KEK Theory Workshop 2011 (March 15, 2011)

Gauged Supergravity, String Theory and Black Holes

HIGH ENERGY ACCELERATOR RESEARCH ORGANIZATION

Tetsuji KIMURA (KEK Theory Center)

Flux Compactifications beyond Calabi-Yau

Why flux compactifications beyond CY in 10D Strings?

1. CY with fluxes → 4D ungauged SUGRA
   → break 10D Eqs. of Motion
2. non-CY with fluxes → 4D gauged SUGRA
   non-CY: $SU(n)$-structure with torsion, generalized geometry, etc.
   gauge fields, matter fields, gauge coupling const., mass parameters...

Flux Compactifications beyond Calabi-Yau

TK's Recent Works

- Heterotic string with H, $\partial\phi$ and torsion

  No-go theorem [hep-th/0605247] w/ P.Yi

  Index theorems [arXiv:0704.2111]

  Intersecting five-branes [arXiv:0905.2185,0912.1334] w/ S.Mizoguchi

- Type II strings with (non)geometric fluxes

  AdS vacua via generalized geometry [arXiv:0810.0937]

  T-duality on doubled geometry with D-branes [arXiv:0806.1783] w/ C.Albertsson & R.A.Reid-Edwards

Flux Compactifications beyond Calabi-Yau

Further Applications

(Non-)SUSY solutions of AdS black hole, dS vacua, etc.

Remark: Naked Singularity appeared in SUSY RN-AdS BH solutions.

- Pure AdS SUGRA (only gravitational multiplet):

  L.J. Romans [hep-th/9203018], M.M. Caldarelli and D. Klemm [hep-th/9808097], etc.

  There exists a SUSY solution of rotating AdS black hole with regular horizon.

- Gauged SUGRA with vector multiplets (without hyper-sector):

  W.A. Sabra, et.al. (electric charges [hep-th/9903143], magnetic/dyonic charges [hep-th/0003213]), etc.

- Found SUSY AdS-BHs with vector multiplets (without hyper-sector):

  [arXiv:0911.4926], [arXiv:1011.2202], [arXiv:1012.3756], [arXiv:1012.4314], etc.

Current Motivation

Find a (non-)rotating AdS-BH solution of Gauged SUGRA with hyper-sector

(embedded into String Theory Compactification Scenarios)

Why?

Hyper-sector carries the information of flux compactification!

(details in later discussions)

Contents

Contents

L. Andrianopoli, M. Bertolini, A. Ceresole, R. D'Auria, S. Ferrara, P. Fré and T. Magri [hep-th/9605032]

R. D'Auria, L. Sommovigo and S. Vaulà [hep-th/0409097]

Supersymmetric multiplets in 4D $\mathcal{N} = 2$ SUGRA:

1 gravitational multiplet: $\{g_{\mu\nu}, A^0_\mu, \psi_{A\mu}\}$ $\mu = 0,1,2,3$ (4D, curved)
$A = 1,2$ ($SU(2)$ R-symmetry)

n_V vector multiplets: $\{A^a_\mu, t^a, \lambda^{aA}\}$ $a = 1, \ldots, n_\text{V}$

t^a in special Kähler geometry (SKG) $\mathcal{SM}$

$n_\text{H} + 1$ hypermultiplets: $\{q^u, \zeta_\alpha\}$ $u = 1, \ldots, 4n_\text{H} + 4$
$\alpha = 1, \ldots, 2n_\text{H} + 2$

q^u in quaternionic geometry (QG) $\mathcal{HM}$

Gauging: *Promote* (subgroups of) global isometry groups on $\mathcal{SM} \times \mathcal{HM}$ to local symmetries

Supersymmetric multiplets in 4D $\mathcal{N}=2$ SUGRA:

1 gravitational multiplet: $\{g_{\mu\nu}, A^0_\mu, \psi_{A\mu}\}$ — $\mu = 0,1,2,3$ (4D, curved); $A = 1,2$ ($SU(2)$ R-symmetry)

n_{V} vector multiplets: $\{A^a_\mu, t^a, \lambda^{aA}\}$ — $a = 1,\ldots,n_{\mathsf{V}}$

t^a in special Kähler geometry (SKG) $\mathcal{SM}$

$n_{\mathsf{H}}+1$ "hyper- + tensor-" multiplets: $\{q^{\widehat{u}}, B^I_{\mu\nu}, \zeta_\alpha\}$ — $\widehat{u} = 1,\ldots,4n_{\mathsf{H}}+4-n_{\mathsf{T}}$; $I = 1,\ldots,n_{\mathsf{T}}$; $\alpha = 1,\ldots,2n_{\mathsf{H}}+2$

$\{q^{\widehat{u}}, B^I_{\mu\nu}\}$ in a certain geometry $\mathcal{HM}' \times \mathcal{TM}$

Gauging: *Promote* (subgroups of) global isometry groups on $\mathcal{SM} \times \mathcal{HM}' \times \mathcal{TM}$ to local symmetries

