Osaka Camp at Kimiidera (紀三井寺), Wakayama: November 07, 2011

String Theories, Flux Compactifications, and $\mathcal{N} = 2$ Gauged Supergravities

KEK
HIGH ENERGY ACCELERATOR RESEARCH ORGANIZATION

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Contents

We are looking for the origin of 4D physics

Physical information

- Particle contents and spectra
- (Broken) symmetries
- Potential, vacuum and cosmological constant

String theory is one of the candidates, though it is defined in 10D.

Extra dimensions $(10 - 4 = 6)$ play a significant role.

In the present stage, we have not understood yet

how to extract the "special directions 4D" from 10D

However, we can investigate physical data

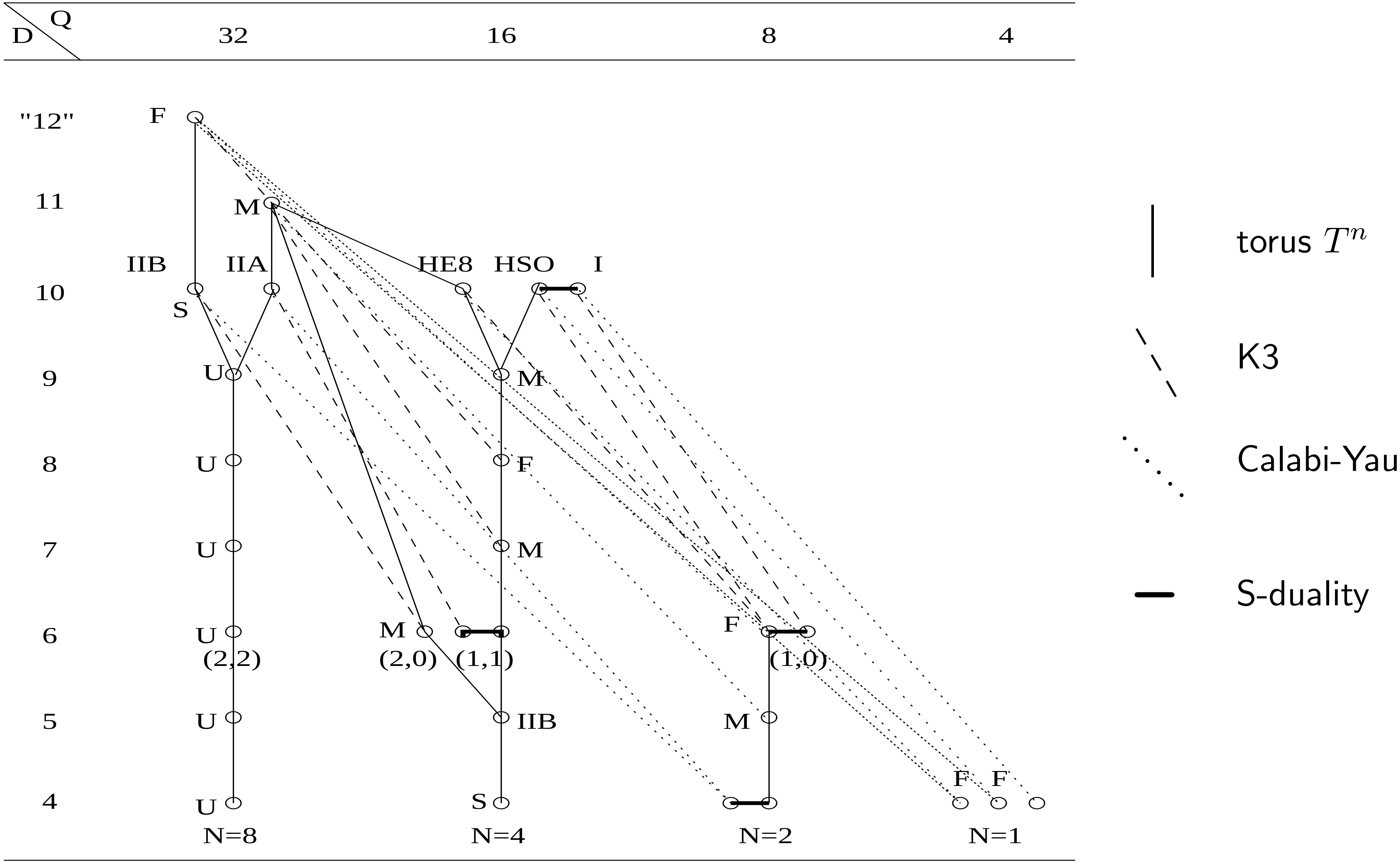
of low energy effective theories reduced from string theories

under a set of assumptions.

ex.)

- ✓ $10 = 4 + 6$ with $4 =$ (A)dS or Minkowski ; $6 =$ internal
- ✓ $\mathcal{N} = 2$ SUSY
- ✓ gauge interactions

... other configurations can be also considerable.



B. de Wit, J. Louis in the Proceedings "NATO Advanced Study Institute on Strings, Branes and Dualities (1997)," hep-th/9801132

WHY $\mathcal{N} = 2$?

- If $\mathcal{N} \geq 4$, matter fields take values in adjoint repr. of gauge symmetry
- $\mathcal{N} = 1$ system is not subject to tight restrictions (just Kähler)
- $\mathcal{N} = 2$ system has mathematically rich structures (suitably tight)

  controllable!!
- Partial SUSY breaking scenarios from $\mathcal{N} = 2$ to $\mathcal{N} = 1$ is now being investigated

HOW 4D $\mathcal{N} = 2$ from 10D ?

- Start from type II string theories (32 SUSY charges)
- Compactification of 6D space vs 4D brane world
- 1/4-SUSY preserving: Calabi-Yau manifold and its generalizations

  (cf: torus T^6 preserves all SUSY $\to$ 4D $\mathcal{N} = 8$ theory)

However... Calabi-Yau is not enough!

Beyond Calabi-Yau

Why beyond CY in 10D Strings?

↓

Back-reactions to CY caused by matter fields!

1. CY with fluxes $\longrightarrow$ 4D ungauged SUGRA

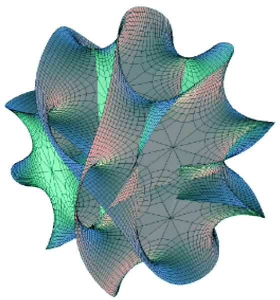
   $\longrightarrow$ break 10D Eqs. of Motion

2. non-CY with fluxes $\longrightarrow$ 4D gauged SUGRA

   non-CY: $SU(n)$-structure with torsion, generalized geometry, etc.

   gauge fields, matter fields, gauge coupling const., mass parameters...

