

Flux Compactifications, $\mathcal{N} = 2$ Gauged Supergravities and Black Holes

based on arXiv:1108.1113 [hep-th]

木村 哲士

素粒子論：4次元時空上の物理現象の起源を探る

理解したい情報

- 場の種類とスペクトラム
- 対称性とその破れ
- 相互作用項、真空、宇宙項、その他

ストリング理論は一つの候補だが、10次元で摂動論的に定義されている
余剰次元の扱いが重要

現在のところ、10次元から特定の4次元を力学的に切り出す方法は確立していない





しかし、一度4次元時空を何かの操作で切り出すことを仮定すれば、
どのような有効理論が登場するかは議論できる

(例)




- ✓ 10次元時空 = 4次元 (A)dS もしくは Minkowski と 6次元余剰次元空間
- ✓ $\mathcal{N} = 2$ 超対称性
- ✓ ゲージ相互作用

... 他の仮定を置いても良い

WHY $\mathcal{N} = 2$?

- 
 $\mathcal{N} \geq 4$ 以上では物質場はゲージ変換に対して随伴表現
(他の表現に再度分解する機構が必要)
- 
 $\mathcal{N} = 1$ では物質場の対称性があまり強くない (時にはこれでも十分)
- 
 $\mathcal{N} = 2$ では物質場は随伴表現も他の表現も持ちながら、
適度に強い(弱い)対称性の縛りがある \implies **controllable!!**
- 
 局所 $\mathcal{N} = 2$ から $\mathcal{N} = 1$ に自発的に破れる機構がある

HOW 4D $\mathcal{N} = 2$ from 10D ?

-  II型ストリング理論から出発する (今回はIIA型)
-  6次元空間をコンパクト化 vs ブレーンワールド
-  超対称性を $1/4$ だけ保つ配位を考える (ex: Calabi-Yau 3-fold, etc.)
(cf: T^6 トーラスコンパクト化では4次元に $\mathcal{N} = 8$ 超対称性が出現)

しかし... Calabi-Yau では十分ではない

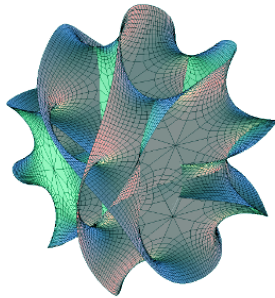
Beyond Calabi-Yau

何故 Calabi-Yau では十分ではないと考えるのか?



CY上の物質場やフラックスの配位がCY空間を歪める (back reactions)

1. CY with fluxes → 4D ungauged SUGRA
→ break 10D Eqs. of Motion
2. non-CY with fluxes → 4D gauged SUGRA
non-CY: $SU(n)$ -structure with torsion, generalized geometry, etc.
gauge fields, matter fields, gauge coupling const., mass parameters...

Calabi-Yau 3-fold \mathcal{M}_{CY} 

Ricci平坦な Kähler 多様体

(トーシヨンなし)

(ホロノミー群は $SU(3) \subset SU(4) \sim SO(6)$)

$$ds_{10D}^2 = \underbrace{\eta_{\mu\nu}(x) dx^\mu dx^\nu}_{4D} + \underbrace{g_{mn}(x, y) dy^m dy^n}_{CY}$$

Levi-Civita 接続の共変微分について共変定数な2形式 (J) と正則3形式 (Ω) :

$$dJ = \nabla_{[m} J_{np]} = 0 \quad d\Omega = \nabla_{[m} \Omega_{npq]} = 0$$

non-CY 3-fold \mathcal{M}_6

Ricci 2-form がゼロでトーシオンを許す (概) 複素幾何

($SU(3)$ -structure 多様体)

$dJ \neq 0$ and/or $d\Omega \neq 0$

CY からのズレを表す :

$$dJ = \frac{3}{2} \text{Im}(\overline{\mathcal{W}}_1 \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3, \quad d\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

$$dJ = \frac{3}{2} \text{Im}(\overline{\mathcal{W}}_1 \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3, \quad d\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

complex (1/4-SUSY Minkowski _{1,3})	hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = 0$
	balanced	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = 0$
	special hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	Kähler	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	conformally CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 + 2\mathcal{W}_5 = 0$
almost complex (1/4-SUSY AdS ₄)	symplectic	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	nearly Kähler	$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	almost Kähler	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	quasi Kähler	$\mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	semi Kähler	$\mathcal{W}_4 = \mathcal{W}_5 = 0$
	half-flat	$\text{Im}\mathcal{W}_1 = \text{Im}\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$

閉形式でない $(dJ, d\Omega)$ を、基底形式の外微分の性質に翻訳する：

NS-NS

$$d \begin{pmatrix} \beta^I \\ \alpha_I \end{pmatrix}_{\Sigma_-} \sim \begin{pmatrix} e_{\Lambda}^I & m^{\Lambda I} \\ e_{\Lambda I} & m^{\Lambda}{}_{I} \end{pmatrix}_{Q^T} \begin{pmatrix} \tilde{\omega}^{\Lambda} \\ \omega_{\Lambda} \end{pmatrix}_{\Sigma_+}$$

e_0^I, e_{0I} : H -flux charges ($H^{\text{fl}} = -e_0^I \alpha_I + e_{0I} \beta^I$)
 e_a^I, e_{aI} : geometric flux charges (トーション)
 $m^{\Lambda I}, m^{\Lambda}{}_{I}$: nongeometric flux charges ($e_{\Lambda}^I, e_{\Lambda I}$ の“磁氣的”双対)

$$\hat{\mathbf{F}} \equiv \hat{F}_0 + \hat{F}_2 + \dots + \hat{F}_{10} \equiv e^{\hat{B}} \hat{\mathbf{G}} \quad (\text{自己双対条件 } \hat{\mathbf{F}} = \lambda(*\hat{\mathbf{F}}), \quad \lambda(\hat{F}_k) \equiv (-)^{\lfloor \frac{k+1}{2} \rfloor} \hat{F}_k)$$

R-R

$$\frac{1}{\sqrt{2}} \hat{\mathbf{G}} = (G_0^{\Lambda} + G_2^{\Lambda} + G_4^{\Lambda}) \omega_{\Lambda} - (\tilde{G}_{0\Lambda} + \tilde{G}_{2\Lambda} + \tilde{G}_{4\Lambda}) \tilde{\omega}^{\Lambda} \\ + (G_1^I + G_3^I) \alpha_I - (\tilde{G}_{1I} + \tilde{G}_{3I}) \beta^I$$

$$G_0^{\Lambda} \equiv p^{\Lambda}, \quad \tilde{G}_{0\Lambda} \equiv q_{\Lambda} - \xi^I e_{\Lambda I} + \tilde{\xi}_{I\Lambda} e^I$$