Standard action (bosonic part) (gauge coupling const. g; index $\Lambda = 0, 1, \dots, n_{\mathsf{V}}$):

$$
\begin{aligned}
S &= \int \mathrm{d}^4 x \sqrt{-g} \Big\{ \frac{1}{2} R - G_{a\bar{b}}(t,\bar{t}) \nabla_\mu t^a \nabla^\mu \bar{t}^{\bar{b}} - h_{uv}(q) \nabla_\mu q^u \nabla^\mu q^v \\
&\qquad + \frac{1}{2} \mathrm{Im} \mathcal{N}_{\Lambda\Sigma}(t,\bar{t}) F^\Lambda_{\mu\nu} F^{\Sigma\mu\nu} + \frac{1}{2} \mathrm{Re} \mathcal{N}_{\Lambda\Sigma}(t,\bar{t}) F^\Lambda_{\mu\nu} (*F^\Sigma)^{\mu\nu} \\
&\qquad - V(t,\bar{t},q) \Big\} \\
V &= \mathrm{g}^2 \Big\{ \Big(G_{a\bar{b}} k^a_\Lambda k^{\bar{b}}_\Sigma + 4 h_{uv} k^u_\Lambda k^v_\Sigma \Big) \overline{L}^\Lambda L^\Sigma + \sum_{x=1}^{3} \Big(G^{a\bar{b}} f^\Lambda_a \overline{f}^\Sigma_{\bar{b}} - 3 \overline{L}^\Lambda L^\Sigma \Big) \mathcal{P}^x_\Lambda \mathcal{P}^x_\Sigma \Big\}
\end{aligned}
$$

$\mathrm{Im}\, \mathcal{N}_{\Lambda\Sigma}$ (generalized kinetic term) $\mathrm{Re}\, \mathcal{N}_{\Lambda\Sigma}$ (generalized θ-term)

$\nabla_\mu t^a = \partial_\mu t^a + \mathrm{g} A^\Lambda_\mu k^a_\Lambda(t)$ $\nabla_\mu q^u = \partial_\mu q^u + \mathrm{g} A^\Lambda_\mu k^u_\Lambda(q)$

(L^Λ, f^Λ_a, etc.: mathematical objects in $\mathcal{N}=2$ SUGRA)

(Gauging a SUGRA with tensor+hyper is also possible.)

(Abelian) gauged SUGRAs can be derived from flux compactification!

Contents

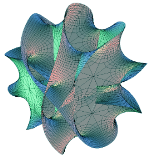
M. Graña [hep-th/0509003]

M. Graña, J. Louis and D. Waldram [hep-th/0505264][hep-th/0612237]

D. Cassani et.al. [arXiv:0707.3125][arXiv:0804.0595][arXiv:0911.2708]

etc. (Keyword: generalized geometry and Hitchin functional)

Calabi-Yau 3-fold $\mathcal{M}_{\text{CY}}$



Ricci-flat, torsionless, (compact) Kähler manifold with $SU(3)$ holonomy group

$$\mathrm{d}s^2_{\text{10D}} = \underbrace{\eta_{\mu\nu}(x)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu}_{\text{4D}} + \underbrace{g_{mn}(x,y)\,\mathrm{d}y^m\,\mathrm{d}y^n}_{\text{CY 3-fold}}$$

Invariant two-form J and three-form Ω on CY w.r.t. Levi-Civita connection:

$$\mathrm{d}J = \nabla_{[m}J_{np]} = 0, \qquad \mathrm{d}\Omega = \nabla_{[m}\Omega_{npq]} = 0$$

This is suitable for 1/4-SUSY condition with vanishing background fields

$\delta_{\text{SUSY}}\psi_{m\pm} = \nabla_m\,\varepsilon^{(\text{10D})}_{\pm} = 0$

$$\varepsilon^{(\text{10D})}_{+} = \varepsilon^{(\text{4D})}_{1+} \otimes \eta^1_+ + (\text{c.c.})\,, \quad \varepsilon^{(\text{10D})}_{-} = \varepsilon^{(\text{4D})}_{2+} \otimes \eta^2_- + (\text{c.c.})$$

$$(\varepsilon^{(\text{4D})}_{1,2+})^* = \varepsilon^{(\text{4D})}_{1,2-}\,, \quad (\eta^{1,2}_-)^* = \eta^{1,2}_+ = \begin{pmatrix} 0 \\ 0 \\ 0 \\ * \end{pmatrix} \; : \; SU(3)\text{-invariant} \subset SU(4) \sim SO(6)$$

$$\eta^1_\pm = \eta^2_\pm \equiv \eta_\pm\,, \qquad J_{mn} = \mp\mathrm{i}\,\eta^\dagger_\pm\,\gamma_{mn}\,\eta_\pm||\eta_+||^{-2}\,, \qquad \Omega = -\mathrm{i}\,\eta^\dagger_-\,\gamma_{mnp}\,\eta_+||\eta_+||^{-2}$$

NS-NS fields in 10D are expanded around CY:

$$\begin{aligned}\phi(x,y) &= \varphi(x)\\ g_{\mathfrak{m}\bar{\mathfrak{n}}}(x,y) &= \mathrm{i}v^a(x)\,(\omega_a)_{\mathfrak{m}\bar{\mathfrak{n}}}(y), \quad g_{\mathfrak{m}\mathfrak{n}}(x,y) = \mathrm{i}\bar{z}^{\bar{\jmath}}(x)\left(\frac{(\overline{\chi}_{\bar{\jmath}})_{\mathfrak{m}\bar{\mathfrak{p}}\bar{\mathfrak{q}}}\Omega^{\bar{\mathfrak{p}}\bar{\mathfrak{q}}}{}_{\mathfrak{n}}}{||\Omega||^2}\right)(y)\\ \widehat{B}_2(x,y) &= B_2(x) + b^a(x)\omega_a(y)\\ t^a &\equiv b^a + \mathrm{i}v^a\end{aligned}$$

