On Static Black Holes in Type IIA on a Nearly-Kähler Coset

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We consider 4D N=2 abelian gauged supergravity with B-field. This is derived from (massive) type IIA string theory compactified on a nearly-Kähler coset space G2/SU(3). Here we discuss static black hole solutions and show there are **NO** non-trivial field configurations.

4D N=2 action from type IIA with consistent truncation

D. Cassani; arXiv:0804.0595

*"Type II supergravity on backgrounds admitting SU(3) x SU(3) structure and general fluxes is considered. Using the generalized geometry formalism, we study dimensional reductions leading to N=2 gauged supergravity in four dimensions, possibly **with tensor multiplets**. In particular, a geometric formula for the **full** N=2 scalar potential is given. Then we implement a truncation ansatz, and derive the **complete** N=2 bosonic action. While the NSNS contribution is obtained via a direct dimensional reduction, the contribution of the RR sector is computed starting from the democratic formulation and demanding consistency with the reduced equations of motion."*

G2/SU(3) as a nearly-Kähler

A.-K. Kashani-Poor; arXiv:0709.4482

$$\mathrm{d}J = \frac{3}{2}\mathrm{Im}(\overline{W}_1\Omega)\,, \quad \mathrm{d}\Omega = W_1 J\wedge J$$

$$\mathrm{d}\omega_\Lambda = e_\Lambda \alpha\,, \quad \mathrm{d}\alpha = 0\,, \quad \mathrm{d}\beta = e_\Lambda \widetilde{\omega}^\Lambda\,, \quad \mathrm{d}\widetilde{\omega}^\Lambda = 0$$

e_Λ : NSNS (geometric) flux charges

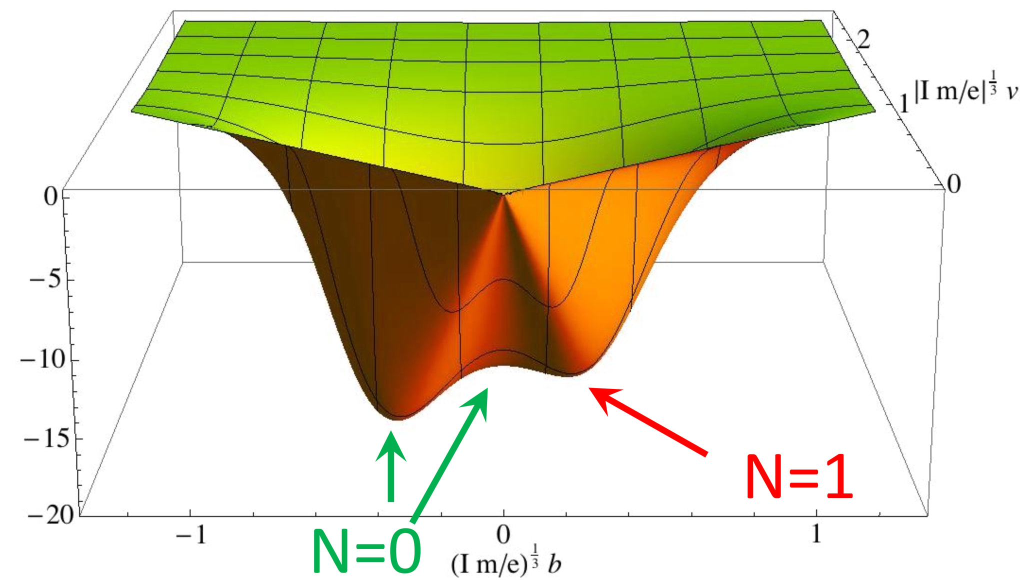
$e_{\mathrm{R}\Lambda}, m_{\mathrm{R}}^\Lambda$: RR flux charges

$$m_{\mathrm{R}}^\Lambda e_\Lambda = 0$$

Geometry with torsion provides gauge interactions in 4D physics, whilst Calabi-Yau geometry does not. The torsion W1 is supported by NSNS fluxes (dilaton, H-flux, and so forth). G2/SU(3) is the simplest 6D coset space containing torsion.

Vacua

Cassani, Kashani-Poor; arXiv:0901.4251



N=2 gauged supergravity from G2/SU(3) has three vacua; one is supersymmetric, the other two are not. They deform Minkowski spaces to AdS, and are dictated by RR fluxes, in particular, by the Romans' mass. They yield the Stückelberg mass deformation in 4D.

