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Flux Compactifications, $\mathcal{N} = 2$ Gauged Supergravities and Black Holes

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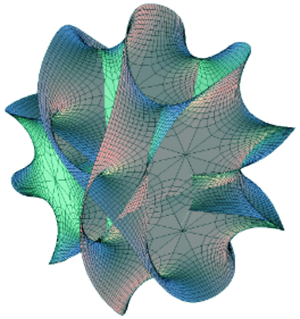
KEK
HIGH ENERGY ACCELERATOR RESEARCH ORGANIZATION

Flux Compactification beyond Calabi-Yau



How important is the flux compactification beyond CY in 10D Strings?

1. CY $\longrightarrow$ 4D ungauged SUGRA
   $\longrightarrow$ Fluxes break 10D Eqs. of Motion
2. non-CY with fluxes $\longrightarrow$ 4D gauged SUGRA
   non-CY: $SU(3)$-structure with torsion, etc.

- In 4D $\mathcal{N} = 2$ ungauged SUGRA $\longrightarrow$ No scalar potential

(Extremal) charged Black holes in asymptotic flat has been investigated
Charges = series of D-branes wrapped on cycles in CY

- In 4D $\mathcal{N} = 2$ gauged SUGRA $\longrightarrow$ Scalar potential turned on

Cosmological constant appears as VEV of scalar potential

3 types of gauged SUGRA

KEK 理論研究会2011：木村の講演スライド を参照
(東北地方太平洋沖地震とそれに伴う東日本大震災のため開催運営を断念)

10D type IIA on non-CY with fluxes

$\downarrow$

4D $\mathcal{N}=2$ abelian gauged SUGRA with scalar potential

and $Q_{\text{NS}} = \begin{pmatrix} e_{\Lambda}{}^{I} & e_{\Lambda I} \\ m^{\Lambda I} & m^{\Lambda}{}_{I} \end{pmatrix}$, $c_{\text{R}} = \begin{pmatrix} m_{\text{R}}^{\Lambda} \\ e_{\text{R}\Lambda} \end{pmatrix}$

$e_{\Lambda}{}^{I}$, $e_{\Lambda I}$: Geometric flux charges (NSNS-flux charges)

$m^{\Lambda I}$, $m^{\Lambda}{}_{I}$: Nongeometric flux charges ("magnetic" NSNS-flux charges)

$e_{\text{R}\Lambda}$, m_{R}^{Λ} : RR-flux charges (with Romans' mass m_{R}^{0})

$0 = m^{\Lambda I} = m^{\Lambda}{}_{I} = m_{\text{R}}^{\Lambda}$	$0 = m^{\Lambda I} = m^{\Lambda}{}_{I}$	generic
Standard Gauged SUGRA	Gauged SUGRA	Gauged SUGRA
n_{V} VMs	n_{V} VMs	n_{V} VMs
n_{H} HMs	n_{H} HMs	$\widetilde{n}_{\text{H}}$ HMs
1 UHM	1 TM	n_{T} TMs
[hep-th/9605032]	[hep-th/0312210]	[hep-th/0409097]

VM : vector multiplet (U)HM : (universal) hypermultiplet TM : tensor multiplet

Anti-de Sitter Black Holes in 4D $\mathcal{N} = 2$ Gauged SUGRA

Comments

☞ AdS-BH with naked singularity in pure AdS SUGRA

L.J. Romans [hep-th/9203018], M.M. Caldarelli and D. Klemm [hep-th/9808097], etc.

SUSY solution of rotating AdS black hole with regular horizon

☞ AdS-BH with regular horizon in Gauged SUGRA with VMs (no HMs)

[hep-th/9903143], [arXiv:0911.4926], [arXiv:1012.4314], etc.

Question

How can we obtain AdS-BH solutions

with hypermultiplets?

— **Setup** and **Result** —

- 4D $\mathcal{N} = 2$ gauged SUGRA with VM and UHM
  from 10D type IIA on non-CY with fluxes
- Ansatze for matter fields
- Regular solution (AdS Black Hole)

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Coset space $G_2/SU(3)$ D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

✓ nearly-Kähler (almost complex geometry)

✓ NSNS-sector : torsion and H-flux

✓ RR-sector : 2-, 4-form and Romans' mass (0-form)

✓ 1 VM with cubic prepotential $\mathcal{F} = \dfrac{X^1 X^1 X^1}{X^0}$

✓ 1 UHM (no other HMs)

nilmanifolds and solvmanifolds: M. Graña, R. Minasian, M. Petrini and A. Tomasiello [hep-th/0609124]
coset spaces with $SU(3)$- or $SU(2)$-structure: P. Koerber, D. Lüst and D. Tsimpis [arXiv:0804.0614]
a pair of $SU(3)$-structures with $(m^{\Lambda I}, m^{\Lambda}{}_I)$: D. Gaiotto and A. Tomasiello [arXiv:0904.3959]

10D type IIA on $G_2/SU(3)$ with fluxes

$\downarrow$

4D $\mathcal{N} = 2$ abelian gauged SUGRA with B-field ($\Lambda = 0, 1$ and $\boldsymbol{\xi^0} \equiv (\xi^0, \widetilde{\xi}_0)^{\mathrm{T}}$)

$$S = \int \Big[\frac{1}{2} R\,(*1) + \frac{1}{2} \mu_{\Lambda\Sigma}\, F^{\Lambda} \wedge *F^{\Sigma} + \frac{1}{2} \nu_{\Lambda\Sigma}\, F^{\Lambda} \wedge F^{\Sigma} - g_{\mathfrak{t}\bar{\mathfrak{t}}}\, \mathrm{d}\mathfrak{t} \wedge *\mathrm{d}\bar{\mathfrak{t}}$$
$$- \mathrm{d}\varphi \wedge *\mathrm{d}\varphi - \frac{\mathrm{e}^{-4\varphi}}{4} \mathrm{d}B \wedge *\mathrm{d}B - \frac{\mathrm{e}^{2\varphi}}{2} \Big(D\xi^0 \wedge *D\xi^0 + D\widetilde{\xi}_0 \wedge *D\widetilde{\xi}_0 \Big) + \mathrm{d}B \wedge \xi^0\, \mathrm{d}\widetilde{\xi}_0$$
$$+ \mathrm{d}B \wedge \big(e_{\mathsf{R}\Lambda} - e_{\Lambda 0}\, \xi^0 \big) A^{\Lambda} - \frac{1}{2} m_{\mathsf{R}}^{\Lambda}\, e_{\mathsf{R}\Lambda}\, B \wedge B - V\,(*1) \Big]$$

