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Flux Compactifications, $\mathcal{N}=2$ Gauged Supergravities and Black Holes

based on arXiv:1108.1113 [hep-th]

木村 哲士, 西中 崇博

HIGH ENERGY ACCELERATOR RESEARCH ORGANIZATION

Question

How can we obtain AdS-BH solutions

with hypermultiplets?

— Consider 4D $\mathcal{N} = 2$ systems from 10D type II strings —

AdS has negative cosmological constant in the system $\longleftarrow$ $\not{C}\not{Y}$

FI parameters support the cosmological const.

HMs forbid the existence of FI parameters

Question

How can we obtain AdS-BH solutions

with hypermultiplets?

— **Setup** and **Result** —

- 4D $\mathcal{N} = 2$ gauged SUGRA with VM and UHM
  from 10D type IIA on non-CY with fluxes
- Ansätze for matter fields
- AdS Black Hole

- Black holes from CY with D-branes :

$$\boxed{\frac{\partial I_1}{\partial \mathsf{z}} = 0}$$

Value of vector modulus z is not fixed at infinity $\rightarrow$ attractor mechanism

BH charges are governed by D-brane charges

BH mass is given by BH charges

- Black holes from non-CY with fluxes :

$$\boxed{\frac{\mathrm{e}^{-4C}}{r^4}\frac{\partial I_1}{\partial \mathsf{z}} + \frac{\partial V}{\partial \mathsf{z}} = 0} \quad \longrightarrow \quad \boxed{\frac{\partial V}{\partial \mathsf{z}} = 0} \quad \text{and} \quad \boxed{\frac{\partial I_1}{\partial \mathsf{z}} = 0}$$

Value of vector modulus z is fixed at infinity $\rightarrow$ moduli stabilization

BH charges are governed by geometric- and RR-flux charges

BH mass is arbitrary

Analysis

Coset space $\dfrac{G_2}{SU(3)}$:

D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

$$\mathrm{d}J \;=\; \frac{3}{2}\mathrm{Im}\big(\overline{\mathcal{W}}_1\Omega\big) \;\neq\; 0\,, \qquad \mathrm{d}\Omega \;=\; \mathcal{W}_1 J\wedge J \;\neq\; 0$$

✓ nearly-Kähler (almost complex geometry)

✓ NSNS-sector : torsion and H-flux

✓ RR-sector : 2-, 4-form and Romans' mass (0-form)

✓ 1 VM with cubic prepotential $\mathcal{F} = \dfrac{X^1X^1X^1}{X^0}$

✓ 1 UHM (no other HMs)

10D massive type IIA on $\frac{G_2}{SU(3)}$ with fluxes

$\downarrow$

4D $\mathcal{N} = 2$ SUGRA with B-field and scalar potential V

- GM : $(g_{\mu\nu}, A_\mu^0)$
- VM : (A_μ^1, z) w/ $F_2^\Lambda = \mathrm{d}A_1^\Lambda + m_{\mathsf{R}}^\Lambda B_2$
- UHM $\rightarrow$ TM : $(\varphi, B_{\mu\nu}, \xi^0, \widetilde{\xi}_0)$
- $D\xi^0 = \mathrm{d}\xi^0 - e_\Lambda{}^0 A_1^\Lambda$, $D\widetilde{\xi}_0 = \mathrm{d}\widetilde{\xi}_0 - e_{\Lambda 0} A_1^\Lambda$
- NSNS-sector ; RR-sector

Precise data on $\frac{G_2}{SU(3)}$:

$$e_{10} \neq 0, \; m_{\mathsf{R}}^0 \neq 0, \; e_{\mathsf{R}0} \neq 0$$

$$e_\Lambda{}^0 = 0 = e_{00}$$

$$m_{\mathsf{R}}^1 = 0 = e_{\mathsf{R}1}$$

$$\mathcal{W}_1 = \frac{\mathrm{i}\, e_{10}\, \mathrm{Im}(\mathsf{z})}{3\sqrt{\mathrm{Vol.}}}$$

D. Cassani [arXiv:0804.0595]

☞ Consider spacetime metric (extremal, static, spherically symmetric $\rightarrow \mathrm{AdS}_2 \times S^2$)

$$\mathrm{d}s^2 \;=\; -\mathrm{e}^{2A(r)}\mathrm{d}t^2 + \mathrm{e}^{-2A(r)}\mathrm{d}r^2 + \mathrm{e}^{2C(r)}r^2\big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\big)$$

