix}_{V,H} = \begin{pmatrix} \mu + \nu\mu^{-1}\nu & -\nu\mu^{-1} \\ -\mu^{-1}\nu & \mu^{-1} \end{pmatrix}_{V,H}, \quad \begin{aligned} \mu_{V,H} &\equiv \text{Im } \mathcal{N}_{V,H} \\ \nu_{V,H} &\equiv \text{Re } \mathcal{N}_{V,H} \end{aligned}$$

Electric/magnetic charges with constraints ( $n_V \leq n_H$ ):

$$Q_\Lambda^I \equiv \begin{pmatrix} e_\Lambda^I & e_{\Lambda I} \\ m^{\Lambda I} & m^\Lambda_I \end{pmatrix}, \quad c \equiv \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}; \quad \mathbb{C}_{V,H} \equiv \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}_{V,H}, \quad \tilde{Q}^{\Lambda_I} \equiv \mathbb{C}_V^T Q \mathbb{C}_H$$

$Q \mathbb{C}_H Q^T = 0 = c^T Q$  : Closure condition of massive tensor gauge transformation (tensors appear!)

Tadpole cancellation in 10D string theory,

or Nilpotency of exterior derivative on  $SU(3) \times SU(3)$  generalized geometry

$Q^T \mathbb{C}_V Q = 0$  : Abelianity condition (of Heisenberg algebra)

also from Nilpotency of exterior derivative

→ possible to non-abelianize?

[Electric Gauging] (ex.) “Heisenberg” directions:

$$\nabla_{\mu} q^u = \partial_{\mu} q^u + g k_{\Lambda}^u A_{\mu}^{\Lambda}, \quad k_{\Lambda} = -[2q_{\Lambda} + e_{\Lambda}^{\mathbb{I}}(\mathbb{C}_{\text{H}\xi})_{\mathbb{I}}] \frac{\partial}{\partial a} - e_{\Lambda}^{\mathbb{I}} \frac{\partial}{\partial \xi^{\mathbb{I}}}$$

[Electric/Magnetic Gauging] (ex.) “Heisenberg” directions:

$$\begin{aligned} \nabla_{\mu} q^u &= \partial_{\mu} q^u + g k_{\Lambda}^u A_{\mu}^{\Lambda} + g \tilde{k}^{u\Lambda} \tilde{A}_{\Lambda\mu} \\ k_{\Lambda} &= -[2q_{\Lambda} + e_{\Lambda}^{\mathbb{I}}(\mathbb{C}_{\text{H}\xi})_{\mathbb{I}}] \frac{\partial}{\partial a} - e_{\Lambda}^{\mathbb{I}} \frac{\partial}{\partial \xi^{\mathbb{I}}} \\ \tilde{k}^{\Lambda} &= -[2p^{\Lambda} + m^{\Lambda\mathbb{I}}(\mathbb{C}_{\text{H}\xi})_{\mathbb{I}}] \frac{\partial}{\partial a} + m^{\Lambda\mathbb{I}} \frac{\partial}{\partial \xi^{\mathbb{I}}} \end{aligned}$$

with commutation relations:

$$[k_{\Lambda}, k_{\Sigma}] = [\tilde{k}^{\Lambda}, \tilde{k}^{\Sigma}] = [k_{\Lambda}, \tilde{k}^{\Sigma}] = 0$$

Killing prepotentials  $\mathcal{P}_x$  in terms of the  $SU(2)$  connection  $\omega_x$ :

$$\mathcal{P}_x = (\omega_x)_u k^u \quad \text{where} \quad k = -\Pi_{\text{V}}^{\text{T}} \mathbb{C}_{\text{V}} (2c + \tilde{Q}\xi) \frac{\partial}{\partial a} - \Pi_{\text{V}}^{\text{T}} Q \frac{\partial}{\partial \xi}$$

e.g., Cassani, et.al. [[arXiv:0911.2708](https://arxiv.org/abs/0911.2708)]

 If  $p^\Lambda$  exists, the axion  $a$  is dualized (back) to NS  $B_2$ -field (magnetically coupled).