Mirror Symmetry: exchange $(t^a, \bar{t}^{\bar{b}}) \leftrightarrow (z^i, \bar{z}^{\bar{\jmath}})$

R-R fields:

$$\begin{aligned}\widehat{C}_1(x,y) &= A_1^0(x)\\ \widehat{C}_3(x,y) &= A_1^a(x)\omega_a(y) + \xi^I(x)\alpha_I(y) - \widetilde{\xi}_I(x)\beta^I(y)\end{aligned}$$

cohomology class on CY	basis	degrees
$H^{(1,1)}$	ω_a	$a = 1,\ldots,h^{(1,1)}$
$H^{(0)} \oplus H^{(1,1)}$	$\omega_\Lambda = (1, \omega_a)$	$\Lambda = 0,1,\ldots,h^{(1,1)}$
$H^{(2,2)} \oplus H^{(6)}$	$\widetilde{\omega}^\Lambda = (\widetilde{\omega}^a, \frac{\text{vol.}}{\lvert\text{vol.}\rvert})$	
$H^{(2,1)}$	χ_i	$i = 1,\ldots,h^{(2,1)}$
$H^{(3)}$	(α_I, β^I)	$I = 0,1,\ldots,h^{(2,1)}$

$$\mathrm{d}\omega_\Lambda = 0 = \mathrm{d}\widetilde{\omega}^\Lambda$$

$$\mathrm{d}\alpha_I = 0 = \mathrm{d}\beta^I$$

10D Type IIA action $S^{(10\text{D})}_{\text{IIA}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}$:

$$\begin{aligned} S_{\text{NS}} &= \frac{1}{2} \int \mathrm{e}^{-2\phi} \Big\{ \widehat{R} * \mathbb{1} + 4 \mathrm{d}\phi \wedge * \mathrm{d}\phi - \frac{1}{2} \widehat{H}_3 \wedge * \widehat{H}_3 \Big\} \\ S_{\text{R}} + S_{\text{CS}} &= -\frac{1}{4} \int \Big\{ \widehat{F}_2 \wedge * \widehat{F}_2 + (\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3) \wedge * (\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3) \Big\} - \frac{1}{4} \int \widehat{B}_2 \wedge \widehat{F}_4 \wedge \widehat{F}_4 \end{aligned}$$

↓

4D $\mathcal{N} = 2$ ungauged SUGRA: Neither gauge couplings, Nor scalar potential

$$S^{(4\text{D})} = \int \Big\{ \frac{1}{2} R * \mathbb{1} - G_{a\bar{b}} \, \mathrm{d}t^a \wedge * \mathrm{d}\bar{t}^{\bar{b}} - h_{uv} \, \mathrm{d}q^u \wedge * \mathrm{d}q^v + \frac{1}{2} \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda} \wedge * F^{\Sigma} + \frac{1}{2} \mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda} \wedge F^{\Sigma} \Big\}$$

gravitational multiplet	$g_{\mu\nu}, A^0_1$		
vector multiplet (VM)	$A^a_1, t^a, \bar{t}^{\bar{b}}$	$t^a \in \text{SKG}_{\text{V}}$	mirror dual: $\text{SKG}_{\text{V}} \leftrightarrow \text{SKG}_{\text{H}}$
hypermultiplet (HM)	$z^i, \bar{z}^{\bar{\jmath}}, \xi^i, \widetilde{\xi}_j$	$z^i \in \text{SKG}_{\text{H}}$	
universal hypermultiplet (UHM)	$\varphi, a, \xi^0, \widetilde{\xi}_0$	$a \leftrightarrow B_2$	Hodge dual

$\mathcal{HM}$ = Special QG

$$\underset{4n_{\text{H}}+4}{\{q^u\}} = \underset{2n_{\text{H}}(\text{SKG}_{\text{H}})}{\{z^i, \bar{z}^{\bar{\jmath}}\}} + \underset{2n_{\text{H}}}{\{\xi^i, \widetilde{\xi}_j\}} + \underset{4\,(\text{UHM})}{\{\varphi, a, \xi^0, \widetilde{\xi}_0\}} = \underset{\text{SKG}_{\text{H}}}{\{z^i, \bar{z}^{\bar{\jmath}}\}} + \{\varphi\} + \underset{\text{``Heisenberg''}}{\{a, \xi^I, \widetilde{\xi}_J\}}$$

non-CY 3-fold $\mathcal{M}_6$

vanishing Ricci 2-form, torsionful, (compact) non-Kähler manifold
with (a pair of) $SU(3)$-structure
$\mathrm{d}J \neq 0$ and/or $\mathrm{d}\Omega \neq 0$

- $\eta^1_{\pm} = \eta^2_{\pm}$ at any points on $\mathcal{M}_6$: $(J_{mn} = \mp \mathrm{i}\, \eta^{\dagger}_{\pm} \gamma_{mn} \eta_{\pm} ||\eta_+||^{-2}, \ \Omega_{mnp} = -\mathrm{i}\, \eta^{\dagger}_{-} \gamma_{mnp} \eta_{+} ||\eta_+||^{-2})$

$$\mathrm{d}J = \frac{3}{2} \mathrm{Im}(\overline{\mathcal{W}}_1 \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \, , \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

- $\eta^1_{\pm} \neq \eta^2_{\pm}$ at a certain point on $\mathcal{M}_6$:

$$\eta^2_+ = c_{||}(y) \eta^1_+ + c_{\perp}(y) (v + \mathrm{i} v')_m \gamma^m \eta^1_-$$

$$(v - \mathrm{i} v')^m \equiv \eta^{1\dagger}_+ \gamma^m \eta^2_- \, , \quad J = j + v \wedge v' \, , \quad \Omega = \omega \wedge (v + \mathrm{i} v')$$

They are given by 1/4-SUSY condition with non-vanishing background fields:

$$\boxed{\delta \psi_{m\pm} = \nabla_m \varepsilon^{(\text{10D})}_{\pm} + (\text{NS-fluxes}) + (\text{RR-fluxes}) = 0}$$

Background fluxes behave as torsion.