$c \equiv (p^{\Lambda}, q_{\Lambda})^T$: R-R flux charges

(p^0 : Romans' mass)

10次元IIA型作用 (democratic formulation) $S_{\text{IIA}}^{(10\text{D})} = S_{\text{NS}} + \tilde{S}_{\text{R}}$:

$$S_{\text{NS}} + \tilde{S}_{\text{R}} = \frac{1}{2} \int e^{-2\phi} \left\{ \hat{R} * \mathbb{1} + 4d\phi \wedge *d\phi - \frac{1}{2} \hat{H}_3 \wedge * \hat{H}_3 \right\} - \frac{1}{8} \int [\hat{\mathbf{F}} \wedge * \hat{\mathbf{F}}]_{10}$$

“自己双対 $\hat{\mathbf{F}} = \lambda(*\hat{\mathbf{F}})$ ” と “場の方程式 $(d + \hat{H} \wedge) * \hat{\mathbf{F}} = 0 \Leftrightarrow (d - \hat{H} \wedge) \hat{\mathbf{F}} = 0$ ”



4次元 $\mathcal{N} = 2$ 可換ゲージ群を持つ超重力理論 (非自明なポテンシャル項付き)
(非可換ゲージ群の実現は今のところ難しい)

3種類のゲージ化された超重力理論

ゲージ化：物質場の住む空間の isometry group をゲージ化すること

ベクトル多重項： special Kähler geometry (SKG)

ハイパー多重項： quaternionic geometry (QG)

特に、コンパクト化で登場する QG には、部分空間として別の SKG を含む

ポテンシャル項

$$V_{\text{NS}} = -2g^2 e^{2\varphi} \left[\bar{\Pi}_H^T \tilde{Q}^T \mathbb{M}_V \tilde{Q} \Pi_H + \bar{\Pi}_V^T Q \mathbb{M}_H Q^T \Pi_V + 4\bar{\Pi}_H^T \mathbb{C}_H^T Q^T (\Pi_V \bar{\Pi}_V^T + \bar{\Pi}_V \Pi_V^T) Q \mathbb{C}_H \Pi_H \right]$$

$$V_R = -\frac{1}{2} g^2 e^{4\varphi} (c + \tilde{Q}\xi)^T \mathbb{M}_V (c + \tilde{Q}\xi)$$

$$V_{\text{NS}} + V_R = V_{4\text{D}} = g^2 \left[4h_{uv} k^u \bar{k}^v + \sum_{x=1}^3 \left(G^{a\bar{b}} D_a \mathcal{P}_x \overline{D_b \mathcal{P}_x} - 3|\mathcal{P}_x|^2 \right) \right] \quad (\text{abelian : } k_\Lambda^a = 0)$$

$$\Pi_V = e^{\mathcal{K}_V/2} (X^\Lambda, \mathcal{F}_\Lambda)^T$$

$$t^a = X^a / X^0$$

$$a = 1, \dots, n_V$$

SKG_V of vector-moduli

$$\mathcal{P}_+ = 2e^\varphi \Pi_V^T Q \mathbb{C}_H \Pi_H$$

$$\mathcal{P}_- = 2e^\varphi \Pi_V^T Q \mathbb{C}_H \bar{\Pi}_H$$

$$\mathcal{P}_3 = e^{2\varphi} \Pi_V^T \mathbb{C}_V (c + \tilde{Q}\xi)$$

$$\Pi_H = e^{\mathcal{K}_H/2} (Z^I, \mathcal{G}_I)^T$$

$$z^i = Z^i / Z^0$$



$$i = 1, \dots, n_H$$

SKG_H of hyper-moduli

$$\mathbb{M}_{V,H} \equiv \begin{pmatrix} 1 & -\text{Re}\mathcal{N} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \text{Im}\mathcal{N} & 0 \\ 0 & (\text{Im}\mathcal{N})^{-1} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\text{Re}\mathcal{N} & 1 \end{pmatrix}_{V,H}$$

where $\mathcal{F}_\Lambda \equiv \mathcal{N}_{\Lambda\Sigma} X^\Sigma$
 $\tilde{Q} \equiv \mathbb{C}_V^T Q \mathbb{C}_H$

$\mathbb{C}_{V,H}$: symplectic invariant metrics

-  $\mathcal{N} = 2$ ゲージ化されていない超重力理論では \rightarrow ポテンシャル項がない
 漸近平坦な (極限) 荷電ブラックホールの解析などではこの理論でも十分だった
-  $\mathcal{N} = 2$ ゲージ化された超重力理論では \rightarrow ポテンシャル項が登場する
 ポテンシャル項の極致が超対称性を (部分的に) 破り、宇宙項を与える

3種類の超重力理論

$$0 = m^{\Lambda I} = m^{\Lambda}{}_I = p^{\Lambda}$$

Standard Gauged SUGRA

n_V 個のベクトル多重項

n_H 個のハイパー多重項

普遍ハイパー多重項

[[hep-th/9605032](https://arxiv.org/abs/hep-th/9605032)]

$$0 = m^{\Lambda I} = m^{\Lambda}{}_I$$

Gauged SUGRA

n_V 個のベクトル多重項

n_H 個のハイパー多重項

1 個のテンソル多重項

[[hep-th/0312210](https://arxiv.org/abs/hep-th/0312210)]

generic

Gauged SUGRA

n_V 個のベクトル多重項

\tilde{n}_H 個のハイパー多重項

n_T 個のテンソル多重項

[[hep-th/0409097](https://arxiv.org/abs/hep-th/0409097)]

スカラー場 $\{a, \xi^I, \tilde{\xi}_I\}$ のいくつかは “磁荷” $\{p^{\Lambda}, m^{\Lambda}{}_I, m^{\Lambda I}\}$ によってテンソル場に双対変換

[[hep-th/0701247](https://arxiv.org/abs/hep-th/0701247)], [[arXiv:0804.0595](https://arxiv.org/abs/0804.0595)]

Anti-de Sitter Black Holes in 4D $\mathcal{N} = 2$ Gauged SUGRA

Comments

- 👉 AdS-BH with naked singularity in **pure** AdS SUGRA
[L.J. Romans \[hep-th/9203018\]](#), [M.M. Caldarelli and D. Klemm \[hep-th/9808097\]](#), etc.
SUSY solution of rotating AdS black hole with regular horizon
- 👉 AdS-BH with regular horizon in Gauged SUGRA with VMs (**no HMs**)
[\[hep-th/9903143\]](#), [\[arXiv:0911.4926\]](#), [\[arXiv:1012.4314\]](#), etc.

Question

How can we obtain AdS-BH solutions
with hypermultiplets?