Calabi-Yau 3-fold $\mathcal{M}_{\mathsf{CY}}$



Ricci-flat, torsionless, (compact) Kähler manifold with $SU(3)$ holonomy group

$$\mathrm{d}s^2_{\mathsf{10D}} \;=\; \underbrace{\eta_{\mu\nu}(x)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu}_{\mathsf{4D}} + \underbrace{g_{mn}(x,y)\,\mathrm{d}y^m\,\mathrm{d}y^n}_{\mathsf{CY}}$$

Invariant two-form J and three-form Ω on CY w.r.t. Levi-Civita connection:

$$\mathrm{d}J = \nabla_{[m}J_{np]} = 0 \qquad \mathrm{d}\Omega = \nabla_{[m}\Omega_{npq]} = 0$$

non-CY 3-fold $\mathcal{M}_6$

vanishing Ricci 2-form, torsionful, (almost) complex manifold

with $SU(3)$-structure

$\mathrm{d}J \neq 0$ and/or $\mathrm{d}\Omega \neq 0$

$$\mathrm{d}J = \frac{3}{2}\,\mathrm{Im}(\overline{\mathcal{W}}_1 \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \,, \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

$$\mathrm{d}J = \frac{3}{2}\,\mathrm{Im}(\overline{\mathcal{W}}_1\Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3\,, \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

complex (1/4-SUSY Minkowski$_{1,3}$)	hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = 0$
	balanced	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = 0$
	special hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	Kähler	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	conformally CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 + 2\mathcal{W}_5 = 0$
almost complex (1/4-SUSY AdS$_4$)	symplectic	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	nearly Kähler	$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	almost Kähler	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	quasi Kähler	$\mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	semi Kähler	$\mathcal{W}_4 = \mathcal{W}_5 = 0$
	half-flat	$\mathrm{Im}\mathcal{W}_1 = \mathrm{Im}\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$

Non-vanishing $(\mathrm{d}J, \mathrm{d}\Omega)$ are parametrized by coefficients of the "non"-closed basis forms:

NS-NS

$$\mathrm{d}\underbrace{\begin{pmatrix} \beta^I \\ \alpha_I \end{pmatrix}}_{\Sigma_-} \sim \underbrace{\begin{pmatrix} e_\Lambda{}^I & m^{\Lambda I} \\ e_{\Lambda I} & m^\Lambda{}_I \end{pmatrix}}_{Q^{\mathrm{T}}} \underbrace{\begin{pmatrix} \widetilde{\omega}^\Lambda \\ \omega_\Lambda \end{pmatrix}}_{\Sigma_+}$$

$e_0{}^I, e_{0I}$:	H-flux charges ($H^{\mathrm{fl}} = -e_0{}^I \alpha_I + e_{0I} \beta^I$)
$e_a{}^I, e_{aI}$:	geometric flux charges (torsion)
$m^{\Lambda I}, m^\Lambda{}_I$:	nongeometric flux charges (magnetic dual of $e_\Lambda{}^I, e_{\Lambda I}$)

R-R

$$\widehat{\mathbf{F}} \equiv \widehat{F}_0 + \widehat{F}_2 + \ldots + \widehat{F}_{10} \equiv \mathrm{e}^{\widehat{B}} \widehat{\mathbf{G}} \qquad \text{with self-dual cond.} \quad \widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}}) \,, \quad \lambda(\widehat{F}_k) \equiv (-)^{[\frac{k+1}{2}]} \widehat{F}_k$$

$$\begin{aligned} \tfrac{1}{\sqrt{2}} \widehat{\mathbf{G}} \ &= \ (G_0^\Lambda + G_2^\Lambda + G_4^\Lambda)\, \omega_\Lambda - (\widetilde{G}_{0\Lambda} + \widetilde{G}_{2\Lambda} + \widetilde{G}_{4\Lambda})\, \widetilde{\omega}^\Lambda \\ &\quad + (G_1^I + G_3^I)\, \alpha_I - (\widetilde{G}_{1I} + \widetilde{G}_{3I})\, \beta^I \end{aligned}$$

$$G_0^\Lambda \equiv p^\Lambda \,, \qquad \widetilde{G}_{0\Lambda} \equiv q_\Lambda - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_\Lambda{}^I$$

$c \equiv (p^\Lambda, q_\Lambda)^{\mathrm{T}}$: R-R flux charges

(p^0: Romans' mass)

10D Type IIA (democratic) action $S_{\rm IIA}^{(10D)} = S_{\rm NS} + \widetilde{S}_{\rm R}$:

$$S_{\rm NS} + \widetilde{S}_{\rm R} = \frac{1}{2} \int {\rm e}^{-2\phi} \Big\{ \widehat{R} * \mathbb{1} + 4{\rm d}\phi \wedge *{\rm d}\phi - \frac{1}{2} \widehat{H}_3 \wedge * \widehat{H}_3 \Big\} - \frac{1}{8} \int \big[\widehat{\mathbf{F}} \wedge * \widehat{\mathbf{F}} \big]_{10}$$

with "constraint $\widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}})$" and "EoM (Bianchi) $({\rm d} + \widehat{H}\wedge) * \widehat{\mathbf{F}} = 0 \Leftrightarrow ({\rm d} - \widehat{H}\wedge)\widehat{\mathbf{F}} = 0$"

$\downarrow$

4D $\mathcal{N} = 2$ abelian gauged SUGRA with non-trivial scalar potential
(non-abelian gauge symmetries: Unknown yet)

$0 = m^{\Lambda I} = m^{\Lambda}{}_I = p^{\Lambda}$	$0 = m^{\Lambda I} = m^{\Lambda}{}_I$	generic
Standard Gauged SUGRA	Gauged SUGRA	Gauged SUGRA
$n_{\rm V}$ VM	$n_{\rm V}$ VM	$n_{\rm V}$ VM
$n_{\rm H}$ HM	$n_{\rm H}$ HM	$\widetilde{n}_{\rm H}$ HM
1 UHM	1 TM	$n_{\rm T}$ TM
[hep-th/9605032]	[hep-th/0312210]	[hep-th/0409097]

Some of $\{a, \xi^I, \widetilde{\xi}_I\}$ are dualized to 2-form fields caused by magnetic charges $\{p^{\Lambda}, m^{\Lambda}{}_I, m^{\Lambda I}\}$:

[hep-th/0701247], [arXiv:0804.0595]

Scalar potential

$$V_{\mathsf{NS}} = -2\,\mathrm{g}^2 e^{2\varphi}\Big[\overline{\Pi}_{\mathsf{H}}^{\mathrm{T}}\,\widetilde{Q}^{\mathrm{T}}\,\mathbb{M}_{\mathsf{V}}\,\widetilde{Q}\,\Pi_{\mathsf{H}} + \overline{\Pi}_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{M}_{\mathsf{H}}\,Q^{\mathrm{T}}\,\Pi_{\mathsf{V}} + 4\overline{\Pi}_{\mathsf{H}}^{\mathrm{T}}\,\mathbb{C}_{\mathsf{H}}^{\mathrm{T}}\,Q^{\mathrm{T}}\big(\Pi_{\mathsf{V}}\overline{\Pi}_{\mathsf{V}}^{\mathrm{T}} + \overline{\Pi}_{\mathsf{V}}\Pi_{\mathsf{V}}^{\mathrm{T}}\big)Q\,\mathbb{C}_{\mathsf{H}}\,\Pi_{\mathsf{H}}\Big]$$

$$V_{\mathsf{R}} = -\frac{1}{2}\,\mathrm{g}^2 e^{4\varphi}(c+\widetilde{Q}\xi)^{\mathrm{T}}\,\mathbb{M}_{\mathsf{V}}\,(c+\widetilde{Q}\xi)$$