Non-vanishing $\mathrm{d}J$ and $\mathrm{d}\Omega$ are caused by "non"-closed basis forms:

NS-NS

$$\widehat{H} = H^{\text{fl}} + \mathrm{d}\widehat{B}\,, \qquad \mathrm{d}_{H^{\text{fl}}} \equiv \mathrm{d} - H^{\text{fl}}\wedge$$

$$\mathrm{d}_{H^{\text{fl}}} \underbrace{\begin{pmatrix} \beta^I \\ \alpha_I \end{pmatrix}}_{\Sigma_-} \sim \underbrace{\begin{pmatrix} e_\Lambda{}^I & m^{\Lambda I} \\ e_{\Lambda I} & m^\Lambda{}_I \end{pmatrix}}_{Q^{\mathrm{T}}} \underbrace{\begin{pmatrix} \widetilde{\omega}^\Lambda \\ \omega_\Lambda \end{pmatrix}}_{\Sigma_+} \quad \text{with} \quad (\mathrm{d}_{H^{\text{fl}}})^2 \equiv 0$$

$e_0{}^I, e_{0I}$:	H-flux charges ($H^{\text{fl}} = -e_0{}^I \alpha_I + e_{0I}\beta^I$)
$e_a{}^I, e_{aI}$:	geometric flux charges (torsion)
$m^{\Lambda I}, m^\Lambda{}_I$:	nongeometric flux charges (magnetic dual of $e_\Lambda{}^I, e_{\Lambda I}$)

R-R

$$\widehat{\mathbf{F}} \equiv \widehat{F}_0 + \widehat{F}_2 + \ldots + \widehat{F}_{10} \equiv \mathrm{e}^{\widehat{B}}\widehat{\mathbf{G}} \qquad \text{with self-dual cond.} \quad \widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}})\,, \quad \lambda(\widehat{F}_k) \equiv (-)^{[\frac{k+1}{2}]}\widehat{F}_k$$

$$\begin{aligned} \frac{1}{\sqrt{2}}\widehat{\mathbf{G}} \;=\; & (G_0^\Lambda + G_2^\Lambda + G_4^\Lambda)\,\omega_\Lambda - (\widetilde{G}_{0\Lambda} + \widetilde{G}_{2\Lambda} + \widetilde{G}_{4\Lambda})\,\widetilde{\omega}^\Lambda \\ & + (G_1^I + G_3^I)\,\alpha_I - (\widetilde{G}_{1I} + \widetilde{G}_{3I})\,\beta^I \end{aligned}$$

$$G_0^\Lambda \equiv p^\Lambda\,, \qquad \widetilde{G}_{0\Lambda} \equiv q_\Lambda - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_\Lambda{}^I$$

$c \equiv (p^\Lambda, q_\Lambda)^{\mathrm{T}}$: R-R flux charges

(p^0: Romans' mass)

10D Type IIA (democratic) action $S^{(10D)}_{\rm IIA} = S_{\rm NS} + \widetilde{S}_{\rm R}$:

$$S_{\rm NS} + \widetilde{S}_{\rm R} \;=\; \frac{1}{2}\int {\rm e}^{-2\phi}\Big\{\widehat{R} * \mathbb{1} + 4{\rm d}\phi \wedge *{\rm d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3\Big\} - \frac{1}{8}\int \big[\widehat{\mathbf{F}} \wedge *\widehat{\mathbf{F}}\big]_{10}$$

with "constraint $\widehat{\mathbf{F}} \;=\; \lambda(*\widehat{\mathbf{F}})$" and "EoM (Bianchi) $({\rm d} + \widehat{H}\wedge) * \widehat{\mathbf{F}} = 0 \Leftrightarrow ({\rm d} - \widehat{H}\wedge)\widehat{\mathbf{F}} = 0$"

↓

4D $\mathcal{N} = 2$ abelian gauged SUGRA with non-trivial scalar potential
(non-abelian gauge symmetries related to isometries of $\text{SKG}_{\rm V,H}$: Unknown yet)

$0 = m^{\Lambda I} = m^{\Lambda}{}_I = p^{\Lambda}$

Standard Gauged SUGRA
$n_{\rm V}$ VM
$n_{\rm H}$ HM
1 UHM
[hep-th/9605032]

$0 = m^{\Lambda I} = m^{\Lambda}{}_I$

Gauged SUGRA
$n_{\rm V}$ VM
$n_{\rm H}$ HM
1 (massive) TM
[hep-th/0312210]

generic

Gauged SUGRA
$n_{\rm V}$ VM
$\widetilde{n}_{\rm H}$ HM
$n_{\rm T}$ (massive) TM
[hep-th/0409097]

Some of $\{a, \xi^I, \widetilde{\xi}_I\}$ are dualized to 2-form fields caused by magnetic charges $\{p^{\Lambda}, m^{\Lambda}{}_I, m^{\Lambda I}\}$:
[hep-th/0701247], [arXiv:0804.0595]