- $g_{\mu\nu}$, $\mathfrak{t}$, $B_{\mu\nu}$, φ; $(e_{\Lambda}{}^0, e_{\Lambda 0})$: NS-NS sector
- A_{μ}^{Λ}, ξ^0, $\widetilde{\xi}_0$; $(m_{\mathsf{R}}^{\Lambda}, e_{\mathsf{R}\Lambda})$: R-R sector
- GM : $(g_{\mu\nu}, A_{\mu}^0)$, VM : $(A_{\mu}^a, \mathfrak{t})$, UHM $\to$ TM : $(\varphi, B_{\mu\nu}, \xi^0, \widetilde{\xi}_0)$
- $D\xi^0 = \mathrm{d}\xi^0 - e_{\Lambda}{}^0 A_1^{\Lambda}$, $\quad D\widetilde{\xi}_0 = \mathrm{d}\widetilde{\xi}_0 - e_{\Lambda 0} A_1^{\Lambda}$
- $F_2^{\Sigma} = \mathrm{d}A_1^{\Sigma} + m_{\mathsf{R}}^{\Sigma} B_2$
- $V(\mathfrak{t}, \varphi, \xi^0) = V_{\mathsf{NS}}(\mathfrak{t}, \varphi) + V_{\mathsf{R}}(\mathfrak{t}, \varphi, \xi^0)$

Precise data on $G_2/SU(3)$:

$$e_{10} \neq 0,\ m_{\mathsf{R}}^0 \neq 0,\ e_{\mathsf{R}0} \neq 0$$
$$e_{\Lambda}{}^0 = 0 = e_{00}$$
$$m_{\mathsf{R}}^1 = 0 = e_{\mathsf{R}1}$$

$\mu_{\Lambda\Sigma} \equiv \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}$, $\nu_{\Lambda\Sigma} \equiv \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}$

D. Cassani [arXiv:0804.0595]

$$
\begin{aligned}
R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} &= \frac{1}{4} g_{\mu\nu}\, \mu_{\Lambda\Sigma} F^{\Lambda}_{\rho\sigma} F^{\Sigma\rho\sigma} - \mu_{\Lambda\Sigma} F^{\Lambda}_{\mu\rho} F^{\Sigma}_{\nu\sigma}\, g^{\rho\sigma} - g_{\mu\nu}\, g_{\mathsf{t}\bar{\mathsf{t}}} \partial_\rho \mathsf{t} \partial^\rho \bar{\mathsf{t}} + 2 g_{\mathsf{t}\bar{\mathsf{t}}}\, \partial_\mu \mathsf{t} \partial_\nu \bar{\mathsf{t}} \\
&\quad - g_{\mu\nu}\, \partial_\rho \varphi \partial^\rho \varphi + 2 \partial_\mu \varphi \partial_\nu \varphi - \frac{\mathrm{e}^{-4\varphi}}{24} g_{\mu\nu}\, H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} + \frac{\mathrm{e}^{-4\varphi}}{4} H_{\mu\rho\sigma} H_{\nu}{}^{\rho\sigma} \qquad (\delta g_{\mu\nu}) \\
&\quad - \frac{\mathrm{e}^{2\varphi}}{2} g_{\mu\nu} \Big(D_\rho \xi^0 D^\rho \xi^0 + D_\rho \widetilde{\xi}_0 D^\rho \widetilde{\xi}_0 \Big) + \mathrm{e}^{2\varphi} \Big(D_\mu \xi^0 D_\nu \xi^0 + D_\mu \widetilde{\xi}_0 D_\nu \widetilde{\xi}_0 \Big) - g_{\mu\nu} V \,,
\end{aligned}
$$

$$
0 = \frac{1}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, \mu_{\Lambda\Sigma} F^{\Sigma\mu\sigma} \Big) - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu \Big(\nu_{\Lambda\Sigma} F^{\Sigma}_{\nu\rho} \Big) + \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu B_{\nu\rho} (e_{\mathsf{R}\Lambda} - \xi^0 e_{\Lambda 0}) - \mathrm{e}^{2\varphi} Q_{\Lambda \mathbf{0}} D^\sigma \xi^{\mathbf{0}} \,, \qquad (\delta A^{\Lambda}_{\mu})
$$

$$
0 = \frac{1}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, g_{c\bar{b}}\, g^{\mu\nu} \partial_\nu \bar{\mathsf{t}} \Big) + \frac{1}{4} \partial_{\mathsf{t}} (\mu_{\Lambda\Sigma}) F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}} \partial_{\mathsf{t}} (\nu_{\Lambda\Sigma}) F^{\Lambda}_{\mu\nu} F^{\Sigma}_{\rho\sigma} - \partial_{\mathsf{t}} g_{\mathsf{t}\bar{\mathsf{t}}}\, \partial_\mu \mathsf{t} \partial^\mu \bar{\mathsf{t}} - \partial_{\mathsf{t}} V \,, \qquad (\delta \mathsf{t})
$$

$$
0 = \frac{2}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \varphi \Big) + \frac{\mathrm{e}^{4\varphi}}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} - \mathrm{e}^{2\varphi} \Big(D_\mu \xi^0 D^\mu \xi^0 + D_\mu \widetilde{\xi}_0 D^\mu \widetilde{\xi}_0 \Big) - \partial_\varphi V \,, \qquad (\delta \varphi)
$$

