Seminar at Nagoya University (July 04, 2011)

Charged Black Hole Solutions in $\mathcal{N} = 2$ Geometric Flux Compactifications

(Improved Version)

KEK
HIGH ENERGY ACCELERATOR RESEARCH ORGANIZATION

Tetsuji KIMURA

Charged Black Hole Solutions in 4D $\mathcal{N} = 2$ Gauged SUGRA

- WHY $\mathcal{N} = 2$ (8-SUSY charges)?
  - ✓ Scalar fields living in highly symmetric spaces
  - ✓ Flux compactification scenarios in string theories

- WHY Charged Black Holes?
  - ✓ Attractive: 4D $\mathcal{N} = 2$ SUGRA $\subset$ Einstein-Yang–Mills-Matters
  - ✓ Non-trivial: if there exists the cosmological constant with matter fields

Flux Compactifications beyond Calabi-Yau

Why flux compactifications beyond CY in 10D Strings?

1. CY $\rightarrow$ 4D ungauged SUGRA
   $\rightarrow$ Fluxes break 10D Eqs. of Motion
2. non-CY with fluxes $\rightarrow$ 4D gauged SUGRA
   non-CY: $SU(3)$-structure with torsion, generalized geometry, etc.
   gauge coupling constants, mass parameters...

- In 4D $\mathcal{N} = 2$ ungauged SUGRA $\longrightarrow$ No scalar potential.

  (Extremal) charged Black holes in asymptotic flat has been investigated.
  Charges $=$ D-branes wrapped on CY: "D0-D4", "D2-D6", "D0-D2-D6", etc.
  (Hypermultiplets are decoupled from the system.)

- In 4D $\mathcal{N} = 2$ gauged SUGRA $\longrightarrow$ Scalar potential is turned on.

  The cosmological constant Λ is given as VEV of the scalar potential V.
  ("mass deformations" of gravitini)

Anti-de Sitter Black Holes

Remark: Naked Singularity appeared in SUSY charged AdS BH solutions.

- Pure AdS SUGRA (only gravitational multiplet):

  L.J. Romans [hep-th/9203018], M.M. Caldarelli and D. Klemm [hep-th/9808097], etc.

  There exists a SUSY solution of rotating AdS black hole with regular horizon.

- Gauged SUGRA with vector multiplets (without hyper-sector):

  W.A. Sabra, et.al. (electric charges [hep-th/9903143], magnetic/dyonic charges [hep-th/0003213]), etc.

- Found SUSY AdS-BHs with vector multiplets (without hyper-sector):

  [arXiv:0911.4926], [arXiv:1011.2202], [arXiv:1012.3756], [arXiv:1012.4314], etc.

Question

How can we obtain charged BH solutions with hypermultiplets in asymptotically (non)-flat spacetime?

— Setup and Result —

- 10D type IIA string on non-CY with $SU(3)$-structure
- 4D $\mathcal{N} = 2$ gauged SUGRA with VMs and UHM
- Impose covariantly constant condition on matter fields
- Regular solutions!?

(U)HM: (universal) hypermultiplet
VM: vector multiplet

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From now on, you meet a tremendous number of equations..

Ready?

Supersymmetric multiplets in 4D $\mathcal{N}=2$ SUGRA:

1 gravitational multiplet: $\{g_{\mu\nu}, A^0_\mu, \psi_{A\mu}\}$ — $\mu = 0,1,2,3$ (4D, curved); $A = 1,2$ ($SU(2)$ R-symmetry)

n_{V} vector multiplets: $\{A^a_\mu, \mathfrak{t}^a, \lambda^{aA}\}$ — $a = 1, \ldots, n_{\mathsf{V}}$

$\mathfrak{t}^a$ in special Kähler geometry (SKG): $\mathcal{SM}$

$n_{\mathsf{H}}+1$ hypermultiplets: $\{q^u, \zeta_\alpha\}$ — $u = 1, \ldots, 4n_{\mathsf{H}}+4$; $\alpha = 1, \ldots, 2n_{\mathsf{H}}+2$

q^u in quaternionic geometry (QG): $\mathcal{HM}$

Two moduli spaces $\mathcal{SM}$ and $\mathcal{HM}$ govern 4D $\mathcal{N}=2$ SUGRA.
Both are highly understood in a mathematical sense.

Prepotential :	$\mathcal{F}$ is a holomorphic function of X^Λ of degree two $(\mathcal{F}_\Lambda = \partial\mathcal{F}/\partial X^\Lambda)$
Kähler potential :	$\mathcal{K}_{\mathsf{V}} = -\log\big[\mathrm{i}\big(\overline{X}^\Lambda \mathcal{F}_\Lambda - X^\Lambda \overline{\mathcal{F}}_\Lambda\big)\big]$
Symplectic section :	$\Pi_{\mathsf{V}} \equiv \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2}\begin{pmatrix} X^\Lambda \\ \mathcal{F}_\Lambda \end{pmatrix} \equiv \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \quad 1 = \mathrm{i}\big(\overline{L}^\Lambda M_\Lambda - L^\Lambda \overline{M}_\Lambda\big)$
Kähler metric :	$g_{a\bar{b}} = \frac{\partial}{\partial \mathrm{t}^a}\frac{\partial}{\partial \bar{\mathrm{t}}^{\bar{b}}}\mathcal{K}_{\mathsf{V}}\,, \quad \mathrm{t}^a = \frac{X^a}{X^0}$
Kähler covariant derivative :	$D_a \Pi_{\mathsf{V}} = \Big(\frac{\partial}{\partial \mathrm{t}^a} + \frac{1}{2}\frac{\partial \mathcal{K}_{\mathsf{V}}}{\partial \mathrm{t}^a}\Big)\Pi_{\mathsf{V}} \equiv \begin{pmatrix} f_a^\Lambda \\ h_{\Lambda a} \end{pmatrix}$
Period matrix :	$\mathcal{N}_{\Lambda\Sigma} = \overline{\mathcal{F}}_{\Lambda\Sigma} + 2\mathrm{i}\frac{(\mathrm{Im}\mathcal{F})_{\Lambda\Gamma}X^\Gamma (\mathrm{Im}\mathcal{F})_{\Sigma\Delta}X^\Delta}{X^\Pi (\mathrm{Im}\mathcal{F})_{\Pi\Xi}X^\Xi}$
Formulae :	$M_\Lambda = \mathcal{N}_{\Lambda\Sigma}L^\Sigma, \quad h_{\Lambda a} = \overline{\mathcal{N}}_{\Lambda\Sigma} f_a^\Sigma$
(Symplectic matrix) :	$(\mathbb{M}_{\mathsf{V}})_{\boldsymbol{\Lambda\Sigma}} = \begin{pmatrix} \mathbb{1} & -\mathrm{Re}\mathcal{N} \\ 0 & \mathbb{1} \end{pmatrix}\begin{pmatrix} \mathrm{Im}\mathcal{N} & 0 \\ 0 & (\mathrm{Im}\mathcal{N})^{-1} \end{pmatrix}\begin{pmatrix} \mathbb{1} & 0 \\ -\mathrm{Re}\mathcal{N} & \mathbb{1} \end{pmatrix}$

In a similar way... $\Pi_{\mathsf{H}} \equiv \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2}\begin{pmatrix} Z^I \\ \mathcal{G}_I \end{pmatrix}, \quad z^i = \frac{Z^i}{Z^0}, \quad \mathcal{K}_{\mathsf{H}} = -\log\big[\mathrm{i}\big(\overline{Z}^I \mathcal{G}_I - Z^I \overline{\mathcal{G}}_I\big)\big]\,,$ etc.

10D type IIA action $S^{(10\text{D})}_{\text{IIA}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}$:

$$S_{\text{NS}} = \frac{1}{2}\int e^{-2\phi}\Big\{\widehat{R} * \mathbb{1} + 4\text{d}\phi \wedge *\text{d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3\Big\}$$

$$S_{\text{R}} + S_{\text{CS}} = -\frac{1}{4}\int \Big\{\widehat{F}_2 \wedge *\widehat{F}_2 + (\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3) \wedge *(\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3)\Big\} - \frac{1}{4}\int \widehat{B}_2 \wedge \widehat{F}_4 \wedge \widehat{F}_4$$

↓ CY

4D $\mathcal{N} = 2$ ungauged SUGRA: *Neither gauge couplings, Nor scalar potential*

$$S^{(4\text{D})} = \int \text{d}^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{4}\text{Im}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}}\text{Re}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma} - g_{a\bar{b}}\,\partial_\mu t^a\partial^\mu \bar{t}^{\bar{b}} - h_{uv}\,\partial_\mu q^u \partial^\mu q^v\right]$$

gravitational multiplet	$g_{\mu\nu}, A^0_1$		
VMs	$A^a_1, t^a, \bar{t}^{\bar{b}}$	$t^a \in$ SKG$_\text{V}$	mirror dual: SKG$_\text{V}$ ↔ SKG$_\text{H}$
HMs	$z^i, \bar{z}^{\bar{j}}, \xi^i, \widetilde{\xi}_j$	$z^i \in$ SKG$_\text{H}$	
UHM	$\varphi, a, \xi^0, \widetilde{\xi}_0$	$a \leftrightarrow B_2$	Hodge dual

$\mathcal{HM}$ → Special QG

$$\underset{4n_\text{H}+4}{\{q^u\}} = \underset{2n_\text{H}(\text{SKG}_\text{H})}{\{z^i, \bar{z}^{\bar{j}}\}} + \underset{2n_\text{H}}{\{\xi^i, \widetilde{\xi}_j\}} + \underset{4\,(\text{UHM})}{\{\varphi, a, \xi^0, \widetilde{\xi}_0\}} = \underset{\text{SKG}_\text{H}}{\{z^i, \bar{z}^{\bar{j}}\} + \{\varphi\}} + \underset{\text{“Heisenberg”}}{\{a, \xi^I, \widetilde{\xi}_J\}}$$

10D type IIA action $S^{(10D)}_{\text{IIA}} = S_{\text{NS}} + \widetilde{S}_{\text{R}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}$: (democratic form)

$$S_{\text{NS}} = \frac{1}{2}\int \mathrm{e}^{-2\phi}\Big\{\widehat{R} * \mathbb{1} + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3\Big\}$$

$$\widetilde{S}_{\text{R}} = -\frac{1}{8}\int \big[\widehat{\mathbf{F}} \wedge *\widehat{\mathbf{F}}\big]_{10}$$ with self-duality of $\widehat{\mathbf{F}}$, and EoMs for $\widehat{F}_n$ and $\widehat{B}_2$

↓ non-CY with $SU(3)$-structure

4D $\mathcal{N} = 2$ abelian gauged SUGRA (with $\xi^{\boldsymbol{I}} \equiv (\xi^I, \widetilde{\xi}_I)^{\mathrm{T}}$):

$$S^{(4D)} = \int \mathrm{d}^4x\sqrt{-g}\,\Big[\frac{1}{2}R + \frac{1}{4}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma} - g_{a\bar{b}}\,\partial_\mu \mathfrak{t}^a\partial^\mu\bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{\jmath}}\,\partial_\mu z^i\partial^\mu\bar{z}^{\bar{\jmath}}$$
$$-\partial_\mu\varphi\partial^\mu\varphi + \frac{\mathrm{e}^{2\varphi}}{2}(\mathbb{M}_{\text{H}})_{\boldsymbol{IJ}}D_\mu\xi^{\boldsymbol{I}}D^\mu\xi^{\boldsymbol{J}} - \frac{\mathrm{e}^{2\varphi}}{4}\big(D_\mu a - \xi^{\boldsymbol{I}}(\mathbb{C}_{\text{H}})_{\boldsymbol{IJ}}D_\mu\xi^{\boldsymbol{J}}\big)^2 - V(\mathfrak{t},\bar{\mathfrak{t}},q)\Big]$$

