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Non-SUSY Solutions of RN-AdS Black Holes in 4D $\mathcal{N} = 2$ Gauged Supergravity

based on JHEP 09 (2010) 061 [arXiv:1005.4607]

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HIGH ENERGY ACCELERATOR RESEARCH ORGANIZATION

Current Interest: AdS Black Hole Solutions in 4D $\mathcal{N} = 2$ SUGRA

- WHY $\mathcal{N} = 2$ (8-SUSY charges)?
  - ✓ Scalar fields living in highly symmetric spaces
  - ✓ (Flux) compactification scenarios in string/M-theory
- WHY AdS Black Holes?
  - ✓ Attractive: 4D $\mathcal{N} = 2$ SUGRA $\subset$ Einstein-Yang–Mills-Matters
  - ✓ Non-trivial: the existence of the cosmological constant with matter fields

- In the framework of 4D $\mathcal{N} = 2$ ungauged SUGRA,

  (Extremal) RN-BH in asymptotically flat spacetime has been investigated.
  BH charges from D-branes wrapped on CY: "D0-D4", "D2-D6", "D0-D2-D6", etc.
  (Hypermultiplets are decoupled from the system.)

- We (have to) study AdS BH in 4D $\mathcal{N} = 2$ system.

  The cosmological constant Λ is given as expectation value of scalar potential V.
  ("mass deformations" of gravitini)

Remark: Naked Singularity appears in SUSY RN-AdS solution.

- ☞ Pure AdS SUGRA (only gravitational multiplet):
  Romans [hep-th/9203018], Caldarelli, et.al. [hep-th/9808097], etc.
  There exists a SUSY solution of rotating AdS black hole with regular horizon.
- ☞ Gauged SUGRA with vector multiplets (without hyper-sector):
  Sabra, et.al. (electric charges [hep-th/9903143], magnetic/dyonic charges [hep-th/0003213]), etc.

Question

How can we obtain non-SUSY solutions with matter fields
in asymptotically AdS spacetime?

— **Setup** and **Result** —

- 4D $\mathcal{N} = 2$ Abelian gauged SUGRA with mutually local electric/magnetic charges
- Truncate out hyper-sector by hand (artificial)
- Import the Attractor Mechanism
- Find a new attractor equation
- Obtain a non-SUSY solution with small $|\Lambda|$ in T^3-model
- Argue "generic descriptions of moduli" in T^3-model and in STU-model

Contents

Contents

"Standard": Andrianopoli, Bertolini, Ceresole, D'Auria, Ferrara, Fré and Magri [hep-th/9605032]

"Magnetically dualized": D'Auria, Sommovigo and Vaulà [hep-th/0409097]

Action (gravitational const. κ; gauge coupling const. g; index $\Lambda = 0, 1, \ldots, n_{\mathsf{V}}$):

$$S = \int \mathrm{d}^4 x \sqrt{-g} \Big\{ \frac{1}{2\kappa^2} R - G_{a\bar{b}}(z,\bar{z}) \nabla_\mu z^a \nabla^\mu \bar{z}^{\bar{b}} - h_{uv}(q) \nabla_\mu q^u \nabla^\mu q^v$$
$$+ \frac{1}{4} \mu_{\Lambda\Sigma}(z,\bar{z}) F^\Lambda_{\mu\nu} F^{\Sigma\mu\nu} + \frac{1}{4} \nu_{\Lambda\Sigma}(z,\bar{z}) F^\Lambda_{\mu\nu} (*F^\Sigma)^{\mu\nu}$$
$$- \mathrm{g}^2 V(z,\bar{z},q)$$
$$+ \text{(fermionic terms)} \Big\}$$

$$\mu_{\Lambda\Sigma} = \mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma} \quad \text{(generalized } -1/\mathrm{g}^2\text{)}, \qquad \nu_{\Lambda\Sigma} = \mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma} \quad \text{(generalized } \theta\text{-angle)}$$

In this analysis...