$$
\begin{aligned}
0 &= \frac{1}{\sqrt{-g}} \partial_\mu \Big(\mathrm{e}^{-4\varphi} \sqrt{-g} H^{\mu\rho\sigma} \Big) + \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} \Big[D_\mu \xi^{\mathbf{0}} (\mathbb{C}_{\mathsf{H}})_{\mathbf{00}} D_\nu \xi^{\mathbf{0}} + (e_{\mathsf{R}\Lambda} - \xi^0 e_{\Lambda 0}) F^{\Lambda}_{\mu\nu} \Big] \qquad (\delta B_{\mu\nu}) \\
&\quad + 2 m^{\Lambda}_{\mathsf{R}} \mu_{\Lambda\Sigma} F^{\Sigma\rho\sigma} - \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} m^{\Lambda}_{\mathsf{R}} \nu_{\Lambda\Sigma} F^{\Sigma}_{\mu\nu} \,,
\end{aligned}
$$

$$
0 = - \frac{2}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, \mathrm{e}^{2\varphi} g^{\mu\nu} D_\nu \xi^{\mathbf{0}} \Big) + \frac{\partial V}{\partial \xi^{\mathbf{0}}} - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu B_{\nu\rho} D_\sigma \xi^{\mathbf{0}} (\mathbb{C}_{\mathsf{H}})_{\mathbf{00}} \,. \qquad (\delta \xi^{\mathbf{0}})
$$

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Vacuum I : $\mathcal{N} = 1$

$$t_* \;=\; -\frac{\pm 1 + \mathrm{i}\sqrt{15}}{2} \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{\mathsf{R}0}}{m_{\mathsf{R}}^0} \right| \right]^{1/3} , \quad \xi_*^0 \;=\; -\frac{2}{5} \left[\frac{2\sqrt{3}\, m_{\mathsf{R}}^0 (e_{\mathsf{R}0})^2}{5\, e_{10}} \right]^{1/3} , \quad \exp(\varphi_*) \;=\; \frac{4}{3} \left[\frac{\sqrt{5}\, e_{10}}{\sqrt{3}\, m_{\mathsf{R}}^0 (e_{\mathsf{R}0})^2} \right]^{1/3}$$

$$V_* \;=\; -\frac{5\sqrt{5}}{2} \left[\frac{5\,(e_{10})^4}{2\sqrt{3}\, |m_{\mathsf{R}}^0 (e_{\mathsf{R}0})^5|} \right]^{1/3} \;\equiv\; \Lambda_{\mathsf{c.c.}}^{\mathrm{I}} \;<\; 0$$

Vacuum II : $\mathcal{N} = 0$

$$t_* \;=\; \big(\pm 1 - \mathrm{i}\sqrt{3} \big) \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{\mathsf{R}0}}{m_{\mathsf{R}}^0} \right| \right]^{1/3} , \quad \xi_*^0 \;=\; \left[\frac{9\, m_{\mathsf{R}}^0 (e_{\mathsf{R}0})^2}{25\, e_{10}} \right]^{1/3} , \quad \exp(\varphi_*) \;=\; \frac{2}{3} \left[\frac{25\, e_{10}}{\sqrt{3}\, m_{\mathsf{R}}^0 (e_{\mathsf{R}0})^2} \right]^{1/3}$$

$$V_* \;=\; -\frac{80}{27} \left[\frac{25\,(e_{10})^4}{\sqrt{3}\, |m_{\mathsf{R}}^0 (e_{\mathsf{R}0})^5|} \right]^{1/3} \;\equiv\; \Lambda_{\mathsf{c.c.}}^{\mathrm{II}} \;<\; 0$$

Vacuum III : $\mathcal{N} = 0$

$$t_* \;=\; -\mathrm{i} \left[\frac{12}{\sqrt{5}\,(e_{10})^2} \left| \frac{e_{\mathsf{R}0}}{m_{\mathsf{R}}^0} \right| \right]^{1/3} , \quad \xi_*^0 \;=\; 0 \,, \quad \exp(\varphi_*) \;=\; \sqrt{5} \left[\frac{5\, e_{10}}{18\, m_{\mathsf{R}}^0 (e_{\mathsf{R}0})^2} \right]^{1/3}$$

$$V_* \;=\; -\frac{25\sqrt{5}}{6} \left[\frac{5\,(e_{10})^4}{18\, |m_{\mathsf{R}}^0 (e_{\mathsf{R}0})^5|} \right]^{1/3} \;\equiv\; \Lambda_{\mathsf{c.c.}}^{\mathrm{III}} \;<\; 0$$

Note: $m_{\mathsf{R}}^0 > 0$; $\widetilde{\xi}_0$ is not fixed ; $\Lambda_{\mathsf{c.c.}}^{\mathrm{II}} < \Lambda_{\mathsf{c.c.}}^{\mathrm{I}} < \Lambda_{\mathsf{c.c.}}^{\mathrm{III}}$

D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

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☞ Consider spacetime metric (extremal, static, spherically symmetric $\to \mathrm{AdS}_2 \times S^2$)

$$\mathrm{d}s^2 \;=\; -\mathrm{e}^{2A(r)}\mathrm{d}t^2 + \mathrm{e}^{-2A(r)}\mathrm{d}r^2 + \mathrm{e}^{2C(r)} r^2 \big(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2\big)$$

☞ Impose (covariantly) constant condition

$$0 \;\equiv\; \partial_\mu \mathfrak{t}\,, \quad 0 \;\equiv\; \partial_\mu \varphi\,, \quad 0 \;\equiv\; D_\mu \xi^0\,, \quad 0 \;\equiv\; D_\mu \widetilde{\xi}_0\,, \quad 0 \;\equiv\; \partial_\mu B_{\nu\rho}$$

☞ Define electromagnetic charges

$$p^\Lambda \;\equiv\; \frac{1}{4\pi}\int_{S^2} F_2^\Lambda\,, \quad q_\Lambda \;\equiv\; \frac{1}{4\pi}\int_{S^2} \widetilde{F}_{\Lambda 2}$$

$$I_1 \;\equiv\; -\frac{1}{2}\Big[p^\Lambda \mu_{\Lambda\Sigma}\, p^\Sigma + (q_\Lambda - \nu_{\Lambda\Gamma} p^\Gamma)(\mu^{-1})^{\Lambda\Sigma}(q_\Sigma - \nu_{\Sigma\Delta} p^\Delta)\Big]$$

$\widetilde{F}_{\Lambda 2} \equiv \nu_{\Lambda\Sigma} F_2^\Sigma + \mu_{\Lambda\Sigma}(* F_2^\Sigma)$