$$V_{\mathsf{NS}} + V_{\mathsf{R}} = V_{\mathsf{4D}} = \mathrm{g}^2\Big[4h_{uv}k^u\overline{k}^v + \sum_{x=1}^{3}\Big(G^{a\bar{b}}D_a\mathcal{P}_x\overline{D_b\mathcal{P}_x} - 3|\mathcal{P}_x|^2\Big)\Big] \qquad (\text{abelian}:\ k_\Lambda^a = 0)$$

$$\Pi_{\mathsf{V}} = e^{\mathcal{K}_{\mathsf{V}}/2}(X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$$
$$\mathfrak{t}^a = X^a/X^0$$
$$a = 1, \ldots, n_{\mathsf{V}}$$
$\mathsf{SKG}_{\mathsf{V}}$ of vector-moduli

$$\mathcal{P}_+ = 2e^{\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{H}}\,\Pi_{\mathsf{H}}$$
$$\mathcal{P}_- = 2e^{\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{H}}\,\overline{\Pi}_{\mathsf{H}}$$
$$\mathcal{P}_3 = e^{2\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,\mathbb{C}_{\mathsf{V}}(c+\widetilde{Q}\xi)$$

$$\Pi_{\mathsf{H}} = e^{\mathcal{K}_{\mathsf{H}}/2}(Z^I, \mathcal{G}_I)^{\mathrm{T}}$$
$$z^i = Z^i/Z^0$$
$$i = 1, \ldots, n_{\mathsf{H}}$$
$\mathsf{SKG}_{\mathsf{H}}$ of hyper-moduli

$$\mathbb{M}_{\mathsf{V,H}} \equiv \begin{pmatrix} 1 & -\mathrm{Re}\mathcal{N} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathrm{Im}\mathcal{N} & 0 \\ 0 & (\mathrm{Im}\mathcal{N})^{-1} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\mathrm{Re}\mathcal{N} & 1 \end{pmatrix}_{\mathsf{V,H}}$$

where

$$\mathcal{F}_\Lambda \equiv \mathcal{N}_{\Lambda\Sigma}X^\Sigma$$
$$\widetilde{Q} \equiv \mathbb{C}_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{H}}$$

$\mathbb{C}_{\mathsf{V,H}}$: symplectic invariant metrics

- In 4D $\mathcal{N} = 2$ ungauged SUGRA $\longrightarrow$ No scalar potential

(Extremal) charged Black holes in asymptotic flat has been investigated
Charges = series of D-branes wrapped on cycles in CY

- In 4D $\mathcal{N} = 2$ gauged SUGRA $\longrightarrow$ Scalar potential is turned on

Cosmological constant appears as VEV of scalar potential

$\boxed{\text{Beyond Calabi-Yau}}$ Torsionful Manifolds / Generalized Geometries / Doubled Geometries

TK's recent works on type II: [arXiv:0806.1783], [arXiv:0810.0937], [arXiv:1005.4607], [arXiv:1108.1113]
on hetetoric: [hep-th/0605247], [arXiv:0704.2111], [arXiv:0905.2185], [arXiv:0912.1334]

Coset space $\dfrac{G_2}{SU(3)}$ D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

- ✓ nearly-Kähler (almost complex geometry)
- ✓ NSNS-sector : torsion and H-flux
- ✓ RR-sector : 2-, 4-form and Romans' mass (0-form)

- ✓ 1 VM with cubic prepotential $\mathcal{F} = \dfrac{X^1 X^1 X^1}{X^0}$
- ✓ 1 UHM (no other HMs)

nilmanifolds and solvmanifolds: M. Graña, R. Minasian, M. Petrini and A. Tomasiello [hep-th/0609124]

coset spaces with $SU(3)$- or $SU(2)$-structure: P. Koerber, D. Lüst and D. Tsimpis [arXiv:0804.0614]

a pair of $SU(3)$-structures with $(m^{\Lambda I}, m^{\Lambda}{}_I)$: D. Gaiotto and A. Tomasiello [arXiv:0904.3959]

10D type IIA on $\dfrac{G_2}{SU(3)}$ with fluxes

$$\downarrow$$

4D $\mathcal{N} = 2$ abelian gauged SUGRA with B-field ($\Lambda = 0, 1$ and $\boldsymbol{\xi^0} \equiv (\xi^0, \widetilde{\xi}_0)^{\mathrm{T}}$)

$$\begin{aligned} S = \int \Big[& \frac{1}{2} R\,(*1) + \frac{1}{2} \mu_{\Lambda\Sigma}\, F^{\Lambda} \wedge *F^{\Sigma} + \frac{1}{2} \nu_{\Lambda\Sigma}\, F^{\Lambda} \wedge F^{\Sigma} - g_{\mathfrak{t}\bar{\mathfrak{t}}}\, \mathrm{d}\mathfrak{t} \wedge *\mathrm{d}\bar{\mathfrak{t}} \\ & - \mathrm{d}\varphi \wedge *\mathrm{d}\varphi - \frac{\mathrm{e}^{-4\varphi}}{4} \mathrm{d}B \wedge *\mathrm{d}B - \frac{\mathrm{e}^{2\varphi}}{2} \Big(D\xi^0 \wedge *D\xi^0 + D\widetilde{\xi}_0 \wedge *D\widetilde{\xi}_0 \Big) + \mathrm{d}B \wedge \xi^0\, \mathrm{d}\widetilde{\xi}_0 \\ & + \mathrm{d}B \wedge \big(e_{\mathsf{R}\Lambda} - e_{\Lambda 0}\, \xi^0 \big) A^{\Lambda} - \frac{1}{2} m_{\mathsf{R}}^{\Lambda}\, e_{\mathsf{R}\Lambda}\, B \wedge B - V\,(*1) \Big] \end{aligned}$$

- $g_{\mu\nu}$, $\mathfrak{t}$, $B_{\mu\nu}$, φ; $(e_{\Lambda}{}^{0}, e_{\Lambda 0})$: NS-NS sector
- A_{μ}^{Λ}, ξ^0, $\widetilde{\xi}_0$; $(m_{\mathsf{R}}^{\Lambda}, e_{\mathsf{R}\Lambda})$: R-R sector
- GM : $(g_{\mu\nu}, A_{\mu}^{0})$, VM : $(A_{\mu}^{a}, \mathfrak{t})$, UHM $\to$ TM : $(\varphi, B_{\mu\nu}, \xi^0, \widetilde{\xi}_0)$
- $D\xi^0 = \mathrm{d}\xi^0 - e_{\Lambda}{}^{0} A_{1}^{\Lambda}$, $\quad D\widetilde{\xi}_0 = \mathrm{d}\widetilde{\xi}_0 - e_{\Lambda 0} A_{1}^{\Lambda}$
- $F_{2}^{\Sigma} = \mathrm{d}A_{1}^{\Sigma} + m_{\mathsf{R}}^{\Sigma} B_{2}$
- $V(\mathfrak{t}, \varphi, \xi^0) = V_{\mathsf{NS}}(\mathfrak{t}, \varphi) + V_{\mathsf{R}}(\mathfrak{t}, \varphi, \xi^0)$