☞ Impose (covariantly) constant condition

$$0 \;\equiv\; \partial_\mu \mathsf{z}\,, \quad 0 \;\equiv\; \partial_\mu \varphi\,, \quad 0 \;\equiv\; D_\mu \xi^0\,, \quad 0 \;\equiv\; D_\mu \widetilde{\xi}_0\,, \quad 0 \;\equiv\; \partial_\mu B_{\nu\rho}$$

☞ Define electromagnetic charges

$$p^\Lambda \;\equiv\; \frac{1}{4\pi}\int_{S^2} F_2^\Lambda\,, \quad q_\Lambda \;\equiv\; \frac{1}{4\pi}\int_{S^2} \widetilde{F}_{\Lambda 2}$$

$$\widetilde{F}_{\Lambda 2} \equiv \nu_{\Lambda\Sigma} F_2^\Sigma + \mu_{\Lambda\Sigma}(*F_2^\Sigma)$$

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = T_{\mu\nu} - g_{\mu\nu} V$$

etc.

$$T_{\mu\nu} \equiv \frac{1}{4} g_{\mu\nu}\, \mu_{\Lambda\Sigma}\, F^{\Lambda}_{\rho\sigma}\, F^{\Sigma\rho\sigma} - \mu_{\Lambda\Sigma}\, F^{\Lambda}_{\mu\rho}\, F^{\Sigma}_{\nu\sigma}\, g^{\rho\sigma}$$

$$T_t{}^t = T_r{}^r = -T_\theta{}^\theta = -T_\phi{}^\phi = -\frac{\mathrm{e}^{-4C}}{r^4} I_1$$

$$I_1 \equiv -\frac{1}{2}\Big[p^{\Lambda}\, \mu_{\Lambda\Sigma}\, p^{\Sigma} + (q_{\Lambda} - \nu_{\Lambda\Gamma}\, p^{\Gamma})(\mu^{-1})^{\Lambda\Sigma}(q_{\Sigma} - \nu_{\Sigma\Delta}\, p^{\Delta}) \Big]$$

The equation of motion for $g_{\mu\nu}$ fixes $C(r)$ and $A(r)$:

$$
\begin{aligned}
e^{2C(r)} &= \frac{e^{2c_2}}{r^2}\big(c_1 r + 1\big)^2 \\
e^{2A(r)} &= \frac{e^{-2c_2}}{(c_1)^2} - \frac{2a_1}{c_1(c_1 r + 1)} + \frac{e^{4c_2} I_1}{(c_1)^2(c_1 r + 1)^2} - \frac{V}{3(c_1)^2}(c_1 r + 1)^2 \\
&\equiv 1 - \frac{2\eta}{r_{\text{new}}} + \frac{\mathcal{Z}^2}{r_{\text{new}}^2} - \frac{\Lambda_{\text{c.c.}}}{3} r_{\text{new}}^2
\end{aligned}
$$

Choosing $c_1 r + 1 \equiv r_{\text{new}}$ (and $c_1 \equiv 1$), we can read the "Black Hole" information:

$e^{-2c_2} \equiv 1$:	scalar curvature of S^2	$a_1 \equiv \eta$:	mass parameter
$I_1 \equiv \mathcal{Z}^2$:	square of charges	$V \equiv \Lambda_{\text{c.c.}}$:	cosmological constant

The equations of motion for z, φ, ξ^0 :

$$0 = \frac{e^{-4C(r)}}{r^4} \frac{\partial I_1}{\partial z} + \frac{\partial V}{\partial z} \quad \longrightarrow \quad \frac{\partial I_1}{\partial z} = 0 = \frac{\partial V}{\partial z}$$

$$0 = \frac{\partial V}{\partial \varphi}, \qquad 0 = \frac{\partial V}{\partial \xi^0}, \qquad 0 = \frac{\partial V}{\partial \widetilde{\xi}_0}$$

Solutions

$$\big\{ z \,,\ \xi^0 \,,\ \varphi \,;\ V \big\}_{\text{BHs}} = \big\{ z_* \,,\ \xi^0_* \,,\ \varphi_* \,;\ \Lambda_{\text{c.c.}} \big\}_{\text{Vacua}}$$

Remark: Three AdS vacua (One $\mathcal{N} = 1$, two $\mathcal{N} = 0$)