- $(e_\Lambda{}^I, e_{\Lambda I})$: geometric flux charges & $e_{\text{R}\Lambda}$: RR-flux charges ⟵ non-CY data
  (with constraints $e_\Lambda{}^I e_{\Sigma I} - e_{\Lambda I}e_\Sigma{}^I = 0$)
- $\mathfrak{t}^a \in \text{SKG}_{\text{V}}$ and $z^i \in \text{SKG}_{\text{H}} \subset \mathcal{HM}$ are ungauged (in general)
- $D_\mu\xi^I = \partial_\mu\xi^I - e_\Lambda{}^I A^\Lambda_\mu$ & $D_\mu\widetilde{\xi}_I = \partial_\mu\widetilde{\xi}_I - e_{\Lambda I}A^\Lambda_\mu$
- $D_\mu a = \partial_\mu a - (2e_{\text{R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_\Lambda{}^I)A^\Lambda_\mu$
- details of $V(\mathfrak{t},\bar{\mathfrak{t}},q)$ in the next slide.. D. Cassani [arXiv:0804.0595]

Scalar potential:

$$V = \mathrm{g}^2 \Big[4h_{uv} k^u \overline{k}^v + \textstyle\sum_{x=1}^3 \Big(g^{a\bar{b}} D_a \mathcal{P}_x D_{\bar{b}} \overline{\mathcal{P}}_x - 3|\mathcal{P}_x|^2 \Big) \Big] \equiv V_{\mathsf{NS}} + V_{\mathsf{R}} \quad \text{(abelian: } k_\Lambda^a = 0\text{)}$$

$$V_{\mathsf{NS}} = g^{a\bar{b}} D_a \mathcal{P}_+ D_{\bar{b}} \overline{\mathcal{P}}_+ + g^{i\bar{\jmath}} D_i \mathcal{P}_+ D_{\bar{\jmath}} \overline{\mathcal{P}}_+ - 2|\mathcal{P}_+|^2$$

$$= -2\,\mathrm{g}^2 \mathrm{e}^{2\varphi} \Big[\overline{\Pi}_{\mathsf{H}}^{\mathrm{T}} \widetilde{Q}^{\mathrm{T}} \mathbb{M}_{\mathsf{V}} \widetilde{Q}\, \Pi_{\mathsf{H}} + \overline{\Pi}_{\mathsf{V}}^{\mathrm{T}} Q\, \mathbb{M}_{\mathsf{H}} Q^{\mathrm{T}} \Pi_{\mathsf{V}} + 4 \overline{\Pi}_{\mathsf{H}}^{\mathrm{T}} \mathbb{C}_{\mathsf{H}}^{\mathrm{T}} Q^{\mathrm{T}} (\Pi_{\mathsf{V}} \overline{\Pi}_{\mathsf{V}}^{\mathrm{T}} + \overline{\Pi}_{\mathsf{V}} \Pi_{\mathsf{V}}^{\mathrm{T}}) Q\, \mathbb{C}_{\mathsf{H}} \Pi_{\mathsf{H}} \Big]$$

$$V_{\mathsf{R}} = g^{a\bar{b}} D_a \mathcal{P}_3 D_{\bar{b}} \overline{\mathcal{P}}_3 + |\mathcal{P}_3|^2$$

$$= -\frac{1}{2} \mathrm{g}^2 \mathrm{e}^{4\varphi} (e_{\mathsf{R}\Lambda} - e_{\Lambda 0} \xi^0 + e_\Lambda{}^0 \widetilde{\xi}_0) (\mathrm{Im}\mathcal{N})^{-1|\Lambda\Sigma} (e_{\mathsf{R}\Sigma} - e_{\Sigma 0} \xi^0 + e_\Sigma{}^0 \widetilde{\xi}_0)$$

$\Pi_{\mathsf{V}} = \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2} (X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$

$\mathfrak{t}^a = X^a / X^0$

$a = 1, \ldots, n_{\mathsf{V}}$

SKG$_{\mathsf{V}}$ of vector-moduli

$$\mathcal{P}_+ \equiv \mathcal{P}_1 + \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\, \Pi_{\mathsf{V}}^{\mathrm{T}} Q\, \mathbb{C}_{\mathsf{H}} \Pi_{\mathsf{H}}$$

$$\mathcal{P}_- \equiv \mathcal{P}_1 - \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\, \Pi_{\mathsf{V}}^{\mathrm{T}} Q\, \mathbb{C}_{\mathsf{H}} \overline{\Pi}_{\mathsf{H}}$$

$$\mathcal{P}_3 = \mathrm{e}^{2\varphi}\, \Pi_{\mathsf{V}}^{\mathrm{T}} \mathbb{C}_{\mathsf{V}} (c + \widetilde{Q}\xi)$$

$\Pi_{\mathsf{H}} = \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2} (Z^I, \mathcal{G}_I)^{\mathrm{T}}$

$z^i = Z^i / Z^0$

$i = 1, \ldots, n_{\mathsf{H}}$

SKG$_{\mathsf{H}}$ of hyper-moduli

$$\mathbb{C}_{\mathsf{V,H}} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}; \quad Q = \begin{pmatrix} e_\Lambda{}^I & e_{\Lambda I} \\ 0 & 0 \end{pmatrix}, \quad \widetilde{Q} = \mathbb{C}_{\mathsf{H}}^{\mathrm{T}} Q\, \mathbb{C}_{\mathsf{V}} \quad c = \begin{pmatrix} 0 \\ e_{\mathsf{R}\Lambda} \end{pmatrix}$$

(This can be generalized more..., skipped!)

Cassani et.al. [arXiv:0804.0595], [arXiv:0911.2708]

$$
\begin{aligned}
R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} \;=\;& \frac{1}{4} g_{\mu\nu} \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\rho\sigma} F^{\Sigma\rho\sigma} - \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\mu\rho} F^{\Sigma}_{\nu\sigma}\, g^{\rho\sigma} - g_{\mu\nu}\, g_{a\bar{b}} \partial_\rho \mathrm{t}^a \partial^\rho \bar{\mathrm{t}}^{\bar{b}} + 2 g_{a\bar{b}}\, \partial_\mu \mathrm{t}^a \partial_\nu \bar{\mathrm{t}}^{\bar{b}} - g_{\mu\nu}\, g_{i\bar{\jmath}} \partial_\rho z^i \partial^\rho \bar{z}^{\bar{\jmath}} + 2 g_{i\bar{\jmath}}\, \partial_\mu z^i \partial_\nu \bar{z}^{\bar{\jmath}} \\
& - g_{\mu\nu}\, \partial_\rho \varphi \partial^\rho \varphi + 2 \partial_\mu \varphi \partial_\nu \varphi + \frac{\mathrm{e}^{2\varphi}}{2} g_{\mu\nu} (\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}} D_\rho \xi^{\boldsymbol{I}} D^\rho \xi^{\boldsymbol{J}} - \mathrm{e}^{2\varphi} (\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}} D_\mu \xi^{\boldsymbol{I}} D_\nu \xi^{\boldsymbol{J}} \\
& - \frac{\mathrm{e}^{4\varphi}}{4} g_{\mu\nu} \Big(D_\rho a - \xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D_\rho \xi^{\boldsymbol{J}} \Big) \Big(D^\rho a - \xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D^\rho \xi^{\boldsymbol{J}} \Big) + \frac{\mathrm{e}^{4\varphi}}{2} \Big(D_\mu a - \xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D_\mu \xi^{\boldsymbol{J}} \Big) \Big(D_\nu a - \xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D_\nu \xi^{\boldsymbol{J}} \Big) - g_{\mu\nu} V \,, \\
0 \;=\;& \frac{1}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^{\Sigma\mu\sigma} \Big) - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu \Big(\mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F^{\Sigma}_{\nu\rho} \Big) - \mathrm{e}^{4\varphi} \Big(D^\sigma a - \xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D^\sigma \xi^{\boldsymbol{J}} \Big) \widetilde{G}_{0\Lambda} + \mathrm{e}^{2\varphi} (\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}} D^\sigma \xi^{\boldsymbol{J}} U^{\boldsymbol{I}}{}_{\Lambda} \,, \\
0 \;=\;& \frac{1}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, g_{c\bar{b}}\, g^{\mu\nu} \partial_\nu \bar{\mathrm{t}}^{\bar{b}} \Big) + \frac{1}{4} \frac{\partial (\mathrm{Im}\mathcal{N}_{\Lambda\Sigma})}{\partial \mathrm{t}^c} F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}} \frac{\partial (\mathrm{Re}\mathcal{N}_{\Lambda\Sigma})}{\partial \mathrm{t}^c} F^{\Lambda}_{\mu\nu} F^{\Sigma}_{\rho\sigma} - \frac{\partial g_{a\bar{b}}}{\partial \mathrm{t}^c} \partial_\mu \mathrm{t}^a \partial^\mu \bar{\mathrm{t}}^{\bar{b}} - \frac{\partial V}{\partial \mathrm{t}^c} \,, \\
0 \;=\;& \frac{1}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, g_{k\bar{\jmath}}\, g^{\mu\nu} \partial_\nu \bar{z}^{\bar{\jmath}} \Big) - \frac{\partial g_{i\bar{\jmath}}}{\partial z^k} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{\jmath}} + \frac{\mathrm{e}^{2\varphi}}{2} \frac{\partial (\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}}}{\partial z^k} D_\mu \xi^{\boldsymbol{I}} D^\mu \xi^{\boldsymbol{J}} - \frac{\partial V}{\partial z^k} \,, \\
0 \;=\;& \frac{2}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \varphi \Big) + \mathrm{e}^{2\varphi} (\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}} D_\mu \xi^{\boldsymbol{I}} D^\mu \xi^{\boldsymbol{J}} - \mathrm{e}^{4\varphi} \Big(D_\mu a - \xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D_\mu \xi^{\boldsymbol{J}} \Big) \Big(D^\mu a - \xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D^\mu \xi^{\boldsymbol{J}} \Big) - \frac{\partial V}{\partial \varphi} \,, \\
0 \;=\;& \partial_\mu \Big[\frac{\mathrm{e}^{4\varphi}}{2} \sqrt{-g} \Big(D^\mu a - \xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D^\mu \xi^{\boldsymbol{J}} \Big) \Big] \,, \\
0 \;=\;& \frac{2}{\sqrt{-g}} \partial_\mu \Big(\mathrm{e}^{2\varphi} \sqrt{-g}\, (\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IK}}\, g^{\mu\nu} D_\nu \xi^{\boldsymbol{I}} \Big) + \frac{1}{\sqrt{-g}} \partial_\mu \Big\{ \frac{\mathrm{e}^{4\varphi}}{2} \sqrt{-g} \Big(D^\mu a - \xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D^\mu \xi^{\boldsymbol{J}} \Big) \xi^{\boldsymbol{L}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{LK}} \Big\} \\
& + \frac{\mathrm{e}^{4\varphi}}{2} \Big(D^\mu a - \xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D^\mu \xi^{\boldsymbol{J}} \Big) D_\mu \xi^{\boldsymbol{L}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{LK}} + \big(\partial V / \partial \xi^{\boldsymbol{K}} \big) \,.
\end{aligned}
$$

too hard to solve... ☹

We have to introduce ansatze to solve the Equations of Motion.