- Set background fermionic fields to zero
- Reduce gauge symmetries to *abelian*: $\nabla_\mu z^a \to \partial_\mu z^a$
- Truncate hyper-sector *by hand, after the formulation* *(later)*

Equations of Motion (abbreviate κ and g, and hyper-sector):

$$\delta g^{\mu\nu} : \quad \Big(R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}\Big) - 2 G_{a\bar{b}}\, \partial_{(\mu} z^a \partial_{\nu)} \overline{z}^{\bar{b}} + G_{a\bar{b}}\, \partial_\rho z^a \partial^\rho \overline{z}^{\bar{b}}\, g_{\mu\nu} \;=\; T_{\mu\nu} - V g_{\mu\nu}$$

$$T_{\mu\nu} \;=\; -\mu_{\Lambda\Sigma}\, F^{\Lambda}_{\mu\rho}\, F^{\Sigma}_{\nu\sigma}\, g^{\rho\sigma} + \frac{1}{4} \mu_{\Lambda\Sigma}\, F^{\Lambda}_{\rho\sigma}\, F^{\Sigma\rho\sigma}\, g_{\mu\nu} \qquad \text{(energy-momentum tensor)}$$

$$\delta z^a : \quad -\frac{G_{a\bar{b}}}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g} g^{\mu\nu} \partial_\nu \overline{z}^{\bar{b}}\Big) - \frac{\partial G_{a\bar{b}}}{\partial \overline{z}^{\bar{c}}}\, \partial_\rho \overline{z}^{\bar{b}}\, \partial^\rho \overline{z}^{\bar{c}}$$

$$= \frac{1}{4} \frac{\partial \mu_{\Lambda\Sigma}}{\partial z^a} F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} + \frac{1}{4} \frac{\partial \nu_{\Lambda\Sigma}}{\partial z^a} F^{\Lambda}_{\mu\nu} (*F^{\Sigma})^{\mu\nu} - \frac{\partial V}{\partial z^a}$$

$$\delta A^{\Lambda}_\mu : \quad \varepsilon^{\mu\nu\rho\sigma} \partial_\nu G_{\Lambda\rho\sigma} \;=\; 0\,, \qquad G_{\Lambda\rho\sigma} \;=\; \nu_{\Lambda\Sigma} F^{\Sigma}_{\rho\sigma} - \mu_{\Lambda\Sigma} (*F^{\Sigma})_{\rho\sigma}$$

electric charge $q_\Lambda \;\equiv\; \dfrac{1}{4\pi} \displaystyle\int_{S^2} G_{2\Lambda}\,,$ magnetic charge $p^\Lambda \;\equiv\; \dfrac{1}{4\pi} \displaystyle\int_{S^2} F^{\Lambda}_{2}$

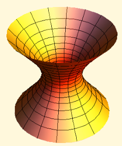
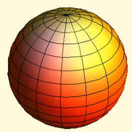
Introduce ansatz for RN(-AdS) BH: Extremal, static, charged, spherically symmetric

$$ds^2 \;=\; -\,\mathrm{e}^{2A(r)}\mathrm{d}t^2 + \mathrm{e}^{2B(r)}\mathrm{d}r^2 + \mathrm{e}^{2C(r)} r^2 \big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\big)$$

Near horizon geometry: $\mathrm{AdS}_2 \times S^2$ (radii: r_{A} and r_{H})

$$A(r) \;=\; \log\frac{r - r_{\mathsf{H}}}{r_{\mathsf{A}}}\,, \qquad B(r) \;=\; -A(r)\,, \qquad C(r) \;=\; \log\frac{r_{\mathsf{H}}}{r}$$

$$R(\mathrm{AdS}_2 \times S^2) \;=\; 2\Big(-\frac{1}{r_{\mathsf{A}}^2} + \frac{1}{r_{\mathsf{H}}^2}\Big)$$