The equation of motion for $B_{\mu\nu}$ and covariantly constant condition:

$$\delta B_{\mu\nu}$$

$$0 \;=\; D_\mu \widetilde{\xi}_0 \quad\longrightarrow\quad 0 \;=\; [\partial_\mu, \partial_\nu]\,\widetilde{\xi}_0 \;=\; e_{\Lambda 0}\, F^{\Lambda}_{\mu\nu}$$

Relation among the charges (p^Λ, q_Σ) and the modulus $\mathsf{z} = x + \mathrm{i}y$:

$$p^1 \;=\; 0\,, \qquad p^0 \;=\; \frac{m_{\mathsf{R}}^0}{e_{\mathsf{R}0}} q_0\,, \qquad (\mu^{-1})^{11} q_1 \;=\; -q_0 \Big[(\mu^{-1})^{10} - \frac{m_{\mathsf{R}}^0}{e_{\mathsf{R}0}} (\mu^{-1}\nu)^1{}_0 \Big]$$

$$(\mu^{-1})^{\Lambda\Sigma} \;=\; -\frac{4\mathrm{i}}{3\mathfrak{I}(\mathsf{z}-\bar{\mathsf{z}})^3} \begin{pmatrix} 6 & 3(\mathsf{z}+\bar{\mathsf{z}}) \\ 3(\mathsf{z}+\bar{\mathsf{z}}) & \mathsf{z}^2 + 4\mathsf{z}\bar{\mathsf{z}} + \bar{\mathsf{z}}^2 \end{pmatrix}$$

$$\nu_{\Lambda\Sigma} \;=\; \frac{\mathfrak{I}(\mathsf{z}+\bar{\mathsf{z}})}{4} \begin{pmatrix} (\mathsf{z}+\bar{\mathsf{z}})^2 & -3(\mathsf{z}+\bar{\mathsf{z}}) \\ -3(\mathsf{z}+\bar{\mathsf{z}}) & 12 \end{pmatrix}$$

It turns out that all the charges are zero : $p^{\Lambda} = 0 = q_{\Sigma}$ (highly non-trivial)

$$\therefore \; I_1 \equiv \mathcal{Z}^2 = 0 \, , \quad F^{\Lambda}_{\mu\nu} = 0$$

- ✓ $A^1_{\mu} = 0$ (gauge-fixing)
- ✓ $2\,\partial_{[\mu} A^0_{\nu]} + m^0_{\mathsf{R}} B_{\mu\nu} = 0$ (gauge-fixing)
- ✓ $0 = \partial_{\mu} \widetilde{\xi}_0$ $(\because e_{00} = 0 = A^1_{\mu})$
- ✓ $\Lambda_{\text{c.c.}} \equiv V < 0$

$\eta \equiv a_1$ is still arbitrary

Schwarzschild-AdS Black Hole

- ☑ Studied : 4D $\mathcal{N} = 2$ gauged SUGRA with VMs and TM(UHM) via flux compactification.
- ☑ Imposed : covariantly constant condition.
- ☑ Found : Schwarzschild-AdS BH.

Different from cases of Calabi-Yau

- ☑ Find charged AdS-BH solutions.
- ☑ Consider a stationary AdS-BH.

Thank you

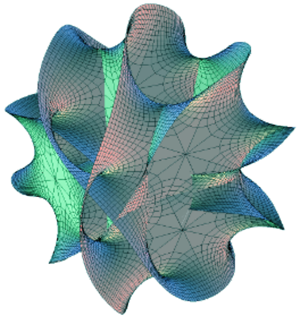
Appendix

- Terminology
- CY compactification in type IIA
- Geometric flux compactifications in type IIA
- Scalar potential
- AdS vacua