— Setup and Result —

- 4D $\mathcal{N} = 2$ gauged SUGRA with VM and UHM
from 10D type IIA on non-CY with fluxes
- Ansatz for matter fields
- Regular solution (AdS Black Hole)

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Coset space $\frac{G_2}{SU(3)}$

D. Cassani and A.K. Kashani-Poor [[arXiv:0901.4251](https://arxiv.org/abs/0901.4251)]

-
- ✓ nearly-Kähler (almost complex geometry)
 - ✓ NSNS-sector : torsion and H -flux
 - ✓ RR-sector : 2-, 4-form and Romans' mass (0-form)
-
- ✓ 1 VM with cubic prepotential $\mathcal{F} = \frac{X^1 X^1 X^1}{X^0}$
 - ✓ 1 UHM (no other HMs)
-

nilmanifolds and solvmanifolds: [M. Graña, R. Minasian, M. Petrini and A. Tomasiello \[hep-th/0609124\]](#)

coset spaces with $SU(3)$ - or $SU(2)$ -structure: [P. Koerber, D. Lüst and D. Tsimpis \[arXiv:0804.0614\]](#)

a pair of $SU(3)$ -structures with $(m^{\Lambda I}, m^{\Lambda}_I)$: [D. Gaiotto and A. Tomasiello \[arXiv:0904.3959\]](#)

10D type IIA on $\frac{G_2}{SU(3)}$ with fluxes



4D $\mathcal{N} = 2$ abelian gauged SUGRA with **B-field** ($\Lambda = 0, 1$ and $\xi^0 \equiv (\xi^0, \tilde{\xi}_0)^T$)

$$S = \int \left[\frac{1}{2} R (*1) + \frac{1}{2} \mu_{\Lambda\Sigma} F^\Lambda \wedge *F^\Sigma + \frac{1}{2} \nu_{\Lambda\Sigma} F^\Lambda \wedge F^\Sigma - g_{\tilde{t}\tilde{t}} d\tilde{t} \wedge *d\tilde{t} \right. \\ \left. - d\varphi \wedge *d\varphi - \frac{e^{-4\varphi}}{4} dB \wedge *dB - \frac{e^{2\varphi}}{2} \left(D\xi^0 \wedge *D\xi^0 + D\tilde{\xi}_0 \wedge *D\tilde{\xi}_0 \right) + dB \wedge \xi^0 d\tilde{\xi}_0 \right. \\ \left. + dB \wedge (e_{R\Lambda} - e_{\Lambda 0} \xi^0) A^\Lambda - \frac{1}{2} m_R^\Lambda e_{R\Lambda} B \wedge B - V (*1) \right]$$

- $g_{\mu\nu}, \mathfrak{t}, B_{\mu\nu}, \varphi; (e_\Lambda^0, e_{\Lambda 0})$: NS-NS sector
- $A_\mu^\Lambda, \xi^0, \tilde{\xi}_0; (m_R^\Lambda, e_{R\Lambda})$: R-R sector
- GM : $(g_{\mu\nu}, A_\mu^0)$, VM : (A_μ^a, \mathfrak{t}) , UHM \rightarrow TM : $(\varphi, B_{\mu\nu}, \xi^0, \tilde{\xi}_0)$
- $D\xi^0 = d\xi^0 - e_\Lambda^0 A_I^\Lambda$, $D\tilde{\xi}_0 = d\tilde{\xi}_0 - e_{\Lambda 0} A_I^\Lambda$
- $F_2^\Sigma = dA_I^\Sigma + m_R^\Sigma B_2$
- $V(\mathfrak{t}, \varphi, \xi^0) = V_{\text{NS}}(\mathfrak{t}, \varphi) + V_{\text{R}}(\mathfrak{t}, \varphi, \xi^0)$

Precise data on $\frac{G_2}{SU(3)}$:

$$e_{10} \neq 0, m_R^0 \neq 0, e_{R0} \neq 0$$

$$e_\Lambda^0 = 0 = e_{00}$$

$$m_R^1 = 0 = e_{R1}$$

$$\mu_{\Lambda\Sigma} \equiv \text{Im}\mathcal{N}_{\Lambda\Sigma}, \nu_{\Lambda\Sigma} \equiv \text{Re}\mathcal{N}_{\Lambda\Sigma}$$

D. Cassani [arXiv:0804.0595]

$$\begin{aligned}
 R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} &= \frac{1}{4}g_{\mu\nu} \mu_{\Lambda\Sigma} F_{\rho\sigma}^{\Lambda} F^{\Sigma\rho\sigma} - \mu_{\Lambda\Sigma} F_{\mu\rho}^{\Lambda} F_{\nu\sigma}^{\Sigma} g^{\rho\sigma} - g_{\mu\nu} g_{\tilde{t}\tilde{t}} \partial_{\rho} \mathfrak{t} \partial^{\rho} \bar{\mathfrak{t}} + 2g_{\tilde{t}\tilde{t}} \partial_{\mu} \mathfrak{t} \partial_{\nu} \bar{\mathfrak{t}} \\
 &\quad - g_{\mu\nu} \partial_{\rho} \varphi \partial^{\rho} \varphi + 2\partial_{\mu} \varphi \partial_{\nu} \varphi - \frac{e^{-4\varphi}}{24} g_{\mu\nu} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} + \frac{e^{-4\varphi}}{4} H_{\mu\rho\sigma} H_{\nu}{}^{\rho\sigma} \\
 &\quad - \frac{e^{2\varphi}}{2} g_{\mu\nu} \left(D_{\rho} \xi^0 D^{\rho} \xi^0 + D_{\rho} \tilde{\xi}_0 D^{\rho} \tilde{\xi}_0 \right) + e^{2\varphi} \left(D_{\mu} \xi^0 D_{\nu} \xi^0 + D_{\mu} \tilde{\xi}_0 D_{\nu} \tilde{\xi}_0 \right) - g_{\mu\nu} V,
 \end{aligned} \tag{\delta g_{\mu\nu}}$$

$$0 = \frac{1}{\sqrt{-g}} \partial_{\mu} \left(\sqrt{-g} \mu_{\Lambda\Sigma} F^{\Sigma\mu\sigma} \right) - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_{\mu} \left(\nu_{\Lambda\Sigma} F_{\nu\rho}^{\Sigma} \right) + \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_{\mu} B_{\nu\rho} (e_{R\Lambda} - \xi^0 e_{\Lambda 0}) - e^{2\varphi} Q_{\Lambda 0} D^{\sigma} \xi^0, \tag{\delta A_{\mu}^{\Lambda}}$$