Precise data on $\dfrac{G_2}{SU(3)}$:

$$e_{10} \neq 0,\ m_{\mathsf{R}}^{0} \neq 0,\ e_{\mathsf{R}0} \neq 0$$

$$e_{\Lambda}{}^{0} = 0 = e_{00}$$

$$m_{\mathsf{R}}^{1} = 0 = e_{\mathsf{R}1}$$

$\mu_{\Lambda\Sigma} \equiv \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}$, $\nu_{\Lambda\Sigma} \equiv \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}$

D. Cassani [arXiv:0804.0595]

Vacuum I : $\mathcal{N} = 1$

$$t_* = -\frac{\pm 1 + i\sqrt{15}}{2} \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3} , \quad \xi_*^0 = -\frac{2}{5} \left[\frac{2\sqrt{3}\, m_R^0 (e_{R0})^2}{5\, e_{10}} \right]^{1/3} , \quad \exp(\varphi_*) = \frac{4}{3} \left[\frac{\sqrt{5}\, e_{10}}{\sqrt{3}\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{5\sqrt{5}}{2} \left[\frac{5\,(e_{10})^4}{2\sqrt{3}\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{I}} < 0$$

Vacuum II : $\mathcal{N} = 0$

$$t_* = (\pm 1 - i\sqrt{3}) \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3} , \quad \xi_*^0 = \left[\frac{9\, m_R^0 (e_{R0})^2}{25\, e_{10}} \right]^{1/3} , \quad \exp(\varphi_*) = \frac{2}{3} \left[\frac{25\, e_{10}}{\sqrt{3}\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{80}{27} \left[\frac{25\,(e_{10})^4}{\sqrt{3}\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{II}} < 0$$

Vacuum III : $\mathcal{N} = 0$

$$t_* = -i \left[\frac{12}{\sqrt{5}\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3} , \quad \xi_*^0 = 0 , \quad \exp(\varphi_*) = \sqrt{5} \left[\frac{5\, e_{10}}{18\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{25\sqrt{5}}{6} \left[\frac{5\,(e_{10})^4}{18\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{III}} < 0$$

Note: $m_R^0 > 0$; $\tilde{\xi}_0$ is not fixed ; $\Lambda_{\text{c.c.}}^{\text{II}} < \Lambda_{\text{c.c.}}^{\text{I}} < \Lambda_{\text{c.c.}}^{\text{III}}$ D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

Anti-de Sitter Black Holes in 4D $\mathcal{N} = 2$ Gauged SUGRA

Comments

- AdS-BH with naked singularity in pure AdS SUGRA

  L.J. Romans [hep-th/9203018], M.M. Caldarelli and D. Klemm [hep-th/9808097], etc.

  SUSY solution of rotating AdS black hole with regular horizon

- AdS-BH with regular horizon in Gauged SUGRA with VMs (no HMs)

  [hep-th/9903143], [arXiv:0911.4926], [arXiv:1012.4314], etc.

Question

How can we obtain AdS-BH solutions

with hypermultiplets?

— **Setup** and **Result** —

- 4D $\mathcal{N} = 2$ gauged SUGRA with VM and UHM
  from 10D type IIA on non-CY with fluxes
- Ansatze for matter fields
- Regular solution (AdS Black Hole)

TK [arXiv:1005.4607], [arXiv:1108.1113]

Current Issue:

How does Non-Abelian gauge symmetry emerge?

Go beyond the “beyond Calabi-Yau”

Thanks!

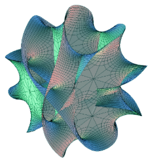
Appendix

- Terminology
- Calabi-Yau compactifications in type IIA
- Geometric flux compactifications in type IIA
- Scalar potential
- Static AdS black hole

Prepotential :	$\mathcal{F}$ is a holomorphic function of X^Λ of degree two $\quad (\mathcal{F}_\Lambda = \partial \mathcal{F}/\partial X^\Lambda)$
Kähler potential :	$\mathcal{K}_{\mathsf{V}} \;=\; -\log\big[\mathrm{i}\big(\overline{X}^\Lambda \mathcal{F}_\Lambda - X^\Lambda \overline{\mathcal{F}}_\Lambda\big)\big]$
Symplectic section :	$\Pi_{\mathsf{V}} \;\equiv\; \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2} \begin{pmatrix} X^\Lambda \\ \mathcal{F}_\Lambda \end{pmatrix} \equiv \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \quad 1 \;=\; \mathrm{i}\big(\overline{L}^\Lambda M_\Lambda - L^\Lambda \overline{M}_\Lambda\big)$
Kähler metric :	$g_{a\bar{b}} \;=\; \dfrac{\partial}{\partial \mathfrak{t}^a} \dfrac{\partial}{\partial \bar{\mathfrak{t}}^{\bar{b}}} \mathcal{K}_{\mathsf{V}}\,, \quad \mathfrak{t}^a = \dfrac{X^a}{X^0}$
Kähler covariant derivative :	$D_a \Pi_{\mathsf{V}} \;=\; \Big(\dfrac{\partial}{\partial \mathfrak{t}^a} + \dfrac{1}{2}\dfrac{\partial \mathcal{K}_{\mathsf{V}}}{\partial \mathfrak{t}^a}\Big)\Pi_{\mathsf{V}} \;\equiv\; \begin{pmatrix} f_a^\Lambda \\ h_{\Lambda a} \end{pmatrix}$
Period matrix :	$\mathcal{N}_{\Lambda\Sigma} \;=\; \overline{\mathcal{F}}_{\Lambda\Sigma} + 2\mathrm{i}\dfrac{(\mathrm{Im}\mathcal{F})_{\Lambda\Gamma} X^\Gamma (\mathrm{Im}\mathcal{F})_{\Sigma\Delta} X^\Delta}{X^\Pi (\mathrm{Im}\mathcal{F})_{\Pi\Xi} X^\Xi}$
Formulae :	$M_\Lambda \;=\; \mathcal{N}_{\Lambda\Sigma} L^\Sigma, \quad h_{\Lambda a} \;=\; \overline{\mathcal{N}}_{\Lambda\Sigma} f_a^\Sigma$
(Symplectic matrix) :	$(\mathbb{M}_{\mathsf{V}})_{\boldsymbol{\Lambda\Sigma}} \;=\; \begin{pmatrix} \mathbb{1} & -\mathrm{Re}\mathcal{N} \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} \mathrm{Im}\mathcal{N} & 0 \\ 0 & (\mathrm{Im}\mathcal{N})^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ -\mathrm{Re}\mathcal{N} & \mathbb{1} \end{pmatrix}$

In a similar way... $\quad \Pi_{\mathsf{H}} \;\equiv\; \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2} \begin{pmatrix} Z^I \\ \mathcal{G}_I \end{pmatrix}, \quad z^i \;=\; \dfrac{Z^i}{Z^0}, \quad \mathcal{K}_{\mathsf{H}} \;=\; -\log\big[\mathrm{i}\big(\overline{Z}^I \mathcal{G}_I - Z^I \overline{\mathcal{G}}_I\big)\big]\,,$ etc.