Prepotential :	$\mathcal{F}$ is a holomorphic function of X^Λ of degree two $(\mathcal{F}_\Lambda = \partial\mathcal{F}/\partial X^\Lambda)$
Kähler potential :	$\mathcal{K}_{\mathsf{V}} = -\log\big[\mathrm{i}\big(\overline{X}^\Lambda \mathcal{F}_\Lambda - X^\Lambda \overline{\mathcal{F}}_\Lambda\big)\big]$
Symplectic section :	$\Pi_{\mathsf{V}} \equiv \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2} \begin{pmatrix} X^\Lambda \\ \mathcal{F}_\Lambda \end{pmatrix} \equiv \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \quad 1 = \mathrm{i}\big(\overline{L}^\Lambda M_\Lambda - L^\Lambda \overline{M}_\Lambda\big)$
Kähler metric :	$g_{a\bar{b}} = \dfrac{\partial}{\partial \mathsf{z}^a}\dfrac{\partial}{\partial \bar{\mathsf{z}}^b}\mathcal{K}_{\mathsf{V}}\,, \quad \mathsf{z}^a = \dfrac{X^a}{X^0}$
Kähler covariant derivative :	$D_a \Pi_{\mathsf{V}} = \Big(\dfrac{\partial}{\partial \mathsf{z}^a} + \dfrac{1}{2}\dfrac{\partial \mathcal{K}_{\mathsf{V}}}{\partial \mathsf{z}^a}\Big)\Pi_{\mathsf{V}} \equiv \begin{pmatrix} f_a^\Lambda \\ h_{\Lambda a} \end{pmatrix}$
Period matrix :	$\mathcal{N}_{\Lambda\Sigma} = \overline{\mathcal{F}}_{\Lambda\Sigma} + 2\mathrm{i}\dfrac{(\mathrm{Im}\mathcal{F})_{\Lambda\Gamma}X^\Gamma (\mathrm{Im}\mathcal{F})_{\Sigma\Delta}X^\Delta}{X^\Pi (\mathrm{Im}\mathcal{F})_{\Pi\Xi}X^\Xi}$
Formulae :	$M_\Lambda = \mathcal{N}_{\Lambda\Sigma}L^\Sigma, \quad h_{\Lambda a} = \overline{\mathcal{N}}_{\Lambda\Sigma} f_a^\Sigma$
(Symplectic matrix) :	$(\mathbb{M}_{\mathsf{V}})_{\mathbf{\Lambda\Sigma}} = \begin{pmatrix} \mathbb{1} & -\mathrm{Re}\mathcal{N} \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} \mathrm{Im}\mathcal{N} & 0 \\ 0 & (\mathrm{Im}\mathcal{N})^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ -\mathrm{Re}\mathcal{N} & \mathbb{1} \end{pmatrix}$

In a similar way... $\Pi_{\mathsf{H}} \equiv \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2}\begin{pmatrix} Z^I \\ \mathcal{G}_I \end{pmatrix}, \quad z^i = \dfrac{Z^i}{Z^0}, \quad \mathcal{K}_{\mathsf{H}} = -\log\big[\mathrm{i}\big(\overline{Z}^I \mathcal{G}_I - Z^I \overline{\mathcal{G}}_I\big)\big]$, etc.

Flux Compactification beyond Calabi-Yau



How important is the flux compactification beyond CY in 10D Strings?

1. CY $\longrightarrow$ 4D ungauged SUGRA
   $\longrightarrow$ Fluxes break 10D Eqs. of Motion
2. non-CY with fluxes $\longrightarrow$ 4D gauged SUGRA
   non-CY: $SU(3)$-structure with torsion, etc.

- In 4D $\mathcal{N}=2$ ungauged SUGRA $\longrightarrow$ No scalar potential

(Extremal) charged Black holes in asymptotic flat has been investigated

Charges = series of D-branes wrapped on cycles in CY

- In 4D $\mathcal{N}=2$ gauged SUGRA $\longrightarrow$ Scalar potential turned on

Cosmological constant appears as VEV of scalar potential

3 types of gauged SUGRA

10D type IIA on non-CY with fluxes

↓

4D $\mathcal{N} = 2$ abelian gauged SUGRA with scalar potential

and $Q_{\rm NS} = \begin{pmatrix} e_{\Lambda}{}^{I} & e_{\Lambda I} \\ m^{\Lambda I} & m^{\Lambda}{}_{I} \end{pmatrix}$, $c_{\rm R} = \begin{pmatrix} m_{\rm R}^{\Lambda} \\ e_{{\rm R}\Lambda} \end{pmatrix}$

$e_{\Lambda}{}^{I}$, $e_{\Lambda I}$: Geometric flux charges (NSNS-flux charges)

$m^{\Lambda I}$, $m^{\Lambda}{}_{I}$: Nongeometric flux charges ("magnetic" NSNS-flux charges)

$e_{{\rm R}\Lambda}$, $m_{\rm R}^{\Lambda}$: RR-flux charges (with Romans' mass $m_{\rm R}^{0}$)

$0 = m^{\Lambda I} = m^{\Lambda}{}_{I} = m_{\rm R}^{\Lambda}$	$0 = m^{\Lambda I} = m^{\Lambda}{}_{I}$	generic
Standard Gauged SUGRA	Gauged SUGRA	Gauged SUGRA
$n_{\rm V}$ VMs	$n_{\rm V}$ VMs	$n_{\rm V}$ VMs
$n_{\rm H}$ HMs	$n_{\rm H}$ HMs	$\widetilde{n}_{\rm H}$ HMs
1 UHM	1 TM	$n_{\rm T}$ TMs
[hep-th/9605032]	[hep-th/0312210]	[hep-th/0409097]