Metric of SQG:

$$h_{uv}\,\mathrm{d}q^u\,\mathrm{d}q^v \;=\; \underbrace{G_{i\bar{\jmath}}\,\mathrm{d}z^i\,\mathrm{d}\bar{z}^{\bar{\jmath}}}_{\text{SKG}_\text{H}} + \underbrace{(\mathrm{d}\varphi)^2}_{\text{4D dilaton}} + \frac{1}{4}\mathrm{e}^{4\varphi}\big(\underbrace{\mathrm{d}a}_{\text{axion}} - \xi^{\mathrm{T}}\,\mathbb{C}_\mathsf{H}\,\mathrm{d}\xi\,\big)^2 - \frac{1}{2}\mathrm{e}^{2\varphi}\,\underbrace{\mathrm{d}\xi^{\mathrm{T}}\,\mathbb{M}_\mathsf{H}\,\mathrm{d}\xi}_{\text{RR-axions}}$$

Scalar potential:

$$V_\mathsf{NS} \;=\; -2\,\mathrm{g}^2\mathrm{e}^{2\varphi}\Big[\overline{\Pi}{}_\mathsf{H}^{\mathrm{T}}\,\widetilde{Q}^{\mathrm{T}}\,\mathbb{M}_\mathsf{V}\,\widetilde{Q}\,\Pi_\mathsf{H} + \overline{\Pi}{}_\mathsf{V}^{\mathrm{T}}\,Q\,\mathbb{M}_\mathsf{H}\,Q^{\mathrm{T}}\,\Pi_\mathsf{V} + 4\overline{\Pi}{}_\mathsf{H}^{\mathrm{T}}\,\mathbb{C}_\mathsf{H}^{\mathrm{T}}\,Q^{\mathrm{T}}\big(\Pi_\mathsf{V}\overline{\Pi}{}_\mathsf{V}^{\mathrm{T}} + \overline{\Pi}_\mathsf{V}\Pi_\mathsf{V}^{\mathrm{T}}\big)Q\,\mathbb{C}_\mathsf{H}\,\Pi_\mathsf{H}\Big]$$

$$V_\mathsf{R} \;=\; -\frac{1}{2}\,\mathrm{g}^2\mathrm{e}^{4\varphi}(c+\widetilde{Q}\xi)^{\mathrm{T}}\,\mathbb{M}_\mathsf{V}\,(c+\widetilde{Q}\xi)$$

$$V \;=\; V_\mathsf{NS} + V_\mathsf{R} \;=\; \ldots \;=\; \mathrm{g}^2\Big[4h_{uv}k^u\bar{k}^v + \sum_{x=1}^{3}\Big(G^{a\bar{b}}D_a\mathcal{P}_x\overline{D_b\mathcal{P}_x} - 3|\mathcal{P}_x|^2\Big)\Big]\,, \qquad (\text{abelian } k_\Lambda^a = 0)$$

$\Pi_\mathsf{V} = \mathrm{e}^{\mathcal{K}_\mathsf{V}/2}(X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$
$t^a = X^a/X^0$
$a = 1, \ldots, n_\mathsf{V}$
SKG$_\mathsf{V}$ of vector-moduli

$$\begin{aligned}
\mathcal{P}_+ \;&\equiv\; \mathcal{P}_1 + \mathrm{i}\mathcal{P}_2 \;=\; 2\mathrm{e}^{\varphi}\,\Pi_\mathsf{V}^{\mathrm{T}}\,Q\,\mathbb{C}_\mathsf{H}\,\Pi_\mathsf{H} \\
\mathcal{P}_- \;&\equiv\; \mathcal{P}_1 - \mathrm{i}\mathcal{P}_2 \;=\; 2\mathrm{e}^{\varphi}\,\Pi_\mathsf{V}^{\mathrm{T}}\,Q\,\mathbb{C}_\mathsf{H}\,\overline{\Pi}_\mathsf{H} \\
\mathcal{P}_3 \;&=\; \mathrm{e}^{2\varphi}\,\Pi_\mathsf{V}^{\mathrm{T}}\,\mathbb{C}_\mathsf{V}(c+\widetilde{Q}\xi)
\end{aligned}$$

$\Pi_\mathsf{H} = \mathrm{e}^{\mathcal{K}_\mathsf{H}/2}(Z^I, \mathcal{G}_I)^{\mathrm{T}}$
$z^i = Z^i/Z^0$
$i = 1, \ldots, n_\mathsf{H}$
SKG$_\mathsf{H}$ of hyper-moduli

$$\mathbb{M}_{\mathsf{V,H}} \;\equiv\; \begin{pmatrix} 1 & -\mathrm{Re}\mathcal{N} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathrm{Im}\mathcal{N} & 0 \\ 0 & (\mathrm{Im}\mathcal{N})^{-1} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\mathrm{Re}\mathcal{N} & 1 \end{pmatrix}_{\mathsf{V,H}} \qquad \widetilde{Q} = \mathbb{C}_\mathsf{V}^{\mathrm{T}}\,Q\,\mathbb{C}_\mathsf{H}$$

$Q\,\mathbb{C}_\mathsf{H}\,Q^{\mathrm{T}} = 0 = Q^{\mathrm{T}}\,\mathbb{C}_\mathsf{V}\,Q = c^{\mathrm{T}}Q$: Nilpotency of exterior derivative $(\mathrm{d}_{H^{\mathsf{fl}}})^2 = 0$