Gauge field strength is replaced:  $F_2^\Lambda \rightarrow \widehat{F}_2^\Lambda = F_2^\Lambda + p^\Lambda B_2$  and  $B_2$  becomes massive.

e.g., D'Auria, et.al. [[hep-th/0409097](#)]

 In addition, if non-vanishing  $(m^{\Lambda I}, m^{\Lambda J})$  are incorporated,

$\rightarrow$   $\left\{ \begin{array}{l} n_V + 1 (\leq n_H + 1) \text{ RR-axions in } (\xi^I, \tilde{\xi}_J) \text{ become massive tensors } (C_{2\widehat{I}}, C_2^{\widehat{J}}) \\ n_V + 1 (\leq n_H + 1) \text{ RR-axions in } (\xi^I, \tilde{\xi}_J) \text{ become massive scalars} \\ \text{Gauge field strength is replaced: } F_2^\Lambda \rightarrow \widehat{F}_2^\Lambda = F_2^\Lambda + m^{\Lambda \widehat{I}} C_{2\widehat{I}} + m^{\Lambda \widehat{J}} C_2^{\widehat{J}} + p^\Lambda B_2 \\ \text{Remaining } 2(n_H - n_V) \text{ RR-axions are flat directions for } V \end{array} \right.$

$I = 0, \dots, n_H; \widehat{I} = 0, \dots, n_V \text{ with } n_V \leq n_H.$

We can choose  $\xi^{\widehat{I}}$  to dualized  $C_{2\widehat{I}}$ , while  $\tilde{\xi}_{\widehat{I}}$  to massive scalars.

When the massive fields are integrated out under the conditions,  
the scalar potential  $V$  carries only  $\Pi_V, \Pi_H$  and  $\varphi$ .

e.g., D'Auria, et.al. [[hep-th/0701247](#)], Cassani [[arXiv:0804.0595](#)]

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$\delta g^{tt} :$ 

$$\begin{aligned}
& -e^{-2B} \left[ \frac{1}{r^2} (1 - e^{2(B-C)}) + C'(3C' - 2B') + \frac{2}{r} (3C' - B') + 2C'' \right] + e^{-2B} G_{a\bar{b}} z^{a'} \bar{z}^{\bar{b}'} \\
& = \frac{e^{-4C}}{r^4} I_S + V
\end{aligned}$$

 $\delta g^{rr} :$ 

$$\begin{aligned}
& -e^{-2B} \left[ \frac{1}{r^2} (1 - e^{2(B-C)}) + C'(C' + 2A') + \frac{2}{r} (C' + A') \right] + e^{-2B} G_{a\bar{b}} z^{a'} \bar{z}^{\bar{b}'} \\
& = \frac{e^{-4C}}{r^4} I_S + V
\end{aligned}$$

 $\delta g^{\theta\theta}, \delta g^{\phi\phi} :$ 

$$\begin{aligned}
& -e^{-2B} \left[ A'' + C'' + A'(A' - B') + C'(A' - B' + C') + \frac{1}{r} (A' - B' + 2C') \right] - e^{-2B} G_{a\bar{b}} z^{a'} \bar{z}^{\bar{b}'} \\
& = -\frac{e^{-4C}}{r^4} I_S + V
\end{aligned}$$

 $\delta z^a :$ 

$$e^{-2B} \left[ G_{a\bar{b}} \bar{z}^{\bar{b}''} + \frac{\partial G_{a\bar{b}}}{\partial \bar{z}^{\bar{c}}} \bar{z}^{\bar{b}'} \bar{z}^{\bar{c}'} + G_{a\bar{b}} \bar{z}^{\bar{b}'} (A' - B' + 2C' + \frac{2}{r}) \right] = \frac{e^{-4C}}{r^4} \frac{\partial I_S}{\partial z^a} + \frac{\partial V}{\partial z^a}$$

Bellucci, et.al. [arXiv:0802.0141]