The equation of motion for $g_{\mu\nu}$:

$$\delta g_{tt} - \delta g_{rr} : \quad e^{2C(r)} = \frac{e^{2c_2}}{r^2}\big(c_1 r + 1\big)^2$$

$$\delta g_{\theta\theta}, \delta g_{\phi\phi} : \quad e^{2A(r)} = e^{-4c_2} \frac{6\, I_1 - e^{4c_2}(c_1 r + 1)}{3\, c_1^2 (c_1 r + 1)^2} \Big[(c_1 r + 1)^3 V + 6\, c_1 \big\{ a_1 - c_1 a_2 (c_1 r + 1) \big\} \Big]$$

$$\delta g_{rr} : \quad a_2 = \frac{1}{2(c_1)^2} e^{2c_2}$$

$C(r)$ and $A(r)$ are expressed in terms of I_1, V and constants of integration $\{a_i, c_i\}$

$$e^{2A(r)} = \frac{e^{-2c_2}}{(c_1)^2} - \frac{2a_1}{c_1(c_1 r + 1)} + \frac{e^{4c_2} I_1}{(c_1)^2 (c_1 r + 1)^2} - \frac{V}{3(c_1)^2}(c_1 r + 1)^2 \equiv 1 - \frac{2\eta}{r_{\text{new}}} + \frac{\mathcal{Z}^2}{r_{\text{new}}^2} - \frac{\Lambda_{\text{c.c.}}}{3} r_{\text{new}}^2$$

Choosing $c_1 r + 1 \equiv r_{\text{new}}$ (and $c_1 \equiv 1$), we can read the "Black Hole" information:

$e^{-2c_2} \equiv 1$:	scalar curvature of S^2	$a_1 \equiv \eta$:	mass parameter
$I_1 \equiv \mathcal{Z}^2$:	square of charges	$V \equiv \Lambda_{\text{c.c.}}$:	cosmological constant

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The equations of motion for $\mathfrak{t}$, φ, ξ^0 :

$$\delta\mathfrak{t}: \quad 0 = \frac{e^{-4C}}{r^4}\frac{\partial I_1}{\partial\mathfrak{t}} + \frac{\partial V}{\partial\mathfrak{t}} \quad \longrightarrow \quad \frac{\partial V}{\partial\mathfrak{t}} = 0 \text{ and } \frac{\partial I_1}{\partial\mathfrak{t}} = 0$$

$$\delta\varphi: \quad 0 = 2\,V_{\mathsf{NS}} + 4\,V_{\mathsf{R}} \quad \longrightarrow \quad V = V_{\mathsf{NS}} + V_{\mathsf{R}} = \frac{1}{2}V_{\mathsf{NS}} = -V_{\mathsf{R}}$$

$$\delta\xi^0: \quad 0 = \frac{\partial V}{\partial\xi^0} = \cancel{\frac{\partial V_{\mathsf{NS}}}{\partial\xi^0}} + \frac{\partial V_{\mathsf{R}}}{\partial\xi^0} \qquad \left(0 = \frac{\partial V}{\partial\tilde{\xi}_0} \text{ is trivial}\right)$$

Regular Solution

$$\{\mathfrak{t}\,,\ \xi^0\,,\ \varphi\,;\ V\}_{\mathsf{BHs}} = \{\mathfrak{t}_*\,,\ \xi^0_*\,,\ \varphi_*\,;\ \Lambda_{\text{c.c.}}\}_{\mathsf{Vacua}}$$

constant in whole region

The equations of motion for $\mathfrak{t}$, $B_{\mu\nu}$:

$$\delta\mathfrak{t}: \quad 0 = \frac{\mathrm{e}^{-4C}}{r^4} \frac{\partial I_1}{\partial \mathfrak{t}} + \frac{\partial V}{\partial \mathfrak{t}} \quad \longrightarrow \quad \frac{\partial V}{\partial \mathfrak{t}} = 0 \text{ and } \frac{\partial I_1}{\partial \mathfrak{t}} = 0$$

$$\delta B_{\mu\nu}: \quad 0 = m_{\mathsf{R}}^{\Lambda}\, \mu_{\Lambda\Sigma} \Big(\frac{\sqrt{-g}}{2}\, \epsilon_{\mu\nu\rho\sigma} F^{\Sigma\rho\sigma} \Big) + m_{\mathsf{R}}^{\Lambda}\, \nu_{\Lambda\Sigma}\, F_{\mu\nu}^{\Sigma} - \big(e_{\mathsf{R}\Lambda} - e_{\Lambda 0}\, \xi^0 \big) F_{\mu\nu}^{\Lambda}$$

$$\text{and} \quad 0 = D_{\mu} \widetilde{\xi}_0 \quad \longrightarrow \quad 0 = [\partial_{\mu}, \partial_{\nu}]\, \widetilde{\xi}_0 = e_{\Lambda 0}\, F_{\mu\nu}^{\Lambda}$$

$$\text{with} \quad F_{\theta\phi}^{\Lambda} = p^{\Lambda} \sin\theta\,, \quad F_{tr}^{\Lambda} = -\frac{1}{r_{\mathsf{new}}^2} (\mu^{-1})^{\Lambda\Sigma} \big(q_{\Sigma} - \nu_{\Sigma\Gamma}\, p^{\Gamma} \big)$$

Solve them to find an appropriate BH charge configuration

It turns out that all the charges are zero : $p^{\Lambda} = 0 = q_{\Lambda}$ (highly non-trivial)

$$\therefore \; I_1 \equiv \mathcal{Z}^2 = 0 \, , \quad F^{\Lambda}_{\mu\nu} = 0$$

✓ $0 = F^1_{\mu\nu} = 2\,\partial_{[\mu} A^1_{\nu]} + \cancel{m^1_{\mathrm{R}}}\, B_{\mu\nu} \rightarrow A^1_{\mu} = \partial_{\mu}\lambda \equiv 0$ (gauge-fixing)

✓ $0 = F^0_{\mu\nu} = 2\,\partial_{[\mu} A^0_{\nu]} + m^0_{\mathrm{R}}\, B_{\mu\nu} \rightarrow 2\,\partial_{[\mu} A^0_{\nu]} = -m^0_{\mathrm{R}}\, B_{\mu\nu} =$ (constant)