Coset spaces with $SU(3)$-structure: D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

$\mathcal{M}_6$	$\frac{G_2}{SU(3)} = S^6$	$\frac{Sp(2)}{S(U(2) \times U(1))} = \mathbb{C}P^3$	$\frac{SU(3)}{U(1) \times U(1)} = \mathbb{F}(1,2;3)$
$\mathcal{SM} = \mathsf{SKG_V}$	$\frac{SU(1,1)}{U(1)}$: t^3	$\Big(\frac{SU(1,1)}{U(1)}\Big)^2$: st^2	$\Big(\frac{SU(1,1)}{U(1)}\Big)^3$: stu
$\mathcal{HM} = \mathsf{SQG}$	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM
$\mathsf{SKG_H} \subset \mathcal{HM}$	—	—	—
matters	1 VM + 1 UHM	2 VM + 1 UHM	3 VM + 1 UHM

Each $\mathsf{SKG_V}$ has a cubic prepotential: $\mathcal{F} = \frac{1}{3!} d_{abc} \frac{X^a X^b X^c}{X^0}$

nilmanifolds and solvmanifolds: M. Graña, R. Minasian, M. Petrini and A. Tomasiello [hep-th/0609124]

coset spaces with $SU(3)$- or $SU(2)$-structure: P. Koerber, D. Lüst and D. Tsimpis [arXiv:0804.0614]

a pair of $SU(3)$-structures with $(m^{\Lambda I}, m^{\Lambda}{}_I)$: D. Gaiotto and A. Tomasiello [arXiv:0904.3959]

A configuration: J.P. Gauntlett, S. Kim, O. Varela and D. Waldram [arXiv:0901.0676]

11D M-theory

$\updownarrow$ SE_7

4D $\mathcal{N}=2$ gauged SUGRA with 1 VM + 1 TM

$\longleftrightarrow$

symplectic rotations

$(X^\Lambda, \mathcal{F}_\Lambda) \to (\widetilde{X}^\Lambda, \widetilde{\mathcal{F}}_\Lambda)$

$B_1 \to \widetilde{B}_1,\ B_2 \to a$

4D $\mathcal{N}=2$ gauged SUGRA with 1 VM + 1 UHM

$$\mathcal{F}(\mathsf{SKG}_{\mathsf{VM}}) = -\frac{(X^1)^3}{X^0}$$

$$\widetilde{\mathcal{F}}(\widetilde{\mathsf{SKG}_{\mathsf{VM}}}) = \sqrt{\widetilde{X}^0(\widetilde{X}^1)^3}$$

A configuration: J.P. Gauntlett, S. Kim, O. Varela and D. Waldram [arXiv:0901.0676]

11D M-theory ⟷ (strong/weak coupling limit) ⟷ 10D Type IIA

SE_7 ↕ (11D M-theory ↔ 4D SUGRA) | $\mathcal{M}_6$ ↕ (10D Type IIA ↔ 4D SUGRA)

(Which $SU(3)$-str. $\mathcal{M}_6$?)

4D $\mathcal{N}=2$ gauged SUGRA with 1 VM + 1 TM ⟷ 4D $\mathcal{N}=2$ gauged SUGRA with 1 VM + 1 UHM

symplectic rotations

$(X^\Lambda, \mathcal{F}_\Lambda) \to (\widetilde{X}^\Lambda, \widetilde{\mathcal{F}}_\Lambda)$

$B_1 \to \widetilde{B}_1, \; B_2 \to a$

$$\mathcal{F}(\mathsf{SKG}_{\mathsf{VM}}) = -\frac{(X^1)^3}{X^0}$$

$$\widetilde{\mathcal{F}}(\widetilde{\mathsf{SKG}_{\mathsf{VM}}}) = \sqrt{\widetilde{X}^0 (\widetilde{X}^1)^3}$$

Comment: $\mathcal{M}_6 \neq \dfrac{G_2}{SU(3)}$

Contents

Consider a gauged SUGRA with 1 VM + 1 HM. TK [arXiv:0810.0937]

Truncate to $\mathcal{N} = 1$ system and evaluate the superpotential $\mathcal{W}_{\mathcal{N}=1} = \mathcal{W}_{\text{R}} + U\mathcal{W}_{\text{NS}}$

and the discriminants $\Delta(\mathcal{W})$, $\Delta(\mathcal{W}_{\text{NS}})$.

SUSY AdS vacuum appears if $\Delta(\mathcal{W}_{\text{R}}) > 0$ and $\Delta(\mathcal{W}_{\text{NS}}) > 0$:

$$\langle V \rangle \equiv 4\Lambda_{\text{cosmo.const.}} = -\frac{4}{[\text{Re}\mathcal{C}\mathcal{G}_0]^2}\sqrt{\frac{\Delta(\mathcal{W}_{\text{NS}})}{3}}$$

$\langle t \rangle$ is fixed (attractor point)

SUSY Minkowski vacuum appears if $\Delta(\mathcal{W}_{\text{R}}) < 0$ and $\Delta(\mathcal{W}_{\text{NS}}) < 0$:

$$\langle V \rangle = 0 \,, \qquad \langle t \rangle \text{ is fixed (attractor point)}$$

SUSY AdS vacuum appears if $\Delta(\mathcal{W}_{\text{R}}) \cdot \Delta(\mathcal{W}_{\text{NS}}) < 0$:

$$\langle V \rangle < 0 \,, \qquad \langle t \rangle \text{ is not fixed}$$

Extremal Reissner-Nordström black holes in the system of 4D $\mathcal{N}=2$ ungauged SUGRA:

asymptotically flat
- ✓ BPS solutions
- ✓ non-BPS solutions
- ✓ multi-center BPS solutions
- ✓ multi-center non-BPS solutions

from 10D Type II strings on CY space

If you generalize CY to a geometry with $SU(3)$-structures?