Contents

- Asymptotically flat BH in ungauged SUGRA (10D type II on CY):

  → (non)-BPS solutions with constant vector moduli

- Asymptotically AdS BH in gauged SUGRA without HM:

  → non-BPS solutions with constant vector moduli supported by FI paramters

- Gauged SUGRA with "constant" VM and HM:

  → EoMs are not gauge invariant (ex: $D_\mu \xi^I = \cancel{\partial_\mu \xi^I} - e_\Lambda{}^I A_\mu^\Lambda$)

↓

1. Impose (Covariantly) Constant Condition:

$$
\begin{array}{cccc}
0 \equiv \partial_\mu \mathrm{t}^a & 0 \equiv \partial_\mu z^i & 0 \equiv D_\mu \xi^i & 0 \equiv D_\mu \widetilde{\xi}_i \\
0 \equiv \partial_\mu \varphi & 0 \equiv D_\mu a & 0 \equiv D_\mu \xi^0 & 0 \equiv D_\mu \widetilde{\xi}_0
\end{array}
$$

2. Focus on systems with VMs + UHM (i.e., absence of $\{z^i, \overline{z}^{\overline{\jmath}}, \xi^i, \widetilde{\xi}_j\} \in \mathrm{SKG_H}$ part)

Non-CY coset spaces with $SU(3)$-structure: D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

$\mathcal{M}_6$	$\frac{G_2}{SU(3)}$ (nearly-Kähler)	$\frac{Sp(2)}{S[U(2)\times U(1)]}$ (half-flat)	$\frac{SU(3)}{U(1)\times U(1)}$ (half-flat)
$\mathcal{SM} = \text{SKG}_\text{V}$	$\frac{SU(1,1)}{U(1)}$: $\mathfrak{t}^3$	$\left(\frac{SU(1,1)}{U(1)}\right)^2$: $\mathfrak{st}^2$	$\left(\frac{SU(1,1)}{U(1)}\right)^3$: $\mathfrak{stu}$
$\mathcal{HM} = \text{SQG}$	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM
$\text{SKG}_\text{H} \subset \mathcal{HM}$	—	—	—
matters	1 VM + 1 UHM	2 VM + 1 UHM	3 VM + 1 UHM

Each SKG_V has a cubic prepotential: $\mathcal{F} = \frac{1}{3!} d_{abc} \frac{X^a X^b X^c}{X^0}$

nilmanifolds and solvmanifolds: M. Graña, R. Minasian, M. Petrini and A. Tomasiello [hep-th/0609124]

coset spaces with $SU(3)$- or $SU(2)$-structure: P. Koerber, D. Lüst and D. Tsimpis [arXiv:0804.0614]

a pair of $SU(3)$-structures with $(m^{\Lambda I}, m^{\Lambda}{}_I)$: D. Gaiotto and A. Tomasiello [arXiv:0904.3959]

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} \;=\; \frac{1}{4} g_{\mu\nu} \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\rho\sigma} F^{\Sigma\rho\sigma} - \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\mu\rho} F^{\Sigma}_{\nu\sigma}\, g^{\rho\sigma} - g_{\mu\nu}\, V \,,$$

$$0 \;=\; \frac{1}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^{\Sigma\mu\sigma} \Big) - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu \Big(\mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F^{\Sigma}_{\nu\rho} \Big) \;\equiv\; - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu G_{\Lambda\nu\rho} \,,$$

$$0 \;=\; \frac{1}{4} \frac{\partial (\mathrm{Im}\mathcal{N}_{\Lambda\Sigma})}{\partial \mathrm{t}^c} F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}} \frac{\partial (\mathrm{Re}\mathcal{N}_{\Lambda\Sigma})}{\partial \mathrm{t}^c} F^{\Lambda}_{\mu\nu} F^{\Sigma}_{\rho\sigma} - \frac{\partial V}{\partial \mathrm{t}^c} \,,$$

$$0 \;=\; - \frac{\partial V}{\partial \varphi} \,,$$

$$0 \;=\; \frac{\partial V}{\partial \xi^0} \;=\; \frac{\partial V}{\partial \widetilde{\xi}_0} \,,$$

$$G_{\Lambda\mu\nu} \;\equiv\; \frac{\sqrt{-g}}{2} \epsilon_{\mu\nu\rho\sigma} \Big(\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}\, F^{\Sigma\rho\sigma} \Big) + \mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F^{\Sigma}_{\mu\nu} \,,$$

$$\Pi_{\mathsf{H}} \;=\; \begin{pmatrix} Z^0 \\ \mathcal{G}_0 \end{pmatrix} \;=\; \begin{pmatrix} 1 \\ -\mathrm{i} \end{pmatrix} , \quad \mathbb{C}_{\mathsf{H}} \;=\; \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} , \quad \mathbb{M}_{\mathsf{H}} \;=\; \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} ,$$

$$\mathcal{P}_+ \;=\; -2 \mathrm{e}^{\varphi} L^{\Lambda} (e_{\Lambda 0} + \mathrm{i} e_{\Lambda}{}^0) \,, \quad \mathcal{P}_+ \;=\; -2 \mathrm{e}^{\varphi} L^{\Lambda} (e_{\Lambda 0} - \mathrm{i} e_{\Lambda}{}^0) \,, \quad \mathcal{P}_3 \;=\; \mathrm{e}^{2\varphi} L^{\Lambda} (e_{\mathsf{R}\Lambda} - e_{\Lambda 0} \xi^0 + e_{\Lambda}{}^0 \widetilde{\xi}_0) \,.$$

Quite easy to solve in an appropriate metric ansatz! ☺

Assume 4D spacetime metric in the following static form:

$$\mathrm{d}s^2 \;=\; -\mathrm{e}^{2A(r)}\mathrm{d}t^2 + \mathrm{e}^{-2A(r)}\mathrm{d}r^2 + \mathrm{e}^{2C(r)}\, r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2\right)$$

Define electromagenetic charges of the system:

$$q_{\Lambda} \;\equiv\; \int_{S^2} G_{\Lambda\mathit{2}} \qquad \text{(electric charges)}$$

$$p^{\Lambda} \;\equiv\; \int_{S^2} F_{\mathit{2}}^{\Lambda} \qquad \text{(magnetic charges)}$$

Configuration of the gauge field strength:

$$F^{\Lambda}_{\theta\phi} \;=\; p^{\Lambda} \sin\theta \,, \qquad F^{\Lambda}_{tr} \;=\; -\frac{\mathrm{e}^{-2C}}{r^2} (\mathrm{Im}\mathcal{N})^{-1|\Lambda\Sigma} \big[q_{\Sigma} - \mathrm{Re}\mathcal{N}_{\Sigma\Gamma}\, p^{\Gamma} \big]$$

Energy-momentum tensor of the gauge fields is given by

$$\begin{aligned} T_{\mu\nu} \;&\equiv\; \frac{1}{4} g_{\mu\nu}\, \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}\, F^{\Lambda}_{\rho\sigma} F^{\Sigma\rho\sigma} - \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}\, F^{\Lambda}_{\mu\rho} F^{\Sigma}_{\nu\sigma}\, g^{\rho\sigma} \\ T_t{}^t \;=\; T_r{}^r \;=\; -T_{\theta}{}^{\theta} \;=\; -T_{\phi}{}^{\phi} \;&\equiv\; -\frac{\mathrm{e}^{-4C}}{r^4} I_1(p,q;\mathfrak{t},\bar{\mathfrak{t}}) \\ I_1(p,q;\mathfrak{t},\bar{\mathfrak{t}}) \;&\equiv\; -\frac{1}{2} \big(p^{\Lambda}, q_{\Lambda} \big) \big(\mathbb{M}_{\mathsf{V}} \big)_{\boldsymbol{\Lambda\Sigma}} \begin{pmatrix} p^{\Sigma} \\ q_{\Sigma} \end{pmatrix} \\ &=\; -\frac{1}{2} \Big[\big(q_{\Lambda} - \mathrm{Re}\mathcal{N}_{\Lambda\Gamma}\, p^{\Gamma} \big) (\mathrm{Im}\mathcal{N})^{-1|\Lambda\Sigma} \big(q_{\Sigma} - \mathrm{Re}\mathcal{N}_{\Sigma\Delta}\, p^{\Delta} \big) \Big] \end{aligned}$$

In addition, parts of EoM for $\mathfrak{t}^a$ are rearranged to

$$\frac{1}{4} \frac{\partial (\mathrm{Im}\mathcal{N}_{\Lambda\Sigma})}{\partial \mathfrak{t}^c} F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}} \frac{\partial (\mathrm{Re}\mathcal{N}_{\Lambda\Sigma})}{\partial \mathfrak{t}^c} F^{\Lambda}_{\mu\nu} F^{\Sigma}_{\rho\sigma} \;=\; -\frac{\mathrm{e}^{-4C}}{r^4} \frac{\partial I_1}{\partial \mathfrak{t}^c}$$

$I_1(p,q;\mathfrak{t},\bar{\mathfrak{t}})$: called the "First symplectic invariant"

The Equations of motion are rewritten as $\left({}' \equiv \frac{\mathrm{d}}{\mathrm{d}r}\right)$

$$\delta g_{tt} : \quad 0 = \mathrm{e}^{2A}\Big[\frac{1}{r^2}(1 - \mathrm{e}^{-2(A+C)}) + \frac{2}{r}(A' + 3C') + C'(2A' + 3C') + 2C''\Big] + \frac{\mathrm{e}^{-4C}}{r^4} I_1 + V$$

$$\delta g_{rr} : \quad 0 = \mathrm{e}^{2A}\Big[\frac{1}{r^2}(1 - \mathrm{e}^{-2(A+C)}) + \frac{2}{r}(A' + C') + C'(2A' + C')\Big] + \frac{\mathrm{e}^{-4C}}{r^4} I_1 + V$$

$$\delta g_{\theta\theta}, \delta g_{\phi\phi} : \quad 0 = \mathrm{e}^{2A}\Big[\frac{2}{r}(A' + C') + 2(A')^2 + C'(2A' + C') + A'' + C''\Big] - \frac{\mathrm{e}^{-4C}}{r^4} I_1 + V$$

$$\delta \mathrm{t}^a : \quad 0 = \frac{\mathrm{e}^{-4C}}{r^4}\frac{\partial I_1}{\partial \mathrm{t}^a} + \frac{\partial V}{\partial \mathrm{t}^a} \quad \longrightarrow \quad \frac{\partial I_1}{\partial \mathrm{t}^a} = 0 = \frac{\partial V}{\partial \mathrm{t}^a}$$

$$\delta \varphi : \quad 0 = 2V_{\mathsf{NS}} + 4V_{\mathsf{R}} \quad \longrightarrow \quad V = \frac{1}{2}V_{\mathsf{NS}} = -V_{\mathsf{R}}$$

$$\delta \xi^0 : \quad 0 = \mathrm{e}^{4\varphi} e_{\Lambda 0} (\mathrm{Im}\mathcal{N})^{-1|\Lambda\Sigma}\big[e_{\mathsf{R}\Sigma} - e_{\Sigma 0}\xi^0 + e_{\Sigma}{}^0 \widetilde{\xi}_0\big] \quad \longrightarrow \quad e_{\mathsf{R}\Sigma} - e_{\Sigma 0}\xi^0 + e_{\Sigma}{}^0 \widetilde{\xi}_0 = 0$$

$$\delta \widetilde{\xi}_0 : \quad 0 = -\mathrm{e}^{4\varphi} e_{\Lambda}{}^0 (\mathrm{Im}\mathcal{N})^{-1|\Lambda\Sigma}\big[e_{\mathsf{R}\Sigma} - e_{\Sigma 0}\xi^0 + e_{\Sigma}{}^0 \widetilde{\xi}_0\big]$$

We can solve $C(r)$ and $A(r)$ in terms of I_1, V and constants of integration.

We can solve $C(r)$ and $A(r)$ in terms of I_1, V and constants of integration $\{a_i, c_i\}$:

$$\mathrm{d}s^2 = -\,\mathrm{e}^{2A(r)}\mathrm{d}t^2 + \mathrm{e}^{-2A(r)}\mathrm{d}r^2 + \mathrm{e}^{2C(r)}\, r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$

$$\mathrm{e}^{2C(r)} = \frac{\mathrm{e}^{2c_2}}{r^2}\left(c_1 r + 1\right)^2$$

$$\mathrm{e}^{2A(r)} = \frac{\mathrm{e}^{-2c_2}}{(c_1)^2} - \frac{2a_1}{c_1(c_1 r + 1)} + \frac{\mathrm{e}^{4c_2} I_1}{(c_1)^2(c_1 r + 1)^2} - \frac{V}{3(c_1)^2}(c_1 r + 1)^2 \equiv 1 - \frac{2\eta}{r_{\text{new}}} + \frac{\mathcal{Z}^2}{r_{\text{new}}^2} - \frac{\Lambda}{3} r_{\text{new}}^2$$

Choosing $c_1 r + 1 \equiv r_{\text{new}}$ (and $c_1 \equiv 1$), we can read the "Black Hole" information:

$\mathrm{e}^{-2c_2} \equiv 1$:	scalar curvature of S^2
$a_1 \equiv \eta$:	mass parameter
$I_1 \equiv \mathcal{Z}^2$:	square of the charges
$V \equiv \Lambda$:	cosmological constant $(\Lambda = -3/\ell^2)$

$$\eta \geq \eta_0 = \frac{\ell}{3\sqrt{6}}\left(\sqrt{1 + 12\mathcal{Z}^2\ell^{-2}} + 2\right)\left(\sqrt{1 + 12\mathcal{Z}^2\ell^{-2}} - 1\right)^{1/2}$$ (condition of the regular horizon)

The remaining task is to find field configurations $\{\mathfrak{t}^a, A_\mu^\Lambda, \varphi, a, \xi^0, \widetilde{\xi}_0\}$

which formulate charged BH information $\{I_1, V\}$.