$$0 = \frac{1}{\sqrt{-g}} \partial_{\mu} \left(\sqrt{-g} g_{\tilde{t}\tilde{t}} g^{\mu\nu} \partial_{\nu} \bar{\mathfrak{t}} \right) + \frac{1}{4} \partial_{\mathfrak{t}} (\mu_{\Lambda\Sigma}) F_{\mu\nu}^{\Lambda} F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}} \partial_{\mathfrak{t}} (\nu_{\Lambda\Sigma}) F_{\mu\nu}^{\Lambda} F_{\rho\sigma}^{\Sigma} - \partial_{\mathfrak{t}} g_{\tilde{t}\tilde{t}} \partial_{\mu} \mathfrak{t} \partial^{\mu} \bar{\mathfrak{t}} - \partial_{\mathfrak{t}} V, \tag{\delta \mathfrak{t}}$$

$$0 = \frac{2}{\sqrt{-g}} \partial_{\mu} \left(\sqrt{-g} g^{\mu\nu} \partial_{\nu} \varphi \right) + \frac{e^{4\varphi}}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} - e^{2\varphi} \left(D_{\mu} \xi^0 D^{\mu} \xi^0 + D_{\mu} \tilde{\xi}_0 D^{\mu} \tilde{\xi}_0 \right) - \partial_{\varphi} V, \tag{\delta \varphi}$$

$$\begin{aligned}
 0 &= \frac{1}{\sqrt{-g}} \partial_{\mu} \left(e^{-4\varphi} \sqrt{-g} H^{\mu\rho\sigma} \right) + \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} \left[D_{\mu} \xi^0 (\mathbb{C}_H)_{00} D_{\nu} \xi^0 + (e_{R\Lambda} - \xi^0 e_{\Lambda 0}) F_{\mu\nu}^{\Lambda} \right] \\
 &\quad + 2m_{\text{R}}^{\Lambda} \mu_{\Lambda\Sigma} F^{\Sigma\rho\sigma} - \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} m_{\text{R}}^{\Lambda} \nu_{\Lambda\Sigma} F_{\mu\nu}^{\Sigma},
 \end{aligned} \tag{\delta B_{\mu\nu}}$$

$$0 = -\frac{2}{\sqrt{-g}} \partial_{\mu} \left(\sqrt{-g} e^{2\varphi} g^{\mu\nu} D_{\nu} \xi^0 \right) + \frac{\partial V}{\partial \xi^0} - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_{\mu} B_{\nu\rho} D_{\sigma} \xi^0 (\mathbb{C}_H)_{00}. \tag{\delta \xi^0}$$

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Vacuum I : $\mathcal{N} = 1$

$$t_* = -\frac{\pm 1 + i\sqrt{15}}{2} \left[\frac{3}{5(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3}, \quad \xi_*^0 = -\frac{2}{5} \left[\frac{2\sqrt{3} m_R^0 (e_{R0})^2}{5 e_{10}} \right]^{1/3}, \quad \exp(\varphi_*) = \frac{4}{3} \left[\frac{\sqrt{5} e_{10}}{\sqrt{3} m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{5\sqrt{5}}{2} \left[\frac{5(e_{10})^4}{2\sqrt{3} |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{I}} < 0$$

Vacuum II : $\mathcal{N} = 0$

$$t_* = (\pm 1 - i\sqrt{3}) \left[\frac{3}{5(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3}, \quad \xi_*^0 = \left[\frac{9 m_R^0 (e_{R0})^2}{25 e_{10}} \right]^{1/3}, \quad \exp(\varphi_*) = \frac{2}{3} \left[\frac{25 e_{10}}{\sqrt{3} m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{80}{27} \left[\frac{25(e_{10})^4}{\sqrt{3} |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{II}} < 0$$

Vacuum III : $\mathcal{N} = 0$

$$t_* = -i \left[\frac{12}{\sqrt{5}(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3}, \quad \xi_*^0 = 0, \quad \exp(\varphi_*) = \sqrt{5} \left[\frac{5 e_{10}}{18 m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{25\sqrt{5}}{6} \left[\frac{5(e_{10})^4}{18 |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{III}} < 0$$

Note: $m_R^0 > 0$; $\tilde{\xi}_0$ is not fixed ; $\Lambda_{\text{c.c.}}^{\text{II}} < \Lambda_{\text{c.c.}}^{\text{I}} < \Lambda_{\text{c.c.}}^{\text{III}}$

D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

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- ☞ Consider spacetime metric (extremal, static, spherically symmetric $\rightarrow \text{AdS}_2 \times S^2$)

$$ds^2 = -e^{2A(r)} dt^2 + e^{-2A(r)} dr^2 + e^{2C(r)} r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

- ☞ Define electromagnetic charges

$$p^\Lambda \equiv \frac{1}{4\pi} \int_{S^2} F_2^\Lambda, \quad q_\Lambda \equiv \frac{1}{4\pi} \int_{S^2} \tilde{F}_{\Lambda 2}$$

$$I_1 \equiv -\frac{1}{2} \left[p^\Lambda \mu_{\Lambda\Sigma} p^\Sigma + (q_\Lambda - \nu_{\Lambda\Gamma} p^\Gamma) (\mu^{-1})^{\Lambda\Sigma} (q_\Sigma - \nu_{\Sigma\Delta} p^\Delta) \right]$$

$$\tilde{F}_{\Lambda 2} \equiv \nu_{\Lambda\Sigma} F_2^\Sigma + \mu_{\Lambda\Sigma} (*F_2^\Sigma)$$

- ☞ Impose (covariantly) constant condition

$$0 \equiv \partial_\mu \mathfrak{t}, \quad 0 \equiv \partial_\mu \varphi, \quad 0 \equiv D_\mu \xi^0, \quad 0 \equiv D_\mu \tilde{\xi}_0, \quad 0 \equiv \partial_{[\mu} B_{\nu\rho]}$$

The equation of motion for $g_{\mu\nu}$:

$$\delta g_{tt} - \delta g_{rr} : e^{2C(r)} = \frac{e^{2c_2}}{r^2} (c_1 r + 1)^2$$

$$\delta g_{\theta\theta}, \delta g_{\phi\phi} : e^{2A(r)} = e^{-4c_2} \frac{6 I_1 - e^{4c_2} (c_1 r + 1)}{3 c_1^2 (c_1 r + 1)^2} \left[(c_1 r + 1)^3 V + 6 c_1 \{ a_1 - c_1 a_2 (c_1 r + 1) \} \right]$$

$$\delta g_{rr} : a_2 = \frac{1}{2(c_1)^2} e^{2c_2}$$

$C(r)$ and $A(r)$ are expressed in terms of I_1 , V and constants of integration $\{a_i, c_i\}$

$$e^{2A(r)} = \frac{e^{-2c_2}}{(c_1)^2} - \frac{2a_1}{c_1(c_1 r + 1)} + \frac{e^{4c_2} I_1}{(c_1)^2 (c_1 r + 1)^2} - \frac{V}{3(c_1)^2 (c_1 r + 1)^2} \equiv 1 - \frac{2\eta}{r_{\text{new}}} + \frac{\mathcal{Z}^2}{r_{\text{new}}^2} - \frac{\Lambda_{\text{c.c.}}}{3} r_{\text{new}}^2$$