Possible to find a static black hole solution with (non-)trivial field configurations?

$$\mathrm{d}s^2 = -\mathrm{e}^{2A(r)}\mathrm{d}t^2 + \mathrm{e}^{-2A(r)}\mathrm{d}r^2 + \mathrm{e}^{2C(r)}r^2\mathrm{d}\Omega^2$$

$$\mathrm{e}^{2A(r)} \equiv 1 - \frac{2\eta}{r} + \frac{Z^2}{r^2} + \frac{r^2}{\ell^2}\,, \quad \mathrm{e}^{2C(r)} \equiv 1\,, \quad \Lambda_{\mathrm{c.c.}} = -\frac{3}{\ell^2}\,, \quad Z^2 = Q_e^2 + Q_m^2$$

Tools

1. Conditions among NSNS flux charges and RR flux charges
2. Time independence, static electric/magnetic fields
3. Electric/magnetic charges defined by the gauge field strengths
4. EoM for gauge fields, B-fields, and Einstein equation

$$p^\Lambda \equiv \int \mathrm{d}\theta\mathrm{d}\phi\, F^\Lambda_{\theta\phi}\,, \quad q_\Lambda \equiv \int \mathrm{d}\theta\mathrm{d}\phi \widetilde{F}_{\Lambda\theta\phi}\,, \quad \widetilde{F}_{\Lambda\mu\nu} \equiv \nu_{\Lambda\Sigma}F^\Sigma_{\mu\nu} + \mu_{\Lambda\Sigma}(*F^\Sigma)_{\mu\nu}$$

$$0 = \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}}\partial_\mu \widetilde{F}_{\Lambda\nu\rho} + \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}}\partial_\mu B_{\nu\rho}(e_{\mathrm{R}\Lambda} - e_\Lambda\xi) - \mathrm{e}^{2\varphi}e_\Lambda D^\sigma\widetilde{\xi}\,,$$

$$0 = \frac{1}{\sqrt{-g}}\partial_\mu\Big(\mathrm{e}^{-4\varphi}\sqrt{-g}H^{\mu\rho\sigma}\Big) + \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}\Big[D_\mu\xi D_\nu\widetilde{\xi} - D_\mu\widetilde{\xi}D_\nu\xi + (e_{\mathrm{R}\Lambda} - e_\Lambda\xi)F^\Lambda_{\mu\nu}\Big] + 2m_{\mathrm{R}}^\Lambda\mu_{\Lambda\Sigma}F^{\Sigma\rho\sigma} - \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}m_{\mathrm{R}}^\Lambda\nu_{\Lambda\Sigma}F^\Sigma_{\mu\nu}\,,$$

$$E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

$$= g_{\mu\nu}\Big[\frac{1}{4}\mu_{\Lambda\Sigma}F^\Lambda_{\rho\sigma}F^{\Sigma\rho\sigma} - g_{z\overline{z}}\,\partial_\rho z\partial^\rho\overline{z} - \partial_\rho\varphi\partial^\rho\varphi - \frac{\mathrm{e}^{-4\varphi}}{24}H_{\rho\sigma\lambda}H^{\rho\sigma\lambda} - \frac{\mathrm{e}^{2\varphi}}{2}\Big(D_\rho\xi D^\rho\xi + D_\rho\widetilde{\xi}D^\rho\widetilde{\xi}\Big) - V\Big] - \Big[\mu_{\Lambda\Sigma}F^\Lambda_{\mu\rho}F^{\Sigma\rho}_{\nu} - 2\partial_\mu z\partial_\nu\overline{z} - 2\partial_\mu\varphi\partial_\nu\varphi - \frac{\mathrm{e}^{-4\varphi}}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} - \mathrm{e}^{2\varphi}\Big(D_\mu\xi D_\nu\xi + D_\mu\widetilde{\xi}D_\nu\widetilde{\xi}\Big)\Big]$$

Electric/magnetic fields

$$F^\Lambda_{\theta\phi} = f^\Lambda(\theta,\phi)\sin\theta\,, \quad F^\Lambda_{tr} = \frac{\mathrm{e}^{-2C(r)}}{r^2}g^\Lambda(\theta,\phi)$$

Eq. of motion for gauge fields

$$D_t\widetilde{\xi} = 0 = D_r\widetilde{\xi}\,, \quad H_{r\theta\phi} = 0 = H_{t\theta\phi}$$

$$H_{tr\theta} = \frac{\mathrm{e}^{-2C}}{r^2}\partial_\theta\Big[e_{\mathrm{R}\Lambda}g^\Lambda\Big]\,, \quad H_{tr\phi} = \frac{\mathrm{e}^{-2C}}{r^2}\partial_\phi\Big[e_{\mathrm{R}\Lambda}g^\Lambda\Big]$$