Contents

Covariantly constant condition on UHM ($\underline{\varphi}$, $\underline{\xi}^0$, $\underline{\widetilde{\xi}}_0$: asymptotic constant values in the vacuum):

$$0 = \partial_\mu \varphi \qquad \rightarrow \qquad \varphi(x) = \underline{\varphi}$$

$$0 = D_\mu \xi^0 = \partial_\mu \xi^0 - e_\Lambda{}^0 A^\Lambda_\mu \qquad \rightarrow \qquad \xi^0(x) = e_\Lambda{}^0 \int^x A^\Lambda_\mu \, \mathrm{d}x'^\mu + \underline{\xi}^0$$

$$0 = D_\mu \widetilde{\xi}_0 = \partial_\mu \widetilde{\xi}_0 - e_{\Lambda 0} A^\Lambda_\mu \qquad \rightarrow \qquad \widetilde{\xi}_0(x) = e_{\Lambda 0} \int^x A^\Lambda_\mu \, \mathrm{d}x'^\mu + \underline{\widetilde{\xi}}_0$$

Constantness condition of $e_{\mathrm{R}\Lambda}$ from EoMs for ξ^0, $\widetilde{\xi}_0$:

$$e_{\mathrm{R}\Lambda} = -e_{\Lambda 0} \xi^0 + e_\Lambda{}^0 \widetilde{\xi}_0 = \left(e_\Lambda{}^0 e_{\Sigma 0} - e_{\Lambda 0} e_\Sigma{}^0 \right) \int^x A^\Sigma_\mu \, \mathrm{d}x'^\mu + \left(e_{\Lambda 0} \underline{\xi}^0 - e_\Lambda{}^0 \underline{\widetilde{\xi}}_0 \right)$$

$$\boxed{0 = e_\Lambda{}^0 e_{\Sigma 0} - e_{\Lambda 0} e_\Sigma{}^0}$$

This is nothing but the consistency condition on 6D internal space $\mathcal{M}_6$!

In addition, the same EoMs tell that $\partial V_{\mathrm{R}} / \partial \mathrm{t}^a = 0$ and $\boxed{V_{\mathrm{R}} = 0 = -V = -\frac{1}{2} V_{\mathrm{NS}}}$

which implies that the spacetime has no cosmological constant!!

Covariantly constant condition on UHM ($\underline{a}$: asymptotic constant value in the vacuum):

$$0 \;=\; D_{\mu} a \;=\; \partial_{\mu} a - (2 e_{\mathsf{R}\Lambda} - e_{\Lambda 0} \xi^0 + e_{\Lambda}{}^0 \widetilde{\xi}_0) A_{\mu}^{\Lambda} \;\rightarrow\; \partial_{\mu} a \;=\; e_{\mathsf{R}\Lambda} A_{\mu}^{\Lambda}$$

$$\therefore \quad a(x) \;=\; e_{\mathsf{R}\Lambda} \int^x A_{\mu}^{\Lambda} \, \mathrm{d}x'^{\mu} + \underline{a}$$

Summary

Asymptotic constant values $\{\underline{\varphi}, \underline{\xi}^0, \underline{\widetilde{\xi}}_0, \underline{a}\}$ will be fixed in the vacuum.

RR-flux charges are fixed by the geometric flux charges.

The consistency condition of the geometric flux charges are derived.

$$e_{\Lambda}{}^0 e_{\Sigma 0} - e_{\Lambda 0} e_{\Sigma}{}^0 = 0 \qquad e_{\mathsf{R}\Lambda} = e_{\Lambda 0} \underline{\xi}^0 - e_{\Lambda}{}^0 \underline{\widetilde{\xi}}_0$$

$$\xi^0(x) = e_{\Lambda}{}^0 \int^x A_1^{\Lambda} + \underline{\xi}^0 \qquad \widetilde{\xi}_0(x) = e_{\Lambda 0} \int^x A_1^{\Lambda} + \underline{\widetilde{\xi}}_0 \qquad a(x) = e_{\mathsf{R}\Lambda} \int^x A_1^{\Lambda} + \underline{a}$$

Asymptotically flat solution: $V = 0$

The configuration of UHM is universal.

Prepotential of VMs will tell us the existence of solutions..

Contents

Let us consider the following three models

associated with the internal coset spaces $\mathcal{M}_6$ with $SU(3)$-structure:

$$\mathrm{T}^3\text{-model}: \quad \text{single modulus model with } \mathcal{F} = \frac{X^1 X^1 X^1}{X^0} \quad \longleftarrow \quad \mathcal{M}_6 = \frac{G_2}{SU(3)}$$

$$\mathrm{ST}^2\text{-model}: \quad \text{two moduli model with } \mathcal{F} = \frac{X^1 X^2 X^2}{X^0} \quad \longleftarrow \quad \mathcal{M}_6 = \frac{Sp(2)}{S[U(2) \times U(1)]}$$

$$\mathrm{STU}\text{-model}: \quad \text{three moduli model with } \mathcal{F} = \frac{X^1 X^2 X^3}{X^0} \quad \longleftarrow \quad \mathcal{M}_6 = \frac{SU(3)}{U(1) \times U(1)}$$

Notice: We consider general situations with non-vanishing ${e_\Lambda}^0$ whilst each coset space has the vanishing ${e_\Lambda}^0$.

The prepotential $\mathcal{F} = (X^1)^3/X^0$ gives the concrete description of everything!

$$\mathsf{t} \equiv \frac{X^1}{X^0}, \quad K_{\mathsf{V}} = -\log\left[\mathrm{i}(\mathsf{t} - \bar{\mathsf{t}})^3\right], \quad g_{\mathsf{t}\bar{\mathsf{t}}} = -\frac{3}{(\mathsf{t} - \bar{\mathsf{t}})^2}, \quad R^{\mathsf{t}}{}_{\mathsf{t}\mathsf{t}\bar{\mathsf{t}}} = -\partial_{\bar{\mathsf{t}}}(g^{\mathsf{t}\bar{\mathsf{t}}}\partial_{\mathsf{t}} g_{\mathsf{t}\bar{\mathsf{t}}})$$

$$V = \frac{4\mathrm{i}\mathrm{e}^{2\varphi}}{3(\mathsf{t} - \bar{\mathsf{t}})^3}\Big[3E_0\overline{E}_0 + 3E_0\overline{E}_1\mathsf{t} + 3\overline{E}_0E_1\bar{\mathsf{t}} + E_1\overline{E}_1(2\mathsf{t}^2 - \mathsf{t}\bar{\mathsf{t}} + 2\bar{\mathsf{t}}^2)\Big] + V_{\mathsf{R}}$$

(For a simple expression, we introduce $E_\Lambda \equiv e_{\Lambda 0} + \mathrm{i}e_\Lambda{}^0$).

Introduce another useful expression:

$$\mathcal{E}_{\Lambda\Sigma} \equiv e_{\Lambda 0}e_{\Sigma 0} + e_\Lambda{}^0 e_\Sigma{}^0, \quad \mathcal{C}_{\Lambda\Sigma} \equiv e_\Lambda{}^0 e_{\Sigma 0} - e_{\Lambda 0}e_\Sigma{}^0 \ (= 0)$$

The solution of $0 = \partial_{\mathsf{t}} V = \partial_{\mathsf{t}} V_{\mathsf{NS}} + \partial_{\mathsf{t}} V_{\mathsf{R}}$ is

$$\mathsf{t}_* = -\frac{\mathcal{E}_{01}}{\mathcal{E}_{11}} - \mathrm{i}\frac{3\mathcal{C}_{01}}{\mathcal{E}_{11}} \quad \longrightarrow \quad V_{\mathsf{NS}*} = -\frac{4\mathrm{e}^{2\varphi}}{27}\frac{\mathcal{E}_{11}}{\mathcal{C}_{01}}$$

or

$$\mathsf{t}_* = -\frac{\mathcal{E}_{01}}{\mathcal{E}_{11}} + \mathrm{i}\frac{3\mathcal{C}_{01}}{5\mathcal{E}_{11}} \quad \longrightarrow \quad V_{\mathsf{NS}*} = \frac{50\mathrm{e}^{2\varphi}}{27}\frac{\mathcal{E}_{11}}{\mathcal{C}_{01}}$$

Both are singular!! (i.e., K_{V} and $R^{\mathsf{t}}{}_{\mathsf{t}\mathsf{t}\bar{\mathsf{t}}}$ are ill-defined.)

If we set $e_\Lambda{}^0 = 0$ ($\mathcal{M}_6 = G_2/SU(3)$) $\longrightarrow$ ill-defined, singular charged solutions.

Prepotentials define diagonal Kähler metrics $g_{a\bar{b}} = \partial_a \partial_{\bar{b}} K_{\mathsf{V}}$:

$$\mathrm{ST}^2: \quad K_{\mathsf{V}} = -\log\big[-\mathrm{i}(\mathfrak{s}-\bar{\mathfrak{s}})(\mathfrak{t}-\bar{\mathfrak{t}})^2\big], \qquad \mathfrak{s} \equiv \frac{X^1}{X^0}, \ \mathfrak{t} \equiv \frac{X^2}{X^0}$$

$$\mathrm{STU}: \quad K_{\mathsf{V}} = -\log\big[-\mathrm{i}(\mathfrak{s}-\bar{\mathfrak{s}})(\mathfrak{t}-\bar{\mathfrak{t}})(\mathfrak{u}-\bar{\mathfrak{u}})\big], \quad \mathfrak{s} \equiv \frac{X^1}{X^0}, \ \mathfrak{t} \equiv \frac{X^2}{X^0}, \ \mathfrak{u} \equiv \frac{X^3}{X^0}$$

These models have extrema $\partial_a V_{\mathsf{NS}} = 0$ only at $\mathfrak{s}-\bar{\mathfrak{s}} = 0 = \mathfrak{t}-\bar{\mathfrak{t}} = \mathfrak{u}-\bar{\mathfrak{u}}$

under the condition $\mathcal{C}_{\Lambda\Sigma} = 0$ from the UHM sector.

↓

Each extrema has ill-defined curvature value.

↓

Singular charged solutions!

Even if $e_{\Lambda}{}^0 = 0$, singular charged solutions.

All models with cubic prepotentials have singular charged solutions

under the covariantly constant conditions..

This is “consistent” with the fact that AdS vacua appear in coset space compactifications.

Consider a single modulus model with $\mathcal{F} = -\mathrm{i} X^0 X^1$.

(I do not know its 10D origin, or the corresponding $\mathcal{M}_6$. Do you know?)

Regard $\{e_{\Lambda 0}, e_{\Lambda}{}^0, e_{\mathsf{R}\Lambda}\}$ as purely the 4D parameters of gauging.

$$\mathfrak{t} \equiv \frac{X^1}{X^0}, \quad K_{\mathsf{V}} = -\log\big[2(\mathfrak{t} + \bar{\mathfrak{t}})\big], \quad g_{\mathfrak{t}\bar{\mathfrak{t}}} = \frac{1}{(\mathfrak{t}+\bar{\mathfrak{t}})^2}, \quad R^{\mathfrak{t}}{}_{\mathfrak{t}\mathfrak{t}\bar{\mathfrak{t}}} = -\partial_{\bar{\mathfrak{t}}}(g^{\mathfrak{t}\bar{\mathfrak{t}}}\partial_{\mathfrak{t}} g_{\mathfrak{t}\bar{\mathfrak{t}}})$$

$$V_{\mathsf{NS}} = \frac{2\mathrm{e}^{2\varphi}}{\mathfrak{t}+\bar{\mathfrak{t}}}\Big[E_0\overline{E}_0 + (E_0\overline{E}_1 + 2\overline{E}_0 E_1)\mathfrak{t} + (2E_0\overline{E}_1 + \overline{E}_0 E_1)\bar{\mathfrak{t}} + E_1\overline{E}_1 \mathfrak{t}\bar{\mathfrak{t}}\Big]$$

The solution of $\partial_{\mathfrak{t}} V_{\mathsf{NS}} = 0$ is

$$\mathfrak{t}_* = \frac{\sqrt{(\mathcal{E}_{01})^2}}{\mathcal{E}_{11}} + \mathrm{i}\,\frac{\mathcal{C}_{01}}{\mathcal{E}_{11}} \qquad \longrightarrow \qquad V_{\mathsf{NS}*} = 2\mathrm{e}^{2\varphi}\Big[-3\mathcal{E}_{01} + \sqrt{(\mathcal{E}_{01})^2}\Big]$$

As far as $\mathcal{E}_{01} \neq 0$, values of K_{V}, $R^{\mathfrak{t}}{}_{\mathfrak{t}\mathfrak{t}\bar{\mathfrak{t}}}$ are regular:

if $\mathcal{E}_{01} > 0$: $\Lambda = \frac{1}{2}V_{\mathsf{NS}*} = -2\mathrm{e}^{2\varphi}\mathcal{E}_{01} < 0$ → AdS-BH

if $\mathcal{E}_{01} < 0$: $\Lambda = \frac{1}{2}V_{\mathsf{NS}*} = -4\mathrm{e}^{2\varphi}\mathcal{E}_{01} > 0$ → dS-BH

However, we have already known that $V = \frac{1}{2}V_{\mathsf{NS}} = -V_{\mathsf{R}} = 0$ in the analysis of UHM.