**Comment:** We have already known  $I_S(z, \bar{z}, p, q)$ ,  $V(z, \bar{z}, p, q)$  and their derivatives as

Attractor Equations [CLICK](#)

- $\partial_a I_S = 0$  appears in the search of extremal RN-BH in  $\mathcal{N} = 2$  SUGRA

$$I_S = |Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}, \quad \partial_a I_S = 2\bar{Z} D_a Z + i C_{abc} G^{b\bar{b}} G^{c\bar{c}} \overline{D_b Z} \overline{D_c Z}$$

- $\partial_a V = 0$  has the same form which appears in the search of (flux) vacua of  $\mathcal{N} = 1$  SUGRA

$$V = -3|Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}, \quad \partial_a V = -2\bar{Z} D_a Z + i C_{abc} G^{b\bar{b}} G^{c\bar{c}} \overline{D_b Z} \overline{D_c Z}$$

$Z$  in  $\mathcal{N} = 1$  SUGRA is not the central charge but the superpotential

$$C_{abc} \equiv \partial_a X^\Lambda \partial_b X^\Sigma \partial_c X^\Gamma \frac{\partial^3 \mathcal{F}}{\partial X^\Lambda \partial X^\Sigma \partial X^\Gamma}$$

A useful formula among the BH charges  $\Gamma = (p^\Lambda, q_\Lambda)^\text{T}$  and the invariant  $I_S(z, \bar{z}, p, q)$

$$\Gamma^\text{T} + i \frac{\partial I_S}{\partial \tilde{\Gamma}} = 2i \bar{Z} \Pi^\text{T} + 2i G^{a\bar{b}} D_a Z \bar{D}_b \Pi^\text{T}$$

$$\tilde{\Gamma} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} \Gamma, \quad \Pi = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \quad Z = L^\Lambda q_\Lambda - M_\Lambda p^\Lambda = \tilde{\Gamma}^\text{T} \Pi$$

This does not (explicitly) depend on the scalar potential  $-g^2 V$ .

This can be applied to any points in the spacetime.

• Single modulus model ( $a = 1$ ):  $\mathcal{F} = \frac{(X^1)^3}{X^0}$

$$Z = e^{K/2} \left( q_0 + qt - 3pt^2 + p^0 t^3 \right), \quad t = \frac{X^1}{X^0}$$

$$e^K = \frac{i}{(t - \bar{t})^3}, \quad G_{t\bar{t}} = -\frac{3}{(t - \bar{t})^2} \equiv e_t^{\hat{1}} e_{\bar{t}}^{\bar{1}} \delta_{\hat{1}\bar{1}}, \quad C_{ttt} = \frac{6i}{(t - \bar{t})^3}$$

Search a solution with  $V = -3|Z|^2 + |D_{\hat{1}}Z|^2 < 0 \rightarrow Z \neq 0$

Consider a non-SUSY solution  $\rightarrow D_{\hat{1}}Z \neq 0$



The generic forms of the central charge and its derivative:

$$Z \equiv -i\rho e^{i(\alpha - 3\phi)}, \quad D_{\hat{1}}Z \equiv \sigma e^{-i\phi} \quad (\rho, \sigma > 0)$$

Kallosh, et.al. [[hep-th/0606263](https://arxiv.org/abs/hep-th/0606263)]



The generic forms:  $Z \equiv -i\rho e^{i(\alpha-3\phi)}$ ,  $D_{\hat{1}}Z \equiv \sigma e^{-i\phi}$  ( $\rho, \sigma > 0$ )

The volume factors  $\rho$  and  $\sigma$  are related via the attractor equation.