Calabi-Yau 3-fold $\mathcal{M}_{\text{CY}}$



Ricci-flat, torsionless, (compact) Kähler manifold with $SU(3)$ holonomy group

Invariant two-form J and three-form Ω on CY w.r.t. Levi-Civita connection:

$$\mathrm{d}J = \nabla_{[m} J_{np]} = 0, \qquad \mathrm{d}\Omega = \nabla_{[m} \Omega_{npq]} = 0$$

This is suitable for 1/4-SUSY condition with vanishing background fields

$$\delta_{\text{SUSY}} \psi_{m\pm} = \nabla_m \, \varepsilon_{\pm}^{(\text{10D})} = 0$$

$$\boxed{\varepsilon_{+}^{(\text{10D})} \;=\; \varepsilon_{1+}^{(\text{4D})} \otimes \eta_{+}^{1} + (\text{c.c.})\,, \qquad \varepsilon_{-}^{(\text{10D})} \;=\; \varepsilon_{2+}^{(\text{4D})} \otimes \eta_{-}^{2} + (\text{c.c.})}$$

$$(\varepsilon_{1,2+}^{(\text{4D})})^c = \varepsilon_{1,2-}^{(\text{4D})}\,, \quad (\eta_{-}^{1,2})^* = \eta_{+}^{1,2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ * \end{pmatrix} \;:\; SU(3)\text{-invariant} \subset SU(4) \sim SO(6)$$

$$\eta_{\pm}^{1} = \eta_{\pm}^{2} \equiv \eta_{\pm}\,, \qquad J_{mn} \sim \mp \mathrm{i}\, \eta_{\pm}^{\dagger} \, \gamma_{mn} \, \eta_{\pm}\,, \qquad \Omega \;\sim\; -\mathrm{i}\, \eta_{-}^{\dagger} \, \gamma_{mnp} \, \eta_{+}$$

NS-NS fields in 10D are expanded around CY :

$$\phi(x,y) = \varphi(x)$$

$$g_{\mathfrak{m}\bar{\mathfrak{n}}}(x,y) = \mathrm{i}v^a(x)\,(\omega_a)_{\mathfrak{m}\bar{\mathfrak{n}}}(y), \quad g_{\mathfrak{m}\mathfrak{n}}(x,y) = \mathrm{i}\,\overline{z}^{\bar{\jmath}}(x)\left(\frac{(\overline{\chi}_{\bar{\jmath}})_{\mathfrak{m}\bar{\mathfrak{p}}\bar{\mathfrak{q}}}\Omega^{\bar{\mathfrak{p}}\bar{\mathfrak{q}}}{}_{\mathfrak{n}}}{||\Omega||^2}\right)(y)$$

$$\widehat{B}_2(x,y) = B_2(x) + b^a(x)\omega_a(y)$$

$$\mathfrak{t}^a(x) \equiv b^a(x) + \mathrm{i}v^a(x)$$

R-R fields :

$$\widehat{C}_1(x,y) = A_1^0(x)$$

$$\widehat{C}_3(x,y) = A_1^a(x)\wedge\omega_a(y) + \xi^I(x)\alpha_I(y) - \widetilde{\xi}_I(x)\beta^I(y)$$

cohomology class on CY	basis	degrees
$H^{(1,1)}$	ω_a	$a = 1,\ldots,h^{(1,1)}$
$H^{(0)}\oplus H^{(1,1)}$	$\omega_\Lambda = (1,\omega_a)$	$\Lambda = 0,1,\ldots,h^{(1,1)}$
$H^{(2,2)}\oplus H^{(6)}$	$\widetilde{\omega}^\Lambda = (\widetilde{\omega}^a, \frac{\mathrm{vol.}}{\lvert\mathrm{vol.}\rvert})$	
$H^{(2,1)}$	χ_i	$i = 1,\ldots,h^{(2,1)}$
$H^{(3)}$	(α_I,β^I)	$I = 0,1,\ldots,h^{(2,1)}$

$$\mathrm{d}\omega_\Lambda = 0 = \mathrm{d}\widetilde{\omega}^\Lambda$$

$$\mathrm{d}\alpha_I = 0 = \mathrm{d}\beta^I$$

10D Type IIA action $S^{(10\text{D})}_{\text{IIA}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}$:

$$\begin{aligned} S_{\text{NS}} &= \frac{1}{2}\int \mathrm{e}^{-2\phi}\Big\{\widehat{R} * \mathbb{1} + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3\Big\} \\ S_{\text{R}} + S_{\text{CS}} &= -\frac{1}{4}\int \Big\{\widehat{F}_2 \wedge *\widehat{F}_2 + (\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3) \wedge *(\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3)\Big\} - \frac{1}{4}\int \widehat{B}_2 \wedge \widehat{F}_4 \wedge \widehat{F}_4 \end{aligned}$$

$$\downarrow$$

4D $\mathcal{N} = 2$ ungauged SUGRA: Neither gauge couplings, Nor scalar potential

$$S^{(4\text{D})} = \int \Big\{\frac{1}{2}R * \mathbb{1} - G_{a\bar{b}}\,\mathrm{d}\mathfrak{t}^a \wedge *\mathrm{d}\bar{\mathfrak{t}}^{\bar{b}} - h_{uv}\,\mathrm{d}q^u \wedge *\mathrm{d}q^v + \frac{1}{2}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^\Lambda \wedge *F^\Sigma + \frac{1}{2}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}F^\Lambda \wedge F^\Sigma\Big\}$$

gravitational multiplet	$g_{\mu\nu}\,,\ A^0_1$		
vector multiplet (VM)	$A^a_1\,,\ \mathfrak{t}^a\,,\ \bar{\mathfrak{t}}^{\bar{b}}$	$\mathfrak{t}^a \in \text{SKG}_\text{V}$	
hypermultiplet (HM)	$z^i\,,\ \bar{z}^{\bar{j}}\,,\ \xi^i\,,\ \widetilde{\xi}_j$	$z^i \in \text{SKG}_\text{H}$	
universal hypermultiplet (UHM)	$\varphi\,,\ a\,,\ \xi^0\,,\ \widetilde{\xi}_0$	$a \leftrightarrow B_2$	(Hodge dual)

$\mathcal{HM}$ = Special QG

$$\underbrace{\{q^u\}}_{4n_\text{H}+4} = \underbrace{\{z^i, \bar{z}^{\bar{j}}\}}_{2n_\text{H}(\text{SKG}_\text{H})} + \underbrace{\{\xi^i, \widetilde{\xi}_j\}}_{2n_\text{H}} + \underbrace{\{\varphi, a, \xi^0, \widetilde{\xi}_0\}}_{4\,(\text{UHM})} = \underbrace{\{z^i, \bar{z}^{\bar{j}}\} + \{\varphi\}}_{\text{SKG}_\text{H}} + \underbrace{\{a, \xi^I, \widetilde{\xi}_J\}}_{\text{“Heisenberg”}}$$