Conflict!

We find a singular solution again!

Impossible to find a regular solution of $\partial_\varphi V = 0$**,** $\partial_{\xi^{\mathbf{0}}} V = 0$ **and** $\partial_{\mathfrak{t}^a} V = 0$ **under** $D_\mu \xi^{\mathbf{0}} = 0$**??**

Contents

- ☑ Studied charged solutions in gauged SUGRA with VMs and UHM.
- ☑ Imposed the covariantly constant condition.
- ☑ Understood that it is impossible to acquire non-vanishing scalar potential.
- ☑ Found singular solutions with cubic prepotentials.
- ☑ Found again a singular solution with non-cubic prepotential.

- ☑ No regular solution at all? (It seems quite strange..)
- ☑ What's happen if we consider a stationary black hole?

Improvement

arXiv:1108.1113

Contents

- 4D $\mathcal{N} = 2$ Gauged SUGRA from 10D type IIA
- Vacua
- Ansatze
- Black holes
- Summary and Discussions

Careful analysis on Coset space $G_2/SU(3)$ D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

✓ nearly-Kähler (almost complex geometry)

✓ NSNS-sector : torsion and H-flux

✓ RR-sector : 2-, 4-form and Romans' mass (0-form)

✓ 1 VM with cubic prepotential $\mathcal{F} = \dfrac{X^1 X^1 X^1}{X^0}$

✓ 1 UHM (no other HMs)

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nilmanifolds and solvmanifolds: M. Graña, R. Minasian, M. Petrini and A. Tomasiello [hep-th/0609124]

coset spaces with $SU(3)$- or $SU(2)$-structure: P. Koerber, D. Lüst and D. Tsimpis [arXiv:0804.0614]

a pair of $SU(3)$-structures with $(m^{\Lambda I}, m^{\Lambda}{}_I)$: D. Gaiotto and A. Tomasiello [arXiv:0904.3959]

10D type IIA on $G_2/SU(3)$ with fluxes

↓

4D $\mathcal{N}=2$ abelian gauged SUGRA with B-field ($\Lambda = 0, 1$ and $\boldsymbol{\xi}^{\mathbf{0}} \equiv (\xi^0, \widetilde{\xi}_0)^{\mathrm{T}}$)

$$S = \int \Big[\frac{1}{2} R\,(*1) + \frac{1}{2} \mu_{\Lambda\Sigma}\, F^{\Lambda} \wedge *F^{\Sigma} + \frac{1}{2} \nu_{\Lambda\Sigma}\, F^{\Lambda} \wedge F^{\Sigma} - g_{\mathfrak{t}\bar{\mathfrak{t}}}\, \mathrm{d}\mathfrak{t} \wedge *\mathrm{d}\bar{\mathfrak{t}}$$
$$- \mathrm{d}\varphi \wedge *\mathrm{d}\varphi - \frac{\mathrm{e}^{-4\varphi}}{4} \mathrm{d}B \wedge *\mathrm{d}B - \frac{\mathrm{e}^{2\varphi}}{2} \Big(D\xi^0 \wedge *D\xi^0 + D\widetilde{\xi}_0 \wedge *D\widetilde{\xi}_0 \Big) + \mathrm{d}B \wedge \xi^0\, \mathrm{d}\widetilde{\xi}_0$$
$$+ \mathrm{d}B \wedge \big(e_{\mathsf{R}\Lambda} - e_{\Lambda 0}\, \xi^0 \big) A^{\Lambda} - \frac{1}{2} m_{\mathsf{R}}^{\Lambda}\, e_{\mathsf{R}\Lambda}\, B \wedge B - V\,(*1) \Big]$$

- $g_{\mu\nu}$, $\mathfrak{t}$, $B_{\mu\nu}$, φ; $({e_{\Lambda}}^0, e_{\Lambda 0})$: NS-NS sector
- A_{μ}^{Λ}, ξ^0, $\widetilde{\xi}_0$; $(m_{\mathsf{R}}^{\Lambda}, e_{\mathsf{R}\Lambda})$: R-R sector
- GM : $(g_{\mu\nu}, A_{\mu}^0)$, VM : $(A_{\mu}^a, \mathfrak{t})$, UHM → TM : $(\varphi, B_{\mu\nu}, \xi^0, \widetilde{\xi}_0)$
- $D\xi^0 = \mathrm{d}\xi^0 - {e_{\Lambda}}^0 A_1^{\Lambda}$, $\quad D\widetilde{\xi}_0 = \mathrm{d}\widetilde{\xi}_0 - e_{\Lambda 0} A_1^{\Lambda}$
- $F_2^{\Sigma} = \mathrm{d}A_1^{\Sigma} + m_{\mathsf{R}}^{\Sigma} B_2$
- $V(\mathfrak{t}, \varphi, \xi^0) = V_{\mathsf{NS}}(\mathfrak{t}, \varphi) + V_{\mathsf{R}}(\mathfrak{t}, \varphi, \xi^0)$

Precise data on $G_2/SU(3)$:

$$e_{10} \neq 0,\ m_{\mathsf{R}}^0 \neq 0,\ e_{\mathsf{R}0} \neq 0$$
$${e_{\Lambda}}^0 = 0 = e_{00}$$
$$m_{\mathsf{R}}^1 = 0 = e_{\mathsf{R}1}$$

$\mu_{\Lambda\Sigma} \equiv \mathrm{Im}\mathcal{N}_{\Lambda\Sigma},\ \nu_{\Lambda\Sigma} \equiv \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}$

D. Cassani [arXiv:0804.0595]

$$R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} = \frac{1}{4}g_{\mu\nu}\,\mu_{\Lambda\Sigma}F^{\Lambda}_{\rho\sigma}F^{\Sigma\rho\sigma} - \mu_{\Lambda\Sigma}F^{\Lambda}_{\mu\rho}F^{\Sigma}_{\nu\sigma}\,g^{\rho\sigma} - g_{\mu\nu}\,g_{\mathrm{t}\bar{\mathrm{t}}}\partial_\rho \mathrm{t}\partial^\rho\bar{\mathrm{t}} + 2g_{\mathrm{t}\bar{\mathrm{t}}}\,\partial_\mu \mathrm{t}\partial_\nu\bar{\mathrm{t}}$$

$$-g_{\mu\nu}\,\partial_\rho\varphi\partial^\rho\varphi + 2\partial_\mu\varphi\partial_\nu\varphi - \frac{\mathrm{e}^{-4\varphi}}{24}g_{\mu\nu}\,H_{\rho\sigma\lambda}H^{\rho\sigma\lambda} + \frac{\mathrm{e}^{-4\varphi}}{4}H_{\mu\rho\sigma}H_{\nu}{}^{\rho\sigma} \qquad (\delta g_{\mu\nu})$$

$$-\frac{\mathrm{e}^{2\varphi}}{2}g_{\mu\nu}\Big(D_\rho\xi^0 D^\rho\xi^0 + D_\rho\widetilde{\xi}_0 D^\rho\widetilde{\xi}_0\Big) + \mathrm{e}^{2\varphi}\Big(D_\mu\xi^0 D_\nu\xi^0 + D_\mu\widetilde{\xi}_0 D_\nu\widetilde{\xi}_0\Big) - g_{\mu\nu}V\,,$$

$$0 = \frac{1}{\sqrt{-g}}\partial_\mu\Big(\sqrt{-g}\,\mu_{\Lambda\Sigma}F^{\Sigma\mu\sigma}\Big) - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}}\partial_\mu\Big(\nu_{\Lambda\Sigma}F^{\Sigma}_{\nu\rho}\Big) + \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}}\partial_\mu B_{\nu\rho}(e_{\mathsf{R}\Lambda} - \xi^0 e_{\Lambda 0}) - \mathrm{e}^{2\varphi}Q_{\Lambda\mathbf{0}}D^\sigma\xi^{\mathbf{0}}\,, \qquad (\delta A^{\Lambda}_{\mu})$$

$$0 = \frac{1}{\sqrt{-g}}\partial_\mu\Big(\sqrt{-g}\,g_{c\bar{b}}\,g^{\mu\nu}\partial_\nu\bar{\mathrm{t}}\Big) + \frac{1}{4}\partial_{\mathrm{t}}(\mu_{\Lambda\Sigma})F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}}\partial_{\mathrm{t}}(\nu_{\Lambda\Sigma})F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma} - \partial_{\mathrm{t}}g_{\mathrm{t}\bar{\mathrm{t}}}\,\partial_\mu\mathrm{t}\partial^\mu\bar{\mathrm{t}} - \partial_{\mathrm{t}}V\,, \qquad (\delta\mathrm{t})$$

$$0 = \frac{2}{\sqrt{-g}}\partial_\mu\Big(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\varphi\Big) + \frac{\mathrm{e}^{4\varphi}}{6}H_{\mu\nu\rho}H^{\mu\nu\rho} - \mathrm{e}^{2\varphi}\Big(D_\mu\xi^0 D^\mu\xi^0 + D_\mu\widetilde{\xi}_0 D^\mu\widetilde{\xi}_0\Big) - \partial_\varphi V\,, \qquad (\delta\varphi)$$

$$0 = \frac{1}{\sqrt{-g}}\partial_\mu\Big(\mathrm{e}^{-4\varphi}\sqrt{-g}H^{\mu\rho\sigma}\Big) + \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}\Big[D_\mu\xi^{\mathbf{0}}(\mathbb{C}_{\mathsf{H}})_{\mathbf{00}}D_\nu\xi^{\mathbf{0}} + (e_{\mathsf{R}\Lambda} - \xi^0 e_{\Lambda 0})F^{\Lambda}_{\mu\nu}\Big] \qquad (\delta B_{\mu\nu})$$

$$+2m^{\Lambda}_{\mathsf{R}}\mu_{\Lambda\Sigma}F^{\Sigma\rho\sigma} - \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}m^{\Lambda}_{\mathsf{R}}\nu_{\Lambda\Sigma}F^{\Sigma}_{\mu\nu}\,,$$

$$0 = -\frac{2}{\sqrt{-g}}\partial_\mu\Big(\sqrt{-g}\,\mathrm{e}^{2\varphi}g^{\mu\nu}D_\nu\xi^{\mathbf{0}}\Big) + \frac{\partial V}{\partial\xi^{\mathbf{0}}} - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}}\partial_\mu B_{\nu\rho}D_\sigma\xi^{\mathbf{0}}(\mathbb{C}_{\mathsf{H}})_{\mathbf{00}}\,. \qquad (\delta\xi^{\mathbf{0}})$$

Contents

Vacuum I : $\mathcal{N} = 1$

$$t_* = -\frac{\pm 1 + i\sqrt{15}}{2} \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3}, \quad \xi_*^0 = -\frac{2}{5} \left[\frac{2\sqrt{3}\, m_R^0 (e_{R0})^2}{5\, e_{10}} \right]^{1/3}, \quad \exp(\varphi_*) = \frac{4}{3} \left[\frac{\sqrt{5}\, e_{10}}{\sqrt{3}\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{5\sqrt{5}}{2} \left[\frac{5\,(e_{10})^4}{2\sqrt{3}\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{I}} < 0$$