VM : vector multiplet (U)HM : (universal) hypermultiplet TM : tensor multiplet

10D type IIA action $S_{\text{IIA}}^{(\text{10D})} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}$:

$$S_{\text{NS}} = \frac{1}{2}\int e^{-2\phi}\Big\{\widehat{R}*\mathbb{1} + 4d\phi\wedge *d\phi - \frac{1}{2}\widehat{H}_3\wedge *\widehat{H}_3\Big\}$$

$$S_{\text{R}} + S_{\text{CS}} = -\frac{1}{4}\int\Big\{\widehat{F}_2\wedge *\widehat{F}_2 + (\widehat{F}_4 - \widehat{C}_1\wedge\widehat{F}_3)\wedge *(\widehat{F}_4 - \widehat{C}_1\wedge\widehat{F}_3)\Big\} - \frac{1}{4}\int \widehat{B}_2\wedge\widehat{F}_4\wedge\widehat{F}_4$$

↓ CY

4D $\mathcal{N}=2$ ungauged SUGRA: *Neither gauge couplings, Nor scalar potential*

$$S^{(4\text{D})} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}R + \frac{1}{4}\text{Im}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}}\text{Re}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma} - g_{a\bar{b}}\,\partial_\mu z^a\partial^\mu\bar{z}^{\bar{b}} - h_{uv}\,\partial_\mu q^u\partial^\mu q^v\Big]$$

gravitational multiplet	$g_{\mu\nu}, A^0_1$		
VMs	$A^a_1, z^a, \bar{z}^{\bar{b}}$	$z^a \in \text{SKG}_{\text{V}}$	mirror dual: $\text{SKG}_{\text{V}} \leftrightarrow \text{SKG}_{\text{H}}$
HMs	$z^i, \bar{z}^{\bar{\jmath}}, \xi^i, \widetilde{\xi}_j$	$z^i \in \text{SKG}_{\text{H}}$	
UHM	$\varphi, a, \xi^0, \widetilde{\xi}_0$	$a \leftrightarrow B_2$	Hodge dual

$\mathcal{HM}$ → Special QG

$$\{q^u\} = \{z^i, \bar{z}^{\bar{\jmath}}\} + \{\xi^i, \widetilde{\xi}_j\} + \{\varphi, a, \xi^0, \widetilde{\xi}_0\} = \{z^i, \bar{z}^{\bar{\jmath}}\} + \{\varphi\} + \{a, \xi^I, \widetilde{\xi}_J\}$$

$\{q^u\}$: $4n_{\text{H}} + 4$; $\{z^i, \bar{z}^{\bar{\jmath}}\}$: $2n_{\text{H}}(\text{SKG}_{\text{H}})$; $\{\xi^i, \widetilde{\xi}_j\}$: $2n_{\text{H}}$; $\{\varphi, a, \xi^0, \widetilde{\xi}_0\}$: 4 (UHM); $\{z^i, \bar{z}^{\bar{\jmath}}\}$: SKG_{H}; $\{a, \xi^I, \widetilde{\xi}_J\}$: "Heisenberg"

10D type IIA action $S^{(10\text{D})}_{\text{IIA}} = S_{\text{NS}} + \widetilde{S}_{\text{R}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}$: (democratic form)

$$S_{\text{NS}} = \frac{1}{2}\int \mathrm{e}^{-2\phi}\Big\{\widehat{R} * \mathbb{1} + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3\Big\}, \quad \widetilde{S}_{\text{R}} = -\frac{1}{8}\int \big[\widehat{\mathbf{F}} \wedge *\widehat{\mathbf{F}}\big]_{10}$$

with "constraint $\widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}})$" and "EoM (Bianchi) $(\mathrm{d} + \widehat{H}\wedge) * \widehat{\mathbf{F}} = 0 \Leftrightarrow (\mathrm{d} - \widehat{H}\wedge)\widehat{\mathbf{F}} = 0$"