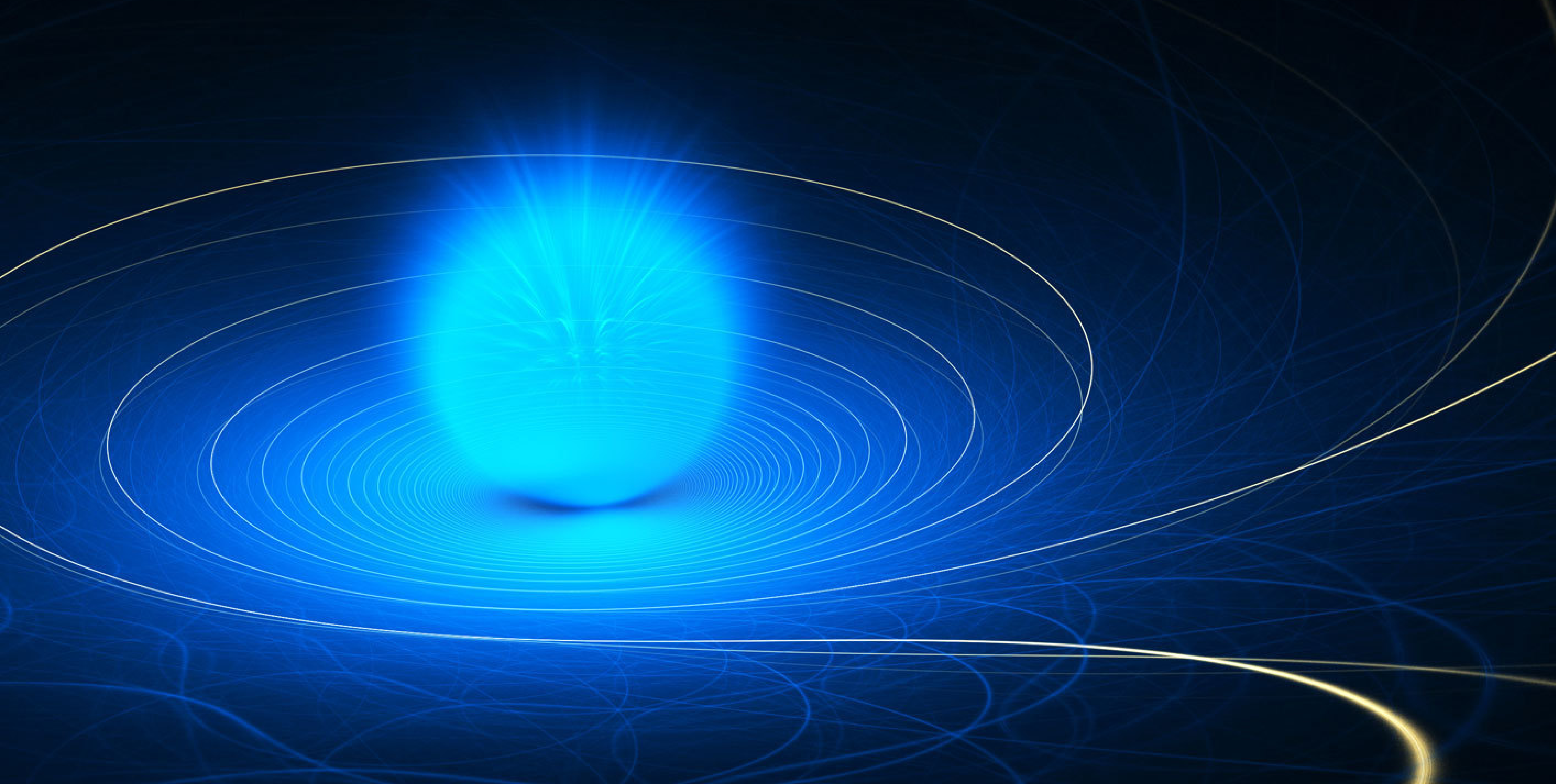
✓ $0 = D_{\mu}\widetilde{\xi}_0 = \partial_{\mu}\widetilde{\xi}_0 - \cancel{e_{00}}\, A^0_{\mu} - e_{10}\, \cancel{A^1_{\mu}} = \partial_{\mu}\widetilde{\xi}_0$ $(\because e_{00} = 0 = A^1_{\mu})$

✓ $\Lambda_{\text{c.c.}} \equiv V < 0$

$\eta \equiv a_1$ is still arbitrary

Schwarzschild-AdS Black Holes!

Schwarzschild-AdS Black Holes!



- Black holes from CY :

$$\boxed{\frac{\partial I_1}{\partial \mathfrak{t}} = 0}$$

Value of vector modulus $\mathfrak{t}$ is not fixed at infinity $\to$ attractor mechanism

BH charges govern value of fields at horizon

BH mass is given by BH charges

- Black holes from non-CY :

$$\boxed{\frac{\partial V}{\partial \mathfrak{t}} = 0} \quad \text{and} \quad \boxed{\frac{\partial I_1}{\partial \mathfrak{t}} = 0}$$

Value of vector modulus $\mathfrak{t}$ is (mostly) fixed at infinity $\to$ moduli stabilization

BH charges are governed by geometric- and RR-flux charges

BH mass is arbitrary

Contents

- ☑ Studied : 4D $\mathcal{N} = 2$ gauged SUGRA with VMs and TM(UHM) via flux compactification.
- ☑ Reconfirmed : Romans' mass is inevitable.
- ☑ Imposed : covariantly constant condition.
- ☑ Found : Schwarzschild-AdS BHs.

Different from cases of Calabi-Yau

- ☑ Find charged AdS-BH solutions.
- ☑ Consider a stationary AdS-BH.
- ☑ Various directions!