$$\sigma = -\frac{\rho}{3} e^{-i\alpha} G_V \quad (G_V \neq 0)$$

The formula leads to the following two equations: ( $\Gamma = (p^0, p, q, q_0)^T$ ):

$$p + \frac{\partial I_S}{\partial q} = -\frac{2\rho}{3\sqrt{3}} e^{-i\alpha} e^{K/2} \left[ (3\sqrt{3} - 2G_V) t - G_V \bar{t} \right]$$

$$p^0 + \frac{\partial I_S}{\partial q_0} = -\frac{2\rho}{3\sqrt{3}} e^{-i\alpha} e^{K/2} (3\sqrt{3} - G_V)$$

$$\rightarrow t = \frac{3\sqrt{3} - 2G_V}{3\sqrt{3} - G_V} \left[ \frac{p + i\frac{\partial I_S}{\partial q}}{p^0 + i\frac{\partial I_S}{\partial q_0}} \right] + \frac{G_V}{3\sqrt{3} - G_V} \left[ \frac{p - i\frac{\partial I_S}{\partial q}}{p^0 - i\frac{\partial I_S}{\partial q_0}} \right] \quad \text{“generic solution”}$$

Difficult to evaluate the explicit solution caused by the complicated functions  $G_V$  and  $I_S$

● Three Moduli model called the STU-model:  $\mathcal{F} = \frac{X^1 X^2 X^3}{X^0}$

(Cartan part of 4D  $\mathcal{N} = 8$   $SO(8)$  gauged SUGRA  $\leftarrow$  IIA/IIB/Heterotic string triality)

$$Z = e^{K/2} \left( q_0 + q_a z^a - p^1 z^2 z^3 - p^2 z^3 z^1 - p^3 z^1 z^2 + p^0 z^1 z^2 z^3 \right), \quad z^a = \frac{X^a}{X^0}$$

$$K = -\log \left[ -i(z^1 - \bar{z}^1)(z^2 - \bar{z}^2)(z^3 - \bar{z}^3) \right]$$

$$G_{a\bar{b}} = -\frac{\delta_{ab}}{(z^a - \bar{z}^{\bar{a}})^2} = e_a^{\hat{a}} e_{\bar{b}}^{\bar{\hat{b}}} \delta_{\hat{a}\bar{\hat{b}}}, \quad C_{\hat{1}\hat{2}\hat{3}} = 1$$

Search a solution with  $V = -3|Z|^2 + |D_{\hat{a}}Z|^2 < 0 \rightarrow Z \neq 0$

Consider a non-SUSY solution  $\rightarrow D_{\hat{a}}Z \neq 0$



The generic forms:  $Z \equiv -i\rho e^{i(\alpha-3\phi)}$ ,  $D_{\hat{a}}Z \equiv \sigma e^{-i\phi}$  ( $\rho, \sigma > 0$ )

Kallosch, et.al. [[hep-th/0606263](https://arxiv.org/abs/hep-th/0606263)]

The generic forms:  $Z \equiv -i\rho e^{i(\alpha-3\phi)}$ ,  $D_{\hat{a}}Z \equiv \sigma e^{-i\phi}$  ( $\rho, \sigma > 0$ )

The volume factors  $\rho$  and  $\sigma$  are related via the attractor equation.

$$\sigma = -\rho e^{-i\alpha} G_V \quad (G_V \neq 0)$$

The formula leads to the following two equations:

$$p^a + \frac{\partial I_S}{\partial q_a} = -2\rho e^{-i\alpha} e^{K/2} \left[ (1 - G_V) z^a - 2G_V \bar{z}^{\bar{a}} \right]$$

$$p^0 + \frac{\partial I_S}{\partial q_0} = -2\rho e^{-i\alpha} e^{K/2} (1 - 3G_V)$$

$$\rightarrow z^a = V_{\text{eff}}^2 \left[ \frac{p^a + i \frac{\partial I_S}{\partial q_a}}{p^0 + i \frac{\partial I_S}{\partial q_0}} \right] + (1 - V_{\text{eff}}^2) \left[ \frac{p^a - i \frac{\partial I_S}{\partial q_a}}{p^0 - i \frac{\partial I_S}{\partial q_0}} \right] \quad \text{“generic solution”}$$

Neither  $V_{\text{eff}} = 1$  nor  $V_{\text{eff}} = 0$

Difficult to evaluate the explicit solution caused by the complicated functions  $G_V$  and  $I_S$