$\downarrow$

The cosmological constant would appear as $\langle V \rangle = 4\Lambda_{\text{cosmo.const.}}$

$\downarrow$

asymptotically AdS series of the above!?

(seems very hard to solve EoM for HM...)

— A Lazy Analysis — TK [arXiv:1005.4607]

Consider abelian gauged SUGRA with VM and HM

with setting HM to zero *in the stage of EoM*.

$$\rightarrow V = \mathrm{g}^2 \Big\{ G^{a\bar{b}} D_a Z \overline{D_b Z} - 3|Z|^2 \Big\}$$

with $h_{uv} = 0, \quad \mathcal{P}_1 = 0 = \mathcal{P}_2, \quad \mathcal{P}_3 = Z = q_\Lambda L^\Lambda - p^\Lambda M_\Lambda$ (central charge)

(Note: removed the (non)geometric flux charges Q...)

Focus on the T^3-model (with restriction to the so-called "D0-D4" system):

A funny non-SUSY AdS-BH solution with near horizon geometry $AdS_2 \times S^2$ is found as

$$Z \neq 0 \,, \quad D_t Z \neq 0 \,, \quad \Lambda_{\text{c.c.}} < 0 < S_{\text{BH}}$$

small q_0: $\Lambda_{\text{c.c.}} \sim -\sqrt{(-p^3 q_0)^3} < 0 \,, \quad S_{\text{BH}} \sim \sqrt{-p^3 q_0} \gg |\Lambda_{\text{c.c.}}| > 0$

large q_0: $\Lambda_{\text{c.c.}} \sim p^3 q_0 < 0 \,, \quad S_{\text{BH}} \sim \sqrt{\frac{5}{6}} \sim$ constant ?

— Serious Analyses — [arXiv:0911.4926], [arXiv:1012.3756], [arXiv:1012.4314],etc.

SUSY AdS-BH solutions with regular horizon

(based on [arXiv:0804.0009], [arXiv:0902.4186], [arXiv:0909.1743], etc.)

They have not involved hyper-sector yet!

(Notice: In asymptotically flat case, the study of UHM contribution has started [arXiv:1005.3650]!)

(Non)geometric flux charges have not contributed to these systems.

skip the details...

Contents

- ☑ 4D $\mathcal{N} = 2$ Gauged SUGRA and Flux Compactifications in Type II Strings
- ☑ Hyper-sector involves (non)geometric flux charges
- ☑ Non-trivial hyper-sector involves (non)geometric flux charges into EoMs!
- ☑ New configurations of SUSY (non-)rotating AdS black holes/branes!?

Study SUSY variations, Bianchi ids, and EoMs carefully!

in particular, explicit forms of $\delta\psi_{A\mu}\,,\ \delta\lambda^{aA}\,,\ \delta\zeta_{\alpha}\,.$

... to be continued

Appendix

$$dJ = \frac{3}{2}\mathrm{Im}(\overline{\mathcal{W}}_1\Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3\,, \qquad d\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

complex ($\frac{1}{4}$-SUSY Minkowski$_{1,3}$)	hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = 0$
	balanced	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = 0$
	special hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	Kähler	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	conformally CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 + 2\mathcal{W}_5 = 0$
almost complex ($\frac{1}{4}$-SUSY AdS$_4$)	symplectic	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	nearly Kähler	$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	almost Kähler	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	quasi Kähler	$\mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	semi Kähler	$\mathcal{W}_4 = \mathcal{W}_5 = 0$
	half-flat	$\mathrm{Im}\mathcal{W}_1 = \mathrm{Im}\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$

(Weighted) symplectic section: $\Pi_{\mathsf{V}} \equiv e^{\mathcal{K}_{\mathsf{V}}/2} \begin{pmatrix} X^\Lambda \\ \mathcal{F}_\Lambda \end{pmatrix} \equiv \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}$ with $t^a = \dfrac{X^a}{X^0}$ and

$$\mathcal{K}_{\mathsf{V}} = -\log\left\{ \mathrm{i}\left(\overline{X}^\Lambda \mathcal{F}_\Lambda - X^\Lambda \overline{\mathcal{F}}_\Lambda \right)\right\}, \quad 1 = \mathrm{i}\left(\overline{L}^\Lambda M_\Lambda - L^\Lambda \overline{M}_\Lambda \right)$$

$$D_a \Pi_{\mathsf{V}} = \left(\partial_a + \frac{1}{2} \partial_a \mathcal{K}_{\mathsf{V}} \right) \Pi_{\mathsf{V}} \equiv \begin{pmatrix} f_a^\Lambda \\ h_{\Lambda a} \end{pmatrix}$$

$$M_\Lambda = \mathcal{N}_{\Lambda\Sigma} L^\Sigma, \quad h_{\Lambda a} = \overline{\mathcal{N}}_{\Lambda\Sigma} f_a^\Sigma$$

$$G_{a\bar{b}} = \frac{\partial}{\partial t^a} \frac{\partial}{\partial \bar{t}^{\bar{b}}} \mathcal{K}_{\mathsf{V}}$$