Thanks for your attention

Appendix

- Terminology
- CY compactification in type IIA
- Geometric flux compactifications in type IIA
- Scalar potential

Prepotential :	$\mathcal{F}$ is a holomorphic function of X^Λ of degree two $(\mathcal{F}_\Lambda = \partial\mathcal{F}/\partial X^\Lambda)$
Kähler potential :	$\mathcal{K}_\mathsf{V} = -\log\big[\mathrm{i}\big(\overline{X}^\Lambda\mathcal{F}_\Lambda - X^\Lambda\overline{\mathcal{F}}_\Lambda\big)\big]$
Symplectic section :	$\Pi_\mathsf{V} \equiv \mathrm{e}^{\mathcal{K}_\mathsf{V}/2}\begin{pmatrix} X^\Lambda \\ \mathcal{F}_\Lambda \end{pmatrix} \equiv \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \quad 1 = \mathrm{i}\big(\overline{L}^\Lambda M_\Lambda - L^\Lambda\overline{M}_\Lambda\big)$
Kähler metric :	$g_{a\bar{b}} = \dfrac{\partial}{\partial t^a}\dfrac{\partial}{\partial\bar{t}^b}\mathcal{K}_\mathsf{V}\,, \quad t^a = \dfrac{X^a}{X^0}$
Kähler covariant derivative :	$D_a\Pi_\mathsf{V} = \Big(\dfrac{\partial}{\partial t^a} + \dfrac{1}{2}\dfrac{\partial\mathcal{K}_\mathsf{V}}{\partial t^a}\Big)\Pi_\mathsf{V} \equiv \begin{pmatrix} f_a^\Lambda \\ h_{\Lambda a} \end{pmatrix}$
Period matrix :	$\mathcal{N}_{\Lambda\Sigma} = \overline{\mathcal{F}}_{\Lambda\Sigma} + 2\mathrm{i}\dfrac{(\mathrm{Im}\mathcal{F})_{\Lambda\Gamma}X^\Gamma(\mathrm{Im}\mathcal{F})_{\Sigma\Delta}X^\Delta}{X^\Pi(\mathrm{Im}\mathcal{F})_{\Pi\Xi}X^\Xi}$
Formulae :	$M_\Lambda = \mathcal{N}_{\Lambda\Sigma}L^\Sigma, \quad h_{\Lambda a} = \overline{\mathcal{N}}_{\Lambda\Sigma}f_a^\Sigma$
(Symplectic matrix) :	$(\mathbb{M}_\mathsf{V})_{\mathbf{\Lambda\Sigma}} = \begin{pmatrix} \mathbb{1} & -\mathrm{Re}\mathcal{N} \\ 0 & \mathbb{1} \end{pmatrix}\begin{pmatrix} \mathrm{Im}\mathcal{N} & 0 \\ 0 & (\mathrm{Im}\mathcal{N})^{-1} \end{pmatrix}\begin{pmatrix} \mathbb{1} & 0 \\ -\mathrm{Re}\mathcal{N} & \mathbb{1} \end{pmatrix}$

In a similar way... $\Pi_\mathsf{H} \equiv \mathrm{e}^{\mathcal{K}_\mathsf{H}/2}\begin{pmatrix} Z^I \\ \mathcal{G}_I \end{pmatrix}, \quad z^i = \dfrac{Z^i}{Z^0}, \quad \mathcal{K}_\mathsf{H} = -\log\big[\mathrm{i}\big(\overline{Z}^I\mathcal{G}_I - Z^I\overline{\mathcal{G}}_I\big)\big]$, etc.

10D type IIA action $S_{\text{IIA}}^{(10\text{D})} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}$:

$$\begin{aligned} S_{\text{NS}} &= \frac{1}{2} \int e^{-2\phi} \Big\{ \widehat{R} * \mathbb{1} + 4 \mathrm{d}\phi \wedge * \mathrm{d}\phi - \frac{1}{2} \widehat{H}_3 \wedge * \widehat{H}_3 \Big\} \\ S_{\text{R}} + S_{\text{CS}} &= -\frac{1}{4} \int \Big\{ \widehat{F}_2 \wedge * \widehat{F}_2 + (\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3) \wedge * (\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3) \Big\} - \frac{1}{4} \int \widehat{B}_2 \wedge \widehat{F}_4 \wedge \widehat{F}_4 \end{aligned}$$

$\downarrow$ CY

4D $\mathcal{N} = 2$ ungauged SUGRA: *Neither gauge couplings, Nor scalar potential*

$$S^{(4\text{D})} = \int \mathrm{d}^4 x \sqrt{-g} \Big[\frac{1}{2} R + \frac{1}{4} \mathrm{Im} \mathcal{N}_{\Lambda\Sigma} F_{\mu\nu}^{\Lambda} F^{\Sigma \mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}} \mathrm{Re} \mathcal{N}_{\Lambda\Sigma} F_{\mu\nu}^{\Lambda} F_{\rho\sigma}^{\Sigma} - g_{a\bar{b}} \, \partial_\mu \mathfrak{t}^a \partial^\mu \bar{\mathfrak{t}}^{\bar{b}} - h_{uv} \, \partial_\mu q^u \partial^\mu q^v \Big]$$

gravitational multiplet	$g_{\mu\nu}, A_1^0$		
VMs	$A_1^a, \mathfrak{t}^a, \bar{\mathfrak{t}}^{\bar{b}}$	$\mathfrak{t}^a \in \text{SKG}_{\text{V}}$	mirror dual: $\text{SKG}_{\text{V}} \leftrightarrow \text{SKG}_{\text{H}}$
HMs	$z^i, \bar{z}^{\bar{j}}, \xi^i, \widetilde{\xi}_j$	$z^i \in \text{SKG}_{\text{H}}$	
UHM	$\varphi, a, \xi^0, \widetilde{\xi}_0$	$a \leftrightarrow B_2$	Hodge dual

$\mathcal{HM} \rightarrow$ Special QG

$$\underbrace{\{q^u\}}_{4n_{\text{H}}+4} = \underbrace{\{z^i, \bar{z}^{\bar{j}}\}}_{2n_{\text{H}}(\text{SKG}_{\text{H}})} + \underbrace{\{\xi^i, \widetilde{\xi}_j\}}_{2n_{\text{H}}} + \underbrace{\{\varphi, a, \xi^0, \widetilde{\xi}_0\}}_{4\,(\text{UHM})} = \underbrace{\{z^i, \bar{z}^{\bar{j}}\} + \{\varphi\}}_{\text{SKG}_{\text{H}}} + \underbrace{\{a, \xi^I, \widetilde{\xi}_J\}}_{\text{"Heisenberg"}}$$