↓ non-CY with $SU(3)$-structure with $m_{\text{R}}^{\Lambda} = 0$

4D $\mathcal{N} = 2$ abelian gauged SUGRA (with $\xi^{\boldsymbol{I}} \equiv (\xi^I, \widetilde{\xi}_I)^{\text{T}}$):

$$S^{(4\text{D})} = \int \mathrm{d}^4 x\sqrt{-g}\,\Big[\frac{1}{2}R + \frac{1}{4}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma} - g_{a\bar{b}}\,\partial_\mu \mathsf{z}^a \partial^\mu \bar{\mathsf{z}}^{\bar{b}} - g_{i\bar{\jmath}}\,\partial_\mu z^i \partial^\mu \bar{z}^{\bar{\jmath}}$$
$$-\partial_\mu\varphi\partial^\mu\varphi + \frac{\mathrm{e}^{2\varphi}}{2}(\mathbb{M}_{\text{H}})_{\boldsymbol{IJ}} D_\mu\xi^{\boldsymbol{I}} D^\mu\xi^{\boldsymbol{J}} - \frac{\mathrm{e}^{2\varphi}}{4}\big(D_\mu a - \xi^{\boldsymbol{I}}(\mathbb{C}_{\text{H}})_{\boldsymbol{IJ}} D_\mu \xi^{\boldsymbol{J}}\big)^2 - V(\mathsf{z}, \bar{\mathsf{z}}, q)\Big]$$

- $(e_{\Lambda}{}^{I}, e_{\Lambda I})$: geometric flux charges & $e_{\text{R}\Lambda}$: RR-flux charges ⟵ non-CY data
  (with constraints $e_{\Lambda}{}^{I} e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}{}^{I} = 0$)
- $\mathsf{z}^a \in \text{SKG}_{\text{V}}$ and $z^i \in \text{SKG}_{\text{H}} \subset \mathcal{HM}$ are ungauged (in general)
- $D_\mu \xi^I = \partial_\mu \xi^I - e_{\Lambda}{}^{I} A_\mu^\Lambda$ & $D_\mu \widetilde{\xi}_I = \partial_\mu \widetilde{\xi}_I - e_{\Lambda I} A_\mu^\Lambda$
- $D_\mu a = \partial_\mu a - (2e_{\text{R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_{\Lambda}{}^{I}) A_\mu^\Lambda$
- $V(\mathsf{z}, \bar{\mathsf{z}}, q)$: scalar potential

D. Cassani [arXiv:0804.0595]

Non-vanishing m_{R}^{Λ} dualizes the axion field a in standard SUGRA to B-field.

4D gauged action is different from the standard one:

$$\begin{aligned} S^{(\mathsf{4D})} \;=\; \int \Big[& \frac{1}{2} R(*\mathbb{1}) + \frac{1}{2} \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F_{2}^{\Lambda} \wedge * F_{2}^{\Sigma} + \frac{1}{2} \mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F_{2}^{\Lambda} \wedge F_{2}^{\Sigma} - g_{a\bar{b}}\, \mathrm{d}\mathsf{z}^{a} \wedge * \mathrm{d}\bar{\mathsf{z}}^{\bar{b}} - g_{i\bar{\jmath}}\, \mathrm{d}z^{i} \wedge * \mathrm{d}\bar{z}^{\bar{\jmath}} \\ & - \mathrm{d}\varphi \wedge * \mathrm{d}\varphi - \frac{\mathrm{e}^{-4\varphi}}{4} H_{3} \wedge * H_{3} - \frac{\mathrm{e}^{2\varphi}}{2} (\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{I}} \wedge * D\xi^{\boldsymbol{J}} - V(*\mathbb{1}) \\ & + \frac{1}{2} \mathrm{d}B \wedge \Big[\xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{J}} + \big(2 e_{\mathsf{R}\Lambda} - \xi^{I} e_{\Lambda I} + \widetilde{\xi}_{I} e_{\Lambda}{}^{I} \big) A_{1}^{\Lambda} \Big] - \frac{1}{2} m_{\mathsf{R}}^{\Lambda} e_{\mathsf{R}\Lambda} B_{2} \wedge B_{2} \Big] \end{aligned}$$

Constraints among flux charges:

$$e_{\Lambda}{}^{I} e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}{}^{I} \;=\; 0, \qquad m_{\mathsf{R}}^{\Lambda} e_{\Lambda}{}^{I} \;=\; 0 \;=\; m_{\mathsf{R}}^{\Lambda} e_{\Lambda I}$$