 × 

$$\to \quad ds^2_{(\text{near horizon})} \;=\; -\left(\frac{r - r_{\mathsf{H}}}{r_{\mathsf{A}}}\right)^2 \mathrm{d}t^2 + \left(\frac{r_{\mathsf{A}}}{r - r_{\mathsf{H}}}\right)^2 \mathrm{d}r^2 + r_{\mathsf{H}}^2\big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\big)$$

$$\;=\; -\frac{\mathrm{e}^{2\tau}}{r_{\mathsf{A}}^2}\mathrm{d}t^2 + r_{\mathsf{A}}^2 \mathrm{d}\tau^2 + r_{\mathsf{H}}^2\big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\big) \qquad (\tau = \log(r - r_{\mathsf{H}}))$$

If the attractor mechanism works (via extremality), the scalar fields are subject to

$$\frac{\mathrm{d}}{\mathrm{d}r} z^a \Big|_{\text{horizon}} = 0\,, \qquad \Big(\frac{\mathrm{d}}{\mathrm{d}r}\Big)^2 z^a \Big|_{\text{horizon}} = 0$$

The EoM are drastically reduced to — Bellucci, et.al. [arXiv:0802.0141]

$$\delta g^{tt},\, \delta g^{rr} : \quad \frac{1}{r_{\mathsf{H}}^2} = \frac{1}{r_{\mathsf{H}}^4} I_S + V \Big|_{\text{horizon}} \quad \longrightarrow \quad r_{\mathsf{H}}^2 = \frac{1 - \sqrt{1 - 4 I_S V}}{2V} \Big|_{\text{horizon}}$$

$$\delta g^{\theta\theta},\, \delta g^{\phi\phi} : \quad \frac{1}{r_{\mathsf{A}}^2} = \frac{1}{r_{\mathsf{H}}^4} I_S - V \Big|_{\text{horizon}} \quad \longrightarrow \quad r_{\mathsf{A}}^2 = \frac{r_{\mathsf{H}}^2}{\sqrt{1 - 4 I_S V}} \Big|_{\text{horizon}}$$

$$\delta z^a : \quad 0 = \frac{1}{r_{\mathsf{H}}^4} \frac{\partial I_S}{\partial z^a} - \frac{\partial V}{\partial z^a} \Big|_{\text{horizon}} \quad \longrightarrow \quad 0 = \frac{1}{r_{\mathsf{H}}^4} (1 - 2 r_{\mathsf{H}}^2 V) \frac{\partial}{\partial z^a} r_{\mathsf{H}}^2 \Big|_{\text{horizon}}$$

(1st) Symplectic Invariant:

$$I_S(z, \overline{z}, p, q) = -\frac{1}{2} \big(p^{\Lambda} \;\; q_{\Lambda} \big) \begin{pmatrix} \mu_{\Lambda\Sigma} + \nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Delta} \nu_{\Delta\Sigma} & -\nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Sigma} \\ -(\mu^{-1})^{\Lambda\Gamma} \nu_{\Gamma\Sigma} & (\mu^{-1})^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} p^{\Sigma} \\ q_{\Sigma} \end{pmatrix} \equiv -\frac{1}{2} \Gamma^{\mathrm{T}} \mathbb{M} \Gamma$$

with $\quad T_t{}^t = T_r{}^r = -T_{\theta}{}^{\theta} = -T_{\phi}{}^{\phi} = -\frac{\mathrm{e}^{-4C}}{r^4} I_S$

BH entropy (and the effective potential) is given as the Area of the event horizon:

$$S_{\mathsf{BH}}(p,q) \;=\; \frac{A_{\mathsf{H}}}{4\pi} \;=\; r_{\mathsf{H}}^2\Big|_{\text{horizon}} \;\equiv\; V_{\text{eff}}(z,\overline{z},p,q)\Big|_{\text{horizon}}$$

$$V_{\text{eff}}(z,\overline{z},p,q) \;=\; \frac{1-\sqrt{1-4I_S V}}{2V} \;=\; I_S + (I_S)^2 V + 2(I_S)^3 V^2 + \mathcal{O}((I_S)^4 V^3)$$