10D type IIA action $S^{(\text{10D})}_{\text{IIA}} = S_{\text{NS}} + \widetilde{S}_{\text{R}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}$: (democratic form)

$$S_{\text{NS}} \;=\; \frac{1}{2}\int \mathrm{e}^{-2\phi}\Big\{\widehat{R} * \mathbb{1} + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3\Big\}, \qquad \widetilde{S}_{\text{R}} \;=\; -\frac{1}{8}\int \big[\widehat{\mathbf{F}} \wedge *\widehat{\mathbf{F}}\big]_{10}$$

with "constraint $\widehat{\mathbf{F}} \;=\; \lambda(*\widehat{\mathbf{F}})$" and "EoM (Bianchi) $(\mathrm{d} + \widehat{H}\wedge) * \widehat{\mathbf{F}} = 0 \Leftrightarrow (\mathrm{d} - \widehat{H}\wedge)\widehat{\mathbf{F}} = 0$"

$\downarrow$ non-CY with $SU(3)$-structure with $m^{\Lambda}_{\text{R}} = 0$

4D $\mathcal{N} = 2$ abelian gauged SUGRA (with $\xi^{\boldsymbol{I}} \equiv (\xi^I, \widetilde{\xi}_I)^{\text{T}}$):

$$S^{(\text{4D})} = \int \mathrm{d}^4x\sqrt{-g}\,\Big[\frac{1}{2}R + \frac{1}{4}\text{Im}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}}\text{Re}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma} - g_{a\bar{b}}\,\partial_{\mu}\mathfrak{t}^a\partial^{\mu}\bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{\jmath}}\,\partial_{\mu}z^i\partial^{\mu}\bar{z}^{\bar{\jmath}}$$
$$-\partial_{\mu}\varphi\partial^{\mu}\varphi + \frac{\mathrm{e}^{2\varphi}}{2}(\mathbb{M}_{\text{H}})_{\boldsymbol{IJ}}D_{\mu}\xi^{\boldsymbol{I}}D^{\mu}\xi^{\boldsymbol{J}} - \frac{\mathrm{e}^{2\varphi}}{4}\big(D_{\mu}a - \xi^{\boldsymbol{I}}(\mathbb{C}_{\text{H}})_{\boldsymbol{IJ}}D_{\mu}\xi^{\boldsymbol{J}}\big)^2 - V(\mathfrak{t}, \bar{\mathfrak{t}}, q)\Big]$$

- $({e_{\Lambda}}^I, e_{\Lambda I})$: geometric flux charges & $e_{\text{R}\Lambda}$: RR-flux charges $\longleftarrow$ non-CY data
  (with constraints ${e_{\Lambda}}^I e_{\Sigma I} - e_{\Lambda I}{e_{\Sigma}}^I = 0$)
- $\mathfrak{t}^a \in \text{SKG}_{\text{V}}$ and $z^i \in \text{SKG}_{\text{H}} \subset \mathcal{HM}$ are ungauged (in general)
- $D_{\mu}\xi^I = \partial_{\mu}\xi^I - {e_{\Lambda}}^I A^{\Lambda}_{\mu}$ & $D_{\mu}\widetilde{\xi}_I = \partial_{\mu}\widetilde{\xi}_I - e_{\Lambda I}A^{\Lambda}_{\mu}$
- $D_{\mu}a = \partial_{\mu}a - (2e_{\text{R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I {e_{\Lambda}}^I)A^{\Lambda}_{\mu}$
- $V(\mathfrak{t}, \bar{\mathfrak{t}}, q)$: scalar potential

D. Cassani [arXiv:0804.0595]

Non-vanishing m_{R}^{Λ} dualizes the axion field a in standard SUGRA to B-field.

4D gauged action is different from the standard one:

$$
\begin{aligned}
S^{(\mathsf{4D})} \;=\; \int \Big[& \frac{1}{2} R(*\mathbb{1}) + \frac{1}{2} \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F_{2}^{\Lambda} \wedge * F_{2}^{\Sigma} + \frac{1}{2} \mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F_{2}^{\Lambda} \wedge F_{2}^{\Sigma} - g_{a\bar{b}}\, \mathrm{d}\mathfrak{t}^{a} \wedge * \mathrm{d}\bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{j}}\, \mathrm{d}z^{i} \wedge * \mathrm{d}\bar{z}^{\bar{j}} \\
& - \mathrm{d}\varphi \wedge * \mathrm{d}\varphi - \frac{\mathrm{e}^{-4\varphi}}{4} H_{3} \wedge * H_{3} - \frac{\mathrm{e}^{2\varphi}}{2} (\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{I}} \wedge * D\xi^{\boldsymbol{J}} - V(*\mathbb{1}) \\
& + \frac{1}{2} \mathrm{d}B \wedge \Big[\xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{J}} + \big(2 e_{\mathsf{R}\Lambda} - \xi^{I} e_{\Lambda I} + \widetilde{\xi}_{I} e_{\Lambda}{}^{I} \big) A_{1}^{\Lambda} \Big] - \frac{1}{2} m_{\mathsf{R}}^{\Lambda} e_{\mathsf{R}\Lambda} B_{2} \wedge B_{2} \Big]
\end{aligned}
$$

Constraints among flux charges:

$$
e_{\Lambda}{}^{I} e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}{}^{I} \;=\; 0, \qquad m_{\mathsf{R}}^{\Lambda} e_{\Lambda}{}^{I} \;=\; 0 \;=\; m_{\mathsf{R}}^{\Lambda} e_{\Lambda I}
$$

Scalar potential from (non)geometric flux compactifications:

$$\begin{aligned}
V &= \mathrm{g}^2\Big[4h_{uv}k^u\overline{k}^v + \sum_{x=1}^{3}\Big(g^{a\overline{b}}D_a\mathcal{P}_xD_{\overline{b}}\overline{\mathcal{P}}_x - 3|\mathcal{P}_x|^2\Big)\Big] = \ldots \equiv V_{\mathsf{NS}} + V_{\mathsf{R}} \quad (\text{abelian: } k^a_\Lambda = 0) \\
V_{\mathsf{NS}} &= g^{a\overline{b}}D_a\mathcal{P}_+D_{\overline{b}}\overline{\mathcal{P}}_+ + g^{i\overline{\jmath}}D_i\mathcal{P}_+D_{\overline{\jmath}}\overline{\mathcal{P}}_+ - 2|\mathcal{P}_+|^2 \\
&= -2\,\mathrm{g}^2\mathrm{e}^{2\varphi}\Big[\overline{\Pi}_{\mathsf{H}}^{\mathrm{T}}\,\widetilde{Q}^{\mathrm{T}}\,\mathbb{M}_{\mathsf{V}}\,\widetilde{Q}\,\Pi_{\mathsf{H}} + \overline{\Pi}_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{M}_{\mathsf{H}}\,Q^{\mathrm{T}}\,\Pi_{\mathsf{V}} + 4\overline{\Pi}_{\mathsf{H}}^{\mathrm{T}}\,\mathbb{C}_{\mathsf{H}}^{\mathrm{T}}\,Q^{\mathrm{T}}(\Pi_{\mathsf{V}}\overline{\Pi}_{\mathsf{V}}^{\mathrm{T}} + \overline{\Pi}_{\mathsf{V}}\Pi_{\mathsf{V}}^{\mathrm{T}})Q\,\mathbb{C}_{\mathsf{H}}\,\Pi_{\mathsf{H}}\Big] \\
V_{\mathsf{R}} &= g^{a\overline{b}}D_a\mathcal{P}_3D_{\overline{b}}\overline{\mathcal{P}}_3 + |\mathcal{P}_3|^2 \\
&= -\frac{1}{2}\,\mathrm{g}^2\mathrm{e}^{4\varphi}\big(e_{\mathsf{R}\Lambda} - e_{\Lambda I}\xi^I + e_\Lambda{}^I\widetilde{\xi}_I\big)(\mathrm{Im}\mathcal{N})^{-1|\Lambda\Sigma}\big(e_{\mathsf{R}\Sigma} - e_{\Sigma I}\xi^I + e_\Sigma{}^I\widetilde{\xi}_I\big)
\end{aligned}$$