10D type IIA action $S^{(10\mathrm{D})}_{\mathrm{IIA}} = S_{\mathsf{NS}} + \widetilde{S}_{\mathsf{R}} = S_{\mathsf{NS}} + S_{\mathsf{R}} + S_{\mathsf{CS}}$: (democratic form)

$$S_{\mathsf{NS}} = \frac{1}{2}\int \mathrm{e}^{-2\phi}\Big\{\widehat{R} * \mathbb{1} + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3\Big\}, \quad \widetilde{S}_{\mathsf{R}} = -\frac{1}{8}\int \big[\widehat{\mathbf{F}} \wedge *\widehat{\mathbf{F}}\big]_{10}$$

with "constraint $\widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}})$" and "EoM (Bianchi) $(\mathrm{d} + \widehat{H}\wedge) * \widehat{\mathbf{F}} = 0 \Leftrightarrow (\mathrm{d} - \widehat{H}\wedge)\widehat{\mathbf{F}} = 0$"

$\downarrow$ non-CY with $SU(3)$-structure with $m^{\Lambda}_{\mathsf{R}} = 0$

4D $\mathcal{N} = 2$ abelian gauged SUGRA (with $\xi^{\boldsymbol{I}} \equiv (\xi^I, \widetilde{\xi}_I)^{\mathrm{T}}$):

$$S^{(4\mathrm{D})} = \int \mathrm{d}^4x\sqrt{-g}\,\Big[\frac{1}{2}R + \frac{1}{4}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma} - g_{a\bar{b}}\,\partial_{\mu}\mathfrak{t}^a\partial^{\mu}\bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{\jmath}}\,\partial_{\mu}z^i\partial^{\mu}\bar{z}^{\bar{\jmath}}$$
$$-\partial_{\mu}\varphi\partial^{\mu}\varphi + \frac{\mathrm{e}^{2\varphi}}{2}(\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}}D_{\mu}\xi^{\boldsymbol{I}}D^{\mu}\xi^{\boldsymbol{J}} - \frac{\mathrm{e}^{2\varphi}}{4}\big(D_{\mu}a - \xi^{\boldsymbol{I}}(\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}}D_{\mu}\xi^{\boldsymbol{J}}\big)^2 - V(\mathfrak{t},\bar{\mathfrak{t}},q)\Big]$$

- $(e_{\Lambda}{}^I, e_{\Lambda I})$: geometric flux charges & $e_{\mathsf{R}\Lambda}$: RR-flux charges $\longleftarrow$ non-CY data
  (with constraints $e_{\Lambda}{}^I e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}{}^I = 0$)
- $\mathfrak{t}^a \in \mathsf{SKG}_{\mathsf{V}}$ and $z^i \in \mathsf{SKG}_{\mathsf{H}} \subset \mathcal{HM}$ are ungauged (in general)
- $D_{\mu}\xi^I = \partial_{\mu}\xi^I - e_{\Lambda}{}^I A^{\Lambda}_{\mu}$ & $D_{\mu}\widetilde{\xi}_I = \partial_{\mu}\widetilde{\xi}_I - e_{\Lambda I}A^{\Lambda}_{\mu}$
- $D_{\mu}a = \partial_{\mu}a - (2e_{\mathsf{R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_{\Lambda}{}^I)A^{\Lambda}_{\mu}$
- $V(\mathfrak{t},\bar{\mathfrak{t}},q)$: scalar potential

D. Cassani [arXiv:0804.0595]

Non-vanishing m_{R}^{Λ} dualizes the axion field a in standard SUGRA to B-field.

4D gauged action is different from the standard one:

$$\begin{aligned} S^{(\mathsf{4D})} \;=\; \int \Big[& \frac{1}{2} R (*\mathbb{1}) + \frac{1}{2} \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F_{\mathit{2}}^{\Lambda} \wedge * F_{\mathit{2}}^{\Sigma} + \frac{1}{2} \mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F_{\mathit{2}}^{\Lambda} \wedge F_{\mathit{2}}^{\Sigma} - g_{a\bar{b}} \, \mathrm{d}\mathfrak{t}^a \wedge * \mathrm{d}\bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{\jmath}} \, \mathrm{d}z^i \wedge * \mathrm{d}\bar{z}^{\bar{\jmath}} \\ & - \mathrm{d}\varphi \wedge * \mathrm{d}\varphi - \frac{\mathrm{e}^{-4\varphi}}{4} H_{\mathit{3}} \wedge * H_{\mathit{3}} - \frac{\mathrm{e}^{2\varphi}}{2} (\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{I}} \wedge * D\xi^{\boldsymbol{J}} - V(*\mathbb{1}) \\ & + \frac{1}{2} \mathrm{d}B \wedge \Big[\xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{J}} + \big(2 e_{\mathsf{R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_{\Lambda}{}^I \big) A_{\mathit{1}}^{\Lambda} \Big] - \frac{1}{2} m_{\mathsf{R}}^{\Lambda} e_{\mathsf{R}\Lambda} B_{\mathit{2}} \wedge B_{\mathit{2}} \Big] \end{aligned}$$

Constraints among flux charges:

$$e_{\Lambda}{}^I e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}{}^I \;=\; 0, \qquad m_{\mathsf{R}}^{\Lambda} e_{\Lambda}{}^I \;=\; 0 \;=\; m_{\mathsf{R}}^{\Lambda} e_{\Lambda I}$$