Choosing $c_1 r + 1 \equiv r_{\text{new}}$ (and $c_1 \equiv 1$), we can read the Black Hole information:

$e^{-2c_2} \equiv 1$: scalar curvature of S^2

$a_1 \equiv \eta$: mass parameter

$I_1 \equiv \mathcal{Z}^2$: square of charges

$V \equiv \Lambda_{\text{c.c.}}$: cosmological constant

The equations of motion for t , φ , ξ^0 :

$$\delta t : 0 = \frac{e^{-4C}}{r^4} \frac{\partial I_1}{\partial t} + \frac{\partial V}{\partial t} \quad \longrightarrow \quad \frac{\partial V}{\partial t} = 0 \quad \text{and} \quad \frac{\partial I_1}{\partial t} = 0$$

$$\delta \varphi : 0 = 2V_{\text{NS}} + 4V_{\text{R}} \quad \longrightarrow \quad V = V_{\text{NS}} + V_{\text{R}} = \frac{1}{2}V_{\text{NS}} = -V_{\text{R}}$$

$$\delta \xi^0 : 0 = \frac{\partial V}{\partial \xi^0} = \frac{\cancel{\partial V_{\text{NS}}}}{\cancel{\partial \xi^0}} + \frac{\partial V_{\text{R}}}{\partial \xi^0} \quad \left(0 = \frac{\partial V}{\partial \tilde{\xi}_0} \text{ is trivial} \right)$$

Regular Solution

$$\{t, \xi^0, \varphi; V\}_{\text{BHs}} = \{t_*, \xi_*^0, \varphi_*; \Lambda_{\text{c.c.}}\}_{\text{Vacua}}$$

constant in whole region

The equations of motion for t , $B_{\mu\nu}$:

$$\delta t : 0 = \frac{e^{-4C}}{r^4} \frac{\partial I_1}{\partial t} + \frac{\partial V}{\partial t} \quad \longrightarrow \quad \frac{\partial V}{\partial t} = 0 \quad \text{and} \quad \frac{\partial I_1}{\partial t} = 0$$

$$\delta B_{\mu\nu} : 0 = m_R^\Lambda \mu_{\Lambda\Sigma} \left(\frac{\sqrt{-g}}{2} \epsilon_{\mu\nu\rho\sigma} F^{\Sigma\rho\sigma} \right) + m_R^\Lambda \nu_{\Lambda\Sigma} F_{\mu\nu}^\Sigma - (e_{R\Lambda} - e_{\Lambda 0} \xi^0) F_{\mu\nu}^\Lambda$$

$$\text{and} \quad 0 = D_\mu \tilde{\xi}_0 \quad \longrightarrow \quad 0 = [\partial_\mu, \partial_\nu] \tilde{\xi}_0 = e_{\Lambda 0} F_{\mu\nu}^\Lambda$$

$$\text{with} \quad F_{\theta\phi}^\Lambda = p^\Lambda \sin \theta, \quad F_{tr}^\Lambda = -\frac{1}{r_{\text{new}}^2} (\mu^{-1})^{\Lambda\Sigma} (q_\Sigma - \nu_{\Sigma\Gamma} p^\Gamma)$$

Solve them to find an appropriate BH charge configuration

It turns out that all the charges are zero : $p^\Lambda = 0 = q_\Lambda$ (highly non-trivial)

$$\therefore I_1 \equiv \mathcal{Z}^2 = 0, \quad F_{\mu\nu}^\Lambda = 0$$

$$\checkmark \quad 0 = F_{\mu\nu}^1 = 2\partial_{[\mu}A_{\nu]}^1 + m_R^1 B_{\mu\nu} \rightarrow A_\mu^1 = \partial_\mu\lambda \equiv 0 \quad (\text{gauge-fixing})$$

$$\checkmark \quad 0 = F_{\mu\nu}^0 = 2\partial_{[\mu}A_{\nu]}^0 + m_R^0 B_{\mu\nu} \rightarrow 2\partial_{[\mu}A_{\nu]}^0 = -m_R^0 B_{\mu\nu} = (\text{constant})$$

$$\checkmark \quad 0 = D_\mu\tilde{\xi}_0 = \partial_\mu\tilde{\xi}_0 - e_{00}A_\mu^0 - e_{10}A_\mu^1 = \partial_\mu\tilde{\xi}_0 \quad (\because e_{00} = 0 = A_\mu^1)$$

$$\checkmark \quad \Lambda_{\text{c.c.}} \equiv V < 0$$

$\eta \equiv a_1$ is still arbitrary

Schwarzschild-AdS Black Holes

- Black holes from CY :

$$\frac{\partial I_1}{\partial t} = 0$$

Value of vector modulus t is not fixed at infinity \rightarrow attractor mechanism

BH charges govern value of fields at horizon

BH mass is given by BH charges

- Black holes from non-CY :

$$\frac{\partial V}{\partial t} = 0$$

and

$$\frac{\partial I_1}{\partial t} = 0$$

Value of vector modulus t is (mostly) fixed at infinity \rightarrow moduli stabilization

BH charges are governed by geometric- and RR-flux charges

BH mass is arbitrary

(covariantly) constant solutions \longrightarrow all the charges are zero!

In order to find a black hole with non-trivial charges,
the covariantly constant condition must be (partially) relaxed.

cf.)

Gauged SUGRA without hypermultiplets \longleftarrow Fayet-Iliopoulos parameters can be involved

There are charged black hole solutions with constant scalars.

Bellucci, Ferrara, Marrani and Yeranyan [[arXiv:0802.0141](https://arxiv.org/abs/0802.0141)]

Contents

- Introduction
- 4D $\mathcal{N} = 2$ Gauged SUGRA from 10D type IIA
- Vacua
- Black holes
- Summary and Discussions

- ✓ Studied : 4D $\mathcal{N} = 2$ gauged SUGRA with VMs and TM(UHM) via flux compactification.
- ✓ Reconfirmed : Romans' mass is inevitable.
- ✓ Imposed : covariantly constant condition.
- ✓ Found : Schwarzschild-AdS BHs.

Different from cases of Calabi-Yau

- ✓ Find **charged** AdS-BH solutions.
- ✓ Consider a **stationary** AdS-BH.
- ✓ Various directions!