$$\Pi_{\mathsf{V}} = \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2}(X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$$
$$t^a = X^a/X^0$$
$$a = 1, \ldots, n_{\mathsf{V}}$$
SKG$_{\mathsf{V}}$ of vector-moduli

$$\begin{aligned}
\mathcal{P}_+ &\equiv \mathcal{P}_1 + \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{H}}\,\Pi_{\mathsf{H}} \\
\mathcal{P}_- &\equiv \mathcal{P}_1 - \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{H}}\,\overline{\Pi}_{\mathsf{H}} \\
\mathcal{P}_3 &= \mathrm{e}^{2\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,\mathbb{C}_{\mathsf{V}}(c_{\mathsf{R}} + \widetilde{Q}\xi)
\end{aligned}$$

$$\Pi_{\mathsf{H}} = \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2}(Z^I, \mathcal{G}_I)^{\mathrm{T}}$$
$$z^i = Z^i/Z^0$$
$$i = 1, \ldots, n_{\mathsf{H}}$$
SKG$_{\mathsf{H}}$ of hyper-moduli

$$\mathbb{C}_{\mathsf{V,H}} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}; \quad Q = \begin{pmatrix} e_\Lambda{}^I & e_{\Lambda I} \\ m^{\Lambda I} & m^\Lambda{}_I \end{pmatrix}, \quad \widetilde{Q} = \mathbb{C}_{\mathsf{H}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{V}} \quad c_{\mathsf{R}} = \begin{pmatrix} m^\Lambda_{\mathsf{R}} \\ e_{\mathsf{R}\Lambda} \end{pmatrix}$$

Cassani et.al. [arXiv:0804.0595], [arXiv:0911.2708]

Coset spaces with $SU(3)$-structure: D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

$\mathcal{M}_6$	$\frac{G_2}{SU(3)} = S^6$	$\frac{Sp(2)}{S(U(2)\times U(1))} = \mathbb{C}P^3$	$\frac{SU(3)}{U(1)\times U(1)} = \mathbb{F}(1,2;3)$
$\mathcal{SM} = \mathsf{SKG_V}$	$\frac{SU(1,1)}{U(1)}$: $\mathfrak{t}^3$	$\left(\frac{SU(1,1)}{U(1)}\right)^2$: $\mathfrak{st}^2$	$\left(\frac{SU(1,1)}{U(1)}\right)^3$: $\mathfrak{stu}$
$\mathcal{HM} = \mathsf{SQG}$	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM
$\mathsf{SKG_H} \subset \mathcal{HM}$	—	—	—
matters	1 VM + 1 UHM	2 VM + 1 UHM	3 VM + 1 UHM

Each $\mathsf{SKG_V}$ has a cubic prepotential: $\mathcal{F} = \frac{1}{3!} d_{abc} \frac{X^a X^b X^c}{X^0}$

nilmanifolds and solvmanifolds: M. Graña, R. Minasian, M. Petrini and A. Tomasiello [hep-th/0609124]

coset spaces with $SU(3)$- or $SU(2)$-structure: P. Koerber, D. Lüst and D. Tsimpis [arXiv:0804.0614]

a pair of $SU(3)$-structures with $(m^{\Lambda I}, m^{\Lambda}{}_I)$: D. Gaiotto and A. Tomasiello [arXiv:0904.3959]

☞ Consider spacetime metric (extremal, static, spherically symmetric $\to \mathrm{AdS}_2 \times S^2$)

$$\mathrm{d}s^2 \;=\; -\mathrm{e}^{2A(r)}\mathrm{d}t^2 + \mathrm{e}^{-2A(r)}\mathrm{d}r^2 + \mathrm{e}^{2C(r)} r^2 \big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\big)$$

☞ Define electromagnetic charges

$$p^{\Lambda} \;\equiv\; \frac{1}{4\pi}\int_{S^2} F_2^{\Lambda}\,, \qquad q_{\Lambda} \;\equiv\; \frac{1}{4\pi}\int_{S^2} \widetilde{F}_{\Lambda 2}$$

$$I_1 \;\equiv\; -\frac{1}{2}\Big[p^{\Lambda}\mu_{\Lambda\Sigma}\,p^{\Sigma} + (q_{\Lambda} - \nu_{\Lambda\Gamma}p^{\Gamma})(\mu^{-1})^{\Lambda\Sigma}(q_{\Sigma} - \nu_{\Sigma\Delta}p^{\Delta})\Big]$$

$\widetilde{F}_{\Lambda 2} \equiv \nu_{\Lambda\Sigma}F_2^{\Sigma} + \mu_{\Lambda\Sigma}(*F_2^{\Sigma})$

☞ Impose (covariantly) constant condition

$$0 \;\equiv\; \partial_{\mu}\mathrm{t}\,, \quad 0 \;\equiv\; \partial_{\mu}\varphi\,, \quad 0 \;\equiv\; D_{\mu}\xi^0\,, \quad 0 \;\equiv\; D_{\mu}\widetilde{\xi}_0\,, \quad 0 \;\equiv\; \partial_{[\mu}B_{\nu\rho]}$$

The equation of motion for $g_{\mu\nu}$:

$$\delta g_{tt} - \delta g_{rr} : \quad \mathrm{e}^{2C(r)} = \frac{\mathrm{e}^{2c_2}}{r^2}(c_1 r + 1)^2$$

$$\delta g_{\theta\theta}, \delta g_{\phi\phi} : \quad \mathrm{e}^{2A(r)} = \mathrm{e}^{-4c_2} \frac{6\, I_1 - \mathrm{e}^{4c_2}(c_1 r + 1)}{3\, c_1^2 (c_1 r + 1)^2} \Big[(c_1 r + 1)^3 V + 6\, c_1 \big\{ a_1 - c_1 a_2 (c_1 r + 1) \big\} \Big]$$

$$\delta g_{rr} : \quad a_2 = \frac{1}{2(c_1)^2} \mathrm{e}^{2c_2}$$

$C(r)$ and $A(r)$ are expressed in terms of I_1, V and constants of integration $\{a_i, c_i\}$

$$\mathrm{e}^{2A(r)} = \frac{\mathrm{e}^{-2c_2}}{(c_1)^2} - \frac{2a_1}{c_1(c_1 r + 1)} + \frac{\mathrm{e}^{4c_2} I_1}{(c_1)^2 (c_1 r + 1)^2} - \frac{V}{3(c_1)^2}(c_1 r + 1)^2 \equiv 1 - \frac{2\eta}{r_{\text{new}}} + \frac{\mathcal{Z}^2}{r_{\text{new}}^2} - \frac{\Lambda_{\text{c.c.}}}{3} r_{\text{new}}^2$$