Eq. of motion for B-field

$$0 = \sin\theta\,\partial_\theta\Big[e_{\mathrm{R}\Lambda}g^\Lambda\Big]\partial_r\Big(\frac{\mathrm{e}^{-4\varphi-2C}}{r^2}\Big) + 2D_r\xi D_\phi\widetilde{\xi}$$

$$0 = \frac{1}{\sin\theta}\partial_\phi\Big[e_{\mathrm{R}\Lambda}g^\Lambda\Big]\partial_r\Big(\frac{\mathrm{e}^{-4\varphi-2C}}{r^2}\Big) - 2D_r\xi D_\theta\widetilde{\xi}$$

Einstein equation

$$g^{tt}E_{tt} = g^{rr}E_{rr} = -\frac{Z^2}{r^4} + \frac{3}{\ell^2}\,, \quad g^{\theta\theta}E_{\theta\theta} = g^{\phi\phi}E_{\phi\phi} = \frac{Z^2}{r^4} + \frac{3}{\ell^2}$$

$$0 = g^{tt}E_{tt} - g^{rr}E_{rr} = -2g^{rr}\Big[g_{z\overline{z}}(\partial_r z)^2 + (\partial_r\varphi)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_r\xi)^2\Big]$$

$$0 = g^{\theta\theta}E_{\theta\theta} - g^{\phi\phi}E_{\phi\phi} = g^{\theta\theta}\Big[g_{z\overline{z}}(\partial_\theta z)^2 + (\partial_\theta\varphi)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_\theta\xi)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_\theta\widetilde{\xi})^2\Big] - g^{\phi\phi}\Big[g_{z\overline{z}}(\partial_\phi z)^2 + (\partial_\phi\varphi)^2 + \frac{\mathrm{e}^{2\varphi}}{2}\Big((D_\phi\xi)^2 + (D_\phi\widetilde{\xi})^2\Big)\Big] + \frac{\mathrm{e}^{-4\varphi}}{2}\Big[H_{tr\theta}H^{tr\theta} + H_{tr\phi}H^{tr\phi}\Big]$$

$$\frac{3}{\ell^2} = g^{rr}E_{rr} + g^{\theta\theta}E_{\theta\theta} = -g^{\phi\phi}\Big[g_{z\overline{z}}(\partial_\phi z)^2 + (\partial_\phi\varphi)^2 + \frac{\mathrm{e}^{2\varphi}}{2}\Big((D_\phi\xi)^2 + (D_\phi\widetilde{\xi})^2\Big)\Big] - V$$

$$-\frac{Z^2}{r^4} = g^{rr}E_{rr} - g^{\theta\theta}E_{\theta\theta} = \frac{\mathrm{e}^{-4C}}{r^4}\mu_{\Lambda\Sigma}\Big[f^\Lambda f^\Sigma + g^\Lambda g^\Sigma\Big] - g^{\theta\theta}\Big[g_{z\overline{z}}(\partial_\theta z)^2 + (\partial_\theta\varphi)^2 + \frac{\mathrm{e}^{2\varphi}}{2}\Big((D_\theta\xi)^2 + (D_\theta\widetilde{\xi})^2\Big)\Big]$$

Result : All the fields are (covariantly) constant!

$$0 = \partial_r z = \partial_r\varphi = D_r\xi\,, \quad H_{tr\theta} = 0 = H_{tr\phi}$$

$$0 = \partial_\theta z = \partial_\theta\varphi = D_\theta\xi = D_\theta\widetilde{\xi}$$

$$0 = \partial_\phi z = \partial_\phi\varphi = D_\phi\xi = D_\phi\widetilde{\xi}$$

$$\to \quad 0 = \partial_\mu z = \partial_\mu\varphi = D_\mu\xi = D_\mu\widetilde{\xi} = H_{\mu\nu\rho}$$

Remark

TK; arXiv:1108.1113

*Starting with the (covariantly) constant condition on all scalar fields with arbitrary A(r) and C(r), we find that the possible solutions are only two: **AdS vacua**, or **Schwarzschild-AdS black hole** with vanishing electric/magnetic charges and arbitrary mass parameter.*

Conclusion

No non-trivial field configurations for static charged Black Holes.
This is caused by the Stückelberg-type mass deformation:

$$F^\Lambda_{\mu\nu} = (\partial_\mu A^\Lambda_\nu - \partial_\nu A^\Lambda_\mu) + m^\Lambda_{\mathrm{R}}B_{\mu\nu}$$