Vacuum II : $\mathcal{N} = 0$

$$t_* = \big(\pm 1 - i\sqrt{3} \big) \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3}, \quad \xi_*^0 = \left[\frac{9\, m_R^0 (e_{R0})^2}{25\, e_{10}} \right]^{1/3}, \quad \exp(\varphi_*) = \frac{2}{3} \left[\frac{25\, e_{10}}{\sqrt{3}\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{80}{27} \left[\frac{25\,(e_{10})^4}{\sqrt{3}\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{II}} < 0$$

Vacuum III : $\mathcal{N} = 0$

$$t_* = -i \left[\frac{12}{\sqrt{5}\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3}, \quad \xi_*^0 = 0\,, \quad \exp(\varphi_*) = \sqrt{5} \left[\frac{5\, e_{10}}{18\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{25\sqrt{5}}{6} \left[\frac{5\,(e_{10})^4}{18\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{III}} < 0$$

Note: $m_R^0 > 0$; $\widetilde{\xi}_0$ is not fixed ; $\Lambda_{\text{c.c.}}^{\text{II}} < \Lambda_{\text{c.c.}}^{\text{I}} < \Lambda_{\text{c.c.}}^{\text{III}}$

D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

Contents

☞ Consider spacetime metric (extremal, static, spherically symmetric $\to \mathrm{AdS}_2 \times S^2$)

$$\mathrm{d}s^2 = -\mathrm{e}^{2A(r)}\mathrm{d}t^2 + \mathrm{e}^{-2A(r)}\mathrm{d}r^2 + \mathrm{e}^{2C(r)}r^2\big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\big)$$

☞ Impose (covariantly) constant condition

$$0 \equiv \partial_\mu \mathfrak{t}\,, \quad 0 \equiv \partial_\mu \varphi\,, \quad 0 \equiv D_\mu \xi^0\,, \quad 0 \equiv D_\mu \widetilde{\xi}_0\,, \quad 0 \equiv \partial_\mu B_{\nu\rho}$$

☞ Define electromagnetic charges

$$p^\Lambda \equiv \frac{1}{4\pi}\int_{S^2} F_2^\Lambda\,, \quad q_\Lambda \equiv \frac{1}{4\pi}\int_{S^2} \widetilde{F}_{\Lambda 2}$$

$$I_1 \equiv -\frac{1}{2}\Big[p^\Lambda \mu_{\Lambda\Sigma}\, p^\Sigma + (q_\Lambda - \nu_{\Lambda\Gamma}p^\Gamma)(\mu^{-1})^{\Lambda\Sigma}(q_\Sigma - \nu_{\Sigma\Delta}p^\Delta)\Big]$$

$$\widetilde{F}_{\Lambda 2} \equiv \nu_{\Lambda\Sigma}F_2^\Sigma + \mu_{\Lambda\Sigma}(*F_2^\Sigma)$$

The equation of motion for $g_{\mu\nu}$:

$$\delta g_{tt} - \delta g_{rr} : \quad \mathrm{e}^{2C(r)} = \frac{\mathrm{e}^{2c_2}}{r^2}(c_1 r + 1)^2$$

$$\delta g_{\theta\theta}, \delta g_{\phi\phi} : \quad \mathrm{e}^{2A(r)} = \mathrm{e}^{-4c_2} \frac{6\, I_1 - \mathrm{e}^{4c_2}(c_1 r + 1)}{3\, c_1^2 (c_1 r + 1)^2} \Big[(c_1 r + 1)^3 V + 6\, c_1 \big\{ a_1 - c_1 a_2 (c_1 r + 1) \big\} \Big]$$

$$\delta g_{rr} : \quad a_2 = \frac{1}{2(c_1)^2} \mathrm{e}^{2c_2}$$

$C(r)$ and $A(r)$ are expressed in terms of I_1, V and constants of integration $\{a_i, c_i\}$

$$\mathrm{e}^{2A(r)} = \frac{\mathrm{e}^{-2c_2}}{(c_1)^2} - \frac{2a_1}{c_1(c_1 r + 1)} + \frac{\mathrm{e}^{4c_2} I_1}{(c_1)^2 (c_1 r + 1)^2} - \frac{V}{3(c_1)^2}(c_1 r + 1)^2 \equiv 1 - \frac{2\eta}{r_{\text{new}}} + \frac{\mathcal{Z}^2}{r_{\text{new}}^2} - \frac{\Lambda_{\text{c.c.}}}{3} r_{\text{new}}^2$$

Choosing $c_1 r + 1 \equiv r_{\text{new}}$ (and $c_1 \equiv 1$), we can read the "Black Hole" information:

$\mathrm{e}^{-2c_2} \equiv 1$:	scalar curvature of S^2	$a_1 \equiv \eta$:	mass parameter
$I_1 \equiv \mathcal{Z}^2$:	square of charges	$V \equiv \Lambda_{\text{c.c.}}$:	cosmological constant

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The equations of motion for $\mathfrak{t}$, φ, ξ^0 :

$$\delta\mathfrak{t} : \quad 0 = \frac{\mathrm{e}^{-4C}}{r^4}\frac{\partial I_1}{\partial \mathfrak{t}} + \frac{\partial V}{\partial \mathfrak{t}} \quad \longrightarrow \quad \frac{\partial V}{\partial \mathfrak{t}} = 0 \text{ and } \frac{\partial I_1}{\partial \mathfrak{t}} = 0$$

$$\delta\varphi : \quad 0 = 2\,V_{\mathsf{NS}} + 4\,V_{\mathsf{R}} \quad \longrightarrow \quad V = V_{\mathsf{NS}} + V_{\mathsf{R}} = \frac{1}{2}V_{\mathsf{NS}} = -V_{\mathsf{R}}$$

$$\delta\xi^0 : \quad 0 = \frac{\partial V}{\partial \xi^0} = \cancel{\frac{\partial V_{\mathsf{NS}}}{\partial \xi^0}} + \frac{\partial V_{\mathsf{R}}}{\partial \xi^0} \qquad \left(0 = \frac{\partial V}{\partial \widetilde{\xi}_0} \text{ is trivial}\right)$$

Regular Solution

$$\{\mathfrak{t}\,,\ \xi^0\,,\ \varphi\,;\ V\}_{\mathsf{BHs}} = \{\mathfrak{t}_*\,,\ \xi^0_*\,,\ \varphi_*\,;\ \Lambda_{\mathsf{c.c.}}\}_{\mathsf{Vacua}}$$

constant in whole region

The equations of motion for $\mathfrak{t}$, $B_{\mu\nu}$:

$$\delta\mathfrak{t}: \quad 0 = \frac{\mathrm{e}^{-4C}}{r^4}\frac{\partial I_1}{\partial\mathfrak{t}} + \frac{\partial V}{\partial\mathfrak{t}} \qquad \longrightarrow \qquad \frac{\partial V}{\partial\mathfrak{t}} = 0 \text{ and } \frac{\partial I_1}{\partial\mathfrak{t}} = 0$$

$$\delta B_{\mu\nu}: \quad 0 = m_{\mathsf{R}}^{\Lambda}\,\mu_{\Lambda\Sigma}\Big(\frac{\sqrt{-g}}{2}\,\epsilon_{\mu\nu\rho\sigma}F^{\Sigma\rho\sigma}\Big) + m_{\mathsf{R}}^{\Lambda}\,\nu_{\Lambda\Sigma}\,F_{\mu\nu}^{\Sigma} - \big(e_{\mathsf{R}\Lambda} - e_{\Lambda 0}\,\xi^0\big)F_{\mu\nu}^{\Lambda}$$

$$\text{and} \qquad 0 = D_{\mu}\widetilde{\xi}_0 \qquad \longrightarrow \qquad 0 = [\partial_{\mu}, \partial_{\nu}]\,\widetilde{\xi}_0 = e_{\Lambda 0}\,F_{\mu\nu}^{\Lambda}$$

$$\text{with} \qquad F_{\theta\phi}^{\Lambda} = p^{\Lambda}\sin\theta\,, \quad F_{tr}^{\Lambda} = -\frac{1}{r_{\mathsf{new}}^2}(\mu^{-1})^{\Lambda\Sigma}\big(q_{\Sigma} - \nu_{\Sigma\Gamma}\,p^{\Gamma}\big)$$

Solve them to find an appropriate BH charge configuration

It turns out that all the charges are zero : $p^{\Lambda} = 0 = q_{\Lambda}$ (highly non-trivial)

$$\therefore \; I_1 \equiv \mathcal{Z}^2 = 0 \,, \quad F^{\Lambda}_{\mu\nu} = 0$$

✓ $0 = F^1_{\mu\nu} = 2\,\partial_{[\mu} A^1_{\nu]} + \not{m}^{1}_{\mathrm{R}} B_{\mu\nu} \;\rightarrow\; A^1_{\mu} = \partial_{\mu}\lambda \equiv 0$ (gauge-fixing)

✓ $0 = F^0_{\mu\nu} = 2\,\partial_{[\mu} A^0_{\nu]} + m^0_{\mathrm{R}} B_{\mu\nu} \;\rightarrow\; 2\,\partial_{[\mu} A^0_{\nu]} = -m^0_{\mathrm{R}} B_{\mu\nu} =$ (constant)

✓ $0 = D_{\mu}\widetilde{\xi}_0 = \partial_{\mu}\widetilde{\xi}_0 - \not{e}_{00}\, A^0_{\mu} - e_{10}\, \not{A}^{1}_{\mu} = \partial_{\mu}\widetilde{\xi}_0$ $(\because e_{00} = 0 = A^1_{\mu})$

✓ $\Lambda_{\text{c.c.}} \equiv V < 0$

$\eta \equiv a_1$ is still arbitrary

Schwarzschild-AdS Black Holes!

- Black holes from CY :

$$\boxed{\frac{\partial I_1}{\partial \mathfrak{t}} = 0}$$

Value of vector modulus $\mathfrak{t}$ is not fixed at infinity $\rightarrow$ attractor mechanism

BH charges govern value of fields at horizon

BH mass is given by BH charges

- Black holes from non-CY :

$$\boxed{\frac{\partial V}{\partial \mathfrak{t}} = 0} \quad \text{and} \quad \boxed{\frac{\partial I_1}{\partial \mathfrak{t}} = 0}$$

Value of vector modulus $\mathfrak{t}$ is (mostly) fixed at infinity $\rightarrow$ moduli stabilization

BH charges are governed by geometric- and RR-flux charges

BH mass is arbitrary

Contents

☑ Studied : 4D $\mathcal{N} = 2$ gauged SUGRA with VMs and TM(UHM) via flux compactification.

☑ Reconfirmed : Romans' mass is inevitable.

☑ Imposed : covariantly constant condition.

☑ Found : Schwarzschild-AdS BHs.

Different from cases of Calabi-Yau

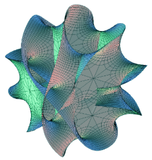
☑ Find charged AdS-BH solutions.

☑ Consider a stationary AdS-BH.

☑ Various directions!