Choosing $c_1 r + 1 \equiv r_{\text{new}}$ (and $c_1 \equiv 1$), we can read the Black Hole information:

$\mathrm{e}^{-2c_2} \equiv 1$:	scalar curvature of S^2	$a_1 \equiv \eta$:	mass parameter
$I_1 \equiv \mathcal{Z}^2$:	square of charges	$V \equiv \Lambda_{\text{c.c.}}$:	cosmological constant

The equations of motion for $\mathfrak{t}$, φ, ξ^0 :

$$\delta\mathfrak{t} : \quad 0 = \frac{e^{-4C}}{r^4}\frac{\partial I_1}{\partial \mathfrak{t}} + \frac{\partial V}{\partial \mathfrak{t}} \quad \longrightarrow \quad \frac{\partial V}{\partial \mathfrak{t}} = 0 \text{ and } \frac{\partial I_1}{\partial \mathfrak{t}} = 0$$

$$\delta\varphi : \quad 0 = 2\,V_{\mathsf{NS}} + 4\,V_{\mathsf{R}} \quad \longrightarrow \quad V = V_{\mathsf{NS}} + V_{\mathsf{R}} = \frac{1}{2}V_{\mathsf{NS}} = -V_{\mathsf{R}}$$

$$\delta\xi^0 : \quad 0 = \frac{\partial V}{\partial \xi^0} = \cancel{\frac{\partial V_{\mathsf{NS}}}{\partial \xi^0}} + \frac{\partial V_{\mathsf{R}}}{\partial \xi^0} \qquad \left(0 = \frac{\partial V}{\partial \widetilde{\xi}_0} \text{ is trivial}\right)$$

Regular Solution

$$\big\{\mathfrak{t}\,,\ \xi^0\,,\ \varphi\,;\ V\big\}_{\mathsf{BHs}} = \big\{\mathfrak{t}_*\,,\ \xi^0_*\,,\ \varphi_*\,;\ \Lambda_{\mathsf{c.c.}}\big\}_{\mathsf{Vacua}}$$

constant in whole region

The equations of motion for $\mathfrak{t}$, $B_{\mu\nu}$:

$$\delta\mathfrak{t} : \quad 0 = \frac{\mathrm{e}^{-4C}}{r^4}\frac{\partial I_1}{\partial \mathfrak{t}} + \frac{\partial V}{\partial \mathfrak{t}} \quad \longrightarrow \quad \frac{\partial V}{\partial \mathfrak{t}} = 0 \text{ and } \frac{\partial I_1}{\partial \mathfrak{t}} = 0$$

$$\delta B_{\mu\nu} : \quad 0 = m_{\mathsf{R}}^{\Lambda}\,\mu_{\Lambda\Sigma}\Big(\frac{\sqrt{-g}}{2}\,\epsilon_{\mu\nu\rho\sigma}F^{\Sigma\rho\sigma}\Big) + m_{\mathsf{R}}^{\Lambda}\,\nu_{\Lambda\Sigma}\,F_{\mu\nu}^{\Sigma} - \big(e_{\mathsf{R}\Lambda} - e_{\Lambda 0}\,\xi^0\big)F_{\mu\nu}^{\Lambda}$$

$$\text{and} \quad 0 = D_{\mu}\widetilde{\xi}_0 \quad \longrightarrow \quad 0 = [\partial_{\mu}, \partial_{\nu}]\,\widetilde{\xi}_0 = e_{\Lambda 0}\,F_{\mu\nu}^{\Lambda}$$

$$\text{with} \quad F_{\theta\phi}^{\Lambda} = p^{\Lambda}\sin\theta\,, \quad F_{tr}^{\Lambda} = -\frac{1}{r_{\mathsf{new}}^2}(\mu^{-1})^{\Lambda\Sigma}\big(q_{\Sigma} - \nu_{\Sigma\Gamma}\,p^{\Gamma}\big)$$

Solve them to find an appropriate BH charge configuration

It turns out that all the charges are zero : $p^{\Lambda} = 0 = q_{\Lambda}$ (highly non-trivial)

$$\therefore \; I_1 \equiv \mathcal{Z}^2 = 0 \, , \quad F^{\Lambda}_{\mu\nu} = 0$$

✓ $0 = F^1_{\mu\nu} = 2\,\partial_{[\mu} A^1_{\nu]} + \cancel{m^1_{\mathsf{R}}}\, B_{\mu\nu} \;\rightarrow\; A^1_{\mu} = \partial_{\mu}\lambda \equiv 0$ (gauge-fixing)

✓ $0 = F^0_{\mu\nu} = 2\,\partial_{[\mu} A^0_{\nu]} + m^0_{\mathsf{R}}\, B_{\mu\nu} \;\rightarrow\; 2\,\partial_{[\mu} A^0_{\nu]} = -m^0_{\mathsf{R}}\, B_{\mu\nu} =$ (constant)

✓ $0 = D_{\mu}\widetilde{\xi}_0 = \partial_{\mu}\widetilde{\xi}_0 - \cancel{e_{00}}\, A^0_{\mu} - e_{10}\, \cancel{A^1_{\mu}} = \partial_{\mu}\widetilde{\xi}_0$ $(\because\, e_{00} = 0 = A^1_{\mu})$

✓ $\Lambda_{\text{c.c.}} \equiv V < 0$

$\eta \equiv a_1$ is still arbitrary

Schwarzschild-AdS Black Holes

- Black holes from CY :

$$\boxed{\frac{\partial I_1}{\partial \mathfrak{t}} = 0}$$

Value of vector modulus $\mathfrak{t}$ is not fixed at infinity $\rightarrow$ attractor mechanism

BH charges govern value of fields at horizon

BH mass is given by BH charges

- Black holes from non-CY :

$$\boxed{\frac{\partial V}{\partial \mathfrak{t}} = 0} \quad \text{and} \quad \boxed{\frac{\partial I_1}{\partial \mathfrak{t}} = 0}$$

Value of vector modulus $\mathfrak{t}$ is (mostly) fixed at infinity $\rightarrow$ moduli stabilization

BH charges are governed by geometric- and RR-flux charges

BH mass is arbitrary

(covariantly) constant solutions $\longrightarrow$ all the charges are zero!

In order to find a black hole with non-trivial charges,

the covariantly constant condition must be (partially) relaxed.

cf.)

Gauged SUGRA without hypermultiplets $\longleftarrow$ Fayet-Iliopoulos parameters can be involved

There are charged black hole solutions with constant scalars.

Bellucci, Ferrara, Marrani and Yeranyan [arXiv:0802.0141]

Fin.