Scalar potential from (non)geometric flux compactifications:

$$V = \mathrm{g}^2 \Big[4 h_{uv} k^u \overline{k}^v + \sum_{x=1}^{3} \Big(g^{a\bar{b}} D_a \mathcal{P}_x D_{\bar{b}} \overline{\mathcal{P}}_x - 3 |\mathcal{P}_x|^2 \Big) \Big] = \ldots \equiv V_{\mathsf{NS}} + V_{\mathsf{R}} \quad (\text{abelian: } k^a_\Lambda = 0)$$

$$V_{\mathsf{NS}} = g^{a\bar{b}} D_a \mathcal{P}_+ D_{\bar{b}} \overline{\mathcal{P}}_+ + g^{i\bar{\jmath}} D_i \mathcal{P}_+ D_{\bar{\jmath}} \overline{\mathcal{P}}_+ - 2 |\mathcal{P}_+|^2$$

$$= -2\, \mathrm{g}^2 \mathrm{e}^{2\varphi} \Big[\overline{\Pi}_{\mathsf{H}}^{\mathrm{T}} \widetilde{Q}^{\mathrm{T}} \mathbb{M}_{\mathsf{V}} \widetilde{Q} \, \Pi_{\mathsf{H}} + \overline{\Pi}_{\mathsf{V}}^{\mathrm{T}} Q \, \mathbb{M}_{\mathsf{H}} Q^{\mathrm{T}} \Pi_{\mathsf{V}} + 4 \overline{\Pi}_{\mathsf{H}}^{\mathrm{T}} \mathbb{C}_{\mathsf{H}}^{\mathrm{T}} Q^{\mathrm{T}} (\Pi_{\mathsf{V}} \overline{\Pi}_{\mathsf{V}}^{\mathrm{T}} + \overline{\Pi}_{\mathsf{V}} \Pi_{\mathsf{V}}^{\mathrm{T}}) Q \, \mathbb{C}_{\mathsf{H}} \Pi_{\mathsf{H}} \Big]$$

$$V_{\mathsf{R}} = g^{a\bar{b}} D_a \mathcal{P}_3 D_{\bar{b}} \overline{\mathcal{P}}_3 + |\mathcal{P}_3|^2$$

$$= -\frac{1}{2} \mathrm{g}^2 \mathrm{e}^{4\varphi} \big(e_{\mathsf{R}\Lambda} - e_{\Lambda I} \xi^I + e_\Lambda{}^I \widetilde{\xi}_I \big) (\mathrm{Im} \mathcal{N})^{-1|\Lambda\Sigma} \big(e_{\mathsf{R}\Sigma} - e_{\Sigma I} \xi^I + e_\Sigma{}^I \widetilde{\xi}_I \big)$$

$\Pi_{\mathsf{V}} = \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2} (X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$

$z^a = X^a / X^0$

$a = 1, \ldots, n_{\mathsf{V}}$

SKG$_{\mathsf{V}}$ of vector-moduli

$$\begin{aligned} \mathcal{P}_+ &\equiv \mathcal{P}_1 + \mathrm{i} \mathcal{P}_2 = 2 \mathrm{e}^{\varphi} \, \Pi_{\mathsf{V}}^{\mathrm{T}} Q \, \mathbb{C}_{\mathsf{H}} \Pi_{\mathsf{H}} \\ \mathcal{P}_- &\equiv \mathcal{P}_1 - \mathrm{i} \mathcal{P}_2 = 2 \mathrm{e}^{\varphi} \, \Pi_{\mathsf{V}}^{\mathrm{T}} Q \, \mathbb{C}_{\mathsf{H}} \overline{\Pi}_{\mathsf{H}} \\ \mathcal{P}_3 &= \mathrm{e}^{2\varphi} \, \Pi_{\mathsf{V}}^{\mathrm{T}} \mathbb{C}_{\mathsf{V}} (c_{\mathsf{R}} + \widetilde{Q} \xi) \end{aligned}$$

$\Pi_{\mathsf{H}} = \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2} (Z^I, \mathcal{G}_I)^{\mathrm{T}}$

$z^i = Z^i / Z^0$

$i = 1, \ldots, n_{\mathsf{H}}$

SKG$_{\mathsf{H}}$ of hyper-moduli

$$\mathbb{C}_{\mathsf{V,H}} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} ; \quad Q = \begin{pmatrix} e_\Lambda{}^I & e_{\Lambda I} \\ m^{\Lambda I} & m^\Lambda{}_I \end{pmatrix} , \quad \widetilde{Q} = \mathbb{C}_{\mathsf{H}}^{\mathrm{T}} Q \, \mathbb{C}_{\mathsf{V}} \quad c_{\mathsf{R}} = \begin{pmatrix} m_{\mathsf{R}}^\Lambda \\ e_{\mathsf{R}\Lambda} \end{pmatrix}$$