Appendix

Calabi-Yau 3-fold $\mathcal{M}_{\text{CY}}$



Ricci-flat, torsionless, (compact) Kähler manifold with $SU(3)$ holonomy group

$$ds^2_{\text{10D}} = \eta_{\mu\nu}(x)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu + \widehat{g}_{mn}(x,y)\,\mathrm{d}y^m\,\mathrm{d}y^n$$

4D CY 3-fold

Invariant two-form J and three-form Ω on CY w.r.t. Levi-Civita connection:

$$\mathrm{d}J = \nabla_{[m}J_{np]} = 0, \quad \mathrm{d}\Omega = \nabla_{[m}\Omega_{npq]} = 0$$

This is suitable for $\frac{1}{4}$-SUSY condition with vanishing fields

$$\delta_{\text{SUSY}} \psi_{m\pm} = \nabla_m \, \varepsilon_{\pm}^{\text{(10D)}} = 0$$

$$\varepsilon_{+}^{\text{(10D)}} = \varepsilon_{1+}^{\text{(4D)}} \otimes \eta_{+}^{1} + (\text{c.c.}) \, , \quad \varepsilon_{-}^{\text{(10D)}} = \varepsilon_{2+}^{\text{(4D)}} \otimes \eta_{-}^{2} + (\text{c.c.})$$

$$(\varepsilon_{1,2+}^{\text{(4D)}})^* = \varepsilon_{1,2-}^{\text{(4D)}} \, , \quad (\eta_{-}^{1,2})^* = \eta_{+}^{1,2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ * \end{pmatrix} \; : \quad SU(3)\text{-invariant} \subset SU(4) \sim SO(6)$$

$$\eta_{\pm}^{1} = \eta_{\pm}^{2} \equiv \eta_{\pm} \, , \qquad J_{mn} = \mp \mathrm{i} \, \eta_{\pm}^{\dagger} \, \gamma_{mn} \, \eta_{\pm} ||\eta_{+}||^{-2} \, , \qquad \Omega = -\mathrm{i} \, \eta_{-}^{\dagger} \, \gamma_{mnp} \, \eta_{+} ||\eta_{+}||^{-2}$$

NS-NS fields in 10D are expanded around CY:

$$\phi(x,y) = \varphi(x)$$

$$\widehat{g}_{\mathfrak{m}\bar{\mathfrak{n}}}(x,y) = \mathrm{i}v^a(x)\,(\omega_a)_{\mathfrak{m}\bar{\mathfrak{n}}}(y), \quad \widehat{g}_{\mathfrak{m}\mathfrak{n}}(x,y) = \mathrm{i}\bar{z}^{\bar{\jmath}}(x)\left(\frac{(\overline{\chi}_{\bar{\jmath}})_{\mathfrak{m}\bar{\mathfrak{p}}\bar{\mathfrak{q}}}\Omega^{\bar{\mathfrak{p}}\bar{\mathfrak{q}}}{}_{\mathfrak{n}}}{||\Omega||^2}\right)(y)$$

$$\widehat{B}_2(x,y) = B_2(x) + b^a(x)\omega_a(y)$$

$$\mathfrak{t}^a \equiv b^a + \mathrm{i}v^a$$

Mirror Symmetry: exchange $(\mathfrak{t}^a, \bar{\mathfrak{t}}^{\bar{b}}) \leftrightarrow (z^i, \bar{z}^{\bar{\jmath}})$

R-R fields are:

$$\widehat{C}_1(x,y) = A_1^0(x)$$

$$\widehat{C}_3(x,y) = A_1^a(x)\omega_a(y) + \xi^I(x)\alpha_I(y) - \widetilde{\xi}_I(x)\beta^I(y)$$

cohomology class on CY	basis	degrees
$H^{(1,1)}$	ω_a	$a = 1,\ldots,h^{(1,1)}$
$H^{(0)} \oplus H^{(1,1)}$	$\omega_\Lambda = (1, \omega_a)$	$\Lambda = 0,1,\ldots,h^{(1,1)}$
$H^{(2,2)} \oplus H^{(6)}$	$\widetilde{\omega}^\Lambda = (\widetilde{\omega}^a, \frac{\mathrm{vol.}}{\lvert\mathrm{vol.}\rvert})$	
$H^{(2,1)}$	χ_i	$i = 1,\ldots,h^{(2,1)}$
$H^{(3)}$	(α_I, β^I)	$I = 0,1,\ldots,h^{(2,1)}$

$$\mathrm{d}\omega_\Lambda = 0 = \mathrm{d}\widetilde{\omega}^\Lambda$$

$$\mathrm{d}\alpha_I = 0 = \mathrm{d}\beta^I$$

10D Type IIA action $S^{(10D)}_{\rm IIA} = S_{\sf NS} + S_{\sf R} + S_{\sf CS}$:

$$S_{\sf NS} = \frac{1}{2}\int e^{-2\phi}\Big\{\widehat{R}*\mathbb{1} + 4 d\phi\wedge *d\phi - \frac{1}{2}\widehat{H}_3\wedge *\widehat{H}_3\Big\}$$

$$S_{\sf R} + S_{\sf CS} = -\frac{1}{4}\int\Big\{\widehat{F}_2\wedge *\widehat{F}_2 + (\widehat{F}_4 - \widehat{C}_1\wedge\widehat{F}_3)\wedge *(\widehat{F}_4 - \widehat{C}_1\wedge\widehat{F}_3)\Big\} - \frac{1}{4}\int\widehat{B}_2\wedge\widehat{F}_4\wedge\widehat{F}_4$$

↓

4D $\mathcal{N}=2$ ungauged SUGRA: Neither gauge couplings, Nor scalar potential

$$S^{(4D)} = \int\Big\{\frac{1}{2}R*\mathbb{1} - g_{a\bar{b}}\,d\mathfrak{t}^a\wedge *d\bar{\mathfrak{t}}^{\bar{b}} - h_{uv}\,dq^u\wedge *dq^v + \frac{1}{2}{\rm Im}\mathcal{N}_{\Lambda\Sigma}F_2^\Lambda\wedge *F_2^\Sigma + \frac{1}{2}{\rm Re}\mathcal{N}_{\Lambda\Sigma}F_2^\Lambda\wedge F_2^\Sigma\Big\}$$

gravitational multiplet	$g_{\mu\nu}, A_1^0$		
vector multiplet (VM)	$A_1^a, \mathfrak{t}^a, \bar{\mathfrak{t}}^{\bar{b}}$	$\mathfrak{t}^a \in {\sf SKG_V}$	mirror dual: ${\sf SKG_V} \leftrightarrow {\sf SKG_H}$
hypermultiplet (HM)	$z^i, \bar{z}^{\bar{\jmath}}, \xi^i, \widetilde{\xi}_j$	$z^i \in {\sf SKG_H}$	
universal hypermultiplet (UHM)	$\varphi, a, \xi^0, \widetilde{\xi}_0$	$a \leftrightarrow B_2$	Hodge dual

$\mathcal{HM}$ = Special QG

$$\underset{4n_{\sf H}+4}{\{q^u\}} = \underset{2n_{\sf H}({\sf SKG_H})}{\{z^i, \bar{z}^{\bar{\jmath}}\}} + \underset{2n_{\sf H}}{\{\xi^i, \widetilde{\xi}_j\}} + \underset{4\,({\sf UHM})}{\{\varphi, a, \xi^0, \widetilde{\xi}_0\}} = \underset{\sf SKG_H}{\{z^i, \bar{z}^{\bar{\jmath}}\}} + \{\varphi\} + \underset{\text{“Heisenberg”}}{\{a, \xi^I, \widetilde{\xi}_J\}}$$

non-CY 3-fold $\mathcal{M}_6$

vanishing Ricci 2-form, torsionful, (compact) non-Kähler manifold
with (a pair of) $SU(3)$-structure

$\mathrm{d}J \neq 0$ and/or $\mathrm{d}\Omega \neq 0$

- $\eta^1_\pm = \eta^2_\pm$ at any points on $\mathcal{M}_6$: $(J_{mn} = \mp \mathrm{i}\, \eta^\dagger_\pm \gamma_{mn} \eta_\pm ||\eta_+||^{-2},\ \Omega_{mnp} = -\mathrm{i}\, \eta^\dagger_- \gamma_{mnp} \eta_+ ||\eta_+||^{-2})$

$$\mathrm{d}J = \frac{3}{2} \mathrm{Im}(\overline{\mathcal{W}}_1 \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \,, \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

- $\eta^1_\pm \neq \eta^2_\pm$ at a certain point on $\mathcal{M}_6$:

$$\eta^2_+ = c_{\parallel}(y) \eta^1_+ + c_{\perp}(y) (v + \mathrm{i} v')_m \gamma^m \eta^1_-$$

$$(v - \mathrm{i} v')^m \equiv \eta^{1\dagger}_+ \gamma^m \eta^2_- \,, \quad J = j + v \wedge v' \,, \quad \Omega = \omega \wedge (v + \mathrm{i} v')$$

They are given by 1/4-SUSY condition with non-vanishing background fields:

$$\delta\psi_{m\pm} = \nabla_m \varepsilon^{(\text{10D})}_\pm + (\text{NS-fluxes}) + (\text{RR-fluxes}) = 0$$

Flux fields behave as torsion.

$$d J = \frac{3}{2}\,\mathrm{Im}(\overline{\mathcal{W}}_1 \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3\,, \qquad d\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

complex ($\frac{1}{4}$-SUSY Minkowski$_{1,3}$)	hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = 0$
	balanced	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = 0$
	special hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	Kähler	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	conformally CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 + 2\mathcal{W}_5 = 0$
almost complex ($\frac{1}{4}$-SUSY AdS$_4$)	symplectic	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	nearly Kähler	$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	almost Kähler	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	quasi Kähler	$\mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	semi Kähler	$\mathcal{W}_4 = \mathcal{W}_5 = 0$
	half-flat	$\mathrm{Im}\mathcal{W}_1 = \mathrm{Im}\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$

Tensor (Fierz) products of $SU(3)$-invariant spinors on 6D internal space $\mathcal{M}_6$:

$$\Phi_+ \equiv 8\,\mathrm{e}^{-b}\,\eta_+^1 \otimes \eta_+^{2\dagger} = X^\Lambda \omega_\Lambda - \mathcal{F}_\Lambda \widetilde{\omega}^\Lambda \qquad \text{even polyform} \qquad (\sim \mathrm{e}^{-b+\mathrm{i}J})$$

$$\Phi_- \equiv 8\,\mathrm{e}^{-b}\,\eta_+^1 \otimes \eta_-^{2\dagger} = Z^I \alpha_I - \mathcal{G}_I \beta^I \qquad \text{odd polyform} \qquad (\sim -\mathrm{i}\Omega)$$

$\Phi_\pm$: called the pure spinors on $T\mathcal{M}_6 \oplus T^*\mathcal{M}_6$

with the even/odd basis-forms and the symplectic metrics $\mathbb{C}_{\mathsf{V,H}}$:

$$\Sigma_+^{\boldsymbol{\Lambda}} \equiv \begin{pmatrix} \widetilde{\omega}^\Lambda \\ \omega_\Lambda \end{pmatrix}, \qquad \int_{\mathcal{M}_6} \langle \Sigma_+^{\boldsymbol{\Lambda}}, \Sigma_+^{\boldsymbol{\Sigma}} \rangle = (\mathbb{C}_{\mathsf{V}}^{-1})^{\boldsymbol{\Lambda\Sigma}}$$

$$\Sigma_-^{\boldsymbol{I}} \equiv \begin{pmatrix} \beta^I \\ \alpha_I \end{pmatrix}, \qquad \int_{\mathcal{M}_6} \langle \Sigma_-^{\boldsymbol{I}}, \Sigma_-^{\boldsymbol{J}} \rangle = (\mathbb{C}_{\mathsf{H}}^{-1})^{\boldsymbol{IJ}}$$

Kähler potentials:

$$\mathcal{K}_{\mathsf{V}} = -\log \mathrm{i} \int_{\mathcal{M}_6} \langle \Phi_+, \overline{\Phi}_+ \rangle, \qquad \mathcal{K}_{\mathsf{H}} = -\log \mathrm{i} \int_{\mathcal{M}_6} \langle \Phi_-, \overline{\Phi}_- \rangle$$

Non-vanishing $\mathrm{d}J$ and $\mathrm{d}\Omega$ are caused by "non"-closed basis forms:

NS-NS

$$\widehat{H} = H^{\mathrm{fl}} + \mathrm{d}\widehat{B}\,, \qquad \mathrm{d}_{H^{\mathrm{fl}}} \equiv \mathrm{d} - H^{\mathrm{fl}}\wedge$$

$$\mathrm{d}_{H^{\mathrm{fl}}} \underbrace{\begin{pmatrix} \beta^I \\ \alpha_I \end{pmatrix}}_{\Sigma_-} \sim \underbrace{\begin{pmatrix} e_\Lambda{}^I & m^{\Lambda I} \\ e_{\Lambda I} & m^\Lambda{}_I \end{pmatrix}}_{Q^{\mathrm{T}}} \underbrace{\begin{pmatrix} \widetilde{\omega}^\Lambda \\ \omega_\Lambda \end{pmatrix}}_{\Sigma_+} \quad \text{with} \quad (\mathrm{d}_{H^{\mathrm{fl}}})^2 \equiv 0$$

$e_0{}^I, e_{0I}$: H-flux charges ($H^{\mathrm{fl}} = -e_0{}^I \alpha_I + e_{0I}\beta^I$)
$e_a{}^I, e_{aI}$: geometric flux charges (torsion)
$m^{\Lambda I}, m^\Lambda{}_I$: nongeometric flux charges (magnetic dual of $e_\Lambda{}^I, e_{\Lambda I}$)