Thanks for your attention

Appendix

- Terminology
- Calabi-Yau compactifications in type IIA
- Geometric flux compactifications in type IIA
- Scalar potential

Prepotential : \mathcal{F} is a holomorphic function of X^Λ of degree two ($\mathcal{F}_\Lambda = \partial\mathcal{F}/\partial X^\Lambda$)

Kähler potential : $\mathcal{K}_V = -\log [i(\bar{X}^\Lambda \mathcal{F}_\Lambda - X^\Lambda \bar{\mathcal{F}}_\Lambda)]$

Symplectic section : $\Pi_V \equiv e^{\mathcal{K}_V/2} \begin{pmatrix} X^\Lambda \\ \mathcal{F}_\Lambda \end{pmatrix} \equiv \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \quad 1 = i(\bar{L}^\Lambda M_\Lambda - L^\Lambda \bar{M}_\Lambda)$

Kähler metric : $g_{a\bar{b}} = \frac{\partial}{\partial t^a} \frac{\partial}{\partial \bar{t}^b} \mathcal{K}_V, \quad t^a = \frac{X^a}{X^0}$

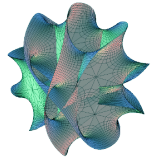
Kähler covariant derivative : $D_a \Pi_V = \left(\frac{\partial}{\partial t^a} + \frac{1}{2} \frac{\partial \mathcal{K}_V}{\partial t^a} \right) \Pi_V \equiv \begin{pmatrix} f_a^\Lambda \\ h_{\Lambda a} \end{pmatrix}$

Period matrix : $\mathcal{N}_{\Lambda\Sigma} = \bar{\mathcal{F}}_{\Lambda\Sigma} + 2i \frac{(\text{Im}\mathcal{F})_{\Lambda\Gamma} X^\Gamma (\text{Im}\mathcal{F})_{\Sigma\Delta} X^\Delta}{X^\Pi (\text{Im}\mathcal{F})_{\Pi\Xi} X^\Xi}$

Formulae : $M_\Lambda = \mathcal{N}_{\Lambda\Sigma} L^\Sigma, \quad h_{\Lambda a} = \bar{\mathcal{N}}_{\Lambda\Sigma} f_a^\Sigma$

(Symplectic matrix) : $(\mathbb{M}_V)_{\Lambda\Sigma} = \begin{pmatrix} \mathbb{1} & -\text{Re}\mathcal{N} \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} \text{Im}\mathcal{N} & 0 \\ 0 & (\text{Im}\mathcal{N})^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ -\text{Re}\mathcal{N} & \mathbb{1} \end{pmatrix}$

In a similar way... $\Pi_H \equiv e^{\mathcal{K}_H/2} \begin{pmatrix} Z^I \\ \mathcal{G}_I \end{pmatrix}, \quad z^i = \frac{Z^i}{Z^0}, \quad \mathcal{K}_H = -\log [i(\bar{Z}^I \mathcal{G}_I - Z^I \bar{\mathcal{G}}_I)], \quad \text{etc.}$

Calabi-Yau 3-fold \mathcal{M}_{CY} 

Ricci-flat, torsionless, (compact) Kähler manifold
with $SU(3)$ holonomy group

Invariant two-form J and three-form Ω on CY w.r.t. Levi-Civita connection:

$$dJ = \nabla_{[m} J_{np]} = 0, \quad d\Omega = \nabla_{[m} \Omega_{npq]} = 0$$

This is suitable for 1/4-SUSY condition with **vanishing** background fields

$$\delta_{SUSY} \psi_{m\pm} = \nabla_m \varepsilon_{\pm}^{(10D)} = 0$$

$$\varepsilon_{+}^{(10D)} = \varepsilon_{1+}^{(4D)} \otimes \eta_{+}^1 + (\text{c.c.}), \quad \varepsilon_{-}^{(10D)} = \varepsilon_{2+}^{(4D)} \otimes \eta_{-}^2 + (\text{c.c.})$$

$$(\varepsilon_{1,2+}^{(4D)})^c = \varepsilon_{1,2-}^{(4D)}, \quad (\eta_{-}^{1,2})^* = \eta_{+}^{1,2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ * \end{pmatrix} : \quad SU(3)\text{-invariant} \subset SU(4) \sim SO(6)$$

$$\eta_{\pm}^1 = \eta_{\pm}^2 \equiv \eta_{\pm}, \quad J_{mn} \sim \mp i \eta_{\pm}^{\dagger} \gamma_{mn} \eta_{\pm}, \quad \Omega \sim -i \eta_{-}^{\dagger} \gamma_{mnp} \eta_{+}$$

NS-NS fields in 10D are expanded around CY :

$$\begin{aligned}\phi(x, y) &= \varphi(x) \\ g_{m\bar{n}}(x, y) &= iv^a(x) (\omega_a)_{m\bar{n}}(y), \quad g_{mn}(x, y) = i \bar{z}^{\bar{j}}(x) \left(\frac{(\bar{\chi}_{\bar{j}})_{m\bar{p}\bar{q}} \Omega^{\bar{p}\bar{q}}}{\|\Omega\|^2} \right) (y) \\ \widehat{B}_2(x, y) &= B_2(x) + b^a(x) \omega_a(y) \\ \mathfrak{t}^a(x) &\equiv b^a(x) + iv^a(x)\end{aligned}$$

R-R fields :

$$\begin{aligned}\widehat{C}_1(x, y) &= A_1^0(x) \\ \widehat{C}_3(x, y) &= A_1^a(x) \wedge \omega_a(y) + \xi^I(x) \alpha_I(y) - \tilde{\xi}_I(x) \beta^I(y)\end{aligned}$$

cohomology class on CY	basis	degrees
$H^{(1,1)}$	ω_a	$a = 1, \dots, h^{(1,1)}$
$H^{(0)} \oplus H^{(1,1)}$	$\omega_\Lambda = (1, \omega_a)$	$\Lambda = 0, 1, \dots, h^{(1,1)}$
$H^{(2,2)} \oplus H^{(6)}$	$\tilde{\omega}^\Lambda = (\tilde{\omega}^a, \frac{\text{vol.}}{ \text{vol.} })$	
$H^{(2,1)}$	χ_i	$i = 1, \dots, h^{(2,1)}$
$H^{(3)}$	(α_I, β^I)	$I = 0, 1, \dots, h^{(2,1)}$

$$d\omega_\Lambda = 0 = d\tilde{\omega}^\Lambda$$

$$d\alpha_I = 0 = d\beta^I$$

10D Type IIA action $S_{\text{IIA}}^{(10\text{D})} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}$:

$$S_{\text{NS}} = \frac{1}{2} \int e^{-2\phi} \left\{ \widehat{R} * \mathbb{1} + 4d\phi \wedge *d\phi - \frac{1}{2} \widehat{H}_3 \wedge * \widehat{H}_3 \right\}$$

$$S_{\text{R}} + S_{\text{CS}} = -\frac{1}{4} \int \left\{ \widehat{F}_2 \wedge * \widehat{F}_2 + (\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3) \wedge * (\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3) \right\} - \frac{1}{4} \int \widehat{B}_2 \wedge \widehat{F}_4 \wedge \widehat{F}_4$$