We read the "cosmological constant Λ" from the scalar curvature:

$$R(\mathrm{AdS}_2 \times S^2) \;=\; 2\Big(-\frac{1}{r_{\mathsf{A}}^2} + \frac{1}{r_{\mathsf{H}}^2}\Big) \;=\; 4V$$

$$V\Big|_{\text{horizon}} \;\equiv\; \Lambda(\text{"cosmological constant"})$$

Attractor Equation

$$0 \;=\; \frac{1}{r_{\mathsf{H}}^4}\,(1-2r_{\mathsf{H}}^2 V)\,\frac{\partial}{\partial z^a}V_{\text{eff}}\Big|_{\text{horizon}} \qquad \to \qquad 0 \;=\; \frac{\partial}{\partial z^a}V_{\text{eff}}(z,\overline{z},p,q)\Big|_{\text{horizon}}$$

If $r_{\mathsf{H}} < \infty$ and $V\Big|_{\text{horizon}} \leq 0$

The "ATTRACTOR EQUATION" which we have to solve is

$$0 = \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \overline{z}, p, q)\Big|_{\text{horizon}}$$

$$= \frac{1}{2V^2\sqrt{1 - 4I_S V}} \left\{ 2V^2 \frac{\partial I_S}{\partial z^a} - \left(\sqrt{1 - 4I_S V} + 2I_S V - 1\right) \frac{\partial V}{\partial z^a} \right\} \Bigg|_{\text{horizon}}$$

Evaluate I_S and V described in terms of the central charge Z

Useful when we consider (non-)SUSY solutions

SUSY variation of gravitini carries definition of Z *and more..*

$$\delta\psi_{A\mu} = D_\mu \varepsilon_A + \epsilon_{AB} T^-_{\mu\nu} \gamma^\nu \varepsilon^B + i\, \mathrm{g}\, \mathcal{S}_{AB} \gamma_\mu \varepsilon^B$$

$$Z = -\frac{1}{2}\left(\frac{1}{4\pi}\int_{S^2} T_2^-\right), \qquad \mathcal{S}_{AB} = \frac{i}{2}\sum_{x=1}^{3} (\sigma^x)_{AB} \mathcal{P}_x$$

Describe Z, I_S and V in terms of $(L^\Lambda, M_\Lambda) = e^{K/2}(X^\Lambda, \mathcal{F}_\Lambda)$ on $\mathcal{SM}$:

$$\begin{aligned} Z &= L^\Lambda q_\Lambda - M_\Lambda p^\Lambda \\ I_S &= |Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z} \\ V &= \sum_{x=1}^{3} \Big(-3|\mathcal{P}_x|^2 + G^{a\bar{b}} D_a \mathcal{P}_x \overline{D_b \mathcal{P}_x} \Big) + 4 h_{uv} k^u \overline{k}^v \end{aligned}$$

$\mathcal{P}_x$: $SU(2)$ triplet of Killing prepotentials in $\mathcal{N} = 2$ SUGRA

(In general, both vector moduli and hyper moduli contribute to $\mathcal{P}_x$)

Hyper-sector is "truncated" $\rightarrow$ $\mathcal{P}_3 = \mathcal{P}_{3,\Lambda} L^\Lambda - \widetilde{\mathcal{P}}_3^\Lambda M_\Lambda$ still remains in V

Further, Identify electric/magnetic FI with charges $(\mathcal{P}_{3,\Lambda}, \widetilde{\mathcal{P}}_3^\Lambda) = (q_\Lambda, p^\Lambda) \rightsquigarrow \mathcal{P}_3 \equiv Z$