In a similar way...

$$\Pi_{\mathsf{H}} \equiv e^{\mathcal{K}_{\mathsf{H}}/2} \begin{pmatrix} Z^I \\ \mathcal{G}_I \end{pmatrix}, \quad z^i = \frac{Z^i}{Z^0}, \quad \mathcal{K}_{\mathsf{H}} = -\log\left\{ \mathrm{i}\left(\overline{Z}^I \mathcal{G}_I - Z^I \overline{\mathcal{G}}_I \right)\right\}, \quad \text{etc.}$$

Tensor (Fierz) products of $SU(3)$-invariant spinors on 6D internal space $\mathcal{M}_6$:

$$\Phi_+ \equiv 8\,\mathrm{e}^{-b}\,\eta_+^1 \otimes \eta_+^{2\dagger} = X^\Lambda \omega_\Lambda - \mathcal{F}_\Lambda \widetilde{\omega}^\Lambda \qquad \text{even polyform} \qquad (\sim \mathrm{e}^{-b+\mathrm{i}J})$$

$$\Phi_- \equiv 8\,\mathrm{e}^{-b}\,\eta_+^1 \otimes \eta_-^{2\dagger} = Z^I \alpha_I - \mathcal{G}_I \beta^I \qquad \text{odd polyform} \qquad (\sim -\mathrm{i}\Omega)$$

$\Phi_\pm$: called the pure spinors on $T\mathcal{M}_6 \oplus T^*\mathcal{M}_6$

with the even/odd basis-forms and the symplectic metrics $\mathbb{C}_{\mathsf{V,H}}$:

$$\Sigma_+^{\boldsymbol{\Lambda}} \equiv \begin{pmatrix} \widetilde{\omega}^\Lambda \\ \omega_\Lambda \end{pmatrix}, \qquad \int_{\mathcal{M}_6} \langle \Sigma_+^{\boldsymbol{\Lambda}}, \Sigma_+^{\boldsymbol{\Sigma}} \rangle = (\mathbb{C}_{\mathsf{V}}^{-1})^{\boldsymbol{\Lambda\Sigma}}$$

$$\Sigma_-^{\boldsymbol{I}} \equiv \begin{pmatrix} \beta^I \\ \alpha_I \end{pmatrix}, \qquad \int_{\mathcal{M}_6} \langle \Sigma_-^{\boldsymbol{I}}, \Sigma_-^{\boldsymbol{J}} \rangle = (\mathbb{C}_{\mathsf{H}}^{-1})^{\boldsymbol{IJ}}$$

Kähler potentials:

$$\mathcal{K}_{\mathsf{V}} = -\log \mathrm{i} \int_{\mathcal{M}_6} \langle \Phi_+, \overline{\Phi}_+ \rangle, \qquad \mathcal{K}_{\mathsf{H}} = -\log \mathrm{i} \int_{\mathcal{M}_6} \langle \Phi_-, \overline{\Phi}_- \rangle$$

[Electric Gauging] (ex.) "Heisenberg" directions:

$$\nabla_\mu q^u = \partial_\mu q^u + \mathrm{g}\, k^u_\Lambda A^\Lambda_\mu \,, \qquad k_\Lambda = -\big[2q_\Lambda + e_\Lambda{}^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}}\xi)_{\boldsymbol{I}}\big] \frac{\partial}{\partial a} - e_\Lambda{}^{\boldsymbol{I}} \frac{\partial}{\partial \xi^{\boldsymbol{I}}}$$

[Electric/Magnetic Gauging] (ex.) "Heisenberg" directions:

$$\begin{aligned} \nabla_\mu q^u &= \partial_\mu q^u + \mathrm{g}\, k^u_\Lambda A^\Lambda_\mu + \mathrm{g}\, \widetilde{k}^{u\Lambda} \widetilde{A}_{\Lambda\mu} \\ k_\Lambda &= -\big[2q_\Lambda + e_\Lambda{}^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}}\xi)_{\boldsymbol{I}}\big] \frac{\partial}{\partial a} - e_\Lambda{}^{\boldsymbol{I}} \frac{\partial}{\partial \xi^{\boldsymbol{I}}} \\ \widetilde{k}^\Lambda &= -\big[2p^\Lambda + m^{\Lambda \boldsymbol{I}} (\mathbb{C}_{\mathsf{H}}\xi)_{\boldsymbol{I}}\big] \frac{\partial}{\partial a} + m^{\Lambda \boldsymbol{I}} \frac{\partial}{\partial \xi^{\boldsymbol{I}}} \end{aligned}$$

with commutation relations:

$$[k_\Lambda, k_\Sigma] = [\widetilde{k}^\Lambda, \widetilde{k}^\Sigma] = [k_\Lambda, \widetilde{k}^\Sigma] = 0$$

Killing prepotentials $\mathcal{P}_x$ in terms of the $SU(2)$ connection ω_x:

$$\mathcal{P}_x = (\omega_x)_u k^u \quad \text{where} \quad k = k_\Lambda L^\Lambda - \widetilde{k}^\Lambda M_\Lambda = -\Pi_{\mathsf{V}}^{\mathrm{T}} \mathbb{C}_{\mathsf{V}} (2c + \widetilde{Q}\xi) \frac{\partial}{\partial a} - \Pi_{\mathsf{V}}^{\mathrm{T}} Q \frac{\partial}{\partial \xi}$$

Cassani, et.al. [arXiv:0911.2708]