R-R

$$\widehat{\mathbf{F}} \equiv \widehat{F}_0 + \widehat{F}_2 + \ldots + \widehat{F}_{10} \equiv \mathrm{e}^{\widehat{B}}\widehat{\mathbf{G}} \qquad \text{with self-dual cond.} \quad \widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}})\,, \quad \lambda(\widehat{F}_k) \equiv (-)^{[\frac{k+1}{2}]}\widehat{F}_k$$

$$\begin{aligned} \tfrac{1}{\sqrt{2}}\widehat{\mathbf{G}} &= (G_0^\Lambda + G_2^\Lambda + G_4^\Lambda)\,\omega_\Lambda - (\widetilde{G}_{0\Lambda} + \widetilde{G}_{2\Lambda} + \widetilde{G}_{4\Lambda})\,\widetilde{\omega}^\Lambda \\ &\quad + (G_1^I + G_3^I)\,\alpha_I - (\widetilde{G}_{1I} + \widetilde{G}_{3I})\,\beta^I \end{aligned}$$

$$G_0^\Lambda \equiv m_{\mathsf{R}}^\Lambda\,, \qquad \widetilde{G}_{0\Lambda} \equiv e_{\mathsf{R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_\Lambda{}^I$$

$c \equiv (m_{\mathsf{R}}^\Lambda, e_{\mathsf{R}\Lambda})^{\mathrm{T}}$: R-R flux charges

(m_{R}^0: Romans' mass)

10D Type IIA (democratic) action $S_{\rm IIA}^{\rm (10D)} = S_{\sf NS} + \widetilde{S}_{\sf R}$:

$$S_{\sf NS} + \widetilde{S}_{\sf R} \;=\; \frac{1}{2}\int \mathrm{e}^{-2\phi}\Big\{\widehat{R} * \mathbb{1} + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3\Big\} - \frac{1}{8}\int \big[\widehat{\mathbf{F}} \wedge *\widehat{\mathbf{F}}\big]_{10}$$

with "constraint $\widehat{\mathbf{F}} \;=\; \lambda(*\widehat{\mathbf{F}})$" and "EoM (Bianchi) $(\mathrm{d} + \widehat{H}\wedge) * \widehat{\mathbf{F}} = 0 \Leftrightarrow (\mathrm{d} - \widehat{H}\wedge)\widehat{\mathbf{F}} = 0$"

↓

4D $\mathcal{N}=2$ abelian gauged SUGRA with non-trivial scalar potential
(non-abelian gauge symmetries related to isometries of SKG$_{\sf V,H}$: Unknown yet)

$0 = m^{\Lambda I} = m^{\Lambda}{}_I = m^{\Lambda}_{\sf R}$	$0 = m^{\Lambda I} = m^{\Lambda}{}_I$	generic
Standard Gauged SUGRA	Gauged SUGRA	Gauged SUGRA
$n_{\sf V}$ VM	$n_{\sf V}$ VM	$n_{\sf V}$ VM
$n_{\sf H}$ HM	$n_{\sf H}$ HM	$\widetilde{n}_{\sf H}$ HM
1 UHM	1 (massive) TM	$n_{\sf T}$ (massive) TM
[hep-th/9605032]	[hep-th/0312210]	[hep-th/0409097]

Some of $\{a, \xi^I, \widetilde{\xi}_I\}$ are dualized to 2-form fields caused by magnetic charges $\{m^{\Lambda}_{\sf R}, m^{\Lambda}{}_I, m^{\Lambda I}\}$:

[hep-th/0701247], [arXiv:0804.0595]

- Electric Gauging of "Heisenberg" directions:

$$\nabla_\mu q^u = \partial_\mu q^u + g\, k^u_\Lambda A^\Lambda_\mu \,, \qquad k_\Lambda = -\big[2e_{\mathsf{R}\Lambda} + e_\Lambda{}^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}}\xi)_{\boldsymbol{I}}\big] \frac{\partial}{\partial a} - e_\Lambda{}^{\boldsymbol{I}} \frac{\partial}{\partial \xi^{\boldsymbol{I}}}$$

- Electric/Magnetic Gauging of "Heisenberg" directions:

$$\begin{aligned} \nabla_\mu q^u &= \partial_\mu q^u + g\, k^u_\Lambda A^\Lambda_\mu + g\, \widetilde{k}^{u\Lambda} \widetilde{A}_{\Lambda\mu} \\ k_\Lambda &= -\big[2e_{\mathsf{R}\Lambda} + e_\Lambda{}^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}}\xi)_{\boldsymbol{I}}\big] \frac{\partial}{\partial a} - e_\Lambda{}^{\boldsymbol{I}} \frac{\partial}{\partial \xi^{\boldsymbol{I}}} \\ \widetilde{k}^\Lambda &= -\big[2m^\Lambda_{\mathsf{R}} + m^{\Lambda\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}}\xi)_{\boldsymbol{I}}\big] \frac{\partial}{\partial a} + m^{\Lambda\boldsymbol{I}} \frac{\partial}{\partial \xi^{\boldsymbol{I}}} \end{aligned}$$

with commutation relations:

$$[k_\Lambda, k_\Sigma] = [\widetilde{k}^\Lambda, \widetilde{k}^\Sigma] = [k_\Lambda, \widetilde{k}^\Sigma] = 0$$

Killing prepotentials $\mathcal{P}_x$ in terms of the $SU(2)$ connection ω_x:

$$\mathcal{P}_x = (\omega_x)_u k^u \quad \text{where} \quad k = k_\Lambda L^\Lambda - \widetilde{k}^\Lambda M_\Lambda = -\Pi_{\mathsf{V}}^{\mathrm{T}} \mathbb{C}_{\mathsf{V}} (2c + \widetilde{Q}\xi) \frac{\partial}{\partial a} - \Pi_{\mathsf{V}}^{\mathrm{T}} Q \frac{\partial}{\partial \xi}$$

Cassani, et.al. [arXiv:0911.2708]

Metric of SQG:

$$h_{uv}\,\mathrm{d}q^u\,\mathrm{d}q^v = \underbrace{g_{i\bar{\jmath}}\,\mathrm{d}z^i\,\mathrm{d}\bar{z}^{\bar{\jmath}}}_{\text{SKG}_\text{H}} + \underbrace{(\mathrm{d}\varphi)^2}_{\text{4D dilaton}} + \frac{1}{4}\mathrm{e}^{4\varphi}\big(\underbrace{\mathrm{d}a}_{\text{axion}} - \xi^{\mathrm{T}}\,\mathbb{C}_\text{H}\,\mathrm{d}\xi\,\big)^2 - \frac{1}{2}\mathrm{e}^{2\varphi}\underbrace{\mathrm{d}\xi^{\mathrm{T}}\,\mathbb{M}_\text{H}\,\mathrm{d}\xi}_{\text{RR-axions}}$$

Scalar potential:

$$V_\text{NS} = -2\,\mathrm{g}^2\mathrm{e}^{2\varphi}\Big[\overline{\Pi}_\text{H}^{\mathrm{T}}\,\widetilde{Q}^{\mathrm{T}}\,\mathbb{M}_\text{V}\,\widetilde{Q}\,\Pi_\text{H} + \overline{\Pi}_\text{V}^{\mathrm{T}}\,Q\,\mathbb{M}_\text{H}\,Q^{\mathrm{T}}\,\Pi_\text{V} + 4\overline{\Pi}_\text{H}^{\mathrm{T}}\,\mathbb{C}_\text{H}^{\mathrm{T}}\,Q^{\mathrm{T}}(\Pi_\text{V}\overline{\Pi}_\text{V}^{\mathrm{T}} + \overline{\Pi}_\text{V}\Pi_\text{V}^{\mathrm{T}})Q\,\mathbb{C}_\text{H}\,\Pi_\text{H}\Big]$$

$$V_\text{R} = -\frac{1}{2}\,\mathrm{g}^2\mathrm{e}^{4\varphi}(c+\widetilde{Q}\xi)^{\mathrm{T}}\,\mathbb{M}_\text{V}\,(c+\widetilde{Q}\xi)$$

$$V = V_\text{NS} + V_\text{R} = \ldots = \mathrm{g}^2\Big[4h_{uv}k^u\bar{k}^v + \sum_{x=1}^{3}\Big(g^{a\bar{b}}D_a\mathcal{P}_x D_{\bar{b}}\overline{\mathcal{P}}_x - 3|\mathcal{P}_x|^2\Big)\Big]\,, \qquad (\text{abelian } k_\Lambda^a = 0)$$

$\Pi_\text{V} = \mathrm{e}^{\mathcal{K}_\text{V}/2}(X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$

$t^a = X^a/X^0$

$a = 1, \ldots, n_\text{V}$

SKG$_\text{V}$ of vector-moduli

$$\mathcal{P}_+ \equiv \mathcal{P}_1 + \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\,\Pi_\text{V}^{\mathrm{T}}\,Q\,\mathbb{C}_\text{H}\,\Pi_\text{H}$$

$$\mathcal{P}_- \equiv \mathcal{P}_1 - \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\,\Pi_\text{V}^{\mathrm{T}}\,Q\,\mathbb{C}_\text{H}\,\overline{\Pi}_\text{H}$$

$$\mathcal{P}_3 = \mathrm{e}^{2\varphi}\,\Pi_\text{V}^{\mathrm{T}}\,\mathbb{C}_\text{V}(c+\widetilde{Q}\xi)$$

$\Pi_\text{H} = \mathrm{e}^{\mathcal{K}_\text{H}/2}(Z^I, \mathcal{G}_I)^{\mathrm{T}}$

$z^i = Z^i/Z^0$

$i = 1, \ldots, n_\text{H}$

SKG$_\text{H}$ of hyper-moduli

$$\mathbb{M}_\text{V,H} \equiv \begin{pmatrix} 1 & -\mathrm{Re}\mathcal{N} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathrm{Im}\mathcal{N} & 0 \\ 0 & (\mathrm{Im}\mathcal{N})^{-1} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\mathrm{Re}\mathcal{N} & 1 \end{pmatrix}_\text{V,H} \qquad \widetilde{Q} = \mathbb{C}_\text{V}^{\mathrm{T}}\,Q\,\mathbb{C}_\text{H} \qquad c = \begin{pmatrix} m_\text{R}^\Lambda \\ e_{\text{R}\Lambda} \end{pmatrix}$$

$Q\,\mathbb{C}_\text{H}\,Q^{\mathrm{T}} = 0 = Q^{\mathrm{T}}\,\mathbb{C}_\text{V}\,Q = c^{\mathrm{T}}Q$: Nilpotency of exterior derivative $(\mathrm{d}_{H^{\text{fl}}})^2 = 0$

Coset spaces with $SU(3)$-structure: D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

$\mathcal{M}_6$	$\frac{G_2}{SU(3)}$ (nearly-Kähler)	$\frac{Sp(2)}{S[U(2)\times U(1)]}$ (half-flat)	$\frac{SU(3)}{U(1)\times U(1)}$ (half-flat)
$\mathcal{SM} = \mathsf{SKG_V}$	$\frac{SU(1,1)}{U(1)}$: $\mathfrak{t}^3$	$\Big(\frac{SU(1,1)}{U(1)}\Big)^2$: $\mathfrak{st}^2$	$\Big(\frac{SU(1,1)}{U(1)}\Big)^3$: $\mathfrak{stu}$
$\mathcal{HM} = \mathsf{SQG}$	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM
$\mathsf{SKG_H} \subset \mathcal{HM}$	—	—	—
matters	1 VM + 1 UHM	2 VM + 1 UHM	3 VM + 1 UHM

Each $\mathsf{SKG_V}$ has a cubic prepotential: $\mathcal{F} = \frac{1}{3!} d_{abc} \frac{X^a X^b X^c}{X^0}$

nilmanifolds and solvmanifolds: M. Graña, R. Minasian, M. Petrini and A. Tomasiello [hep-th/0609124]

coset spaces with $SU(3)$- or $SU(2)$-structure: P. Koerber, D. Lüst and D. Tsimpis [arXiv:0804.0614]

a pair of $SU(3)$-structures with $(m^{\Lambda I}, m^{\Lambda}{}_I)$: D. Gaiotto and A. Tomasiello [arXiv:0904.3959]

Thanks for your attention.