Cassani, Ferrara, Marrani, Morales and Samtleben [arXiv:0911.2708]

$$V \equiv -3|Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}$$

The "ATTRACTOR EQUATION" (with a non-trivial factor $-1 \leq G_{\mathsf{V}} \equiv \dfrac{1 - V_{\text{eff}}^2}{1 + V_{\text{eff}}^2} \leq 1$)

$$
\begin{aligned}
0 &= \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \overline{z}, p, q)\Big|_{\text{horizon}} \\
&= \frac{1}{2V^2 \sqrt{1 - 4I_S V}} \Big\{ 2V^2 \frac{\partial I_S}{\partial z^a} - \big(\sqrt{1 - 4 I_S V} + 2 I_S V - 1 \big) \frac{\partial V}{\partial z^a} \Big\} \Big|_{\text{horizon}} \\
&= \frac{1 + V_{\text{eff}}^2}{\sqrt{1 - 4 I_S V}} \Big\{ 2\, G_{\mathsf{V}}\, \overline{Z} D_a Z + i\, C_{abc}\, G^{b\overline{b}} G^{c\overline{c}}\, \overline{D_b Z}\, \overline{D_c Z} \Big\} \Big|_{\text{horizon}}
\end{aligned}
$$

Solve the equation $0 = 2\, G_{\mathsf{V}}\, \overline{Z} D_a Z + i\, C_{abc}\, G^{b\overline{b}} G^{c\overline{c}}\, \overline{D_b Z}\, \overline{D_c Z} \Big|_{\text{horizon}}$

under the condition $V < 0$, $1 - 4 I_S V > 0$, $\partial_a I_S \neq 0$, $\partial_a V \neq 0$, $D_a Z \neq 0$

- ✍ If $V < 0$ and $D_a Z = 0$ (SUSY) $\rightarrow$ Naked Singularity $\rightarrow$ Search non-SUSY solution $D_a Z \neq 0$
- ✍ If $\partial_a I_S = 0$ $(G_{\mathsf{V}} = 1)$ or $\partial_a V = 0$ $(G_{\mathsf{V}} = -1)$ $\rightarrow$ asymptotically flat $V = 0$ or Empty Hole $Z = 0$

Contents

- Consider the Single Modulus Model of $\Gamma = (0, p, 0, q_0)$ (analogous to "D0-D4" system)

  with cubic prepotential $\mathcal{F} = \frac{(X^1)^3}{X^0}$, $t = \frac{X^1}{X^0}$ $\longrightarrow$ Kähler potential $\mathrm{e}^K = \frac{i}{(t - \bar{t})^3}$

  (as the "large volume limit of Calabi-Yau")

The ATTRACTOR EQUATION and its solution ($t = 0 + iy$, $y < 0$):

$$p(y^2)^3 + (q_0 - 18p^3 q_0^2)(y^2)^2 - 12p^2 q_0^3 (y^2) - 2p q_0^4 \; = \; 0$$

with $p \neq 0\,, \quad q_0 \neq 0\,, \quad p q_0 < 0$

$$y^2 \;=\; \mathfrak{A} + \mathfrak{B} \quad \text{or} \quad \mathfrak{A} + \omega^{\pm}\mathfrak{B} \qquad (\omega^3 = 1)$$

$$\mathfrak{A} \;=\; \frac{q_0}{3p}(18\, p^3 q_0 - 1)\,, \qquad \mathfrak{B} \;=\; \frac{1}{3p}\left(\mathfrak{C}^{1/3} + \frac{q_0^2}{4}\frac{1 + (18\, p^3 q_0)^2}{\mathfrak{C}^{1/3}}\right)$$

$$\mathfrak{C} \;=\; -q_0^3\left[1 - 27\, p^3 q_0 - (18\, p^3 q_0)^3 - 3\sqrt{3}\sqrt{-2\, p^3 q_0 - 9(p^3 q_0)^2 - 432(p^3 q_0)^3}\right]$$