Scalar potential from (non)geometric flux compactifications:

$$V = \mathrm{g}^2 \Big[4 h_{uv} k^u \overline{k}^v + \sum_{x=1}^{3} \Big(g^{a\overline{b}} D_a \mathcal{P}_x D_{\overline{b}} \overline{\mathcal{P}}_x - 3 |\mathcal{P}_x|^2 \Big) \Big] = \ldots \equiv V_{\mathsf{NS}} + V_{\mathsf{R}} \quad (\text{abelian: } k_\Lambda^a = 0)$$

$$\begin{aligned} V_{\mathsf{NS}} &= g^{a\overline{b}} D_a \mathcal{P}_+ D_{\overline{b}} \overline{\mathcal{P}}_+ + g^{i\overline{j}} D_i \mathcal{P}_+ D_{\overline{j}} \overline{\mathcal{P}}_+ - 2 |\mathcal{P}_+|^2 \\ &= -2\, \mathrm{g}^2 \mathrm{e}^{2\varphi} \Big[\overline{\Pi}_{\mathsf{H}}^{\mathrm{T}} \, \widetilde{Q}^{\mathrm{T}} \, \mathbb{M}_{\mathsf{V}} \, \widetilde{Q} \, \Pi_{\mathsf{H}} + \overline{\Pi}_{\mathsf{V}}^{\mathrm{T}} \, Q \, \mathbb{M}_{\mathsf{H}} \, Q^{\mathrm{T}} \, \Pi_{\mathsf{V}} + 4 \overline{\Pi}_{\mathsf{H}}^{\mathrm{T}} \, \mathbb{C}_{\mathsf{H}}^{\mathrm{T}} \, Q^{\mathrm{T}} (\Pi_{\mathsf{V}} \overline{\Pi}_{\mathsf{V}}^{\mathrm{T}} + \overline{\Pi}_{\mathsf{V}} \Pi_{\mathsf{V}}^{\mathrm{T}}) Q \, \mathbb{C}_{\mathsf{H}} \, \Pi_{\mathsf{H}} \Big] \\ V_{\mathsf{R}} &= g^{a\overline{b}} D_a \mathcal{P}_3 D_{\overline{b}} \overline{\mathcal{P}}_3 + |\mathcal{P}_3|^2 \\ &= -\frac{1}{2} \mathrm{g}^2 \mathrm{e}^{4\varphi} \big(e_{\mathsf{R}\Lambda} - e_{\Lambda I} \xi^I + e_\Lambda{}^I \widetilde{\xi}_I \big) (\mathrm{Im} \mathcal{N})^{-1|\Lambda\Sigma} \big(e_{\mathsf{R}\Sigma} - e_{\Sigma I} \xi^I + e_\Sigma{}^I \widetilde{\xi}_I \big) \end{aligned}$$

$\Pi_{\mathsf{V}} = \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2} (X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$

$t^a = X^a / X^0$

$a = 1, \ldots, n_{\mathsf{V}}$

$\mathsf{SKG}_{\mathsf{V}}$ of vector-moduli

$$\begin{aligned} \mathcal{P}_+ &\equiv \mathcal{P}_1 + \mathrm{i} \mathcal{P}_2 = 2 \mathrm{e}^{\varphi} \, \Pi_{\mathsf{V}}^{\mathrm{T}} \, Q \, \mathbb{C}_{\mathsf{H}} \, \Pi_{\mathsf{H}} \\ \mathcal{P}_- &\equiv \mathcal{P}_1 - \mathrm{i} \mathcal{P}_2 = 2 \mathrm{e}^{\varphi} \, \Pi_{\mathsf{V}}^{\mathrm{T}} \, Q \, \mathbb{C}_{\mathsf{H}} \, \overline{\Pi}_{\mathsf{H}} \\ \mathcal{P}_3 &= \mathrm{e}^{2\varphi} \, \Pi_{\mathsf{V}}^{\mathrm{T}} \, \mathbb{C}_{\mathsf{V}} (c_{\mathsf{R}} + \widetilde{Q} \xi) \end{aligned}$$

$\Pi_{\mathsf{H}} = \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2} (Z^I, \mathcal{G}_I)^{\mathrm{T}}$

$z^i = Z^i / Z^0$

$i = 1, \ldots, n_{\mathsf{H}}$

$\mathsf{SKG}_{\mathsf{H}}$ of hyper-moduli

$$\mathbb{C}_{\mathsf{V,H}} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} ; \quad Q = \begin{pmatrix} e_\Lambda{}^I & e_{\Lambda I} \\ m^{\Lambda I} & m^\Lambda{}_I \end{pmatrix} , \quad \widetilde{Q} = \mathbb{C}_{\mathsf{H}}^{\mathrm{T}} \, Q \, \mathbb{C}_{\mathsf{V}} \quad c_{\mathsf{R}} = \begin{pmatrix} m_{\mathsf{R}}^\Lambda \\ e_{\mathsf{R}\Lambda} \end{pmatrix}$$

Cassani et.al. [arXiv:0804.0595], [arXiv:0911.2708]

Non-CY coset spaces with $SU(3)$-structure: D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

$\mathcal{M}_6$	$\frac{G_2}{SU(3)}$ (nearly-Kähler)	$\frac{Sp(2)}{S[U(2)\times U(1)]}$ (half-flat)	$\frac{SU(3)}{U(1)\times U(1)}$ (half-flat)
$\mathcal{SM} = \text{SKG}_\text{V}$	$\frac{SU(1,1)}{U(1)}$: $\mathfrak{t}^3$	$\left(\frac{SU(1,1)}{U(1)}\right)^2$: $\mathfrak{s}\mathfrak{t}^2$	$\left(\frac{SU(1,1)}{U(1)}\right)^3$: $\mathfrak{s}\mathfrak{t}\mathfrak{u}$
$\mathcal{HM} = \text{SQG}$	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM
$\text{SKG}_\text{H} \subset \mathcal{HM}$	—	—	—
matters	1 VM + 1 UHM	2 VMs + 1 UHM	3 VMs + 1 UHM

Each SKG_V has a cubic prepotential: $\mathcal{F} = \frac{1}{3!} d_{abc} \frac{X^a X^b X^c}{X^0}$

Each AdS vacuum requires non-vanishing Romans' mass! (Vanishing Romans' mass $\rightarrow$ Reduction to CY)

nilmanifolds and solvmanifolds: M. Graña, R. Minasian, M. Petrini and A. Tomasiello [hep-th/0609124]

coset spaces with $SU(3)$- or $SU(2)$-structure: P. Koerber, D. Lüst and D. Tsimpis [arXiv:0804.0614]

a pair of $SU(3)$-structures with $(m^{\Lambda I}, m^{\Lambda}{}_I)$: D. Gaiotto and A. Tomasiello [arXiv:0904.3959]

One for All,

ひとりはみんなのために、みんなは日本のために。

All for Japan.

Remember 3.11

東日本大震災