4D $\mathcal{N} = 2$ ungauged SUGRA: **Neither gauge couplings, Nor scalar potential**

$$S^{(4\text{D})} = \int \left\{ \frac{1}{2} R * \mathbb{1} - G_{a\bar{b}} dt^a \wedge *d\bar{t}^{\bar{b}} - h_{uv} dq^u \wedge *dq^v + \frac{1}{2} \text{Im} \mathcal{N}_{\Lambda\Sigma} F^\Lambda \wedge *F^\Sigma + \frac{1}{2} \text{Re} \mathcal{N}_{\Lambda\Sigma} F^\Lambda \wedge F^\Sigma \right\}$$

gravitational multiplet	$g_{\mu\nu}, A_1^0$	
vector multiplet (VM)	$A_1^a, \mathfrak{t}^a, \bar{\mathfrak{t}}^{\bar{b}}$	$\mathfrak{t}^a \in \text{SKG}_V$
hypermultiplet (HM)	$z^i, \bar{z}^{\bar{j}}, \xi^i, \tilde{\xi}_j$	$z^i \in \text{SKG}_H$
universal hypermultiplet (UHM)	$\varphi, a, \xi^0, \tilde{\xi}_0$	$a \leftrightarrow B_2$ (Hodge dual)

$\mathcal{HM} = \text{Special QG}$

$$\underbrace{\{q^u\}}_{4n_H + 4} = \underbrace{\{z^i, \bar{z}^{\bar{j}}\}}_{2n_H(\text{SKG}_H)} + \underbrace{\{\xi^i, \tilde{\xi}_j\}}_{2n_H} + \underbrace{\{\varphi, a, \xi^0, \tilde{\xi}_0\}}_{4(\text{UHM})} = \underbrace{\{z^i, \bar{z}^{\bar{j}}\}}_{\text{SKG}_H} + \{\varphi\} + \underbrace{\{a, \xi^I, \tilde{\xi}_J\}}_{\text{"Heisenberg"}}$$

10D type IIA action $S_{\text{IIA}}^{(10\text{D})} = S_{\text{NS}} + \tilde{S}_{\text{R}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}$: (democratic form)

$$S_{\text{NS}} = \frac{1}{2} \int e^{-2\phi} \left\{ \widehat{R} * \mathbb{1} + 4d\phi \wedge *d\phi - \frac{1}{2} \widehat{H}_3 \wedge * \widehat{H}_3 \right\}, \quad \tilde{S}_{\text{R}} = -\frac{1}{8} \int [\widehat{\mathbf{F}} \wedge * \widehat{\mathbf{F}}]_{10}$$

with “constraint $\widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}})$ ” and “EoM (Bianchi) $(d + \widehat{H} \wedge) * \widehat{\mathbf{F}} = 0 \Leftrightarrow (d - \widehat{H} \wedge) \widehat{\mathbf{F}} = 0$ ”

↓ non-CY with $SU(3)$ -structure with $m_{\text{R}}^{\Lambda} = 0$

4D $\mathcal{N} = 2$ abelian gauged SUGRA (with $\xi^I \equiv (\xi^I, \tilde{\xi}_I)^{\text{T}}$):

$$S^{(4\text{D})} = \int d^4x \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{4} \text{Im} \mathcal{N}_{\Lambda\Sigma} F_{\mu\nu}^{\Lambda} F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}} \text{Re} \mathcal{N}_{\Lambda\Sigma} F_{\mu\nu}^{\Lambda} F_{\rho\sigma}^{\Sigma} - g_{a\bar{b}} \partial_{\mu} t^a \partial^{\mu} \bar{t}^{\bar{b}} - g_{i\bar{j}} \partial_{\mu} z^i \partial^{\mu} \bar{z}^{\bar{j}} \right. \\ \left. - \partial_{\mu} \varphi \partial^{\mu} \varphi + \frac{e^{2\varphi}}{2} (\mathbb{M}_{\text{H}})_{IJ} D_{\mu} \xi^I D^{\mu} \xi^J - \frac{e^{2\varphi}}{4} (D_{\mu} a - \xi^I (\mathbb{C}_{\text{H}})_{IJ} D_{\mu} \xi^J)^2 - V(t, \bar{t}, q) \right]$$

- $(e_{\Lambda}^I, e_{\Lambda I})$: geometric flux charges & $e_{\text{R}\Lambda}$: RR-flux charges
(with constraints $e_{\Lambda}^I e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}^I = 0$) ← non-CY data
- $t^a \in \text{SKG}_{\text{V}}$ and $z^i \in \text{SKG}_{\text{H}} \subset \mathcal{H}\mathcal{M}$ are ungauged (in general)
- $D_{\mu} \xi^I = \partial_{\mu} \xi^I - e_{\Lambda}^I A_{\mu}^{\Lambda}$ & $D_{\mu} \tilde{\xi}_I = \partial_{\mu} \tilde{\xi}_I - e_{\Lambda I} A_{\mu}^{\Lambda}$
- $D_{\mu} a = \partial_{\mu} a - (2e_{\text{R}\Lambda} - \xi^I e_{\Lambda I} + \tilde{\xi}_I e_{\Lambda}^I) A_{\mu}^{\Lambda}$
- $V(t, \bar{t}, q)$: scalar potential

D. Cassani [arXiv:0804.0595]

Non-vanishing m_{R}^{Λ} dualizes the axion field a in standard SUGRA to B-field.

4D gauged action is different from the standard one:

$$\begin{aligned}
 S^{(4\text{D})} = \int & \left[\frac{1}{2}R(*\mathbb{1}) + \frac{1}{2}\text{Im}\mathcal{N}_{\Lambda\Sigma}F_2^{\Lambda} \wedge *F_2^{\Sigma} + \frac{1}{2}\text{Re}\mathcal{N}_{\Lambda\Sigma}F_2^{\Lambda} \wedge F_2^{\Sigma} - g_{a\bar{b}}dt^a \wedge *d\bar{t}^{\bar{b}} - g_{i\bar{j}}dz^i \wedge *d\bar{z}^{\bar{j}} \right. \\
 & - d\varphi \wedge *d\varphi - \frac{e^{-4\varphi}}{4}H_3 \wedge *H_3 - \frac{e^{2\varphi}}{2}(\mathbb{M}_{\text{H}})_{IJ}D\xi^I \wedge *D\xi^J - V(*\mathbb{1}) \\
 & \left. + \frac{1}{2}dB \wedge \left[\xi^I(\mathbb{C}_{\text{H}})_{IJ}D\xi^J + (2e_{\text{R}\Lambda} - \xi^I e_{\Lambda I} + \tilde{\xi}_{I\Lambda} e_{\Lambda}^I)A_I^{\Lambda} \right] - \frac{1}{2}m_{\text{R}}^{\Lambda}e_{\text{R}\Lambda}B_2 \wedge B_2 \right]
 \end{aligned}$$