Various values at the event horizon:

$$Z\Big|_{\text{horizon}} = \frac{q_0 + 3p\,y^2}{2}\sqrt{-\frac{1}{2y^3}} \neq \mathbf{0}\,, \qquad D_t Z\Big|_{\text{horizon}} = \frac{3i(q_0 - p\,y^2)}{4y}\sqrt{-\frac{1}{2y^3}} \neq \mathbf{0}$$

$$I_S\Big|_{\text{horizon}} = \frac{q_0^2 + 3p^2 y^4}{-2y^3} > \mathbf{0}$$

$$\Lambda = \frac{6(pq_0)^2(q_0 + 3p\,y^2)^2}{y^5} < \mathbf{0}$$

$$S_{\text{BH}} = \frac{-y}{12(pq_0)^2(q_0 + 3p\,y^2)^2}\left\{-y^4 + \sqrt{y^8 + 12(pq_0)^2(q_0 + 3p\,y^2)^2(q_0^2 + 3p^2\,y^4)}\right\} > \mathbf{0}$$

their asymptotic behaviors?

Look at the Small q_0 limit:

The dominant part of the Modulus $t = 0 + iy$ $(y < 0)$ is

$$y \sim -\sqrt{-\frac{q_0}{p}} + \text{(sub-leading orders)}$$

The dominant parts of various values are

$$Z\Big|_{\text{horizon}} \sim -q_0 \left(-\frac{p^3}{q_0^3}\right)^{1/4} + \ldots, \qquad D_t Z\Big|_{\text{horizon}} \sim ip \left(-\frac{p}{q_0}\right)^{1/4} + \ldots$$

$$I_S\Big|_{\text{horizon}} \sim \sqrt{-p^3 q_0} + \ldots$$

$$\Lambda \sim -\sqrt{(-p^3 q_0)^{\mathbf{3}}} + \ldots$$

$$S_{\text{BH}} \sim \sqrt{-p^3 q_0} + \ldots$$

$$I_S \sim S_{\text{BH}} \gg -\Lambda > 0$$

very small $|\Lambda|$ compared to others: similar to the non-BPS RN-BH,

but Never connected to RN-BH with $\Lambda = 0$!

Argue the Large q_0 limit:

The dominant part of the Modulus $t = 0 + iy$ $(y < 0)$ is

$$y \sim pq_0 + \text{(sub-leading orders)}$$

The dominant parts of various values are

$$Z\Big|_{\text{horizon}} \sim \sqrt{-p^3 q_0} + \ldots \neq 0\,, \qquad D_t Z\Big|_{\text{horizon}} \sim \frac{-i}{pq_0}\sqrt{-p^3 q_0} + \ldots \neq 0$$

$$I_S\Big|_{\text{horizon}} \sim -p^3 q_0 + \ldots > 0$$

$$\Lambda \sim p^3 q_0 + \ldots < 0 \qquad \text{(same magnitude to } I_S \text{ !)}$$

$$S_{\text{BH}} \sim \sqrt{\frac{5}{6}} + \ldots > 0 \qquad \text{(Why constant ??)}$$

Pathological behaviors of Λ and S_{BH}: *not yet completely understood the reason*
incorrect expansions? and/or breaking $SU(2)$ by truncating hyper-sector?

Contents

☑ Studied Extremal RN-AdS Black Hole solution in Abelian gauged SUGRA

☑ Obtained the analytic description of non-SUSY solution in T^3-model

! Different behaviors from the ones of non-BPS RN black hole in asympt.-flat spacetime:

- ✓ Should we expand the solution in terms of "Λ" rather than "q_0"?
- ✓ Consider the contribution of (electrically/magnetically coupled) hyper-sector!?
  - D'Auria, et.al. [hep-th/0409097] (massive tensors)
  - D'Auria, et.al. [hep-th/0701247], Cassani, et.al. [arXiv:0911.2708] (generalized geometry)
  - Hristov, Looyestijn and Vandoren [arXiv:1005.3650] (gauged SUGRA to ungauged SUGRA via Higgs mechanism)

☞ Argue more general solutions in more general setups (rotating, non-extremal, etc..)