Cassani et.al. [arXiv:0804.0595], [arXiv:0911.2708]

Non-CY coset spaces with $SU(3)$-structure: D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

$\mathcal{M}_6$	$\frac{G_2}{SU(3)}$ (nearly-Kähler)	$\frac{Sp(2)}{S[U(2)\times U(1)]}$ (half-flat/nearly-Kähler)	$\frac{SU(3)}{U(1)\times U(1)}$ (half-flat/nearly-Kähler)
$\mathcal{SM} = \mathsf{SKG_V}$	$\frac{SU(1,1)}{U(1)}$: z^3	$\left(\frac{SU(1,1)}{U(1)}\right)^2$: $\mathfrak{s}\mathsf{z}^2$	$\left(\frac{SU(1,1)}{U(1)}\right)^3$: $\mathfrak{s}\mathsf{z}\mathfrak{u}$
$\mathcal{HM} = \mathsf{SQG}$	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM
$\mathsf{SKG_H} \subset \mathcal{HM}$	—	—	—
matters	1 VM + 1 UHM	2 VMs + 1 UHM	3 VMs + 1 UHM

Each $\mathsf{SKG_V}$ has a cubic prepotential: $\mathcal{F} = \frac{1}{3!} d_{abc} \frac{X^a X^b X^c}{X^0}$

Each AdS vacuum requires non-vanishing Romans' mass! (Vanishing Romans' mass → Reduction to CY)

nilmanifolds and solvmanifolds: M. Graña, R. Minasian, M. Petrini and A. Tomasiello [hep-th/0609124]

coset spaces with $SU(3)$- or $SU(2)$-structure: P. Koerber, D. Lüst and D. Tsimpis [arXiv:0804.0614]

a pair of $SU(3)$-structures with $(m^{\Lambda I}, m^{\Lambda}{}_I)$: D. Gaiotto and A. Tomasiello [arXiv:0904.3959]

Vacuum I : $\mathcal{N} = 1$

$$z_* = -\frac{\pm 1 + i\sqrt{15}}{2} \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3} , \quad \xi_*^0 = -\frac{2}{5} \left[\frac{2\sqrt{3}\, m_R^0 (e_{R0})^2}{5\, e_{10}} \right]^{1/3} , \quad \exp(\varphi_*) = \frac{4}{3} \left[\frac{\sqrt{5}\, e_{10}}{\sqrt{3}\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{5\sqrt{5}}{2} \left[\frac{5\,(e_{10})^4}{2\sqrt{3}\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{I}} < 0$$

Vacuum II : $\mathcal{N} = 0$

$$z_* = \big(\pm 1 - i\sqrt{3} \big) \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3} , \quad \xi_*^0 = \left[\frac{9\, m_R^0 (e_{R0})^2}{25\, e_{10}} \right]^{1/3} , \quad \exp(\varphi_*) = \frac{2}{3} \left[\frac{25\, e_{10}}{\sqrt{3}\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{80}{27} \left[\frac{25\,(e_{10})^4}{\sqrt{3}\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{II}} < 0$$

Vacuum III : $\mathcal{N} = 0$

$$z_* = -i \left[\frac{12}{\sqrt{5}\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3} , \quad \xi_*^0 = 0 , \quad \exp(\varphi_*) = \sqrt{5} \left[\frac{5\, e_{10}}{18\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{25\sqrt{5}}{6} \left[\frac{5\,(e_{10})^4}{18\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{III}} < 0$$

Note: $m_R^0 > 0$; $\widetilde{\xi}_0$ is not fixed ; $\Lambda_{\text{c.c.}}^{\text{II}} < \Lambda_{\text{c.c.}}^{\text{I}} < \Lambda_{\text{c.c.}}^{\text{III}}$ D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

Anti-de Sitter Black Holes in 4D $\mathcal{N} = 2$ Gauged SUGRA

Comments

- ☞ AdS-BH with naked singularity in pure AdS SUGRA

  L.J. Romans [hep-th/9203018], M.M. Caldarelli and D. Klemm [hep-th/9808097], etc.

  SUSY solution of rotating AdS black hole with regular horizon

- ☞ AdS-BH with regular horizon in Gauged SUGRA with VMs (no HMs)

  [hep-th/9903143], [arXiv:0911.4926], [arXiv:1012.4314], etc.