Constraints among flux charges:

$$e_{\Lambda}^I e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}^I = 0, \quad m_{\text{R}}^{\Lambda} e_{\Lambda}^I = 0 = m_{\text{R}}^{\Lambda} e_{\Lambda I}$$

Scalar potential from (non)geometric flux compactifications:

$$V = \mathbf{g}^2 \left[4h_{uv} k^u \bar{k}^v + \sum_{x=1}^3 \left(g^{a\bar{b}} D_a \mathcal{P}_x D_{\bar{b}} \bar{\mathcal{P}}_x - 3|\mathcal{P}_x|^2 \right) \right] = \dots \equiv V_{\text{NS}} + V_{\text{R}} \quad (\text{abelian: } k_{\Lambda}^a = 0)$$

$$\begin{aligned} V_{\text{NS}} &= g^{a\bar{b}} D_a \mathcal{P}_+ D_{\bar{b}} \bar{\mathcal{P}}_+ + g^{i\bar{j}} D_i \mathcal{P}_+ D_{\bar{j}} \bar{\mathcal{P}}_+ - 2|\mathcal{P}_+|^2 \\ &= -2 \mathbf{g}^2 e^{2\varphi} \left[\bar{\Pi}_{\text{H}}^{\text{T}} \tilde{Q}^{\text{T}} \mathbb{M}_{\text{V}} \tilde{Q} \Pi_{\text{H}} + \bar{\Pi}_{\text{V}}^{\text{T}} Q \mathbb{M}_{\text{H}} Q^{\text{T}} \Pi_{\text{V}} + 4\bar{\Pi}_{\text{H}}^{\text{T}} \mathbb{C}_{\text{H}}^{\text{T}} Q^{\text{T}} (\Pi_{\text{V}} \bar{\Pi}_{\text{V}}^{\text{T}} + \bar{\Pi}_{\text{V}} \Pi_{\text{V}}^{\text{T}}) Q \mathbb{C}_{\text{H}} \Pi_{\text{H}} \right] \end{aligned}$$

$$\begin{aligned} V_{\text{R}} &= g^{a\bar{b}} D_a \mathcal{P}_3 D_{\bar{b}} \bar{\mathcal{P}}_3 + |\mathcal{P}_3|^2 \\ &= -\frac{1}{2} \mathbf{g}^2 e^{4\varphi} (e_{\text{R}\Lambda} - e_{\Lambda I} \xi^I + e_{\Lambda}^I \tilde{\xi}_I) (\text{Im}\mathcal{N})^{-1|\Lambda\Sigma} (e_{\text{R}\Sigma} - e_{\Sigma I} \xi^I + e_{\Sigma}^I \tilde{\xi}_I) \end{aligned}$$

$$\Pi_{\text{V}} = e^{\mathcal{K}_{\text{V}}/2} (X^{\Lambda}, \mathcal{F}_{\Lambda})^{\text{T}}$$

$$t^a = X^a / X^0$$

$$a = 1, \dots, n_{\text{V}}$$

SKG_V of vector-moduli

$$\mathcal{P}_+ \equiv \mathcal{P}_1 + i\mathcal{P}_2 = 2e^{\varphi} \Pi_{\text{V}}^{\text{T}} Q \mathbb{C}_{\text{H}} \Pi_{\text{H}}$$

$$\mathcal{P}_- \equiv \mathcal{P}_1 - i\mathcal{P}_2 = 2e^{\varphi} \Pi_{\text{V}}^{\text{T}} Q \mathbb{C}_{\text{H}} \bar{\Pi}_{\text{H}}$$

$$\mathcal{P}_3 = e^{2\varphi} \Pi_{\text{V}}^{\text{T}} \mathbb{C}_{\text{V}} (c_{\text{R}} + \tilde{Q}\xi)$$

$$\Pi_{\text{H}} = e^{\mathcal{K}_{\text{H}}/2} (Z^I, \mathcal{G}_I)^{\text{T}}$$

$$z^i = Z^i / Z^0$$

$$i = 1, \dots, n_{\text{H}}$$

SKG_H of hyper-moduli

$$\mathbb{C}_{\text{V,H}} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}; \quad Q = \begin{pmatrix} e_{\Lambda}^I & e_{\Lambda I} \\ m^{\Lambda I} & m^{\Lambda}_I \end{pmatrix}, \quad \tilde{Q} = \mathbb{C}_{\text{H}}^{\text{T}} Q \mathbb{C}_{\text{V}} \quad c_{\text{R}} = \begin{pmatrix} m_{\text{R}}^{\Lambda} \\ e_{\text{R}\Lambda} \end{pmatrix}$$

Cassani et.al. [arXiv:0804.0595], [arXiv:0911.2708]

Coset spaces with $SU(3)$ -structure: [D. Cassani and A.K. Kashani-Poor \[arXiv:0901.4251\]](#)

\mathcal{M}_6	$\frac{G_2}{SU(3)} = S^6$	$\frac{Sp(2)}{S(U(2) \times U(1))} = \mathbb{C}P^3$	$\frac{SU(3)}{U(1) \times U(1)} = \mathbb{F}(1, 2; 3)$
$\mathcal{SM} = \text{SKG}_V$	$\frac{SU(1, 1)}{U(1)} : \mathfrak{t}^3$	$\left(\frac{SU(1, 1)}{U(1)}\right)^2 : \mathfrak{st}^2$	$\left(\frac{SU(1, 1)}{U(1)}\right)^3 : \mathfrak{stu}$
$\mathcal{HM} = \text{SQG}$	$\frac{SU(2, 1)}{U(2)} : \text{UHM}$	$\frac{SU(2, 1)}{U(2)} : \text{UHM}$	$\frac{SU(2, 1)}{U(2)} : \text{UHM}$
$\text{SKG}_H \subset \mathcal{HM}$	—	—	—
matters	1 VM + 1 UHM	2 VM + 1 UHM	3 VM + 1 UHM

Each SKG_V has a cubic prepotential: $\mathcal{F} = \frac{1}{3!} d_{abc} \frac{X^a X^b X^c}{X^0}$

nilmanifolds and solvmanifolds: [M. Graña, R. Minasian, M. Petrini and A. Tomasiello \[hep-th/0609124\]](#)

coset spaces with $SU(3)$ - or $SU(2)$ -structure: [P. Koerber, D. Lüst and D. Tsimpis \[arXiv:0804.0614\]](#)

a pair of $SU(3)$ -structures with $(m^{\Lambda I}, m^{\Lambda}_I)$: [D. Gaiotto and A. Tomasiello \[arXiv:0904.3959\]](#)

Fin.